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Can Wage and Price Stickiness Account for Sizeable Costs of Business Cycle Fluctuations?

by

Matthias Paustian

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Can Wage and Price Stickiness Account for Sizeable Costs of Business Cycle Fluctuations?∗

Matthias Paustian †

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Abstract

This paper asks the following two questions: First, can a model with nominal rigidities in wage and price setting account for the average welfare costs of business cycle fluctuations identified in Gali, Gertler, and Lopez-Salido (2003)? Second, do we need to agree on a particular scheme for nominal rigidities to answer that question? We compute a quadratic approximation to agents expected lifetime utility and evaluate welfare for different modeling schemes of nominal rigidities that all have the same average duration of contracts. Calvo (1983) wage and price contracts can deliver sizeable welfare costs, but other contracts of the same average stickiness cannot. Calvo (1983) contracts can imply welfare costs that are up to 4 times higher than those implied by overlapping contracts in the spirit of Taylor (1980) or Wolman (1999). Furthermore, the sticky information framework of Mankiw and Reis (2002) may generate welfare costs that are even smaller. This paper calls for more research into the origins of wage and price stickiness.

JEL codes: E52, E32
Keywords: welfare, Calvo, Taylor, sticky information, costs of nominal rigidities

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1 Introduction

The welfare costs of business cycle fluctuations have been at the center of macroeconomic research since the seminal paper by Lucas (1987). Lucas (2003) focused on the variability of consumption and has argued that the costs of these fluctuations are very small, roughly 0.07 percent of steady state consumption. His conclusion is that macroeconomists should set their research priorities on improving economic growth rather than on fine tuning of the cycle. Recently, Gali, Gertler, and Lopez-Salido (2003) have built a measure of the costs of business cycle fluctuations centering around the gap between the marginal product of labor and the marginal rate of substitution between consumption and leisure, the so-called "efficiency gap". They calibrate a small number of parameters and take that measure to U.S. data. Gali, Gertler, and Lopez-Salido (2003) show that the average welfare costs of business cycle fluctuations could very well be higher than what Lucas (2003) computes. Costs in the benchmark calibration of Gali, Gertler, and Lopez-Salido (2003) are 0.28 per cent of steady state consumption and range up to 0.75 per cent.

Variations in the efficiency gap arise endogenously in models with wage and price stickiness such as in the seminal work by Erceg, Henderson, and Levin (2000). As pointed out by Gali, Gertler, and Lopez-Salido (2003), there are however a number of other frictions that could also contribute to the variance of this gap.

This paper asks the following two questions: First, can a model with nominal rigidities in wage and price setting account for the average welfare costs of business cycle fluctuations identified in Gali, Gertler, and Lopez-Salido (2003)? Second, do we need to agree on a particular scheme for nominal rigidities to answer that question?

Addressing the first question helps to understand the sources of fluctuations in the efficiency gap and delimits the welfare gains from improved monetary policy. The second question needs a bit more justification. Most monetary dynamic general equilibrium models with optimizing agents are silent about the origins of wage and price stickiness. Instead, the empirical observation of infrequent nominal adjustment is used to motivate monetary models with mostly ad-hoc features of nominal rigidities. Since we have little theoretical guidance as to how to in-

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\footnote{It should be noted that such exogenous imposition of nominal rigidities does not square well with a main reason for using structural macroeconomic models: The hope to obtain relationships which are invariant to policy by explicitly modeling how agents optimize in the environment they are facing. For instance, in the framework of Calvo (1983) wage and price setting, it may be doubted that the probability of price and wage adjustment is invariant to the costs of non-adjustment. That costs depends crucially on the weight the central bank puts on the wage versus price inflation in an interest rate rule and is therefore not at all invariant to policy.}
troduce nominal stickiness, we compute the welfare costs of nominal rigidities across a host of popular modeling devices for stickiness. Should we find similar welfare costs across these devices, one could argue that agreeing on a particular scheme for nominal rigidities is not of major importance.

This paper is related to recent work by Canzoneri, Cumby, and Diba (2004) as well as Ascari (2004) and Kiley (2002). Canzoneri, Cumby, and Diba (2004) compute the cost of nominal inertia in a more elaborate New Neoclassical Synthesis model with Calvo (1983) contracts. They show that the cost of nominal inertia can be much larger than the welfare costs of business cycles identified by Lucas (2003). However, Ascari (2004) and Kiley (2002) have shown that the choice of a modeling device for nominal rigidities may have serious consequences for the steady state effects of inflation. Monetary general equilibrium models that feature steady state inflation often assume that agents can index their contracts to the trend inflation rate, thereby completely bypassing the point made in Ascari (2004). It is an open question, to what extent the findings of Kiley (2002) and Ascari (2004) are also relevant for the costs of nominal rigidities over the business cycle. We are motivated by these three papers to undertake a comparison of welfare costs of wage and price stickiness across different popular modeling devices for nominal rigidities.

We build on the seminal paper by Erceg, Henderson, and Levin (2000) featuring nominal stickiness in wage and price setting. The models of nominal rigidities we consider are the following: Random price adjustment of Calvo (1983), $N$ period overlapping contracts as in Taylor (1980), a more general scheme with time varying adjustment probabilities as suggested by Wolman (1999) and finally a variant of the sticky information framework of Mankiw and Reis (2002). The choice of these four modeling devices is motivated by the following facts. First, a fair comparison between all of these models is possible by requiring an equal average duration over which contracts or information sets are fixed. In such a way no model implies more exogenous stickiness on average. Second, welfare costs of all schemes can be easily computed using modifications of the linear-quadratic framework of Rotemberg and Woodford (1997). Third, the considered schemes cover most of the devices used in monetary models. Fourth, all of these models imply that the welfare cost of nominal rigidities stem from the dispersion of differentiated goods across producers or of differentiated labor across workers. Thus we consider a homogenous family of modelling devices for nominal rigidities.

The schemes suggested by Taylor (1980), Wolman (1999) and Mankiw and Reis (2002) are relevant for the costs of nominal rigidities over the business cycle. We are motivated by these three papers to undertake a comparison of welfare costs of wage and price stickiness across different popular modeling devices for nominal rigidities.

*An often used scheme missing from this comparison is Rotemberg (1982) quadratic adjustment costs. We do not consider it here, since it does not belong to the family of rigidities that imply dispersion of output across firms or hours across workers. Furthermore, the derivation of welfare based loss function is not as easily done as with the other schemes.*
Reis (2002) all relax at least one unpleasant assumption of the frequently used Calvo (1983) contracts. Taylor (1980) contracts truncate the infinite horizon present in the expectation formation of price and wage setters in the scheme of Calvo (1983). Wolman (1999) additionally relaxes the assumption that adjustment probabilities are independent of time since last adjustment. Finally, the Mankiw and Reis (2002) scheme provides a tractable behavioral foundation for stickiness by considering limited capacity to process information. We derive purely quadratic utility based loss function from the model corresponding to these four different schemes of nominal rigidities and computes welfare based on a linear approximation to the equilibrium conditions.

To preview our main results briefly. Large costs wage and price stickiness arise almost exclusively under the assumption of Calvo (1983) contracts, while the costs implied by the other contracts are much smaller. It therefore appears that the main welfare relevant choice a modeler is faced with is whether to assume the infinite horizon scheme of Calvo (1983) or to use any one of the finite horizon schemes.

The paper proceeds as follows. Section 2 describes the setup of the model, which is similar to the basic framework in Erceg, Henderson, and Levin (2000). In section 3 we introduce Calvo (1983) wage and price setting as our baseline model of nominal rigidities. The reader familiar with both the model of Erceg, Henderson, and Levin (2000) and the setup of Calvo (1983) may skip these sections and proceed to section 4 where the log-linearized necessary conditions for equilibrium are collected and the model’s baseline calibration is presented. Section 5 computes the welfare costs of nominal rigidities for the Calvo (1983) scheme. In section 6 we make an important robustness checks in our baseline model. Taking up recent criticism of the rental market assumption by Danthine and Donaldson (2002) and others, we allow capital to be firm specific rather than assuming that it can be costlessly and instantaneously reallocated across firms. In section 7 we depart from Calvo (1983) and allow for the above mentioned alternative plausible assumptions about price and wage setting. Finally, section 8 summarizes the findings and concludes. A technical appendix with derivations and proofs is available from the author upon request.

2 Model

The model we consider bases its key building blocks on Erceg, Henderson, and Levin (2000). In particular, capital is in fixed supply in the aggregate. We abstract from aggregate capital accumulation, because the derivation of a welfare based loss function becomes extremely cumbersome with aggregate capital accu-
In the baseline model, we assume that there exists an economy wide rental market that allows capital to move freely between firms. This assumption is relaxed in a robustness check. We assume further that subsidies exist that completely offset the effects of monopolistic competition in the steady state in order to avoid highly involved derivations of the loss function that would arise with an inefficient steady state. Finally, following Erceg, Henderson, and Levin (2000) we assume that households can fully insure against the idiosyncrasies in income streams arising from nominal wage rigidities. While this may bias the welfare costs of nominal rigidities, it greatly facilitates the analysis. In the next subsection we start with a discussion of the households problem.

2.1 Households

There is a continuum of households with unit mass indexed by $h$. Households are infinitely lived, supply labor $N_t(h)$ and receive nominal wage $W_t(h)$, consume final goods $C_t(h)$, and purchase state contingent securities $B_t(h)$. Furthermore, they are subject to lump sum transfers $T_t$, hold nominal money balances $M_t(h)$ and receive profits $\Gamma_t(h)$ from the monopolistic retailers. The utility function is assumed to be separable in consumption, real money balances and leisure. The representative household’s problem is:

$$
\max_{B_{t+1},M_{t+1},C_t,N_t} E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[ U(C_{t+i}(h)) + H \left( \frac{M_{t+i}(h)}{P_{t+i}} \right) + V(N_{t+i}(h)) \right]
$$

s.t. $C_t(h) = \delta_{t+1,t} B_t(h) - B_{t-1}(h) + (1 + \tau_w) \frac{W_t(h)}{P_t} N_t(h) + T_t(h) + \Gamma_t(h) - M_{t+1}(h) - M_t(h)$.

Here $B_t$ is a row vector of state contingent bonds, where each bond pays one unit in a particular state of nature in the subsequent period. The column vector $\delta_{t+1,t}$ represents the price of these bonds. Therefore, the inner product gives total expenditures for state contingent bonds. $\tau_w$ is a wage subsidy used to offset the steady state effects of monopolistic competition. The first order conditions for consumption and state contingent bond holdings give rise to the standard Euler equation. Note that consumption is perfectly insured against idiosyncratic labor income and


[4] This task has recently accomplished by Benigno and Woodford (2004).
therefore consumption is no longer indexed by \( h \).

\[
U_C(C_t) = E_t \beta \left\{ \frac{P_t}{P_{t+1}} U_c(C_{t+1}) \right\} R^n_t, \tag{1}
\]

Here, \( R^n_t \) is the nominal interest rate on a non-contingent bond. We follow the standard practice to omit the first-order condition for money holdings as this equation merely serves to back out the quantity of money that supports a given nominal interest rate.

The following functional forms are used in later parts of the analysis

\[
U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \tag{2}
\]

\[
V(N_t) = -\frac{N_t^{1+\chi}}{1+\chi} \tag{3}
\]

A continuum of households supply differentiated labor \( N_t(h) \), which is aggregated according to the Dixit-Stiglitz form:

\[
L_t = \int_0^1 [N_t(h)]^{\frac{\epsilon-1}{\epsilon}} dh \left[ \frac{\epsilon-1}{\epsilon} \right], \quad k > 1. \tag{4}
\]

The demand function for differentiated labor is:

\[
N_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{-\kappa} L_t \tag{5}
\]

Here \( W_t \) is the Dixit-Stiglitz wage index.

### 2.2 Production

Firms in the final good sector produce a homogeneous good, \( Y_t \), using intermediate goods, \( Y_t(z) \), as input in production. There is a continuum of intermediate goods of measure unity. The production functions that transforms intermediate goods into final output is given by

\[
Y_t = \left[ \int_0^1 Y_t(z)^{\frac{1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}} \tag{6}
\]

where \( \epsilon > 1 \). The solution to the problem of optimal factor demand yields the following constant price elasticity demand function for variety \( z \).

\[
Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t \tag{7}
\]
A continuum of monopolistically competitive intermediate goods firms indexed by \( z \in [0, 1] \) and owned by consumers uses both labor \( L_t(z) \) and capital \( K_t(z) \) to produce output according to the following constant returns technology:

\[
Y_t(z) = A_t L_t(z)^{1-\alpha} K_t(z)^\alpha
\]  

(8)

\( A_t \) denotes total factor productivity, which follows an exogenous stochastic process. Capital is freely mobile across firms rather than being firm specific. Firms rent capital from households in a competitive market on a period by period basis after they observe the productivity shock. Firm \( z \) chooses \( L_t(z) \) and \( K_t(z) \) to minimize total cost subject to meeting demand

\[
\min_{K_t(z), L_t(z)} w_t^r L_t(z) + Z_t K_t(z) \quad \text{s.t.} \quad A_t L_t(z)^{1-\alpha} K_t(z)^\alpha - Y_t = 0.
\] 

(9)

Here, \( w_t^r \) is the real wage and \( Z_t \) the real rental rate for capital. Let \( X_t(z) \) denote the Lagrange multiplier with respect to the constraint. The first order conditions with respect to \( L_t(z) \) and \( K_t(z) \) are given by

\[
w_t^r = (1 - \alpha) X_t(z) A_t K_t(z)^\alpha L_t(z)^{-\alpha}
\] 

(10)

\[
Z_t = \alpha X_t(z) A_t K_t(z)^{\alpha-1} L_t(z)^{1-\alpha}
\] 

(11)

The first order conditions imply that all firms choose the same capital to labor ratio, therefore marginal cost \( X_t \) is equalized across firms. This will not be true for the case of firm specific capital, where the immobility of capital across firms prevents firms from choosing equal capital to labor ratios.

2.3 The efficiency gap

In order to compute welfare, it is useful to consider the solution under perfectly flexible wages and prices. Up to a first order approximation in log deviations, flexible price output is given by

\[
\tilde{Y}_t^* = \left[ \frac{1 + \omega_2}{\omega_2 + \alpha - (1 - \alpha) \omega_1} \right] \tilde{A}_t
\] 

(12)

Here \( \omega_1 \equiv \frac{U_{CCC}}{U_C} \) is the elasticity of the marginal utility of consumption evaluated at the steady state and \( \omega_2 \equiv \frac{V_{NNN}}{V_N} \) is the elasticity of the marginal utility of labor. The subutility function \( U(C_t) \equiv \frac{1-\sigma}{1-\sigma} c_t^{1-\sigma} \) implies \( \omega_1 = -\sigma \) and for \( V(N_t) \equiv -\frac{N_t^{1+\chi}}{1+\chi} \)

---

5Throughout this text, for any variable \( X_t \), \( \bar{X} \) denotes its steady value and \( \hat{X}_t \equiv \log X_t - \log \bar{X} \)
we have $\omega_2 = \chi$. Given these functional forms, the natural level of output in log-deviation is given by

$$\hat{Y}_t^* = \left[ \frac{1 + \chi}{\chi + \alpha + (1 - \alpha)\sigma} \right] \hat{A}_t$$

(13)

One can use this equation together with the firm’s first-order condition for labor demand, to derive a key equation for this model. That equation links marginal cost to the output gap and the gap between the average marginal rate of substitution between consumption and labor and the real wage.

$$\hat{X}_t = \left[ \frac{\chi + \alpha}{1 - \alpha + \sigma} \right] \left( \hat{Y}_t - \hat{Y}_t^* \right) - \left[ \chi \hat{L}_t + \sigma \hat{Y}_t - \hat{w}_t \right]$$

(14)

When there are no nominal rigidities in the labor market, the marginal rate of substitution between consumption and labor is equal to the real wage and the last term in brackets vanishes. We then recover the condition from sticky price models that marginal cost is log-linearly related to the output gap. When prices are perfectly flexible as well, the marginal product of labor is equal to the real wage, i.e. log marginal cost is zero. It follows that the output gap is zero. These two gaps, the difference between the real wage and the marginal product of labor and the difference between the real wage and the marginal rate of substitution are combined into a single efficiency gap in the welfare analysis of Gali, Gertler, and Lopez-Salido (2003).

3 Calvo wage and price setting

In this subsection, we outline Calvo (1983) wage and price setting as our reference model of nominal rigidities. In our baseline Calvo (1983) model, we assume that firms are fully rational and purely forward looking, while we allow for backward looking elements in price setting as a robustness check. In any given period, a firm faces a constant probability $\theta$ of receiving a signal that allows that firm to reset its price. Firms that do not receive the signal, carry on the prices posted in the last period and satisfy any demand at that price. The problem of a firm that receives a signal to change its price in period $t$ is to maximize expected real profits as valued by the household in those states of the world where the price remains fixed through choice of the optimal nominal price $P_t^\beta$.

$$\max_{P_t^\beta} \mathbb{E}_t \sum_{i=0}^{\infty} (\theta \beta)^i \Lambda_{t+i} \left\{ (1 + \tau_p) \left[ \frac{P_t^\beta (z)}{P_{t+i}} \right]^{1-\epsilon} Y_{t+i} - X_{t+i} \left[ \frac{P_t^\beta (z)}{P_{t+i}} \right]^{-\epsilon} Y_{t+i} \right\}$$

(15)

Here we have made use of properties of the Cobb-Douglas Production function, rewriting total cost as marginal cost times production.
Here, $\Lambda_{t+i}$ is the households marginal utility of consumption in period $t+i$ and $\tau_t$ is sales subsidy suitably chosen as to offset the steady state effects of monopolistic competition $(1 + \tau_p = \frac{\epsilon}{\epsilon-1})$. The first order condition is

$$E_t \sum_{i=0}^{\infty} (\theta \beta)^i \Lambda_{t+i} \left\{ (1 + \tau_p) P^*_t(z) (1 - \epsilon) [P_{t+i}]^{1-\epsilon} Y_{t+i} + \epsilon X_{t+i} P_{t+i}^* Y_{t+i} \right\} = 0.$$  \hspace{1cm} (16)

As a robustness check, we allow for backward looking elements in the wage and price setting as proposed by Gali and Gertler [1999] and Gali, Gertler, and Lopez-Salido [2001], since that allows to match the observed persistence in inflation rates. In particular, we assume that price setters that do receive a signal to re-set their price belong to one of two groups. The first group sets prices in a purely forward-looking manner as outlined above. The second group of measure $\omega$ is comprised of backward looking firms that set their price according to the following rule of thumb:

$$P_t^b = \pi_{t-1} \left( P_{t-1}^* \right)^{1-\omega} \left( P_{t-1}^b \right)^{\omega}.$$  \hspace{1cm} (17)

This rule posits that backward looking firms adjust prices according a geometric average of prices changed last period adjusted for last periods inflation rate. The consumption based price index is given by

$$P_t \equiv \left[ \int_0^1 P_t(z)^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}}.$$  \hspace{1cm} (18)

Since the fraction of firms that can change the price is chosen randomly and by the law of large numbers, the aggregate price index evolves as

$$P_{t-\epsilon}^{1-\epsilon} = \theta P_{t-\epsilon} + (1 - \theta)(1 - \omega)(P_t^*)^{1-\epsilon} + (1 - \theta)\omega (P_t^b)^{1-\epsilon}.$$  \hspace{1cm} (19)

This setup gives rise to the following hybrid new Keynesian Philips curve

$$\hat{\pi}_t = \frac{(1 - \omega)(1 - \theta)(1 - \beta \theta)}{\zeta} \hat{X}_t + \frac{\beta \theta}{\zeta} E_{t+1} \hat{\pi}_{t+1} + \frac{\omega}{\zeta} \hat{\pi}_{t-1}.$$  \hspace{1cm} (20)

Here $\zeta \equiv \theta + \omega[1 - \theta(1 - \beta)]$ and $\pi_t \equiv \frac{P_t}{P_{t-1}}$.

Wage setting is modeled in exactly the same way as price setting. The forward looking household that receives a signal to change the wage maximizes expected utility through choice of the nominal wage $W_t^*$ subject to the demand curve and the budget constraint. The FOC for this problem is:

$$E_t \sum_{j=0}^{\infty} (\theta w \beta)^j N_{t+j}(h) U_C(C_{t+j}) \left[ (1 + \tau_w) \frac{W_t^*(h)}{P_{t+j}} + \frac{\kappa}{\kappa - 1} \frac{V_N(N_{t+j}(h))}{U_C(C_{t+j})} \right] = 0.$$  \hspace{1cm} (21)
We again assume that those households that do receive a signal to reset their wages belong to one of two groups. A measure $\varphi$ of backward looking households set their wage according to the following rule of thumb

$$W^b_t = \pi_t^w (W^*_t)^{1-\varphi} (W^b_{t-1})^\varphi$$  \hspace{1cm} (22)

The wage index is defined as

$$W_t \equiv \left[ \int_0^1 W_t(h)^{1-\kappa} \, dh \right]^{\frac{1}{1-\kappa}}$$  \hspace{1cm} (23)

Since the fraction of wage setters that receive the signal to change their wage is randomly chosen and by the law of large numbers, the aggregate wage index evolves according to the formula

$$W_t^{1-\kappa} = \theta_w W_{t-1}^{1-\kappa} + (1 - \theta_w)(1 - \varphi) (W^*_t)^{1-\kappa} + (1 - \theta_w) \varphi (W^b_t)^{1-\kappa}$$  \hspace{1cm} (24)

This setup gives rise to a hybrid new Keynesian wage Philips curve

$$\hat{\pi}_t^w = (1-\varphi)(1-\theta_w)(1-\beta\theta_w) \hat{\mu}_t + \frac{\beta\theta_w}{\zeta_w} E_t \hat{\pi}_{t+1}^w + \frac{\varphi}{\zeta_w} \hat{\pi}_{t-1}^w$$  \hspace{1cm} (25)

Here $\zeta_w = \theta_w + \varphi [1 - \theta_w (1 - \beta)]$, $\hat{\mu}_t \equiv \chi \hat{L}_t + \sigma \hat{C}_t - \hat{w}_t^r$ and $\pi_t^w \equiv \frac{W_t}{W_{t-1}}$.

4 The key equations and model calibration

In this section, we collect necessary conditions that must be satisfied by equilibrium sequences of allocations and prices for the case of Calvo (1983) wage and price setting. The model has 7 endogenous variables: price inflation $\pi_t$, wage inflation $\pi^w_t$, labor $L_t$, output $Y_t$, marginal cost $X_t$, nominal interest rate $R^n_t$ and the real wage $w^r_t$. The model’s equilibrium conditions are summarized in the following box. The monetary policy rule necessary to close the model is not listed as it will be varied in our calculations of welfare costs of wage and price stickiness.
\[ Y_t = E_t \hat{Y}_{t+1} - \sigma^{-1} \left( \hat{P}^w_t - \hat{\pi}_{t+1} \right) \]  
(26)

\[ \hat{\pi}_t = \frac{(1 - \varphi)(1 - \theta)(1 - \beta \theta w)}{(1 + \kappa \chi) \zeta w} \hat{\pi}_t + \frac{\beta \theta w}{\zeta w} E_t \hat{\pi}_{t+1} + \frac{\varphi}{\zeta w} \hat{\pi}_{t-1} \]  
(27)

\[ \hat{\pi}_t = \frac{(1 - \omega)(1 - \theta)(1 - \beta \theta)}{\zeta} \hat{\pi}_t + \frac{\beta \theta}{\zeta} E_t \hat{\pi}_{t+1} + \frac{\omega}{\zeta} \hat{\pi}_{t-1} \]  
(28)

\[ \hat{\mu}_t = \chi \hat{L}_t + \sigma \hat{Y}_t - \hat{w}^r_t \]  
(33)

\[ \hat{\mu}_t \equiv \chi \hat{L}_t + \sigma \hat{Y}_t - \hat{w}^r_t \]  
(26)

\[ \hat{\zeta}_w \equiv \theta_w + \varphi [1 - \theta_w (1 - \beta)] \]  
(33)

\[ \hat{\zeta} \equiv \theta + \omega [1 - \theta (1 - \beta)] \]  
(33)

Upper case letters denote the aggregate of the respective lower case variables. (26) is the consumption Euler equation. (27) and (28) are the wage and price Philips curves. (29) is the log-linearized production function, which deserves some explanation. It has been pointed out by Yun (1996) that the full non-linear aggregate production function depends on a price dispersion term.

\[ Y_t = \frac{A_t}{D_t} K^\alpha L_t^{1-\alpha} \]  
with: \[ D_t \equiv \int_0^1 \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} dz \]  
and \[ \hat{D}_t = \theta \hat{D}_{t-1} \]  
(34)

Christiano, Eichenbaum, and Evans (2001) have shown that the price dispersion term can be ignored for a log-linear analysis around a steady state that features zero price dispersion. One can further show that this term evolves as a univariate AR(1) regardless of the fraction of backward looking price setters by log-linearizing the price index and the price dispersion term. Therefore, we can safely ignore this term in our log-linear analysis. (30) is the firm’s labor demand function. (31) is an identity defining the change of the real wage. (31) links marginal cost to the output gap and the “wage gap”. It is not a necessary equation for general equilibrium, but is needed to compute the loss function. The wage gap is the
difference between the average marginal rate of substitution between consumption and labor on the hand and the real wage on the other, see proposition ?? in the appendix. Finally, the last equation is the exogenous stochastic process for total factor productivity.

We use the baseline calibration from the sticky price model of Pappa (2004) and assume symmetric price and wage setting parameters. In particular, the markup is 14 \% in both goods and labor markets, resulting in \( \kappa = \epsilon = 7.88 \). Furthermore, we assume that prices and wages are fixed on average 4 quarters, such that \( \theta = \theta_w = \frac{3}{4} \). In our baseline calibration, we set the fraction of backward looking agents in both price and wage setting to 0 and vary this parameter in a robustness check. The labor share in production \( 1 - \alpha \) is set to 0.65. We set the coefficient of relative risk aversion \( \sigma \) to 2 and assume a Frisch (constant marginal utility of wealth) elasticity of labor supply of \( \frac{1}{3} \), implying \( \chi = 3 \). The exogenous process for technology follows an AR(1) with autoregressive parameter equal to 0.906. The innovation has standard deviation equal to 0.00852. Finally, the time preference rate is matched to yield an annual real interest rate of 1.03, i.e. \( \beta = 1.03^{-0.25} \).

5 Welfare costs with Calvo contracts

This section computes welfare costs of business cycle fluctuations stemming from nominal rigidities in wage and price setting. The welfare measure is the expected discounted lifetime utility of a randomly drawn household. As is common in the literature, we neglect the arbitrarily small utility flow from real money balances.

\[
E_0 \sum_{j=0}^{\infty} \beta^j \mathcal{W}_{t+j} \equiv E_0 \sum_{j=0}^{\infty} \beta^j \left\{ U(C_{t+j}) + \int_0^1 V(N_{t+j}(h))dh \right\}. \tag{35}
\]

\( E_0 \) is an expectation conditional on a particular initial state vector or an assumption on the distribution of the initial state vector. Let \( \mathcal{W}_t^{*} \) denote period utility under perfectly flexible wages and prices. The consumption equivalent welfare measure \( \mathcal{L} \equiv - \sum_{t=0}^{\infty} \beta^t \left( \mathcal{W}_t - \mathcal{W}_t^{*} \right) / \left( U_C \bar{C} \right) \) can be approximated up to second order by the following weighted sum of second moments.

\[
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \lambda_0 \delta^2_t + \lambda_1 \left( \hat{Y}_t - \hat{Y}_t^{*} \right)^2 + \lambda_2 \left( \Delta \hat{\pi}_t \right)^2 + \lambda_3 \left( \pi^w_t \right)^2 + \lambda_4 \left( \Delta \pi^w_t \right)^2 \right] \tag{36}
\]

\( \mathcal{L} \) gives the onetime increase in consumption, expressed as a percentage of period consumption in the steady state, necessary to make agents as well off in a world with nominal rigidities as in a world with perfectly flexible wages and prices.
Since this loss function is free of first moments, it can be accurately evaluated by considering a linear approximation to the model's equilibrium conditions. Here, the weights are given by

\[ \tilde{\lambda}_0 = 0.5 \epsilon \frac{\theta}{(1 - \theta)(1 - \theta \beta)} \]  

(37)

\[ \tilde{\lambda}_1 = 0.5 \left( \frac{\chi + \alpha}{1 - \alpha} + \sigma \right) \]  

(38)

\[ \tilde{\lambda}_2 = \frac{\omega}{(1 - \omega) \theta} \tilde{\lambda}_0 \]  

(39)

\[ \tilde{\lambda}_3 = 0.5 \kappa^2 (1 - \alpha)(\kappa^{-1} + \chi) \frac{\theta_w}{(1 - \theta_w)(1 - \theta_w \beta)} \]  

(40)

\[ \tilde{\lambda}_4 = \frac{\varphi}{(1 - \varphi) \theta_w} \tilde{\lambda}_3 \]  

(41)

Schmitt-Grohé and Uribe (2004b) have noted that the ranking of monetary policy rules can in principle depend on the assumed distribution of the initial state vector. We have considered two cases. First we condition the initial state vector to have zero variance, i.e. the economy is in the deterministic steady state at time zero. Second we condition the covariance matrix of the initial state vector to be equal to its long run unconditional covariance matrix. In that case the welfare measure is equal to \((1 - \beta)^{-1}\) times the unconditional expectation of period utility. We found that both measures give roughly the same welfare costs and therefore only report the second measure in our tables. The model’s parameters give rise to the following weights in the loss function:

\[ \tilde{\lambda}_0 = 46.26, \quad \tilde{\lambda}_1 = 3.57, \quad \tilde{\lambda}_3 = 740.87 \]  

(42)

Note that the weight on wage inflation is roughly 16 times larger than on price inflation for our benchmark calibration despite the fact that the average duration of wage contracts is the same as for price contracts. This is a result of a low wage elasticity. With sticky wages labor supply is demand determined. The inverse of the labor supply elasticity signals how much compensation in terms of real wage the household requires for supplying an extra unit of labor. When wages are sticky households are induced to vary their labor supply without any such compensation taking place. Therefore, it is clear that the inverse of the labor supply elasticity is closely related to the welfare cost of wage inflation. For instance, setting \(\chi = 1\) brings the weight on wage inflation relative to price inflation down to 3.1 for our benchmark calibration. Another important parameter determining the relative weight is the wage elasticity of labor demand \(\kappa\). The higher this parameter, the more substitutable are different varieties of labor in production. Differences in relative quantities of labor demanded by the labor aggregator are a function
of differences in relative wages posted and that function is increasing in the substitutability ($\kappa$) of labor varieties in the aggregator. For instance, reducing the markup in both labor and goods market to 10% ($\kappa = \epsilon = 11$) increases the relative weight on wage inflation stabilization to 22.

We now turn to the computation of the welfare costs given a variety of interest rate rules of the form:

$$\hat{i}_t = (1 - \alpha_2)\alpha_0 \left( \hat{Y}_t - \hat{Y}_t^* \right) + (1 - \alpha_2)\alpha_1 \hat{\pi}_t + \alpha_2 \hat{i}_{t-1}$$

(43)

We consider the following interest rate rules, whose coefficients are summarized in Table 1. Rule 1 is our benchmark rule estimated on quarterly U.S. data over the sample 1980:1-2004:2. Rule 2 is an estimated rule where the response to the output gap is restricted to zero. The reason is that the output gap is typically only marginally significant or even insignificant and that the adverse welfare effects of reacting to a wrong measure of the gap can be very large, see Schmitt-Grohé and Uribe (2004a). We finally consider the classical Taylor (1993) rule. It has been shown by Gerlach-Kristen (2004) that a Wald test cannot reject the null of a cointegrating vector with such coefficients for the Euro area data. Thus, such an interest rate rule can be seen as a very crude approximation to ECB behavior.

In Table 2 on the next page the welfare measure is displayed for various fractions of forward and backward looking wage and price setters given our three considered rules.

### Table 1: the considered rules

<table>
<thead>
<tr>
<th>Rule</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 1: estimated</td>
<td>0.09</td>
<td>2.13</td>
<td>0.78</td>
</tr>
<tr>
<td>Rule 2: estimated, no output gap</td>
<td>0.00</td>
<td>1.90</td>
<td>0.77</td>
</tr>
<tr>
<td>Rule 3: Taylor (1993)</td>
<td>0.50</td>
<td>1.50</td>
<td>0.00</td>
</tr>
</tbody>
</table>

---

7All data is taken from the FRED2 database. The interest rate is the end of quarter federal funds rate available at http://research.stlouisfed.org/fred2/series/FEDFUNDS/downloaddata/FEDFUNDS.txt. It is converted to quarterly decimal rate by dividing by 400. The output gap is constructed as the log difference between quarterly real GDP and potential real GDP as constructed by the Congressional Budget Office. The series are available at http://research.stlouisfed.org/fred2/series/GDPC96/downloaddata/GDPC96.txt and http://research.stlouisfed.org/fred2/series/GDPPOT2/downloaddata/GDPPOT2.txt, respectively. Inflation is constructed as the log of the first difference of the GDP price index available at http://research.stlouisfed.org/fred2/series/GDPCTPI/downloaddata/GDPCTPI.txt. All data except potential GDP are seasonally adjusted. All coefficients are significant at standard levels.
Table 2: Welfare costs of nominal rigidities with mobile capital

<table>
<thead>
<tr>
<th>((\omega, \varphi))</th>
<th>L(rule 1)</th>
<th>L(rule 2)</th>
<th>L(rule 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>23.262</td>
<td>35.415</td>
<td>18.559</td>
</tr>
<tr>
<td>(0.5, 0)</td>
<td>23.446</td>
<td>35.153</td>
<td>19.492</td>
</tr>
<tr>
<td>(0, 0.5)</td>
<td>23.453</td>
<td>34.204</td>
<td>23.793</td>
</tr>
<tr>
<td>(0.5, 0.5)</td>
<td>23.542</td>
<td>33.805</td>
<td>24.873</td>
</tr>
</tbody>
</table>

The table shows that the welfare costs of nominal rigidities vary across the assumed monetary policy rules and across the degree of backward looking wage and price setters. For our baseline choice of purely forward looking price and wage setters as well as the estimated monetary policy rule 1, the welfare costs are equivalent to a one time increase in consumption of roughly 23 per cent of steady state consumption. Given our calibration for the discount factor \(\beta\) this is equivalent to a compensating variation of 0.17 per cent of steady state consumption in every period. That number is roughly comparable in magnitude to the benchmark estimate in Gali, Gertler, and Lopez-Salido (2003) for the welfare costs of business cycles. Backward looking elements in wage and price can have an ambiguous effect on the welfare costs of nominal rigidities. As pointed out in Amato and Laubach (2003) (for backward looking price setters only), the Phillips curve changes in two ways. First it becomes more inertial, since lagged inflation enters that equation. Ceteris paribus, any given shock implies a longer lasting effect on the price and wage level and therefore more dispersion of relative prices and relative wages. At the same time, the slope of the Philips curve with respect to marginal cost decreases, implying that any given shock to marginal costs has a smaller effect on inflation. Which effect dominates is a priori ambiguous and depends on the policy rule in place.

We conclude that allowing for backward looking agents alters the welfare costs of nominal rigidities, but not by huge amounts. When considering alternative schemes of nominal rigidities, we do not include a discussion of backward looking elements in order to keep the comparison small. We next turn to checking the robust of our results when varying an important assumption about the real side of the economy.

---

Amato and Laubach (2003) shows that the welfare loss of optimal policy increases with the degree of backward looking price setters. For a given non-optimal policy rule that we consider, this need not be the case.
6 Robustness checks for the baseline Calvo model

So far we have assumed that capital is freely mobile across sectors and can be reallocated as to equalize the shadow value of capital across firms. It has been argued by Danthine and Donaldson (2002), Woodford (2003, p.166) and others that capital cannot be instantaneously be relocated across firms. Danthine and Donaldson (2002) view it as unreasonable that it is too costly to post a new price tag, but that it is costless to unbolt machinery and ship it between firms. Furthermore, Eichenbaum and Fischer (2004) show that departing from the assumption of perfect capital mobility is necessary to reconcile the Calvo (1983) model with the data. Sveen and Weinke (2004) further discuss the implications of modeling capital for the equilibrium dynamics in sticky prices models.

For the purpose of business cycle analysis, capital might better be modeled as being firm specific. In this subsection, we solve the firms price setting problem when capital is fixed at the firm level. The problem of the firm is now to choose its optimal nominal price \( P^*_t(z) \) subject to the demand curve and the production function to maximize

\[
\max_{P^*_t(z)} \mathbb{E}_t \sum_{i=0}^{\infty} (\theta \beta)^i \Lambda_{t+i} \left( 1 + \tau_p \right) \left[ \frac{P^*_t(z)}{P_{t+i}} \right]^{1-\epsilon} Y_{t+i} - Z_{t+i} K(z) - w^*_{t+i} L_{t+i}(z) \right].
\]

Noting that \( \frac{\partial L_{t+i+j}(z)}{\partial P_t(z)} = \frac{\partial L_{t+i+j}(z)}{\partial Y_{t+i+j}(z)} \frac{\partial Y_{t+i+j}(z)}{\partial P_t(z)} = -\epsilon \frac{L_{t+i+j}(z)}{P_t(z)} \), the first order condition for this problem is

\[
\mathbb{E}_t \sum_{i=0}^{\infty} (\theta \beta)^i \Lambda_{t+i} \left( 1 + \tau_p \right)(1 - \epsilon) \left[ \frac{P^*_t(z)}{P_{t+i}} \right]^{1-\epsilon} Y_{t+i} + \frac{\epsilon}{1 - \alpha} + w^*_{t+i} L_{t+i}(z) \right].
\]

With firm specific capital, proposition ?? in the appendix shows that the Philips Curve is given by

\[
\pi_t = \frac{(1 - \omega)(1 - \theta)(1 - \beta \theta)}{(1 - \alpha - \epsilon \theta)(1 - \theta \beta)} \frac{(1 - \alpha)}{(1 - \alpha + \alpha \epsilon)} \hat{X}^u_t + \frac{\beta \theta}{\zeta} \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\omega}{\zeta} \hat{\pi}_{t-1}
\]

Here, \( \hat{X}^u_t \) is average marginal cost. Since capital is fixed at the firm level, the capital-labor ratio differs across firms and so does marginal cost.

For the case of immobile capital, the coefficient \( \lambda_0 \) in the loss function changes \( \lambda_0 = \frac{1}{2} \left[ \frac{1 - \alpha - \epsilon \theta}{1 - \alpha - \epsilon} \right] \epsilon^2 \frac{\theta}{(1 - \theta)(1 - \theta \beta)} \) and \( \lambda_2 \) adjust accordingly. When we assume that capital is fixed at the level of an individual firm, the weight on price inflation in the loss function rises roughly by a factor 5 to 242.536, while the weights on the variability of the output gap and wage inflation remain the same. Firm
specific capital implies that a given dispersion in relative quantities results in a much bigger dispersion of labor across firms. Capital is fixed at the firm level, the firm can only adjust labor to vary production. Since labor has decreasing marginal product in production at the level of the individual firm, the dispersion of labor across firms is welfare reducing. Therefore, the weight attached to price inflation rises strongly with firm specific capital.

A greater coefficient on price inflation variability in the loss function does not imply that the considered interest rate rules will lead to a higher loss with firm specific capital than with mobile capital. The reason is that the structural equations change, too. In particular, the slope of the price Philips curve also falls by roughly factor 5 from 0.0852 to 0.0162. A given disturbance to marginal cost results in much less price inflation with firm specific capital. To understand why the equilibrium variance of inflation falls, consider the problem of a firm that receives a signal to change its price. By setting a lower price than the fixed price firms it can attract additional demand. But with firm specific capital marginal cost depends on the firms’ own level of production while it only depends on the aggregate production level with a common rental market. Therefore, the firm will choose a relative price that deviates from unity by less when marginal cost depends on own output and therefore on its relative price. As a result the variance of price inflation is smaller with firm specific capital. Table 3 displays the welfare costs associated with our simple rules for the case of immobile capital.

Table 3: Welfare costs of nominal rigidities with immobile capital

<table>
<thead>
<tr>
<th>(ω, ϕ)</th>
<th>L(rule 1)</th>
<th>L(rule 2)</th>
<th>L(rule 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>18.950</td>
<td>30.353</td>
<td>14.872</td>
</tr>
<tr>
<td>(0.5, 0)</td>
<td>19.858</td>
<td>30.668</td>
<td>17.143</td>
</tr>
<tr>
<td>(0, 0.5)</td>
<td>19.140</td>
<td>29.906</td>
<td>17.945</td>
</tr>
<tr>
<td>(0.5, 0.5)</td>
<td>19.777</td>
<td>29.747</td>
<td>20.532</td>
</tr>
</tbody>
</table>

Whether we model capital as fixed at the firm level or assume a rental market does not have a large effect for the benchmark calibration of this model. Price

9If production were linear in labor, the weights attached to price inflation variability would be the same across mobile and firm specific capital. Dispersion of labor across firms would still be welfare reducing, but only because it is identical to the dispersion of output across firms. Since each variety has decreasing marginal product in the bundler, dispersion of output is again welfare reducing.

10See Kimball (1995) for a discussion of assumption on factor markets on price setting and Ball and Romer (1990) for the seminal paper on real rigidities.
dispersion is more costly with fixed capital, but the variance of inflation is also smaller. The two effects can roughly offset each other. This result depends crucially on the concavity of the production function with respect to labor, i.e. on \( \alpha \). The welfare loss with firm specific capital increases monotonically as \( \alpha \) approaches unity, but remains bounded when we assume a rental market. For \( \alpha \) close to unity the welfare costs with firm specific capital can therefore be much larger than under the rental market assumption. The table shows that the loss with firm specific capital is generally somewhat lower for the policy rules considered, but the difference is not huge. In broad terms, the picture emerging from Table 2 is similar to that from Table 3. Therefore, we proceed with the case of mobile capital only.

7 How special is Calvo pricing?

The weakest element in New Keynesian models is clearly the modeling of nominal rigidities. There is strong disagreement on why prices or wages are sticky or how to best model price and wage setting behavior. Despite this lack of consensus, the apparatus of Calvo (1983) and Yun (1996) has emerged as an often unquestioned standard in the analysis of welfare effects of monetary policy. It is used in important contributions such as Pappa (2004), Kollmann (2004), Kollmann (2002), Erceg, Henderson, and Levin (2000) or Rotemberg and Woodford (1997).

The apparatus of Calvo (1983) has been criticized by a number of authors. Danthine and Donaldson (2002) consider the possibility of rationing. For firms who last changed their price a long time ago, that price may no longer be above marginal cost given today’s demand. For those firms the individual rationality condition that positive profits are earned at the marginal units sold may be violated and they might be better off by simply not producing the extra units.

Recently, Ascari (2004) and Kiley (2002) have pointed out that Calvo (1983) pricing implies much more price dispersion induced by steady state inflation than Taylor (1980) pricing. Ascari (2004) considers a standard New Keynesian model with trend inflation and Calvo pricing, where those firms that are not re-optimizing their price cannot adjust for trend inflation. He computes that a trend inflation rate of 5\% generates a steady state loss\(^1\) of output relative to the zero inflation case of 11.5\% with Calvo (1983) pricing, but only by 0.5\% with Taylor (1980) pricing of the same average duration over which prices are fixed. The reason is that trend inflation translates into much more price dispersion with Calvo (1983) pricing. We are motivated by these findings to undertake a systematic quantitative comparison

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\(^{11}\)The loss reported here refers to the case firm-specific capital and a markup of 10\%. The loss is only 3\% when there is an economy wide rental market for capital.
of welfare effects of monetary policy across different assumptions about wage and price setting. We go beyond the analysis of Ascari (2004) and Kiley (2002) and improve upon them, by undertaking a full welfare analysis rather than computing output dispersion in the steady state.

Following Wolman (1999), we nest all considered models of price and wage setting as special cases of the following set up. Limited ability to reset prices is described by a vector $\alpha$. The $j$-th element of that vector is the probability that a firm adjusts its price in period $t$, conditional on the previous adjust having occurred in period $t - j$. The vector $\alpha$ can be specified in any way the researcher chooses. One can deduce from $\alpha$ a vector $\omega$, denoting the fraction of firms charging prices set in period $t - j$. The Wolman (1999) scheme is very flexible, it encompasses Taylor (1980) overlapping contracts as a special case and can approximate Calvo pricing for large $J$.

There are two key differences between the Calvo (1983) price setting scheme and the scheme proposed by Wolman (1999). The first is that Calvo (1983) has an infinite tail: for any integer $J$ there is a nonzero fraction of firms that last adjusted their price $J$ periods ago. In the scheme considered here, $J$ is finite. Furthermore, the probability that a firm adjusts its price in any given period is independent of the time elapsed since last adjustment under Calvo (1983) pricing. A firm that has not adjusted its price for say 20 years is just as likely to keep the price fixed in period $t$ as a firm that has last adjusted its price in period $t - 1$.

Both the infinite tail and the constant adjustment probabilities of Calvo (1983) are often considered to be unrealistic. Nevertheless, the Calvo (1983) scheme is often used under the implicit assumption that these particularities do not influence welfare computations by much.

In the following subsection, we derive the models equilibrium equations for the case of Wolman (1999) wage and price contracts. Taylor (1980) contracts are a special case of the former. Therefore, a separate discussion of equilibrium with Taylor (1980) contracts is not necessary.

### 7.1 Wage and Price setting with the Wolman (1999) scheme

For wage setting according to the Wolman (1999) scheme, the first order condition for choice of the optimal nominal wage $W_t^*$ is:

$$E_t \sum_{j=0}^{J_w} \phi_j^w \beta^j \left\{ N_{t+j}(h) U_C(C_{t+j})(1 + \tau_w) \frac{W_t^*(h)}{P_{t+j}} + \frac{\kappa}{\kappa - 1} V_N(N_{t+j}(h)) \right\}$$

(47)

\(^{12}\)Of course, the fraction of firms that have not adjusted their 20 years ago is small.
We denote by \( \phi_j^w \) the probability that a wage set in period \( t \) is still in place in period \( t + j \). We can rewrite this condition using stationary variables \( w_t^* = \frac{W^*}{w^*_t} \) as

\[
w_t^* = \left( \frac{E_t \sum_{j=0}^{J-1} \phi_j^w \beta_j^j \left( \pi_{t,j}^w \right)^{\kappa(1+\kappa)} N_{t+j}^{1+\kappa}}{E_t \sum_{j=0}^{J-1} \phi_j^w \beta_j^j \left( \pi_{t,j}^w \right)^\kappa N_{t+j} C_{t+j}^{1-\sigma} \pi_{t,j}^w} \right)^\frac{1}{1+\kappa} \tag{48}
\]

Here, \( \pi_{t,j}^w \equiv \Pi_{k=0}^{j} \pi_{t+k}^w \), i.e wage inflation between periods \( t \) and \( t + j \), i.e inflation between periods \( t \) and \( t + j \). At any point in time there are \( J \) such conditions determining the aggregate wage index, corresponding to the FOC of wage setters in the current and previous \( J - 1 \) periods.

For the case of a general wage setting scheme with \( J \) cohorts, the first order conditions log-linearized around a steady state with zero wage inflation is:

\[
\hat{w}_t^* = E_t \left( \frac{1}{1 + \kappa \chi} \right) \sum_{j=0}^{J-1} \beta_j^j \phi_j^w \hat{\mu}_t + \sum_{j=1}^{J-1} \nu_{t+j}^w \frac{1}{1 + \kappa \chi} \tag{49}
\]

with: \( \nu_j^w = \sum_{j=0}^{J-1} \beta_j^j \phi_j^w \), \( \gamma_j^w = \sum_{k=j}^{J-1} \beta_k^j \phi_k^w \) \tag{50}

Recall that we had introduced the notation \( \hat{\mu}_t \) for the difference between the marginal rate of substitution and the real wage: \( \hat{\mu}_t = \chi \hat{L}_t + \sigma \hat{C}_t - \hat{w}_t^r \). We can see that the wage chosen by households in period \( t \) differs from the aggregate wage index to the extent that agents expect gaps between the marginal rate of substitution and the real wage and to the extent that they expect wage inflation. Agents look at most \( J - 1 \) periods into the future and if \( \phi_j \) is strictly decreasing in \( j \) then they attach a smaller weight to variables farther into the future.

From the definition of the wage index we obtain

\[
0 = \sum_{j=0}^{J-1} \omega_j^w \hat{w}_{t-j} - \sum_{j=0}^{J-2} \theta_j^w \hat{w}_{t-j} \quad \text{with:} \quad \theta_j^w = \sum_{k=j+1}^{J-1} \omega_k^w \tag{51}
\]

Similarly, for price setting we have that the following first order conditions for the optimal nominal price \( P_t^* \) expressed in terms of the stationary variable \( p_t^* = \frac{P_t^*}{P_t^*} \)

\[
p_t^* = \frac{E_t \sum_{j=0}^{J-1} \phi_j^p \beta_j^j \Lambda_t \hat{X}_{t+j} (\pi_{t+j}^e)^{\gamma} Y_{t+j}}{E_t \sum_{j=0}^{J-1} \phi_j^p \beta_j^j \Lambda_t \hat{X}_{t+j} (\pi_{t+j}^e)^{\gamma-1} Y_{t+j}} \tag{52}
\]

Here, \( \pi_{t,j} \equiv \Pi_{k=0}^{j} \pi_{t+k} \), i.e inflation between periods \( t \) and \( t + j \). For the case of a general price setting scheme with \( J \) cohorts, the first order condition log-
linearized around a steady state with zero price inflation is:

\[
\hat{p}_t^* = E_t \sum_{j=0}^{J^p-1} \frac{\beta^j \phi^p_j}{\nu^p_j} \hat{X}_{t+j} + \sum_{j=1}^{J^p-1} \gamma^p_j \hat{\pi}_{t+j}
\]

with: \( \nu^p = \sum_{j=0}^{J^p-1} \beta^j \phi^p_j \), \( \gamma^p_j = \sum_{k=j}^{J^p-1} \beta^k \phi^p_k \) (54)

Firms charge a price that is different from the current price level, if the expect log marginal cost \( \hat{X}_t \) differs from zero and if price inflation is expected. From the definition of the price index we obtain

\[
0 = \sum_{j=0}^{J^p-1} \omega^p_j \hat{p}^*_t - j - \sum_{j=0}^{J^p-2} \theta^p_j \hat{\pi}_t - j
\]

with: \( \theta^p_j = \sum_{k=j+1}^{J^p} \omega^p_k \) (55)

For the case of Wolman (1999) rigidities, one can derive a relation between aggregate inputs and aggregate output that again depends on a price dispersion term

\[
Y_t = \frac{A_t}{D_t} \bar{K}^\alpha L_t^{1-\alpha}
\]

with: \( D_t \equiv \sum_{j=0}^{J^p-1} \omega^p_j \left( \frac{P^*_t - j}{P_t} \right)^{-\epsilon} \) (56)

For this pricing scheme it is also possible to show that the price dispersion term can be ignored for a log-linear analysis around a steady state with zero price dispersion. One can log-linearize the price dispersion term together with the aggregate price index to show that \( \bar{D}_t = 0 \) up to first order.

Equilibrium sequences for allocations and prices in the model with Wolman (1999) rigidities must then satisfy (26), (29) - (33), the monetary policy rule as well as \( J^p \) equations of the type in (53) and of \( J^w \) equations of the type (49) together with (55) and (51).

The loss function is given by

\[
L_0 = \sum_{t=0}^{\infty} \beta^t \left[ \lambda_0 \sum_{j=0}^{J^p-1} \omega^p_j (\hat{p}^t_{t-j} - \hat{p}^*_t)^2 + \lambda_1 \left( \hat{Y}_t - \hat{Y}^*_t \right)^2 + \lambda_3 \sum_{j=0}^{J^w-1} \omega^w_j (\hat{w}^t_{t-j} - \hat{w}^*_t)^2 \right]
\]

with: \( \lambda_0 = \frac{1}{2} \epsilon, \) \( \lambda_1 = \frac{1}{2} \frac{\chi + \alpha + \sigma}{1 - \alpha}, \) \( \lambda_3 = \frac{1}{2} (1 - \alpha)(\kappa^{-1} + \omega_2)\kappa^2 \) (57)

Here, \( \hat{p}^t_{t-j} \equiv \hat{P}^*_t - \hat{P}_t \) and similarly for \( \hat{w}^t_{t-j} \). For the case of Taylor (1980) contracts, the sum of the weights on price and wage dispersion is 3.94 and 63.11, respectively. The weight on the output gap remains at 3.57 as for the case with
Calvo (1983) contracts. It is evident that the weights attached to price inflation and wage inflation in the loss function fall by roughly factor 11 relative to the Taylor (1980)\(^{13}\) setting. However, by inspecting how the weights in the loss function change, one cannot infer much about the change in equilibrium welfare when switching from Calvo (1983) to Taylor (1980). The reason is that while weights in the loss function go down, equilibrium variances generally go up.

### 7.2 Welfare costs of nominal rigidities with the Wolman 1999 scheme

We require that the average duration over which a contract is fixed is the same across all considered schemes of price and wage setting, respectively. Using the general notation above the average duration, \( D \), of the contract is

\[
D \equiv \frac{1}{c} \sum_{k=1}^{J} k \alpha_k \Pi_{j=0}^{k-1} (1 - \alpha_j) \quad \text{with:} \quad \alpha_0 \equiv 0, \quad c \equiv \sum_{k=1}^{J} \alpha_k \Pi_{j=0}^{k-1} (1 - \alpha_j) \quad (58)
\]

For Calvo (1983) pricing, \( J = \infty \), \( \alpha_j = (1 - \theta) \) for \( j = 1, 2, \ldots \) and \( c = \theta \). The duration is given by

\[
D = \frac{(1 - \theta)}{\theta} \sum_{k=1}^{\infty} \theta^k k = \frac{1}{1 - \theta} \quad (59)
\]

Prices and wages are fixed for at most 4 periods in all cases considered, i.e. \( \alpha_4 = 1 \). We consider contracts that all have an average duration of 3 quarters: 3 period Taylor contracts (\( \alpha_j = 0 \) for \( j = 1, 2 \) and \( \alpha_3 = 1 \)), truncated Calvo (\( \alpha_j = 0.19 \) for \( j = 1, 2, 3 \)) and a more general upward sloping scheme in the spirit of Wolman (1999) (\( \alpha_1 = 0.05, \alpha_2 = 0.255, \alpha_3 = 0.52 \)).

The welfare losses relative to the flexible price allocation is displayed in Table 4. Table 4 shows that for our two estimated rules, Calvo (1983) price and wage setting implies much larger welfare losses than any of the schemes with finite horizon. Note that these differences arise despite the fact that the average duration of wage and price contracts is equal across all considered schemes. For the estimated interest rate rule, the welfare costs of price and wage stickiness are several times higher with the Calvo (1983) scheme than with any of the finite horizon adjustment schemes. It therefore appears that price and wage setters last adjusting their

\(^{13}\)It is clear that welfare cannot be expressed as a function of the variance of wage and price inflation and the output gap alone. However, the variance of wage inflation is a good summary statistic for the variance of the relative contract wages entering the loss function.
Table 4: Comparison of rules across wage and price setting schemes

<table>
<thead>
<tr>
<th>interest rate rule</th>
<th>L (Calvo)</th>
<th>L (tr. Calvo)</th>
<th>L (Taylor)</th>
<th>L (Wolman)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 1</td>
<td>17.458</td>
<td>4.822</td>
<td>4.296</td>
<td>4.543</td>
</tr>
<tr>
<td>Rule 2</td>
<td>24.109</td>
<td>5.643</td>
<td>4.845</td>
<td>5.242</td>
</tr>
<tr>
<td>Rule 3</td>
<td>24.526</td>
<td>15.513</td>
<td>15.901</td>
<td>15.318</td>
</tr>
</tbody>
</table>

contract long ago matter for welfare despite the fact that they have increasingly smaller share in their respective population. Whether we assume Taylor (1980) or truncated Calvo (1983) or the more flexible scheme of Wolman (1999) does not matter much for welfare computations.

7.3 A variant of Mankiw and Reis (2002): Sticky information

The last modeling device for price and wage stickiness we examine is a variant of the sticky information framework of Mankiw and Reis (2002). These authors build an alternative framework around the notion that agents have limited capacity to process information. In particular, they assume that in any period only a randomly chosen fraction of firms updates their information set. Firms are free to change their price in any period based on the information they have available. The price level is therefore determined by a weighted average of expectations of current optimal price based on information sets at all past periods in time. As with Calvo (1983) pricing, the weights attached to these prices decay geometrically.

We build a $N$-period version of the model by Mankiw and Reis (2002). In particular, we assume that no firm or household operates with information outdated longer than one year, i.e. $N = 4$. As with Wolman (1999) contracts, the arrival of new information is described by a vector $\alpha$. The generic entry $\alpha_j$ denotes the probability that a firm that last updated its information set in $t - j$ periods ago receives a signal to update the information set in period $t$. Similarly, one can deduce from $\alpha$ a vector $\omega$ giving the fractions of firms charging prices based on the information sets in periods $t - j$, for $j = 0, 1, \ldots, N - 1$. In order to keep the amount of exogenous stickiness comparable to the earlier models, we require that the average duration of an information set is again equal to 3 quarters.

Such a price and wage setting scheme has several attractive distinctions relative the Calvo (1983) model. First, steady state inflation is completely neutral in this setup, therefore avoiding the huge costs of steady state inflation of the Calvo (1983) model pointed out in Ascari (2004). Second, while Calvo (1983) price set-
ters must choose one price which does well on average over the infinite weighted future, sticky information firms can change their price in any period. Even firms that do not receive new information will change their price in any period, as long as the state variables evolve in an autoregressive manner\(^\text{14}\).

Equilibrium sequences for allocations and prices in the model with sticky information must then satisfy (26), (29) - (33), the monetary policy rule as well as the following conditions stemming from wage and price setting. Let \(W^*_{t,t-j}\) and \(P^*_{t,t-j}\) denote the nominal optimal wage and nominal price chosen at time \(t\) based on an information set last updated at time \(t-j\). We define the optimality in conditions in terms of stationary variables \(\hat{w}^*_{t,t-j} \equiv W^*_{t,t-j} - \hat{W}_{t-j}\) and \(\hat{p}^*_{t,t-j} \equiv P^*_{t,t-j} - \hat{P}_{t-j}\).

\[
\hat{w}^*_{t,t-j} = E_{t-j} \sum_{k=0}^{j-1} \hat{w}_t - \hat{w}_t^{\omega} + \chi \hat{L}_t + \sigma \hat{Y}_t \quad \text{for } j = 0, 1, \ldots, N^w - 1 \tag{60}
\]

\[
\hat{p}^*_{t,t-j} = E_{t-j} \sum_{k=0}^{j-1} \hat{p}_t - \hat{p}_t^{\omega} + \hat{X}_t \quad \text{for } j = 0, 1, \ldots, N^p - 1 \tag{61}
\]

The log-linearized price and wage indices (normalized by the current price level and wage, respectively) are

\[
0 = \sum_{j=0}^{N^w-1} \omega_j^w \hat{w}^*_{t,t-j} + \sum_{k=1}^{N^w-2} \sum_{j=k}^{N^w-1} \omega_j^w \hat{w}_t^{\omega} \tag{62}
\]

\[
0 = \sum_{j=0}^{N^p-1} \omega_j^p \hat{p}^*_{t,t-j} + \sum_{k=1}^{N^p-2} \sum_{j=k}^{N^p-1} \omega_j^p \hat{p}_t^{\omega} \tag{63}
\]

\(^{60}\) indicates that the \(N^w\) cohorts of households set their nominal wage such that in conditional expectation based on the respective information set\(^{15}\) the real wage equals the marginal rate of substitution between consumption and leisure. Similarly, \(^{61}\) indicates that \(N^p\) cohorts of firms set their price such that in conditional expectation based on their respective information sets, price equals nominal marginal costs. It is again easy to show that the price dispersion term affecting the non-linear aggregate production function is zero up to first-order and can therefore be ignored.

\(^{14}\)Since the exogenous stochastic process is highly autocorrelated this is clearly the case in our model.

\(^{15}\)Expectations based on past information sets do not fit the standard setup for solving linear rational expectations models. Using a simple trick outlined in [McCallum (2001)] it is easy to cast this problem in the form necessary for standard solution codes.
The loss function for the sticky information model is of the form (57). In that formula, one must simply substitute \( \hat{p}_{t-j} \) with the expectation based on the information \( t - j \) periods ago of the optimal price and \( \hat{w}_{t-j} \) with the expectation based on the information \( t - j \) periods ago of the optimal wage.

In the following table we display the welfare losses associated with different monetary policy rules and different assumptions about the arrival rates of new information. Recall that the j-th entry in the vector \( \alpha \) denotes the probability that any given price or wage setter receives the most recent information set, conditional on last having received an information update \( j \) periods ago. The column \textit{truncated} refers to a model where \( \alpha_j = 0.19 \) for \( j = 1, 2, 3 \), the column \textit{staggered} \( \alpha_j = 0 \) for \( j = 1, 2 \) and \( \alpha_3 = 1 \) and finally the column \textit{increasing} refers to \( \alpha_1 = 0.05, \alpha_2 = 0.255, \alpha_3 = 0.52 \). Note that this is completely analogous to the cases considered for Wolman (1999) pricing.

Table 5: Comparison of rules across different sticky information schemes

<table>
<thead>
<tr>
<th>interest rate rule</th>
<th>( L(\text{truncated}) )</th>
<th>( L(\text{staggered}) )</th>
<th>( L(\text{increasing}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 1</td>
<td>4.968</td>
<td>4.237</td>
<td>4.912</td>
</tr>
<tr>
<td>Rule 2</td>
<td>3.107</td>
<td>2.690</td>
<td>3.097</td>
</tr>
<tr>
<td>Rule 3</td>
<td>11.406</td>
<td>11.500</td>
<td>11.663</td>
</tr>
</tbody>
</table>

One can see that the welfare costs of Mankiw and Reis (2002) price and wage setting are the smallest of all schemes considered.\(^\text{16}\) Relative to the standard Calvo (1983) contract with average duration of 3 periods, the Mankiw and Reis (2002) scheme implies welfare costs that are only roughly a quarter as big. In particular, the welfare costs of nominal inertia are of a similar order of magnitude as the welfare costs of business cycle fluctuations that Lucas (2003) suggested. Comparing the costs of truncated Calvo (1983) with those of truncated Mankiw and Reis (2002), one finds that sticky information implies sizeably smaller welfare costs. This is important since ad-hoc models of nominal rigidities are often justified by arguing that agents have imperfect capacities to constantly compute and post the optimal price. Our comparison suggests it may well make a difference for welfare comparisons whether one explicitly models imperfect ability to process information as in Mankiw and Reis (2002) or resorts to more ad-hoc models of price setting.

\(^{16}\) Whether this is a general result that holds for any monetary policy rule considered is an open question.
7.4 Why is the Calvo scheme so special?

Apparently, the Calvo (1983) scheme for nominal rigidities has quite different implications for the welfare costs of business cycle fluctuations than the finite horizon schemes. To explain why this is the fact, we construct a 20 period approximation to the infinite horizon model of Calvo (1983). The probabilities that a contract is fixed for $j = 1,\ldots,19$ periods are identical to the Calvo model. All contracts that have not been updated in any of these periods are updated in period 20. This implies that we cut the Calvo (1983) tail after covering 99.98 percent of wage and price setters. By constructing such an approximation to the Calvo (1983) scheme, we can quantify the contribution of particular cohorts of wage and price setters to the overall welfare costs.

In the following figures, we plot the percentage contribution of each cohort of wage and price setters to the overall welfare loss, defined by the expression $\tilde{\lambda}_0 \omega_j^p (\hat{p}_t^o - j)^2 / L100$ and $\tilde{\lambda}_3 \omega_j^w (\hat{w}_t^o - j)^2 / L100$. In the same figure we also plot the weights $\omega_j^p$ and $\omega_j^w$, representing the share of firms or workers having contracts that were last updated $j$ periods ago. For better visibility, we have scaled these weights by a constant such that the first weight is equal to the contribution of cohort 1 to welfare.

Figure 1: Welfare decomposition by price setter cohorts

Figure 1 shows that the contribution of the cohorts of price setters to welfare does not follow the pattern of the weights, i.e. their population share. The fraction of firms that last adjusted their price $j$ periods ago, $\omega_j^p$ decays geometrically. However, the variance of the relative price $\hat{P}_t^{o-j} - P_t$ generally increases with $j$. In particular, cohorts 5 to roughly 15 contribute more than proportionally to the
welfare loss. The Calvo (1983) scheme is often defended by arguing that the admitted implausibly assumption that certain contracts remain in place for a very long time does not matter much, because these contracts make up only a very small fraction of the total. Our point is to stress that contracts that have been fixed 5 – 15 periods ago contribute significantly to welfare and contribute much more than can be deduced by just looking at their population share.

Figure 2: welfare decomposition by wage setter cohorts

The same pattern is evident for wage setters. Again, the contributions to welfare does not follow the pattern of the population share of individual cohorts. Here, the discrepancy is even more pronounced, as indicated by the strong hump shaped pattern for the contracts of intermediate duration. The two figures taken together also point out that nominal wage rigidities account for a larger share of the overall welfare costs than price rigidities.

8 Summary and conclusion

This paper has asked two questions. First, can a simple general equilibrium model with sticky wages and prices account for sizeable welfare costs of business cycle fluctuations as identified in Gali, Gertler, and Lopez-Salido (2003)? The answer is a qualified yes. Assuming Calvo (1983) wage and price contracts with average duration of 4 quarters our benchmark calibration delivers welfare costs that range from 0.14 to 0.23 per cent of period consumption as can be deduced from the first row in Table 2 on page 15 after multiplying by (1-\beta). That is smaller than the benchmark estimate of the cost of business cycles Gali, Gertler, and Lopez-Salido.
of 0.28, but roughly on the same order of magnitude. The answer is only a qualified yes, since Calvo (1983) pricing is special. What scheme for nominal rigidities we assume matters strongly for the welfare costs of our considered monetary policy rules. Calvo (1983) contracts can deliver welfare costs that are up to 4 times higher than finite horizon schemes of the form suggested in Taylor (1980) or Wolman (1999). It is again stressed that the differences arise despite the fact that all schemes assumes an equal average duration of exogenous stickiness. Assuming a sticky information scheme similar to Mankiw and Reis (2002) with the same degree of average duration of the information set delivers welfare costs that are even smaller. The finding that Calvo (1983) pricing involves much larger welfare losses favours the view that state dependent pricing rules such as Dotsey, King, and Wolman (1999) should receive much more attention.

The apparent differences in the welfare costs of rigidities across the various schemes point to the answer to the second question. Agreement on how to model nominal rigidities seems is necessary to make precise quantitative statements about the welfare costs of wage and price stickiness. Our relatively simple exercise that only changes the distribution of price and wage duration already shows large differences in welfare across scenarios.

We conclude by discussing our findings in more detail and relating it to the existing literature. Comparing Taylor (1980) and Wolman (1999) contracts to the Calvo (1983) scheme, we find that the fact that wage and price setters look infinitely far into the future in the Calvo (1983) model matters a lot. Despite the fact that both schemes imply the same average duration of price and wage contracts, welfare costs in the Calvo (1983) model can be up to 3 times higher than those under Taylor (1980) contracts. This result arises, because the equilibrium dispersion of output across producers and labor across households is strongly influenced by wages and prices set far out in the tail of the distribution of relative prices and wages, respectively. Therefore, truncating the tail of that distribution as done by Taylor (1980) has important consequences for welfare. Our analysis seems to suggest that the exact distribution of price and wage contract duration does not matter much once the tail is cut. Therefore, it appears that the main modeling choice is whether to follow the widely used Calvo (1983) scheme or not. A point similar to ours is brought fourth by Kiley (2002) and Ascarì (2004). These authors considered only steady state inflation, not the welfare costs of business cycle variations. In fact, their analysis suggests costs that are roughly 10 higher in the infinite horizon scheme of Calvo (1983) than under the finite scheme of Taylor (1980). We show that when one is concerned with the welfare costs of nominal rigidity over the cycle rather than in the steady state, these numbers fall strongly. The reason is that trend inflation is unidirectional such that firms farther and farther out in the distribution of the Calvo (1983) tail necessarily charge a nominal
price that is more and more eroded in real terms. With zero steady state inflation and business cycle shocks this need not be true. Shocks are positive and negative and it may happen that the price of a firm having last optimized its price a long time ago is close to the one of a firm that is currently optimizing. Therefore, it is intuitively clear that business cycle shocks generate less relative price dispersion than trend inflation. This feature explains why in our study, welfare costs of Calvo (1983) contracts exceed the costs of Taylor (1980) contracts, but not by as much as in the analysis of Kiley (2002) and Asciari (2004).

Turning to the case of Mankiw and Reis (2002) sticky information we find further implications for the welfare costs of nominal rigidities. Mankiw and Reis (2002) model incomplete capabilities of agents to process information in highly stylized and mechanical fashion as an infrequent update of information sets. Our welfare analysis suggests that for the monetary policy rules considered here, the costs of nominal rigidities introduced in such a fashion are smaller than those of similar Taylor (1980) or Wolman (1999) contracts. The reason is that wage and price setters generally choose to change their wages and prices even if they do not receive any new information due to the autoregressive nature of the state vector. Therefore, sticky information schemes of a given average duration typically involve less price and wage stickiness than Taylor (1980) contracts of the same duration.

Finally, we note some limitations of the current analysis and directions for future research. This paper has not taken the different schemes for nominal rigidities to the data. It appears highly desirable to spell out the implications for the model’s second moments of different schemes of nominal rigidities. Which scheme is more realistic can probably not be determined without the need to consult micro data on price and wage adjustment as well as macro data on aggregate time series. Furthermore, it is not clear that our main finding that finite horizon schemes imply smaller welfare costs than the Calvo (1983) scheme is general. Our results are dependent on the particular monetary policy rules chosen. Finally, one would like to know whether our main points would also go through in a more complex model set-up. We currently answer the same set of questions in a more realistic model with capital accumulation, habit formation, etc. That analysis requires the use of numerical second-order solution of the type suggested in Schmitt-Grohé and Uribe (2004c). Preliminary results suggests that our findings still hold broadly in such a setting.
References


