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Discussion Paper 32/2005

On Delegation under Relational Contracts

by

Oliver Gürtler

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On Delegation under Relational Contracts

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Abstract
The benefits and costs of different forms of job design have been analyzed in the literature yet. The focus has thereby mostly been on job designs under formal contracts between the parties. However, in the real world relational contracts - informal agreements sustained by the value of future relationships - play a role as important as formal ones. This paper therefore considers the advantages and disadvantages of two different kinds of job design, partial delegation and complete delegation with specialization, when the parties make use of both, formal and informal agreements. It is found that many of the results derived in the absence of informal contracts will no longer hold, if these contracts become available.

Key words: Job design, relational contracts, formal contracts, delegation.

JEL classification: D82, J33, L23, M52, M54

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1 Introduction

In each multi-person organization, tasks have to be divided between the organization’s members. Specifically, the manager of the organization (henceforth called the principal) has to decide about which tasks to handle himself and which tasks to delegate to his subordinates (henceforth called the agents). In other words, the principal has to install an appropriate job design (i.e. a grouping of tasks into jobs). This decision is essential for the organization’s success in the market. A firm with an inefficient internal organization is likely to produce at higher costs than its better organized competitors and so faces an important comparative disadvantage.  

Besides the obvious reason that a principal is usually time-constrained and thus cannot handle all tasks himself, the economic literature gives two main reasons for delegation of a task. The first reason is based on the assumption that an agent is more appropriate for a certain task, either because

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1In order to see, what consequences an inefficient internal organization might have, consider the Deutsche Telekom AG, Germany’s biggest telecommunications company. Until 2004, the company was subdivided into four divisions, T-Online (responsible for Internet services), T-Mobile (mobile telecommunications services), T-Systems (Information Technology), and T-Com (fixed network). This structure proved to be very problematic. The single divisions acted in an uncoordinated way and partly spent resources to alienate customers from each other. Telekom’s CEO Kai-Uwe Ricke finally realized that Telekom was inefficiently organized and restructured the company. Presently, it consists of three divisions, one responsible for business customers, the second covering the market for mobile telecommunications services, and the third dealing with broadband and fixed network services.

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he is in possession of relevant information that cannot be easily transferred or because he has important abilities the principal has not. The second reason builds on incentive considerations. Aghion & Tirole (1997) e.g. argue that delegation of formal responsibility leads to increased initiative at lower layers of a hierarchy. If formal responsibility is delegated to an agent, the principal commits not to overrule the agent, yielding higher incentives for the agent. On the other hand, delegation of a task and hence of responsibility may also entail incentive problems, as the agent is likely to pursue different goals than the firm. These incentive problems might outbalance the benefits from delegation. Concentrating on delegation as a means to ensure an efficient use of decentralized information, Melumad & Reichelstein (1987) and Melumad et al. (1995, 1997) extensively deal with these incentive problems. Making use of different revelation mechanisms, they demonstrate under what circumstances the problems may or may not be eliminated.

Further, in a model, where only aggregate output on several tasks is measurable, Itoh (1994, 2001) analyzes different modes of delegation. Assuming that the principal is exogenously forced to delegate at least one task, Itoh found that three effects mainly influenced the optimal allocation of tasks. With risk-averse agents, the principal seeks to do some task himself or to assign all tasks to a single agent in order to save on risk premiums. However, assigning all tasks to a single agent might lead to an overload of that agent and, hence, to high effort costs the agent must be compensated for. Finally, if the principal decides to handle some task himself, there will arise some kind of free-rider problem that cannot be eliminated by means of incentive
pay.

While Itoh mentioned important aspects of the job design decision, his analysis is incomplete in that the principal solely relies on formal incentive contracts to motivate the agents. In many firms, however, incentives are not solely provided via formal contracts, but also via long-lasting relational agreements. Employees are often paid contingent on contractible measures (such as sales volume), but also on subjective assessments that are not verifiable by a third party. An example is Nokia, the world’s leading mobile phone supplier. While Nokia makes extensive use of formal incentives (e.g. payments based on project/program-success), every year, there is one subjective performance evaluation of all employees. Based on this evaluation, wages are increased or not. Hence, an important question is how (or whether) the appropriateness of a job design will change, if incentives are provided by a combination of formal and relational contracts.

This question is tried to be answered in the current paper. I therefore combine the model of Itoh (1994) with a model of Baker et al. (1994). Baker et al. consider a principal-agent relationship, where the principal remunerates the agent contingent on both, the realizations of contractible and non-contractible performance measures. The authors particularly focus on the interaction of these forms of compensation. This paper applies the Baker

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2 Relational contracts are also referred to as informal, implicit or self-enforcing contracts. Throughout the paper, I use relational contracts and informal contracts as synonyms.

3 For further examples see Gibbons (2005), who reports on several other firms tying their employees’ compensations to subjective performance measures.

4 Other papers analyzing the interaction of formal and relational contracts include
et al. model to situations, in which two tasks have to be dealt with and the principal is not able to handle all the tasks himself.

Two kinds of job design, partial delegation (one task is handled by the principal and one task by an agent) and complete delegation with specialization (each task is dealt with by a different agent), are compared in the absence as well as in the presence of relational agreements. Formal contracts are based on an imperfect measure of joint contribution to firm value on the two tasks. In the absence of relational contracts, the job design decision is then determined by a trade-off of two countervailing effects. Due to the imperfection of the measure, the agents’ efforts are usually distorted with respect to desired effort. This effect is more distinctive under complete delegation with specialization, since, in this case, two agents instead of one are involved in the production process. On the other hand, as compensation is based on a measure of joint performance, a free-rider problem is present. While this problem can be effectively mitigated under complete delegation with specialization by means of incentive pay, under partial delegation, it cannot. Assuming that the performance measure is positively correlated to total contribution to firm value, providing the agent with higher incentives automatically yields lower incentives for the principal (and vice versa).


5Itoh considered a third job design, namely complete delegation without specialization, where a single agent handles both tasks. However, in this paper this job design is always dominated by complete delegation with specialization. The reason is that Itoh considered risk-averse agents, whereas the current paper deals with risk-neutral ones.
Considering both effects, a simple, intuitive condition is derived: Partial delegation will be optimal, if and only if the distortion in an agent’s effort is relatively high.

It is further assumed that an agent’s true contribution to firm value is observable (but not contractible). Hence, in a model with infinitely repeated interaction, relational agreements may become feasible. The principal may pay an agent a bonus based on the realization of his true contribution to firm value. If the principal could choose the bonus arbitrarily, the first-best could be achieved. However, the principal is tempted to renege on the bonus and this temptation increases in the bonus size. Hence, he may credibly commit not to renege on the bonus, only if this bonus is rather low so that first-best efforts are not achieved. The quality of a certain job design therefore mainly depends on three factors. First, it depends on the size of the respective relational bonus that can be sustained. Second, it depends on the job designs’ relative needs for relational bonuses, i.e., the relative profit increase after a replacement of formal by relational contracts. Third, it depends on the respective status quo points, that is, the profits that can be achieved in the absence of relational agreements. The interplay of these three arguments decides about the optimality of a job design. It is found that the introduction of relational agreements has a crucial influence on the model results. A job design being optimal under formal contracts is likely to be relatively worse when relational contracts are available. The reason is that relational contracts may be extremely effective especially in situations where formal contracts are not. A job design’s weakness in providing good
formal incentives may become its strength under relational agreements.

There additionally exists a recent, complementary paper by Schöttner (2004). She also discusses the benefits and costs of several kinds of job design under interplay of formal and relational contracts. Whereas this paper mainly treats the question of whether or not to delegate a task, Schöttner, in a slightly different setting, focuses on several forms of complete delegation. That is, in her paper, the decision to delegate all tasks is exogenously given, and then the best form of complete delegation is derived.

The paper is organized as follows: Section 2 presents the optimal job design in the absence of relational agreements, while section 3 extends the analysis to a combined use of formal and relational contracts. Section 4 contains a discussion of several model assumptions and section 5 concludes.

2 Job design in the absence of relational contracts

As mentioned before, the current model combines the models of Itoh (1994) and Baker et al. (1994). Consider a principal and two identical agents, all assumed to be risk-neutral. In the organization, two tasks have to be dealt with, tasks $a$ and $b$. Because of e.g. time restrictions the principal is able to handle at most one task, whereas an agent may handle both tasks.\footnote{The model also (and perhaps better) refers to a situation, where more than two tasks have to be handled, but only two tasks may be delegated by the principal. Since he has to handle all remaining tasks, the principal has to delegate at least one task.} The
principal can therefore decide to assign either both tasks to a single agent (complete delegation without specialization), task a to one agent and task b to another (complete delegation with specialization), or to delegate one task and handle the remaining task himself (partial delegation). With risk-neutral agents and tasks being substitutes, it can be shown that complete delegation without specialization is always dominated by complete delegation with specialization. The paper therefore focuses on a comparison of complete delegation with specialization (henceforth CDS) and partial delegation (henceforth PD).

The person in charge of task \(i = a, b\) exerts unobservable effort \(e_i \geq 0\) that stochastically determines an observable, but unverifiable output \(y_i\). This output measures contribution to firm value on task \(i\) and equals either one or zero. Let the probability that output equals one be given by \(\text{Prob}\{y_i = 1|e_i\} = \min\{e_i, 1\}\). Total output is given by \(y = y_a + y_b\).

\(\text{It is implicitly assumed that task sharing is impossible. One reason for this assumption could be that each task requires the use of a machine that cannot be operated by two people at the same time.}\)

\(\text{The proof of this statement is available from the author upon request.}\)

\(\text{As pointed out by Malcomson (1984), a rank-order tournament between the agents could be arranged, even if output is unverifiable by a third party. With the assumptions made in this paper (in particular, risk neutrality and unlimited liability of the agents) such a tournament would always yield a first-best solution, in the static as well as in the dynamic case. However, a tournament scheme may also lead to serious problems such as collusion between the agents (see e.g. Dye (1984)) or sabotage (see e.g. Lazear (1989), Konrad (2000) or Chen (2003)). Throughout the paper it is assumed that these problems are so severe that the tournament scheme is never desired.}\)
Efforts $e_a$ and $e_b$ additionally affect a second performance measure $p$ that is contractible and therefore may be the basis of an enforceable contract. $p$ is an imperfect measure of joint contribution to firm value on the two tasks and also equals either one or zero. The probability of a measure realization of one is given by $\Pr \{p = 1|e_a, e_b\} = \min \{\mu_a e_a + \mu_b e_b, 1\}$. The realization of each parameter $\mu_i$ is unknown, when the principal determines the job design and when the agents are offered a wage contract. Thereafter, it is revealed to the respective person in charge, that is, the person in charge for task $a$ ($b$) privately learns the realization of $\mu_a$ ($\mu_b$). The parameter $\mu_i$ characterizes the actual difference between the effect of $e_i$ on $y$ and its effect on $p$. Following Baker et al. $\mu_i$ can be interpreted as follows: There are days (i.e., values of $\mu_i$), where high effort spent on task $i$ leads to similar increases in $y$ and $p$ ($\mu_i$ around one), days, where high effort increases $y$ but not $p$ ($\mu_i$ near zero) and days, where small effort increases $p$ but not $y$ ($\mu_i$ much larger than one).

It is further assumed that the mean of $\mu_i$ equals one so that, in expectation, the measure $p$ is an unbiased measure of total output $y$. This assumption allows to characterize the expected difference of $p$ from $y$ by a single measure, namely the variance $\Var[\mu_i] = E_{\mu_i}[(\mu_i)^2] - 1$. The parameters $\mu_a$ and $\mu_b$ are independently, identically distributed (i.i.d. assumption), i.e., $\Var[\mu_a] = \Var[\mu_b] =: \Var[\mu]$. Their distribution is common knowledge.

Effort entails costs, which, to derive several closed-form solutions, are assumed to be quadratic and given by $C(e_i) = \frac{c}{2} (e_i)^2$, with $c > 0$. Throughout the paper, it is assumed that the parameter $c$ and the support of $\mu_i$ are such that, in equilibrium, $e_i < 1$ and $\mu_a e_a + \mu_b e_b < 1$. Since under CDS as well
as under PD, no person handles both tasks, the formula of effort costs does not show a term indicating the degree of cost substitutability between the two tasks.\footnote{This is the case under complete delegation without specialization in Itoh (1994). Under that job design, costs, entailed by effort on a task, are likely to increase in the effort level on the other task. It is this cost substitutability between the tasks that causes the inexpediency of complete delegation without specialization.} Finally, each agent is supposed to have an outside option that leads to an expected utility of $\bar{U}$. I assume that $\bar{U} = 0$. This assumption is relaxed in section 4.

The timing of the model is as follows: At stage 1, the principal determines a job design. At stage 2, he offers a wage contract to one or two agents, respectively. At stage 3, the agent(s) accept(s) or reject(s) the offer. An agent rejecting the offer as well as an agent not being offered a wage contract realizes his outside option. If all wage offers are rejected, the principal handles one task himself, whereas the other task is not handled at all. At stage 4, the person in charge of task $i$ learns the realization of $\mu_i$, while efforts are chosen at stage 5. At stage 6, $p$ and $y$ are realized and payments are made.

Before the model is solved, consider the first-best solution, in which efforts are contractible. In the first-best solution, the principal could hire two agents and determine their efforts to maximize

$$R = e_a + e_b - \frac{\gamma}{2} (e_a)^2 - \frac{\gamma}{2} (e_b)^2.$$  

First-best efforts are therefore given by $e^{fb}_a = e^{fb}_b = \frac{1}{c}$.

Let us now solve the model. As in this section a one-period model is considered, relational contracts are not feasible and the principal solely relies on formal contracts to motivate the agents. The wage contracts are therefore given by $w^{CDS,f}_i = \alpha_{oi}^{CDS,f} + \alpha_{1i}^{CDS,f}p$ and $w^{PD,f}_i = \alpha_{oi}^{PD,f} + \alpha_{1i}^{PD,f}p$, where
the $f$ should indicate the isolated consideration of formal contracts. While
the agents always receive a fixed wage of $\alpha_{oi}$, they will receive the variable
component $\alpha_{1i}$, only if the joint performance measure $p$ equals one.

The model is solved by backward induction. I start with the CDS case.
After observing the realization of $\mu_a$, the agent working on task $a$ chooses
his effort to maximize expected utility. This expected utility is given by (1).

$$EU^{CDS,f,ep}_a = \alpha^{CDS,f}_{aa} + \alpha^{CDS,f}_{ia} (e_a \mu_a + E_{\mu_b}[e_b \mu_b]) - \frac{c}{2} (e_a)^2$$  \hspace{1cm} (1)

$E_{\mu_i} [\cdot]$ denotes the expectation operator with respect to $\mu_i$ and $ep$ stands for ex post, since (1) denotes the expected utility after observing the parameter $\mu_a$.

The optimal effort satisfies $e_a = \frac{\alpha^{CDS,f}_{1a}}{c} \mu_a$. Similarly, the agent working
on task $b$ exerts effort $e_b = \frac{\alpha^{CDS,f}_{1b}}{c} \mu_b$. The agents’ ex ante expected utilities,
that is, their expected utilities before observing the signals are given by

$$EU^{CDS,f,ea}_a = \alpha^{CDS,f}_{aa} + \frac{\left(\alpha^{CDS,f}_{1a}\right)^2}{2c} E \left[ \left( \mu_a \right)^2 \right] + \frac{\alpha^{CDS,f}_{1a} \alpha^{CDS,f}_{1b}}{c} E \left[ \left( \mu_b \right)^2 \right]$$ \hspace{1cm} (2)

$$EU^{CDS,f,ea}_b = \alpha^{CDS,f}_{ab} + \frac{\left(\alpha^{CDS,f}_{1b}\right)^2}{2c} E \left[ \left( \mu_b \right)^2 \right] + \frac{\alpha^{CDS,f}_{1a} \alpha^{CDS,f}_{1b}}{c} E \left[ \left( \mu_a \right)^2 \right]$$ \hspace{1cm} (3)

In equations (2) and (3) as well as in the following, notation is simplified
by writing $E_{\mu_i} [\cdot] = E \left[ \cdot \right]$. The principal determines the parameters
$\alpha^{CDS,f}_{1a}$ and $\alpha^{CDS,f}_{1b}$ such that his expected profit is maximized, while the fixed
wages are set such that the agents’ participation constraints are binding. The principal’s expected profit thus equals

$$E\pi_{CDS,f} = \frac{\alpha_{1a}^{CDS,f} + \alpha_{1b}^{CDS,f}}{c} - \left(\alpha_{1a}^{CDS,f}\right)^2 E\left[\left(\mu_a\right)^2\right] + \left(\alpha_{1b}^{CDS,f}\right)^2 E\left[\left(\mu_b\right)^2\right]$$

(4)

Maximizing (4) yields the solution $\alpha_{1a}^{CDS,f} = \frac{1}{E[\mu_a^2]}$ and $\alpha_{1b}^{CDS,f} = \frac{1}{E[\mu_b^2]}$.

Using the i.i.d. assumption, the solution becomes $\alpha_{1a}^{CDS,f} = \alpha_{1b}^{CDS,f} = \frac{1}{E[\mu^2]}$.

The principal’s expected profit is given by $E\pi_{CDS,f} = \frac{1}{cE[\mu^2]}$.

The optimal formal contract under PD can be derived analogously. Suppose in this case, without loss of generality, that the principal delegates the second task and handles the first task himself. The principal’s and the agent’s optimal effort are then $e_a = \frac{1-a_{1a}^{PD,f}}{c} \mu_a$ and $e_b = \frac{a_{1b}^{PD,f}}{c} \mu_b$. The optimal incentive parameter satisfies $\alpha_{1b}^{PD,f} = \frac{1}{E[\mu_a^2] + E[\mu_b^2]}$, or with the i.i.d. assumption, $\alpha_{1b}^{PD,f} = \frac{1}{2E[\mu^2]}$. The principal achieves an expected profit of $E\pi_{PD,f} = \frac{2E[\mu^2] + 1}{4cE[\mu^2]}$.

A comparison of $E\pi_{CDS,f}$ and $E\pi_{PD,f}$ immediately yields the following proposition:

**Proposition 1** The principal chooses CDS (PD, is indifferent between both job designs) if and only if $E[\mu^2] < (>, =) 1.5$.

Let me explain the intuition behind proposition 1. Compared to CDS, PD exhibits one important advantage and one important disadvantage. The disadvantage stems from the free-rider problem. Under both kinds of job design, the agents are compensated contingent on the realization of some aggregate performance measure. Hence, they receive only part of their marginal
product, whereas they bear the complete effort costs. As a consequence, they decide to choose inefficiently low efforts. This free-rider problem can be mitigated effectively under CDS by installing high-powered incentives, i.e., by increasing both variable components. Under PD, on the other hand, providing the principal and the agent with high-powered incentives is impossible.

The joint performance measure is positively correlated to total output. Thus, if the principal provides the agent with high incentives, he will automatically decrease his marginal payoff from exerting effort. That is, installing high incentives for the agent leads to low incentives for the principal and vice versa.\textsuperscript{11} As a consequence, under PD the free-rider problem is still present.

However, as seen in proposition 1, PD may also be the preferred choice of job design. There exists a second negative effect that is less severe under PD than under CDS. As mentioned before, the measure $p$ is only an imperfect measure of total contribution to firm value. Due to this imperfection, an agent’s behavior shows distortions with respect to desired behavior. This distortion depends on the realization of $\mu_i$. For $\mu_i < 1$, the agent responsible for task $i$ exerts undesirably low effort. On the contrary, for $\mu_i > 1$, the actual effort is undesirably high. Since the principal must compensate the agents for their effort costs such distortions from desired effort are costly. Under PD, this distorting behavior is clearly less serious. There is only one agent behaving inefficiently. The principal focuses on the realization of output and

\textsuperscript{11}As shown by Holmström (1982), the free-rider problem might be solved by introducing a third party being able to “break the budget”. Since such a solution entails new complications (see e.g. Eswaran and Kotwal (1984)) it is not considered in this paper.
therefore chooses a more desired effort.

The cut-off in proposition 1 is a result of the interaction of these two effects. As is clear from the preceding argumentation, PD will be preferable, if the distortion in an agent’s effort with respect to the desired effort is very high. Since, in expectation, $p$ is an unbiased measure of $y$, the agents’ efforts will be highly distorted if $Var[\mu]$ is high. This variance can be rewritten as $Var[\mu] = E[\mu^2] - 1$. Hence, this variance as well as the relative advantage of PD compared to CDS is strictly increasing in $E[\mu^2]$. Consequently, there exists a cut-off value for $E[\mu^2]$, where the optimal job design changes.

3 Job design when formal and relational contracts interact

In the one-period model in section 2, no relational agreement could be sustained, since every such agreement would be reneged on. In order to analyze the interaction of formal and relational contracts, I therefore consider an infinitely repeated version of the model from section 2.

In this infinite horizon model, some additional assumptions have to be introduced. First, it is assumed that the principal, besides the wage payment specified in section 2, offers an agent a bonus payment contingent on the agent’s contribution to firm value. As in Baker et al. (1994), to avoid complications by creating a temptation for the agents to break the implicit contract, the relational bonus is assumed to be non-negative. This assumption should be fulfilled in most real world settings.
The principal discounts future profits. The discount rate is \( r \), i.e., a one-unit profit in the next period is worth \( \frac{1}{1+r} \) units in the present one. The discount rate \( r \) could, for example, represent the interest rate, to which the principal could lend or borrow money. The agents are assumed to discount future utility at a rate \( r_a \), which may or may not differ from \( r \). The purpose of this discount rate is simply to appropriately define the infinitely repeated game. It does not affect the model results, as the following argumentation will show. In the model, discounting solely affects a party’s temptation to renege on a relational contract. However, as discussed before, an agent is not interested in refusing the payment of a relational bonus, since the bonus accrues to him. Moreover, he could breach the relational contract by deviating from the agreed effort. As this effort is expected utility maximizing, such a deviation is also not desirable for the agent.

All players, i.e., the principal and the agents, are assumed to follow a modified grim trigger strategy. Roughly speaking, they start by cooperating (that is, by honoring the relational agreement) and continue cooperation unless one player defects, in which case they refuse to cooperate forever after. Referred to the model this means that, after the informal agreement was reneged on once, no player will ever honor some informal contract and the parties will rely on the formal contracts derived in section 2.\footnote{Two remarks are necessary: First, in the literature on infinite games, it is sometimes argued that the game remaining after one party defects coincides with the game as a whole. As a consequence, equilibria being available in the game as a whole should also be available after the relational agreement was broken. Hence, the parties should be able to renegotiate from punishment to a different equilibrium with higher payoffs. I abstract from this} Moreover, un-
der CDS, these strategies imply that if the principal reneges on the relational bonus of only one agent, both agents lose trust in the principal.

Finally, suppose that the change from a certain job design to another entails considerable fix costs, so that the principal always maintains the job design he initially has chosen. This assumption has implications for payments off the equilibrium path. It ensures that, in case the principal reneges on the relational contract, that is, when relational contracts are no longer available, he does not change the job design. Although mainly made to simplify calculations, this assumption seems to map practice very well, for firms seem to change their organizational structure very rarely. The assumption is cancelled in section 4.

In order to derive the optimal combination of formal and relational contracts, I search for a subgame perfect equilibrium of the game. I consider only stationary contracts, under which the principal in every period offers the same wage contract and the agents choose the same efforts on the equilibrium path. This is, as shown by Levin (2003), without loss of generality.

possibility by assuming that renegotiation costs are too high, either because renegotiation causes too high monetary costs or because it simply takes too long. In the latter case, renegotiation would prevent the parties from working on their tasks so that renegotiation gains would be outweighed by the loss in production. Second, Abreu (1988) showed that highest equilibrium payoffs are supported by the strongest credible punishments. However, in the current model, grim trigger strategies may not yield strongest credible punishment (i.e. the grim trigger strategies may not form an optimal penal code). Again, it could be argued that the elaboration of an optimal penal code would be too costly so that relying on the (relatively simple) grim trigger strategies is preferred.
The wage payment to the agent dealing with task $i$ is in each period given by

$$w_{i}^{CDS,r} = \alpha_{oi}^{CDS,r} + \alpha_{1i}^{CDS,r} p + \beta_{i}^{CDS,r} y_i \quad \text{or} \quad w_{i}^{PD,r} = \alpha_{oi}^{PD,r} + \alpha_{1i}^{PD,r} p + \beta_{i}^{PD,r} y_i,$$

where $r$ indicates the combined use of formal and relational contracts. The term $\beta_{i}y_i$ corresponds to an informal promise of the principal to pay the agent a bonus depending on the realization of unverifiable output.\footnote{One could also assume that the wage payment contains a further element $\gamma_i y_j$, where $\gamma_i$ is a payment the agent will receive, if the contribution of the person in charge for the other task equals one. With the restriction $\gamma_i \geq 0$, the principal will always set $\gamma_i$ equal to zero. The element $\gamma_i y_j$ is therefore not considered in the wage contract.} Since such an informal promise cannot be enforced by a court, it must be self-enforcing.

Consider first the CDS case. The incentives provided by a relational contract depend on whether or not the agents believe that the principal will honor the contract. If, in a given period, they trust the principal, the agents will choose their efforts, after observing $\mu_i$, to maximize expected ex post utilities given by (5) and (6), respectively:

$$EU_a^{CDS,r, ep} = \alpha_{oa}^{CDS,r} + \alpha_{1a}^{CDS,r} \left( e_a \mu_a + E_{\mu_b} [e_b \mu_b] \right) + \beta_a^{CDS,r} e_a - \frac{c}{2} (e_a)^2 \quad \text{(5)}$$

$$EU_b^{CDS,r, ep} = \alpha_{ob}^{CDS,r} + \alpha_{1b}^{CDS,r} \left( e_b \mu_b + E_{\mu_a} [e_a \mu_a] \right) + \beta_b^{CDS,r} e_b - \frac{c}{2} (e_b)^2 \quad \text{(6)}$$

The optimal efforts therefore satisfy $e_a = \frac{\alpha_{oa}^{CDS,r} \mu_a + \beta_a^{CDS,r}}{\alpha_{1a}^{CDS,r} + \beta_a^{CDS,r}}$ and $e_b = \frac{\alpha_{ob}^{CDS,r} \mu_b + \beta_b^{CDS,r}}{\alpha_{1b}^{CDS,r} + \beta_b^{CDS,r}}$.\footnote{One could also assume that the wage payment contains a further element $\gamma_i y_j$, where $\gamma_i$ is a payment the agent will receive, if the contribution of the person in charge for the other task equals one. With the restriction $\gamma_i \geq 0$, the principal will always set $\gamma_i$ equal to zero. The element $\gamma_i y_j$ is therefore not considered in the wage contract.}
The agents’ expected ex ante utilities are

\[
EU_{a}^{CDS_r,ea} = \alpha_{a}^{CDS_r} + \frac{\left(\alpha_{1a}^{CDS_r}\right)^2 E \left(\left(\mu_a\right)^2\right) + \left(\beta_{a}^{CDS_r}\right)^2}{2c} 
\]

\[
EU_{b}^{CDS_r,ea} = \alpha_{ob}^{CDS_r} + \frac{\left(\alpha_{1b}^{CDS_r}\right)^2 E \left(\left(\mu_b\right)^2\right) + \left(\beta_{b}^{CDS_r}\right)^2}{2c} 
\]

The principal again determines the fixed wages such that the agents’ participation constraints are binding. His expected profit is then given by

\[
E\pi_{CDS_r} = \frac{CDS_r^a + \alpha_{1a}^{CDS_r} + \beta_{a}^{CDS_r} + \beta_{b}^{CDS_r}}{c} 
\]

\[
- \frac{\left(\alpha_{1a}^{CDS_r}\right)^2 E \left(\left(\mu_a\right)^2\right) + \left(\beta_{a}^{CDS_r}\right)^2}{2c} 
\]

\[
- \frac{\left(\alpha_{1b}^{CDS_r}\right)^2 E \left(\left(\mu_b\right)^2\right) + \left(\beta_{b}^{CDS_r}\right)^2}{2c} 
\]

\[
- \frac{\alpha_{1b}^{CDS_r} \beta_{a}^{CDS_r}}{c} 
\]

Note that the principal will honor the relational contract, only if the discounted additional future profits arising from the combined use of formal and relational agreements exceed the present gain from not paying the two relational bonuses. The non-reneging constraint is therefore given by

\[
(E\pi_{CDS_r} - E\pi_{CDS_f}) \frac{1}{r} \geq \beta_{a}^{CDS_r} + \beta_{b}^{CDS_r} 
\]

It can easily be seen that this constraint is more likely to be satisfied, the higher the additional profit from relying on relational agreements, the lower the discount rate \( r \), and the lower the relational bonus to be paid. This is intuitive. If the principal does gain very much from the use of relational
contracts and if he is rather patient (that is, future profits are hardly dis-
counted), the benefit from not paying the relational bonuses will probably be
outweighed by the loss in future profits. On the other hand, the gain from
not paying the bonus and, hence, the reneging temptation certainly increases
in the size of the bonuses.

Consider now the optimal choice of the incentive parameters. While deter-
mining $\alpha_{1a}^{CDS,r}$, $\alpha_{1b}^{CDS,r}$, $\beta_a^{CDS,r}$ and $\beta_b^{CDS,r}$, the principal maximizes $E\pi_{CDS,r}$
subject to the non-reneging constraint. Using the i.i.d. assumption, the
Lagrangian to the maximization-problem is given by

$$L = \frac{1 + \lambda}{c} \left[ \alpha_{1a}^{CDS,r} + \alpha_{1b}^{CDS,r} - 0.5 \left( \left( \alpha_{1a}^{CDS,r} \right)^2 + \left( \alpha_{1b}^{CDS,r} \right)^2 \right) \right] E \left[ \mu^2 \right]$$

The first-order conditions to the maximization-problem are

$$\frac{\partial L}{\partial \alpha_{1a}^{CDS,r}} = \frac{1 + \lambda}{c} \left( 1 - \beta_a^{CDS,r} - \alpha_{1a}^{CDS,r} E \left[ \mu^2 \right] \right) = 0$$
$$\frac{\partial L}{\partial \alpha_{1b}^{CDS,r}} = \frac{1 + \lambda}{c} \left( 1 - \beta_b^{CDS,r} - \alpha_{1b}^{CDS,r} E \left[ \mu^2 \right] \right) = 0$$
$$\frac{\partial L}{\partial \beta_a^{CDS,r}} = \frac{1 + \lambda}{c} \left( 1 - \beta_a^{CDS,r} - \alpha_{1a}^{CDS,r} \right) - \lambda r = 0$$
$$\frac{\partial L}{\partial \beta_b^{CDS,r}} = \frac{1 + \lambda}{c} \left( 1 - \beta_b^{CDS,r} - \alpha_{1b}^{CDS,r} \right) - \lambda r = 0$$

These conditions lead to a symmetric solution, the principal chooses same
wage contracts for the two agents, i.e. $\alpha_{1a}^{CDS,r} = \alpha_{1b}^{CDS,r} = \alpha_1^{CDS,r}$, and
$\beta_a^{CDS,r} = \beta_b^{CDS,r} = \beta_1^{CDS,r}$. Using this symmetry, the first-order conditions
simplify to
\[
\frac{1 + \lambda}{c} \left( 1 - \beta^{CDS,r} - \alpha_1^{CDS,r} E[\mu^2] \right) = 0 \quad (16)
\]
\[
\frac{1 + \lambda}{c} \left( 1 - \beta^{CDS,r} - \alpha_1^{CDS,r} \right) - \lambda r = 0 \quad (17)
\]

If, in the optimum, the non-reneging constraint is non-binding (i.e. \( \lambda = 0 \)), the solution is \( \alpha_1^{CDS,r} = 0 \) and \( \beta^{CDS,r} = 1 \). That is, if the principal is sufficiently patient, a first-best relational contract will be installed. Each agent bases his effort decision solely on the realization of output and, as a consequence, no distorting behavior will arise.

Of more interest is the case, in which the principal is less patient so that the non-reneging constraint binds in the optimum. From (16) and the binding condition (10), the second-best relational bonus and the second-best expected profit can be derived. The possible values of relational bonus and expected profit are given by (18) and (19), respectively:

\[
\beta^{CDS,r} = \begin{cases} 
1 & \text{for } r \leq \tilde{r}^c \\
2 - \frac{2rcE[\mu^2]}{E[\mu^2]-1} & \text{for } \tilde{r}^c > r > \tilde{r}^c \\
0 & \text{for } r \geq \tilde{r}^c 
\end{cases} \quad (18)
\]

\[
E\pi_{CDS,r} = \begin{cases} 
\frac{1}{c} & \text{for } r \leq \tilde{r}^c \\
\frac{(E[\mu^2]-1)(1+4rcE[\mu^2]) - 4r^2c^2(E[\mu^2])^2}{E[\mu^2](E[\mu^2]-1)c} & \text{for } \tilde{r}^c > r > \tilde{r}^c \\
\frac{1}{cE[\mu^2]} & \text{for } r \geq \tilde{r}^c 
\end{cases} \quad (19)
\]

with \( \tilde{r}^c = \frac{E[\mu^2]-1}{2cE[\mu^2]} \) and \( \tilde{r}^c = \frac{E[\mu^2]-1}{cE[\mu^2]} \).

The derivation of the optimal combination of formal and relational contract in the PD case is analogous. The optimal relational bonus in this case
is given by (20), the optimal expected profit by (21).

\[
\beta^{PD,r} = \begin{cases} 
1 & \text{for } r \leq \hat{r}^p \\
2 - \frac{4rcE[\mu^2]}{2E[\mu^2]-1} & \text{for } \hat{r}^p > r > \check{r}^p \\
0 & \text{for } r \geq \check{r}^p 
\end{cases} \tag{20}
\]

\[
E_{\pi_{PD,r}} = \begin{cases} 
\frac{1}{c} & \text{for } r \leq \hat{r}^p \\
\frac{4(E[\mu^2])^2 - 1 + 8rcE[\mu^2](2E[\mu^2](1-r)c - 1)}{4E[\mu^2](2E[\mu^2]-1)c} & \text{for } \hat{r}^p > r > \check{r}^p \\
\frac{2E[\mu^2] + 1}{4cE[\mu^2]} & \text{for } r \geq \check{r}^p 
\end{cases} \tag{21}
\]

with \(\hat{r}^p = \frac{2E[\mu^2] - 1}{4cE[\mu^2]}\) and \(\check{r}^p = \frac{2E[\mu^2] - 1}{2cE[\mu^2]}\).

Let us now compare the expected profits to see, which kind of job design the principal prefers. When comparing the profits, it is convenient to distinguish between the cases \(E[\mu^2] < 1.5\), \(E[\mu^2] > 1.5\) and \(E[\mu^2] = 1.5\). I start with the first one.

As shown in proposition 1, CDS is optimal in the absence of relational agreements. With \(E[\mu^2] < 1.5\), conditions \(\hat{r}^c < \check{r}^c < \hat{r}^p < \check{r}^p\) hold. Proposition 2 shows the principal’s optimal job design in this case.

**Proposition 2** Suppose that \(E[\mu^2] < 1.5\). (i) For \(r \leq \hat{r}^c\), both job designs lead to the first-best solution. The principal is in this case indifferent between the two job designs. (ii) For \(\hat{r}^c < r \leq \hat{r}^p\), PD yields the first-best solution, whereas CDS does not. PD is thus preferred. (iii) For \(\hat{r}^p < r \leq \check{r}^p\), there exists a cut-off \(\check{r}\) with \(\check{r}^p < \check{r} < \check{r}^p\) such that PD is preferred only if \(r \in [\check{r}^p, \check{r}]\). (iv) For \(r > \check{r}^p\), CDS is preferred.

Proof: See Appendix.
Note first that $r > \bar{r}^p$ corresponds to the case, where the principal is so impatient or the interest rate is so high that any informal contract would be reneged on. Put differently, in this case only formal contracts are available. As should be clear, the model analyzed in section 2 is only a special case of the interaction of formal and relational contracts. Of great interest is the result that the optimal job design if only formal contracts are available need no longer be optimal when the principal uses some combination of formal and relational contract to compensate his agents. On the contrary, when formal and relational contracts interact, this job design is often the less preferred one. In other words, the results derived in the less general model in section 2 are not robust to an introduction of relational agreements.

Let me explain this result in more detail. The principal would always prefer to rely on informal contracts rather than on formal contracts, since, in this way, distortions in the agents’ efforts are mitigated. However, as condition (10) indicates, the principal may be unable to commit not to renege on relational bonuses so high that the first-best solution would be achieved. He therefore uses some combination of formal and relational contracts as incentive device. The appropriateness of a job design in this case roughly depends on three factors. First, it depends on the job designs’ needs for relational agreements, i.e., the relative profit increase under each job design when formal incentives are replaced by relational ones. Second, it depends on the relative size of the relational bonuses that can be sustained under each job design. Third, it depends on the respective status quo point, that is the respective profits, if relying solely on formal contracts. While the third point
has already been treated in section 2, in this section, the first two points are analyzed.

I start with the first one. When comparing the principal’s benefit from the introduction of a relational contract with a fixed bonus under PD and under CDS, there are two countervailing effects. On the one hand, a relational contract seems to be more beneficial under PD. From (16), we see that formal and relational incentives are substitutes. That is, the introduction of relational contracts leads to lower remuneration based on the realization of the contractible measure \( p \). Under PD, this effect mitigates the free-rider problem, since, as explained in section 2, lower formal incentives for the agent yield higher incentives for the principal. This advantage is absent under CDS. CDS, on the other hand, especially benefits from the introduction of relational agreements since, under that job design, distortions in effort behavior of two agents are mitigated.

Is any of these effects dominant? In order to answer this question, it is convenient to calculate, for a fixed relational bonus \( \beta \), the difference between profit in the presence and absence of relational agreements under each job design. Denote by \( \Delta^{CDS} \) and \( \Delta^{PD} \) these differences. Using (9), the symmetry of the solution and (16), one can show that \( \Delta^{CDS} = \frac{1}{2E[\mu^2]} (-2\beta + \beta^2 + 2E[\mu^2] \beta - E[\mu^2] \beta^2) \). Similarly, \( \Delta^{PD} \) can be demonstrated to equal \( \frac{-2\beta + \beta^2 + 4E[\mu^2] \beta - 2E[\mu^2] \beta^2}{2E[\mu^2]} \). It is then straightforward to show that \( \Delta^{CDS} > \Delta^{PD} \iff E[\mu^2] > 1.5 \). The condition says that the job design performing relatively worse in the absence of relational contracts benefits more strongly from their introduction. In case \( E[\mu^2] < 1.5 \)
this means that the first effect is dominant, i.e., under PD, the principal benefits more strongly from the introduction of relational agreements.

Let us now analyze, under which job design higher relational bonuses can be sustained. The possibility to remunerate an agent with a certain relational bonus depends on the level of the bonus, the number of bonuses to be paid, the difference in profit in the presence and absence of relational contracts, and the discount rate. Hence, the comparison of the two job designs with respect to the maximum relational bonus they may implement depends on the respective difference in profits when relational contracts are honored and when they are not as well as on the number of bonuses to be paid. Recall that for $E[\mu^2] < 1.5$, $\Delta^{PD} > \Delta^{CDS}$. This should lead to the implementation of higher relational bonuses under PD, since, under that job design, the principal is more heavily punished for reneging on the relational contract. Moreover, under PD, a higher relational bonus should be sustained since this bonus has to be paid only for one agent and not for two as under CDS. On account of this, the principal’s benefit from reneging on the relational contract should be lower under PD.

Since these effects are enforcing, PD should always lead to higher relational bonuses than CDS. This can be confirmed comparing (18) and (20). Not only is the second-best relational bonus higher, but there are also parameter constellations, where the first-best bonus can only be implemented under PD. Similarly, there are parameter constellations, for which no relational contract is feasible under CDS, whereas there are relational agreements under PD. Further, since $\tilde{r}^C < \tilde{r}^P$, there exists no range of parameter values,
where, under both kinds of job design, there is a combined use of formal and relational contracts.

As a consequence, PD allows a much wider use of relational contracts than CDS. It is therefore preferred for many values of the discount parameter \( r \). Since CDS is optimal in the absence of relational agreements, there exists a clear cut-off \( \tilde{r} \), where the optimal job design changes. For \( r < \tilde{r} \), the principal under PD makes extensive use of informal agreements, whereas formal contracts are of major importance under CDS. Hence, for \( r < \tilde{r} \), PD is optimal. For \( r > \tilde{r} \), under both job designs relational agreements are rather unimportant. In this case, CDS is preferred since it is very effective in mitigating the free-rider problem and suffers only little from distortion in efforts.

It is worth emphasizing the relation between a job design’s appropriateness under formal contracts and its suitability under a combined use of formal and relational contracts. This relation is namely very helpful in explaining the arising discrepancy between the optimal job design in the absence and presence of relational agreements. Loosely speaking, a job design performing poorly in the absence of relational contracts is likely to do (relatively) better in their presence. Let me explain this in more detail. A job design performing poorly in the absence of relational agreements may sustain a relatively high relational bonus, as the principal’s punishment in case of reneging on the relational contract is relatively high. The principal is therefore less tempted to renege on the relational agreement. Similarly, as mentioned before, a poorly performing job design benefits relatively more from the introduction of re-
lational agreements, as there are more inefficient actions to be mitigated by relational contracts. To summarize, a job design’s weaknesses in the absence of relational agreements may become its strengths under a combined use of formal and relational contracts.

Consider now the case $E[\mu^2] > 1.5$. In this case, conditions $\hat{r}_c < \hat{r}_p < \tilde{r}_c < \tilde{r}_p$ hold. Proposition 3 describes the principal’s optimal choice.

**Proposition 3** Suppose that $E[\mu^2] > 1.5$. (i) For $r \leq \hat{r}_c$, both job designs lead to the first-best solution. The principal is in this case indifferent between the two job designs. (ii) For $r > \hat{r}_c$, PD is always preferred.

Proof: See Appendix.

The effects at work in this setting are the same as for $E[\mu^2] < 1.5$. On the one hand, the relational bonus to be sustained as well as the status quo point is higher under PD. Under CDS, on the other hand, the principal benefits relatively more from the introduction of relational contracts. As the first two effects always outweigh the third, for all values of the discount rate $r$, PD is (weakly) preferred.

Finally, suppose that $E[\mu^2] = 1.5$. In this case, we have $\hat{r}_c < \hat{r}_p = \tilde{r}_c < \tilde{r}_p$. Again, the same effects as in the first two cases determine the optimal job design. In proposition 4, I therefore only present the optimal job design, without further explaining the intuition behind the results.

**Proposition 4** With $E[\mu^2] = 1.5$, the following results hold: (i) For $r \leq \hat{r}_c$, both job designs lead to the first-best solution. The principal is in this case indifferent between the two job designs. (ii) For $\hat{r}_c < r < \hat{r}_p$, PD is always preferred.
preferred. (iii) For \( r \geq r^p \), the principal is indifferent between the two job designs.

Proof: Obvious and omitted.

4 Discussion

Up to this point, two assumptions were made facilitating the analysis, namely that a change in organizational structure entails considerable fixed costs and that the agents’ reservation utilities equal zero. I now relax these assumptions. Since the model becomes extremely complicated once the assumptions are cancelled, I only discuss the effects that such a cancellation entails. I begin with the assumption concerning the fixed costs.

If a change in job design is totally costless, the principal will, after reneging on the relational contract, always switch to that job design being optimal in the absence of relational agreements. Hence, for \( E[\mu^2] > 1.5(E[\mu^2] < 1.5) \), he will choose PD (CDS). This may lead to a higher profit off the equilibrium path. To be concrete, for \( E[\mu^2] > 1.5(E[\mu^2] < 1.5) \), the principal’s CDS (PD) profit in the absence of relational contracts increases, whereas the PD (CDS) profit does not change. This change in profit has impacts on the non-reneging constraint. A ceteris paribus increase in profit, when the principal solely relies on formal contracts, yields a (weakly) lower relational bonus that can be sustained. Hence, compared to the results in proposition 2, CDS should become more preferable since the relational bonus being sustained under PD decreases. Similarly, compared to the results in proposition
3, PD should become even more dominant.

Although it is with some related implications, the introduction of a reservation utility different from zero entails more complex effects. First, PD should always become more preferable, since, under PD, only one agent has to be compensated for $\bar{U}$. In order to determine the implications on the relational bonus to be sustained and the profits under relational agreements, it is convenient to make some case distinction. In the first case, the reservation utility is rather small. In particular, it is so small that, under both job designs, the profit that can be realized in the absence of relational contracts remains positive. In this case, the model results with respect to the relational bonus do not change at all. With positive reservation utilities both, the profit, when formal and relational contracts interact as well as the profit when the principal solely relies on formal contracts, are decreased by the same amount. The bonus that can be sustained therefore does not change. In the second case, the reservation utilities adopt intermediate values so that, in the absence of relational agreements, one job design leads to a positive profit and the other one to zero profit.\footnote{\emph{An increase in reservation utility then affects only three and not four profits. While it decreases all positive profits, the zero profit is unaffected. As a consequence, the bonus under the job design, where both profits are positive does not change. On the contrary, the bonus under the other job design (weakly) decreases. Hence, the first job design should (relatively) become more desirable. In case three, the reservation utility is assumed that the firm would close down before it made negative profit. Therefore, the worst possible outcome for the principal is a profit of zero.}} An increase in reservation utility then affects only three and not four profits. While it decreases all positive profits, the zero profit is unaffected. As a consequence, the bonus under the job design, where both profits are positive does not change. On the contrary, the bonus under the other job design (weakly) decreases. Hence, the first job design should (relatively) become more desirable. In case three, the reservation utility is assumed that the firm would close down before it made negative profit. Therefore, the worst possible outcome for the principal is a profit of zero.
Ities are so high that both job designs lead to zero profits in the absence of relational contracts. An increase in reservation utility then affects both job designs, since the relational bonus (weakly) decreases under PD as well as under CDS. However, the absolute change in bonus and its impact on the profit may be different.

5 Concluding Remarks

This paper started by comparing two different job designs in a static environment. A very nice and intuitive condition was derived indicating when each job design is optimal, respectively. Thereafter, a model with infinite horizon was considered. The purpose was to allow the principal to use both, formal and informal contracts, as incentive device. It was shown that the introduction of relational contracts has a crucial impact on a job design’s appropriateness. Particularly, a job design being optimal in the absence of relational contracts need no longer be optimal, if these contracts are available.

The reason is that a job design performing very poorly in the absence of relational agreements allows the principal to install high-powered informal incentives, since his punishment from defecting is very high. Moreover, the principal benefits from the introduction of relational contracts more strongly in settings, where relying solely on formal contracts is not very profitable. The interplay of these two effects may overturn the results derived in the absence of relational agreements.

This observation is particularly very interesting, since most economic
models are static ones. As seen in this paper, the results derived in static scenarios need to be handled with care. It should thus be of great interest how the introduction of dynamic and, hence, of informal agreements affects other model results. Future research should deal with this question.

Appendix

Proof of proposition 2:

The proof of parts (i), (ii) and (iv) is obvious and therefore omitted. It remains to prove part (iii). For $\hat{r}^p < r \leq \check{r}^p$, PD leads to a mixture of formal and relational contracts, whereas under CDS only formal contracts are available. The profits to be compared thus are $E\pi_{PD,r} = \frac{4(E[\mu^2])^2 - 1 + 8rcE[\mu^2]((2E[\mu^2] - 1) - 2rcE[\mu^2])}{4E[\mu^2](2E[\mu^2] - 1)c}$ and $E\pi_{CDS,r} = \frac{1}{cE[\mu^2]}$. PD is the preferred choice of job design if the following condition holds: $4(E[\mu^2])^2 - 1 + 8rcE[\mu^2]((2E[\mu^2] - 1) - 2rcE[\mu^2]) > 8E[\mu^2] - 4$. Simplifying yields $z(r) := 4(E[\mu^2] - 1)^2 - 1 + 16rc(E[\mu^2])^2 - 8rcE[\mu^2] - 16r^2c^2(E[\mu^2])^2 > 0$. The derivative of $z$ with respect to $r$ will be positive, only if $2E[\mu^2] - 1 - 4rcE[\mu^2] > 0$. For $r = \hat{r}^p$, the left-hand-side of the inequality is zero. Consequently, it as well as the derivative of $z$ with respect to $r$ is negative for $r > \hat{r}^p$. Since $z$ is positive for $r = \hat{r}^p$ (PD achieves the first-best solution) and negative for $r = \check{r}^p$ (under both job designs relational contracts are not available), there must be a cut-off $\check{r}$, with $\hat{r}^p < \check{r} < \check{r}^p$, at which the optimal job design changes. This proves part (iii) of proposition 2.

Proof of proposition 3:

The proof of part (i) is again obvious and so omitted. The proof of
part (ii) is obvious except for the range of parameter values, where $\hat{r}_p < r \leq \hat{r}_c$. In this case, both job designs lead to a mixture of formal and relational contracting. Hence, condition $E\pi_{CDS,r} > E\pi_{PD,r}$ is equivalent to 

$$
\left[ (E[\mu^2] - 1) \left( 1 + 4rcE[\mu^2] \right) - 4r^2c^2(E[\mu^2])^2 \right] 4(2E[\mu^2] - 1) \text{ strictly exceeding} \left[ 4(E[\mu^2])^2 - 1 + 8rcE[\mu^2] \left( (2E[\mu^2] - 1) - 2rcE[\mu^2] \right) \right] (E[\mu^2] - 1).
$$

Simplifying this condition yields $y(r) := U(r) + V > 0$, with $U(r) = 16rc^2(E[\mu^2])^2 + 24(2E[\mu^2])^2 - 16(E[\mu^2])^3 - 8E[\mu^2]$ and $V = 4(E[\mu^2])^3 - 12(E[\mu^2])^2 + 11E[\mu^2] - 3$. The function $y$ is strictly convex in $r$. It has two nulls, $r_1 = \frac{1}{4c(E[\mu^2])^2} \left( -3E[\mu^2] + 2(E[\mu^2])^2 + 1 - \sqrt{-3E[\mu^2] + 2(E[\mu^2])^2 + 1} \right)$, $r_2 = \frac{1}{4c(E[\mu^2])^2} \left( -3E[\mu^2] + 2(E[\mu^2])^2 + 1 + \sqrt{-3E[\mu^2] + 2(E[\mu^2])^2 + 1} \right)$. In order to show that PD will perform better than CDS, if $\hat{r}_p < r \leq \hat{r}_c$, it suffices to show that the right null $r_2$ is smaller than $\hat{r}_p$. The right null will be smaller than $\hat{r}_p$, if and only if $-3E[\mu^2] + 2(E[\mu^2])^2 + 1 + \sqrt{-3E[\mu^2] + 2(E[\mu^2])^2 + 1} < 2(E[\mu^2])^2 - E[\mu^2]$. Rearranging this condition leads to $2(E[\mu^2])^2 - E[\mu^2] > 0$, which is always fulfilled. Hence, part (ii) of proposition 3 is proved.

References


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