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A Theory of Debt Maturity and Innovation

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Discussion Paper

A Theory of Debt Maturity and Innovation

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A Theory of Debt Maturity and Innovation*

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Abstract

I propose a theory of debt maturity as an incentive device to motivate innovation when contracts are fundamentally incomplete and shaped by ex-post renegotiation. The financing of innovative firms must balance two goals. On the one hand, since innovation is inherently risky, the entrepreneur must receive adequate protection after failure. Simultaneously, the firm must be liquidated when its assets can be redeployed more efficiently elsewhere. Meeting these two goals can be especially challenging when contracts are incomplete. I show how an appropriate choice of debt maturity, together with ex-post contract renegotiation, embeds a "put option" into the firm's capital structure. The put is exercised when liquidation is efficient, and it partially insures the entrepreneur against failure and thus motivates innovation. The theory has novel empirical implications for the financing patterns of innovative firms.

Keywords: Innovation, Debt maturity, Incomplete contracts, Renegotiation

JEL Codes: C78, D82, D86, G32, G33, O31, O32

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1 Introduction

There is a rapidly growing body of evidence suggesting that banks play a central role in financing innovative firms.¹ This new evidence has reversed the earlier consensus in the literature, which pointed against banks' role (and debt) in innovation financing (see Hall and Lerner, 2010). In fact, in their review of the literature on the topic, Kerr and Nanda (2015) highlight the surprising importance of banks as a source of finance for innovative firms and indicate that this is an important and underexplored area of research.

A significant difference between bank debt and publicly-issued debt is that bank debt is easier to restructure than public debt (Gertner and Scharfstein, 1991). Bank debt is usually held by a single bank or a syndicate of banks. The typical bank loan is renegotiated multiple times, with major aspects of the loan (pricing, maturity, amount, and covenants) being significantly modified (Roberts and Sufi, 2009b, Roberts, 2015). On the other hand, publicly-issued debt is typically spread out among many creditors and is plagued by free-rider problems that make it more difficult to restructure.²

In this paper, I show how firms can use their bank debt's maturity as an incentive device to motivate innovation. The mechanism I propose relies on debt renegotiation, which implies that banks have a comparative advantage as a financing source for young innovative firms.

I illustrate the core mechanism in a stylized model.³ A firm makes an externally financed investment that delivers stochastic payoffs. The investment is made on date 0, the state of the project is realized on date 1, and project payoffs are realized on date 2. The state of the project on date 1 determines whether it is efficient to continue or liquidate it. The firm can choose between two types of projects: a low-risk standard project or a high-risk novel project. The novel project is interpreted as a risky attempt to innovate, which is

¹See Cornaggia et al (2015), Robb and Robinson, (2014), Chava et al (2013), Mann (2018), Hochberg et al (2018) and Nanda and Nichola (2014) among others.

²There are legal restrictions as well, such as the Trust Indenture Act of 1939 in the US, which prevents many forms of publicly-issued debt from being effectively restructured.

³The model is in the tradition of Hart and Moore (1989), Diamond (1993), Rajan (1992), Dewatripont and Maskin (1995), Bolton and Scharfstein (1996), and Diamond (2004). They study how re-contracting and ex-post bargaining shapes the optimal debt structure.

more likely to fail, but it can also deliver a high payoff conditional on success.⁴ I assume that it is efficient to undertake the novel project, but only if it is liquidated after failure. That is, efficient risk-taking requires that failed firms are closed down, and their assets (and managers) redeployed more productively elsewhere.

Two key frictions, which are likely to plague a typical firm engaged in innovation, are critical for my analysis. First, there is a moral hazard problem: the bank can observe the project's choice but cannot establish in court that the firm was shirking on its opportunities to innovate.⁵ The moral hazard problem implies that the firm must be provided with the incentive to innovate. Second, there is a risk-shifting problem: the firm would not liquidate voluntarily because of limited liability. The risk-shifting problem implies that debt renegotiation may be necessary to avoid inefficient outcomes.

Debt maturity in this setup is defined in terms of the arrival of the firm's cash flows relative to the debt repayment date. Short-term debt matures before the firm's cash flows arrive and must be refinanced at terms that depend on the firm's state. Long-term debt matures at the same time as the cash flow from the firm assets arrives.

Since the project generates cash flows on date 2, short-term debt issued on date 0 must be refinanced on date 1. Short-term debt can be refinanced from the existing or new lenders on terms that depend on the project's state. Specifically, short-term debt is reprised favorably after the arrival of good news about the project's prospects. This serves as a reward for success and creates an incentive for the firm to undertake the high-risk novel project.

On the other hand, after the realization of bad news, the firm cannot repay the short-term debt in full. As a result, short-term lenders obtain control rights. They can remove the firm's manager from operating the project by selling the project's assets or replacing him with another (more efficient) manager. That is, short-term debt exhibits low tolerance for

⁴The novel project can represent activities such as R&D, identifying new clients, customizing products, or changing the marketing and distribution methods of the firm.

⁵Many innovative activities emerge organically within the firm and, unlike more routine tasks, cannot be mapped in advance, much less contracted upon (see Aghion and Tirole, 1994 and Hellmann and Thiele, 2011).

failure but a high reward for success.⁶

If the firm is financed with long-term debt, then in states where the project has negative NPV, the bank has no control rights: the firm has not missed a payment, and therefore, is not in default. As a result, the long-term debt must be renegotiated, and the firm “bribed” to liquidate the project voluntarily. This is accomplished by lowering the interest or by writing-off some of the debt.

Since the firm must be bribed to liquidate when termination is efficient, the bank would set a higher face value of long-term debt to break-even on its loan. That is, the expected bribe is priced in the face value of the long-term debt. The higher value of long-term debt implies that the firm must share a greater fraction of the profits with the bank when project continuation is efficient. Thus, long-term debt has a higher tolerance for failure but a lower reward for success.

The model implies that innovative firms optimally choose their maturity structure by trading-off the tolerance for failure associated with long-term debt against the reward for success related to short-term debt. When the novel project is very risky, then tolerating failure is more important than rewarding success. In this case, long-term debt provides the correct incentives for the firm to undertake the novel project. On the other hand, when the novel project is not as risky, rewarding success is more critical than tolerating failure, and the optimal maturity is short-term.

The model also implies that innovative firms’ debt maturity increases with the project’s liquidation value and the firm’s bargaining power during debt renegotiations. A higher liquidation value implies that there is a greater surplus to be gained after efficient termination. As a result, the bank is willing to give a larger bribe to the firm in exchange for liquidation. Simultaneously, holding the liquidation value fixed, greater bargaining power allows the firm to extract a greater share of the surplus during debt renegotiation. Both factors enhance the tolerance for failure associated with long-term debt and make it more attractive as a source

⁶I assume that the firm is not locked into its original creditors. Some new lenders are informed about the state of the firm and will be willing to lend as long as they can break-even in expectation.

of funds for innovative projects.

In this setting, debtor-friendly bankruptcy regimes would lead firms to lengthen their maturity since the banks will be willing to make more concessions in private workouts with the firm (i.e., the bargaining power of the firm will be higher). The reason is that the firm will reject any workout, which leaves it worse-off compared to the outcome from filing for bankruptcy. Thus, the model predicts a novel complementarity between long-term debt and debtor-friendly bankruptcy regimes.⁷

Related literature. Empirical studies have documented that failure tolerant venture capitalists and institutional investors tend to fund more innovative firms. Furthermore, legal systems that are more “forgiving” have been shown to promote entrepreneurial activity.⁸

The incentives for innovation from an optimal contracting perspective are studied in Manso (2011), who incorporates the trade-off between exploitation (applying a conventional method) and exploration (using a novel method) into a standard principal-agent model and shows that the optimal schemes to motivate innovation feature high tolerance for failure, in addition, to reward for success. Manso (2011) argues that such incentive schemes can be implemented with debtor-friendly bankruptcy regimes, but he focuses on complete contracts and does not consider the effect of debt maturity.

The mechanism in this paper is similar to that in Bebchuk and Fershtman (1994). In their model, the firm’s ability to sell the firm’s shares before bad information becomes public acts as a put option which provides insurance for the firm. In my model, the put option is provided by the renegotiation of long-term debt and the firm’s ability to extract rents in the process. This put option is priced ex-ante in the face value of long-term debt, which makes this source of funds more expensive in states where continuation is efficient.

⁷Although the model is cast in terms of debt maturity, one should keep in mind that what ultimately matters is the allocation of control rights. An alternative interpretation of short-term debt is as allowing the lender to call the loan at any time. Similarly, the contract can embed a provision that gives the bank the option to ask for collateral before the loan matures even if the borrower has not missed a payment (Gorton and Kahn, 2000).

⁸See Tian and Wang (2014) and Acharya and Subramanian (2009), among others.

This paper is also related to Diamond and He (2014), who studies the relationship between maturity and debt-overhang and shows that a greater overhang can characterize shorter maturities. There are significant differences, however. First, Diamond and He do not allow for debt renegotiation, whereas it plays a major role in my analysis. Second, they consider only projects that marginally change the payoff profile while leaving the value of existing debt approximately the same. In contrast, I consider large risk-shifting changes in the risk profiles, which redistribute value across stake-holders.

The rest of the paper is organized as follows. Section 2 presents the baseline model. Section 3 shows how innovation can be motivated with state-contingent debt. Section 4 analyzes the role of debt in boosting innovation when contracts are incomplete. Section 5 studies the factors shaping debt maturity. Section 6 draws empirical implications. Section 7 offers a discussion. Finally, Section 7 concludes. All proofs are contained in the appendix.

2 The model

I will consider an economy with a single entrepreneur and a continuum of investors. All agents are risk neutral and do not discount future payoffs. The firm has a project which requires an investment of I . The firm is penniless and must borrow the entire set-up cost. The investors are *Bertrand competitors* and would lend as long as they expect to break-even.⁹ Investors have access to a constant-returns-to-scale technology with gross per-period return normalized to one, and therefore, their opportunity cost of funds is also one.

The economy lasts for two periods and three dates: 0, 1 and 2. The investment is made at date 0. The state of the project is realized at date 1. The state of the project determines the expected payoff at date 2. In addition, at date 1 the project can either be liquidated and its asset sold or continued until the final date. Liquidation yields a payoff of L . If the project is continued then it generates a random payoff at date 2 which is given by¹⁰

⁹The assumption that the credit market is perfectly competitive at date 0 implies that loans carry zero NPV and therefore any inefficiency is ultimately borne by the entrepreneur.

¹⁰If the project is liquidated at date 1, then the cash flow at date 2 will be zero.

$$\begin{cases} Y & \text{with probability } p_i \\ 0 & \text{with probability } 1 - p_i \end{cases}$$

Thus, when the state on date 1 is i , then the expected payoff on date 2 is $p_i Y$. I assume that liquidation at date 2 yields a payoff of zero i.e. the project's assets are worthless at the final date. Thus, the decision to continue or liquidate at date 1 is characterized as follows¹¹

$$p_i Y \begin{cases} > \\ = \\ < \end{cases} L \quad \text{then} \quad \begin{cases} \text{continue} \\ \text{continue or liquidate} \\ \text{liquidate} \end{cases} \quad (1)$$

The liquidation value L is the best alternative use of the project's assets at date 1. When $L > p_i Y$ the most efficient use of the project's assets is outside of the firm and the efficient action is to liquidate the project in order to redeploy the assets elsewhere. The value of the project in state i equals $\Pi_i \equiv \max\{p_i Y, L\}$. Note that the uncertainty whether the project should be continued or liquidated is resolved at date 1. However, uncertainty remains until the final date since cash flows are random.¹²

For simplicity, I assume that the project's state can be either low p_1 , middle p_2 or high p_3 where

$$0 \leq p_1 < p_2 < p_3 \leq 1 \quad (2)$$

I further assume that the following holds

$$p_1 Y < L < p_2 Y < p_3 Y \quad (3)$$

Thus, project liquidation is efficient in the low state whereas project continuation is

¹¹If liquidation yields the same payoff as continuation I assume that the project is continued.

¹²The assumption that the liquidation value of the project is known with certainty at date 0 is for simplicity. The same results would be obtained if the liquidation value is state-contingent as long as the optimal liquidation and continuation decisions in date 1 remain unchanged.

efficient in the middle and in the high state. I will say that the project *fails* if the low state occurs in which case, liquidation is the efficient action. On the other hand, I will say that the project *succeeds* if the high state occurs (the middle state is treated as a reference where the project performs as expected). Further, I assume that $L < I$. Hence, if the firm issues debt, then the proceeds from liquidation in the low state are not enough to cover the cost of the investment. This assumption implies that the firm is financed by risky debt and makes the model interesting.

Project types. The entrepreneur can develop two types of projects α and β . Project α is the *standard project* and project β is the *novel project*. If the firm develops project α the probability of state i is $\alpha_i \in [0, 1]$. On the other hand, if the firm develops project β the probability of state i is $\beta_i \in [0, 1]$. I assume that

$$\beta_1 > \alpha_1, \quad \beta_2 < \alpha_2 \quad \text{and} \quad \beta_3 > \alpha_3 \quad (4)$$

The novel project is more likely to fail and lead to liquidation. At the same time, conditional on project continuation being efficient (i.e. the middle or the high state is realized), (4) implies the high state is more likely. That is,

$$\frac{\beta_3}{\beta_2 + \beta_3} > \frac{\alpha_3}{\alpha_2 + \alpha_3}$$

Observe that the novel project is riskier: β is dominated by α in the *second-order stochastic sense*. The assumption of three states (i.e. low, middle and high) is for clarity of exposition. Similar results hold more generally when there are more than three states.¹³

I assume that the entrepreneur incurs a private non-pecuniary cost of developing project β equal to c . The private cost of developing project α is normalized to zero. This cost can

¹³What is important, however, is that there are *at least* three states. The reason is that the critical ingredient of the model is that project β is riskier in the sense of second-order stochastic dominance: a notion of risk which requires at least three states. On the other hand, with only two states, project β , by being more likely to fail $\beta_1 > \alpha_1$, is dominated by α in the first-order stochastic sense since.

be interpreted as the foregone private benefits of exploiting the conventional project α or the additional effort and time necessary to manage the novel project. The private cost of managing the novel project can also include the greater risk of losing the benefit of control in states where the project must be liquidated.

I assume that, taking into account the efficient project liquidation decisions in (1), project β carries greater expected surplus than project α . That is,

$$\sum_{i=1}^N \beta_i \max\{p_i Y, L\} - I - c > \sum_{i=1}^N \alpha_i \max\{p_i Y, L\} - I \quad (5)$$

The efficient therefore action is to develop project β . Finally, I assume that

$$\sum_{i=1}^N \beta_i p_i < \sum_{i=1}^N \alpha_i p_i \quad (6)$$

Combined with (5) the above parameter restriction implies that the novel project will be ex-ante efficient only when it is liquidated conditional on the realization of the low state.

Information. The value of the project in any state i , and whether it should be continued or liquidated, is independent of the project type (i.e. novel or standard). Ex-ante, however, the choice of project type determines the probability distribution over different states, and therefore, the expected value of the project. The choice of project is private information known only to the entrepreneur. As a result, contracts cannot be contingent on the project type and must provide the correct incentives for the entrepreneur to undertake the novel project. On the other hand, the realization of the project state is observed by both the entrepreneur and the investor and the two parties have symmetric information from this point on.

Timeline. The sequence of events is depicted on Figure 1. At date 0 the firm offers a funding contract to the investor on a take-it-or-leave-it basis. If the contract is accepted, the start-up funds are provided and the project started. After the start-up funds have been

sunk, but before the state realized, the firm takes an action which is to develop either the standard or the novel project. On date 1 the state of the project is realized and observed by the firm and the investor. The project is then either liquidated or continued. On date 2 the project's cash flows are realized (if the firm was not liquidated on date 1) and payments allocated between the firm and the investor. The project's assets are worthless at the final date.

Remarks. First, the choice of project type (standard or novel) is private information of the entrepreneur and cannot be contracted upon. Equivalently, the project's state cannot be independently verified by third parties. Second, risk-taking (i.e. undertaking the novel project) is efficient. That is, shirking in this case represents taking the safer project. Third, it is efficient to liquidate the project in the low state and to continue it in the middle and in the high state. Fourth, I focus on parameters such that the novel project will be ex-ante efficient only when it is liquidated conditional on the low state as implied by (5) - (6).

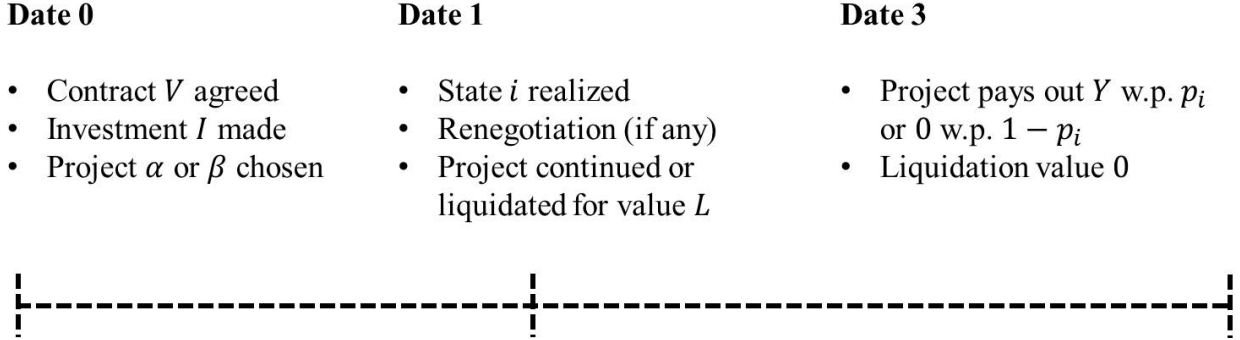
First-best. The first best outcome in this environment is straightforward: the firm undertakes the novel project on date 0 and subsequently liquidates the project on date 1 if and only if the low state is realized (which occurs with probability β_1). Otherwise, the project is continued until the final date.

Feasible contracts. I will consider two contracting environments depending on whether the financing contract between the entrepreneur and the investor can be made state-contingent (Section 3) or not (Sections 4 and 5). In both contracting environments I assume that neither the cash flows at date 2 nor the proceeds from project liquidation at date 1 can be diverted by the entrepreneur. In addition, the project's assets cannot be diverted by the entrepreneur and can be seized by the investor in the event of default.

In Section 3 I assume that the state of the project at date 1 is verifiable, and therefore, contracts can be made contingent on this information. In this case the allocation of control

rights is irrelevant since the distribution of payoffs and continuation/liquidation decision can be specified directly in the original contract. In contrast in Sections 4 I assume that the state cannot be verified and therefore contracting is incomplete. In this case, debt maturity will be shown to play a critical role.

Figure 1: Timeline.



3 Performance-sensitive debt

In this section, I assume that contracts *can* be made contingent on the state of the project. A state-contingent contract specifies date 1 and date 2 payouts from the entrepreneur to the investor in state i together with a rule of whether to continue or to liquidate the project in this particular state. Let $D_{1,i}^C(x) \geq 0$ and $D_{2,i}^C(x) \geq 0$ denote the payment from the firm to the investor in date 1 and date 2 respectively if the project is continued in state i and the realized cash flow at date 2 is $x \in \{0, Y\}$. The expected payoff to the investor in state i if the project is continued is

$$p_i [D_{1,i}^C(Y) + D_{1,i}^C(Y)] + (1 - p_i) [D_{1,i}^C(0) + D_{1,i}^C(0)] \leq p_i Y \quad (7)$$

where the weak inequality follows from limited liability: the entrepreneur has no wealth, and therefore, the expected payment to the investor cannot exceed the expected cash flow (recall that the project's assets are worthless at date 2). Next, let $D_{1,i}^L(L) \geq 0$ and $D_{2,i}^L(L) \geq 0$ denote the payment from the firm to the investor in date 1 and date 2 if the project is

liquidated in state i . The expected payoff to the investor in state i if the project is liquidated is

$$D_{1,i}^L(L) + D_{2,i}^L(L) \leq L \quad (8)$$

where, by limited liability, the payment to the investor cannot exceed the proceeds from liquidating the project at date 1, which equal L . Without loss of generality, I will set $D_{2,i}^L(L) = 0$. That is, all payments from the firm to the investor are made at date 2 even if the firm was liquidated at date 1. The contract between the firm and the investor is then summarized by

$$D \equiv \left\{ \left\{ D_{1,i}^C(x), D_{1,i}^C(x) \right\}_{x=0,Y}, D_{1,i}^C(L) \right\}_{i=1}^3 \quad (9)$$

Efficiency requires that the project is liquidated in the low state and continued in the middle state and in the high state. The expected payoff to the investor in state i therefore equals

$$V_i = \left\{ \begin{array}{l} p_i [D_{1,i}^C(Y) + D_{1,i}^C(Y)] + (1 - p_i) [D_{1,i}^C(0) + D_{1,i}^C(0)] \\ D_{1,i}^L(L) \end{array} \right\} \quad \text{as} \quad \frac{L}{Y} \left\{ \begin{array}{l} \leq \\ > \end{array} \right\} p_i$$

where V_i is the value of the firm's debt in state i . Observe that debt maturity becomes irrelevant: continuation and liquidation rules can be stipulated directly into the contract between the investor and the entrepreneur.

We can simplify the contract by observing the following. First, the entrepreneur has no wealth and cash flows (if any) are realized only at date 2. Then limited liability implies

$$D_{1,i}^C(0) = D_{1,i}^C(Y) = D_{2,i}^C(0) = 0 \quad \text{and} \quad D_{2,i}^C(Y) \leq Y \quad (10)$$

Henceforth, I will drop the time index and denote the payment from the firm to the

investor in state i by D_i . The contract between the firm and the investor can then be summarized by three numbers $D \equiv \{D_i\}_{i=1}^3$. In order to invest in the firm, the investor must be promised at least I in expectation. That is,

$$\sum_{i=1}^N \beta_i V_i \geq I \quad (11)$$

Since the credit market is competitive, (11) will hold with equality in equilibrium. The objective of the entrepreneur is to develop the novel project since it carries greater surplus than the standard project. Since the choice of project type is not observed by the investor, the contract in (9) must satisfy an incentive constraint which ensures that the firm has an incentive to develop the novel project after borrowing the funds from the investor. That is,

$$\sum_{i=1}^N \beta_i (\Pi_i - V_i(D)) - I - c \geq \sum_{i=1}^N \alpha_i (\Pi_i - V_i(D)) - I \quad (12)$$

Hence, the firm can finance project β if and only if there is a contract jointly satisfying (7) - (8) and (10) - (12).

3.1 The optimal contract

The incentive-compatibility constraint in (12) can be equivalently expressed as

$$\underbrace{\left\{ \sum_{i=1}^N (\beta_i - \alpha_i) \max \{p_i Y, L\} - c \right\}}_{\Delta S} - \underbrace{\left\{ \sum_{i=1}^N (\beta_i - \alpha_i) V_i \right\}}_{T(V)} \geq 0 \quad (13)$$

where $\{V_i\}_{i=1}^N$ is the payoff profile generated by the contract between the entrepreneur and the firm. I will say that a given contract is *optimal* if it maximizes the left-hand side of (13), which is equivalent to minimizing $T(V)$. That is,

$$\min_{\{V_i\}_{i=1}^N} \sum_{i=1}^N (\beta_i - \alpha_i) V_i \quad (14)$$

subject to the investor's break-even condition when the firm develops project β

$$\sum_{i=1}^N \beta_i V_i = I \quad (15)$$

and the limited liability constraints

$$0 \leq V_i \leq \max \{p_i Y, L\} \quad (16)$$

A payoff profile which solves the program in (14) - (16) will be denoted $V^* \equiv \{V_i^*\}_{i=1}^N$. Henceforth, I will refer to the payoff profile V^* and the contract D^* which generates it interchangeably. Let $l_i \equiv \frac{\beta_i}{\alpha_i}$ denote the *likelihood ratio* of state i . We have the following result.

Proposition 1. The payoff profile V^* must jointly satisfy the following set of conditions.

For any $l_i < l_j$

- (i) $V_i^* = 0$ and $V_j^* = 0$,
- (ii) $V_i^* = \Pi_i$ and $V_j^* \in [0, \Pi_j]$ and
- (iii) $V_i^* \in (0, \Pi_i)$ and $V_j^* = 0$.

If the likelihood ratio of state i is greater than the likelihood ratio of state j then the occurrence of state i is more indicative that the firm developed project β than the occurrence of state j . The payoff profile must then penalize state j relative to state i . This can be achieved by reducing the payoff to the investor in state i by $\frac{\epsilon}{\beta_i}$ and increasing the payoff to the investor in state j by $\frac{\epsilon}{\beta_j}$. Such a perturbation, whenever feasible (i.e. when it does not violate limited liability in (16)) leaves the expected payoff to the investor equal to I , while it will also lower the objective function in (14).

If V^* is optimal, such a perturbation should not be possible which is true if and only if the conditions in (i) - (iii) are jointly satisfied. Stated differently, the conditions in (i) - (iii) imply that when $l_i < l_j$ it must not be possible to *cross-subsidize* from state i to state j . That is, it is not possible to decrease the payoff in state i and increase it in state j in such

a way that the expected payoff to the investor remains unchanged.

We can use the optimality conditions in (i) - (iii) to derive the optimal state-contingent payoff plan for any possible ranking of the likelihood ratios generated by projects α and β . Specifically, if the likelihood ratios satisfy the *Monotone Likelihood Ratio Property (MLRP)*, $l_1 < l_2 < l_3$, then the optimal payoff profile takes the Live-Or-Die form of Innes (1990):

$$V_1^* = \min \left\{ \frac{I}{\beta_1}, L \right\}, \quad V_2^* = \min \left\{ \frac{I - \beta_1 V_1^*}{\beta_2}, p_2 Y \right\}, \quad V_3^* = \frac{I - \beta_1 V_1^* - \beta_2 V_2^*}{\beta_3}. \quad (17)$$

This payoff profile performs cross-subsidization: from the low to the middle state, from the middle to the high state and from the low to the high state. In other words, the contract in (17) provides maximum reward for success and maximum penalty for failure among all payoffs that satisfy limited liability and ensure that the investor breaks-even in expectation. The reason such a contract is optimal in this case is that higher states are uniformly more likely to occur under project β when the likelihood ratio satisfies the MLRP.

However, the probability distribution over states generated by the novel and the standard project satisfy the relation in (4) which implies that α dominates β in the second-order stochastic sense, and therefore, the MLRP does not hold. As a result, the contract in (17) is no longer optimal. In particular, we have $l_2 < \min \{l_1, l_3\}$.¹⁴ First, if $l_2 < l_1 < l_3$ then optimal contract is given by

$$V_1^* = \min \left\{ \frac{I - \beta_2 V_2^*}{\beta_1}, L \right\}, \quad V_2^* = \min \left\{ \frac{I}{\beta_2}, p_2 Y \right\}, \quad V_3^* = \frac{I - \beta_1 V_1^* - \beta_2 V_2^*}{\beta_3}. \quad (18)$$

I will refer to (19) as the *Reward for Success* contract and denote it by V_{RS} . Such a contract performs cross-subsidization from the middle to the low state, from the middle to the high state and from the low to the high state. On the other hand, if $l_2 < l_3 < l_1$, the optimal contract is

¹⁴For example, let $\alpha = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, then case $l_2 < l_1 < l_3$ occurs for $\beta = (\frac{1}{3} + \epsilon, \frac{1}{3} - 3\epsilon, \frac{1}{3} + 2\epsilon)$ whereas $l_2 < l_3 < l_1$ occurs for $\beta = (\frac{1}{3} + 2\epsilon, \frac{1}{3} - 3\epsilon, \frac{1}{3} + \epsilon)$ for $\epsilon \in (0, \frac{1}{9})$.

$$V_1^* = \frac{I - \beta_2 V_2^* - \beta_3 V_3^*}{\beta_1}, \quad V_2^* = \min \left\{ \frac{I}{\beta_2}, p_2 Y \right\}, \quad V_3^* = \min \left\{ \frac{I - \beta_2 V_2^*}{\beta_3}, p_3 Y \right\}. \quad (19)$$

I will refer to (19) as the *Tolerance for Failure* contract and denote it by V_{TF} . This contract performs cross-subsidization from the middle to the low state, from the middle to the high state and from the high to the low state. The discussion in this section is summarized in the following proposition.

Proposition 2. The contract is optimal if it generates a payoff profile for the investor V^* characterized by

$$V^* = \left\{ \begin{array}{c} V_{RS} \\ \in \{V_{RS}, V_{TF}\} \\ V_{TF} \end{array} \right\} \quad \text{as} \quad l_1 \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} l_3 \quad (20)$$

where V_{RS} is given by (18) and V_{TF} is given by (19).

When choosing the optimal contract the entrepreneur trades-off tolerating failure on the one hand and rewarding success on the other. This choice is non-trivial since the contracts in (18) and (19) imply

$$\underbrace{L - V_{1,RS} < L - V_{1,TF}}_{TF \text{ is better after failure}} \quad \text{and} \quad \underbrace{p_3 Y - V_{3,RS} > p_3 Y - V_{3,TF}}_{RS \text{ is better after success}}$$

First note the common elements between (18) and (19): they both perform cross-subsidization from the middle to the high state and from the middle to the low state. This can be contrasted with the contract in (17) which performs cross-subsidization from lower to higher states. The reason for this difference is straightforward: the middle state has the lowest likelihood ratio, and therefore, its realization is most indicative that the firm selected project α . At the same time, the contract in (18) performs cross-subsidization from the low to the high state because $l_1 < l_3$ whereas the contract which generates the payoff profile in (19) performs cross-subsidization from the high to the low state because $l_1 > l_3$.

4 Debt maturity

In this section, I assume that state-contingent contracts are not feasible. As pointed out in the Introduction, the projects of innovative firms tend to be opaque which makes them especially difficult to verify by outside parties. In addition, the financiers and the firm might be unwilling to take actions which reveal information about the project publicly for fear it will reduce the value of the firm. In this case explicit performance-sensitive debt contracts in (18) and (19) will not be feasible.

I show how the firm can structure its debt maturity so as to provide incentives to undertake the efficient action. Specifically, the firm's choice of debt maturity must meet two objectives. First, the project must be liquidated when this is efficient. Second, the entrepreneur must have an incentive to develop the novel project.

In the analysis to follow, one must keep in mind the following. *(i)* Since the entrepreneur has limited liability he will not voluntarily liquidate the project unless the investor intervenes. In order to prevent inefficient continuation the investor must either force the entrepreneur to liquidate (i.e. by refusing to renew credit in case of short-term debt) or by bribing the entrepreneur to liquidate by writing-off debt (in case of long-term debt). *(ii)* Since the credit market is competitive at date 0 the loan issued to the entrepreneur has zero NPV because of the investor's break-even condition, Hence, any ex-post inefficiencies are borne by the entrepreneur. Next, I derive the payoff profiles associated with short-term and long-term debt.

4.1 Short-term and long-term debt

The debt contract specifies that the investor transfers funds I to the firm upfront in exchange for a future repayment either on date 1 or on date 2. Debt holders are given control rights conditional on the contracted upon repayment not being met i.e. when the firm defaults on its payment. Thus, in the event of default the control rights are transferred to the investor

who has the right to liquidate the project's assets in order to collect repayment. At the same time, as long as repayments on the debt are being met, the firm retains control right.

Short-term debt is debt issued at date 0 and maturing at date 1 with face value R_{01} . The repayment on date 1 comes either from refinancing at an interest rate contingent on the state of the firm or from the proceeds of project liquidation. The face value of short-term debt R_{01} is set so that the expected repayment to an investor who lends an amount I have an expected payoff equal to I . A refinanced short-term debt is debt issued at date 1 and maturing at date 2 with face value which is contingent on the state of the firm at date-1. I assume that, whenever possible, short-term debt is refinanced at terms which ensure that the expected payoff on short-term debt is equal to R_{01} . If the firm cannot repay in full at date 1, then I assume that all decisions in this case are made in the interest of short-term debt owners since they have a control right to force project liquidation.

Long-term debt is debt issued at date-0 and maturing at date 2 after the firm's cash flows (if any) have been realized. The face value of long-term debt R_{02} is agreed upon at date-0 and set so that the investor who lends an amount I have an expected payoff equal to I . I assume that the investor and the firm can freely renegotiate any aspect of their debt contract such as interest rate, face value, maturity and so on.

4.2 Payoff profiles

Suppose the firm borrows I at date 0 by issuing short-term debt with face value R_{01} . At date 1 the state of the project is realized and observed by the investor and the firm. If $p_1Y < L$ project liquidation is efficient. The investor forces liquidation by refusing to roll-over credit and the firm is closed down. The investor gets L and the firm gets 0.

If $L \leq p_iY < R_{01}$ project continuation is efficient, but the firm is insolvent. That is, all future cash flows are pledged to the investor. The firm gets 0 and the investor gets p_iY . Third, if $R_{01} < p_iY$ short-term debt is rolled over for a new face value of $R_{12}(p_i)$ which is selected so that the expected payoff to the investor equals R_{01} , that is, $R_{01} = p_iR_{12}(p_i)$. The

expected payoff to the firm is $p_i Y - R_{01}$ and the expected payoff to the investor is R_{01} . The value of short-term debt at date 1 is equal to

$$V_{i,ST} = \min \{R_{01}, \max \{L, p_i Y\}\} \quad (21)$$

where the face value of short-term debt R_{01} ensures that the investor breaks-even in expectation $\sum_{i=1}^3 \beta_i V_{i,ST} = I$ and is given by

$$R_{01} = \left\{ \begin{array}{l} \frac{I - \beta_1 L}{\beta_2 + \beta_3} \\ \frac{I - \beta_1 L - \beta_2 p_2 Y}{\beta_3} \end{array} \right\} \quad \text{as} \quad I \left\{ \begin{array}{l} \leq \\ > \end{array} \right\} \beta_1 L + (\beta_2 + \beta_3) p_2 Y. \quad (22)$$

Next, suppose the firm borrows I at date 0 by issuing long-term debt which promises to repay R_{02} at date 2. At date 1 the state of the project is realized and observed by the investor and the firm. If $L \leq p_i Y$ project continuation is efficient. The investor gets $p_i R_{02}$ and the firm gets $p_i(Y - R_{02})$ as per the original contract. On the other hand, if $L > p_i Y$ project liquidation is efficient. However, the investor has no control rights and cannot unilaterally force termination without the approval of the firm. If the firm liquidates (under the terms of the original contract), the proceeds L accrue solely to the investor. On the other hand, if the firm continues, then $p_i(Y - R_{02}) > 0$ as long as $p_i > 0$ and $R_{02} < Y$.

In other words, there is a *risk-shifting problem* since all benefits from efficient liquidation go to the investor, which implies that the firm has no incentive to take the efficient action. In this case, the debt contract must be renegotiated, and the firm bribed, to ensure that the project is closed down. Renegotiation creates a surplus of $L - p_i Y$ since it avoids inefficient project continuation. This surplus must be allocated between the firm and the investor through bilateral bargaining.

4.3 Renegotiation

I apply the generalized Nash bargaining solution where the firm and the investor have weights μ and $1 - \mu$ respectively. The firm gets $\mu(L - p_i Y)$ of the surplus whereas the investor gets $(1 - \mu)(L - p_i Y)$, where $\mu \in [0, 1]$ is the bargaining power of the firm. After renegotiation the firm is liquidated and the payoffs to the entrepreneur and the investor are given by

$$\text{Firm} = p_i(Y - R_{02}) + \mu(L - p_i Y) \quad (23)$$

$$\text{Investor} = p_i R_{02} + (1 - \mu)(L - p_i Y) \quad (24)$$

The first term in each expression is the payoff under the original contract, the second term is the allocation of the surplus $L - p_i Y$ from taking the efficient decision. Thus, in states where project termination is efficient the long-debt contract is renegotiated and the post-renegotiation payoffs for each party are given by (23) - (24).¹⁵ The value of long-term debt at date 1, taking into account debt renegotiation, is equal to

$$V_{i,LT} = p_i R_{02} + (1 - \mu) \max \{L - p_i Y, 0\} \quad (25)$$

Observe that renegotiation takes place only in the low state since this is when liquidation is efficient $L > p_1 Y$. The face value of long-term debt R_{02} ensures that the investor breaks-even in expectation $\sum_{i=1}^3 \beta_i V_{i,LT} = I$ and it equals

$$R_{02}(\mu) = \frac{I - \beta_1(1 - \mu)(L - p_1 Y)}{\sum_{i=1}^3 \beta_i p_i}, \quad (26)$$

Since the credit market is competitive at date 0 the face value of long-term debt adjusts

¹⁵Following Hart and Moore (1998) one can assume that the entrepreneur gets to make a take-it-or-leave-it offer to the investor with probability μ whereas the investor gets to make a take-it-or-leave-it offer to the entrepreneur with the complement probability. The outcome will be the same and still given by (23) - (24). In this paper, I take the division of bargain power as exogenous. Harris and Raviv (1995) provide an analysis of the design of bargaining games.

so that the expected payoff to the investor equals the loan amount. That is, any ex-post rents accruing to the investor (as a result of ex-post bargaining) will be fully competed away by a lower face value of long-term debt. Next, the face value of long-term debt is an increasing function of μ .

$$\frac{dR(\mu)}{d\mu} > 0.$$

If the entrepreneur has greater bargaining power, then the post-negotiating payoff of the investor in states where termination is efficient will be lower. In order to break-even, the payoff to the investor in states where project continuation is efficient must be higher, which is accomplished by raising the face value of long-term debt. That is, an entrepreneur who extracts larger rent during renegotiation in bad states must also share a larger share of the profits with the investors in good states. Stated differently, a strong bargaining position for the entrepreneur creates a cross-subsidization from states where the project is continued to states where it is liquidated.

Proposition 3. Short-term debt rewards the entrepreneur for success (where success is the realization of the high state):

$$p_3Y - V_{3,ST} > p_3Y - V_{3,LT}.$$

At the same time long-term debt tolerates failure (where failure is the realization of the low state where the project is liquidated):

$$L - V_{1,ST} < L - V_{1,LT}.$$

To summarize: project continuation and liquidation decisions will be efficient for each debt maturity. However, different maturities attain efficiency in different ways. Long-term debt attains efficiency by bribing the entrepreneur to liquidate (i.e. renegotiating). Short-term debt attains efficiency by refusing to renew credit. By favorably refinancing in states

where project continuation is efficient, short-term debt rewards the entrepreneur for success. At the same time, by ensuring that the entrepreneur prefers to liquidate voluntarily in states where liquidation is efficient, long-term debt provides insurance for failure.

5 Factors shaping maturity choices

In this section I analyze how different factors shape the firm's choice of maturity. First, I examine how the choice of maturity is affected by the riskiness of the novel project. Second, I analyze how the bargaining power of the firm affects maturity choices. Third, I examine how the liquidation value of the project's assets (i.e. their tangibility) shapes the maturity preferences of the firm. Fourth, I show why equity tends to be dominated by either short-term or long-term debt in this environment. Finally, I show how firms can improve their capacity to finance innovation by using mixed maturity debt.

5.1 Project quality

I define the *quality of the novel project* as the ratio of the increase in the probability of the high state relative to the increase in the probability of the low state induced by project β relative to project α . That is, $q \equiv \frac{\beta_3 - \alpha_3}{\beta_1 - \alpha_1} \geq 0$. Novel projects with higher q will be said to be of higher quality since for the same increase in the probability of the low state they deliver a greater increase in the probability of the high state. The next result characterizes the firm's maturity choices as a function of q .

Proposition 4. Suppose the firm finances the novel project, then there exists a cutoff \tilde{q} with the following property. When $q < \tilde{q}$ then the firm is either indifferent between the two maturities or prefers long-term debt. On the other hand, when $q > \tilde{q}$ then the firm is either indifferent between the two maturities or prefers short-term debt.

Firms financing the novel project must balance two goals: rewarding success on the one hand and tolerating failure on the other. If the novel project has relatively low quality q (i.e.

high risk) then tolerating failure is more important, given that the novel project is relatively likely to fail. As a result, long-term debt relaxes the firm's incentive constraint to a greater extent than short-term debt $IC_{LT}(q|\beta) > IC_{ST}(q|\beta)$. On the other hand, if the novel project has relatively high quality q (i.e. low risk) then rewarding success becomes more important given that failure is not very likely. As a result, short-term debt relaxes the firm's incentive constraint to a greater extent than long-term debt $IC_{LT}(q|\beta) < IC_{ST}(q|\beta)$.

The effect of project quality on the maturity preferences of the firm is illustrated on Figure 3(a). On the horizontal line is the quality of the novel project q . On the vertical line is the incentive-constraint associated to short-term and long-term as a function of q . For q sufficiently small (in particular, below the vertical line on Figure 3(a)) the novel project is inefficient $\Delta S < 0$ since its quality is too low and the firm chooses to develop the standard project. In this case, the standard project can be financed only with short-term debt since long-term debt is not incentive-compatible when it comes to the financing of the standard project $IC_{ST}(q|\alpha) > 0 > IC_{LT}(q|\alpha)$. For intermediate ranges of q the novel project is efficient, but only long-term debt is incentive-compatible $IC_{LT}(q|\beta) > 0 > IC_{ST}(q|\beta)$. Thus, firms use long-term debt to finance the novel project for values of q in this intermediate interval. Finally, if q is large enough then both short-term and long-term debt can be used to finance the novel project and the choice of debt maturity becomes irrelevant.

5.2 Effect of bargaining power

How does the bargaining power of the firm μ affect its choice of debt maturity? Since bargaining occurs only when debt is renegotiated on date 1, and renegotiation can take place only when the firm is financed with long-term debt, short-term debt is not affected by μ . On the other hand, the firm's preference for long-term debt depends on μ .

Specifically, a higher value of μ has two effects on the incentive-compatibility constraint for a firm financed with long-term debt. On the one hand it relaxes the incentive constraint of the firm by allowing it to capture a larger share of the surplus generated by debt renegotiation

in the low state. At the same time, since higher μ lowers the payoff to the investor in the low state, the face value of long-term debt must increase to ensure the investor's break-even condition holds. This second effect hardens the incentive constraint. Thus, higher bargaining power for the entrepreneur makes long-term both more tolerant of failure but also less rewarding of success.

Proposition 5. There exist cutoffs $\tilde{\mu}_1$ and $\tilde{\mu}_2$ such that: (i) long-term debt is incentive-compatible iff $\mu \geq \tilde{\mu}_1$ and (ii) long-term debt satisfy the investor's break-even constraint iff $\mu \leq \tilde{\mu}_2$. As a result, long-term debt can be used to finance the novel project iff $\mu \in (\tilde{\mu}_1, \tilde{\mu}_2)$.

Long-term debt can be used to finance project β if two conditions jointly hold. First, the payoff profile from long-term debt must be incentive-compatible, i.e. $IC_{LT} \geq 0$. This will be the case whenever μ is above a cutoff $\tilde{\mu}_1$ given by

$$\tilde{\mu}_1 \equiv 1 - \frac{I \left(\frac{\sum_i \alpha_i p_i}{\sum_i \beta_i p_i} - 1 \right) + \Delta S}{(L - p_1 Y) \frac{\sum_i \alpha_i p_i}{\sum_i \beta_i p_i}} \quad (27)$$

Second, the investor must break-even in expectation (i.e. his individual rationality constraint must hold). This will be the case when μ is below a cutoff $\tilde{\mu}_2$ given by

$$\tilde{\mu}_2 \equiv \frac{\beta_1 L + \beta_2 p_2 Y + \beta_3 p_3 Y - I}{\beta_1 (L - p_1 Y)} \quad (28)$$

Thus, higher μ helps with the firm's incentive-compatibility constraint but it hurts with the investor's individual rationality constraint. The reason is that higher value of μ lowers the payoff to the investor in states where the project is liquidated. As a result, long-term debt can be used to finance the novel project iff $\mu \in (\tilde{\mu}_1, \tilde{\mu}_2)$. One can easily find parameter values for which

$$0 < \tilde{\mu}_1 < \tilde{\mu}_2 < 1$$

The effect of μ can be illustrated by comparing Figure 3(a) and Figure 3(b). The bar-

gaining power on Figure 3(a) is equally spread out among the firm and the investor $\mu = \frac{1}{2}$. On the other hand, the investor has a greater bargaining power on Figure 3(b). For values of q below 0.4 the novel project is inefficient and the firm undertakes the standard project. In this case, the standard project can be financed with both maturities. For q above 0.4 but below about 0.5 the novel project is efficient but neither short-term nor long-term debt is incentive-compatible. For q above 0.5 and below about 1.2 the novel project is efficient and can be financed with short-term but not with long-term debt. Therefore, a firm with a novel project in this range issues short-term debt. Finally, for q above 1.2 both maturities can be used to finance the novel project.

5.3 Effect of liquidation value

What is the effect of L on the maturity preferences of innovative firms? Figure 3(c) shows the incentive-constraint associated to short-term and long-term debt under the same parameters used to construct Figure 3(a), but assuming that the project's assets have a lower liquidation value. Projects with low liquidation value L can be interpreted as having low asset-tangibility since they cannot be redeployed in alternative use without a significant loss of value.

Whereas on Figure 3(a) (where the project was characterized by a high liquidation value) long-term debt could be used to finance the novel project for intermediate values of q on Figure 3(c) this is no longer feasible. Specifically, for q below 0.4 the novel project is inefficient and the firms finance the standard project. The standard project in this case can be financed with both maturities. For values of q between about 0.4 and 0.8 the novel project is efficient but neither maturity can be used to finance it. For values of q between about 0.8 and 1.1 only short-term debt can be used to finance the novel project. Finally, for values of q above 1.1 both maturities can finance the novel project.

The implication is that the firm's preference for long-term debt is *increasing* in the liquidation value of the project. The reason is the following. First, when L is high then the amount that can be pledged to the investor in states where liquidation is optimal increases.

As a result, the payoff to the investor in states where continuation is efficient will be lower, and therefore, the reward for success for the entrepreneur rises. Second, higher value for L implies that long-term debt will be characterized by a greater tolerance for failure (i.e. the payoff to the firm in the low state is increasing in L) while short-term debt still leaves the entrepreneur with a payoff of zero conditional on liquidation. This feature of long-term debt emerges because the surplus allocated between the investor and the entrepreneur after renegotiation is increasing in L , which allows the entrepreneur to capture greater rents.

Finally, for relatively low values of q (i.e. high risk of failure) tolerating failure is more important than rewarding success, which implies that long-term debt would relax the incentive-constraint of the firm to a greater extent than short-term debt. This additional advantage conferred on long-term debt implies that, other things being equal, firms with higher liquidation value prefer to lengthen their maturity.

5.4 Long-term debt vs equity

Suppose the firm can issue equity in addition to debt. Let $1 - \rho$ be the fraction of equity in the firm retained by the entrepreneur and $\rho \in [0, 1]$ the fraction of equity held by the investor where ρ is such that the investor breaks-even in expectation.

$$\beta_1 \rho L + \beta_2 \rho p_2 Y + \beta_3 p_3 Y = I$$

where the above assumes efficient continuation and liquidation decisions on date 1. With equity, the question arises of who decides whether the project is to be continued or liquidated on date 1 (that is, who has the controlling share). I assume that neither the entrepreneur nor the investor can commit to carry out treats not in his best interest. As a result, regardless of which party has control rights on date 1, the other party would refuse to make any concessions since, under the original equity contract, both parties are strictly better-off when the efficient action is being taken. That is,

$$\rho \max \{L, pY\} > \rho p_1 Y \quad \text{and} \quad \rho \max \{L, pY\} > \rho L.$$

$$(1 - \rho) \max \{L, pY\} > (1 - \rho) p_1 Y \quad \text{and} \quad (1 - \rho) \max \{L, pY\} > (1 - \rho) L.$$

This implies that the original allocation of control rights is irrelevant and it will not be necessary to bribe the entrepreneur in order to attain efficient liquidation. This can be contrasted with the case of long-term debt where in the low state the entrepreneur is better off continuing under the terms of the original contract, and therefore, his treat is *credible*. The next result shows that, as long as the firm has a high bargaining power, equity is an inferior funding arrangement in this environment.

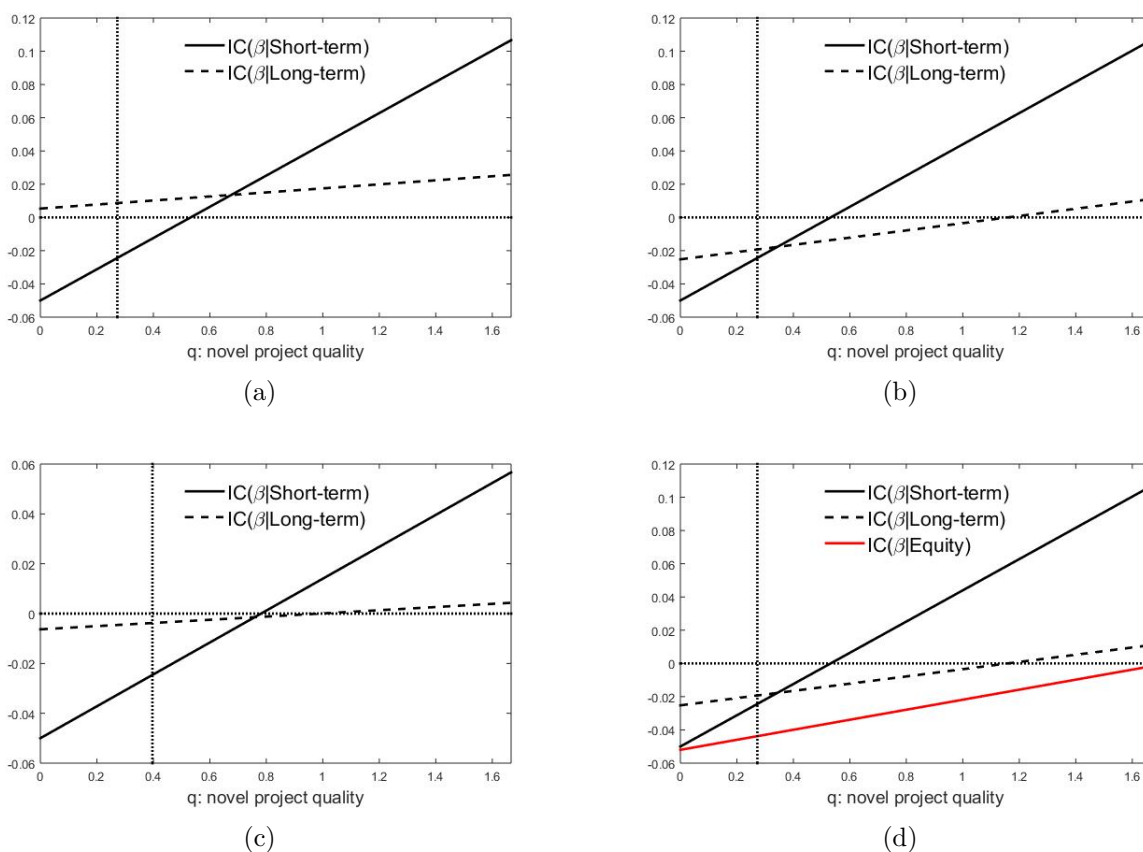
Proposition 6. There exists a cutoff $\mu^* \in [0, 1]$ such that for any $\mu > \mu^*$ the equity contract is dominated by either long-term or short-term debt when it comes to financing the novel project.

This result is illustrated on Figure 3(d) which shows the incentive-compatibility constraint associated with equity in addition to that of long-term and short-term debt. We can see from the figure that equity is dominated by both short-term and long-term debt when it comes to the financing of the novel project. In fact, even high quality novel projects cannot be funded with equity for the particular example since the incentive-compatibility condition does not hold. The reason is that the equity contract is inferior to long-term debt when it comes to tolerating failure since the entrepreneur cannot extract a bribe in order to liquidate the firm. At the same time, equity is dominated by short-term debt when it comes to rewarding success since the entrepreneur must share some of the surplus with the investors when project continuation is efficient.

The implication of this section is a form of pecking order where innovative firms rely more on debt rather than equity financing. The importance of debt - and especially bank

debt with its scope for renegotiation - for the financing of innovation is supported by recent empirical findings which underscore the critical role of bank debt in the funding of young and innovative firms.

Figure 2: Comparison of different maturity structures.



5.5 Mixed maturity and callable debt

In some cases, neither long-term nor short-term debt can be used to finance the novel project. This is the case on Figure 3(b) where for intermediate values of q the novel project is efficient, but cannot be financed in an incentive-compatible way by either maturity. In this section I show how the firm can use mixed maturity structure to improve its debt capacity.

The mixed debt maturity structure is modeled as in Diamond (1993). Specifically, the

contract between the firm and its lenders can now include a payment either on date 1, on date 2 or both. As before, the face value of debt issued on date 0 and maturing on date 1 is R_{01} , the face value of debt issued on date 0 and maturing on date 2 is R_{02} and the face value of debt issued on date 1 in state i and maturing on date 2 is R_{12}^i . If the firm does not pay the amount specified in the loan contract on date 1, the lender has the right to take control and then either liquidate for L or to take control over the cash flows on date 2.

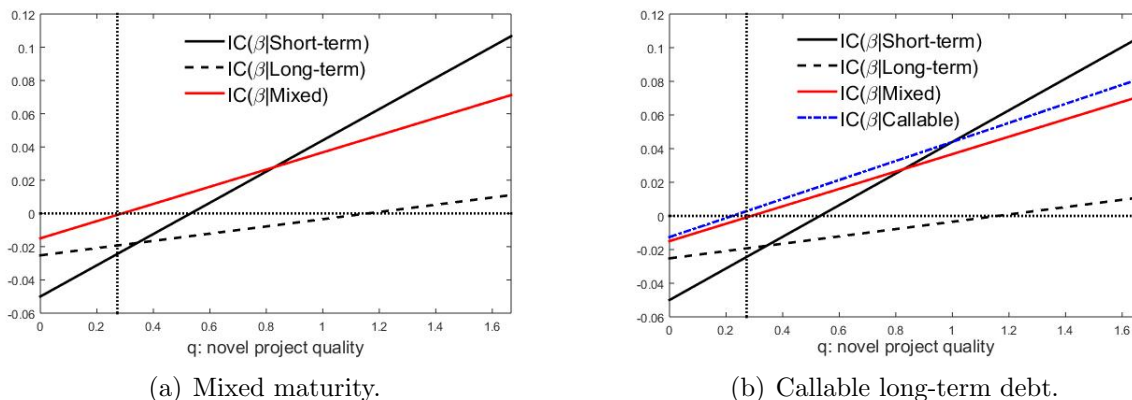
Debt maturing on date 1 is paid out either by liquidating the project or by refinancing. Refinancing can be done by either rolling over debt from the existing creditors or by issuing debt to new creditors to pay off the existing old creditors. The new lenders will be willing to buy the debt of the firm as long as they expect to break-even in expectation. As before, I assume that the state of the project is observed by the investors (old and new) and the firm, but cannot be independently verified by outsider such as courts, and therefore, contracts cannot be based on this information.

Suppose new debt with face value R_{12}^i is issued at date 1, which along with the existing long-term debt, implies that the firm must pay $R_{12}^i + R_{02}$ on date 2. The long-term debt contract features a covenant which restricts both the amount and the priority of new debt that can be issued on date 1. I will follow Diamond (1993) in modeling the covenant as follows. The long-term debt contract specifies a maximum payment of P that can be promised to new lenders on date 1. Thus, the face value of debt issued on date 1 and maturing on date 2 is restricted to satisfy $R_{01}^i \leq P \leq Y$. Thus, if $P \leq Y - R_{02}$ it follows that only junior new debt is allowed, whereas if $P > Y - R_{02}$ then the long-term debt allows for some new debt senior to it.

A result similar to Diamond (1993) holds in this setting: if the firm is using both maturities then short-term debt is risk-free. In addition, the long-term debt contract allows the firm to issue new senior short-term debt. The firm will structure its mixed maturity so that its long-term debt is renegotiated when liquidation is efficient and all concessions (during debt renegotiations) are made by long-term debt. Figure 4(a) shows that mixed

debt maturity dominates only long-term or only short-term debt for values of q below about 0.8 whereas above this value short-term debt is dominant. Thus, for intermediate quality of the novel project, the firm will use both types of maturities. Finally, Figure 4(b) shows that the firm can alternatively finance novel projects of intermediate quality by issuing long-term debt with a pre-specified call price.

Figure 3: Mixed maturity and callable debt.



6 Empirical implications

The model delivers several empirical predictions, which are summarized in this section. First, other things being equal, an efficient renegotiation process renders long-term debt more attractive. On the other hand, if the long-term debt cannot be renegotiated or this process involves a significant delay, the firms prefer to shorten their maturity.

- *Prediction 1*: Innovative firms prefer shorter (longer) maturity when debt renegotiation is less (more) efficient.

Second, factors that decrease the financiers ex-post bargaining power relative to the entrepreneurs would push firms towards longer maturities.

- *Prediction 3*: Innovative firms prefer shorter (longer) maturity when their bargaining power during debt renegotiation is low (high).

For example, debtor-friendly bankruptcy regimes would lead firms to lengthen their maturity since the creditors will be willing to make more concessions in private workouts. Specifically, the entrepreneur will reject any workout, which leaves him worse-off compared to the outcome from filing for bankruptcy. Hence, one can think of debtor-friendly bankruptcy regimes as giving greater bargaining power to the entrepreneur, leading to longer maturities. Third, if the firm's assets have high tangibility and can be easily redeployed, the firm long-term debt becomes more attractive.

- *Prediction 4*: Innovative firms prefer shorter (longer) maturity when their assets have low (high) tangibility.

The reason is that the gains from successful renegotiation in states where project liquidation is efficient will be larger, which allows the entrepreneur to extract a greater payment in order to approve liquidation. Fourth, the optimal maturity depends on the riskiness of the novel project.

- *Prediction 5*: Innovative firms prefer shorter (longer) maturity when the novel project has a low (high) failure risk.

One interpretation is that high failure risk corresponds to radical innovation, whereas low failure risk to incremental innovation. Then, incrementally innovative projects will be financed with shorter maturities, whereas radically innovative projects with longer maturities. Moreover, to the extent that new firms are more likely to have radical, innovative opportunities than older firms, the model predicts that high-risk projects combined with high bargaining power for the firm lead to more significant lengthening of maturities among the younger firms.

7 Discussion

Rajan (1992). It is useful to link the results in this section with those in Rajan (1992). In Rajan, greater bargaining power for the entrepreneur generates more significant incentive distortion and, as a result, creates a preference for shorter maturities. This result is driven by the fact that exerting effort in Rajan's model is assumed to shift the probability distribution over the project's state in the first-order stochastic sense (i.e., it uniformly decreases risk). In contrast, I show that innovative firms prefer to lengthen their maturity in response to greater anticipated bargaining power.

Thus, the analysis here complements Rajan (1992) by highlighting the critical role of the incentives to innovate and how this could lead to a heterogeneous effect of the allocation of bargaining power on maturity choices. In particular, one can synthesize the approach in both papers by deriving the prediction that factors increasing the firm's bargaining power relative to the investor lead to lengthening debt maturity among innovative firms and shortening debt maturity among conventional firms.

Diamond (1991a, 1993). In the example on Figure 3(a), firms financing the standard project issue short-term debt, firms financing high risk novel projects (intermediate values of q) issue long-term debt whereas firms financing lower risk novel projects (high values of q) are indifferent between short-term and long-term debt. This cross sectional pattern of debt maturity is similar to the one Diamond (1991a) where long-term debt is issued by intermediate quality firms whereas short-term debt is issued by either the highest quality or the lowest quality firms.

The underlying mechanism giving rise to this pattern, however, is different. Specifically, in Diamond, the firm's maturity choices are shaped by trading-off liquidity risk after bad news with favorable refinancing after good news. The critical assumption in Diamond is of limited pledgeability, which implies that a firm financed with short-term debt can be liquidated by its lenders even when if the firm is more valuable as a going concern. On the other hand,

I abstract from inefficient liquidation and instead focus on maturity choices shaped by the firm's desire to commit to innovating. The analysis in this section complements Diamond (1991a) by showing that it can arise more broadly and even in the absence of liquidity risk.

At the same time, in the environment I consider the implication that firms prefer long-term debt for intermediate values of project quality does not always hold as can be seen from Figures 3(b) - 4(b). In such cases, it is optimal to either issue mixed maturity or to issue long-term callable debt.

Debt-overhang. Following Myers (1977), short-term debt is seen as a way to minimize debt-overhang problems. The idea is that if debt matures before new investments are undertaken, then the firm will not be subject to debt-overhang since a valuable new investment allows it to refinance at more favorable terms. This observation has led to the prediction that growth firms (which correspond to firms with novel projects) should shorten their debt maturity to avoid debt-overhang. Simultaneously, the empirical literature has found mixed results on the relationship between debt maturity and growth opportunities.¹⁶ These findings are consistent with my model, which predicts that debt maturity varies even among innovative firms based on their projects' riskiness, the liquidation value of their assets, and their bargaining power during debt renegotiation. The model also implies that other things being equal, greater bargaining power for the firms relative to the financiers leads to longer maturities even among growth firms.

8 Conclusion

I analyzed the relation between debt maturity choices and the firms' incentives to undertake innovative projects. The firm's capital structure must provide risk-taking incentives that balance two components: rewarding success on the one hand and tolerating failure on the

¹⁶Barclay and Smith (1995) and Guedes and Opler (1996) find a negative relation between debt maturity and growth opportunities, whereas Stohs and Mauer (1996) and Johnson (2003) document a positive relation after controlling for factors such as firm leverage.

other. Tolerating failure serves as insurance and ensures that the entrepreneur finds the (efficient) risky project more attractive. Ideally, one would design a contract that rewards the entrepreneur after success while it also protects him after failure through appropriate severance pay or similar measures.

However, if state-contingent contracts are difficult to write and enforce - a problem most likely to plague young innovative firms - one must find other ways to ensure the correct incentives. One approach will be to rely on the legislative environment to complete the contracts between the creditor and the debtor. For example, studies have shown that so-called debtor-friendly bankruptcy regimes promote entrepreneurial activity. However, bankruptcy regimes have their limitations since firms cannot write their own rules for bankruptcy, and therefore, must rely on the procedure provided by the government.

In this environment, debt maturity emerges as another way to promote risk-taking and innovation. The core idea is to recognize that debt maturity can be seen as generating a trade-off between rewarding success and tolerating failure. Short-term debt has a low tolerance for failure but a high reward for success. Long-term debt, in contrast, has a high tolerance for failure but a low reward for success. The model implies that firms with increased bargaining power, high asset tangibility, and novel projects which are highly risky (but still efficient) prefer longer debt maturity.

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Appendix: proof of selected propositions

Proof of Proposition 1.

Proof. Consider the program of maximizing the payoff to the investor if the firm develops project α subject to the constraint that the expected payoff to the investor if the firm develops project β equals I (i.e. the investor breaks-even) and subject to the limited liability constraints in each state. That is,

$$\max_{\{V_i\}_{i=1}^N} \sum_i \alpha_i V_i \tag{29}$$

subject to

$$\sum_{i=1}^N \beta_i V_i = I \quad (30)$$

$$0 \leq V_i \leq \max \{p_i Y, L\} \quad (31)$$

Let $\{V_i^*\}_{i=1}^N$ be the solution to this program of maximizing (29) subject to (30) - (31). In order to characterize this solution, suppose there are two states i and j , $i \neq j$, such that

$$0 < V_i^* < \Pi_i \quad \text{and} \quad 0 < V_j^* < \Pi_j$$

That is, the payoff to the investor in i and j is interior. Consider a small perturbation, $\epsilon_i > 0$ and $\epsilon_j > 0$, around the optimal loan-contract V^* . That, is

$$V_i^* + \epsilon_i < \Pi_i \quad \text{and} \quad V_j^* - \epsilon_j > 0$$

such that the expected payoff to the investor remains the same $\epsilon_i \beta_i - \epsilon_j \beta_j = 0$, that is $\epsilon_j = \beta_i / \beta_j$. The perturbed payoff $\{V_i\}_{i=1}^N$ satisfies (31) and (30). Let $\Delta P(\epsilon_i, \epsilon_j)$ denote the change in (29) as a result of the perturbation $\epsilon \equiv (\epsilon_i, \epsilon_j)$

$$\begin{aligned} \Delta P(\epsilon) &= \alpha_i \epsilon_i - \alpha_j \epsilon_j \\ &= \epsilon_i \alpha_i \left(1 - \frac{\beta_i \alpha_j}{\beta_j \alpha_i} \right) \\ &= \epsilon_i \alpha_i \left(1 - \frac{L_i}{L_j} \right) \end{aligned}$$

where $L_i = \beta_i / \alpha_i$ and $L_j = \beta_j / \alpha_j$ denote the likelihood ratios in state i and j respectively.

We have

$$\Delta P(\epsilon) \begin{cases} > \\ = \\ < \end{cases} 0 \quad \text{as} \quad L_i \begin{cases} < \\ = \\ > \end{cases} L_j$$

Since V^* was assumed to be optimal, we must have

$$\Delta P(\epsilon) \leq 0$$

Hence, the following must hold: if $L_i < L_j$, then V^* satisfies one of the following relations:

$$(i) \quad V_i^* = 0 \quad \text{and} \quad V_j^* = 0$$

$$(ii) \quad V_i^* = \Pi_i \quad \text{and} \quad V_j^* = \Pi_j$$

$$(iii) \quad V_i^* \in (0, \Pi_i) \quad \text{and} \quad V_j^* = 0$$

If (i) - (iii) does not hold, then one can construct a perturbation ϵ around V^* which satisfies (31) - (30) and for which $\Delta P(\epsilon) > 0$, is a contradiction. Note that for any other payoff profile $\{V_i\}_{i=1}^3$ which satisfies limited liability (31) and the investors break-even condition (30) we must have

$$\sum_i (\beta_i - \alpha_i) V_i^* \leq \sum_i (\beta_i - \alpha_i) V_i$$

At the same time, the payoff profile $\{V_i^*\}_{i=1}^3$ is incentive-compatible if and only if

$$\Delta S \geq \sum_i (\beta_i - \alpha_i) V_i^* \tag{32}$$

Therefore, if V satisfies incentive-compatibility then so does V^* , but the reverse is not always true. That is, project β can be funded if and only if the payoff profile $\{V_i^*\}_{i=1}^3 = \{V_{i,RS}\}_{i=1}^3$ is incentive-compatible. ■

Proof of Proposition 2.

Proof. The probability distribution over states generated by the novel and the standard project is given by

$$\beta_1 > \alpha_1, \quad \beta_2 < \alpha_2 \quad \text{and} \quad \beta_3 > \alpha_3$$

which implies that $L_2 < \min \{L_1, L_3\}$. We need to consider two cases.

Case (i): suppose that $L_2 < L_1 < L_3$. Thus implies that the payoff profile $\{V_i^*\}_{i=1}^3$ must jointly satisfies the following

$$V_2^* \in [0, \Pi_2) \quad \Rightarrow \quad V_1^* = V_3^* = 0 \quad (33)$$

$$\text{and} \quad V_1^* \in [0, \Pi_1) \quad \Rightarrow \quad V_3^* = 0. \quad (34)$$

From (33) - (34) it follows immediately that V^* equals *Reward for Success* contract in (18).

That is,

$$V_2^* = \min \left\{ \frac{I}{\beta_2}, p_2 Y \right\}, \quad V_1^* = \min \left\{ \frac{I - \beta_2 V_2^*}{\beta_1}, L \right\} \quad \text{and} \quad V_3^* = \min \left\{ \frac{I - \beta_1 V_1^* - \beta_2 V_2^*}{\beta_3}, p_3 Y \right\}.$$

Thus, $\{V_i^*\}_{i=1}^3$ performs (i) cross-subsidization from the middle to the high and the low state and (ii) cross-subsidization from the low to the high state. Finally, the maximum level of I (the investment at date 0) which ensures that the incentive-compatibility condition in (32) holds equals

Case (ii): suppose that $L_2 < L_3 < L_1$. This ordering of the likelihood ratios implies that the payoff profile $\{V_i^*\}_{i=1}^3$ must jointly satisfy the following

$$V_2^* \in [0, \Pi_2) \quad \Rightarrow \quad V_1^* = V_3^* = 0 \quad (35)$$

$$\text{and } V_3^* \in [0, \Pi_1) \quad \Rightarrow \quad V_1^* = 0. \quad (36)$$

which implies that $\{V_i^*\}_{i=1}^3$ equals the *Tolerance for Failure* contract in (19). That is,

$$V_2^* = \min \left\{ \frac{I}{\beta_2}, p_2 Y \right\}, \quad V_3^* = \min \left\{ \frac{I - \beta_2 V_2^*}{\beta_3}, p_3 Y \right\} \quad \text{and} \quad V_1^* = \min \left\{ \frac{I - \beta_2 V_2^* - \beta_3 V_3^*}{\beta_1}, L \right\}.$$

Thus $\{V_i^*\}_{i=1}^3$ performs (i) cross-subsidization from the middle to the high and the low state and (ii) cross-subsidization from the high to the low state. Finally, the same steps as in Case (i) establish that project β can be funded if and only if $\{V_i^*\}_{i=1}^3$ is incentive-compatible which is the case if and only if the amount of the investment is below the cutoff

$$\tilde{I}_2 \equiv \beta_1 \left(L - \frac{c}{\beta_1 - \alpha_1} \right) + \beta_2 p_2 Y + \beta_3 p_3 Y. \quad (37)$$

■

Proof of Proposition 3.

Proof. We must show that the payoff profiles generated by short-term and long-term debt satisfy

$$V_{1,ST} > V_{1,LT} \quad \text{and} \quad V_{3,ST} < V_{3,LT}.$$

The first inequality is interpreted as long-term debt as being more tolerant of failure the second inequality is interpreted as short-term debt as being more rewarding of success. The first inequality follows from

$$V_{1,ST} = L > p_1 R_{01} + (1 - \mu)(L - p_1 Y) = V_{1,LT}$$

which holds for any $\mu \in [0, 1]$ and $R_{01} \leq Y$ as long as termination is efficient in the low

state $L > p_1 Y$, which holds by assumption. To show the second inequality I proceed by contradiction. I assume

$$V_{3,ST} \leq V_{3,LT}$$

Then since $V_{3,ST} = p_3 R_{02}$ and $V_{3,LT} = R_{01}$ we must have

$$V_{2,ST} = p_2 R_{02} < p_3 R_{02} \quad \text{and} \quad V_{2,ST} = \min \{R_{01}, p_2 Y\} < R_{01}$$

which implies that the the payoff profiles satisfy

$$V_{1,ST} > V_{1,LT}, \quad V_{2,ST} \geq V_{2,LT} \quad \text{and} \quad V_{3,ST} \geq V_{3,LT}$$

As a result, the expected payoff to the investor is strictly greater with short-term debt

$$\sum_{i=1}^3 \beta_i V_{i,ST} > \sum_{i=1}^3 \beta_i V_{i,LT}$$

Therefore, we must have

$$\sum_{i=1}^3 \beta_i V_{i,LT} < I \quad \text{or} \quad \sum_{i=1}^3 \beta_i V_{i,ST} > I$$

That is, either the investor cannot break-even when the firm is financed with long-term debt $\sum_{i=1}^3 \beta_i V_{i,LT} < I$ or the expected payoff to the investor when the firm is financed with short-term debt exceeds I that is $\sum_{i=1}^3 \beta_i V_{i,ST} > I$. But this is a contradiction, since in equilibrium both type of debt would keep the investor at his break-even point. Hence, we must have $V_{3,ST} < V_{3,LT}$ as desired. ■

Proof of Proposition 4.

Proof. The incentive-compatibility constraints associated to the payoff profile $V \equiv \{V_i\}_{i=1}^3$ can be expressed as

$$IC(q) = \Delta S - (\beta_1 - \alpha_1) \left(q + (1+q) \frac{\beta_3}{\beta_2} \right) \left(n(q)V_1 + V_3 - (1+q) \frac{I}{\beta_2} \right),$$

where the above substitutes for V_2 by using the investor's break-even condition, which implies

$$V_2 = \frac{I - \beta_1 V_1 - \beta_3 V_3}{\beta_3},$$

and where $q \equiv \frac{\beta_3 - \alpha_3}{\beta_1 - \alpha_1}$ is the project's quality β and the function $n(q)$ is defined as

$$n(q) \equiv \frac{1 + (1+q) \frac{\beta_1}{\beta_2}}{q + (1+q) \frac{\beta_3}{\beta_2}}.$$

Note that $n(q)$ satisfies

$$\frac{dn(q)}{dq} < 0, \quad n(0) = \frac{\beta_1 + \beta_2}{\beta_3} \quad \text{and} \quad \lim_{q \rightarrow \infty} n(q) = \frac{\beta_1}{\beta_2 + \beta_3}$$

Next, we have

$$IC_{LT}(q) \geq IC_{ST}(q) \quad \Leftrightarrow \quad n(q) \leq \frac{V_{3,LT} - V_{3,ST}}{V_{1,ST} - V_{3,LT}} > 0$$

where the strict inequality follows from Proposition 3 which implies $V_{1,ST} > V_{1,LT}$ and $V_{3,ST} < V_{3,LT}$. ■