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Entrepreneur-Investor Information Design

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Entrepreneur-Investor Information Design*

Oleg Muratov

August 21, 2020

Abstract

I consider an environment in which the entrepreneur generates information about the quality of the projects prior to contracting with the investor. The investor faces a moral hazard problem, since the entrepreneur may divert the funding for private consumption. When the investor bargains with the entrepreneur, I find that the efficient amount of information is generated if and only if the bargaining power of the entrepreneur is high enough. I interpret this result in terms of investors' concentration, competitiveness, and generosity measures. I show that the investor prefers a non-absolute bargaining power when the project costs are high enough.

1 Introduction

Start-up entrepreneurship financed by venture capital (VC) has been the fuel of economic innovations for several decades, most notably boosting the growth of innovations in computer hardware and software.¹ Kortum and Lerner (2001) show that VC-backed firms are approximately three times more efficient in generating innovation than corporate research. Moreover, VC industry has grown at an impressive rate from \$610 million to more than \$84 billion within a span of three decades.²

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¹For example, Venture Capital investors were among the first to finance Apple, Microsoft, and Google (Kaplan and Lerner (2010))

²<https://pitchbook.com/news/articles/the-state-of-the-us-venture-industry-in-15-charts>

Profitability of start-up projects is highly uncertain and because of that start-ups rarely obtain funding from traditional banks. Instead, they turn to VC investors, making high levels of risk a distinctive feature of this industry. Much of the VC-provided capital is ultimately lost because of it. According to Da Rin et al. (2013) this issue is universal across all investment types and stages.

Generating pre-investment information could decrease the levels of risk. Even though the investment decision is a joint one, the information generation is often controlled solely by the entrepreneur. For instance, a firm can order a marketing research by a consulting agency and the choice of the agency will determine the informativeness of that research. Developers of software and new technologies can work on alpha-/beta-versions or prototypes of their products. Different designs would lead to more or less informativeness. Another example of information generation is a crowdfunding campaign. By observing the number of pre-ordered items the entrepreneur can make conclusions about the underlying true demand and the future profitability.

Although information is generated by the entrepreneur, if there was no conflict of interest, the information provision would be efficient. Good projects would be funded and bad projects would be discarded. However, entrepreneurs might prefer to implement even the bad projects, which creates incentives for insufficient information provision.

This paper investigates the amount of information generated about a project prior to signing the financial agreement between the two agents, the entrepreneur (he) and the investor (she). To focus on the strategic incentives of the entrepreneur, I assume that the entrepreneur can choose among all quantities of information to be generated, that he can publicly commit to his choice, and that information generation is free of cost. In a setting like that the full amount of information is efficient, since the full information maximizes the sum of payoffs net of information provision cost, and that cost is zero. I consider the specific source of conflict of interest: after contracting with the investor the entrepreneur can secretly divert the funds. This moral hazard problem may distort the entrepreneur's information generation choice. My main interest is under what conditions will the entrepreneur choose to generate the full information.

My main result is that when the entrepreneur negotiates the financing terms with the investor the efficient amount of information is generated if the entrepreneur's bargaining power is high enough.³ Moreover, the quantity of generated information is increasing in the bargaining power of the entrepreneur. The full information fails to be generated when the entrepreneur's bargaining power is not sufficiently high, even though the information is free. I also characterize the environments in which the quantity of information is monotonic in the cost of the investment and/or the severeness of the moral hazard problem.

³For dichotomous project quality and in continuous uniform case the condition becomes *if and only if*.

I interpret my results in terms of VC-capital market structure. High concentration measures of the investment funds can lead to investors having higher bargaining power, and thus, to the inefficient informativeness. There are investment hubs where corporations with VC-subsidaries play dominant roles in regional start-up markets.⁴ For sufficiently high investment costs, such entities might find it beneficial to split their investing operations between multiple independent subsidiaries: even though the immediate effect of lowering the bargaining power is negative, it also results in higher start-ups' motivation to generate information, making the overall effect for the investor positive.

Bargaining power can also be interpreted in terms of the scarcity/abundance of the investment money. During the bust phases of a business cycle, it is usually harder to obtain funding. Hence, those few projects that manage to secure financing are likely to have provided less information, because of the diminished bargaining power of the entrepreneurs. This can lead to a lower success rate in the start-up industry.

Bargaining power has also been empirically linked to the sizes of the VC-funds in Cumming and Dai (2011). Interestingly, Cumming and Dai (2011) find the VC-funds can have diminishing returns from higher bargaining power, with the fund overall performance being a concave inverse U-shaped function with respect to its bargaining power. The results of my paper offer the inefficient pre-contract information generation as a plausible source for diminishing returns from the bargaining power. I find that because of entrepreneur's incentives to generate information the investor prefers a non-absolute bargaining power for herself when the project costs are high enough. Moreover, I find a setting in which the investor's payoff with respect to her bargaining power is inverse U-shaped.

My results also explain why the investors might have incentives to maintain the reputation for being generous, since it allows to limit their bargaining power. Although this way their share decreases, the surplus for division increases. Hence, the total effect can be positive for the investor.

In order to grasp the intuition behind the results of my model, consider the two extreme scenarios. First, suppose the investor has all the bargaining power. If the new information is favorable enough, she will provide the funding to the entrepreneur, and vice versa. The investor-preferred financing terms make the entrepreneur indifferent between the proper productive spending on the one hand and hiding the investment money on the other. The entrepreneur's payoff from diverting the money is the same, irrespective of what the expected quality of the project is. Thus, the payoff from the contract will also be constant. In the opposite event of unfavorable information, the investor simply does not provide the funding to the entrepreneur. As a result, if the entrepreneur has no bargaining power, the only thing he cares about is how often the favorable enough news arrive. Of course he wants good news to arrive more often and bad news to arrive less

⁴For example, Samsung Ventures.

often. Hence, there is a bound on how much information the entrepreneur will generate. The entrepreneur will provide just enough information to make the investor indifferent.

Second, suppose the entrepreneur has all the bargaining power. In this case, when making an offer to the investor, the entrepreneur will only need to compensate her for the cost of the funding, so that in expectation the investor's payoff from investing into the project is zero. The smaller the uncertainty, the smaller the cost of the funding. Thus, the entrepreneur, being the residual claimant to the returns from the project, prefers to implement a project if and only if it is of the good quality. This is achieved by generating as much information as possible.

Finally, consider the case of the intermediate bargaining power. In that case the entrepreneur trades off two forces. On the one hand - the frequency of funding force, on the other - the quality force. The two forces counteract each other. The bargaining power is what determines the importance of the quality force relative to the frequency of funding force.

Crowdfunding as a way to generate information Crowdfunding platforms, such as Kickstarter and Indiegogo, provide ways for creators of art, novel products and gadgets to raise money for their endeavors. The process of collecting the money is the following. First, the creators of the crowdfunded product announce their campaign. The creators need to specify the details of the campaign: the period during which the contributions are going to be accepted; the menu of the rewards and contributions;⁵ the target amount of money; the promised date of the delivery in case of successful campaign; and the project features and specifics. Members of the public, observing the campaign, decide to contribute or not. If the monetary goal is met by the time the campaign ends, the money is transferred to the project creators and they start working on producing the goods to be delivered to the contributors. Otherwise, the contributions are returned to the crowd.⁶

Crowdfunding is a means of collecting money for entrepreneurial needs. But besides the money collecting purpose it also allows to learn and demonstrate some underlying characteristics of the product. For example, when the project creator and outside observers are uncertain about how high the demand for the novel product is, learning the amount of contributed money and the number of pre-ordered items allows to better judge the actual demand. If the crowdfunding campaign is successful and a lot of money is collected, it is more likely that the product will be well received in the market.

By adjusting the characteristics of the campaign such as its duration, monetary target, or the menu of pledges and rewards, the creator is able to generate different levels of informativeness. For example, observing a project that fails to achieve the goal of

⁵Often the rewards are items or bundles of the product being crowdfunded.

⁶In practice, the creators sometimes fail to meet the delivery deadlines, and in some extreme cases, fail to deliver whatsoever. Such events are not a subject of this paper.

\$50,000 with a 5 days-duration of the campaign might seem like unfavorable news about the project perspectives. But it is less unfavorable than observing the same project failing to reach the same \$50,000 goal within a campaign lasting for 60 days.

Anecdotal evidence also suggests the importance of the informational content of the crowdfunding campaigns results. There are multiple cases of entrepreneurs being approached by the venture capitalists after successful crowdfunding campaigns. A notable example is the campaign of Pebble watch. Its creators had an idea for smart watch with electronic ink display but initially failed to attract external funding. After running a Kickstarter campaign and dramatically exceeding the monetary goal, they were funded by the VC investors, who were impressed by the project's performance on the Kickstarter.

The setting of my paper can be related to those projects, whose creators expect to seek additional outsider financing after the crowdfunding stage. The results of my analysis establish that creators with a high bargaining power are likely to design a crowdfunding campaign such that the results will carry a high informational content. That is, the menu of prices, the target sum, and the campaign length will lead to profiles of contributions that will make it clear, whether the demand for that good is actually high or low.

Related literature This work is largely related to the field of information design, particularly to the setting where the uninformed sender chooses and commits to the structure of information revealing experiments, as in Kamenica and Gentzkow (2011), and often referred to as Bayesian Persuasion. Different papers study extensions to the baseline model. In particular, Perez-Richet (2014), Hedlund (2017), Alonso and Câmara (2018) relax the assumption of the uninformed sender; Kolotilin et al. (2017), relax the uninformed receiver assumption; Wang (2013) introduces multiple receivers; Alonso and Câmara (2016) relax the common prior assumption; Perez-Richet and Skreta (2018) and Lipnowski et al. (2019) relax the power of sender's commitment. My work compliments the Bayesian Persuasion literature by allowing for a bargaining stage between the sender and the receiver after the realization of the experiment outcome. My work stresses the importance of the sender's bargaining power for the precision of the information stemming out of the experiment design.

Another modification of the analysis in my work is the presence of moral hazard in the form of unobserved/uncontractible action by the sender after the realization of experiment outcome. Boleslavsky and Kim (2018) also study Bayesian Persuasion in the presence of Moral Hazard problem. In their work the hidden action takes place before the experiment outcome and influences the distribution over the true states, but not the outcome directly. In my work the hidden action takes place after the experiment outcome realization and also after the receiver's action. Moreover, in my work it is the sender, who takes the covert action, whereas in the work of Boleslavsky and Kim (2018) it is a third player, who takes a covert action.

Works by Boleslavsky and Cotton (2018), Au and Kawai (2019), and Au and Kawai (2020) consider the cases which can be interpreted, using the lenses of the current paper, as multiple entrepreneurs competing for the funds of a single the investor, with the investor having all the bargaining power. Their results suggest importance of high-quality competitiveness for informational efficiency.

Section 3 studies the case of the continuous project quality. Earlier papers that study persuasion under continuous state are Gentzkow and Kamenica (2016), Kolotilin et al. (2017), and Kolotilin (2018). Gentzkow and Kamenica (2016) assume state-independent sender's utility. Kolotilin et al. (2017) assume that both players' payoffs are linear in the state and moreover that the sender's payoff with respect to the state is the affine transformation of the receiver's payoff with respect to the state. Section 3 of my work considers an example of interaction of the two players, where sender's preferences are state-dependent, and, moreover, sender's utility is not an affine transformation of the receiver's utility. Results of that analysis are similar to the interval revelation results in Kolotilin (2018), where the results are derived under a set of slightly different assumptions: the state of the world is two-dimensional, for both the receiver's and the sender's types; the utility function of the receiver is linear.

Most closely related is a recent paper by Azarmsa and Cong (2020), who also apply the Bayesian Persuasion framework to study entrepreneur's financing. In their paper they look at the interaction of the three players: the entrepreneur, the insider investor, and the outsider investor. The entrepreneur and the insider observe the experiment outcome deterministically, while the outsider investor observes it with a non-unit probability. When the financial contract offered by the entrepreneur is a simple share, the experiment design and the investment decision are inefficient. When the entrepreneur also designs the financial security contract, the efficiency is restored, with the security contract being a non-linear function of project payoff. My findings compliment the analysis of Azarmsa and Cong (2020). I focus on the bargaining power. In order to restore efficiency, in my model the entrepreneur does not need the commitment power to design any sophisticated non-linear security before designing the experiment structure. Instead, it is sufficient that he has a high enough bargaining power. My model also does not rely on the presence of the outsider investor.

Strausz (2009) also studies the effect of information on entrepreneurial financing. In his work, the investor has private information about the probability of the project being profitable. Strausz (2009) shows that when the entrepreneur makes a take-it-or-leave-it financial contracting offer to the investor, the menu of contracts that separates low probability projects and high probability projects are suboptimal in general. In other words, full information is not optimal for the entrepreneur. Moreover, the optimal mechanism in that setting is such that for medium project costs the investor reveals his private information only partially. My results in the case of the investor having the

bargaining power are similar to the conclusion of Strausz (2009), even though the exact settings differ: in my paper there is no asymmetry of information; the advantage that the investor has, when non-precise information is optimal, is in terms of the bargaining power, not the knowledge; in some sense, in my paper it is the entrepreneur, who has the informational advantage, because Similarly to my work, where information design stage is followed by the contract design, Bergemann and Välimäki (2002) study the information design stage followed by mechanism design stage. In their work information is costly, is acquired by each agent, and experiment results are private. They show that in the mechanism design setting if players' values are private each player purchases the efficient amount of private information; and if players' values are common the information is either under- or over-acquired. In my work the information is costless, public, and controlled by the single agent - the entrepreneur, acting as the sender. My results complement and, to a certain extent, contrast the findings of Bergemann and Välimäki (2002), because I show that public information is efficient only if the payoff of the agent, acquiring the information, is sufficiently co-aligned with the payoff of the other agent. Moreover, since in my work any information is free, the efficiency/inefficiency is not influenced by the cost of information, but rather solely by the nature of the agents' interaction.

Works by Bergemann and Hege (1998), Bergemann and Hege (2005), Halac et al. (2016), and Drugov and Macchiavello (2014) study entrepreneurial incentives in presence of moral hazard in the dynamic setting. Similarly to my work, we can think of the settings, studied in those papers, as of endogenously emerging information in the principal-agent/entrepreneur-investor settings. Each time in those settings an agent exerts effort/shirks (or pulls one of the bandit arms) a new piece of information arrives. In my work, however, the information-related choice and the effort-putting/shirking choice by the agent are disentangled both in terms of the actions and the time those actions take place. Moreover, I assume away the absolute bargaining power of either of players.

I have mentioned the crowdfunding campaign as an example of an information design technology. Strausz (2017) also studies the crowdfunding. In Strausz (2017), however, crowdfunding, as a mechanism, plays multiple roles - it allows to learn information about the crowd's willingness to pay, to collect the seed investment money from that crowd, and to incentivize the proper action by the entrepreneur through the means of deferring part of the crowd's payment. In my model, the information generation decision and the investment decision are separate in time.⁷

The remainder of the paper is organized as follows. The version of the model with the two states of the world is introduced in the Section 2, where in subsection Section 2.2 the entrepreneur has all the bargaining power, in the subsection 2.3 the investor has all the bargaining power, and in the subsection 2.4 the bargaining power is intermediate.

⁷For further details of the institutional background of VC and corporate finance see Kaplan and Strömberg (2003), Da Rin et al. (2013), Bottazzi et al. (2016), Gompers et al. (2019).

Section 3 expands the analysis to the case of continuous infinity number of states. Section 5 concludes. The appendices A and B include proofs and technical details.

2 Two states of the world

2.1 General Description

There is an agent, the entrepreneur, with an idea of a project. The project costs c to implement. The entrepreneur seeks the funding in the size of c from another agent, the investor. The state of the world, ω captures the project quality:

$$\omega \in \{\text{good}, \text{bad}\}.$$

Nature chooses the project quality according to

$$\mathbb{P}\{\omega = \text{good}\} = \alpha_0.$$

A good project yields a return of 1, if and only if an amount c is invested into it, otherwise it yields 0. A bad projects always returns 0. Throughout the paper it is assumed that $c < 1$.

In the beginning the entrepreneur chooses the experiment structure and both players observe the outcome of the experiment.⁸ As a result of observing the experiment outcome the two players form a posterior belief $\hat{\alpha}$. After that the entrepreneur approaches the investor and the two negotiate the terms of financing.

The financing terms, negotiated by the two agents, specify the way the possible returns of the project are shared, once and if they are realized. The entrepreneur gets share s of the returns, and the investor - share $1 - s$. The three possible bargaining procedures will be specified in the sections 2.2, 2.3, 2.4.

The investor always has an option of saying “no” to the entrepreneur and terminally stepping out of the negotiations of the financing terms. So, the investor takes an action $a^I \in \{0, 1\}$, where $a^I = 1$ corresponds to her providing the funding to the entrepreneur.

If and once the entrepreneur and the investor reach an agreement on the contract terms, the investor transfers the sum c to the entrepreneur.

Following the seminal paper of Hölmstrom (1979), I assume that their interaction is affected by the moral hazard: instead of investing the sum c properly the entrepreneur can also divert it to his private benefit. If the entrepreneur chooses to divert the money the project yields nothing, regardless of whether $\omega = \text{good}$ or $\omega = \text{bad}$, but the entrepreneur enjoys the private benefit of size δc , with $\delta < \frac{1-c}{c}$.⁹ The decision of whether to invest the

⁸The experiment choice will be described in a greater detail in the text further below.

⁹ δ is an exogenously given share of money that the entrepreneur can recover. This way the aggregate

money properly is denoted by $i \in \{0, 1\}$, where $i = 1$ corresponds to the proper money investment.¹⁰

After the entrepreneur makes the money spending decision the outcome of the project is observed. If the return is positive, it is shared between the parties in accordance with the pre-specified contract.

The players' payoffs are:

- if the investor provides funding,

$$V^E(\omega, a, i) = \begin{cases} s, & \text{if } E \text{ spends money properly and } \omega = \text{good} \\ 0, & \text{if } E \text{ spends money properly but } \omega = \text{bad} \\ \delta c, & \text{if } E \text{ hides the money} \end{cases}$$

$$V^I(\omega, a, i) = \begin{cases} 1 - s - c, & \text{if } E \text{ spends money properly and } \omega = \text{good} \\ -c, & \text{if } E \text{ hides the money, or } \omega = \text{bad} \end{cases}$$

- if the investor does not provide funding, both players get their reservation payoffs 0.

Except for the experiment design stage at the start of the play, the game described above is relatively familiar.¹¹ Suppose now that there is an experiment with two possible outcomes - high and low.¹² The outcome is publicly observed before the negotiations. The entrepreneur chooses the probabilities of each outcome conditional on the underlying true project quality by setting the numbers $(x, y) \in (0, 1)^2$, as in the following matrix:

outcome \ state	Good	Bad
	High	Low
High	x	y
Low	$1 - x$	$1 - y$

For example, if the entrepreneur chooses $x = 1$ and $y = 0$, he induces the experiment structure, which will be hereafter referred to as the precise experiment. After learning

welfare loss by the society from such action of the entrepreneur is $(1 - \delta)c$. δ could be interpreted as an inverse measure of transparency of the accounting system. The higher the transparency, the less the entrepreneur will be able to “hide”. Alternatively, higher δ could mean less trust between economic agents.

¹⁰Rather than in the direct “runaway” interpretation, the entrepreneur might be contemplating between exerting costly investment effort and assuming a more “laid back” approach which results in the money being inefficiently spent. Sometimes, however, hidden action can take the form of outright fraud, as in the infamous case of Theranos company, <https://www.nytimes.com/2018/06/15/health/theranos-elizabeth-holmes-fraud.html>

¹¹In particular, a dynamic repeated version of such environment is studied for example in Bergemann and Hege (1998).

¹²In a two state version of the model it is without loss to consider two outcome experiments.

the results of such experiment, an observer can always state with certainty, whether the project is good, or bad. Since the experiment outcome is public knowledge before the negotiations, the precise experiment induces investing into good projects and discarding the bad ones.

The precise experiment is an extreme example of experiment structure, since it generates the largest amount of information possible.

Another extreme design of the experiment is achieved by setting $x = y \in (0, 1)$. For such experiments observing any outcome is not informative at all. Such experiments will be referred to as the noisy experiment.

For an arbitrary choice of $\{x, y\} \in (0, 1)^2$ the updated beliefs after observing the high outcome is

$$\mathbb{P}\{\text{“good”} | \text{“high”}\} = \frac{\alpha_0 \times x}{\alpha_0 \times x + (1 - \alpha_0) \times y},$$

and after observing the low outcome -

$$\mathbb{P}\{\text{“good”} | \text{“low”}\} = \frac{\alpha_0 \times (1 - x)}{\alpha_0 \times (1 - x) + (1 - \alpha_0) \times (1 - y)}.$$

The beliefs of the entrepreneur and the investor naturally affect the results of negotiations. Hence, the funding terms depend on the realized outcome of the experiment, and through that - on the experiment design.

The interests of the entrepreneur and the investor are misaligned. They both would like the good projects to be implemented. However, the entrepreneur, unlike the investor, also wants the contract to be signed as often as possible, because this way he would have the investment funds to use for his own discretion. This makes his payoff from signing the contract bounded from below by the payoff from diverting the funds, whereas if the contract is not signed, the entrepreneur’s payoff is zero.

The experiment structure that maximizes the sum of payoffs is the precise experiment. Given the conflict of the entrepreneur’s and the investor’s interest, and the fact that it is solely the entrepreneur who chooses the structure of the experiment, a natural question arises: will he choose the precise experiment? And if he does not, how informative will the chosen experiment be?

The cases of each of the agents making a take-it-or-leave-it offer to the respective counter-party are considered in the two subsections below. The two cases represent the extreme bargaining powers: the case of the entrepreneur making a take-it-or-leave-it offer corresponds to the entrepreneur having the absolute bargaining power; and vice versa, when the investor is the proposer, she has the absolute bargaining power. These cases show the crucial role that the bargaining power plays for the amount of information generated: when the entrepreneur chooses both the experiment structure and the contract, he prefers the precise experiment. When it is the investor who proposes the contract, the

entrepreneur sees the precise experiment as suboptimal.

The two extreme settings are followed by the subsection that considers the case of intermediate bargaining powers. It shows how the bargaining power can adjust the forces of the two conflicting interests.

2.2 The entrepreneur makes a take-it-or-leave-it offer

Throughout this subsection assume that after the experiment outcome is observed the entrepreneur makes a take-it-or-leave-it contract offer to the investor, denoted s^E . Other than that the players' actions and payoffs are as described in the previous subsection.

The timing is the following: first, the entrepreneur chooses the experiment design by setting x and y . Then the Nature chooses the experiment outcome according to the conditional probabilities, (x, y) and the probability of the project being good:

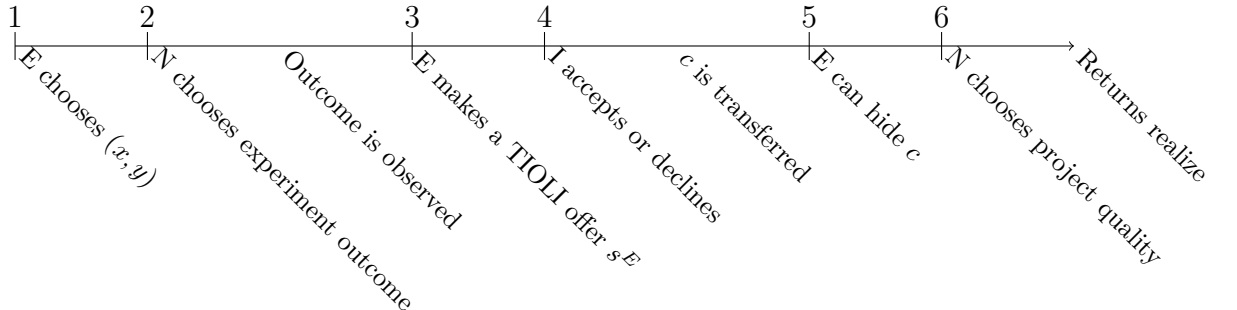
$$\mathbb{P}\{\text{"high"}\} = \alpha_0 x + (1 - \alpha_0)y.$$

After both players observe the outcome, the entrepreneur proposes the contract s^E to the investor. If the latter accepts the offer, $a^I = 1$, the game continues and the entrepreneur receives the investment sum c . He now chooses whether to spend it properly on the project, or to spend it for private benefit. In the end the Nature chooses the project quality according to currently held belief, denoted by $\hat{\alpha}$,¹³

$$\hat{\alpha} = \begin{cases} \frac{\alpha_0 x}{\alpha_0 x + (1 - \alpha_0)y}, & \text{if the outcome was high,} \\ \frac{\alpha_0(1-x)}{\alpha_0(1-x) + (1 - \alpha_0)(1-y)}, & \text{if the outcome was low,} \end{cases}$$

and the return realizes according to project quality and the spending decision by the entrepreneur, i.e. return = 1 if and only if the quality is good and the entrepreneur has spent the money properly.

Below is the stylized depiction of the timeline:



¹³The actual frequency of good projects needs to coincide with the ex-ante probability of the good project, α_0 . In a timeline above this happens if the Nature's eventual choice of the project quality is consistent with the previously realized experiment outcome, i.e. it happens according to the posterior beliefs.

It is worthwhile to mention that the analysis with the timeline as above is essentially the same as the analysis in case of the Nature covertly choosing the project quality before the start of the play, according to the probability of project being good α_0 .¹⁴

Analysis This is a dynamic game of symmetric information. A natural concept to use for finding the equilibria in this game is the Subgame Perfect Nash Equilibrium. Hence, the backward induction can be applied to solve for the SPNE in this game. Players' behavioral strategies are naturally mappings from the current histories into current actions.

In the equilibrium the entrepreneur will spend the money properly if his share of the updated expected return from the project is higher than payoff from “hiding” the money: $\hat{\alpha}s^E \geq \delta c$; he will offer a share contract that makes the investor exactly indifferent $(1 - s^E)\hat{\alpha} = c$; anticipating the possibility of improper money spending, the investor will only fund the project if after observing the experiment outcome it holds that $\hat{\alpha}s^E \geq \delta c$. The latter condition, together with the entrepreneur-preferred contract implies that a project gets funded if and only if the belief after the experiment outcome is

$$\hat{\alpha} \geq (1 + \delta)c.$$

Taking the above observations into account, we can write down the entrepreneur's payoff conditional on the posterior as:

$$V^E(\hat{\alpha}) = \begin{cases} (\hat{\alpha} - c), & \text{if } \hat{\alpha} \geq (1 + \delta)c \\ 0, & \text{otherwise.} \end{cases}$$

Figure 1 depicts this function. It can be seen that the payoff is a two-piece linear function being flat for $\alpha < (1 + \delta)c$ and increasing with a slope of 1 for $\alpha \geq (1 + \delta)c$.

Figure 1 is helpful for finding the optimal conditional probabilities, x and y . Call the entrepreneur's payoff from the realized posterior the ex-post payoff, and the payoff at the stage of choosing the experiment design – the ex-ante payoff. Choosing different values of x and y corresponds to choosing a pair of values of posteriors, $\hat{\alpha}^h$ and $\hat{\alpha}^l$, above and below the prior α_0 , respectively. Evaluating the ex-post payoff at those posteriors and weighting them proportionally to $\frac{\alpha_0 - \hat{\alpha}^l}{\hat{\alpha}^h - \hat{\alpha}^l}$ and $\frac{\hat{\alpha}^h - \alpha_0}{\hat{\alpha}^h - \hat{\alpha}^l}$ would give the entrepreneur's ex-ante payoff.

Result Suppose the entrepreneur chooses the precise experiment, setting $x = 1$ and $y = 0$. Then, the ex-ante payoff of the entrepreneur would be $\alpha_0(1 - c)$. Regardless of what the value of α_0 is, such payoff is an improvement over the payoff from generating

¹⁴Such timeline, however, would make the analysis more cumbersome due to the absence of proper subgames, and the need to apply a solution concept like the Perfect Bayes Equilibrium. The conclusions, nevertheless, would be essentially the same.

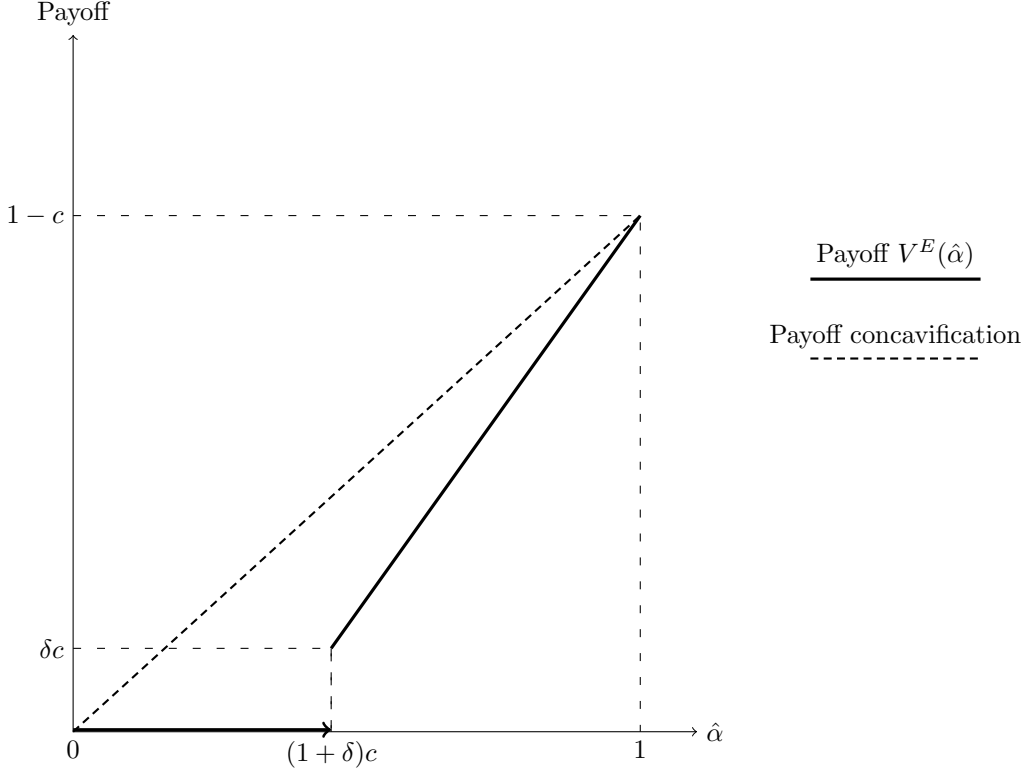


Figure 1: The entrepreneur's payoff when he makes a take-it-or-leave-it offer

no information, $(\alpha_0 - c) \times \mathbb{I}_{\{\alpha_0 \geq (1+\delta)c\}}$. So, it is safe to state that the entrepreneur at least prefers the most informative, precise experiment, to generating no new information. In fact, the following technique, called *concavification*, can be applied: find the smallest concave function, that is everywhere weakly greater than the Entrepreneur's payoff $V^E(\hat{\alpha})$.¹⁵ Evaluating this function at $\hat{\alpha} = \alpha_0$ would give the Entrepreneur's payoff from the optimal experiment design, in this case, $\alpha_0(1 - c)$.

Thus, a stronger statement can be made: the precise experiment is optimal for the entrepreneur among all experiments, not only the two-outcome experiments:

Proposition 1. *If the entrepreneur makes a take-it-or-leave-it offer to the investor, he will choose the two-outcome precise experiment ($x = 1, y = 0$).*

The proof with the reference to the results of Kamenica and Gentzkow (2011) are in the subsection A.1 of appendix A.

The intuition behind the result can be traced through the analogy with the corporate finance. For the region of posterior beliefs $\hat{\alpha} \geq (1 + \delta)c$, where the investment is possible, the entrepreneur effectively acts as the single shareholder of the project-enterprise. In the same region the investor is the single lender. The effective cost of the project funding is $(1 + \delta)c$ - the actual cost of investment, c , plus the cost of proper the entrepreneur's incentives, δc . There is no lending by the investor when the posterior belief is below

¹⁵It can be seen that in this case the concavification of $V^E(\hat{\alpha})$ is the linear function that passes through the points $(0, 0)$ and $(1, 1 - c)$.

the effective cost of financing, which leads to the project not being undertaken. The entrepreneur has a zero payoff when in expectation his project is non-viable. So, over the whole space of the beliefs the entrepreneur is effectively the residual claimant to the returns of the project. The precise experiment is the efficient choice, and the entrepreneur, acting as the residual claimant, prefers the efficient outcome.

2.3 The investor makes a take-it-or-leave-it offer

Let us now look at the case of the investor having the absolute bargaining power in negotiating the terms of the contract. In this version the investor decides whether to make an offer to the entrepreneur or not, and which offer to make.

Setup Overall the actions and the timeline are similar to those in the case of the entrepreneur having the absolute bargaining power with the addition of the investor's choice of whether to offer any financing to the entrepreneur. The remaining actions need the proper relabelling - now $s^I \in [0, 1]$ is the investor's choice of which share contract to offer; whereas $a^E \in \{0, 1\}$ is the entrepreneur's choice of whether to accept it.

Analysis As before, the posterior belief $\hat{\alpha}$ depends on the realized experiment outcome and the pair (x, y) . The only few departures from the previous analysis are: the entrepreneur will accept any offer by the investor; the investor will offer the smallest s^I among $s^I \times \hat{\alpha} \geq \delta \times c$, i.e. among contracts satisfying the entrepreneur's proper incentives, so $s^I = \delta c / \hat{\alpha}$. As before, the investor will only make an offer if $\hat{\alpha} \in [(1 + \delta)c, 1]$. Otherwise, the investor would not be able to recoup the cost of investment and the cost of providing incentives to the entrepreneur.

Anticipating such continuation from the ensuing play the entrepreneur at the stage of choosing the experiment design can compute his payoff from any posterior belief:

$$V^E(\hat{\alpha}) = \begin{cases} \delta c, & \text{if } \hat{\alpha} \geq (1 + \delta)c \\ 0, & \text{otherwise.} \end{cases}$$

Figure 2 depicts this payoff function.

Result In this setting the payoff from the precise experiment is $\alpha_0 \times \delta c$. If the prior belief about the entrepreneur's project quality is high enough, $\alpha_0 \geq (1 + \delta)c$, he can attract financing without generating any information.¹⁶ That means that for him the choice of the precise experiment is suboptimal, since providing no information yields a greater payoff of $\delta \times c$.

¹⁶Which is achieved by the noisy experiment, $x = y \in (0, 1)$.

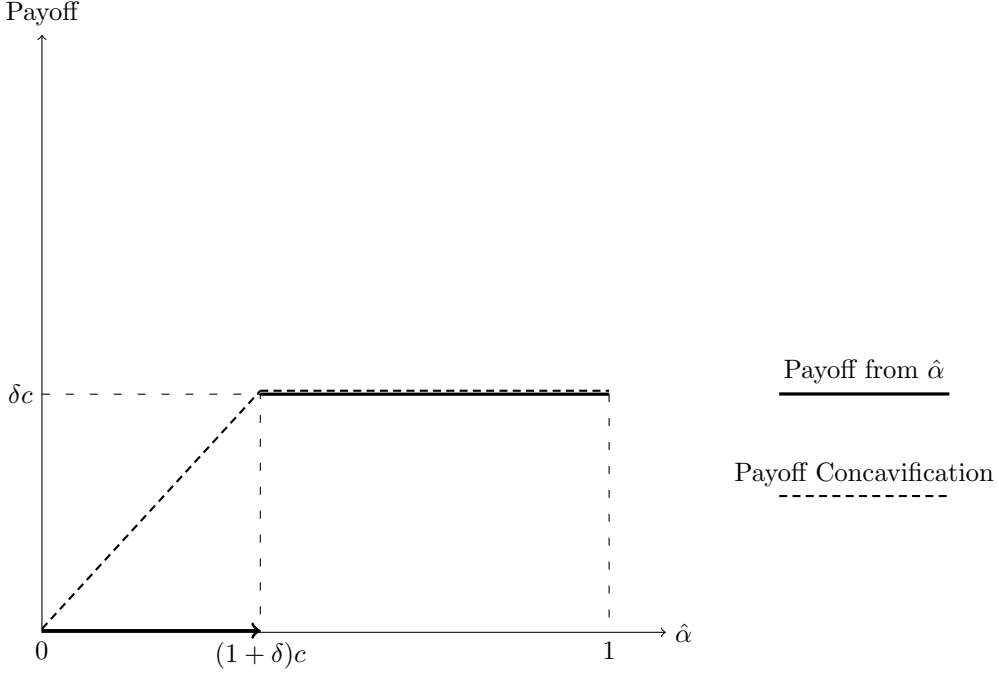


Figure 2: The entrepreneur's payoff when the investor makes a take-it-or-leave-it offer

If the entrepreneur is not able to attract financing without new information, $\alpha_0 < (1 + \delta)c$, the precise experiment is also suboptimal. Consider the following experiment structure: $x = 1$, $y = \frac{\alpha_0}{1 - \alpha_0} \left(\frac{1}{(1 + \delta)c} - 1 \right)$. Under such experiment design agents are certain that the project is bad if they observe the low outcome. If they observe the high outcome the agents believe that the project is good with probability $(1 + \delta)c$. The high experiment outcome happens with frequency $1 - \frac{\alpha_0}{(1 + \delta)c}$, and the low - with frequency $\frac{\alpha_0}{(1 + \delta)c}$. This allows the entrepreneur to achieve the ex-ante payoff of $\frac{\alpha_0}{(1 + \delta)c} \times \delta c$, because all the times the high outcome realizes, the investor is willing to offer a contract. Since $(1 + \delta)c < 1$ the entrepreneur prefers this experiment structure to the precise experiment. In fact, choosing such experiment design if $\alpha_0 < (1 + \delta)$ and generating no information otherwise is optimal for the entrepreneur.¹⁷

Proposition 2. *If the investor makes a take-it-or-leave-it offer,*

- *in case the entrepreneur cannot attract financing without an informative experiment ($\alpha_0 < (1 + \delta)c$), an experiment with two outcomes and conditional probabilities $x = 1$, $y = \frac{\alpha_0}{1 - \alpha_0} \left(\frac{1}{(1 + \delta)c} - 1 \right)$ is optimal*
- *in case the entrepreneur can attract financing without an informative experiment ($\alpha_0 \geq (1 + \delta)c$), generating no information is optimal.*

The proof of this result is in the Appendix A.2.

¹⁷In this case it can be seen that the concavification of $V^E(\hat{\alpha})$ is a two-piece linear function $\min \left\{ \hat{\alpha} \frac{\delta}{(1 + \delta)}; \delta c \right\}$. Evaluating this function at $\hat{\alpha} = \alpha_0$ for different values of α_0 shows the optimality of payoffs, achieved through the described experiments.

Note that in this case it is the investor who is the residual claimant to the returns of the project. When the investment is feasible, the entrepreneur is effectively an employee working for the constant wage of $\delta \times c$. The investor “employs” the entrepreneur to carry out the spending effort. The entrepreneur, being unable to affect the wage through any kind of negotiation, simply prefers to be “employed” as often as possible. This is achieved through some informational obfuscation.

2.4 Intermediate Bargaining Powers

Observing how the conclusion about experiment informativeness qualitatively depends on the identity of the agent who has the bargaining power, a question emerges: what is the level of the entrepreneur’s bargaining power at which the precise experiment becomes suboptimal? Besides, are there any other experiment structures that can be supported as optimal for various bargaining powers of the two parties? Studying the case of intermediate bargaining power is essential for answering these questions.

Different bargaining protocols allow for intermediate bargaining powers. One such specific protocol is the Nash Bargaining Solution. Though it takes a reduced form, it embodies several other bargaining protocols.

Recall that the terms of a contract are determined after observing the outcome of the experiment. Therefore, the ex-post expected surplus from the project, $\hat{\alpha} - c$, is going to be divided during the bargaining stage. When searching for the Nash Bargaining Solution we need to account for the entrepreneur’s incentive compatibility constraint, $\hat{\alpha} \times s \geq \delta \times c$, and the investor’s individual rationality constraint: $\hat{\alpha}(1 - s) \geq (1 + \delta)c$, as we did in the previous subsections. Hence, the space of contracts among which the Nash Bargaining Solution will search for the optimal contract is $s \in [\delta c / \hat{\alpha}, 1 - c / \hat{\alpha}]$. The solution to the problem can be called the Nash Bargaining Solution on the space of incentives compatible payoffs, or the constrained Nash Bargaining Solution.

Note that the space of contracts is a non-empty set for $\hat{\alpha} \geq (1 + \delta)c$. For $\hat{\alpha} < (1 + \delta)c$ no incentives-compatible mutually profitable contract can be signed, hence parties’ payoffs will be their reservation payoffs, zeros.

Let β denote the entrepreneur’s bargaining power. The problem statement is:

$$s(\hat{\alpha}) = \arg \max_{\frac{\delta c}{\hat{\alpha}} \leq x \leq 1 - \frac{c}{\hat{\alpha}}} \{(\hat{\alpha}x)^\beta (\hat{\alpha}(1 - x) - c)^{1-\beta}\}.$$

It is straightforward to check that when the solution is interior it is given by

$$s(\hat{\alpha}) = \beta(1 - c / \hat{\alpha}).$$

The interior solution is increasing in the posterior belief, $\hat{\alpha}$, and is always lower than the upper constraint, $1 - c / \hat{\alpha}$. So, if the solution is corner, it can only be the lower corner,

$s = \delta c / \hat{\alpha}$. This happens if the posterior belief is low enough, $\hat{\alpha} < (1 + \delta/\beta)c$. Taking into account also that the Nash Bargaining problem is not defined for $\hat{\alpha} \in [0, (1 + \delta)c)$, we can write down the above observations as

$$s(\hat{\alpha}) = \max \left\{ \frac{\delta c}{\hat{\alpha}}, \beta \left(1 - \frac{c}{\hat{\alpha}} \right) \right\} \mathbb{I}_{\{\hat{\alpha} \in [(1+\delta)c, 1]\}}.$$

Having this expression we can write down the entrepreneur's payoff as a function of posterior belief:

$$\begin{aligned} V^E(\hat{\alpha}) &= \hat{\alpha} \times s(\hat{\alpha}) \times \mathbb{I}_{\{\hat{\alpha} \in [(1+\delta)c, 1]\}} = \max \{ \delta c, \beta(\hat{\alpha} - c) \} \times \mathbb{I}_{\{\hat{\alpha} \in [(1+\delta)c, 1]\}} = \\ &= \begin{cases} 0, & \text{if } \hat{\alpha} < (1 + \delta/\beta)c \\ \delta c, & \text{if } (1 + \delta)c \leq \hat{\alpha} < (1 + \delta/\beta)c \\ \beta(\hat{\alpha} - c), & \text{if } (1 + \delta/\beta)c \leq \hat{\alpha}. \end{cases} \end{aligned}$$

The figure 3 provides the graphic depiction of the entrepreneur's payoff as a function of posteriors.

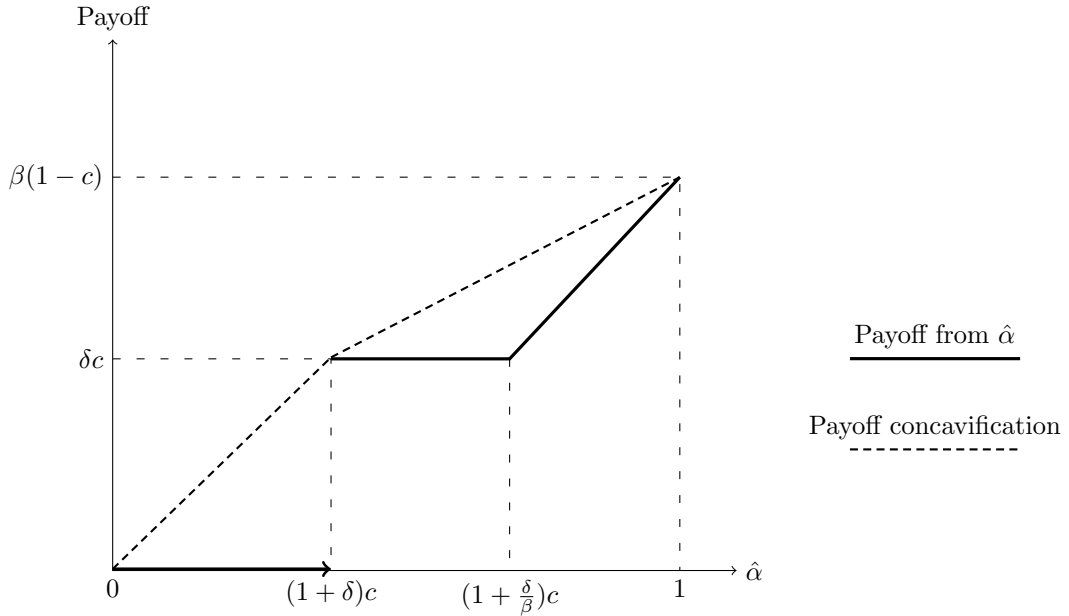


Figure 3: The entrepreneur's payoff in case of intermediate bargaining power

Three regions can be distinguished with respect to the values of posterior beliefs. For low values of the posterior the two agents are unable to sign the financing contract: the investor anticipates that she will not be able to simultaneously incentivize the proper action by the entrepreneur and recoup the cost of investment. This leads to the entrepreneur's ex-post payoff of 0.

In the middle region of the posteriors the project is feasible, but in order for the entrepreneur's incentives to be satisfied he gets a flat-level wage as an "employee" of the

investor - the constant payoff, which is greater than what would have been his “shareholder” payoff.

For the high range of beliefs the entrepreneur’s payoff is increasing in the belief value. This is where the entrepreneur is effectively a shareholder, with the size of his share determined by the bargaining power.

Note that the shape of the payoff function depends on the value of the entrepreneur’s bargaining power, β . The larger β , the more his payoff as the shareholder is sensitive to the perceived project quality. Also the higher bargaining power the lower belief-threshold, at which the entrepreneur’s payoff as a shareholder exceeds his payoff as the employee. Overall, the entrepreneur’s bargaining power increases the entrepreneur’s desire for high quality projects relative to the entrepreneur’s wish to be financed as often as possible. In other words, the bargaining power increases the entrepreneur’s role as the shareholder and decreases his role as the employee.

As a result of its influence on the entrepreneur’s incentives, the value of β has qualitative consequences for the design of the experiment that the entrepreneur prefers. For high values of the entrepreneur’s bargaining power, $\beta \geq \frac{\delta}{(1+\delta)(1-c)}$, the precise experiment structure is optimal, similarly to the case of the entrepreneur having all the bargaining power. The case of low entrepreneur’s bargaining power, $\beta < \frac{\delta c}{1-c}$, is qualitatively similar to the case of the investor making a take-it-or-leave-it offer: the region of posteriors in which the entrepreneur’s payoff is flat is prevalent.

For medium values of the bargaining power, however, unlike in the no bargaining power case, if the prior belief is high, $\alpha_0 > (1+\delta)c$, the entrepreneur has strong incentives to conduct an informative experiment. But unlike the absolute bargaining power case, the entrepreneur does not prefer the precise experiment. This is because conditionally on being financed the entrepreneur wants the beliefs about the project to be more optimistic: this will result in a larger “pie”. He does not want, however, to risk losing the financing whatsoever by generating too much information. So he wants the low outcome of the experiment to still result in contract being signed. He therefore prefers the high outcome of the experiment to result in the belief of 1, leading to the largest pie; and the low outcome to result in the belief $(1+\delta)c$, the lowest under which financing is feasible, since decreasing the low outcome belief while still securing financing allows to increase the probability of the high outcome.¹⁸

Which experiment structure allows to achieve such lottery on posteriors? In order to have the belief being equal to 1 in case of the high outcome, and $(1+\delta)c$ in the low

¹⁸We can also think that confined to the region of posteriors $[(1+\delta)c, 1]$ the entrepreneur is risk-loving on the lotteries on posterior realizations. He therefore prefers the lottery with the largest variance.

In terms of the payoff concavification, one of three functions is the concavification, depending on what the value of β is. For high β , the concavification is the linear function $\hat{\alpha}\beta(1-c)$. For medium β , it is a two-piece linear function $\min \left\{ \hat{\alpha} \frac{\delta}{1+\delta}; \frac{\beta(1-c)(\hat{\alpha}-(1+\delta)c)+\delta c(1-\hat{\alpha})}{1-(1+\delta)c} \right\}$. For low β - $\min \left\{ \hat{\alpha} \frac{\delta}{1+\delta}; \delta c \right\}$. For the proposition in this subsection the case of medium β is relevant.

outcome, the conditional probabilities need to be $x = \frac{\alpha_0 - (1+\delta)c}{\alpha_0(1-(1+\delta)c)}$ and $y = 0$. So that after observing the high experiment outcome the agents are certain that there can be no bad projects. And after observing a low experiment outcome the agents are unsure: they understand that there is a mixture of good and bad projects.

These findings can be summarized as follows:

Proposition 3. *In case of medium level of the entrepreneur's bargaining power, $\frac{\delta c}{(1-c)} \leq \beta < \frac{\delta}{(1+\delta)(1-c)}$, the entrepreneur-preferred experiment structure depends on the prior belief about the quality of the project. For $\alpha_0 < (1+\delta)c$ the optimal conditional probabilities are $x = 1$, $y = \frac{\alpha_0}{1-\alpha_0} \left(\frac{1}{(1+\delta)c} - 1 \right)$. For $\alpha_0 \geq (1+\delta)c$ the optimal conditional probabilities are $x = \frac{\alpha_0 - (1+\delta)c}{\alpha_0(1-(1+\delta)c)}$, $y = 0$.*

The proof is in the subsection A.3 of appendix A.

Investor-Preferred Bargaining Power β denotes the entrepreneur's bargaining power and is inversely related to the investor's bargaining power. Because the informativeness and the equilibrium surplus increases with β the investor might benefit from not having the absolute bargaining power. It is easy to see if the project's ex-ante quality is below the effective cost of financing, $\alpha_0 < (1+\delta)c$. If in this case the entrepreneur's bargaining power is low, $\beta < \frac{\delta}{(1+\delta)(1-c)}$, the investor gets the expected payoff of zero even when the project gets funded. But if the investor's bargaining power decreases, so that $\beta = \frac{\delta}{(1+\delta)(1-c)}$, her payoff jumps up to $\alpha_0(1-\beta)(1-c) = \alpha_0 \frac{(1-(1+\delta)c)}{1+\delta}$, because the precise experiment becomes optimal for the entrepreneur.

Even if the ex-ante quality is above the effective cost, $\alpha_0 \geq (1+\delta)c$, the investor can benefit from a limited bargaining power. Suppose that when the entrepreneur is indifferent between generating no information and choosing the higher shaded experiment, $\hat{\alpha} \in \{(1+\delta)c, 1\}$, the investor-preferred equilibrium is played. Then, among all bargaining powers such that $\beta < \frac{\delta}{(1+\delta)(1-c)}$, the investor prefers $\beta = 0$. She gets the expected payoff of $\alpha_0 - (1+\delta)c$.¹⁹ Comparing it to the payoff from $\beta = \frac{\delta}{(1+\delta)(1-c)}$, we get that the investor prefers the limited bargaining power whenever $\alpha_0 \leq \frac{(1+\delta)^2 c}{(2-c)\delta}$. Note that number is greater than $(1+\delta)c$.

The corollary summarizes these observations:

Corollary 1. *The investor prefers the limited bargaining power, $\beta = \frac{\delta}{(1+\delta)(1-c)}$, if the ex-ante project quality is not too high, $\alpha_0 \leq \frac{(1+\delta)^2 c}{(2-c)\delta}$. If $\alpha_0 > \frac{(1+\delta)^2 c}{(2-c)\delta}$ and, also, the investor-preferred equilibrium is played, she prefers the absolute bargaining power, $\beta = 0$.*

Comparative statics of informativeness Note that for medium and low bargaining powers if the project quality does not allow to finance the project without information,

¹⁹This happens because with probability $\frac{1-\alpha_0}{1-(1+\delta)c}$ the posterior $\hat{\alpha} = (1+\delta)c$ occurs, bringing the investor the payoff 0, and with the complement probability - the posterior $\hat{\alpha} = 1$, leading to the payoff of $1 - (1+\delta)c$.

that is if $\alpha_0 < (1+\delta)c$, the entrepreneur prefers the support of posteriors to be $\{0, (1+\delta)c\}$. Note also that for the combination of medium bargaining power and high value of the prior, $\alpha_0 \geq (1+\delta)c$, he prefers the support to be $\{(1+\delta)c, 1\}$. Call the experiment leading to posteriors $\{0, (1+\delta)c\}$ the lower shaded experiment, and to posteriors $\{(1+\delta)c, 1\}$ the higher shaded experiment.

Consider the following heuristic approach to measuring the informativeness of the shaded experiments. The precise experiment, $(x, y) = (1, 0)$, maximizes the sum of the payoffs of the two parties. For lower shaded experiment, $(x, y) = \left(1, \frac{\alpha_0}{1-\alpha_0} \left(\frac{1}{(1+\delta)c} - 1\right)\right)$, for higher shaded experiment - $(x, y) = \left(\frac{\alpha_0 - (1+\delta)c}{\alpha_0(1-(1+\delta)c)}, 0\right)$. Thus, for a shaded experiment only one conditional probability is different compared to the precise experiment. Thus, a plausible way to comparing the differing conditional probabilities.

For the lower shaded experiment probabilities of high outcomes in case of good state of the world coincide probabilities of high outcome in case of bad state of the world are different by $\frac{\alpha_0}{1-\alpha_0} \left(\frac{1}{(1+\delta)c} - 1\right)$. This difference disappears as the value of $(1+\delta)c$ approaches 1. This can be viewed as the informativeness of the lower shaded experiment structure being increasing in δ and c .

Exactly the opposite holds for the higher shaded experiment: probabilities of high outcomes in case of bad state of the world coincide, whereas probabilities of high outcomes in case of good state of the world differ by $\frac{\alpha_0 - (1+\delta)c}{\alpha_0(1-(1+\delta)c)}$. This difference goes down to zero as the value of $(1+\delta)c$ decreases.

To sum up, in case of the higher shaded experiment the informativeness is decreasing in both the cost parameter, c , and the moral hazard parameter, δ . In case of the lower shaded experiment the informativeness is increasing in these parameters. This means that if the entrepreneur is not able to attract financing without generating information, he will have to generate more information if the project is more costly and/or if the moral hazard problem is more severe.

A formal approach to compare informativeness is to apply the Blackwell Informativeness Criterion. In our case the experiment A is Blackwell more informative than the experiment B, if A results in the distribution of posteriors which is Second-Ordered Stochastically Dominated by that of the experiment B. So, if the two points of posterior support move further away in response to the change of the parameter, the experiment is becoming more informative, and vice versa. Applying that logic to the changes in δ and c we can formally state the following result:

Corollary 2. *For the case of medium (low) bargaining power, if the ex-ante expected project quality is low, $\alpha_0 < (1+\delta)c$ the informativeness of the entrepreneur-preferred experiment is increasing with respect to the investment cost, c , and severity of the moral hazard problem, δ . For high ex-ante expected project quality, $\alpha_0 \geq (1+\delta)c$, the informativeness of the entrepreneur-preferred experiments is decreasing with respect to c and*

δ .

For the proof, please see the subsection A.3 of appendix A.

3 Continuous Project Quality

This section analyzes the case of project qualities being continuously distributed. This section shows that the insights obtained in the setting with the two possible qualities of the project hold under a richer environment of a continuously distributed project quality. Namely, that the amount of information generated by the entrepreneur is increasing his bargaining power. Moreover, for a range of parameters the amount of information is increasing continuously with respect to the bargaining power, a conclusion which cannot be derived under the assumption of the two project qualities.

Assume that the return from the project if the money is invested into it, ω , is distributed in the interval $[0, 1]$ according to an atomless distribution with the CDF $F_\omega()$. Similarly to the previous section, the project of quality ω brings a return of ω if and only if the investment of c is spent on that project. The entrepreneur does not have this money, so he has to approach the investor. As before, neither the entrepreneur nor the investor know the realization of ω in the beginning. Let the entrepreneur choose the experiment, and denote the updated distribution after observing the experiment realization by $\hat{F}_\omega()$. After the entrepreneur had chosen the experiment structure and the two parties observe the result of the experiment they can negotiate the way they would split any positive returns after the investment is made. In case the contract is signed and the sum c is transferred to the entrepreneur, he again has a choice of whether to properly invest it, or to divert it and enjoy the payoff of δc .

Before analyzing the experiment that the entrepreneur chooses in equilibrium, it is useful to understand the properties of the socially optimal experiments. Because there are only two possible levels of investment in the project, 0 and c , the society would be best-off investing into all projects, for which the actual quality is at least c , and discarding all projects, with the actual quality less than c . Since agents are risk neutral, the exact structure of an experiment is irrelevant, as long as it separates the projects with $\omega \geq c$ from the projects with $\omega < c$. Therefore, there are multiple socially efficient experiments. For example, the experiment, which would perfectly reveal the true quality, and the experiment, which would only tell whether the actual quality is greater than, or less than c , both lead to the socially efficient investment decision. The sum of the players' payoffs from both of those experiments is $\int_c^1 (x - c) dF_\omega(x)$. Thus, the set of socially efficient experiment includes but is not limited to the perfectly revealing one.

As for the equilibrium analysis, let us focus throughout the remainder of the section on the generalized bargaining powers of the two parties, as this allows to embody the

two extreme cases of one of the parties making a take-it-or-leave-it offers, and the intermediate bargaining powers. Allow the share contract to be chosen with the help of Nash Bargaining Solution on the space of Incentive Compatible Payoffs. The solution to the bargaining problem is a function $s()$, which is a mapping from the current updated distribution over returns \hat{F}_ω and the actual return ω into the portion of the returns that goes to the entrepreneur (with the remaining portion of the returns going to the investor). Given risk-neutrality of both parties, it is without loss to look only at the shares s^* , which are constant with respect to the actual return, ω , and vary only with the ex-interim expected return.²⁰

Denote the updated expected value of the return from the project after observing the experiment outcome by $\hat{\omega}$, which is given by $\hat{\omega} = \int_0^1 x d\hat{F}_\omega(x)$. Then, the constrained Nash-Bargaining problem can be written as:

$$\begin{aligned} \max_{s^*} \{s^* \hat{\omega}\}^\beta \{(1 - s^*) \hat{\omega} - c\}^{1-\beta} \\ \text{s.t. } s^* \hat{\omega} \geq \delta c. \end{aligned}$$

Similarly to the section 2.4, the solution is

$$s^*(\hat{\omega}) = \max\{\delta c / \hat{\omega}; \beta(1 - c / \hat{\omega})\} \mathbb{I}_{\{\hat{\omega} \geq (1+\delta)c\}},$$

and the entrepreneur's payoff from the updated expected return is

$$V^E(\hat{\omega}) = \max\{\delta c; \beta(\hat{\omega} - c)\} \mathbb{I}_{\{\hat{\omega} \geq (1+\delta)c\}}.$$

As before, the entrepreneur's payoff from the updated expected value of project returns is piece-wise linear, and there are three regions. If the expected project return is low, the entrepreneur gets nothing. If the expected return is medium, the entrepreneur gets “flat wage” payoff, which is equal to the payoff from diverting the investor's money. If the expected project return is high, the entrepreneur gets a share of the return net of the investment cost.

Before the experiment outcome is observed and the contract is determined, the entrepreneur has to choose the experiment. Following the approach of Gentzkow and Kamenica (2016), the constraints on the experiment structures available to the entrepreneur are the following. First, the value of the integral of CDF of updated expectations $\int_0^t \mathbb{P}\{\hat{\omega} \leq x\} dx, t \in (0, 1)$ must lie between the integral of the CDF of means in case

²⁰To see this, suppose, for example, that there is an optimal contract, that is contingent on the actual state, ω . Write down the entrepreneur's payoff, $\int_0^1 s(t, \hat{F}_\omega) d\hat{F}_\omega(t)$. To that contract corresponds the following simple share contract, $s^* := \frac{\int_0^1 s(t, \hat{F}_\omega) d\hat{F}_\omega(t)}{\int_0^1 t d\hat{F}_\omega(t)}$, which gives the same payoff to the entrepreneur: $s^* \mathbb{E}[\omega | \omega \sim \hat{F}_\omega] = \int_0^1 s(t, \hat{F}_\omega) d\hat{F}_\omega(t)$. By risk-neutrality, the two contracts are also payoff equivalent for the investor.

of the fully precise signal, that is $\int_0^t F_\omega(x)dx, t \in (0, 1)$ from above; and the integral of the CDF of means in case of no information, given by $(t - \int_0^1 x dF_\omega)_+, t \in (0, 1)$, from below. In other words, the distribution of updated means must be a mean-preserving spread of the original distributions of project qualities.²¹

The second constraint is that the integral of the CDF of updated means must be convex. This follows simply from the fact that any CDF must be non-decreasing.²²

Denote the CDF of updated expected values by $G_{\hat{\omega}}(x) = \mathbb{P}\{\hat{\omega} \leq x\}$. Then, the entrepreneur's problem of choosing the experiment structure corresponds to choosing the CDF of posterior means, $G_{\hat{\omega}}$, subject to the restrictions described above. Denote the integral of that CDF by $\psi(t) = \int_0^t G_{\hat{\omega}}(x)dx$. The entrepreneur's problem can be written as

$$V^E = \max_{G_{\hat{\omega}}} \left\{ \int_0^1 \max\{\delta c, \beta(x - c)\} \mathbb{I}_{\{x \geq (1+\delta)c\}} dG_{\hat{\omega}}(x) \right\} \quad (\text{OF})$$

$$\text{s.t. } \psi(t) = \int_0^t G_{\hat{\omega}}(x)dx \in \left[(t - \mathbb{E}\omega)_+, \int_0^t F(z)dz \right], \quad (\text{MPC})$$

$$\psi(t) = \int_0^t G_{\hat{\omega}}(x)dx \text{ is convex.} \quad (\text{CC})$$

The problem of finding the optimal $G_{\hat{\omega}}$ can be simplified because of its useful property established in the proposition below:

Proposition 4. *The optimal experiments are characterized by the two points, $\kappa \in [0, (1 + \delta)c]$ and $\tau \in ((1 + \delta/\beta)c, 1]$. At those two points the integral of the updated means of an optimal experiment, $\psi(t) = \int_0^t G_{\hat{\omega}}(x)dx$, must be tangent to the integral of the updated means in case of fully informative experiment, $\bar{\psi}(t) = \int_0^t F(x)dx$. For the states between those two points the optimal experiment only reveals that $\omega \in (\kappa, \tau)$. Moreover, $\mathbb{E}[\omega | \kappa < \omega < \tau] = (1 + \delta)c$.*

The results of the above proposition imply that the entrepreneur's problem can be restated in terms of finding the two numbers, κ and τ , at which $\psi(t)$ is tangent to $\bar{\psi}(t)$. The proposition does not, however, specify the behavior of $\psi(t)$ outside of the region $[\kappa, \tau]$. This is because there exist multiple optimal experiments, which allow for the tangency conditions from the proposition. One particular way which allows for this tangency is to have $\psi(t) = \bar{\psi}(t)$, $t \notin (\kappa, \tau)$. This is achieved by the experiment which reveals the true quality ω for $\omega \notin (\kappa, \tau)$. Furthermore, such experiment structure is the most informative among the optimal experiment structure. These observations lead to the following result:

Corollary 3. *The most informative optimal experiment reveals the state ω completely if $\omega \in [0, \kappa] \cup [\tau, 1]$, and only reveals that $\omega \in (\kappa, \tau)$ otherwise.*

Intuitively, the reason for the condition $\mathbb{E}[\omega | \kappa < \omega < \tau] = (1 + \delta)c$ is for the invest-

²¹This constraint is referred to as (MPC), as in mean-preserving spread constraint.

²²This constraint is referred to as (CC), as in convexity constraint.

ment to take place if the experiment reveals the quality to be medium, $\omega \in (\kappa, \tau)$. This allows for a belief right at the threshold of implementability, when the quality is medium.

Overall, we now need to fully characterize the solution in terms of expressions for κ and τ . Depending on how high the ex-ante expected value of the returns is, two mutually exclusive sets of results hold.

High expected returns, $\mathbb{E}\omega \geq (1 + \delta)c$. Let us first look at the case of high ex-ante returns.²³

Proposition 5. *Suppose that $\mathbb{E}\omega \geq (1 + \delta)c$. Then if the entrepreneur's bargaining power β is high enough to satisfy*

$$\mathbb{E} \left[\omega | \omega < (1 + \delta)c \frac{\beta}{\beta - \delta(1 - \beta)} \right] \leq (1 + \delta)c, \quad (\text{BP})$$

the optimal κ and τ follow from

$$\begin{aligned} \kappa(\lambda) &= (1 + \delta)c - \frac{\delta c}{\beta - \lambda}, \\ \tau(\lambda) &= (1 + \delta)c + \frac{\delta c(1 - \beta)}{\lambda}, \end{aligned}$$

where λ is such that

$$\mathbb{E} [\omega | \kappa(\lambda) < \omega < \tau(\lambda)] = (1 + \delta)c.$$

If β is not high enough to satisfy the condition (BP), then $\kappa = 0$ and τ follows from $\mathbb{E} [\omega | \omega < \tau] = (1 + \delta)c$.

The expressions for κ and τ follow from the first-order condition (FOC) of the associated Lagrangian (L), with the conditions (BP) which are all derived in the appendix B.2.

The proposition states that if the bargaining power of the entrepreneur is high enough, there are distinct non-trivial thresholds above and below of which the true quality of the project is perfectly revealed by the outcome of the entrepreneur-preferred experiment. Between the two thresholds the experiment “pools” together the projects and reports their average value. The “pooled” project qualities on average form such an expected quality that makes the investor indifferent between providing the funding to the entrepreneur and not. If the bargaining power of the entrepreneur is low, the lower threshold is 0, which means that the low quality and the average quality projects are pulled together so that they all receive financing. This happens because the entrepreneur values the safety option of being a fixed wage employee relatively so much, that he does not want to risk losing it in favor of higher probability of being an actual shareholder.

²³Recall that in this case the entrepreneur is able to secure financing without generating any information.

In terms of investments, the experiment outcomes that reveal the quality to be low, below the threshold κ , will lead to the projects being discarded. The medium experiment outcomes, revealing the quality to lie somewhere between κ and τ , will lead to the investment being undertaken and a contract that makes the entrepreneur the fixed-wage employee. When the outcome of the experiment is high the investment is made and both the entrepreneur and the investor are effectively shareholders, with payoffs determined by the respective bargaining powers.

The investment is always made, if the bargaining power of the entrepreneur is not high enough, because the experiment structure “pools” together the lowest quality project with the average quality ones. So the projects with the quality in the range $[0, c]$ are being inefficiently invested into.

The intuition behind the result is similar to the setting with the dichotomous project quality. There are two conflicting goals that the entrepreneur pursues. On the one hand, as a potential shareholder, he would like only those projects with the actual quality being high enough implemented. On the other hand, as a potential fixed-wage employee, he would like the project to be implemented as often as possible. The first goal could be achieved by an experiment, revealing whether the actual state is above the threshold, but that would decrease the probability of funding, therefore hurting the second goal. The second goal in the setting with high ex-ante expected quality could be achieved by revealing no new information, but that would lead to low quality project being possibly implemented, potentially hurting the entrepreneur’s shareholder profits. When the bargaining power of the entrepreneur is sufficiently low, he does not care so much about the shareholder profits, resulting in a diminished informativeness of the experiment. And vice versa.

As for the efficiency of the equilibrium outcome, consider the case the entrepreneur’s bargaining power being equal to 1. Notice that generally even in this case the two thresholds, κ and τ are not equal to each other. This means that even as the entrepreneur’s bargaining power becomes absolute, there will be a non-empty measure of projects, the quality of which will not be revealed. Instead, they will be pulled together. So, for a general distribution of project qualities F_ω , the entrepreneur-preferred experiment will not reveal all the information perfectly.

The perfect knowledge of each project quality is, however, not necessary for the efficient investment decision. The efficient outcome is achieved by investing into all projects, whose actual quality is above the investment cost, $\omega \geq c$, and discarding projects otherwise. So, for the experiment structure to induce the efficient outcome we only need to know whether the actual quality is high enough, or not, every time we observe an experiment outcome. Among the entrepreneur-preferred experiments, from their outcomes we can always infer whether the actual quality is below κ , between κ and τ , or above τ . So, if one of those two points was equal to c , it would allow for the efficient investment decision.

Note that the optimal τ is always greater than $(1 + \delta/\beta)c$, as stated in Proposition 4, so τ cannot be equal to c for any set of parameters. κ , however, in principle can be equal to c . Recall the expression for κ :

$$\kappa = (1 + \delta)c - \delta \times c/(\beta - \lambda).$$

Hence, κ is equal to c if $\beta - \lambda$ is equal to 1. Since λ , being the shadow-price of the constraint, is non-negative, it follows that $\beta \leq 1 + \lambda$. So $\beta = 1$ is the necessary condition for the efficient investment decision. What is also needed, is that λ is equal to 0 in the limit, as β approaches 1. It holds, for instance, for the case of uniform distribution, as shown in the example below. For a general distribution, however, it can be that $\lambda > 0$ even as β goes to 1.

Low expected returns, $\mathbb{E}\omega < (1 + \delta)c$. For the case of lower expected return, define first the following values: let λ_{\max} follow from the condition

$$\begin{aligned} \mathbb{E}[\omega] - \mathbb{E} \left[\omega | \omega < (1 + \delta)c - \frac{\delta c}{\beta - \lambda_{\max}} \right] F_{\omega} \left((1 + \delta)c - \frac{\delta c}{\beta - \lambda_{\max}} \right) \\ = (1 + \delta)c \left(1 - F_{\omega} \left((1 + \delta)c - \frac{\delta c}{\beta - \lambda_{\max}} \right) \right). \quad (\lambda_{\max} \text{ def}) \end{aligned}$$

Define λ_{\min} as $\lambda_{\min} = \frac{(1-\beta)\delta c}{1-(1+\delta)c}$. Let $\kappa(\lambda)$ and $\tau(\lambda)$ be, as before, $\kappa(\lambda) = (1 + \delta)c - \frac{\delta}{\beta - \lambda}$, and $\tau(\lambda) = (1 + \delta)c + \frac{\delta c(1-\beta)}{\lambda}$. Define also the following conditions:

Condition 1. $\lambda_{\max} > \lambda_{\min}$

Condition 2. $\mathbb{E}[\omega | \kappa(\lambda_{\min}) < \omega] > (1 + \delta)c$

Condition 3. $\mathbb{E}[\omega | \kappa(\lambda_{\max}) < \omega < \tau(\lambda_{\max})] < (1 + \delta)c$

Then the result is:

Proposition 6. *Suppose that $\mathbb{E}\omega < (1 + \delta)c$. If the Conditions 1, 2, 3 hold, then the optimal κ and τ follow from*

$$\begin{aligned} \kappa(\lambda) &= (1 + \delta)c - \frac{\delta c}{\beta - \lambda}, \\ \tau(\lambda) &= (1 + \delta)c + \frac{\delta c(1 - \beta)}{\lambda}, \end{aligned}$$

where λ is such that

$$\mathbb{E}[\omega | \kappa(\lambda) < \omega < \tau(\omega)] = (1 + \delta)c.$$

If the conditions do not hold jointly, the optimal κ follows from

$$\int_0^{\kappa} F_{\omega}(x) dx + F_{\omega}(\kappa)((1 + \delta)c - \kappa) = (1 + \delta)c - \mathbb{E}\omega,$$

and optimal τ follows from

$$\mathbb{E}[\omega|\kappa < \omega < \tau] = (1 + \delta)c.$$

Other than the technical differences between the subcases of $\mathbb{E}[\omega] \geq (1 + \delta)c$ and $\mathbb{E}[\omega] < (1 + \delta)c$, which lead to different restrictions on the parameters for the interior solution, i.e. $\kappa > 0, \tau < 1$, the two subcases are similar to each other. Hence, a similar intuition applies here. The difference is that the entrepreneur will not be able to induce the investment to always occur. What he could do, however, when his bargaining power is low, is to induce a higher probability of investment, by sacrificing his shareholder's payoff in the favor of "employee" payoff. This would lead to $\tau = 1$.

It is a matter of algebra to check that when the optimal κ and τ are internal, the expression for κ is increasing in β and the expression for τ - decreasing in β . Since, for the most informative among optimal information structures, such changes mean larger sets of states of the world being revealed, we can state the following result:

Corollary 4. *Suppose that the bargaining power of the entrepreneur, β , is high enough so that (BP) is satisfied strictly in case of high expected returns, or conditions 1, 2, 3 hold in case of low expected returns. Then in the continuous project quality version of the model the optimal amount of information generated by the entrepreneur is strictly increasing in his bargaining power. $\beta = 1$ is the necessary condition for the efficient equilibrium outcome.*

Investor-preferred bargaining power The observation that the investor might prefer a limited bargaining power also holds for the continuous project qualities. For low ex-ante expected returns, $\mathbb{E}[\omega] < (1 + \delta)c$, if β is not high enough to satisfy either of the conditions 1-3, the entrepreneur-optimal experiment reveals only if the expected return is above, or below the effective cost of financing, $(1 + \delta)c$. So, even if the project is funded, the investor gets a 0 expected payoff from funding the project. On the contrary, if β is high and satisfies all of the conditions 1-3, the experiment outcome reveals if the true quality is above a threshold, $\tau \in ((1 + \delta/\beta)c, 1)$. So the investor's expected payoff in equilibrium is $F(\tau)(1 - \beta)(\mathbb{E}[\omega|\omega > \tau] - c)$.

For high ex-ante expected returns, consider first the boundary case, $\mathbb{E}[\omega] = (1 + \delta)c$. Let β^* such that (BP) is satisfied exactly: $\mathbb{E}\left[\omega|\omega < (1 + \delta)c\frac{\beta^*}{\beta^* - \delta(1 - \beta^*)}\right] = (1 + \delta)c$. For $\beta < \beta^*$ the corner solution is optimal, hence the condition $\mathbb{E}[\omega|\omega < \tau] = (1 + \delta)c$ implies $\tau = 1$. So, the investor's payoff is zero for $\beta < \beta^*$. On the other hand, for $\beta \geq \beta^*$ the investor's payoff can be shown to increase with β in a neighborhood of β^* : taking a full derivative of $\tau(\lambda)$ with respect to β , taking into account the constraint $\mathbb{E}[\omega|\kappa(\lambda) < \omega < \tau(\lambda)] = (1 + \delta)c$, and evaluating the derivative at $\beta = \beta^*$ we get $\frac{d\tau}{d\beta}|_{\beta=\beta^*} = -\frac{(1+\delta)c(2-\beta)}{1-\beta} < 0$. Hence, for $\beta > \beta^*$ there is a non-empty region of states,

$\omega \in (\tau, 1)$, $\tau \neq 1$, that get revealed, and that lead to the positive payoff of the investor. Overall, for $\mathbb{E}[\omega] = (1 + \delta)c$ the investor prefers some $\tilde{\beta} > \beta^*$ to all $\beta < \beta^*$. By continuity, for c being not too low, but low enough to satisfy $\mathbb{E}[\omega] \geq (1 + \delta)c$ the investor prefers some non-absolute bargaining power.

Corollary 5. *For continuous project quality the investor prefers non-absolute bargaining power $\beta > 0$ if the project cost c is not too low.*

Uniformly distributed project quality Let us apply the results of Propositions 5 and 6 to the case of uniformly distribute project quality, $\omega \sim \mathcal{U}([0, 1])$. First, consider the case of high expected return: $\mathbb{E}[\omega] = 0.5 \geq (1 + \delta)c$. The condition of proposition 5 for the bargaining power to be high enough becomes

$$\beta \geq \frac{2\delta}{1 + 2\delta}.$$

After solving for λ ,

$$\begin{aligned}\lambda &= \frac{(1 - \beta)\beta}{2 - \beta} \\ \kappa &= \left(1 - \frac{2(1 - \beta)}{\beta}\delta\right)c \\ \tau &= \left(1 + \frac{2\delta}{\beta}\right)c.\end{aligned}$$

In case the bargaining power is not high enough, the optimal corner solution is $\kappa = 0$ and $\tau = 2(1 + \delta)c$.

As for the case of low expected return, $\mathbb{E}[\omega] = 0.5 < (1 + \delta)c$, let us first determine the values for λ_{\max} and λ_{\min} . From the equation (λ_{\max} def), $\lambda_{\max} = \beta - \frac{\delta c}{1 - (1 + \delta)c}$, $\lambda_{\min} = \frac{(1 - \beta)\delta c}{1 - (1 + \delta)c}$. Then, conditions 1 – 3 hold for the same range $\beta \geq \frac{2\delta c}{1 - c}$. The expressions for the optimal internal κ and τ are the same as in the case of high expected returns. In case the bargaining power is not high enough, the optimal corner solution is $\kappa = 2(1 + \delta)c - 1$, $\tau = 1$.

Note that with respect to δ the informativeness is globally decreasing: for the corner solutions, an increase in δ leads to tighter ranges for β ; for the internal solution, an increase in δ leads to a decrease in κ and an increase in τ . As for the comparative statics with respect to c , there is no unequivocal conclusion, since an increase in c causes both the internal κ and τ to increase.

Consider the figures 4 and 5. They depict how the integral of equilibrium CDF of updated means, $\psi(t) = \int_0^t G_{\bar{\omega}}(x)dx$, change with respect to changes in the bargaining power of the entrepreneur. We can observe that as the entrepreneur's bargaining power increases the optimal $\psi(t)$ becomes closer to $\bar{\psi}(t)$, the integral in case of the fully revealing experiment. So, this is a visual demonstration of increasing informativeness with respect

to the increasing β . These graphs are plotted for the uniformly distributed project quality but similar behavior holds for a general distribution of the quality.

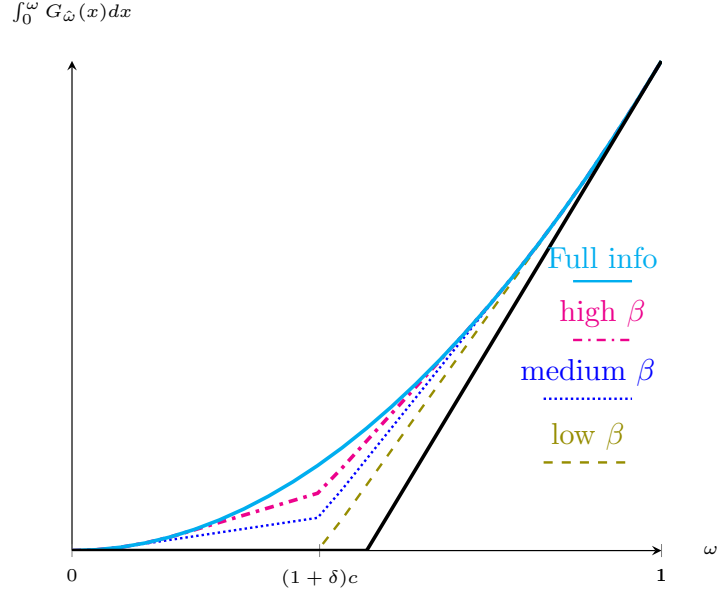


Figure 4: Integrals over distributions of posterior means, $\mathbb{E}\omega > (1 + \delta)c$

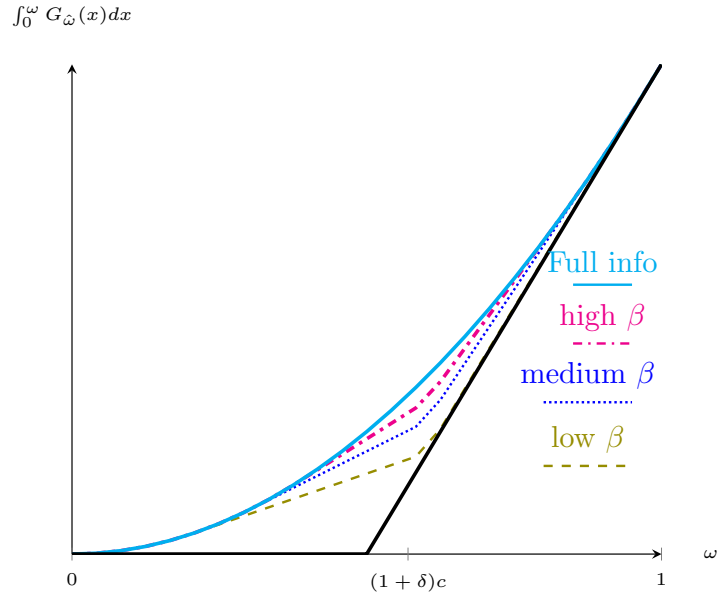


Figure 5: Integrals over distributions of posterior means, $\mathbb{E}\omega < (1 + \delta)c$

Figure 6 illustrates the Proposition 5 for the uniform example by comparing the investor's equilibrium payoff as a function of the bargaining power parameter β for different values of the project cost, c . For high and medium project costs c , the investor prefers non-absolute bargaining power, $\beta > \frac{2\delta}{1+2\delta}$. However, for project cost sufficiently low, she prefers the absolute bargaining power, $\beta = 0$.

The investor's Payoff

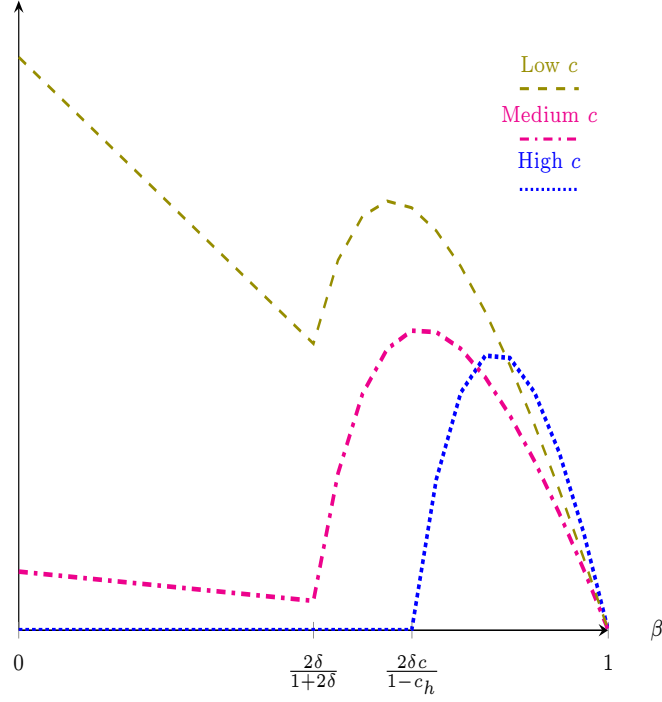


Figure 6: The scaled investor's payoffs from β , for different values of c

4 Discussion

Why do the different levels of the entrepreneur's bargaining power lead to such different qualitative results? Going back to the dichotomous project quality case, if the entrepreneur has no bargaining power the contract that the investor offers to him compensates only for the incentives to not hide the money. Regardless of the exact belief value, as long as the two agents are able to sign the contract, the entrepreneur expects to earn the same amount of money. The investor, in this case acting as the residual claimant, would like to leave to the entrepreneur as little as possible, as long as the former would still prefer to commit to the proper action. The more optimistic the beliefs of the agents, the less monetary motivation is needed for the entrepreneur's proper action to be induced. Thus, being more optimistic about the project quality does not result in a larger payoff for the entrepreneur. Therefore the entrepreneur who can attract the funding for his project without generating new information does not need to provide any additional information. The entrepreneur with initial belief about the project quality outside the feasibility region, on the other hand, does need to provide *some* information. This information takes the form of two reports: high and low. The level of optimism in case of a high report makes the investor exactly indifferent: higher levels of optimism are more costly for the entrepreneur because of the lower probabilities with which they

occur. The level of optimism in case of low report cannot be above the initial optimism, hence it needs to be as low as possible: lower optimism in case of low report increases the frequency of high report, other things equal.

As for the entrepreneur having all the bargaining power, he also cares about the likelihood of being financed in this case, too. But now, conditionally on getting financed, he would like the level of optimism to be as large as possible. This is because he now acts as the residual claimant. In other words, the contract the two agents sign can be viewed as the debt contract. For every value of the posterior belief the entrepreneur promises to return, on average, exactly the cost of investment, keeping the rest to himself. The lower the belief, the larger the riskiness of the project. The entrepreneur needs to promise the investor higher return to compensate for the higher risk in case of lower optimism. The entrepreneur therefore prefers higher values of the posterior to lower ones within the region, in which he is able to convince the investor to sign the contract. In the region of posterior beliefs where the entrepreneur is unable to attract financing he is indifferent between what the exact value of the belief is: his payoff will be 0 regardless. The entrepreneur would like the highest possible level of optimism to occur as often as possible. To achieve higher frequency of high optimism, the entrepreneur chooses the lowest level of optimism in case the low report happens.

The implication of the above results is that information about the project is efficiently learned and revealed only if the bargaining power of the entrepreneur is high enough. If the bargaining power of the entrepreneur is low the following form of inefficiency takes place: a project which can be financed without the new information generated might remain as risky as it is ex-ante; a project which cannot attract financing without the new information will result in the entrepreneur generating just enough information to get it financed, which is, however, not the full information. This particular information structure “pools” together good projects and some of the bad projects. This leads to projects financed being inefficiently risky.

Comparing the assumptions and the results of my paper to that of Azarmla and Cong (2020), my analysis compliments their findings in the following way. First, when the entrepreneur does not design the security contract in their setting, the results of Azarmla and Cong (2020) show that the information and investment decision is always inefficient. In their model the outsider learns the experiment outcome with a non-unit probability. They interpret this probability as the measure of the investor’s bargaining power. Given this interpretation, in that setting the informativeness is non-monotonic in the bargaining power; moreover, no financing can occur for the intermediate levels of bargaining power. In my paper the informativeness is monotonic in the bargaining power and the financing occurs for all sets of parameters with positive probability. Second, to restore efficiency, in my model the entrepreneur does not need the commitment power to design any sophisticated non-linear security before designing the experiment structure.

Instead, to restore efficiency in the general continuous quality case it is necessary that he has a high enough bargaining power, and in dichotomous and uniform project quality case it is also sufficient. In the continuous project quality case of my paper the contract is chosen after the experiment outcome. Thus, due to the risk neutrality it is without loss to consider simple share contracts. In a way, because of the simplicity of financial contract, the optimal experiment structure is richer: there are two thresholds, so that if the quality is between them, it is only revealed that the quality is medium, while if the quality is above the higher threshold, it gets revealed completely. Another important difference is that Azarmsa and Cong (2020) assume that the insider investor cannot influence the experiment even though the bond has already been formed in exchange for seed financing, whereas in my model the investor does not influence the experiment design because the parties have not met, which in some settings might seem more natural.

The bargaining power can be interpreted in several ways. At a micro-level, it is usually assumed that more experienced entrepreneurs have a larger say when the terms of the contract are negotiated. This means that more experienced the entrepreneurs are more likely to learn and reveal as much information about their projects as it is efficiently required. The less experienced entrepreneurs are more likely to learn less information and try to finance riskier projects. That can form a vicious circle at a micro level: the less experienced entrepreneurs will start risky projects, enjoy success less often, and in the future will have less evidence of success to back up their claims of experience, which will again result in riskier projects undertaken.²⁴

At a more macro level bargaining power is usually inversely associated with the market tightness. For example, there is more competition in the IT-industry than there is in healthcare startups.²⁵ Or, alternatively, higher bargaining power of entrepreneurs can be associated with a larger number of investors.

What the results of the model say is that in the environments rich with investment money and numerous VC-investors only projects that are certain to be good get implemented. On the other hand, in the environments with little money, or, with a small number of investors, it is likely that a lot of projects implemented are going to be risky because of being not well researched. Consider the following plausible scenario: suppose that a small number of VC-investors control a large share of the investment money in the economy. They are likely to have a lot of bargaining power. Therefore, they will only be able to invest in risky projects. More money than necessary will be lost, because of failures of some of the projects. Some of the VC-firms may have to go out of business due to this losses resulting in even fewer investors absorbing even more bargaining power.

The two regions, which could be contraposed to each other in terms of the number of

²⁴Another documented source of increased bargaining power is project being at a later stage of development, see Gompers et al. (2019).

²⁵See also Gompers et al. (2019).

players controlling the investment money are, for examples, the Silicon Valley and South Korea. In the former there are numerous independent investment funds, in the latter VC investments are controlled by the respective branches of the industrial giants, like Samsung Ventures, and Samsung NEXT.

As has been mentioned, there is empirical evidence in Cumming and Dai (2011) that a higher bargaining power of the VC-fund can lead to a lower performance of that fund. The results of my paper offer the entrepreneur's inefficient information generation as a plausible source for dis-economy from the bargaining power.

The analysis of the investor-preferred bargaining power shows that the investor prefer non-absolute bargaining power if the cost of the project is not too low. Thus, the investor might have incentives prefer to establish the reputation of being generous: leaving enough "on the table" for the entrepreneur might be beneficial in terms of investment performance. It has been documented by Bengtsson and Ravid (2015) that different U.S. states offer different terms of contracting. In particular, California-based investors offer less harsh terms to the entrepreneurs. The results of my model might offer an explanation for such phenomenon.

Plausible Calibration Exercise Recall that the non-corner solution of the continuous project quality version of the model predicts that the most informative among the entrepreneur-preferred experiments are going to reveal the project state exactly if it is below a certain threshold κ , and also if it is above a different threshold τ , $0 < \kappa < \tau < 1$. There is also going to be a mass of results pulled together somewhere at (κ, τ) .

One could think about the following calibration exercise. If we assume parametrized distributions of qualities and costs, that would induce the distribution of κ 's, τ 's, and therefore, a distribution of experiment results. Fitting the induced results distribution to some observed experiments results data would identify the parameters β and δ . Having a calibrated estimate of β would then allow to perform welfare analysis, for example, whether investors can increase their payoffs by limiting their bargaining powers and thus decreasing β .

A plausible candidate for experiment results data is some crowdfunding campaign results data.

5 Conclusion

I have studied the environment in which the entrepreneur can costlessly generate information about the investment project before spending the money on it. I have focused on the case of the entrepreneur bargaining with the investor in order to attract the outside financing. I characterize the optimal amount of information that the entrepreneur generates, depending on what his bargaining power is. In the presence of post-contractual

moral hazard, I have shown that the amount of information the entrepreneur would like to generate is increasing in his bargaining power.

The intuition for such interaction between the bargaining power and the informativeness is the following. If the new information brings great news about the quality it results in a contract that makes the entrepreneur effectively a shareholder. If the news about the project quality is average, but the project is still feasible to be financed, the entrepreneur effectively becomes a fixed-wage employee. If the bargaining power of the entrepreneur is low, the fixed-wage employee region of the beliefs dominates, and when the bargaining power is high, the shareholder region dominates. The entrepreneur in the role of an employee would like the project to get funded as often as possible, while the shareholder would prefer only the good enough projects to be funded. These two conflicting interests determine the choice of the informativeness.

The implications of our findings on the micro-level are that markets, characterized by the dominant role of investors (investors' side is the short side, and there are few powerful players) are more likely to result in the inefficient investments due to the informational channel. The informational channel can also create incentives for the investors to commit to slightly decreasing their "greed" during the negotiations, since that can lead to a greater size of the "pie" to share and, therefore, a greater payoff.

On the macro-level the informational channel has a potential to exacerbate the effects of the business cycle: during the bust phase, when the investment money is scarce, the entrepreneurs' bargaining power is likely to decrease, leading to the projects which are financed being less efficient. This may, in principle, affect the rate of the recovery.

Appendices

A Optimal experiment structure

A.1 The entrepreneur makes a take-it-or-leave-it offer

Suppose that at the stage preceding contract negotiations the entrepreneur is facing a problem that is reached, than choosing probabilities of observing high and low experiment outcomes. Let him choose a finite space of experiment realizations, S , and for each state of the world, $\omega \in \{\text{bad}, \text{good}\}$, a conditional distribution on the space of experiment realizations: $\{\mathbb{P}\{s|\omega\}\}_{s \in S}$. Note that in the main body of the text for the case of single the entrepreneur, he is only allowed to choose the two families of conditional distributions for a fixed realization space, $S = \{\text{low}, \text{high}\}$.

Recall that for each value of the posterior we know what the consequent payoff of the entrepreneur will be, since the subgame equilibrium outcome is unique. Hence, we can write down the entrepreneur's payoff as a function of the posterior value, denoting it $V^E(\hat{\alpha})$. So the problem that the entrepreneur faces at the beginning of the game is

$$\begin{aligned} & \max_{S, \{\mathbb{P}\{s|\omega\}\}} \left\{ \sum_{\omega} \left(\sum_{s \in S} V^E \left(\frac{\mathbb{P}\{s|\omega\} \times \mathbb{P}\{\omega\}}{\sum_{\omega'} \mathbb{P}\{s|\omega'\} \times \mathbb{P}\{\omega'\}} \right) \mathbb{P}\{s|\omega\} \right) \mathbb{P}\{\omega\} \right\} = \\ & \max_{S, \{\mathbb{P}\{s|\omega\}\}} \left\{ (1 - \alpha_0) \sum_{s \in S} V^E \left(\frac{(1 - \alpha_0) \mathbb{P}\{s|\text{bad}\}}{\alpha_0 \mathbb{P}\{s|\text{good}\} + (1 - \alpha_0) \mathbb{P}\{s|\text{bad}\}} \right) \right. \\ & \left. + \alpha_0 \sum_{s \in S} V^E \left(\frac{\alpha_0 \mathbb{P}\{s|\text{good}\}}{\alpha_0 \mathbb{P}\{s|\text{good}\} + (1 - \alpha_0) \mathbb{P}\{s|\text{bad}\}} \right) \right\}. \end{aligned}$$

Using the results of Kamenica and Gentzkow (2011), the problem described above is equivalent to the one, where the entrepreneur chooses the finite discrete distribution over posterior beliefs, $\mathbf{G} \in \Delta(\Delta(\Omega))$, which is Bayes-plausible, i.e. such that $\mathbb{E}_{\mathbf{G}} \hat{\alpha} = \sum_{\hat{\alpha}: \mathbf{g}(\hat{\alpha}) > 0} \hat{\alpha} \mathbf{g}(\hat{\alpha}) = \alpha_0$, where \mathbf{g} is the correspondent probability mass function. The reformulated problem is:

$$\max_{\{\hat{\alpha}: \mathbf{g}(\hat{\alpha}) > 0\}, \mathbf{g}} \sum_{\hat{\alpha}: \mathbf{g}(\hat{\alpha}) > 0} V^E(\hat{\alpha}) \mathbf{g}(\hat{\alpha})$$

s.t.

$\mathbf{g} : [0, 1] \rightarrow [0, 1]$ non-decreasing

$$\sum_{\hat{\alpha}: \mathbf{g}(\hat{\alpha}) > 0} \mathbf{g}(\hat{\alpha}) = 1$$

$$\sum_{\hat{\alpha}: \mathbf{g}(\hat{\alpha}) > 0} \hat{\alpha} \mathbf{g}(\hat{\alpha}) = \alpha_0.$$

Denote the value of the entrepreneur's problem as $V^{E*}(\alpha_0)$. Another result from Kamenica and Gentzkow (2011) states that

$$V^{E*}(\alpha_0) = \sup\{x | (x, \alpha_0) \in co(V^E)\},$$

where $co(V^E)$ is the convex closure of the graph of the entrepreneur's payoff from the posterior belief. In other words, V^{E*} is the smallest concave function, which is weakly greater than V^E . Aumann et al. (1995) call the result of applying such operator to a function its *concavification*. So, in order to determine the optimal distribution of posteriors one can first find the value from this optimal distribution. For that we would need to find the concavification of $V^E(\hat{\alpha})$.

Since the original function is a piecewise linear function, its concavification is also a piecewise linear function. In this particular case of the entrepreneur making a take-it-or-leave-it offer it actually can be seen that the function that we are looking for is the linear function connecting the two points, $(0, 0)$ and $(1, \beta(1 - c))$. This function is expressed as

$$V^{\text{Linear}}(\hat{\alpha}) = (1 - c)\hat{\alpha}.$$

Check that this is indeed the concavification. Being linear, it is concave. The values of this linear function and of the payoff function coincide at the end-points, at $\hat{\alpha} = 0$ and at $\hat{\alpha} = 1$. Thus, any function below this linear function will either fail to be concave, or fail to be weakly above the payoff function. Figure 7 provides the graphic illustration. A

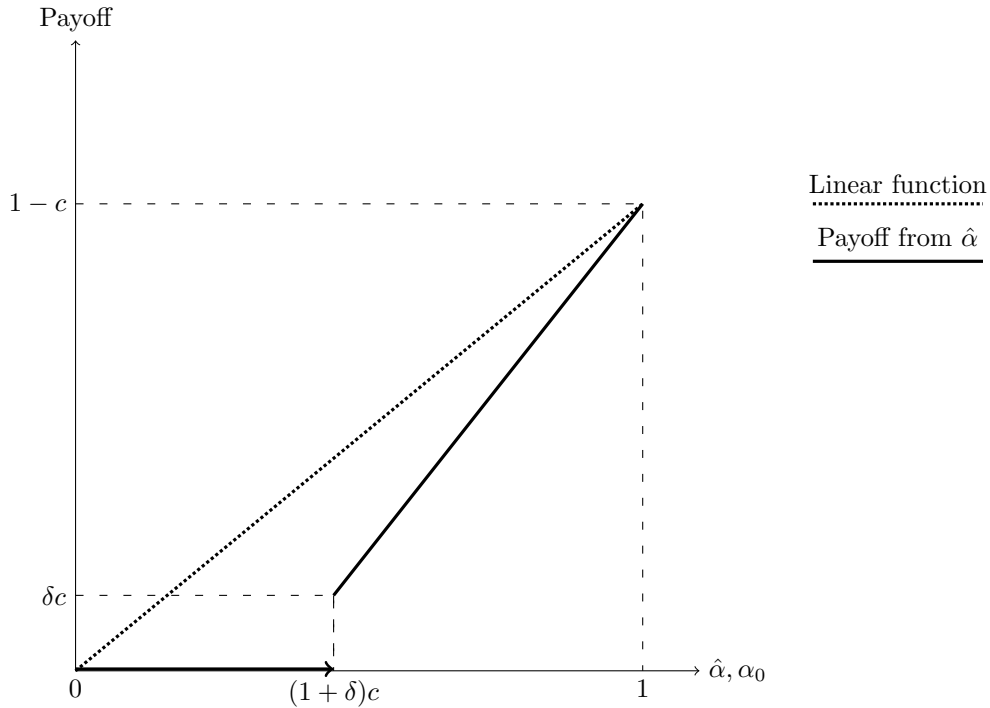


Figure 7: The entrepreneur makes a TIOLI-offer: The straight line is the concavification

simple intermediate result can be stated

Lemma A.1. *Consider the case of the entrepreneur making a take-it-or-leave-it offer and the correspondent the entrepreneur's payoff function from the realized posterior*

$V^E(\hat{\alpha}) = (\hat{\alpha} - c) \times \mathbb{I}_{\{\hat{\alpha} \geq (1+\delta)c\}}$. *Then the concavification of $V^E(\hat{\alpha})$ is a linear function $V^{Linear}(\hat{\alpha}) = \hat{\alpha}(1 - c)$*

Once the concavification of $V^E(\hat{\alpha})$ has been established, the support of the optimal distribution over posteriors can be deduced. The support of posteriors which allows for the ex-ante expected value to be on the linear function are the end-points, 0 and 1. Indeed, if only the two beliefs, 0 and 1, are the possible outcomes after observing the results of the experiment, it should hold that the posterior $\hat{\alpha} = 1$ should occur with frequency α_0 and the posterior $\hat{\alpha} = 0$ - with frequency $1 - \alpha_0$, in order for Bayes-plausibility to hold. When the belief is $\hat{\alpha} = 1$ the subgame equilibrium payoff of the entrepreneur is $1 - c$, when belief is $\hat{\alpha} = 0$ the payoff is zero. On average the entrepreneur gets $\alpha_0 \times V^E(1) + (1 - \alpha_0) \times V^E(0) = \alpha_0 \times (1 - c)$, which is exactly the value of linear function $V^{Linear}(x) = (1 - c) \times x$ evaluated at the prior ($x = \alpha_0$).

These observations help to establish the following:

Lemma A.2. *If the entrepreneur has the ability to choose arbitrary Bayes-plausible distribution of posteriors and makes a take-it-or-leave-it offer to the investor, he chooses the*

distribution of the posteriors to be
$$\begin{cases} 0 & \text{with probability } 1 - \alpha_0 \\ 1 & \text{with probability } \alpha_0 \end{cases}$$

After establishing this, it is easy to see that the two-outcome precise experiment, i.e. The one for which the high outcome only happens in the good state of the world and the low outcome only happens in the bad state of the world leads exactly to the distribution of posteriors described above. Indeed, setting $x = 1$ and $y = 0$, the high outcomes happens with ex-ante probability $\alpha_0 \times x + (1 - \alpha_0) \times y = \alpha_0$ and leads to the posterior belief $\mathbb{P}\{\text{good}|\text{high}\} = \frac{\alpha_0 \times x}{\alpha_0 \times x + (1 - \alpha_0) \times y} = 1$, whereas the low outcomes happens with probability $\alpha_0 \times (1 - x) + (1 - \alpha_0) \times (1 - y) = 1 - \alpha_0$ and results in the posterior belief of $\mathbb{P}\{\text{good}|\text{low}\} = \frac{\alpha_0 \times (1 - x)}{\alpha_0 \times (1 - x) + (1 - \alpha_0) \times (1 - y)} = 0$, exactly as desired by the entrepreneur in this case.

A.2 The investor makes a take-it-or-leave-it offer

It can be seen that in case of the investor making a take-it-or-leave-it offer the subgame²⁶ equilibrium the entrepreneur's payoff, is a two-piece linear function, consisting of two "flat" parts. It takes the value of 0 for $\hat{\alpha} \in [0, (1 + \delta)c)$ and the value of δc for $\hat{\alpha} \in [(1 + \delta)c, 1]$. So the function drawn with the dash-dotted line on the figure is the concavification

²⁶to each posterior $\hat{\alpha} \in [0, 1]$ corresponds a subgame

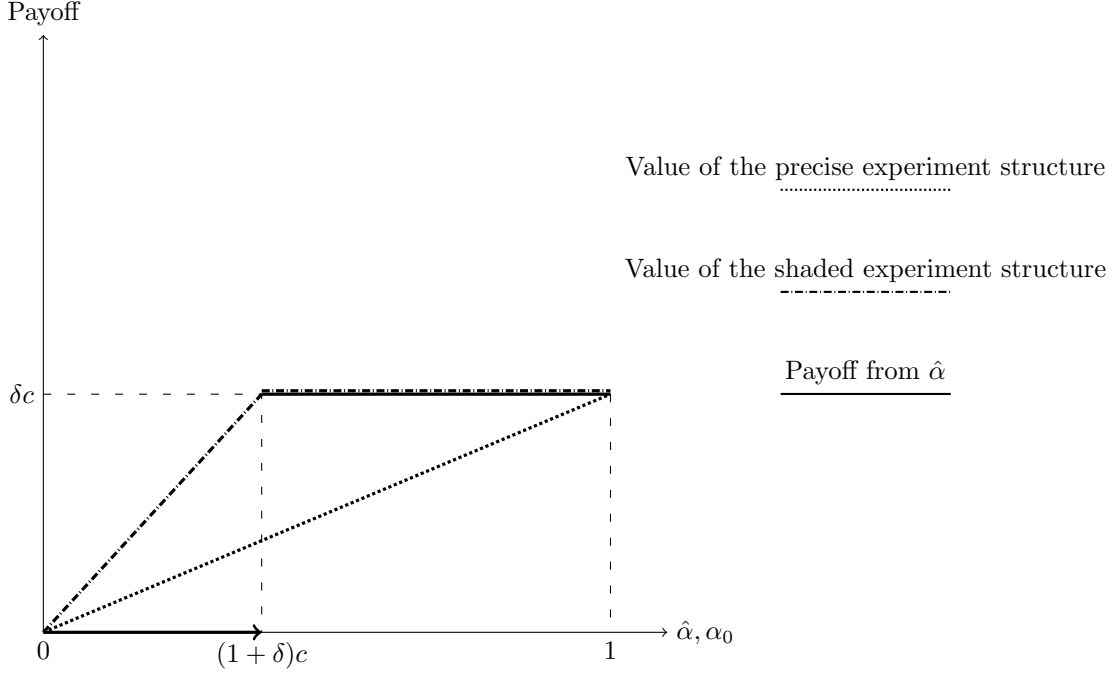


Figure 8: The investor makes a take-it-or-leave-it offer

we are looking for. The algebraic expression for that function is

$$V^{\text{2-part linear}}(\hat{\alpha}) = \frac{\delta}{(1+\delta)} \hat{\alpha} \times \mathbb{I}_{\{\hat{\alpha} < (1+\delta)c\}} + \delta c \times \mathbb{I}_{\{\hat{\alpha} \geq (1+\delta)c\}}.$$

Note that for values of the prior belief α_0 below $(1+\delta)c$ the entrepreneur would like to choose an experiment structure that would induce the support of beliefs to be $\{0, (1+\delta)c\}$. Such structure of the experiment would yield an ex-ante expected payoff on dash-dotted line.

For values of the prior α_0 above the threshold $(1+\delta)c$ the entrepreneur is indifferent between any experiment structure, which induce the support of posteriors $\subseteq [(1+\delta)c, 1]$, since any such experiment structure would result in the expected payoff of δc . Note also that an uninformative experiment can also be chosen. So, the entrepreneur does not have strong incentives to reveal any new information if the prior α_0 is high enough.

Call an experiment structure *shaded* if: the induced support of the posterior beliefs consists of two points, one of the points is always $(1+\delta)c$ and the other points is the extreme belief (0 for $\alpha_0 < (1+\delta)c$, 1 for $\alpha_0 \geq (1+\delta)c$).

The following result holds:

Lemma A.3. *If the entrepreneur has the ability to choose arbitrary Bayes-plausible distribution of posteriors and but the investor makes a take-it-or-leave-it offer after they observe the posterior realization, the entrepreneur with $\alpha_0 < (1+\delta)c$ prefers the distri-*

bution over posteriors to be

$$\hat{\alpha} = \begin{cases} 0, & \text{with probability } 1 - \frac{\alpha_0}{(1+\delta)c} \\ (1+\delta)c, & \text{with probability } \frac{\alpha_0}{(1+\delta)c}. \end{cases}$$

the entrepreneur with $\alpha_0 \geq (1+\delta)c$ is indifferent between Bayes-plausible distribution over posteriors with support $\subseteq [(1+\delta)c, 1]$. He does not have strong incentives to choose any exact one of those distributions.

After establishing the entrepreneur's preferred distribution over posteriors, it is a matter of straightforward computation to see that for $\alpha_0 < (1+\delta)c$ the structure of the two-outcome experiment proposed in the 2, namely, $x = 1, y = \frac{\alpha_0}{1-\alpha_0} \left(\frac{1}{(1+\delta)c} - 1 \right)$ results in the desired distribution over posteriors, as described above.

A.3 Nash-Bargaining Solution

Consider first the three graphs 9, 10, and 11, which help to characterize the qualitatively different results, depending on the value of the entrepreneur's bargaining power, β . Note

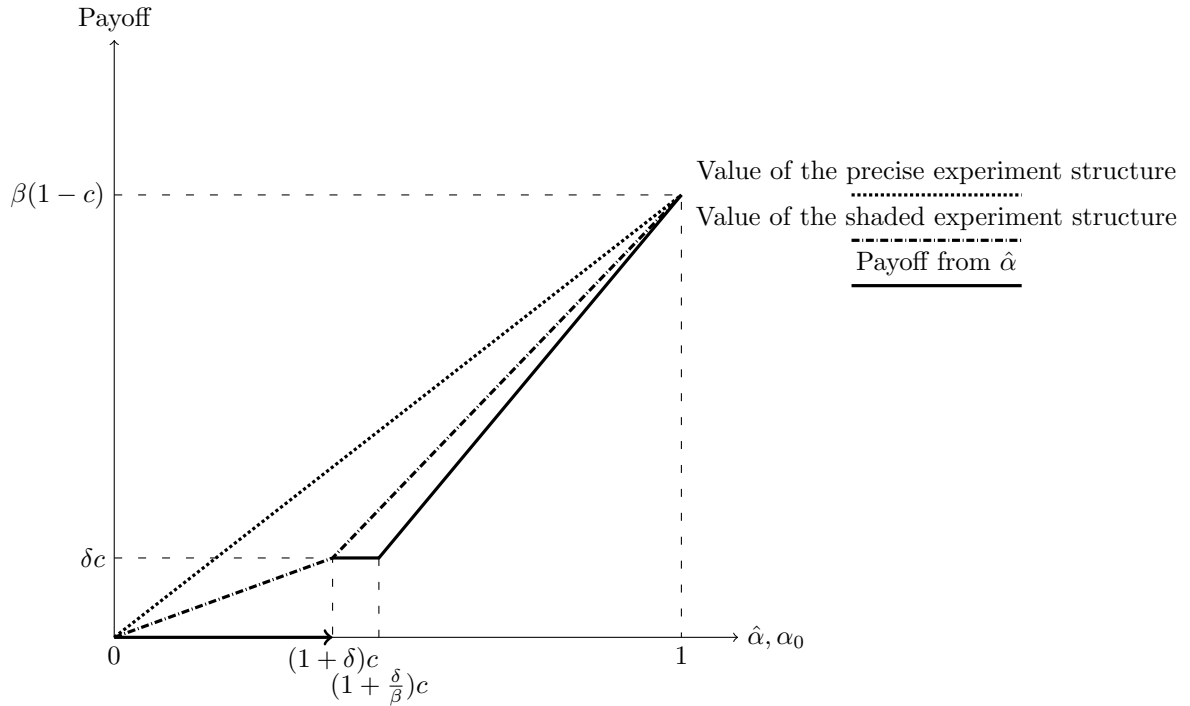


Figure 9: The precise experiment structure is optimal

that the shape of the entrepreneur's payoff from the realized posterior varies with the changes of β . The only two candidates for the concavification are: the linear function, which connects the points $(0, 0)$ and $(1, \max\{\beta(1-c), \delta c\})$; and the two-part linear, which connects the three points $(0, 0)$, $((1+\delta)c, \delta c)$ and $(1, \max\{\beta(1-c), \delta c\})$. For high values of β the former function is weakly above the payoff from the realized posteriors, which is

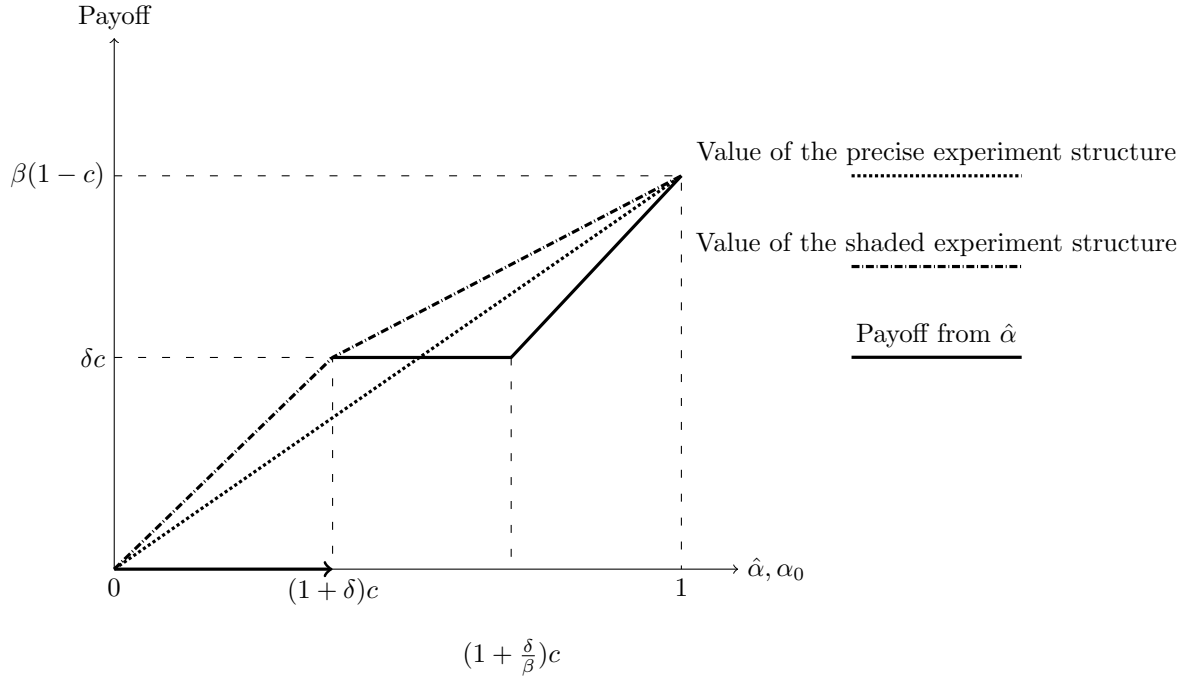


Figure 10: The shaded experiment structure is optimal

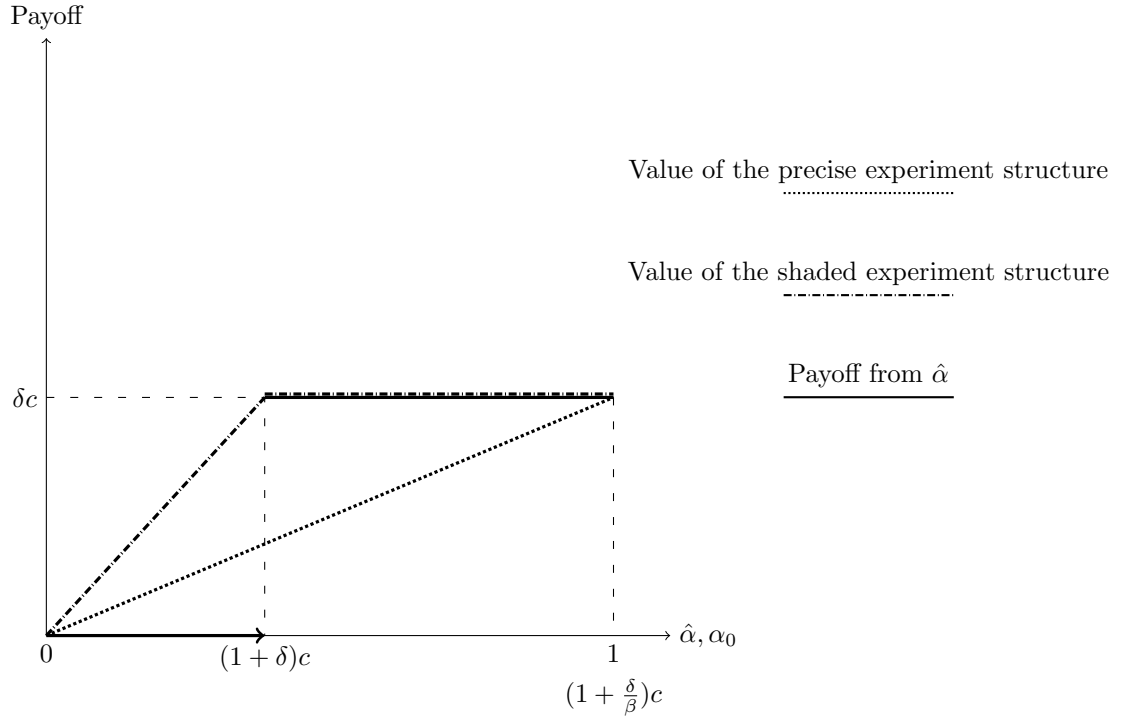


Figure 11: Case $\beta \leq \frac{\delta c}{1-c}$ is outcome equivalent to investor making a take-it-or-leave-it offer

sufficient for being the concavification; for low values of β the latter function is concave, which is in turn sufficient for this function to be the concavification. The switch happens at $\beta = \frac{\delta}{(1+\delta)(1-c)}$. At this value of β the slopes of both the linear function and the two-part linear function coincide. Also for low enough values of β the two-part linear function is flat for high values of prior. This is because the kink of the positive part of the payoff from realized posterior moves to the right of 1. This leads to the similarity with the case of the investor making a take-it-or-leave-it offer.

Altogether this establishes the following result:

Lemma A.4. - *The case of high bargaining power of the entrepreneur, $\beta \in \left[\frac{\delta}{(1+\delta)(1-c)}, 1\right]$, is equivalent to the case of the entrepreneur making a TIOLI-offer. Hence the optimal distribution over posteriors is*

$$\begin{cases} 0, & \text{with probability } 1 - \alpha_0 \\ 1, & \text{with probability } \alpha_0 \end{cases}$$

- *For the case of medium bargaining power of the entrepreneur, $\beta \in \left(\frac{\delta c}{1-c}, \frac{\delta}{(1+\delta)(1-c)}\right)$, the optimal distribution over posterior is:*

$$\begin{aligned} & - \text{If } \alpha_0 < (1 + \delta)c, \begin{cases} 0, & \text{with probability } 1 - \frac{\alpha_0}{(1+\delta)c} \\ (1 + \delta)c, & \text{with probability } \frac{\alpha_0}{(1+\delta)c} \end{cases} \\ & - \text{If } \alpha_0 \geq (1 + \delta)c, \begin{cases} (1 + \delta)c, & \text{with probability } \frac{1-\alpha_0}{1-(1+\delta)c} \\ 1, & \text{with probability } \frac{\alpha_0 - (1+\delta)c}{1-(1+\delta)c} \end{cases} \end{aligned}$$

- *The case of low bargaining power of the entrepreneur, $\beta \in \left[0, \frac{\delta c}{1-c}\right)$, is equivalent to the case of the investor making a TIOLI-offer. Hence, for $\alpha_0 < (1 + \delta)c$ the optimal distribution over posteriors is*

$$\begin{cases} 0, & \text{with probability } 1 - \frac{\alpha_0}{(1+\delta)c} \\ (1 + \delta)c, & \text{with probability } \frac{\alpha_0}{(1+\delta)c}, \end{cases}$$

and for $\alpha_0 \geq (1 + \delta)c$ the entrepreneur is indifferent between Bayes-plausible distributions over posteriors with a support $\subseteq [(1 + \delta)c, 1]$.

Note that the distribution of posteriors in the precise experiment is second order stochastically dominated by the distribution of posteriors in either case of the shaded experiment. This implies that the former experiment is more informative in the Blackwell sense, Blackwell and Girshick (1979), Borgeers (2009). Also note that for the lower shaded experiment, as the c and δ parameters increase, the informativeness of the experiment increases, and for the shaded experiment, as those parameters increase, the informativeness of the experiment decreases, because of exactly the same reasons.

B Omitted proofs for continuous project returns

B.1 Objective Function Transformation

Note that after a series of transformations the objective function (OF) subject to maximization in the entrepreneur's continuous problem in the section 3 can be rewritten:

$$\begin{aligned}\mathbb{E}_{G_{\hat{\omega}}} V^E(\hat{\omega}) &= \int_0^1 \max\{\delta c, \beta(x - c)\} \mathbb{I}_{\{x \geq (1+\delta)c\}} dG_{\hat{\omega}}(x) \\ &= \delta c + \beta(1 - c) - \beta \int_0^1 G_{\hat{\omega}}(x) dx \\ &\quad - \delta c \lim_{z \downarrow 0} G_{\hat{\omega}}((1 + \delta)c - z) + \beta \int_0^{(1+\delta/\beta)c} G_{\hat{\omega}}(x) dx.\end{aligned}$$

Note also that the value of $\psi(1) = \int_0^1 G_{\hat{\omega}}(x) dx$ is always equal to $1 - \mathbb{E}\omega$, as it is bound between $t - \mathbb{E}\omega$, and $\int_0^t F(x) dx$, evaluated at $t = 1$. So the expression subject to maximization can be divided into the term which depends on $G_{\hat{\omega}}$ and the term which is constant with respect to $G_{\hat{\omega}}$:

$$\begin{aligned}\mathbb{E}_{G_{\hat{\omega}}} V^E(\hat{\omega}) &= \text{const} - \delta c \lim_{z \downarrow 0} G_{\hat{\omega}}((1 + \delta)c - z) + \beta \int_0^{(1+\delta/\beta)c} G_{\hat{\omega}}(x) dx \\ &= \text{const} - \delta c \lim_{z \downarrow 0} \psi'((1 + \delta)c - z) + \beta \psi((1 + \delta/\beta)c),\end{aligned}$$

where the second equality uses the fact that $\psi(t) = \int_0^t G_{\hat{\omega}}(x) dx$.

Absent of the constraints (CC), and (MPC) the entrepreneur would like to minimize the value of the left derivative of $\psi()$ at point $(1 + \delta)c$; and, on the other hand, he would like to maximize the value of $\psi()$ at $(1 + \delta/\beta)c$.

B.2 Problem stated as two-variable optimization

Two observations are worth pointing out. First, when the value of $\lim_{z \downarrow 0} \psi'((1 + \delta)c - z)$ is set to its minimum, 0, the maximal possible value for $\psi((1 + \delta/\beta)c)$ can be found from the following procedure: find a linear function, tangent to $\int_0^t F_{\omega}(x) dx$, and which is equal to 0 at $(1 + \delta)c$. The value of this linear function evaluated at $(1 + \delta/\beta)c$ would yield the maximum possible value for $\psi((1 + \delta/\beta)c)$, when the constraints are accounted for. For larger candidate values of $\psi((1 + \delta/\beta)c)$ the convexity requirement of ψ will not hold.

Second, when the value of $\psi((1 + \delta/\beta)c)$ is set to its maximum, $\int_0^{(1+\delta/\beta)c} F(x) dx$, the minimum possible value for $\lim_{z \downarrow 0} \psi'((1 + \delta)c - z)$ is found from the following procedure: find a linear function, tangent to $\int_0^t F_{\omega}(x) dx$ at $(1 + \delta/\beta)c$; evaluate it $(1 + \delta)c$; find a second linear function tangent to $\int_0^t F_{\omega}(x) dx$ at a point below $(1 + \delta)c$ such that the value of this second linear function at $(1 + \delta)c$ is equal to the value of first linear function at

$(1 + \delta)c$. The slope of the second linear function would yield the smallest possible value for the left derivative of $\psi()$ at $(1 + \delta)c$. Smaller candidate values for $\lim_{z \downarrow 0} \psi'((1 + \delta)c - z)$ will not satisfy the convexity of $\psi()$.

More generically, the following series of lemmas is useful for transforming the entrepreneur's experiment design problem into a two-variable constrained optimization problem:

Lemma B.1. *An optimal function $\psi()$ is such that there is a $\kappa \in (0, (1 + \delta)c)$ and $\psi(\kappa)$ is tangent to $\int_0^\kappa F_\omega(x)dx$ and linear with the slope $F_\omega(\kappa)$ in the region $(\kappa, (1 + \delta)c)$.*

Proof 1. *Suppose this is not true. Then, $\forall t \in (0, (1 + \delta)c)$, $\psi(t) < \int_0^t F_\omega(x)dx$. Then, by convexity of $\psi()$ there is a $t^* < (1 + \delta)c$ such that a function ψ^* , which is equal to $\psi()$ everywhere, except $(t^*, (1 + \delta)c)$, and is equal to a line segment, connecting $\psi(t^*)$ and $\psi((1 + \delta)c)$, is an improvement. The left derivative of this function ψ^* at $(1 + \delta)c$ is smaller than that of $\psi()$.*

Lemma B.2. *An optimal function $\psi()$ is linear in the region $((1 + \delta)c, ((1 + \delta/\beta)c)$.*

Proof 2. *This is because the objective, given a value of left derivative at $(1 + \delta)c$, is to maximize the value at $\psi()$ at $(1 + \delta/\beta)c$. Any function with the same value at $(1 + \delta/\beta)c$, which is non-linear in the region $((1 + \delta)c, (1 + \delta/\beta)c)$ would limit the space of continuations of $\psi()$ above $(1 + \delta/\beta)c$, since it would have a higher left derivative at $(1 + \delta/\beta)c$.*

Lemma B.3. *An optimal function $\psi()$ is such that there is a $\tau \in [(1 + \delta/\beta)c, 1)$ and $\psi(\tau)$ is tangent to $\int_0^\tau F_\omega(x)dx$ and linear with the slope $F_\omega(\tau)$ in the region $((1 + \delta/\beta)c, \tau)$.*

Proof 3. *Suppose this is not true. Then, $\forall t \in [(1 + \delta/\beta)c, 1)$, $\psi(t) < \int_0^t F_\omega(x)dx$. Choose a point $t^* \in ((1 + \delta/\beta)c, 1)$: $\psi'(t^*) > \lim_{z \downarrow 0} \psi'((1 + \delta/\beta)c - z)$ in such a way that a line segment, connecting $\psi((1 + \delta)c)$ and $\psi(t^*)$ lies within the constraints. Then, replacing the function $\psi()$ in the region $((1 + \delta)c, t^*)$ with the line segment above will be an improvement, since it will yield a higher value at $(1 + \delta/\beta)c$.*

If we were to express the value of the left derivative of $\psi()$ at $(1 + \delta)c$ it would be equal to $F_\omega(\kappa)$. The value of $\psi()$ at $(1 + \delta/\beta)c$, expressed in terms of κ and τ is $\int_0^\kappa F_\omega(x)dx + F_\omega(\kappa)((1 + \delta)c - \kappa) + F_\omega(\tau)((1 + \delta/\beta)c - (1 + \delta)c)$. There are the following constraints on the values of κ and τ : $\kappa \in [0, (1 + \delta)c]$; $\tau \in [(1 + \delta/\beta)c, 1]$; κ and τ are connected by the equation $\int_0^\kappa F_\omega(x)dx + F_\omega(\kappa)((1 + \delta)c - \kappa) + F_\omega(\tau)(\tau - (1 + \delta)c) =$

$\int_0^\tau F_\omega(x)dx$. After some transformations, the Lagrangian is:

$$\begin{aligned}
\mathcal{L} = & -\delta c F_\omega(\kappa) \\
& + \beta \left(\int_0^\kappa F_\omega(x)dx - F_\omega(\kappa)\kappa + F_\omega(\tau)(1 + \delta/\beta)c - (1 + \delta)c(F_\omega(\tau) - F_\omega(\kappa)) \right) \\
& + \lambda \left(\int_\kappa^\tau F_\omega(x)dx - (1 + \delta)c(F_\omega(\kappa) - F_\omega(\tau)) - F_\omega(\tau)\tau + F_\omega(\kappa)\kappa \right) \\
& + \lambda_\kappa^1 \kappa + \lambda_\kappa^2 ((1 + \delta)c - \kappa) \\
& + \lambda_\tau^1 (\tau - (1 + \delta/\beta)c) + \lambda_\tau^2 (1 - \tau).
\end{aligned} \tag{L}$$

Non-corner solution The First-Order Condition for the non-corner solution is:

$$\begin{aligned}
\kappa(\lambda) &= (1 + \delta)c - \frac{\delta c}{\beta - \lambda} \\
\tau(\lambda) &= (1 + \delta)c + \frac{\delta c(1 - \beta)}{\lambda},
\end{aligned} \tag{FOC}$$

and λ follows from the constraint equation²⁷,

$$\begin{aligned}
\int_{\kappa(\lambda)}^{\tau(\lambda)} F_\omega(x)dx &= (1 + \delta)c(F_\omega(\kappa(\lambda)) - F_\omega(\tau(\lambda))) \\
&+ F_\omega(\tau(\lambda)) \times \tau(\lambda) - F_\omega(\kappa(\lambda)) \times \kappa(\lambda).
\end{aligned}$$

For the equation above it is straightforward to check that the derivative with respect to λ of the left-hand side of the equation is smaller than derivative of the right-hand side of the equation. For the λ as the solution of that equation to exist we would need to check that for the smallest possible λ the left-hand side is greater and for the greatest possible λ it is smaller.

Conditions for non-corner solution: $\mathbb{E}\omega > (1 + \delta)c$ What are the smallest and the greatest possible values of λ ? For the solution to be interior, κ needs to be greater than zero. This holds whenever $\lambda \leq \lambda_{\max} = \beta - \frac{\delta}{1+\delta}$. Also, τ needs to be less than 1, which in turn holds whenever $\lambda \geq \lambda_{\min} = \frac{\delta c(1-\beta)}{1-(1+\delta)c}$. The condition, for which the λ_{\max} is actually greater than λ_{\min} is $\beta \geq \frac{\delta}{(1+\delta)(1-c)}$.

After plugging in the values of λ_{\max} and λ_{\min} into the constraint equation, we get

²⁷This equation is the same as the condition $\mathbb{E}[\omega | \kappa(\lambda) < \omega < \tau(\lambda)] = (1 + \delta)c$

the conditions on the distribution and the parameters for the solution to be interior:

$$\mathbb{E} \left[\omega | \omega < (1 + \delta)c \frac{\beta}{\beta - \delta(1 - \beta)} \right] < (1 + \delta)c$$

$$\mathbb{E} [\omega | \omega > \kappa(\lambda_{\min})] > (1 + \delta)c.$$

Note that the latter of the two conditions holds trivially, since we are studying a case of $\mathbb{E}\omega > (1 + \delta)$, which is a stronger restriction. Note also that the former condition implies $\beta \geq \frac{\delta}{(1+\delta)(1-c)}$, that we required in the previous paragraph. This is because, by ω having the support $[0, 1]$, the condition $\mathbb{E} \left[\omega | \omega < (1 + \delta)c \frac{\beta}{\beta - \delta(1 - \beta)} \right] < (1 + \delta)c$ together with $\mathbb{E}[\omega] > (1 + \delta)c$ implies $(1 + \delta)c \frac{\beta}{\beta - \delta(1 - \beta)} < 1 \Rightarrow \beta > \frac{\delta}{(1+\delta)(1-c)}$. This establishes the constraint (BP). Because the condition for interior τ is always satisfied, the only corner solution possible in the current case is $\kappa = 0$.

Conditions for non-corner solution: $(1 + \delta)c \geq \mathbb{E}\omega$. It can be shown that in this case there is a positive number $\underline{\kappa} > 0$, such that it is dominated for the entrepreneur to choose $\kappa < \underline{\kappa}$. To see this, recall that the entrepreneur would like to increase the value of the integral of updated means $\psi(t) = \int_0^t G_{\hat{\omega}}(x)dx$ evaluated at $t = (1 + \delta/\beta)c$ on the one hand; and to decrease the value of the CDF of updated means, $G_{\hat{\omega}}(t)$, evaluated at $t = (1 + \delta)c$, on the other hand. Because the kink-point of the integral of updated means CDF for the least informative experiment $\underline{\psi}(t) = (t - \mathbb{E}\omega)_+$ is the average quality, $\mathbb{E}\omega$, the points $(1 + \delta/\beta)c$ and $(1 + \delta)c$ lie above the kink-point. Hence, there is a constant positive value of $\psi(1 + \delta/\beta)c = (1 + \delta/\beta)c - \mathbb{E}\omega$ which is attained for all κ in some range $\kappa \in [0, \underline{\kappa}]$. Moreover, the value of CDF of updated means at $(1 + \delta)c$ in this two-variable problem is equal to $\int_0^{\kappa} F_{\omega}(x)dx + F_{\omega}(\kappa)((1 + \delta)c - \kappa)$, which is increasing in κ . Therefore, increasing κ as long as it is in the range $[0, \underline{\kappa}]$ increases the value of the CDF of updated means at $(1 + \delta)c$, $G_{\hat{\omega}}((1 + \delta)c)$ and does not increase the value of the integral of the CDF at $(1 + \delta/\beta)c$, $\psi((1 + \delta/\beta)c) = \int_0^{(1 + \delta/\beta)c} G_{\hat{\omega}}(x)dx$. So it is suboptimal to set κ to be below $\underline{\kappa}$.

How can we establish the value of $\underline{\kappa}$? It is the smallest κ that satisfies $\int_0^{\kappa} F_{\omega}(x)dx + F_{\omega}(\kappa)((1 + \delta)c - \kappa) \geq (1 + \delta)c - \mathbb{E}[\omega]$. For graphical intuition please refer to the figure 13: Establishing $\underline{\kappa}$. Thus the only major thing that changes in the analysis is that now, instead of the constraint $\kappa \geq 0$, κ must be high enough so that $\int_0^{\kappa} F_{\omega}(x)dx + F_{\omega}(\kappa)((1 + \delta)c - \kappa) \geq (1 + \delta)c - \mathbb{E}[\omega]$. Other than replacing one of the constraints and modifying the Lagrangian, the solution remains the same.

Redefine the values λ_{\max} and λ_{\min} as in the main text. Namely, λ_{\max} follows from

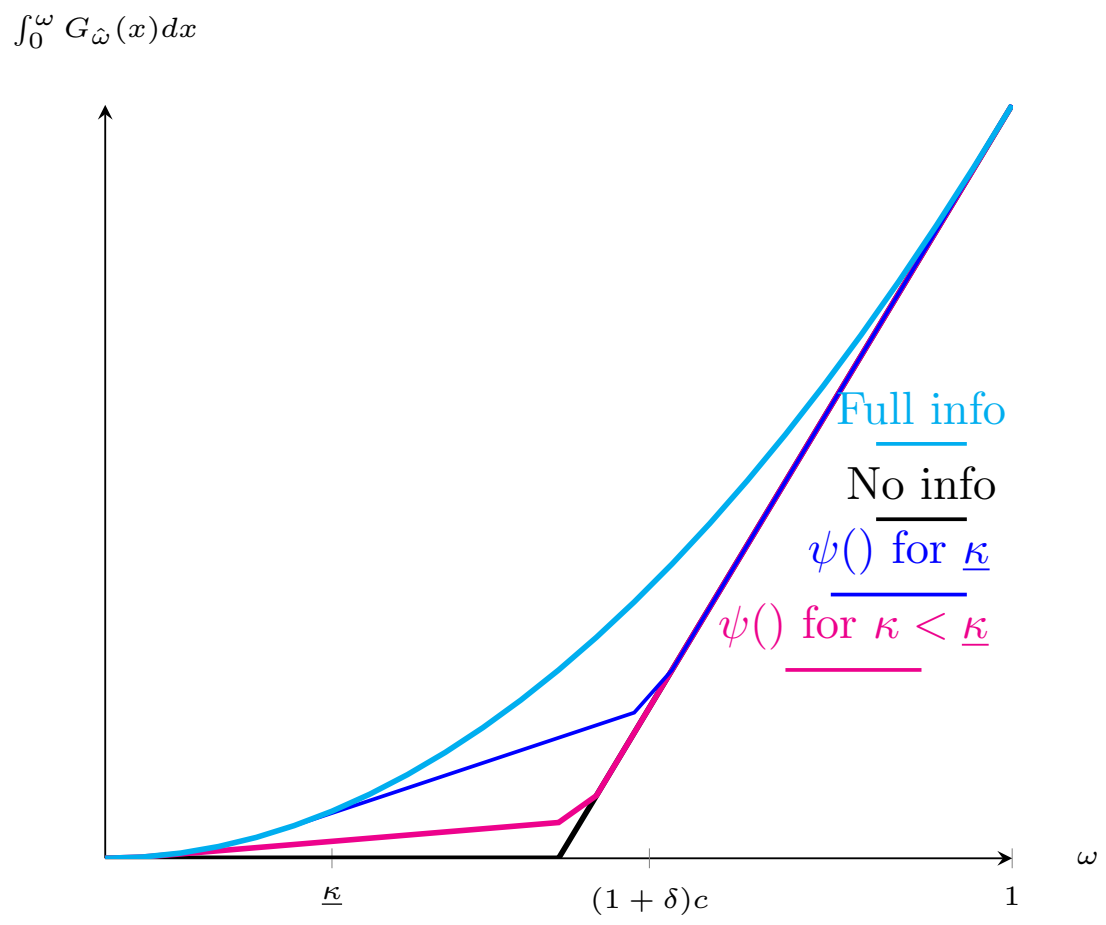


Figure 12: Establishing $\underline{\kappa}$

the expression

$$\begin{aligned}\mathbb{E}[\omega] - \mathbb{E}\left[\omega \mid \omega < (1+\delta)c - \frac{\delta c}{\beta - \lambda_{\max}}\right] &= F_{\omega}\left((1+\delta)c - \frac{\delta c}{\beta - \lambda_{\max}}\right) \\ &= (1+\delta)c \left(1 - F_{\omega}\left((1+\delta)c - \frac{\delta c}{\beta - \lambda_{\max}}\right)\right).\end{aligned}$$

Define λ_{\min} as $\lambda_{\min} = \frac{(1-\beta)\delta c}{1-(1+\delta)c}$. Let $\kappa(\lambda)$ and $\tau(\lambda)$ be, as before, $\kappa(\lambda) = (1+\delta)c - \frac{\delta}{\beta-\lambda}$, and $\tau(\lambda) = (1+\delta)c + \frac{\delta c(1-\beta)}{\lambda}$. Define the following conditions:

Condition 1. $\lambda_{\max} > \lambda_{\min}$

Condition 2. $\mathbb{E}[\omega \mid \kappa(\lambda_{\min}) < \omega] > (1+\delta)c$

Condition 3. $\mathbb{E}[\omega \mid \kappa(\lambda_{\max}) < \omega < \tau(\lambda_{\max})] < (1+\delta)c$

Comparative Statics with respect to β Applying the implicit function theorem to the constraint equation, we can get

$$\frac{d\lambda}{d\beta} = \frac{f_{\omega}(\kappa) \frac{\delta c}{(\beta-\lambda)^2} ((1+\delta)c - \kappa) + f_{\omega}(\tau) \left(-\frac{\delta c}{\lambda}\right) (\tau - (1+\delta)c)}{f_{\omega}(\kappa) \frac{\delta c}{(\beta-\lambda)^2} ((1+\delta)c - \kappa) + f_{\omega}(\tau) \left(\frac{\delta c(1-\beta)}{\lambda^2}\right) (\tau - (1+\delta)c)}.$$

It is straightforward to see that this value is less than 1. Moreover, it can be shown, that if the value of this expression is negative, the absolute value of this expression is smaller than $\frac{\lambda}{1-\beta}$. Then, differentiating $\kappa()$ and $\tau()$ with respect to parameter β we get:

$$\begin{aligned}\kappa(\lambda)'_{\beta} &= \frac{\delta c}{(\beta - \lambda)^2} \left(1 - \frac{d\lambda}{d\beta}\right) > 0 \\ \tau(\lambda)'_{\beta} &= \frac{\delta c}{\lambda} \left(-1 - \frac{1-\beta}{\lambda} \frac{d\lambda}{d\beta}\right) < 0.\end{aligned}$$

Since with the increase of κ and the decrease of τ the most informative of the functions²⁸ $\psi()$ gets closer to $\int_0^t F(x)dx$, which corresponds to full information.

²⁸There are multiple optimal $\psi()$ because multiple functions can achieve the same κ and τ , and their behavior below κ and above τ is irrelevant for the entrepreneur's payoff

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