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**International Evidence on  
Long-Run Money Demand**

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**DISCUSSION PAPERS**

# International Evidence on Long-Run Money Demand

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## Abstract

We explore the long-run demand for M1 based on a dataset comprising 38 countries and relatively long sample periods, extending in some cases to over a century. The evidence supports the existence of a stable long-run relationship between the ratio of M1 to GDP and a short-term interest rate for a large majority of the countries. The log-log specification provides a good characterization of the data, with the exception of periods featuring very low interest rates. An extension of the theory that imposes limits on the amount households can borrow results in a truncated log-log specification, which is in line with what we observe in the data. We estimate the interest rate elasticity to be between 0.3 and 0.6.

*Keywords:* Long-run money demand, Cointegration

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## 1. Introduction

This paper describes and analyzes a new dataset containing annual measurements of money supplies, both real and nominal output (GDP) and thus price levels, and short-term nominal interest rates for 38 countries, for periods that go from three decades to over a century. The framework we use for organizing these data is a money demand function that relates the money that the public and private sectors of the economy choose to hold to the rate of production of goods and the short-term interest rate,

$$M_t = P_t y_t \phi(r_t), \tag{1}$$

where  $M_t$  is the monetary aggregate,  $P_t$  is the price level,  $y_t$  is total real production,  $r_t$  is a short-term nominal interest rate, and  $\phi$  is a decreasing function of  $r_t$ .

The formula (1) contains some strong implications. One is that, if  $r_t$  is stationary,  $M_t$  and  $P_t y_t$  should grow at a common rate in the long run, for any continuous function  $\phi$ . If, on the other hand,  $r_t$  has a unit root—possibly because inflation is driven in part by permanent shocks—then  $M_t$  and  $P_t y_t$  should grow at a common rate in the long run, once we control for the impact of permanent shocks to  $r_t$ . Another implication is that it should be possible to use both cross-country and within-country time series to trace out the function  $\phi$ . This is the agenda carried out in this paper.

In recent years, many economists and central bankers have come to doubt

22 the usefulness of measures of the money supply (such as M1) in the conduct  
23 of monetary policy. What was thought to be a central pillar of the monetary  
24 policies of the newly established European Central Bank in 1999 has come to  
25 be seen as too unreliable to be of any use. These concerns were not without  
26 an empirical basis.

27 First, since shocks to real money demand have historically been volatile,  
28 money-based rules may induce substantial volatility in policy outcomes. In  
29 contrast, as interest rate rules are immune to those shocks, they isolate the  
30 economy from them. The remarkable success that all central banks in the  
31 developed world (and many in the developing world) have had in maintaining  
32 inflation rates very close to target over the last decades, using interest rate  
33 rules, provides strong empirical support for this notion. More recently, how-  
34 ever, with real interest rates at historically low values, the threat of the zero  
35 lower bound (ZLB) became a serious constraint on monetary policy. With  
36 the policy rate constrained by the ZLB, it is conceivable that a money-based  
37 rule may perform better, as recently shown by Belongia and Ireland (2019b).  
38 A maintained assumption in Belongia and Ireland (2019b) is that a stable  
39 relationship such as the one described in (1) exists. We document this is  
40 indeed the case for a large set of countries.

41 Second, shocks to real money demand exhibit very high persistence. This  
42 means that deviations of the data from the theoretical expression in (1) are  
43 long-lived. A large literature has developed to understand this fact, assuming

44 that agents make transactions in physically segmented markets.<sup>1</sup> A recent  
45 example is provided in Alvarez and Lippi (2014). Heterogeneous access to  
46 markets implies that monetary policy affects agent's decisions with lags, so  
47 that the aggregate real money demand responds with lags to disturbances.  
48 Alvarez and Lippi (2014) show how this modification to the standard model  
49 can successfully account for several of the short-run facts. On the other  
50 hand, their theory cannot quantitatively reproduce the long-run movements  
51 exhibited by the data. Our empirical exploration will abstract from these  
52 short run fluctuations. In order to to so, we use the methods of cointegration  
53 and apply them uniformly to a wide variety of countries. The virtue of these  
54 methods for our purposes is that they make precise what we mean by long  
55 run relations. These methods also characterize the short run behavior, by  
56 estimating the moments of the cointegration errors - which as expected in  
57 light of the previous discussion, are quite persistent. This quantification is  
58 useful to discipline future attempts to integrate the long run component we  
59 focus in this paper with the short run evidence that the work of Alvarez and  
60 Lippi (2014) can rationalize.

61 Finally, the argument has been made that as the formula (1), which was  
62 shown to perform very well empirically by Meltzer (1963) and Lucas (1988),  
63 has broken down in the last decades, so it cannot be a reliable guide for  
64 policy. This has indeed been the case in the United States for M1, which

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<sup>1</sup>See Grossman and Weiss (1983) and Rotemberg (1984) for early contributions.

65 adds cash to demand deposits. But several papers (e.g., Carlson, Hoffman,  
 66 Keen and Rasche (2000), Teles and Zhou (2005), Serletis and Gogas (2014),  
 67 Judson, Schlusche and Wong (2014), Barnett (2016), Belongia and Ireland  
 68 (2019a), Anderson, Bordo and Duca (2017)) have argued that it has not  
 69 been the case for other measures, such as MZM (or Money Zero Maturity),  
 70 or by aggregating the components of money using Divisia indexes. Others  
 71 (e.g., Lucas and Nicolini (2015)) have shown that accounting for regulatory  
 72 changes that allowed for newly created deposits that are very close substi-  
 73 tutes to checking accounts, and that occurred precisely at the time of the  
 74 breakdown can restore a stable money demand function.<sup>2</sup> This literature has  
 75 forcefully argued that the apparent breakdown of real money demand in the  
 76 United States is just the result of regulatory changes that made the measure  
 77 of M1 reported by the Federal Reserve an unreliable measure of means of  
 78 transactions.

79 With 38 different datasets, some covering more than a century, we can  
 80 expect surprises and there will indeed be a few. But first we want to explain  
 81 why in 37 countries we apply  $M_t$  as M1 and only for the United States we also  
 82 apply “New M1,” a short hand for M1 plus money market demand accounts,  
 83 or more briefly  $\text{NewM1} = \text{M1} + \text{MMDA}$ . The addition of MMDA appears  
 84 only in the U.S. and only because the 1933 Glass-Steagall Act imposed the  
 85 Regulation Q that prohibited interest payments on checking accounts. For

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<sup>2</sup>See also Ireland (2008) for a related discussion.

the early years of Glass-Steagall checking accounts remained, with free checking and other services, fairly close to what competitive banking would have done without Regulation Q. But by the late 1970s and early 1980s returns to bankers were on the order of 8 percent or more on deposits. Thus, in 1982, banks were allowed to issue the newly created MMDA. Large cash holders substituted away from checking accounts and into MMDAs, so M1 continued to fall. Here we chose  $\text{NewM1} = \text{M1} + \text{MMDA}$ , the monetary aggregate proposed by Lucas and Nicolini (2015), to capture this.<sup>3</sup> For the 37 others without Regulation Q no such NewM1 was needed.

Central banks in the developed world are revising their policy frameworks to adapt them to a world with very low real interest rates. None of them are seriously considering money-based rules, and that is understandable: money demand models are not yet be ready for prime time. We need to better understand the behavior of monetary aggregates in the countries that lack solid evidence. We also need to integrate into a common theoretical framework the low-frequency components analyzed in this paper with the high frequency behavior that Alvarez and Lippi (2014) successfully analyzed. Such an integrated framework, assuming that it passes the quantitative tests, would provide support for policy evaluation exercises such as those in Belongia and Ireland (2019b).<sup>4</sup> The evidence described in this paper provides sizable em-

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<sup>3</sup>Results using other aggregates as MZM or adding Mutual Money Market Funds to NewM1 are very similar to the ones obtained using NewM1.

<sup>4</sup>Belongia and Ireland (2019b) model the lagged response of real money demand as adjustment costs in the utility function.



106 pirical support for the notion of a stable real money demand. This is a  
107 required first input into the analysis of money rules as a policy option when  
108 real interest rates become very low.

109 We take pains to ensure that terms such as “short-term interest rate” and  
110 “money” are measures of the same thing (or almost!) in different countries  
111 and over time within countries. The set of countries is highly heterogeneous  
112 in terms of size, income per capita, and world region. More importantly, the  
113 countries’ respective monetary histories also differ substantially : our sam-  
114 ple includes countries that experienced hyperinflation as well as countries  
115 in which inflation has been almost always within a single digit. The pe-  
116 riods covered include very different growth experiences, different monetary  
117 arrangements, and different degrees of integration within the world markets.  
118 We will explicitly ignore all those differences and will look at this diverse set  
119 of countries through the lens of an extremely simple model. The high degree  
120 of variation in nominal interest rates across countries and over time within  
121 each country is what we will exploit in building our case that the basic fea-  
122 tures of the demand function for money are in general quite solid for a large  
123 set of countries.

124 A particular expression for the function  $\phi(r_t)$  is the well-known squared  
125 root formula that Baumol (1952) and Tobin (1956) derived over half a cen-  
126 tury ago. In this paper, we use a theory that generalizes this Baumol-Tobin  
127 expression along a couple of dimensions. The first generalization allows for a  
128 technology to transform bonds into money that encompasses the linear tech-

129 nology assumed by Baumol and Tobin, but which also allows for nonlinear  
 130 relationships. The second generalization is the consideration of borrowing  
 131 constraints, which affects the behavior of money demand at very low interest  
 132 rates.

133 We address the elements of agents' decision problems in detail in Sec-  
 134 tion 2, where we derive an equation like (1) that generalizes the familiar  
 135 Baumol-Tobin specification. In Section 3, we plot the implied predictions of  
 136 a particular case of the model against the data and let the graphics speak for  
 137 themselves. The methods of cointegration are described in Sections 4 and  
 138 5, where we discuss the methodology that we use throughout the paper and  
 139 discuss the results from cointegration analysis. In Section 6, we discuss some  
 140 extensions, and Section 7 concludes.

## 141 **2. A Model of Money Demand**

142 We study a labor-only, representative agent economy in which making  
 143 transactions is costly. Preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t),$$

144 where  $\beta < 1$ ,  $x_t$  is consumption at date  $t$ , and the function  $U$  is differentiable,  
 145 increasing, and concave. The agent is endowed in each period with a unit  
 146 of time, with  $l_t$  allocated to goods production and  $1 - l_t$  used to carry out  
 147 transactions. The goods production technology is given by  $y_t = x_t = z_t l_t$ ,  
 148 where  $z_t$  is an exogenous stochastic process.

149 We assume that households choose the number  $n$  of “trips to the bank” in  
 150 the manner of the classic Baumol-Tobin model. At the beginning of a period,  
 151 a household begins with some nominal wealth that can be allocated to money  
 152  $M_t$  or to risk-free government bonds  $B_t$ . During the first of the  $n$  subperiods,  
 153 one member of the household uses money to buy consumption goods. During  
 154 this same initial subperiod, another member of the household produces and  
 155 sells goods in exchange for money. At the end of the subperiod, producers  
 156 transfer to the bank the proceeds from their transactions. Thus, the situation  
 157 at the beginning of the second subperiod exactly replicates the situation at  
 158 the beginning of the first. This process is repeated  $n$  times during the period.  
 159 The choice of this variable  $n$  will be the only economically relevant decision  
 160 made by households. Purchases over a period are then subject to a cash-in-  
 161 advance constraint,  $P_t x_t \leq M_t n_t$ .

162 Notice that  $n$  is the velocity of money, and its inverse in equilibrium is the  
 163 money-to-output ratio, or the demand for real money, which is the concept  
 164 that we care about. Baumol-Tobin assumed that the cost of carrying out  
 165 these transactions increases linearly in the number of trips. We consider a  
 166 more general specification in which the total cost of making transactions,  
 167 measured in units of time, is given by

$$\theta(n_t, \nu_t) = \gamma n_t^\sigma \nu_t, \tag{2}$$

169 where  $\gamma$  and  $\sigma$  are positive constants and  $\nu_t$  is an exogenous stochastic pro-

cess. The natural interpretation of the stochastic shock  $\nu_t$  is aggregate disturbances in intermediation technologies. This random component is important for motivating the econometric analysis at the core of the paper. The expression in (2) becomes the Baumol-Tobin linear case when we set the curvature parameter  $\sigma$  equal to 1.

Equilibrium in the labor and goods markets implies

$$x_t = z_t l_t = z_t(1 - \gamma n_t^\sigma \nu_t),$$

so the equilibrium real wage is equal to  $z_t$ .

At the beginning of each period, the agent starts with wealth in real terms  $w_t$ , which can be allocated to money  $m_t$  or interest-bearing bonds  $b_t$ , both also measured in real terms. We can then write this constraint as

$$m_t + b_t \leq w_t. \tag{3}$$

In addition, we impose a productivity-adjusted borrowing constraint for the agent, in the sense that its bond holdings,  $b_t$ , cannot be too negative. Specifically, we impose

$$b_t \geq z_t b^* \tag{4}$$

for some arbitrary value of  $b^*$ .<sup>5</sup> Below we discuss how the equilibrium money

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<sup>5</sup>Our dataset contains long samples of over a century for a few countries. All of the countries had substantial increases in productivity over the length of the sample. We find

186 demand relationship is affected by this constraint.

187 The agent takes the nominal return on short-term bonds,  $r_t$ , as exoge-  
 188 nously given. We do not need to take a stand on how monetary policy is  
 189 executed. Our framework allows for  $r_t$  to be a process determined by mon-  
 190 etary policy, in which case the behavior of the growth rate of the money  
 191 supply is restricted by other equilibrium conditions. But it also allows for  
 192 policy to be described as money rules, in which case the nominal interest  
 193 rate will be given by a Fisher equation.<sup>6</sup>

194 So far, we have been silent with respect to what our measure of money  
 195 accounts for. For the theoretical analysis, one can allow for money to pay  
 196 interest.<sup>7</sup> In what follows, we consider the case in which the interest on money  
 197 is zero. We discuss this choice below in choosing the empirical counterpart  
 198 of the monetary aggregate.

199 The agent's wealth next period, contingent on the actions taken in the  
 200 current period and the realization of the exogenous shock, is given by

$$201 \quad w_t^{t+1} \leq \frac{m_t + b_t(1 + r_t) + [1 - \gamma n_t^\sigma \nu_t] z_t - x_t}{\pi_t^{t+1}} + \tau_t^{t+1}, \quad (5)$$

202 where  $\pi_t^{t+1}$  denotes the gross inflation rate between period  $t$  and period  $t + 1$   
 203 in that particular state, and  $\tau^{t+1}$  is the real value of the monetary transfer the

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it natural to assume that the borrowing constraint depends on the level of technology.

<sup>6</sup>Details are provided in Online Appendix B.

<sup>7</sup>That more general case and its details are discussed in the working paper version of this paper (Benati et al. (2017)).

204 government makes to the representative agent. Finally, the cash-in-advance  
 205 constraint can be written in real terms as

$$206 \quad x_t \leq m_t n_t. \quad (6)$$

207 We now consider the decision problem of a single atomistic agent who takes  
 208 as given the price level, the inflation rates  $\pi_t^{t+1}$ , the interest rate  $r_t$ , the real  
 209 wage  $z_t$ , and the shock  $\nu_t$ . Given the initial wealth  $w_t$ , the agent chooses  
 210 consumption  $x_t$ , the number of trips  $n_t$ , and the assets  $m_t$ ,  $b$ . These choices,  
 211 restricted by equations (3), (4), (5), and (6), determine the wealth  $w_t^{t+1}$   
 212 carried into the next period.

213 In Online Appendix B, we show that as long as the borrowing constraint  
 214 (4) does not bind, the equilibrium number of portfolio adjustments  $n$  depends  
 215 on the interest rate  $r_t$  according to

$$216 \quad r_t = \sigma \gamma n_t^{\sigma+1} \frac{\nu_t}{1 - \gamma n_t^\sigma \nu_t}. \quad (7)$$

217 The solution involves an extended quadratic formula for the equilibrium value  
 218 of  $n$ . Using the cash-in-advance constraint (6), the following relationship  
 219 must hold in equilibrium:

$$220 \quad \frac{m_t}{x_t} = \frac{1}{n_t(r_t, \nu_t)}, \quad (8)$$

221 which is a particular case of (1).

222 This solution has several empirical implications. First, notice that the

223 solution for the money-to-output ratio does not depend on the technology  
224 level,  $z_t$ . In the model, income growth must be associated with a positive  
225 trend in technology. This implies an income elasticity of the real demand for  
226 money that is equal to one, which is the specification we will study.<sup>8</sup> Second,  
227 the stochastic properties of the money-to-output ratio,  $m_t/x_t$ , are inherited  
228 from the stochastic properties of  $r_t$  and  $\nu_t$ . This has testable implications as  
229 long as  $\nu_t$  is stationary, as we will assume throughout the paper. Specifically,  
230 if  $r_t$  is stationary,  $m_t/x_t$  should be too, whereas if  $r_t$  has a unit root,  $m_t/x_t$   
231 should have a unit root as well. Our cointegration analysis below will address  
232 these issues.

## 233 *2.1. Characterization of the solution*

234 We first characterize the solution for the case in which the borrowing  
235 constraint (4) is not binding. Then, we provide a general characterization.

### 236 *2.1.1. When the borrowing constraint does not bind*

237 A simple inspection of equation (7) reveals that velocity is an increasing  
238 and continuous function of the interest rate  $r_t$ . As (8) makes clear, the ratio  
239 of money to income is then negatively related to  $r_t$ .

240 To obtain a simple parametric form that we can take to the data, we  
241 discuss one approximation. Note that  $\gamma\nu_t n_t^\sigma$  represents the welfare cost of

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<sup>8</sup>In Online Appendix B, we allow for a more general specification that does not restrict the income elasticity to be one, and where we are able to test this unitary income elasticity implication.

inflation as a ratio of maximum potential output, which is arbitrarily close to zero when the interest rate  $r_t$  is small. For moderate interest rates, computations of the welfare cost are negligible. Even for interest rates as high as 20%, estimates of the welfare cost of inflation are barely above 4%, so the denominator in the expression above would range from 1 to 0.96.<sup>9</sup> We then use the approximation  $1 - \gamma\nu_t n_t^\sigma \simeq 1$  and write the solution as  $n_t^{\sigma+1} \sigma \gamma \nu_t \simeq r_t$ . Taking logs and using the fact that  $m/x = 1/n$ , we then obtain

$$\ln(m_t/x_t) = \frac{1}{(\sigma+1)} [\ln \sigma \gamma - \ln r_t + \ln \nu_t], \quad (9)$$

which is the log-log function typically used in the literature, with an interest rate elasticity of  $1/(1+\sigma)$ . The Baumol-Tobin case is the one obtained when  $\sigma = 1$ , so the elasticity is equal to  $1/2$ .

Notice that a property of this specification is that real money balances over output,  $m_t/x_t$ , go to infinity when the nominal interest rate differential,  $r_t$ , goes to zero. How can this be a solution for a representative agent with finite wealth? Inspecting the budget constraint (3) suggests that bond holdings must therefore be approaching negative infinite. In this closed economy model, this means that agents are borrowing unbounded amounts from the government. Imagine, then, that agents could run away with the cash they hold and keep some fraction of it. The borrowing constraints (4) would

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<sup>9</sup>The approximation error in a model calibrated to match US data is very small, below 2%, even for interest rates as high as 50% per year.



261 naturally arise from an optimal contracting problem with enforceability con-  
 262 straints.

263 We next turn to considering the case of a binding borrowing constraint.

### 264 2.1.2. When the borrowing constraint binds

265 When the borrowing constraint binds,  $b = z_t b^*$ , the solution is trivial,  
 266 since there is really no economic problem to be decided by the agent. Note  
 267 that the budget constraint (3) implies

$$m_t = w_t - z_t b^*,$$

268 which fully determines the real quantity of money. The values for  $x_t$  and  
 269  $n_t$  are then determined by the equilibrium conditions  $x_t = z_t(1 - \theta(n_t))$  and  
 270  $x_t = n_t m_t$ .

### 271 2.1.3. A full characterization

272 To provide a full characterization of the relationship between money bal-  
 273 ances to output and the interest rate differential, given any value for the state  
 274 variable  $w$ , it is useful to state the following lemma, proved in the Online  
 275 Appendix.

276 **Lemma 1.** *Given values for  $w$  and  $b^*$ , if constraint (4) binds for an arbitrary*  
 277 *interest rate  $r_0$ , then it also binds for any  $r < r_0$ . In addition, if constraint*  
 278 *(4) is not binding for an arbitrary interest rate  $r_1$ , then neither does it bind*  
 279 *for any  $r > r_1$ .*

280 The previous lemma implies that for any value of real initial wealth,  $w$ ,  
 281 there exists a value for the net interest rate  $\tilde{r}$  such that the money demand

equation is given by the solution to (7), for all  $r \geq \tilde{r}$ , whereas it is equal to  $\frac{m}{x}(\tilde{r})$  for all  $r \leq \tilde{r}$ . Such a money demand is depicted as the dotted blue line, labeled A in Figure 1.

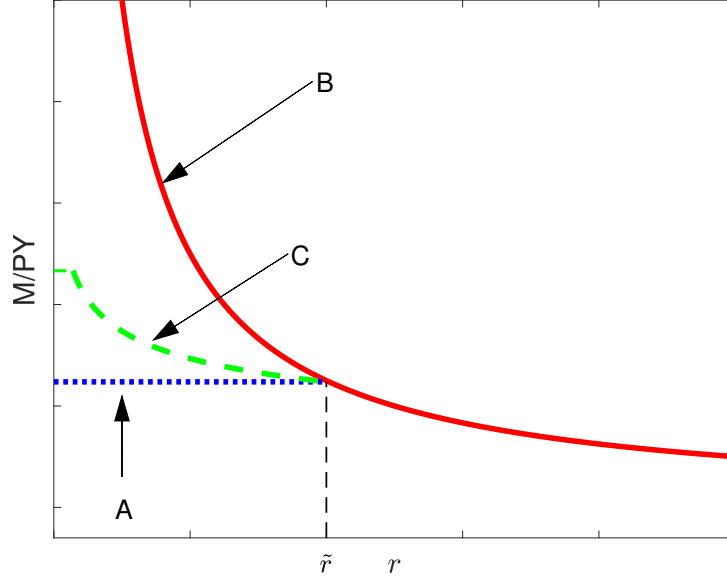


Figure 1 Borrowing constraint and money demand when interest rate is near 0

The location of the kink in the money demand depends on the value for  $w - zb^*$ . The larger that value, the lower will be the value for the interest rate  $\tilde{r}$ . Note also that if we let  $b^*$  go to negative infinity, real money balances also go to infinity as the interest rate goes to zero. Such a money demand is depicted as the solid red line, labeled B in Figure 1.

This sharp characterization at near-zero interest rates depends on the representative agent assumption, and it is not robust to sensible generalizations. Thus, we may not want to take the flat portion of the money demand in schedule A of Figure 1 literally when pursuing our empirical analysis. We

294 discuss this issue next.

295 *2.1.4. Connecting the theory to the data*

296 Consider a model like the one above, with a continuum of agents that are  
297 alike in all respects except that they differ in their productivity. To be more  
298 specific, assume that idiosyncratic productivity for agent  $j$  is equal to  $\xi^j z_t$ ,  
299 where  $\xi^j \in [\xi_l, \xi^h]$ , and where the mean of  $\xi^j$  is equal to one. It would be  
300 natural in this environment to impose agent-specific borrowing constraints,  
301 since agents' ability to pay would vary across types.

302 We explore such an economy in Online Appendix B. There we show that  
303 under certain conditions, there will be a threshold interest rate  $\hat{r}$  such that  
304 for interest rates higher than  $\hat{r}$ , no agent is constrained, so all individual  
305 money demand functions are well approximated by the log-log specification.  
306 It follows that the aggregate money demand function is also log-log. For  
307 interest rates lower than  $\hat{r}$ , the aggregate money demand is a combination  
308 of two types of agents. For the first type, the constraint binds, so their  
309 aggregate demand is insensitive with respect to the interest rate. For the  
310 second type, the constraint does not bind, and their elasticity is given by  
311 the log-log specification. As the interest rate keeps going down, the fraction  
312 of agents for which the elasticity is positive goes down, so the aggregate  
313 elasticity also goes down. The aggregate money demand is decreasing for  
314 this range, but with an interest rate elasticity that is lower than the log-log  
315 specification. Such a money demand is the dashed green line, labeled C in

316 Figure 1.<sup>10</sup>

317 In light of this discussion, in our empirical strategy, we will follow two  
318 complementary approaches. First, we will ignore the borrowing constraints  
319 and use the log-log functional form implied by the theory. We expect this  
320 strategy to work well in countries that did not experience low interest rates.  
321 Second, we will use a parametric form that is observationally very similar to  
322 the log-log specification for interest rates that are not too small, and which  
323 differs from that specification at very low levels of interest rates in a way that  
324 closely resembles the behavior of the real money demand with the borrowing  
325 constraints described above. This parametric form, used by Selden (1956)  
326 and Latané (1960), is given by

$$327 \quad \frac{m_t}{x_t} = \frac{1}{a + br_t}. \quad (10)$$

328 In Figure 2, we plot two curves relating real money balances to the interest  
329 rate. The range of short-term interest rates is the relevant one for the United  
330 States in the last century: between 0% and 15%. The blue circles correspond  
331 to a log-log specification, with an elasticity equal to 1/2, as implied by the  
332 Baumol-Tobin linear technology discussed above. The constant in that equa-  
333 tion has been chosen so that the ratio of money to output is close to 25% when  
334 the interest rate is 6%, which matches the US data reasonably well. The solid

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<sup>10</sup>Similar results for the aggregate money demand arise in a model in which agents get the few first portfolio transactions for free. Such a model is developed and estimated in Alvarez and Lippi (2009).

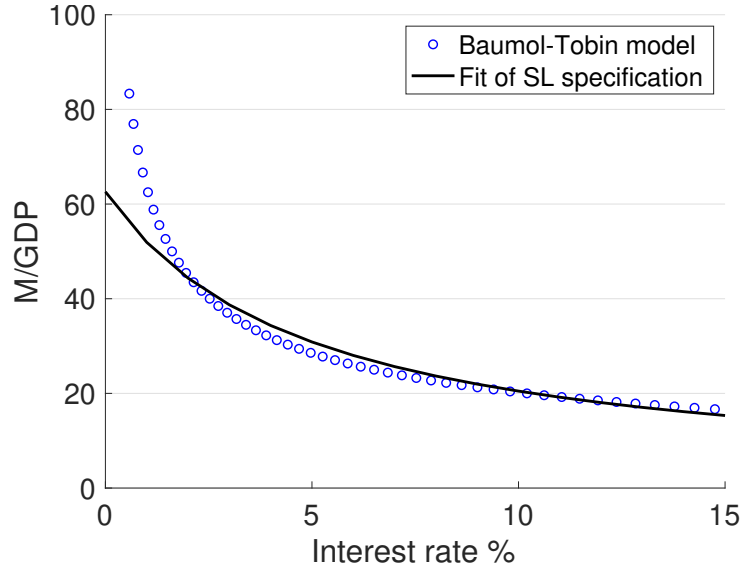


Figure 2 Fitting Baumol-Tobin model with Selden-Latané specification

black line corresponds to the best fit of equation (10) to the blue circles in the figure.<sup>11</sup>

As the figure makes clear, both functional forms behave in a remarkably similar way for interest rates between 15% and 2%. They are so similar, in fact, that it appears that the ability to identify one functional form from the other would require a gigantic amount of data for interest rates within that range. On the other hand, the two functional forms do behave very differently at interest rates between 2% and 0%. To the extent that borrowing constraints are relevant, this formulation ought to work better in countries that did experience low interest rates in several periods. By comparing the empirical performance of the two specifications, we will be able to draw some

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<sup>11</sup>The two parameters of the solid black line were calibrated using OLS.

346 conclusions regarding the quantitative relevance of the borrowing constraints.

347 *2.1.5. The role of regulation and technology*

348 The literature has long recognized that changes in regulation or technol-  
349 ogy can change the equilibrium relationship between interest rates and real  
350 money balances. For instance, Lucas and Nicolini (2015) argue that regula-  
351 tory changes introduced in the United States in the early 1980s can explain  
352 the apparent instability of real money demand in the United States. Alvarez  
353 and Lippi (2009) show that advances in banking technology are important  
354 in explaining their household level data on cash holdings. The theoretical  
355 implications of such changes can be analyzed with this model. In the working  
356 paper version of this paper (Benati et al. (2017)), we show that when money  
357 pays an interest rate  $r_t^m > 0$ , the solution for the number of trips to the bank,  
358  $n_t$ , is an equation equal to (7), except that on the right-hand side,  $r_t$  must be  
359 replaced by  $r_t - r_t^m$ . Thus, if banks are allowed to compete by paying interest  
360 on deposits, the optimal choices of  $n_t$  would change even if the interest rate  
361  $r_t$  is unchanged. In addition, a change in the level parameter  $\gamma$  changes the  
362 optimal value for money balances, again keeping  $r_t$  constant.

363 As mentioned in the Introduction, we will ignore both regulatory and  
364 technological changes in this paper. As it turns out, the data analysis shows  
365 that by and large, these theoretical considerations have little empirical rele-  
366 vance: the relationship in (8) derived from the model is confirmed by the data  
367 for a large set of countries, even though we cover samples that are several

368 decades long. As we document below, we found just a handful of cases for  
 369 which M1 real money demand appears to have breaks that are suggestive of  
 370 further analysis. Our general conclusion is that the apparent breakdown in  
 371 the real money demand relationship observed in the United States, requiring  
 372 a detailed and country-specific analysis, is an exception rather than the rule.

### 373 **3. A First Look at the Data**

374 In this section, we present the data and provide a visual comparison with  
 375 the theory. To begin, we discuss how we map our theoretical construct  $M_t$  to  
 376 the data. This choice is associated with the discussion of its nominal return.  
 377 We have no data on the interest rate paid by deposits, so we choose to work  
 378 with M1, which includes cash and only checking accounts. In deciding to set  
 379 the interest rate on money to zero in the theory, we implicitly assumed that  
 380 checking accounts pay no interest. This is a questionable assumption, but  
 381 it is certainly more appropriate for M1 than for broader aggregates, which  
 382 typically include interest-paying deposits.<sup>12</sup> Accordingly, we identify money  
 383 in the model with M1.<sup>13</sup>

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<sup>12</sup>Deposits did pay interest in the United States after Regulation Q was modified in the early 1980s. Also, some deposits included in M1 did pay interest in very high-inflation countries such as Argentina and Brazil.

<sup>13</sup>We have data on interest rates paid on deposits for the United States. As a robustness check, we use it to compute the opportunity cost of New M1 as the difference between the 3-month Treasury bill rate and money's own rate(details on the computation can be found in Online Appendix A.1.31). Based on Johansen's tests, evidence is qualitatively the same as that based on the Treasury bill rate, with p-values equal to 0.066 for Selden-Latane', and 0.394 based on log-log. Based on Wright's tests, results are the opposite, with the null of cointegration not being rejected based on the log-log, and being rejected for the

Online Appendix A describes the data and the data sources in detail. All of the series are standard, with the single exception of the United States, where we also consider the NewM1 monetary aggregate proposed in Lucas and Nicolini (2015). Specifically, we add to the standard M1 aggregate the money market deposit accounts (MMDAs) that were created in 1982. We call this aggregate New M1.<sup>14</sup> Our simple theory abstracts from investment, so output and consumption are the same. It also abstracts from money demand by firms, which raises the question of whether total output is a better measure than consumption. We chose to use output as our measure of economic activity, as it is customary in the literature.<sup>15</sup>

We first present the raw data in the form of cross plots between the short-term interest rate and the ratio of money to nominal income. The data were not manipulated in any way. Figures 3a to 3c are scatterplots of the short rate and the ratio between nominal M1 and nominal GDP (i.e., the inverse of M1 velocity). We also plot the theoretical curve that corresponds to equation (9), specialized to the case in which the elasticity is equal to 1/2. We allow the per-unit cost,  $\gamma^j$ , to be different across countries. Thus, for each country

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Selden-Latane' specification.

<sup>14</sup>The results are the same with an alternative aggregate in which currency has been adjusted along the lines of Judson (2017) to take into account the sizable expansion in the fraction of US currency held by foreigners since the early 1990s. See Benati (2019a) for details.

<sup>15</sup>There is ample evidence that output and consumption are cointegrated, so this choice is likely to be of little relevance. We have checked this for all countries for which we could find consumption data, and results are qualitatively the same. These results are reported in Online Appendix K



401  $j$ , we plot the curve

$$402 \quad \frac{M_t^j}{Y_t^j} = \frac{\gamma^j}{\sqrt{r_t^j}}, \quad (11)$$

403 where  $Y_t^j$  is nominal income at time  $t$  in country  $j$  and  $M_t^j$  is M1, except for  
404 the United States, where we use New M1, as mentioned above. We calibrate  
405 the single free parameter for each country to be the one that minimizes  
406 the mean squared errors between the curve and the data, but imposing the  
407 elasticity to be  $1/2$ .

408 The criteria used to group the countries follow the results of the tests  
409 performed below. Details will be discussed then, but a rough approximation  
410 is that we start with the countries for which the evidence of comovement  
411 between velocity and the interest rate is very strong and show at the end  
412 the countries for which the evidence is weak or nonexistent. In our view,  
413 it is surprising how well this simple theory, which allows for a single free  
414 parameter per country, performs in this first inspection. Our own summary is  
415 the following. For the 12 countries in Figure 3a, the evidence is very strong,  
416 with two important caveats: for both Brazil and Argentina, a few points  
417 clearly lie below the theoretical line. In both cases, those points correspond  
418 to the years following the successful stabilization of hyperinflations.<sup>16</sup> The  
419 model in the previous section is too simple to be used to address these cases,

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<sup>16</sup>Neither Bolivia nor Israel, which also experienced hyperinflations, exhibits the same puzzling behavior. But for those cases, we do not have data for many years before the hyperinflation, so they are not completely comparable.

420 so we leave that topic for future research. The following 14 countries in Figure  
 421 3b also exhibit clear evidence of a negative relationship, though in some cases  
 422 (such as Bahrain, Barbados, and Thailand), the slope seems to be different  
 423 from the  $1/2$  implied by the linear technology. The 6 countries in the top  
 424 row of Figure 3c also provide good evidence, though in some cases (such as  
 425 Finland and West Germany), the picture is not as clear as before. Finally,  
 426 the 6 countries in the bottom row of Figure 3c are the most problematic.  
 427 Portugal and to a lesser extent Paraguay and Norway, also seem to display  
 428 evidence of a negative relationship, but it is far from the theoretical curve.  
 429 The last three cases are, from the viewpoint of the cross plots, blunt failures.

430 The plots in Figures 3a through 3c depict the data in a way that became  
 431 traditional in empirical studies of money demand. In addition, the data can  
 432 be visually compared with the theoretical curve indicated by the theory (11).  
 433 But depicted in this way, the plots conceal the behavior of the variables over  
 434 time, and they thus fail to show the persistence exhibited by both series,  
 435 particularly how the persistent components of the two series have moved  
 436 together over time. We find that information also very valuable as a visual  
 437 motivation for the cointegration methods that we use in the rest of the paper.  
 438 The figures also add the time dimension, which helps to explain some of the  
 439 apparent failures discussed above.

440 Thus, in Figures 4a to 4c, we present the time series for both the short

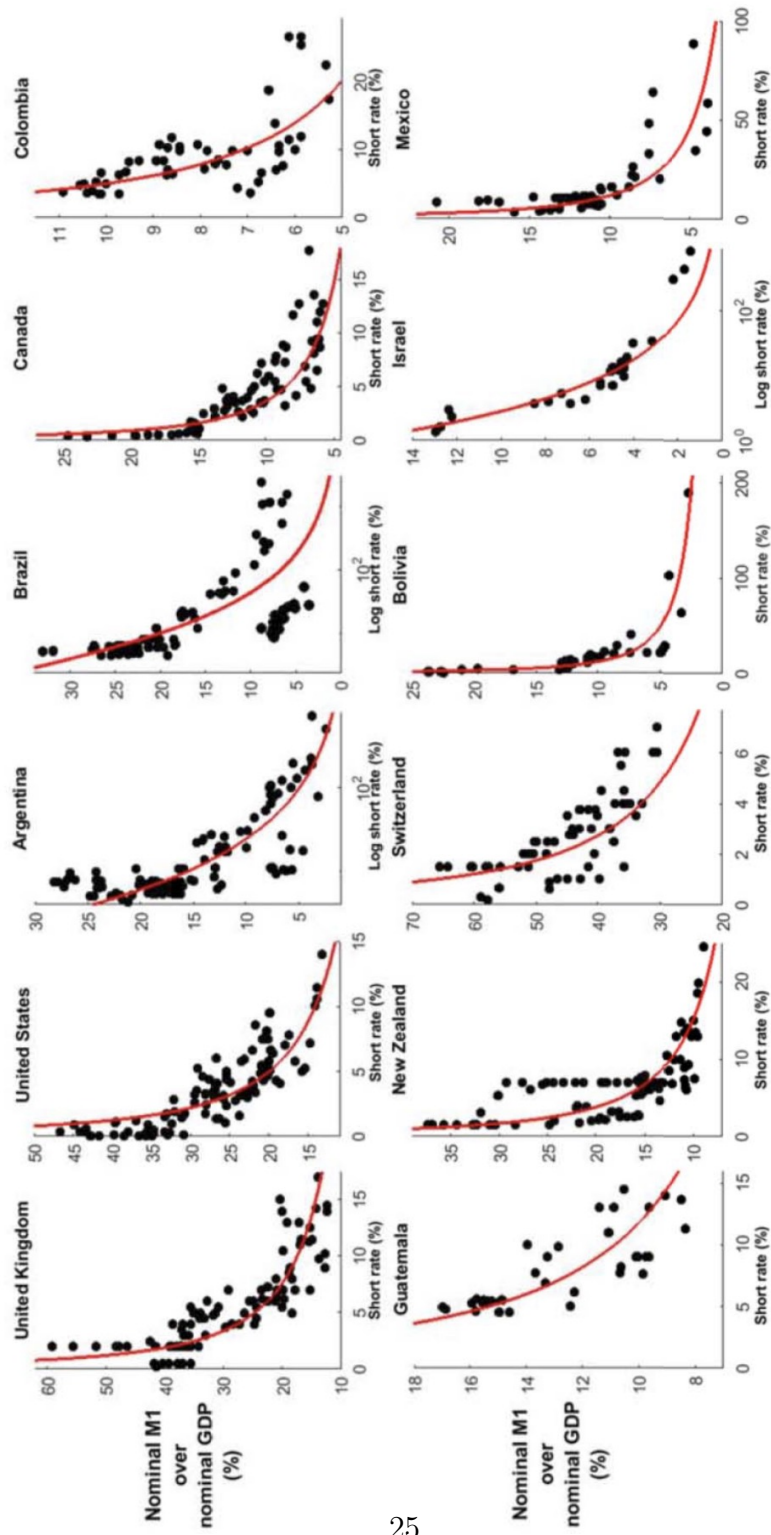


Figure 3a The raw data: scatterplots of the short rate and M1 as a fraction of nominal GDP

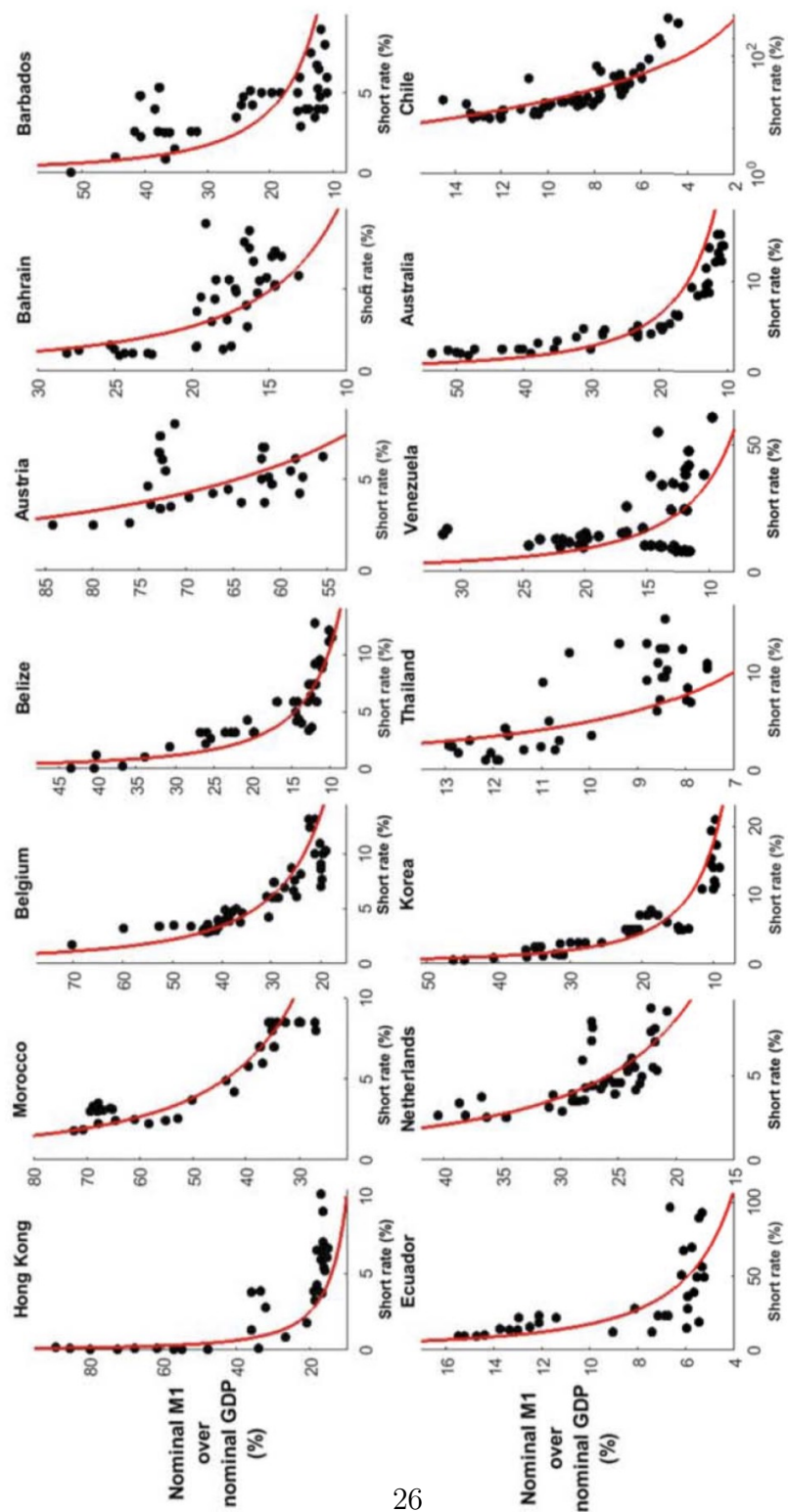


Figure 3b The raw data: scatterplots of the short rate and M1 as a fraction of nominal GDP

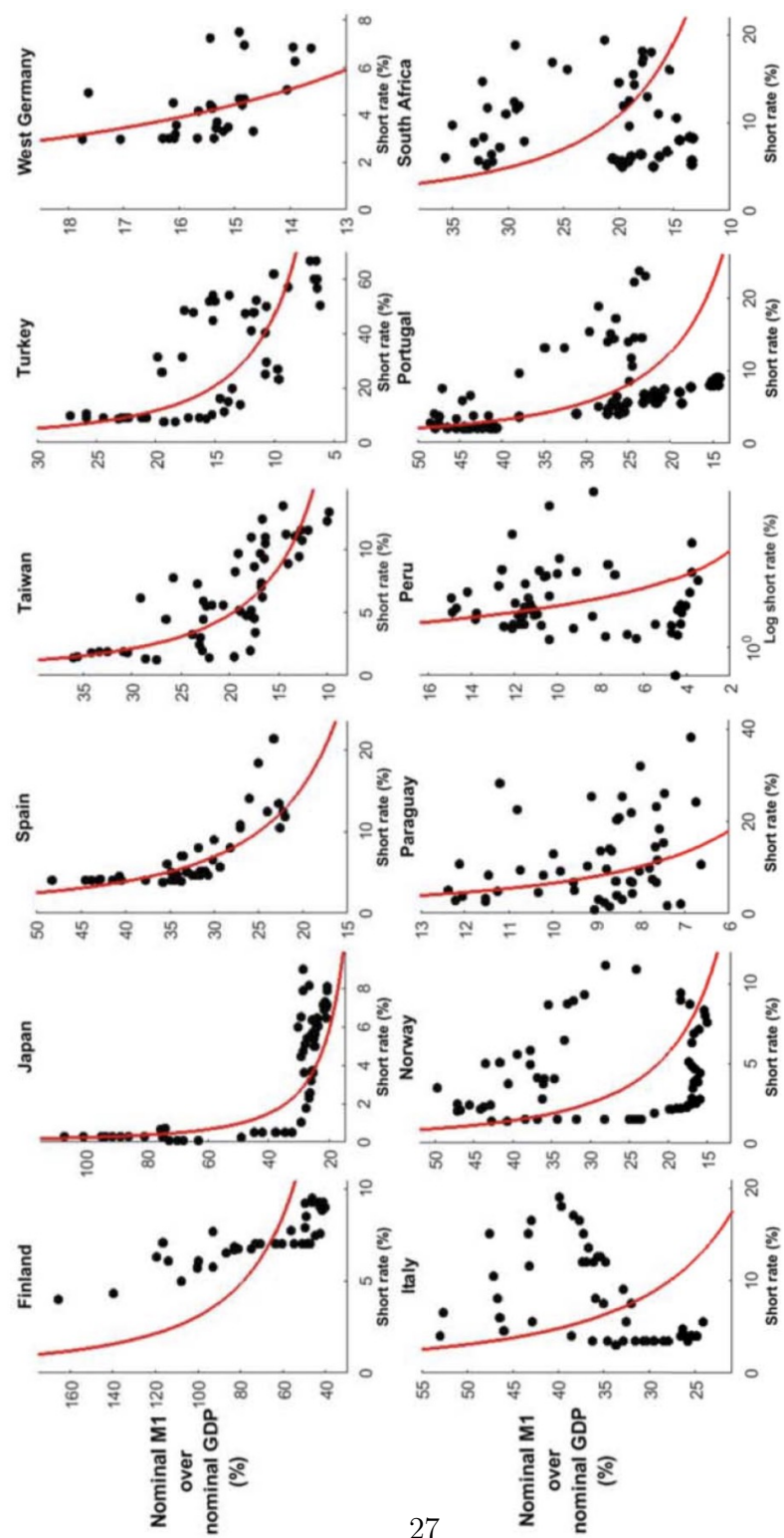


Figure 3c The raw data: scatterplots of the short rate and M1 as a fraction of nominal GDP

term interest rate and velocity.<sup>17</sup> In this case, we did some –minimal–  
manipulation by plotting the two variables along different axes. This manip-  
ulation amounts to making a linear transformation of one of the variables,  
which is consistent with the theoretical constructs in (9) and (10). We also  
go beyond the previous comparison with the theory, where we used the log-  
log specification for all countries. The theory suggests that the formulation  
(10) is more likely to be a better description of the data when the borrowing  
constraints are quantitatively relevant. Thus, we classify countries into two  
groups. For the first group of countries, we plot the log of the interest rate  
and the log of velocity, as specified in equation (9). For the second group  
of countries, we plot only the levels of the same variables, as specified in  
equation (10). This second group of countries comprises those that never  
had their interest rates too high and even had several years of interest rates  
close to zero, as the theory suggests.<sup>18</sup>

For the 26 countries in Figures 4a and 4b, the comovement between veloc-  
ity and the interest rate is very strong. The same caveats regarding Argentina  
and Brazil apply, and notice that in both Israel and Bolivia, our data mostly  
cover the years following the stabilization. Three more puzzles appear clearly  
in the time series: Venezuela, and to a lesser extent Ecuador, seem to exhibit

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<sup>17</sup>We find it convenient to show velocity – the inverse of real money demand according to the model – since it ought to be positively related to the interest rate.

<sup>18</sup>We use the following criterion: we present the data in levels for all countries with some observations below 5% and no observations beyond 100%, and we present the data in logs for all other countries.

different behavior during the first half of the sample relative to the second. In addition, the data for Chile show, as do the data for Argentina and Brazil mentioned above, that the behavior right after the stabilization of a very high inflation does not conform to the theory. The countries in the top row of Figure 4c also exhibit solid evidence of comovement between velocity and the interest rate, though it is less clear-cut than the previous cases. Finally, among the group of countries in Figure 4c, 4 of them suggest blunt failure, whereas both Norway and Portugal seem to conform to the theory in the last few decades, but not before.

To summarize, the first data inspection suggests the following: 26 countries exhibit remarkable evidence of comovement between velocity and the interest rate, 6 countries offer good evidence, and for the final group of 6 countries, the evidence is either weak or nonexistent. In addition, an inspection of the countries that experienced extremely high inflation suggests that in some cases, it takes several years after nominal interest rates have returned to normal for real money demand to recover to its previous levels. Finally, we identify only three candidates (Venezuela, Norway and Portugal) that seem to exhibit a breakdown in the money demand relationship that could justify further analysis of whether regulation may have played a role—an analysis along the same lines discussed in Lucas and Nicolini (2015) for the United States.

Despite the attractiveness of looking at simple plots, however, the previous analysis has several limitations. We would like to formally test whether,

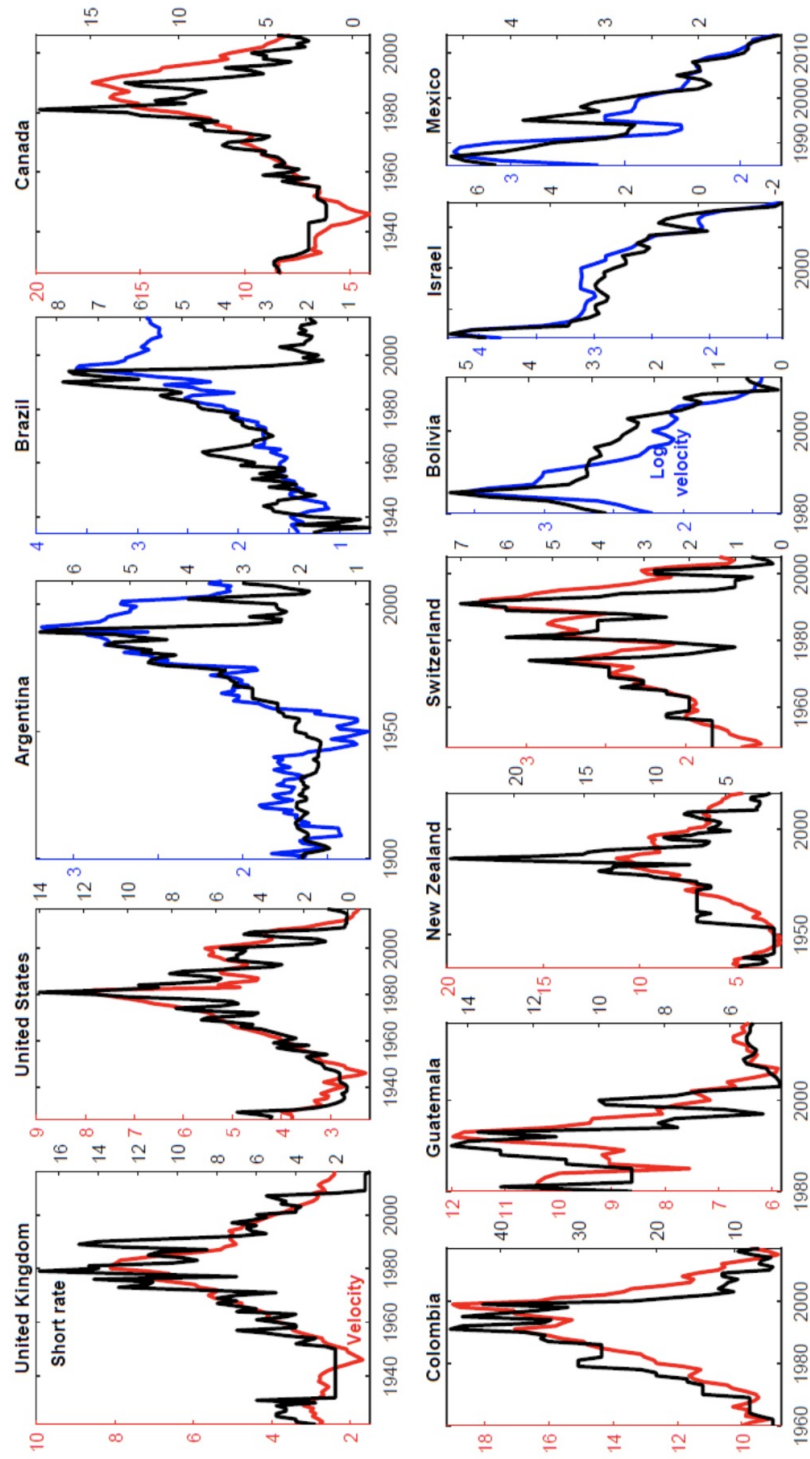


Figure 4a The raw data: M1 velocity and the short rate



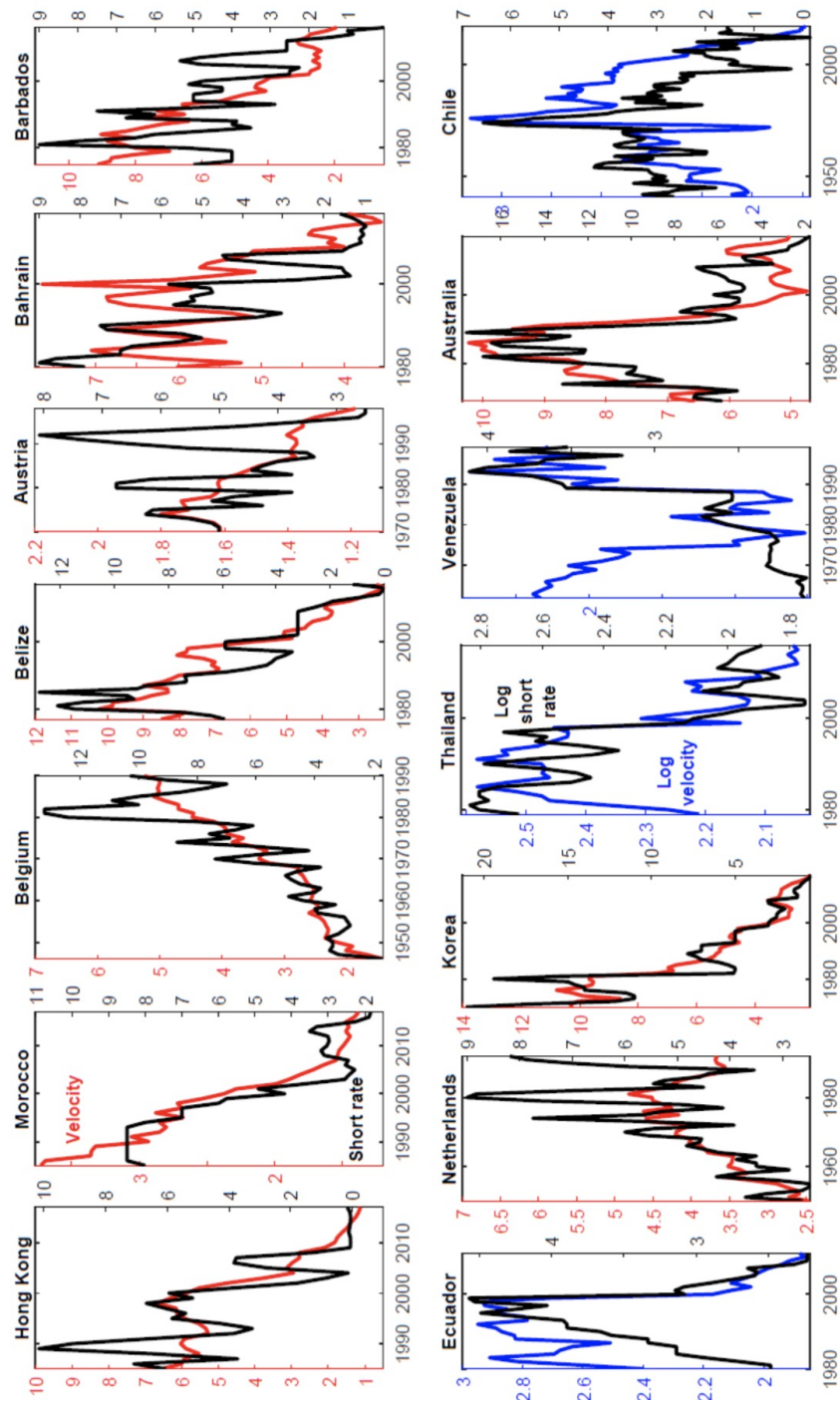


Figure 4b The raw data: M1 velocity and the short rate

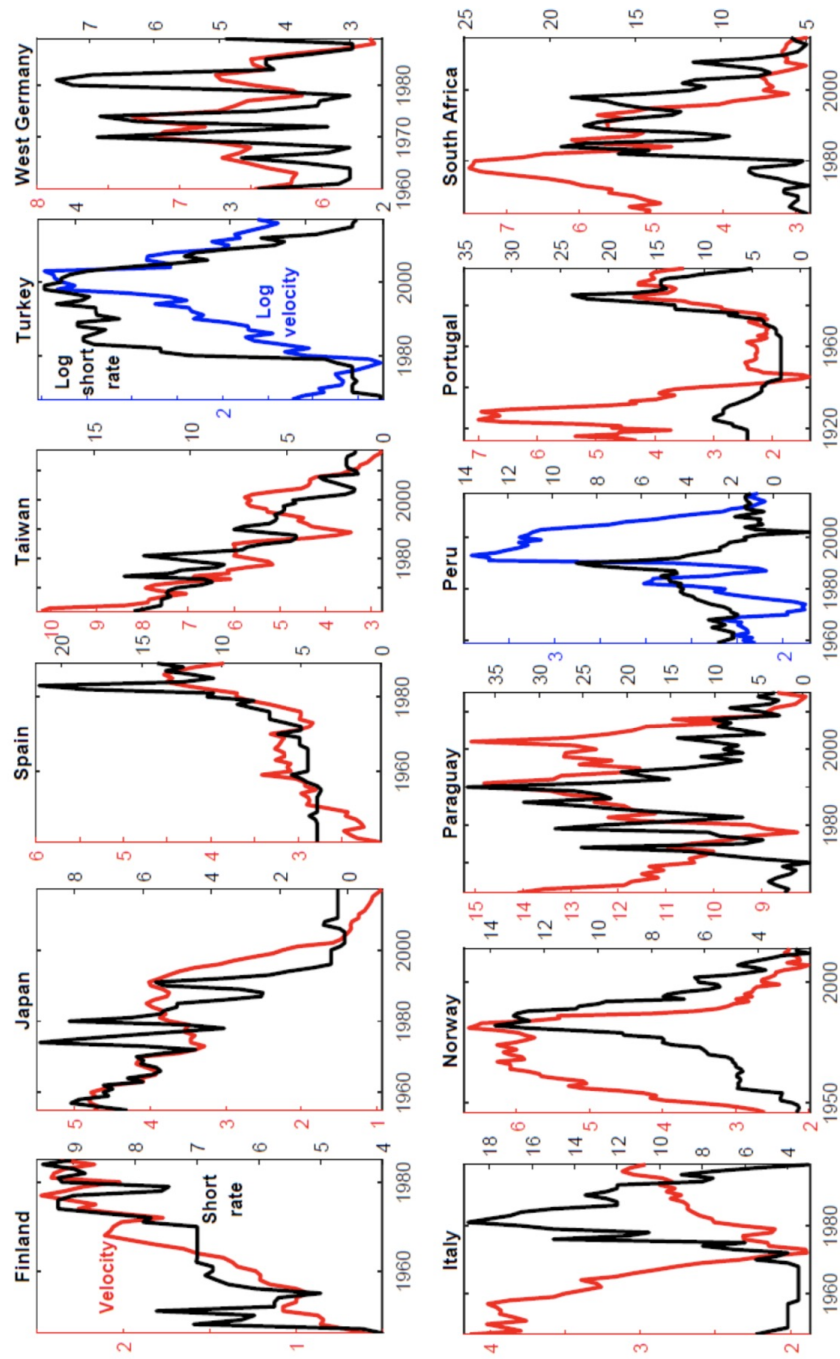


Figure 4c The raw data: M1 velocity and the short rate

483 as the simple model above implies, the ratio between real money balances and  
484 output inherits a unit root when the short-term interest rate exhibits a unit  
485 root. We also want to formally estimate the interest rate elasticities and see  
486 how good  $1/2$  is as an approximation, the value implied by the linear technol-  
487 ogy in the original Baumol-Tobin specification. We would also like to let the  
488 data indicate the quantitative effect of the borrowing constraints when inter-  
489 est rates are very low, as it has been in countries such as Japan, the United  
490 States, and the United Kingdom. If this were the case, the Selden-Latané  
491 specification ought to deliver better results.

492 The plots in Figures 4a through 4c show how persistent the series are and  
493 provide support for the use of the cointegration methods that we use below.  
494 In fact, the statistical tests overwhelmingly identify unit roots both in the  
495 ratio of money to output and in the interest rates (see Online Appendix C). In  
496 spite of the results of the unit root tests, one may have theoretical reasons to  
497 believe that the interest rate, being a policy variable, ought to be stationary.  
498 One such reason is that in most monetary models used to justify inflation  
499 targeting policies, the policies that stabilize the economy around the inflation  
500 target deliver a stationary series for the equilibrium interest rate, even if the  
501 economy is subject to unit root shocks, as long as the real interest rate is  
502 stationary. And these models approximate very well the behavior of inflation  
503 rates in the data in countries that have successfully managed to keep inflation  
504 very close to its target. Clearly, temporary but persistent deviations from  
505 that policy, as one may interpret the US experience from 1965 to 1985, may

506 imply very persistent movements in the interest rate. And it is indeed quite  
507 difficult to distinguish that behavior from a unit root, statistically speaking,  
508 given the our sample size.

509 The good news is that those speculations are of little quantitative rele-  
510 vance: a crucial feature of one of the methods we use in Section 6.1, owing  
511 to Wright (2000), is that they perform very well for highly persistent series,  
512 even if they are not exactly unit roots. We illustrate this property in the  
513 specific context of the model described above. There we run Monte Carlo  
514 experiments with model-generated data. We simulate stationary but very  
515 persistent series and show that these methods identify the true parameters  
516 very well. It is because we are handling very persistent series that we are  
517 fully comfortable embracing the cointegration techniques that follow.

518 We now turn to a brief discussion of the main features of our approach  
519 and several methodological issues.

## 520 **4. Main Features of Our Approach**

521 The cointegration techniques we use were justified above on statistical  
522 grounds: the series we work with are highly persistent, to the point where,  
523 in nearly all cases, it is not possible to reject the null of a unit root. The  
524 notion of cointegration boils down to the existence of a long-run relationship  
525 between series driven by permanent shocks: those shocks are the source of  
526 identification of the relationship between the short rate and velocity. The  
527 existence of the cointegration relationship implies that, in the long run, any

528 permanent increase in the interest rate maps into a corresponding permanent  
529 increase in velocity and therefore a decrease in real money balances; the exact  
530 amount will be captured by the cointegration vector. Further, any deviation  
531 of the two series from their long-run relationship—that is, the cointegration  
532 residual—is transitory and bound to disappear in the long run. The persis-  
533 tence of the residual is therefore a measure of how long-lived these short-run  
534 deviations are. As we will show, these estimated residuals are indeed very  
535 persistent themselves. Our analysis therefore leaves unexplained a substan-  
536 tial fraction of the dynamic interactions between the short-term interest rate  
537 and the money-to-output ratio. As mentioned in the introduction, models  
538 with segmented markets, such as the one in Alvarez and Lippi (2014) have  
539 been developed to account for these persistent short-run deviations. The  
540 statistical properties of the cointegration errors we obtain can be used to  
541 discipline those models. Developing models that can successfully integrate  
542 long-run with short-run behavior are left for future research.

543 We present the analysis in two steps. In the first step, discussed in this  
544 section, we take the results of the unit root test literally—that is, we interpret  
545 the lack of rejection of the null hypothesis as evidence that the series contain  
546 *exact* unit roots—and present evidence from Johansen’s cointegration tests,  
547 which take no cointegration as the null hypothesis. Then, in a second step,  
548 we present the results from Wright’s (2000) tests, which take cointegration

549 as the null hypothesis.<sup>19</sup> There are (at least) two reasons for also considering  
 550 the Wright test, in addition to the Johansen test. First, although the over-  
 551 whelming majority of the papers in the money demand literature have been  
 552 based on Johansen’s procedure, there is no reason to regard no cointegration  
 553 as the “natural null hypothesis.” Rather, it might be argued that, since we  
 554 are searching here for a long-run money demand for *transaction* purposes,  
 555 cointegration should be the natural null.<sup>20</sup> Second, as discussed by Wright  
 556 (2000), Wright’s test works equally well both when the data contain exact  
 557 unit roots, and when they are local-to-unity. On the other hand, as shown  
 558 by Elliot (1998), when the data are local-to-unity, tests (such as Johansen’s)  
 559 that are predicated on the assumption that the data contain exact unit roots  
 560 can perform poorly.

561 Once cointegration is detected, we can use standard methods to estimate  
 562 the parameter that governs the elasticity of the money demand relationship.  
 563 For reasons of robustness, we consider Johansen’s just-mentioned procedure,  
 564 as well as Stock and Watson’s (1993). We also compare the results of using  
 565 the log-log specification and the Selden-Latané one, and use those results to

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<sup>19</sup>The Wright (2000) test searches across the parameter space for all the values of  $\beta$  in the normalized cointegration vector  $[1 - \beta]'$  such that the null hypothesis that  $[1 - \beta]'.y_t$  is  $I(0)$  cannot be rejected, where  $y_t$  features the levels of velocity and the short rate for the Selden-Latané specification, and their logarithms for the log-log. A  $(1 - \alpha)\%$ -coverage confidence interval for  $\beta$  is computed as the set of all values of  $\beta$  for which the null hypothesis that  $[1 - \beta]'.y_t$  is  $I(0)$  cannot be rejected at the  $\alpha\%$  level.

<sup>20</sup>Basic economics logic suggests that, up to fluctuations in the opportunity cost of money, the nominal quantity of money demanded should be proportional to the nominal volume of transactions (proxied by nominal GDP).

566 discuss the behavior of real money demand at very low interest rates.

567 *4.1. Integration properties of the data*

568 Online Appendix C reports evidence from our extensive investigation of  
569 the integration properties of the data based on the unit root tests in Elliot,  
570 Rothenberg and Stock (1996). Our main results can be summarized as follows.

571 First, there is overwhelming evidence of unit roots in the vast majority  
572 of the series, with the bootstrapped  $p$ -values being near-uniformly greater  
573 than the 10% threshold, which, throughout the entire paper, we take as our  
574 benchmark significance level, and in most cases markedly so. In the very few  
575 instances in which this is not the case, we eschew the relevant specifications  
576 (e.g., if we can reject the null of a unit root for the logarithm of the short rate  
577 but not for the level, we eschew the log-log specification, and we uniquely  
578 focus on the Selden-Latané specification).

579 Second, for both the first difference and the log-difference of either veloc-  
580 ity or the short rate, the null of a unit root can be rejected almost uniformly.  
581 In the few instances in which this is not the case—so that the relevant series  
582 should be regarded, according to the tests in Elliot, Rothenberg and Stock  
583 (1996), as  $I(2)$ —we do not run cointegration tests.<sup>21</sup> As for nominal M1  
584 and especially nominal GDP, on the other hand, the opposite is true, with  
585 the null of a unit root not being rejected most of the time. In all of these

---

<sup>21</sup>Both Johansen's and Wright's tests are predicated on the assumption that the series contain (near) unit roots, but that their order of integration is at most one.

586 cases, we will therefore eschew unrestricted specifications for the logarithms  
587 of nominal M1, nominal GDP, and a short rate.

#### 588 *4.2. Methodological issues pertaining to cointegration tests*

589 In this section, we briefly discuss three issues. First, we describe the  
590 bootstrapping procedures we use. Second, we describe a series of Monte  
591 Carlo experiments that help to interpret the test results. Finally, we discuss  
592 evidence related to the persistency of the cointegration residual.

##### 593 *4.2.1. Issues pertaining to bootstrapping*

594 All model statistics in this paper are bootstrapped. In this section, we  
595 briefly discuss details of the bootstrapping procedures we use and how such  
596 procedures perform. In our discussion, we extensively refer to online Ap-  
597 pendix D, which contains the Monte Carlo evidence motivating both our  
598 choices and the way in which we will interpret the evidence based on the  
599 actual data.

600 As for Johansen's tests, we bootstrap trace and maximum eigenvalue  
601 statistics via the procedure proposed by Cavaliere, Rahbek and Taylor (2012;  
602 henceforth, CRT). In a nutshell, for tests of the null of no cointegration  
603 against the alternative of one or more cointegrating vectors, the model that  
604 is being bootstrapped is a simple, noncointegrated VAR in differences (for  
605 the maximum eigenvalue tests of  $h$  versus  $h+1$  cointegrating vectors, on the  
606 other hand, the model that ought to be bootstrapped is the VECM estimated  
607 under the null of  $h$  cointegrating vectors). All of the technical details can be



608 found in CRT, to which the reader is referred. We select the VAR lag order  
609 as the maximum<sup>22</sup> between the lag orders chosen by the Schwartz and the  
610 Hannan-Quinn criteria<sup>23</sup> for the VAR in levels.

611 As for the Wright (2000) test, since it has been designed to be equally  
612 valid for data-generation processes (DGPs) featuring either exact or near unit  
613 roots, we consider two alternative bootstrapping procedures, corresponding  
614 to either of the two possible cases. (In practice, as a comparison between the  
615 results reported in Table 2 in the text and Table E.1 in online Appendix E  
616 makes clear, the two procedures produce near-identical results.) The former  
617 procedure involves bootstrapping—as detailed in CRT, and briefly recounted  
618 in the previous paragraph—the cointegrated VECM estimated (based on Jo-  
619 hansen’s procedure) under the null of one cointegration vector. This boot-  
620 strapping procedure is the correct one if the data feature exact unit roots.  
621 For the alternative possible case in which velocity and the short rate are  
622 near unit root processes, we proceed as follows. Based on the just-mentioned  
623 cointegrated VECM estimated under the null of one cointegration vector, we  
624 compute the implied VAR in levels, which, by construction, features one, and  
625 only one eigenvalue equal to 1. Bootstrapping this VAR would obviously be

---

<sup>22</sup>We consider the maximum between the lag orders chosen by the SIC and HQ criteria because the risk associated with selecting a lag order smaller than the true one (model misspecification) is more serious than the one resulting from choosing a lag order greater than the true one (overfitting).

<sup>23</sup>On the other hand, we do not consider the Akaike Information Criterion since, as discussed by Luetkepohl (1991), for example, for systems featuring  $I(1)$  series, the AIC is an inconsistent lag selection criterion, in the sense of not choosing the correct lag order asymptotically.

equivalent to bootstrapping the underlying cointegrated VECM, that is, it would be correct if the data featured exact unit roots. Since, on the other hand, here we want to bootstrap under the null of a near unit root DGP, we turn such an exact unit root VAR in levels into its corresponding near unit root, by shrinking down the single unitary eigenvalue to  $\lambda=1-0.5\cdot(1/T)$ , where  $T$  is the sample length.<sup>24</sup> The bootstrapping procedure we implement for the second possible case, in which the processes feature near unit roots, is based on bootstrapping such a near unit root VAR.

We now turn to discussing Monte Carlo evidence on the performance of the two bootstrapping procedures.

#### 4.2.2. Monte Carlo evidence

Tables D.1 and D.2 in online Appendix D report extensive Monte Carlo evidence on the performance of the bootstrapping procedures, which is discussed in detail in Sections D.1.1 and D.1.2 of online Appendix D. We perform the Monte Carlo experiments based on two types of DGPs, featuring no cointegration and cointegration, respectively. For either DGP, we consider several alternative sample lengths, from  $T = 50$  to  $T = 1,000$ . For the DGP

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<sup>24</sup>In particular, we do that via a small perturbation of the parameters of the VAR matrices  $B_j$ 's in the cointegrated VECM representation  $\Delta Y_t = A + B_1\Delta Y_{t-1} + \dots + B_p\Delta Y_{t-p} + GY_{t-1} + u_t$ , where  $Y_t$  collects (the logarithms of) M1 velocity and the short rate, and the rest of the notation is obvious. By only perturbing the elements of the VAR matrices  $B_j$ 's—leaving unchanged the elements of the matrix  $G$  (and therefore both the cointegration vector and the loading coefficients)—we make sure that both the long-run equilibrium relationship between velocity and the short rate, and the way in which disequilibria in such a relationship map into subsequent adjustments in the two series, remain unchanged.

643 featuring cointegration, we also consider several alternative values for the  
 644 persistence of the cointegration residual, which we model as an AR(1). Fi-  
 645 nally, for the experiments pertaining to Johansen’s tests, we only consider  
 646 DGPs with exact unit roots, but for those pertaining to Wright tests we also  
 647 consider the corresponding DGPs with roots local-to-unity, which we obtain  
 648 by replacing, in the former DGPs, the exact unit root with  $\lambda=1-0.5\cdot(1/T)$ .  
 649 In the case of cointegrated DGPs featuring exact unit roots, we bootstrap  
 650 Wright’s test statistics based on the first procedure discussed in the previous  
 651 subsection (that is, based on bootstrapping the VECM estimated conditional  
 652 on one cointegration vector, as in CRT). In the case of cointegrated DGPs  
 653 featuring near unit roots, on the other hand, we bootstrap the tests via the  
 654 alternative procedure, based on bootstrapping the corresponding near unit  
 655 root VAR in levels.

656 Our main results can be summarized as follows.

657 As for the Johansen test, if the true DGP features no cointegration, CRT’s  
 658 procedure performs remarkably well irrespective of sample size, with empiri-  
 659 cal rejection frequencies (ERFs) very close to the nominal size. This is in  
 660 line with the Monte Carlo evidence reported in CRT’s Table I, p. 1731, and  
 661 with the analogous evidence reported in Benati (2015). If, however, the true  
 662 DGP features cointegration, the tests perform well only if the persistence of  
 663 the cointegration residual is sufficiently low, the sample size is sufficiently  
 664 large, or both: if the residual is persistent, the sample is short, or both, the  
 665 tests fail to detect cointegration a nonnegligible fraction of the time. This is

in line with some of Engle and Granger's (1987) evidence, and it has a simple explanation: as the residual becomes more and more persistent, it gets closer and closer to a random walk (in which case there would be no cointegration), so that the procedure needs larger and larger samples to detect the truth (i.e., that the residual is highly persistent but ultimately stationary).

As for the Wright test, evidence is qualitatively the same, and quantitatively very close, in the case of either exact or near unit root DGPs. Specifically, if the true DGP features cointegration, the procedure works remarkably well if the sample size is sufficiently long, the persistence of the cointegration residual is sufficiently low, or both, with ERFs very close to the tests' nominal size. As the sample size decreases and/or the persistence of the cointegration residual increases, however, the ERFs increase systematically, to the point where, for example, for  $T = 50$  and the autoregressive parameter of the cointegration residual equal to 0.95, the test incorrectly rejects the null at about twice the nominal size. The explanation for this is straightforward, and it has to do, once again, with Engle and Granger's (1987) previously mentioned point: when the cointegration residual is highly persistent, only sufficiently long samples allow the test to detect the truth (i.e., that the deviation between the two series is ultimately transitory, so that they are in fact cointegrated). But, under these circumstances, the shorter the sample period, the more likely it will be to mistakenly infer that the deviation between the series is permanent, so that they are not, in fact, cointegrated. If, however, the true DGP features no cointegration, the test tends to reject

689 the null at about twice the nominal size, essentially irrespective of sample  
690 length.

691 These results can be summarized as follows. If the Johansen test detects  
692 cointegration, we should have a reasonable presumption that cointegration  
693 is there. If, however, it does not detect it, a possible explanation is that the  
694 sample is too short, the cointegration residual is highly persistent, or both.  
695 As for the Wright test, lack of rejection of cointegration does not represent  
696 very strong evidence that cointegration truly is there, as this also happens  
697 with a comparatively high frequency for DGPs featuring no cointegration.

698 We now turn to the issue of how persistent cointegration residuals in fact  
699 are.

#### 700 4.2.3. *Evidence on the persistence of cointegration residuals*

701 Tables H.1 and H.2 in Online Appendix H report Hansen’s (1999) “grid  
702 bootstrap” median-unbiased (henceforth, MUB) estimates of the sum of the  
703 AR coefficients in AR(2) representations for the “candidate cointegration  
704 residuals” in our dataset.<sup>25</sup> By “candidate cointegration residual” (hence-  
705 forth, CCR), we mean the linear combination of the variables in the system  
706 that will indeed be regarded as a cointegration residual *if* cointegration is de-  
707 tected.<sup>26</sup> For reasons of robustness, for either the Selden-Latané specification

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<sup>25</sup>Results are based on 2,000 bootstrap replications for each possible value of the sum of the AR coefficients in the grid. Bootstrapping has been performed as in Diebold and Chen (1996). For reasons of robustness, we report results based on two alternative estimators of the cointegration vector, Johansen’s and Stock and Watson’s (1993).

<sup>26</sup>We label it as a candidate cointegration residual because, as the Monte Carlo evidence in the previous section has shown, if a residual is highly persistent, cointegration might

708 (Table H.1) or the log-log specification (Table H.2) we consider two alterna-  
 709 tive estimators of the cointegration residual: either Johansen’s or Stock and  
 710 Watson’s (1993).

711 Evidence points toward both a nonnegligible extent of persistence of the  
 712 CCRs and a wide extent of heterogeneity across countries. Focusing on results  
 713 based on the log-log specification, the MUB estimate based on Johansen’s  
 714 estimator of the cointegration vector—let’s label it as  $\hat{\rho}_{MUB}^J$ —ranges from a  
 715 minimum of 0.27 for Belize to a maximum of 1.17 for the Barbados islands.  
 716 By classifying the  $\hat{\rho}_{MUB}^J$ ’s, in an admittedly arbitrary fashion, as “highly per-  
 717 sistent” ( $\hat{\rho}_{MUB}^J \geq 0.8$ ); “moderately persistent” ( $0.4 < \hat{\rho}_{MUB}^J < 0.8$ ); and “not  
 718 very persistent” ( $\hat{\rho}_{MUB}^J \leq 0.4$ ), we end up with 22  $\hat{\rho}_{MUB}^J$ ’s in the first group,  
 719 14 in the second, and 4 in the third. Results based on Stock and Watson’s  
 720 estimator are qualitatively the same, with the three groups comprising, re-  
 721 spectively, 25, 13, and 2 countries.

722 Under these circumstances, statistical tests will often have a hard time  
 723 detecting cointegration even if it truly is present, especially when  $\hat{\rho}_{MUB}$  is  
 724 high and the sample period is comparatively short. This implies that results  
 725 from cointegration tests should not be taken strictly at face value, but rather  
 726 should be interpreted in the light of the previously mentioned Monte Carlo  
 727 evidence in Tables D.1 and D.2 in Online Appendix D.

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well not be detected even if it is present, which would prevent the candidate from being identified as a true cointegration residual.

## 728 5. Results

729 In presenting the results, we first discuss the cointegration tests and then  
730 show the parameter estimates. We finish with a comparison between the  
731 log-log and Selden-Latané specifications.

### 732 5.1. Cointegration tests

733 In this section, we discuss the results from bivariate systems for velocity  
734 and the short rate, as implied by equation (7).<sup>27</sup> Table 1 reports results  
735 from Johansen’s maximum eigenvalue test of 0 versus 1 cointegration vec-  
736 tors, together with the Monte Carlo-based ERFs computed conditional on  
737 the null of one cointegration vector. We highlight in yellow all  $p$ -values for  
738 maximum eigenvalue tests smaller than 10% and all ERFs smaller than 50%,  
739 corresponding to a less-than-even chance of detecting cointegration if it is  
740 truly present in the data.

741 The table reports the cointegration test results for only one of the func-  
742 tional forms when the unit root tests for either the level or the log of one  
743 of the variables were rejected. For two of the countries, Morocco and Hong  
744 Kong, the series were identified as  $I(2)$  for both the level and the log of  
745 the variables, so no cointegration tests are reported. Thus, Table 1 reports  
746 results for only 36 countries.

747 We ordered the countries according to the test results. Within each cate-

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<sup>27</sup>In Online Appendix F, we discuss test results for unrestricted specifications between the log of the interest rate, the log of nominal output, and the log of M1.

Table 1 Bootstrapped $p$ -values <sup>a</sup> for Johansen's maximum eigenvalue <sup>b</sup> test and empirical rejection frequencies of the tests under the null					
<i>Country</i>	<i>Period</i>	I: Bootstrapped $p$ -values		II: Empirical rejection frequencies	
		<i>Selden-Latané</i>	<i>Log-log</i>	<i>Selden-Latané</i>	<i>Log-log</i>
United Kingdom	1922-2016	<b>0.003</b>	0.793	0.975	0.661
US – M1 + MMDAs	1915-2017	<b>0.063</b>	0.212	0.859	0.625
US – M1	1915-2017	0.869	0.218	<b>0.099</b>	<b>0.275</b>
Argentina	1914-2009	–	<b>0.031</b>	–	0.789
Brazil	1934-2014	–	<b>0.093</b>	–	<b>0.341</b>
Canada	1967-2017	<b>0.015</b>	<b>0.028</b>	0.965	0.939
	1926-2006	<b>0.007</b>	0.361	0.968	0.628
Colombia	1960-2017	<b>0.032</b>	<b>0.027</b>	0.648	0.593
Guatemala	1980-2017	<b>0.007</b>	<b>0.038</b>	0.536	<b>0.448</b>
New Zealand	1934-2017	<b>0.099</b>	<b>0.030</b>	0.690	0.819
Switzerland	1948-2005	<b>0.000</b>	<b>0.017</b>	0.923	0.769
Bolivia	1980-2013	<b>0.053</b>	0.681	0.686	<b>0.125</b>
Israel	1983-2016	<b>0.000</b>	0.252	0.767	<b>0.197</b>
Mexico	1985-2014	<b>0.007</b>	0.505	0.537	<b>0.200</b>
Belgium	1946-1990	0.361	<b>0.062</b>	0.699	0.721
Belize	1977-2017	0.704	<b>0.007</b>	<b>0.107</b>	<b>0.394</b>
Austria	1970-1998	0.203	0.180	<b>0.220</b>	<b>0.265</b>
Bahrain	1980-2017	0.401	0.335	<b>0.082</b>	<b>0.085</b>
Barbados	1975-2016	0.542	0.677	<b>0.348</b>	<b>0.335</b>
Ecuador	1980-2011	0.838	0.686	<b>0.043</b>	<b>0.053</b>
Netherlands	1950-1992	0.349	0.568	<b>0.463</b>	<b>0.325</b>
South Korea	1970-2017	0.364	0.955	<b>0.086</b>	<b>0.169</b>
Thailand	1979-2016	0.101	0.212	<b>0.101</b>	<b>0.091</b>
Venezuela	1962-1999	0.776	0.922	<b>0.087</b>	<b>0.059</b>
Australia	1969-2017	0.134	0.960	0.720	<b>0.425</b>
	1941-1989	0.642	0.722	<b>0.168</b>	<b>0.198</b>
Chile	1941-2017	0.442	0.307	0.824	0.628
	1940-1995	0.133	0.175	<b>0.111</b>	0.859
Finland	1946-1985	0.246	–	<b>0.286</b>	–
Japan	1955-2017	0.567	0.142	<b>0.363</b>	0.590
Spain	1941-1989	0.120	0.196	0.636	<b>0.205</b>
Taiwan	1962-2017	–	0.909	–	<b>0.139</b>
Turkey	1968-2017	0.460	–	<b>0.229</b>	–
West Germany	1960-1989	–	0.352	–	<b>0.219</b>
Italy	1949-1996	0.171	–	0.629	–
Norway	1946-2014	<b>0.035</b>	<b>0.043</b>	0.749	0.789
Paraguay	1962-2015	<b>0.074</b>	0.168	<b>0.443</b>	<b>0.366</b>
Peru	1959-2017	<b>0.003</b>	0.171	0.992	<b>0.389</b>
Portugal	1914-1998	0.857	<b>0.047</b>	<b>0.048</b>	<b>0.059</b>
South Africa	1965-2015	0.116	0.157	0.563	<b>0.329</b>
<sup>a</sup> Based on 10,000 bootstrap replications. <sup>b</sup> Null of 0 versus 1 cointegration vectors.					



748 gory, we ordered the countries alphabetically. For each country, we first men-  
749 tion the time period for which we have consistent data. In some cases (Aus-  
750 tralia, Canada, and Chile), we have two different datasets, for long enough  
751 periods, but they do not completely overlap. The series are not exactly the  
752 same, so they cannot be used to construct a single series that can suitably be  
753 analyzed using cointegration. We report the results using both series. The  
754 third and fourth columns report the  $p$ -values of the tests for both the Selden-  
755 Latané and the log-log specifications. Finally, we show the ERF of the Monte  
756 Carlo exercises for both the Selden-Latané and the log-log specifications.

757 We first report the results for the United Kingdom and the United States,  
758 for which we have close to a century of data. For the case of the United States,  
759 we use both M1 and New M1 (the monetary aggregate proposed in Lucas and  
760 Nicolini 2015). The second group of countries contains the ones for which  
761 both  $p$ -values (or the only one that we could run) are below 10%. The next  
762 two groups include countries for which one and only one of the  $p$ -values is  
763 below 10%. The fifth and sixth groups contain countries for which both  $p$ -  
764 values are above 10%, but either the two ERFs are below 50% (fifth group)  
765 or only one is below 50% (sixth group).<sup>28</sup> Finally, the last group includes the  
766 six countries for which we believe the evidence is weak or nonexistent based  
767 on the visual evidence, in spite of the test results.

768 We first discuss how to interpret the United States and United Kingdom

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<sup>28</sup>In classifying countries for which we have more than one set of series, we chose the one that contains the most recent data.

769 results in detail. The other numbers in the table are to be interpreted in  
 770 a similar way. Recall that for both countries, the evidence displayed in the  
 771 simple graphs in Figures 3 and 4 is quite remarkable. The results of the  
 772 tests confirm that notion. In using M1 for the United Kingdom and New  
 773 M1 for the United States, the  $p$ -values for the Selden-Latané specification  
 774 are below 10%, but the ones for the log-log specification are both above  
 775 10%. For both countries, the ERFs are substantially larger than 50%. This  
 776 strong preference of the data for the Selden-Latané specification is consistent  
 777 with the fact that both countries had several periods with interest rates very  
 778 close to zero. Taken together, these results provide evidence of a satiation  
 779 point at zero in the aggregate real money demand. Finally, when using the  
 780 standard M1 aggregate for the United States, both  $p$ -values are higher than  
 781 10%, although the ERFs are below 50% for both specifications, indicating  
 782 that the power of the test is low.

783 With the exception of Hong Kong and Morocco—for which we could not  
 784 run the tests—the order of the countries in the table is the same as the order  
 785 of the countries in the figures in Section 3. The first four groups contain 14  
 786 countries in total. For all of them, the tests detect cointegration in at least  
 787 one of the specifications, even though in several cases the ERFs are low. The  
 788 next two groups contain a total of 16 countries for which cointegration is not  
 789 detected, but the ERFs are low in the two tests (8 countries in group 5) or in  
 790 one test (8 countries in group 6). For these 30 countries, the visual evidence  
 791 is very good – with the caveat of the behavior right after the stabilization of

792 very high inflations, as in Argentina and Brazil.

793 The final group contains 6 countries for which the visual evidence was  
794 problematic or nonexistent. Two problematic cases are Norway and Portu-  
795 gal. In both cases, the tests do detect cointegration in at least one of the  
796 specifications. However, the visual evidence suggests a different behavior  
797 over time, somewhat similar to what occurred with M1 in the United States.  
798 Exploring whether regulation could explain those 2 cases seems to be worth  
799 an avenue pursuing, but not in this paper. For the remaining 4 countries,  
800 the visual evidence does not suggest such a pattern (or any other pattern!).  
801 Even though in 2 of those 4 cases the tests do detect cointegration, we can  
802 only classify those 4 countries as failing to behave as the theory implies.

803 To summarize: we find supporting evidence for 32 out of the 38 countries  
804 analyzed (the 30 countries in groups 1 to 6, plus Hong Kong and Morocco).  
805 Of the remaining 6 countries, regulatory changes may explain the behavior  
806 of real money demand in 2, whereas the other 4 are blunt failures.

807 In Table 2, we present the results for the Wright test. We report 90%  
808 confidence intervals for the second element of the normalized cointegration  
809 vector  $(1 - \beta)$ . As mentioned above, they represent the set of all values of  
810  $\beta$  for which the null hypothesis that  $(1 - \beta)'y_t$  is  $I(0)$  cannot be rejected at  
811 the  $\alpha\%$  level, where  $y_t$  is a vector that contains either the levels or the logs  
812 of the short rate and velocity. The order of the countries is the same as in  
813 Table 1. In those cases in which cointegration is not detected, the entry in  
814 the table is NCD. We highlight in yellow the cases in which the confidence

<b>Table 2 Results from the Wright (2000) test: 90% coverage confidence intervals for the second element of the normalized cointegration vector</b>			
<i>Country</i>	<i>Period</i>	Bootstrapped process: cointegrated VECM	
		<i>Selden-Latané</i>	<i>Log-log</i>
United Kingdom	1922-2016	<b>[-0.529; -0.417]</b>	NCD
US - M1 + MMDAs	1915-2017	<b>[-0.613; -0.393]</b>	<b>[-0.352; -0.108]</b>
US - M1	1915-2017	NCD	<b>[-0.506; -0.029]</b>
Argentina	1914-2009	<b>[-0.107; -0.087]</b>	<b>[-0.513; -0.245]</b>
Brazil	1934-2014	<b>[-0.065; -0.009]</b>	[-1.366; 0.276]
Canada	1926-2006	<b>[-1.490; -1.053]</b>	<b>[-0.719; -0.607]</b>
	1967-2017	<b>[-0.578; -0.494]</b>	<b>[-0.389; -0.345]</b>
Colombia	1960-2017	NCD	NCD
Guatemala	1980-2017	<b>[-0.752; -0.448]</b>	<b>[-0.678; -0.414]</b>
New Zealand	1934-2017	NCD	<b>[-0.589; -0.312]</b>
Switzerland	1948-2005	NCD	NCD
Bolivia	1980-2013	<b>[-0.369; -0.193]</b>	<b>[-0.520; -0.388]</b>
Israel	1983-2016	NCD	<b>[-0.388; -0.320]</b>
Mexico	1985-2014	<b>[-0.260; -0.184]</b>	<b>[-0.422; -0.314]</b>
Belgium	1946-1990	<b>[-0.465; -0.289]</b>	<b>[-1.146; -0.710]</b>
Belize	1977-2017	<b>[-0.840; -0.692]</b>	<b>[-2.567; 1.433]</b>
Austria	1970-1998	[-0.601; 0.080]	[-1.040; 0.618]
Bahrain	1980-2017	NCD	<b>[-0.254; -0.194]</b>
Barbados	1975-2016	<b>[-2.006; -0.748]</b>	<b>[-2.899; 0.101]</b>
Ecuador	1980-2011	NCD	NCD
Netherlands	1950-1992	<b>[-0.394; -0.290]</b>	<b>[-0.483; -0.331]</b>
South Korea	1970-2017	<b>[-0.617; -0.521]</b>	<b>[-0.639; -0.338]</b>
Thailand	1979-2016	NCD	<b>[-0.498; -0.386]</b>
Venezuela	1962-1999	NCD	[-0.249; 0.287]
Australia	1941-1989	<b>[-0.691; -0.526]</b>	<b>[-0.808; -0.704]</b>
	1969-2017	<b>[-0.484; -0.404]</b>	<b>[-0.506; -0.314]</b>
Chile	1940-1995	<b>[-0.140; -0.028]</b>	<b>[-0.382; -0.278]</b>
Finland	1946-1985	<b>[-0.530; -0.414]</b>	<b>[-2.693; -1.780]</b>
Japan	1955-2017	<b>[-0.520; -0.312]</b>	<b>[-0.513; -0.125]</b>
Spain	1941-1989	<b>[-0.163; -0.159]</b>	NCD
Taiwan	1962-2017	<b>[-0.449; -0.341]</b>	<b>[-0.453; -0.253]</b>
Turkey	1968-2017	NCD	NCD
West Germany	1960-1989	[-0.963; 0.931]	[-0.489; 0.692]
Italy	1949-1996	[0.032; 0.204]	[0.159; 0.511]
Norway	1946-2014	[-0.961; 0.985]	[-0.227; 1.043]
Paraguay	1962-2015	[-0.328; 0.125]	<b>[-0.200; -0.024]</b>
Peru	1959-2017	[-0.042; 0.026]	[-0.493; 0.692]
Portugal	1914-1998	[-0.340; 0.433]	[-0.018; 0.210]
South Africa	1965-2015	[-0.170; 0.427]	[-0.052; 1.065]
NCD = No cointegration detected.			

interval lies entirely in the range of negative numbers, so that cointegration is detected in the data, and furthermore the relationship is negative, as it is in the theory. The results in Table 2 are in general even stronger than the ones in Table 1, but the results are consistent. For the 6 countries we had identified as having weak or nonexistent evidence, the results are also weak in this case. On the other hand, for the 16 countries in groups 5 and 6 of Table 1, where the Johansen test failed to detect cointegration (the  $p$ -values were above 10%) but where at least one of the ERFs were low, the Wright test identifies cointegration in 15 of them (West Germany being the only exception). For the 12 countries for which we did identify cointegration using the Johansen test, there is conflicting evidence for only one country: results for Switzerland are very strong in Table 1 but not in Table 2.<sup>29</sup>

## 5.2. *The estimated coefficients on the short rate*

The coefficients can be estimated using both Johansen's and Stock and Watson's procedures. In addition, the Wright test also delivers confidence intervals for the coefficient on the short rate. The full set of results is presented in Online Appendix I, where we show the estimation using the three procedures for the two specifications. We will focus the discussion in this section on the estimates of the elasticity in the log-log specification using

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<sup>29</sup>In light of the low power of the tests, one could repeat the analysis using quarterly data. For nine of the countries in our sample, the tests can be found in Benati (2020), using data since WWII. We could find quarterly (also since WWII) for an additional set of seven countries. Appendix J contains the result. In all the sixteen cases the results are in line with the ones we report in Tables 1 and 2.

834 Stock and Watson’s procedure. The reason for focusing on the log-log speci-  
835 fication is twofold. First, the coefficient on the log of the interest rate has the  
836 natural interpretation of an elasticity. Second, and most importantly, we can  
837 directly relate it to the transactions technology that is the key component of  
838 the theory. In that respect, our reference value has an elasticity equal to 0.5,  
839 which corresponds to the case of a linear technology, as in Baumol and Tobin.  
840 Higher values for the elasticity imply that the exponent  $\sigma$  in equation (2) is  
841 lower than 1, which implies that the marginal cost of making transactions is  
842 decreasing with the number of transactions. The opposite is true when the  
843 elasticity is lower than 0.5.

844 The reason for focusing on Stock and Watson’s estimates is that they  
845 are based on a single equation, whereas Johansen’s is based on estimating  
846 an entire cointegrated VAR, so there are many more coefficients. In small  
847 samples this approach may produce less precise results. In almost all cases,  
848 Johansen’s estimates are broadly in line with Stock and Watson’s but typi-  
849 cally deliver larger standard errors. In 4 cases the results are different, and  
850 we conjecture that this result might be a small-sample issue. For details, see  
851 Online Appendix I. We are aware that as long as the data-generating process  
852 corresponds to the model with the borrowing constraint, the estimate of the  
853 elasticity will be biased downward in countries with several observations of  
854 interest rates near zero. The reason is that the procedure will try to match  
855 those observations with low interest rates that all lie below the log-log curve  
856 that has a good fit with the observations for moderate and high interest rates.

857 However, given that the number of observations at very low rates is not a  
858 very large fraction of the total sample, we expect this bias to be small.

859 In Figure 5, we present the results of the estimation for 33 countries  
860 using the Stock and Watson procedure. As explained above, for 2 countries  
861 (Hong Kong and Morocco), we could not run the tests, and for 3 of them  
862 (Finland, Italy, and Turkey), the test did not detect cointegration for the log-  
863 log specification. The horizontal axis represents the value for the estimator  
864 of the elasticity for the corresponding country, ranging from -1 to 1. For each  
865 country, we report the point estimator with a black rhombus and the 90%  
866 confidence interval with a dotted red line. We order the countries according  
867 to the point estimate, starting with the lowest one. Finally, in the figure  
868 we plot two vertical lines: one at zero, which corresponds to the null of no  
869 relationship between the log of the interest rate and the log of real money  
870 demand, and one at the value 0.5, which corresponds to the linear technology  
871 assumed by Baumol and Tobin.<sup>30</sup>

872 As a summary of the figure, the confidence interval includes zero for only  
873 4 out of the 33 countries. Two of them (Norway and South Africa) belong  
874 to the group with either weak or nonexistent evidence. In no case is the  
875 estimate statistically greater than zero. Finally, for around 20 countries, the  
876 confidence interval includes 0.5 or is remarkably close to it. Table 2, together  
877 with the country plots in Figures 3a to 3c, gives very strong support to a

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<sup>30</sup>For the 3 countries for which we had two different sets of data, here we report the set that includes the latest observations.

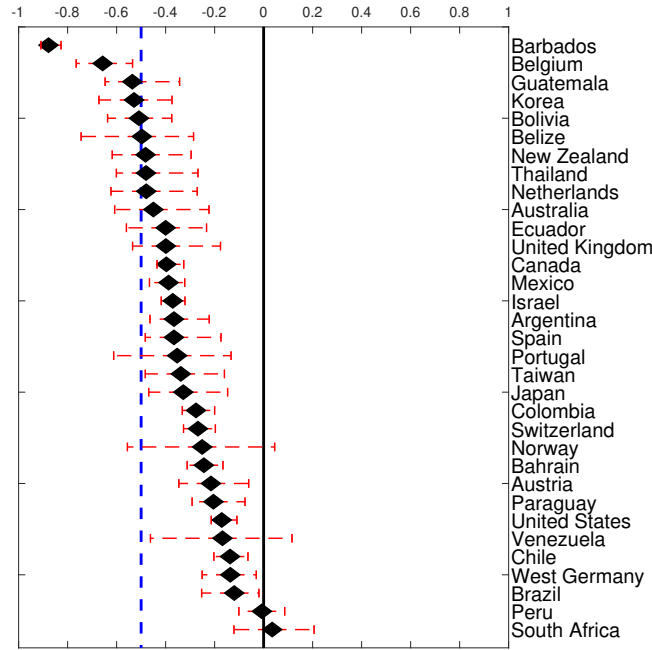


Figure 5 Estimation results using the procedure from Stock and Watson (1993)

rather simple theory that, in essence, was developed over half a century ago.

### 5.3. Evidence on the functional form

Figure 6 provides simple, informal evidence on which specification – the Selden-Latané or the log-log – provides the most plausible description of the data at both low and high interest rates. For selected low-inflation and high-inflation countries, the top row shows the levels of M1 velocity and the short rate, and the bottom row shows the logarithms of the two series. By plotting the series with different axes, we search for a linear relationship between either the levels or the logs. The evidence in the top row therefore corresponds to a Selden-Latané specification and the bottom row to the log-log specification.



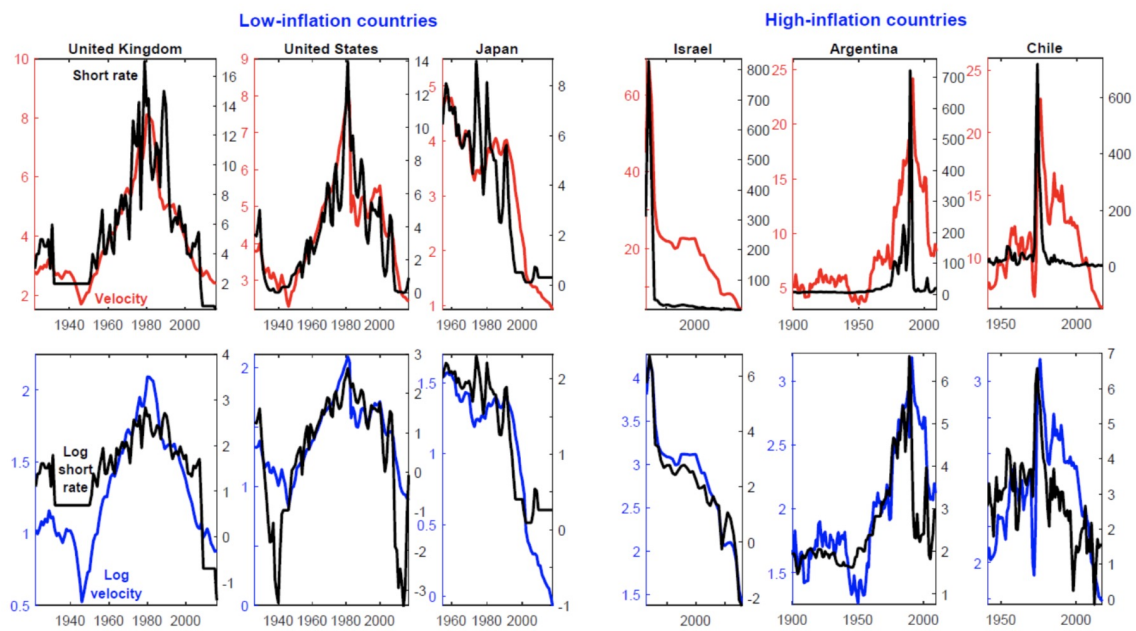


Figure 6 Comparing the Selden-Latané and log-log specifications: selected evidence for low-inflation and high-inflation countries

Two broad patterns emerge from Figure 6. First, for the low-inflation countries, both formulations do a very good job at capturing the rise and fall of both velocity and interest rates in the United Kingdom and the United States, and the persistent fall of both in Japan. The figure clearly shows, however, that the log-log specification is substantially worse when interest rates are close to zero for the three countries. This result is in line with our discussion of the borrowing constraints in the theoretical section. Second, for the high-inflation countries, the opposite is true: the specification in logs appears to deliver a linear relationship, whereas the specification in levels does not.<sup>31</sup> This overall pattern is consistent with the theory, where borrowing constraints play an important role in low interest rates.

We did not select countries in Figure 6 randomly; rather, they are the ones that either had interest rates close to zero for many periods or had very high inflation rates. The full comparison is presented in Online Appendix I, and while the general message is similar, the conclusion there is not as striking as the examples shown here.

## 6. Two Additional Issues

We now discuss two additional issues. First, we show that the estimators of the elasticity of money demand we used in Section 5.1—which are

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<sup>31</sup>These results are in line with the evidence produced by Benati (2019b) based on either monthly or weekly data for 20 cases of hyperinflation, from the French Revolution to Venezuela’s episode: in nearly all cases, econometric evidence shows a clear and often overwhelming preference for the log-log specification.

908 predicated on the assumption that the series feature exact unit roots—work  
 909 equally well for series that are local-to-unity. Then, we discuss tests for the  
 910 stability of the money demand cointegration relationship.

911 *6.1. Robustness of the estimates of the elasticity of money demand to near*  
 912 *unit roots*

913 Table 3 reports results from the following Monte Carlo experiment. We  
 914 simulate the following DGP for the logarithms of the short rate,  $R_t$ , and of  
 915 M1 velocity,  $V_t$ :

916 
$$\ln R_t = \lambda \ln R_{t-1} + \epsilon_t, \text{ with } \lambda = 1 - 0.5 \cdot (1/T), \epsilon_t \sim i.i.d. N(0, 1) \quad (12)$$

917

918 
$$u_t = \rho u_{t-1} + v_t, \text{ with } 0 \leq \rho < 1, v_t \sim i.i.d. N(0, 1). \quad (13)$$

919

920 
$$\ln V_t = \alpha_0 - \alpha_1 \ln R_t + u_t \quad (14)$$

921 We set  $\alpha_0 = 1$  and  $\alpha_1$  equal to Baumol and Tobin's benchmark value of 0.5.  
 922 As for  $\rho$ , we consider six possible values ranging from 0 to 0.95, correspond-  
 923 ing to alternative extents of persistence of the cointegration residual. Finally,  
 924 we consider four possible values for the sample length,  $T$ , ranging from 50  
 925 to 1,000. For each possible combination of values for  $T$  and  $\rho$ , we simu-  
 926 late (12)-(14) 10,000 times, and based on each artificial sample, we estimate  
 927 the elasticity of money demand as we did in Section 5.1, based on either  
 928 Johansen's or Stock and Watson's (1993) procedures. Table 3 reports the  
 929 mean of the Monte Carlo distribution for the estimates of  $\alpha_1$  based on Stock

$\rho$	0	0.25	0.5	0.75	0.9	0.95
T = 50	0.5002	0.4983	0.5018	0.4978	0.4889	0.4992
T = 100	0.5007	0.4996	0.4995	0.5002	0.5025	0.5010
T = 200	0.5000	0.4999	0.4997	0.4989	0.4990	0.4982
T = 1000	0.5000	0.5000	0.5000	0.4998	0.5002	0.5004

Table 3: Mean of Monte Carlo distribution for alternative values of T and  $\rho$

and Watson’s procedure (results based on Johansen’s procedure are qualitatively the same, and they are available upon request). The evidence in the table speaks for itself and shows that the estimates of the elasticity of money demand we discussed in Section 5.1 are in fact robust to the series being local-to-unity, rather than featuring exact unit roots.

## 6.2. Testing for stability in cointegration relationships

We test for stability in cointegration relationships based on the three tests discussed by Hansen and Johansen (1999): two Nyblom-type tests for stability in the cointegration vector and the vector of loading coefficients, respectively; and a fluctuation test, which is essentially a joint test for time variation in the cointegration vector and the loadings. In either case, we bootstrap the test statistics via CRT’s procedure, based on the VECM estimated conditional on one cointegration vector, and not featuring any break or time variation of any kind.

Table H.1 in the Online Appendix reports Monte Carlo evidence on the performance of the tests conditional on bivariate cointegrated DGPs, for alternative sample lengths and alternative degrees of persistence of the cointegration residual, which is modeled as an AR(1). The main results can be

948 summarized as follows. The two Nyblom-type tests exhibit an overall rea-  
 949 sonable performance, incorrectly rejecting the null of no time variation, most  
 950 of the time, at roughly the nominal size. Crucially, this is the case irrespec-  
 951 tive of the sample length and of the persistence of the cointegration residual.  
 952 The fluctuation test, on the other hand, exhibits good performance only if  
 953 the persistence of the cointegration residual is low. The higher the residual's  
 954 persistence, however, the worse the performance, so that for example, when  
 955 the AR root of the residual is equal to 0.95, for a sample length  $T = 50$ ,  
 956 the test rejects at twice the nominal size. This result is clearly problematic  
 957 since, as previously discussed, residuals are typically moderately to highly  
 958 persistent. In what follows, we therefore focus on the results from the two  
 959 Nyblom-type tests, but we eschew instead results from the fluctuation test  
 960 (these results are reported in Tables H.2 and H.5 in the Online Appendix).

961 Tables H.2 and H.3 in the Online Appendix report results from Hansen  
 962 and Johansen's (1999) Nyblom-type tests for stability in either the cointe-  
 963 gration vector or the vector of loading coefficients. The key finding in the  
 964 two tables is that evidence of breaks in either the cointegration vector or the  
 965 loading coefficients vector is weak to nonexistent. Specifically, for the United  
 966 States, based on the Selden-Latané specification, the null of no breaks in  
 967 either feature is never rejected. Based on the log-log specification, stabil-  
 968 ity in the cointegration vector is also never rejected, whereas breaks in the  
 969 loadings are detected. Evidence for other countries is qualitatively the same.  
 970 For instance, based on the Selden-Latané specification, stability in the coin-

971 tegration vector is rejected in three cases, whereas stability in the loadings is  
972 rejected in six cases. Results for the log-log specification are along the same  
973 lines.

## 974 **7. Conclusions**

975 Our review of real money demand behavior leads us to reach the following  
976 conclusions. First, for about 26 of the countries, there is substantial evidence  
977 of a long-run relationship between the ratio of money to nominal income  
978 and the short-term interest rate; the evidence is weaker for 6 countries and  
979 nonexistent for the remaining 6. Second, for the set of countries for which the  
980 evidence is strong, the log-log specification that implies a constant elasticity  
981 is a very good representation of the data, except when interest rates are very  
982 close to zero. Third, there is evidence of a satiation point at zero interest  
983 rates, implying that the elasticity of real balances with respect to the interest  
984 rate is lower in that range, approaching zero as interest rates go to zero.

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