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Limited Pledgeability, Asset Prices, and Macroeconomic Fluctuations

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Limited Pledgeability, Asset Prices, and Macroeconomic Fluctuations

by

Haiping Zhang

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Limited Pledgeability, Asset Prices, and Macroeconomic Fluctuations

Haiping Zhang*

April 2005

Abstract

According to the theory of incomplete contracts, given nonverifiable entrepreneurial project choices together with divergent objectives between an entrepreneur and its outside financier, the entrepreneur can credibly pledge only part of its project outcome for external funding. Meanwhile, entrepreneurial net worth must be put as down payment to ameliorate agency costs.

In a real dynamic general equilibrium model with heterogeneous agents and nonverifiable project choices, endogenous agency costs significantly change the business-cycle pattern in the sense that the model can replicate an important empirical fact, the amplified hump-shaped output behavior. Furthermore, variable asset prices can affect entrepreneurial net worth and then subsequently change the dynamic features of aggregate output along business cycles.

JEL Classification: E32, E44, G3

Keywords: Asset Prices, Business Cycles, Credit Constraints, Hump-Shaped Output Dynamics, Nonverifiable Project Choice.

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1 Introduction

1.1 Current Literature

The balance sheet condition of indebted firms as well as indebted households is important for the macroeconomic activity. A large body of literature derives credit constraints from information frictions at different stages of financial contracting. The borrower has information advantages on his project outcome or his own choices of project as well as effort, in comparison with the outside financier. As those costs incurred could be so high that it might be economically inefficient for the lender to collect such information in all states, she would like to induce the borrower to tell the truth or choose good projects or exert efforts by providing him with a reasonable share of the cake. Given nonverifiable actions and divergent objectives of contracting parties, the project value is below the first-best level\(^1\) if each agent just maximizes its own objective. As the borrower can credibly pledge only part of the project outcome for external funding, his net worth must be put as down payment to ameliorate agency costs. Our paper is related with three lines of research that incorporate moral hazards incurred at different stages of financial contracting into the dynamic general equilibrium (hereafter, DGE) framework and study the macroeconomic consequences of agency costs.

By bringing the problem of costly state verification à la Townsend (1979) into a real business cycle framework with overlapping generations, Bernanke and Gertler (1989) show qualitatively that borrowers’ net worth plays an essential role in the optimal financial contracting under asymmetric information. The aggregate effects of shocks to borrowers’ net worth can amplify and persist. Carlstrom and Fuerst (1997, 1998) embed this mechanism in a tractable real DGE framework for the quantitative analysis of the effects of agency costs on business-cycle dynamics. Their models replicate the empirical fact of hump-shaped output behavior as documented in Cogley and Nason (1993), because households postpone their investment several periods after the shock when agency costs are at their lowest. Bernanke, Gertler, and Gilchrist (1999) further bring in money and price stickiness and show how credit market frictions may influence the transmission of monetary policy in this financial accelerator model.

Another line of research is initiated by Kiyotaki and Moore (1997) and Kiyotaki (1998).\(^1\)The difference in the project value between the cases with and without information frictions is normally called “agency costs”.

1
who derive simple collateral constraints from the theory of inalienable human capital à la Hart and Moore (1994). A positive productivity shock improves the borrower’s revenue and then net worth, which enables these more productive agents to increase leveraged investment in durable assets. Given fixed stock, the excess asset demand of entrepreneurs pushes up asset prices. It further improves the borrower’s net worth contemporaneously and the enhanced credit boom helps allocate asset towards those more productive borrowers. They show how the interaction of asset prices and credit limits becomes a powerful transmission mechanism by which the output effects of shocks persist, amplify, and spill over to other sectors. By introducing the standard cash-in-advance constraint, Cordoba and Ripoll (2004) reveal the role of collateral constraints in transforming small monetary shocks into large persistent output fluctuations. Following the modeling strategy of Bernanke, Gertler, and Gilchrist (1999), Vlieghe (2004) puts collateral constraint in a standard new Keynesian sticky price monetary framework. As some of output variability is due to credit frictions, it may not be optimal for monetary policy to try and achieve the flexible-price level of output.

A third line of research assumes that hidden actions of the entrepreneur, e.g., his nonverifiable choices of the project or the effort level, can affect project outcomes. Be specific, if he chooses a bad project or shirk, the success probability of the project will be low but the entrepreneur can have high private benefits. As shown in Holmstrom and Tirole (1997, 1998), the lender has to leave the borrower a reasonable share of the cake in order to induce him to choose the project with high success probability or exert efforts to raise the success probability of the same project. As a result, the borrower cannot credibly pledge all of the project outcome for external funding. By extending the two-period model of Holmstrom and Tirole (1997) to the infinite time horizon and adding durable asset of fixed total supply, Chen (2001) brings banks into the simple borrower-lender relationship and explains why banking crises often coincide with depression in the asset markets. Aikman and Paustian (2005) incorporate the bank model of Chen (2001) in a standard dynamic new Keynesian framework and show optimal monetary policy in the environment with credit frictions. After assuming liquidity problem of Holmstrom and Tirole (1998) in capital production, Kato (forthcoming) shows the depressed hump-shaped output behavior similar as in Carlstrom and Fuerst (1997) and pro-cyclical corporate demand for liquidity.
1.2 Structure

This paper brings together first two lines of research in a fundamentally modified model of Chen (2001) and makes three contributions to the literature. First, we provide a real DGE model with heterogeneous agents in which credit constraints are derived from nonverifiability of entrepreneurial project choice, as in Holmstrom and Tirole (1997) and Chen (2001). In contrast to depressed hump-shaped output behavior generated by Carlstrom and Fuerst (1997) and Kato (forthcoming), our model produces amplified hump-shaped output behavior due to capital reallocation among agents with different productivity.

In order to show that asset prices can have amplifying effects on aggregate activities, many models, including Kiyotaki and Moore (1997), Kiyotaki (1998), Chen (2001), Vlieghe (2004), and Aikman and Paustian (2005), stress the demand effect on asset prices by assuming fixed asset supply for analytical convenience. The common results are that the rise in asset prices in booms significantly improves the net worth of credit constrained agents and reduces agency costs. As a result, aggregate output responds more strongly than in the first-best case. In contrast, we assume that depreciable capital can be reproduced and its upward sloping supply curve are modeled by introducing ad hoc adjustment costs, see Faia (2004). As the second contribution, we show that, in addition to amplifying effects, variable asset prices greatly change the dynamic features of output behavior. That is, aggregate output reaches its peak earlier, if capital adjustment is more costly so that asset prices respond more strongly to any excess demand. However, this result depends crucially on the specific assumption of capital price formation. By combining Kato (forthcoming) with the current model, Zhang (2005b) introduces variable asset price by assuming nonverifiability of project choice in capital adjustment process instead of ad hoc adjustment costs. The model with dual limited pledgeabilities generates more delayed and amplified output responses to a productivity shock. In this sense, the modeling strategy on capital price formulation does matter for the understanding of macroeconomic fluctuations.

As the third contribution, this paper makes many fundamental improvements over its predecessors, Chen (2001) and Aikman and Paustian (2005). The contracts in their models are neither debt contracts nor inherently consistent. In order to consist with the standard features of debt contracts and the risk aversion of some agents, we change the unit of account in the loan contracts and assume that the capital stock is fully destroyed in a failed project and. Section 3 discusses these issues in detail.
The internal mechanism is briefly shown as follows. The risky projects of entrepreneurs are expected to be more productive than those safe projects of households, but their project choices are nonverifiable. Entrepreneurial net worth has to be provided as down payment in order to guarantee the lender’s interests. Due to credit frictions, a fraction of capital is allocated inefficiently to those less productive agents, i.e., households. If there is an exogenous positive shock on aggregate productivity (hereafter, TFP), the rise in the entrepreneur’s revenue improves his net worth, which reduces agency costs for loans. Entrepreneurs can increase their capital holding by leveraged investment. However, it still takes time for entrepreneurs to gradually accumulate net worth before they can fully explore the profitability of their projects. In addition to the TFP shock, the tilting of capital allocation towards more productive agents also have amplifying effects on aggregate output. The speed of capital reallocation actually determines the shape of output dynamics. As a result, aggregate output reaches the peak in a delayed fashion due to endogenous agency costs and gradually-built entrepreneurial net worth. In order to illustrate the internal mechanism more explicitly, we shut off asset prices by assuming no capital adjustment costs. The one-to-one transformation of consumption goods to capital goods guarantees constant asset prices over time.

In order to study how asset prices can affect the dynamic features of output behavior, we model the upward sloping capital supply curve by introducing ad hoc adjustment costs. A positive TFP shock raises entrepreneurial revenue and spurs their investment. Although the excess demand raises asset prices, the entrepreneurial user cost of capital goods actually falls thanks to leveraged investment. In the meantime, the rise in asset prices improves the contemporaneous entrepreneurial net worth. These two effects jointly result in more capital reallocation towards entrepreneurs with higher productivity in the shock period and hence aggregate output reaches its peak relatively earlier than in the case without adjustment costs. In this sense, asset prices not only amplify the effects of exogenous shocks on aggregate output but also change the its dynamic features.

The paper is organized as follows. Section 2 starts with an overview of the model economy. Credit constraints are derived from nonverifiability of entrepreneurial project choice. Agents’ optimization and relevant market clearing conditions jointly describe the competitive equilibrium in our model economy. Section 3 revisits some essential contractual issues and lays out the benchmark model without moral hazard. After calibration, Section 4 simulates the responses of the model economy with respect to TFP shocks under different scenarios. Section 5 concludes by collecting major findings.
2 The Model

2.1 Overview

Consider a discrete-time closed real economy with a continuum of heterogeneous agents of unit mass, entrepreneurs and households. The population of entrepreneurs is \( \eta \). Although agents have a common discount factor, households are risk averse and infinitely lived, while entrepreneurs are risk neutral and each has a constant probability \( \pi \) of surviving to the next period. At the end of each period, new entrepreneurs of mass \( \eta(1 - \pi) \) are born and replace those dying, which keeps the population of entrepreneurs constant.\(^2\)

There are three goods: a capital good, an intermediate good, and a consumption good. The intermediate good is produced from the households’ safe projects as well as from the entrepreneurs’ risky projects by using capital as the sole input. A continuum of competitive firms have the constant-return-to-scale technology to produce the consumption good by employing the intermediate good and the household labor. Capital depreciates at the rate \( \delta \) and can be reproduced from the consumption good. The moral hazard arises only in the entrepreneurial project choice. The consumption good is chosen as the numeraire. There are four factor prices in the economy, the price of capital \( q_t \), the price of the intermediate good \( v_t \), the wage rate \( w_t \) and the deposit rate \( r_t \) for households.

The time sequence of events is as follows. At the beginning of period \( t \), intermediate goods are generated from the projects installed at the end of period \( t - 1 \). After the realization of aggregate TFP shocks on the consumption good production, competitive firms purchase intermediate goods at the price \( v_t \) and employ households labor at the wage rate \( w_t \) to produce consumption goods. Those entrepreneurs with successful projects repay the predetermined debt amount; as there is no output and capital is assumed fully destroyed in a failed project, those unlucky entrepreneurs are simply released from their debt obligations.\(^3\) After receiving the signal of exiting the economy, the entrepreneurs of mass

\(^2\)Because of agency costs, the entrepreneur prefer to accumulate net worth and postpone consumption until no external funding is needed. Carlsrom and Fuerst (1996) and Bernanke, Gertler, and Gilchrist (1999) handle this problem by imposing a constant death probability, where dying means liquidating the net worth, consuming the proceeds, and exiting the economy. As an alternative, Carlsrom and Fuerst (1997, 1998), Kiyotaki and Moore (1997) and Kato (forthcoming) assume that the infinitely-lived entrepreneurs have a higher discount rate than households. Carlsrom and Fuerst (2001) explore the difference in macroeconomic implications of two assumptions.

\(^3\)Ex post verification of project outcomes incurs no costs and so does debt enforcement as long as there is positive outcomes.
$1 - \pi$ consume all wealth and are replaced by newcomers of the same mass. Competitive
capital producing firms, owned by households, transform consumption goods to capital
and sell at the prevailing price $q_t$. Any profit incurred is lump-sum transferred to house-
holds. Each of the surviving entrepreneurs and newcomers receives a tiny endowment$^4$
e and decide how much to consume, invest, and borrow. As the expected rate of return
of entrepreneurial net worth always exceeds the entrepreneurial rate of time preference,
risk-neutral entrepreneurs prefer to borrow to the limit, invest all net worth in the project,
and postpone consumption until they receive the signal of exiting the economy. In the
meantime, households make decisions on consumption, leisure, investment, and deposits
at the financial intermediary.

2.2 Limited Pledgeability and one-period loan contracts

 Entrepreneurs can invest capital in one of two risky projects at the end of period $t$. At
the beginning of period $t + 1$, both projects, “Good” and “Bad”, can yield $R$ units of
intermediate goods per unit of capital invested if succeed, and zero if fail. Two projects
differ in the probability of success and private benefits for the entrepreneur$^5$, as shown in
the following table,

<table>
<thead>
<tr>
<th>Project</th>
<th>Good</th>
<th>Bad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of Success</td>
<td>$p_G$</td>
<td>$p_B$</td>
</tr>
<tr>
<td>Private Benefits</td>
<td>$b_G$</td>
<td>$b_B$</td>
</tr>
</tbody>
</table>

where $0 < p_B < p_G < 1$ and $b_B > b_G > 0$ imply that project “Bad” is riskier but yields
more private benefits for entrepreneurs than project “Good”. The capital in a failed
project is fully destroyed.

Entrepreneur $i$ use his net worth $n_{i,t}$ and bank loans $z_{i,t}$ to purchase capital $k_{i,t}^e$ and

$^4$The entrepreneurs must provide positive net worth as down payment of loans.
$^5$It is a simplified version of the Principal-Agent setting in Holmstrom and Tirole (1997). In order to
model the divergent objectives between entrepreneurs and outside financiers, the negative relationship
between private benefits for the entrepreneur and the project success probability are commonly assumed
in the incomplete-contract literature. Private benefits may refer to any noneconomic as well as economic
benefits of running a project, e.g., ego, career concerns, large offices or luxury cars. See Hart (1995) and
Tirole (1999) for relevant discussions. An alternative way is to assume that the entrepreneur’s effort is
costly to herself but can raise the success probability of a project.
invest in one of two projects. Entrepreneurial project choice is nonverifiable but irreversible. Expected productivity of project “Good” is higher than that of household’s home technology and only project “Good” is socially preferable, where \( r_t \) is the gross rate of deposit and \( E_t \) is the expectation operator based on information available at period \( t \),

\[
p_G E_t[R v_{t+1} + (1 - \delta) q_{t+1}] + b_G - q_t > 0 > p_B E_t[R v_{t+1} + (1 - \delta) q_{t+1}] + b_B - q_t
\]

The bank provides loan \( z_{i,t} \) to entrepreneur \( i \) at the end of period \( t \) against the promise of repaying \( R_b k^c_{i,t} \) at period \( t + 1 \) if the project succeeds. If the project fails, zero project outcome and fully destroyed capital imply zero return for both parties. In order to induce the entrepreneur to choose project “Good”, the bank must provide enough incentives,

\[
(p_G - p_B) E_t[R v_{t+1} k^c_{i,t} + (1 - \delta) q_{t+1} k^c_{i,t} - R_b k^c_{i,t}] \geq (b_B - b_G) k^c_{i,t}
\]

which is simplified as

\[
R_b = E_t[R v_{t+1} + (1 - \delta) q_{t+1}] - \frac{\Delta b}{\Delta p}
\]

where \( \Delta p \equiv p_G - p_B > 0 \) and \( \Delta b \equiv b_B - b_G > 0 \). The expected return per unit of capital invested in a successful project is \( E_t[R v_{t+1} + (1 - \delta) q_{t+1}] \). According to Equation , the entrepreneur can credibly pledge only \( R_b k^c_{i,t} \) per unit of capital invested to the outside lender, which is independent of his net worth or asset holding. Any promise more than \( R_b k^c_{i,t} \) is not trustworthy because the entrepreneur will deliberately choose project “Bad”.

Competitive banks act here as a simple device to pool the idiosyncratic risk of entrepreneurial projects and we assume no moral hazards in banking sector. The bank can perfectly diversify her loan portfolio so that her ex post collected repayment coincides with the expectation, which guarantees a sure rate of return for her risk-averse depositors, \( r_t \). The bank’s expected break-even condition actually generates the credit constraint for entrepreneur \( i \),

\[
r_t z_{i,t} = p_G R_b k^c_{i,t}.
\]

The bank charges a bankruptcy premium \( \frac{1}{p_G} \) over the deposit rate \( r_t \).

The credit constraint takes a similar form as the collateral constraint in Kiyotaki and Moore (1997). In their model, borrower’s inalienable human capital is unique and essential for the project output. After the financial contracting, he can always threaten to withdraw

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6Entrepreneurs are heterogenous and indexed by \( i \in [0, 1] \).
7In fact, one can assume that the risk-averse household can also perfectly diversify her direct lending to entrepreneurs so that banks are not even needed.
8Chen (2001) studies bank capital by making an extreme assumption on the bank’s portfolio.
his human capital so as to renegotiate the debt repayment down to the collateral value. In this sense, any promised repayment exceeding the collateral value is not enforceable. In contrast, we assume the irreversible project choice and costless debt enforcement as well as costless verification of the ex post project outcome. The underlying moral hazard problem arises only from the nonverifiable project choice.

Were the project choice perfectly verifiable, the entrepreneur would be able to pledge all of the project outcome \( E_t[(1 - \delta)q_{t+1} + Rv_{t+1}] \) for 100% external funding so that no down payment is needed. As a result, capital would be all allocated to the entrepreneurs of higher productivity and our model collapses to the standard RBC model. See Section 3 for detailed discussion on the first-best economy.

### 2.3 Efficiency Conditions

#### 2.3.1 Households

The problem of risk-averse households is quite conventional. Each period, a household is endowed with a unit of labor. The household sells her output of intermediate goods \( G(k_{t-1}^h) \) from the safe home project with capital \( k_{t-1}^h \) invested at the end of last period, supplies labor \( l^h_t \) to the production of consumption goods, receives lump-sum profits from capital production sector \( \Pi_t \), invests capital \( k_t^h \) in the home project, deposits \( d_t \) at the bank for a secured rate of return \( r_t \), and consumes \( c_t^h \). The household maximizes the expected utility with respect to consumption and leisure,

\[
\max_{\{c_t^h, d_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[ \frac{(c_{t+s}^h)^{1-\sigma}}{1-\sigma} + \frac{\chi(1 - l_{t+s}^h)^{1+\psi}}{1+\psi} \right]
\]

where \( 0 < \beta < 1 \) denotes the discount factor, subject to her period-by-period budget constraint,

\[
q_t k_t^h + d_t + c_t^h = v_t G(k_{t-1}^h) + (1 - \delta)q_t k_{t-1}^h + w_t l_t^h + r_{t-1}d_{t-1} + \Pi_t
\]

The household’s optimization over \( \{c_t^h, l_t^h, k_t^h, d_t\} \) gives the equilibrium conditions,

\[
w_t = \chi(1 - l_t^h)^{\psi}(k_t^h)^{\sigma}
\]

\[
\beta r_t = \mathbb{E}_t \left( \frac{c_{t+1}^h}{c_t^h} \right)^{\sigma}
\]

\[
r_t q_t = \mathbb{E}_t [(1 - \delta)q_{t+1} + v_{t+1} G'(k_t^h)]
\]

---

\(^9\) An unsatisfying feature of their model is the extremely high leverage ratio.

\(^{10}\) As households are homogenous, we use lower-case letters to denote relevant quantities of a representative household.
2.3.2 Entrepreneurs

Risk-neutral entrepreneur $i$ maximizes the expected utility with respect to consumption $c^e_i$ and private benefits

$$E_t \sum_{s=0}^{\hat{T}} \beta^s [c^e_{i,t+s} + Bk^e_{i,t+s-1}]$$

where $\hat{T}$ is the stochastic time of death and $B \in \{b_G, b_B\}$ is private benefit per unit of capital invested, subject to his period-by-period budget constraints and credit constraints,

$$q_t k^e_{i,t} - z_{i,t} = N_{i,t} - c^e_{i,t}$$

$$r_t z_{i,t} = p_G R^b_t k^e_{i,t}$$

where $N_{i,t}$ denotes the post-repayment wealth of entrepreneur $i$. $N_{i,t} = [(1 - \delta)q_t + Rv_t - R_{t-1}^b]k^e_{i,t-1}$ if the project succeeds and $N_{i,t} = 0$ if fails. Due to the linear form of the project technology and private benefits as well as the entrepreneur’s linear preference, the capital investment of surviving entrepreneurial $i$ is linearly related with his post-repayment wealth plus the endowment $N_{i,t} + e$. It facilitates aggregation among heterogeneous entrepreneurs and only the first moment of the distribution of entrepreneurial net worth has effects on aggregate economy. The heterogeneity in entrepreneurial net worth does not matter.

As the expected rate of return of entrepreneurial net worth exceeds their rate of time preference, they always borrow to the limit with accumulated wealth and postpone consumption to the period of death. We use lower-case letters without index $i$ to denote per capita quantities of heterogeneous entrepreneurs. Aggregate consumption and net worth for investment as well as credit constraints and budget constraints of the entrepreneurial sector are respectively,

$$c^e_i = (1 - \pi)p_G[Rv_t + (1 - \delta)q_t - R_{t-1}^b]k^e_{i,t-1}$$  \hspace{1cm} (2.5)

$$n_t = \pi p_G[Rv_t + (1 - \delta)q_t - R_{t-1}^b]k^e_{i,t-1} + e$$  \hspace{1cm} (2.6)

$$r_t z_t = p_G R^b_t k^e_t$$  \hspace{1cm} (2.7)

$$q_t k^e_i - z_t = n_t$$  \hspace{1cm} (2.8)

Furthermore, we define

$$\Omega_t \equiv \frac{q_t k^e_{i,t}}{q_t k^e_{i,t} - z_{i,t}} = \frac{r_t q_t}{r_t q_t - p_G R^b_t}$$

$$\mu_t \equiv \frac{q_t - z_{i,t}}{k^e_{i,t}} = \frac{p_G R^b_t}{r_t} = \frac{q_t}{\Omega_t}$$

$$\Psi_t \equiv \frac{\beta p_G E_t[(1 - \delta)q_{t+1} + Rv_{t+1} - R_{t+1}^b]k^e_{i,t+1}}{n_{i,t}} = \frac{\beta p_G \Delta d}{\mu_t \Delta p}$$
as the entrepreneurial leverage ratio, the entrepreneurial user cost of capital, and the
discounted expected gross rate of return of entrepreneurial net worth by leveraged in-
vestment (entrepreneurial profitability) respectively, which are all independent of entre-
preneur’s identity. Given the calibration of relevant parameters in Subsection 4.1, the
entrepreneurial profitability significantly exceeds the cost of external funds, $\Psi_t > \beta r_t$.

### 2.3.3 Production of Consumption Goods and Capital Goods

A continuum of perfectly competitive firms have a constant-return-to-scale technology of
producing consumption goods,

$$Y_t = A_t M_t^{\alpha} L_t^{(1-\alpha)}$$

(2.9)

$$\log A_t = \rho \log A_{t-1} + \epsilon_t$$

(2.10)

where $M_t$ and $L_t = (1-\eta)l_t^h$ denote aggregate input of intermediate goods and house-
holds labor respectively, while an exogenous productivity $\epsilon_t$ has persistent effects on TFP
$A_t$ whose steady state value is normalized at unity. Without moral hazards, aggregate
production of consumption goods always takes place at the efficient level and both factors
are priced at their marginal products,

$$v_t M_t = \alpha Y_t$$

(2.11)

$$w_t (1-\eta)l_t^h = (1-\alpha)Y_t$$

(2.12)

Capital depreciates at the rate $\delta$ and can be reproduced from consumption goods by
a continuum of competitive firms owned by households,

$$K_t - J_t = I_t - \phi \left[ \frac{I_t - (1-p_G)\eta k_t^c}{J_t} - \frac{\delta}{1-\delta} \right]^2 J_t$$

(2.13)

$$K_t = \eta k_t^c + (1-\eta)k_t^h$$

(2.14)

$$J_t = (1-\delta)(p_G\eta k_{t-1}^c + (1-\eta)k_{t-1}^h)$$

(2.15)

where $K_t$ denotes aggregate capital stock at the end of period $t$, $J_t$ denotes aggregate
stock of the remaining capital after the production of intermediate good at the beginning
of period $t$, and $I_t$ denotes aggregate input of consumption goods. Similar as in Faia
(2004), $\phi \left[ \frac{I_t - (1-p_G)K_{t-1}^c}{J_t} - \frac{\delta}{1-\delta} \right]^2 J_t$ specifies capital adjustment costs. Free from moral
hazard problems, these firms maximize the profit with respect to $I_t$,

$$\max_{\{I_t\}} \quad \Pi_t \equiv q_t(K_t - J_t) - I_t$$
and in equilibrium,
\[
\frac{1}{q_t} = 1 - 2\phi \left[ \frac{I_t - (1 - p_G)\eta k_{t-1}^e}{J_t} - \frac{\delta}{1 - \delta} \right].
\] (2.16)

Any profit is transferred to households in a lump-sum form.¹

2.4 Market Equilibrium

Market clearing conditions for intermediate goods, consumption goods, capital goods, labor, and credit market are respectively,

\[
M_t = (1 - \eta)G(k_{t-1}^h) + \eta p_G R k_{t-1}^e
\] (2.17)

\[
Y_t + e = (1 - \eta)c_t^h + \eta c_t^e + I_t
\] (2.18)

\[
K_t - J_t = I_t - \phi \left[ \frac{I_t - (1 - p_G)\eta k_{t-1}^e}{J_t} - \frac{\delta}{1 - \delta} \right]^2 J_t
\] (2.19)

\[
L_t = (1 - \eta)l^h_t
\] (2.20)

\[
\eta z_t = (1 - \eta)d_t
\] (2.21)

Definition 2.1. Competitive equilibrium is a set of allocations \(\{k_t^e, k_t^h, K_t, I_t, J_t, z_t, l_t^h, M_t, Y_t, c_t^h, c_t^e, n_t\}\) together with a set of prices \(\{v_t, q_t, w_t, r_t, R_t^b\}\) and the exogenous process \(\{A_t\}\) satisfying equations (2.1)–(2.18).

3 Loan Contracts Revisited and the Benchmark Case

With respect to the contractual setting, our model differs fundamentally from Chen (2001) and Aikman and Paustian (2005) in the following three aspects.

First, Chen (2001) and Aikman and Paustian (2005) assume fixed capital stock and no depreciation. The contract between a bank and an entrepreneur is specified as follows.

The entrepreneur must deliver a predetermined share of the project outcome to the bank in the form of intermediate goods and keep for himself the rest of the project outcome as well as capital if the project succeeds; if fails, the project yields nothing and the bank just captures the entrepreneur’s capital and hands over to the household. If we calibrate their models so that the entrepreneurial leverage ratio is at the reasonable level, e.g., 2 as in Bernanke, Gertler, and Gilchrist (1999), the ex post price of capital in the non-stochastic steady state is so high that the liquidation value of capital in a failed project

¹In the case of no adjustment costs \(\phi = 0\), the price of capital is constant at unity and there is no profit; while in the case of costly adjustment \(\phi > 0\), the price of capital is variable over time.
even exceeds debt obligations. In other words, according to their contractual setting, a bank actually gets more from a failed project than from a successful project, which seems rather weird and is lack of sound arguments. According to the standard debt contract, an entrepreneur with the failed project could repay the predetermined debt by just liquidating the remaining physical assets and retain anything left, while the lender should not get more than predetermined debt amount from liquidation. Dominated by a standard debt contract, the contract in their models might not exist in equilibrium. In contrast, as capital in a failed project is assumed fully destroyed in our model, the entrepreneur just declares bankruptcy and is released from debt obligations.

Second, the contract in their models specifies the share of the project outcome in terms of the intermediate good for motivating entrepreneurs and banks to behave in a desirable way. In fact, the entrepreneur can not only pledge his expected project outcome but also his capital stock for external funding. In contrast, we specify loan repayment in terms of the consumption good. According to our calibration, the entrepreneurial share of the project outcome is in fact negative, $R - \frac{R^b}{v} < 0$ in the non-stochastic steady state. As both entrepreneurs and households care only about consumption, it is the total pledgeable value in terms of the consumption good that matters.

Third, according to the contractual setting in Chen (2001) and Aikman and Paustian (2005), banks and entrepreneurs are provided with some shares of the project outcome in terms of the intermediate good as incentives. As a result, households get the rest of the project outcome as the deposit return. In the stochastic setting, variable prices of capital and intermediate goods make the ex post deposit return in terms of the consumption good differ from the expected return in the deposit period. It does not matter in the economy with all risk-neutral agents, e.g., in Chen (2001), but does matter in the economy with some risk-averse agents, e.g., households in Aikman and Paustian (2005). As risk-averse households only care about consumption goods, the deposit contract written in the consumption good must dominate that written in the intermediate good.

According to the loan contract in our model, banks provide loans of $z_{i,t-1}$ at the end of period $t - 1$ to entrepreneur $i$ for capital investment $k_{e,i,t-1}$. If his project succeeds, he repays the bank the predetermined debt amount $\frac{r_t - 1}{p_{t-1}} z_{i,t-1} = R^b_{t-1} k_{e,i,t-1}$ after selling his output of intermediate goods at the beginning of period $t$. His post-repayment wealth is $N_{i,t} = [Rv_t + (1 - \delta)q_t - R^b_{t-1}] k_{e,i,t-1}$. If his project fails, he gets nothing $N_{i,t} = 0$. At the aggregate level, the banks with perfectly diversified portfolio get total repayment from entrepreneurs $\eta p G R^b_{t-1} k_{e,t-1}$ with certainty, which guarantees the predetermined rate
of deposit return for risk-averse households,

\[(1 - \eta)r_{t-1}d_{t-1} = \eta p_G R^b_{t-1} k^e_{t-1}\]

Consider the per capita post-repayment entrepreneurial wealth,

\[N_t = p_G[(1 - \delta)q_t + Rv_t]k^e_{t-1} - r_{t-1}z_{t-1}\]

\[= p_G[(1 - \delta)q_t + R^b_{t-1}]k^e_{t-1}\]

\[= p_G[\frac{\Delta b}{\Delta p} + (1 - \delta)(q_t - E_{t-1}q_t) + R(v_t - E_{t-1}v_t)]k^e_{t-1}\]

Variable prices of capital goods and intermediate goods imply that \(q_t \neq E_{t-1}q_t\) and \(v_t \neq E_{t-1}v_t\) in the stochastic environment. In fact, the post-repayment wealth of risk-neutral entrepreneurs acts as a buffer to insure banks and then their risk-averse depositors against any aggregate shocks through loan contracts written in terms of the consumption good\(^{12}\).

Consider the first-best economy without moral hazards, where the entrepreneurial project choice is perfectly verifiable at the date of contracting. As the entrepreneur can now credibly pledge all project outcome for external funding, he does not have to provide any down-payment. In equilibrium, all entrepreneurs invest in project “Good” and get 100% external funding. Furthermore, as the expected productivity of project “Good” exceeds that of household home project, capital is fully allocated to entrepreneurs and there is no heterogeneity in investment among them, \(k^e_{i,t} = k^e_t = \frac{K_t}{\eta}\). By fully pledging the project outcome and the remaining capital stock each period, the entrepreneur just consumes his endowment each period and enjoys private benefits from running project “Good”, \(b_G k^e_t\). However, the contractual relationships among agents in the first-best economy is substantially different from those in our model economy with moral hazard.

If an entrepreneur had pledged all project outcome to acquire external funding via the loan contract \(R^b_{t-1} = E_{t-1}[(1 - \delta)q_t + Rv_t]\) for leveraged investment in project “Good” at period \(t - 1\), his post-repayment wealth at period \(t\) could be negative in the stochastic framework even if his project succeeds, \(N_t = [(1 - \delta)(q_t - q_{t-1}) + R(v_t - v_{t-1})]k^e_{t-1}\). As entrepreneurs even with successful projects would be unable to repay the predetermined debt amount in those states with negative shocks, equity contracts replace debt contracts in financing the entrepreneurs’ projects. Be specific, the household invest \(d_{t-1}\) in banks.

\(^{12}\)The study on business cycles normally assumes that aggregate shocks are relative small so that the economy always fluctuates around its non-stochastic steady state. In our model, the stochastic shocks are always so small that the entrepreneurs with successful projects can still repay even in the worst case.
at the end of period $t - 1$ for an contingent rate of return $r_t$ instead of the predetermined rate of return $r_{t-1}$ after the project outcomes are realized at the beginning of period $t$. Actually, banks provide $(1 - \eta)d_{t-1}$ to entrepreneur who invests capital $k_{t-1}^e = \frac{(1-\eta)d_{t-1}}{q_{t-1}}$ in project “Good”. In return, banks have the right of collecting all of the project outcome as well as the remaining capital if the project succeeds but zero if fails. After collecting from entrepreneurs at the beginning of period $t$, banks transfer $\eta p_G[(1 - \delta)q_t + R_v]k_{t-1}^e$ to households. The ex post rate of return for the household’s financial investment is

$$r_t = \frac{\eta p_G[(1 - \delta)q_t + R_v]k_{t-1}^e}{(1 - \eta)d_{t-1}}.$$

Here, risk-neutral entrepreneurs are unable to provide insurance for risk-averse households against aggregate risks via standard debt contracts any more. In fact, the first-best economy is quite similar as a standard RBC model economy with some minor difference. The following equation system governs market equilibrium in the first-best economy,

\begin{align*}
\beta E_t r_{t+1} &= E_t \left( \frac{c_t^{h+1}}{c_t^h} \right)^\sigma \\
q_t &= E_t \frac{p_G R_{t+1}^b}{r_{t+1}} \\
w_t &= \chi (1 - l_t^h)^\psi (c_t^h)^\sigma \\
R_t^b &= (1 - \delta)q_t + R_v \\
r_tz_{t-1} &= p_G R_{t+1}^b k_{t-1}^e \\
M_t &= p_G \eta k_{t-1}^e \\
Y_t &= A_t M_t^\alpha [(1 - \eta)l_t^h]^{1-\alpha} \\
v_t M_t &= \alpha Y_t \\
(1 - \eta)l_t^h + I_t &= Y_t \\
K_t &= \eta k_{t}^e \\
J_t &= (1 - \delta)p_G \eta k_{t-1}^e \\
K_t - J_t &= I_t - \phi \left[ \frac{I_t - (1 - p_G)\eta k_{t-1}^e}{J_t} - \frac{\delta}{1 - \delta} \right]^2 J_t \\
\frac{1}{q_t} &= 1 - 2\phi \left[ \frac{I_t - (1 - p_G)\eta k_{t-1}^e}{J_t} - \frac{\delta}{1 - \delta} \right] \\
\log A_t &= \rho \log A_{t-1} + \epsilon_t \\
q_t k_t^e &= z_t
\end{align*}
4 Dynamic Analysis

4.1 Calibration

Let us start from households. The home production function takes the linear form\(^13\)

\[
G(k^h_t) = \frac{1}{2} \left( 1 + \frac{\eta k^e_t}{K_t} \right) k^h_t, \tag{4.1}
\]

and the marginal productivity is \(G'(k^h_t) = \frac{1}{2} \left( 1 + \frac{\eta k^e_t}{K_t} \right)\). The quarterly discount factor \(\beta = 0.99\) corresponds to an annual rate of interest of 4%; households labor supply in the steady state \(L = \frac{1}{3}\) requires \(\chi = 2.9\); following Carlstrom and Fuerst (1997, 1998) and Kato (forthcoming), we set \(\sigma = 1\) and \(\psi = 0\).

The share of intermediate goods in consumption good production \(\alpha\) is set at 0.36, and \(\rho\) is chosen at 0.95 as in Carlstrom and Fuerst (1997, 1998) and Bernanke, Gertler, and Gilchrist (1999). Capital depreciates at a quarterly rate of \(\delta = 2.5\%\). As in Carlstrom and Fuerst (1997), the quarterly rate of business failure at 1% implies \(p_G = 0.99\).

The proportion of entrepreneurs in the whole population is \(\eta = 0.5\). Aggregate capital stock \(K = 1.4\) and the price of capital goods \(q = 1\) in the non-stochastic steady state. The values of \(\{\pi = 0.6, R = 5.3, \frac{\Delta b}{\Delta p} = 0.83\}\) are jointly chosen to satisfy the following conditions in the non-stochastic steady state. Entrepreneurs’ aggregate capital holding accounts for half of total stock; the entrepreneurial leverage ratio \(\Omega = 2\), as in Bernanke, Gertler, and Gilchrist (1999); entrepreneurial pledgeability is \(\frac{R_b}{(1-\delta)q+\nu R} = 38\%\).

4.2 Simulation Results

We log-linearize the equation system governing the market equilibrium around its non-stochastic steady state and employ the solution method provided by Schmitt-Grohe and Uribe (2004). Assuming no adjustment costs, subsubsection 4.2.1 compares the impulse responses of our moral hazard model economy with those of the first-best economy and shows how gradually-built entrepreneurial net worth and capital reallocation among households and entrepreneurs can explain the amplified hump-shaped output behavior after a productivity shock. Under various degrees of adjustment costs, Subsubsection

\(^{13}\)This function form implies that there is production externality from the entrepreneurs. The higher the proportion of entrepreneurs’ aggregate capital holding is, the household’s marginal productivity will be higher, too.
4.2.2 compares the impulse responses of two model economies and analyzes how variable asset prices can affect the dynamic features of aggregate output.

4.2.1 Constant Asset Prices: No Capital Adjustment Costs $\phi = 0$

The impulse responses of the moral hazard economy (MH) and the first-best economy (RBC) with respect to an exogenous TFP shock are shown in Figure 1.

As mentioned in Section 3, the first-best economy can be essentially described as a standard RBC model. When an exogenous productivity shock hits the economy, households prefer to smooth consumption by accumulating capital. As there is no adjustment costs, capital is one-to-one transformed from consumption goods so that asset prices are constant at unity. As quite standard in RBC model, aggregate output reaches its peak at the shock period and converges to its steady state level monotonically.

While, our moral hazard model replicates an empirical business cycle fact, the hump-shaped output behavior. The mechanism is as follows. At the shock period $t$, the exogenous rise in TFP of consumption goods production increases aggregate demand for intermediate goods. Given the fixed aggregate supply due to predetermined investment, $k_{e,t-1}$ and $k_{h,t-1}$, the price of intermediate goods rises to clear the market ($v_t > E_{t-1}v_t$), which improves the post-repayment wealth of entrepreneur $i$ with a successful project, given constant asset prices $q_t = E_{t-1}q_t = 1$,

$$N_{i,t} = \left[ \frac{\Delta b}{\Delta p} + R(v_t - E_{t-1}v_t) \right] k_{e,i,t}^{c} > \frac{\Delta b}{\Delta p} k_{i,t-1}^{c}. \quad (4.2)$$

Per capita entrepreneurial net wealth for investment $N_t = \pi N_t + \epsilon$ improves, too. By leveraged investment, entrepreneurs have lower user cost per unit of capital invested than households, $\mu_t = q_t - \frac{z_t}{k_t} = 1 - \frac{\text{FV} R_t^b}{r_t} < 1$. In the shock period, the entrepreneurial user cost falls because the rise in entrepreneurial pledgeability $R_t^b$ exceeds that in the loan rate $r_t$. The fall in user costs together with the improvement in entrepreneurial net worth enables them to increase the capital holding. The boom in the credit demand pushes up loan rate. In the meantime, banks raise deposit rate and attract more deposits to such an extent that the capital holding of households even falls.

The initial rise in entrepreneurial profitability $\Psi_t$ implies that entrepreneurial net worth can yield higher expected return in the next period than that in the steady state. The fact that the entrepreneurial profitability stays above its steady state level in the following four periods encourage entrepreneurs to invest more capital. As the entrepreneurial user cost is below its steady state level until the fourth period after the shock, it
Figure 1: Moral Hazard Model vs. RBC Model: Constant Asset Prices $\phi = 0$
is relatively cheap for entrepreneurs to invest capital goods in the project. As a result, aggregate capital holding of entrepreneurs grows gradually to reach its peak in the fourth period after the shock. Afterwards, the user cost rises above the steady state level, which depresses aggregate entrepreneurial investment.

In Carlstrom and Fuerst (1997) and Kato (forthcoming), moral hazard arises in the intraperiod loan for capital production. As capital is an input for aggregate production, the delayed capital production due to moral hazard results in the hump-shaped output behavior but at a depressed magnitude relative to that in a RBC model. In contrast, there are two forces driving aggregate output in our model: the exogenous shock with persistent effects on TFP and capital reallocation among agents with heterogeneous productivity. Capital is allocated relatively more to entrepreneurs, as their net worth improves gradually. Although TFP in the third period after the shock is lower than that in the initial shock period, aggregate output reaches its peak, because the contribution of the rise in aggregate entrepreneurial capital holding to consumption goods production overcompensates the effects of the gradual fall in TFP. In this sense, our model generates the amplified hump-shaped output behavior via an extra channel of capital reallocation.

4.2.2 Variable Asset Prices: Costly Capital Adjustment $\phi > 0$

A large body of literature initiated by Kiyotaki and Moore (1997) shows that a rise in asset prices can improve the net worth of those credit-constrained firms and then enables them to make more leveraged investment. By assuming fixed supply of durable assets, these models over-stress the responses of asset prices with respect to the change in asset demand. Meanwhile, these models do not generate the hump-shaped output behavior.

In the previous experiment, we shut off the effects of asset price on model dynamics by assuming that depreciable capital can be reproduced one-to-one from consumption goods. We now analyze the impulse responses of the model economy under different degrees of adjustment costs and show how variable asset prices can influence the dynamic features of aggregate output. Given the degree of adjustment costs $\phi = 1$, Figure 2 compares the impulse responses of our moral hazard economy and the first-best economy.

In the shock period, the post-repayment wealth of entrepreneurs per capita is

$$N_t = p_G \left[ \frac{\Delta b}{\Delta p} + (1 - \delta)(q_t - E_{t-1}q_t) + R(v_t - E_{t-1}v_t) \right] k_{t-1}^e.$$  (4.3)

\[ \text{14} \] That is, the maximum response of output is less than that in the standard RBC model.
Figure 2: Moral Hazard Model vs. RBC Model: Costly Asset Adjustment $\phi = 1$
As mentioned above, a positive shock on TFP increases aggregate demand for the intermediate good, which pushes up its price above the expectation, \((v_t > E_{t-1}v_t)\). The excessive demand of entrepreneurs for capital pushes up its price due to costly adjustment \((q_t > E_{t-1}q_t)\), which further improves entrepreneurial net worth, i.e., the collateral effect as stressed by Kiyotaki and Moore (1997).

As long as entrepreneurial profitability stays above its steady state level, entrepreneurs prefer to invest more capital in their projects in order to take advantage of higher expected return of net worth. Compared with the case of constant asset prices, the improvement in entrepreneurial net worth due to the additional effects of asset prices is much larger in the shock period so that more loans are granted for entrepreneurs. The dramatic fall in the profitability of net worth in the following period implies that the profitability has been significantly explored in the initial period. In this sense, the soaring asset prices in the shock period actually change entrepreneurs’ investment pattern and consequently the dynamic pattern of aggregate output via the mechanism of capital reallocation mentioned before. That is, aggregate output reaches its peak in the second period after the shock, earlier than in the case of constant asset prices.

For a better understanding of this mechanism, Figure 3 shows the impulse responses of two economies in the case of \(\phi = 10\), where capital supply is more inelastic than in the case of \(\phi = 1\). Asset price rises so high at the shock period that entrepreneurial net worth improves greatly. The significant improvement in entrepreneurial net worth enables them to increase capital holding to such an extent that the price of the intermediate good in the following period is depressed due to the huge rise in its supply. The entrepreneurial profitability peaks in the shock period and falls below the steady state thereafter, which implies that the profitability is fully explored in the shock period. As a result, the capital holding of entrepreneurs peaks in the shock period and aggregate output just one period after the shock. Our results hold under various calibrations of structure parameters.

5 Conclusion

We provide a real DGE model with heterogenous agents in which nonverifiability of entrepreneurial project choice justifies credit constraints. We bring together two strands of literature concerning credit market imperfections. In the literature explaining the hump-shaped output dynamics, e.g., Carlstrom and Fuerst (1997) and Kato (forthcoming), credit frictions arise in the capital production.
Figure 3: Moral Hazard Model vs. RBC Model: Costly Asset Adjustment $\phi = 10$
It takes time for those credit-constrained agents, who produce depreciable capital, to gradually accumulate net worth and ameliorate agency costs for leveraged investment. As capital, an input for aggregate production, is then produced in the delayed fashion, the responses of aggregate output to a persistent TFP shock has a depressed hump-shaped form. In our moral hazard model without capital adjustment costs, aggregate output is here driven both by TFP and by capital reallocation among agents with heterogenous productivity. In fact, the gradually-built net worth of those credit-constrained agents has the effects on the process of capital reallocation and explains the amplified hump-shaped output behavior here.

In the literature on the role of asset prices in amplifying the effects of a small shock, e.g., Kiyotaki and Moore (1997) and Chen (2001), aggregate stock of durable assets is assumed to be fixed for analytical convenience. The amplifying effects of asset price on the transmission mechanism in a credit-constrained economy is well explored, while the role of asset prices in changing the dynamic pattern of output behavior has not yet been studied. We choose costly capital adjustment as a vehicle to model the upward-sloping capital supply curve. Asset prices have now strong effects on the net worth of credit-constrained agents. Given an exogenous positive TFP shock, the rise in asset prices improves the net worth of those constrained agents in addition to the positive revenue effect. It takes less time for them to invest at the maximum. As a result, aggregate output dynamics are less delayed.

Following the modeling strategy of Carlstrom and Fuerst (1997), Kato (forthcoming) derives credit constraints for those capital producing agents from nonverifiability of project choice, too. Zhang (2005b) combines Kato (forthcoming) with the moral hazard model here and study the interactions between intertemporal and the intratemporal loan contracts financing the production of intermediate goods and capital goods respectively. The model with dual limited pledgeabilities and variable asset prices actually generates even more delayed hump-shaped output behavior. In this sense, one cannot give a clear-cut answer to whether variable asset prices can speed up or delay the responses of aggregate output in our framework with limited pledgeability. It depends substantially on the specific mechanisms of capital supply. If one models asset price by just assuming fixed asset supply, he might miss the whole view of what asset prices can do.
References


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