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Distribution of Trading Activity across Strike Prices in the DAX Index Options Market

by

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April 2004

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Distribution of Trading Activity across Strike Prices in the DAX Index Options Market *

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April 20, 2004

Abstract

In this paper we propose and estimate an econometric model for the distribution of trading activity across options written on the DAX index. The model is based on the observation that in this market options with strike prices ending on 000, 200, 400, 600 and 800 (the class of 200-strike options) are more traded than options with strike prices ending on 100, 300, 500, 700 and 900 (the class of 100-strike contracts). We assume that market participants who would like to trade a continuum of contracts have to choose between the options listed by the exchange. When they have to choose between two neighboring 200- and 100-strike contracts, they prefer the 200-strike contract if the degree of substitution between these two options is high. We derive an equation which links the trading volumes of the 200- and 100-strike options and the degree of substitution between them. This equation has convenient analytical properties and can be readily estimated from the data. The estimation results confirm the hypothesised effect of the degree of substitution on the distribution of trading between 200- and 100-strike contracts. Additionally, we are able to derive some quantitative estimates of the percentage of trades attracted to the 200-strike contracts.

JEL classification: C31; C32; G10
Keywords: Market Microstructure; Options Volume

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1 Introduction

According to most derivative pricing models, any option can be replicated with a trading strategy that involves the underlying instrument and a few liquid options. However, in organized derivative exchanges there are usually many listed call and put options written on the same underlying asset. What are the reasons for the existence of so many different contracts and how is the trading distributed among them? Surprisingly, this question has received little if any attention in the theoretical literature about derivative instruments, which is concerned mainly with pricing issues rather than with the determinants of trading.

Ross (1976) discusses the importance of options as instruments which complete the market. His results are further refined by Green and Jarrow(1987) and Nachman (1988). These papers provide the rationale for the existence of the myriad of derivatives contracts in the market, but they say nothing about the demand and trading in such instruments. Leland (1980) and Brennan and Solanki (1981) show that differences in agents' utility functions can lead to buying or selling options by some of the market participants. Franke, Stapleton, and Subrahmanyam (1998) prove that demand for options can arise even if market agents have homogenous utility functions, given that a non-hedgeable background risk is present in the economy. While these papers provide insight into the characteristics of the market participants who are trading options, they do not say anything about the actual amount of trading in options.

To our knowledge, the only piece of research that deals explicitly with the demand for options is the recent paper by Judd and Dietmar (2003). The goal of this paper is to give a theoretical explanation for the observed robust pattern of the open interest which peaks for the at-the-money option and gradually dissipates for in- and out-of-the money contracts. The authors look at the shape of the open interest of different options with the same time to maturity. They work in a simple equilibrium setting with two trading periods, one asset and a finite number of options written on that asset. In their paper, the demand for options arises through the skew-preference of the agents, which are defined via the third derivative of their utility functions. The authors derive approximate analytical solutions for the open interest across options with different strike prices and identical time to maturity. Their results show that the shape of the open interest as a function of the strike price is very sensitive to the distributional assumptions about the process for the underlying and to the choice of the strike price structure. Only in some special cases is possible to obtain the robust inverted U-shape pattern observed in the market. The authors point out that their main contribution

is to show the limitations of the risk-sharing arguments in explaining the actual demand for options.

In this paper we provide some empirical insight into the determinants of option demand reflected by the trading in options. We do this by focusing on a special phenomenon observed for the DAX index options traded on the EUREX derivatives exchange. In this market options are issued with strike prices falling on a prespecified grid set. Listed contracts with time to maturity exceeding one year have strike prices ending on 000, 200, 400, 600, or 800. We call this type of contracts the class of 200-strike options. Options with time to maturity between six months and one year are either 200-strike options or have strike prices ending on 100, 300, 500, 700, or 900. The latter type of contracts we call the class of 100-strike options. For maturities less than six months there exist 200- and 100-strike options as well as options with strike prices ending on 50 which we refer to as the class of 50-strike options. Inspection of the historical data shows that 200-strike and 100-strike are much more traded than the 50-strike options. Additionally, the comparison of the trading activity between the 200- and 100-strike options reveals that 200-strike options are more traded than the 100-strike options, although the differences in trading volumes is not that big as in the previous case.

What are the reasons for the irregular distribution of the trading activity across options differing only in the last three digits of their strike prices? One explanation can be found in the sequential nature of introducing new contracts. 200-strike options are introduced first and consequently they have accumulated greater open interest than options belonging to the other two classes. Correspondingly, the 100-strike options are introduced earlier than the 50-strike options and therefore, have accumulated greater open interest than the 50-strike options. Since the open interest is a good indicator for the future trading on a contract, this could explain the differences in the trading volumes among the three types of options. Additionally, the strike prices of the more liquid contracts could serve as focal points where traders coordinate trades to achieve greater liquidity. Koch and Lazarov (2003) further hypothesize that given the established attractiveness of 200- vs 100-strike options and the attractiveness of 100- vs 50-strike options, the distribution of trading activity among different contracts is determined by the degree of substitution between them. For example, if two neighboring 200- and a 100-strike options are good substitutes, then the trading will concentrate on the 200-strike contract, which has already been established as a focal point and generally has greater liquidity. Overall, when the degree of substitution between options is high, it can be expected that the more attractive contracts will witness higher trading volume since the traders will coordinate their trades on them. Thus, the degree of concentration of trading on the more attractive options

will depend on the degree of substitution between options.
To test this hypothesis, Koch and Lazarov(2003) define a measure of clustering of trading activity between two neighboring options as the log ratio of their trading volumes measured by the number of transactions. They regress this measure on a set of variables that correspond to various factors which determine the degree of substitution. The regression results as well as various robustness checks support the hypothesized relation between the irregular distribution in trading and the degree of substitution.
In the current paper we formulate and estimate an econometric model for the distribution of the trading of trading activity across options with different strike prices. This extends the results of Koch and Lazarov(2003) in two main directions. First, we perform more formal and complete analysis which models the trading across all options simultaneously. This avoids the drawback of the simple regression approach which focus separately on the pairs of neighboring options without taking into account the impact of the remaining options. Second, in our setting it is possible to get quantitative estimates for the percentage of the trades that are attracted to 200-strike options from their 100-strike neighbors. Getting such quantitative estimate might be important for the design of derivatives markets. Organized options exchanges make their profits from collecting fees from every traded contract and therefore they are interested in maximizing the overall trading volume. Virtually in all derivatives markets, options are introduced with strike prices equally distant to each other, such that this grid size is fixed by the exchanges rules and is not changed. However, as the market conditions change the initially chosen grid size might not serve optimally the exchanges profit maximizing goals. Intuitively, if the degree of substitution between options increases significantly, there will be too many contracts which are good substitutes. The trading will dissipate among different options, which could result in low liquidity and consequently, decreased trading. Alternatively, if the degree of substitution between options decreases significantly, there will be too few contracts available, and some traders will not be able to find a suitable options to trade. This could again result in decreased trading.
In our model we consider only the 200-strike and 100-strike options. The 50-strike options have very low liquidity and additionally including them in the model will lead to unnecessary complications.[1] The starting point of the model is the observation that investors may want to trade options with strike prices that are not available. More specifically, we assume that for

[1]We discuss in Section 4 that the existence of 50-strike options does not impact the derivation of our model under some mild assumptions. For comparison of the trading on 50-strike options versus the trading on 200- and 100-strike options, see Koch and Lazarov (2003).

each pair of two neighboring 200-strike and 100-strike options some traders would like to execute transactions on contracts with strike prices which fall between the two strike prices of these options. These kind of investors are faced with a choice between the two available contracts. Additionally, we assume that there exists a function with values between zero and unity, which determines how the trades are distributed between the two options. This function gives what percentage of the trades are executed on the 200-strike option and what percentage of the trades are executed on the 100-strike option. When it equals unity, all investors trade the 200-strike option and when it equals zero, all investors trade the 100-strike option instead. This function reflects the degree of substitution between the neighboring options. When it has value one, this can be interpreted as the case of a perfect substitution between the neighboring 200-strike and 100-strike options, and in this case all corresponding trades are executed on the more attractive 200-strike contract. Note that we do not restrict the function to take always values greater than one half, which is equivalent of assuming that the 200-contract always attracts more volume. The reason is that sometimes a 100-strike option can have open interest which is much greater than that of its 200-strike neighbor, and correspondingly the 100-strike option may witness more trades. Since the ratio of the open interests of the two neighboring options enters as a parameter in the function, it will drive the function's value below one half.
Using these substitution functions, we can express the trading volume of each option as a function of the degree of substitution and the number of trades that market participants would like to execute on options with strike prices between the strike prices of the two neighboring contracts of the option. All such expression can be combined into one equation which links the trading volumes of 200-strike and 100-strike options and the degree of substitution between them. This equation has convenient analytical properties and can be estimated from the data. The estimation results confirm the hypothesis for the impact of the degree of substitution on the distribution of trading activity and allows us to quantitatively gauge the shift in trading activity to the more attractive 200-strike.
The rest of the paper is organized as follows. Section 2 gives a short overview of the DAX index options market. In Section 3 summary statistics and regression analysis for the distribution of trading across options with different strike prices are presented . Formal derivation of the model for the distribution of trading between 200- and 100-strike options is given in Section 4. The econometric estimation of this model is presented in Section 5. The last section of the paper summarizes the results and concludes.

2 Overview of the DAX Index Options Market

Dax index options are traded on the electronic trading platform EUREX and are all European style and cash settled on the DAX 30 stock index. On every business day options with eight different maturity dates (classes) are available. All options expire on the third Friday of the corresponding maturity month, and if this is a holiday, on the last prior trading day. Options belonging to the first three maturity classes expiry in the first three succeeding months. Contract belonging to maturity classes four, five, and six expiry in the succeeding three expiration months in the cycle (March, June, September and December). Maturity classes seven and eight comprise the succeeding two expiration months in the cycle (June, December).
The exchange introduces options with strike prices that are 50, 100 or 200 points apart. According to the exchange rules, options with time to maturity more than 12 months are all 200-strike options. Contracts with time to maturity between 6 months and 12 months are either 200-strike or 100-strike options. Finally, for maturities less than 6 months, a full menu of 50-strike, 100-strike and 200-strike options exists. The exchange continuously maintains a sufficient number of available contracts around the at-the-money point at each point of time. The minium number of options in each maturity class ranges from five for maturities greater than 6 months and nine for maturities shorter than 6 months. New options series are introduced if the closing level of the DAX index exceeded (dropped below) the average of the third and the second highest (lowest) existing strike prices on the two preceding trading days.

3 Preliminary Characterization of the Distribution of Trading Activity Across 200- and 100-strike Options

Detailed summary statistics for the clustering of trading across 200-strike, 100-strike and 50-strike options are presented in Koch and Lazarov (2003). Since we are dealing with the phenomenon of trade clustering from a different angle than Koch and Lazarov(2003), we will give a new set of summary statistics which are more relevant to our approach of modelling the observed irregular distribution of trading. The data that is used for all estimations spans all transactions on DAX index options for the period 4 January 1999 to 31 July 2002, which makes a total of 908 trading days. We consider only

the options in the first four maturity classes. For higher maturity classes, the liquidity is insignificant and additionally, it is not possible to get reliable estimates of the options' deltas, which are used further in the paper.
Assume a trading day and a maturity class. Denote the strike price of the nearest to the at-the-money point out-of-the money call/put option by s_0^2. Let s_{-1}^2,s_{-2}^2,...,s_{-6}^2 are the next six out-of-the money 200-strike call/put option strikes and s_{-1}^1,s_{-2}^1,...,s_{-6}^1 are the next six out-of-the money 100-strike call/put option strikes. In a similar way, we denote the strike prices of the corresponding in-the-money 200-strike and 100-strike contracts by s_1^2,s_2^2,...,s_6^2 and s_1^1,s_2^1,...,s_6^1. Tables 1 and 2 show the average number of trades on the call and put options with strike prices $s_{-k}^2, s_0^2, s_k^2, s_{-k}^1, s_k^1$, $k = 1, ..., 6$ for each maturity class. It can be seen that for most pairs of neighboring 200-strike and 100-strike options, the 200-strike options have higher average trading volume. This pattern is much more pronounced for the out-of-the-money options, where there are only two exceptions in the first maturity class for the call options and one exception in the first maturity class for the put options. For in-the-money options there are a few exceptions for pairs near to the at-the-money point in the first and the second maturity class, respectively.
Note that each 200-strike option has two 100-strike neighbors, one of which is closer to the at-the-money point. Since option liquidity decreases with moneyness, the 100-strike contract can have a larger volume because of this liquidity advantage.[2] In all of the cases where a 100-strike option has average trading volume higher than that of its 200-strike neighbor, the 100-strike option is closer to the at-the-money point. Tables 3 and 4 show that the open interest exhibits a similar pattern of clustering. In contrast to the case of the trading volume, the average open interest on each 200-strike contract is always higher than the open interest of its neighboring 100-strike contracts.
We also perform a formal test to see if after accounting for the factors that affect option liquidity such as moneyness, time to maturity and open interest, the 200-strike contracts still witness higher trading. For each option we regress the number of transactions on that contract on the following variables

$$tran = \alpha + \beta_1 \cdot ttm + \beta_2 \cdot ttm^2 + \beta_3 \cdot mon + \beta_4 \cdot mon^2 + \beta_5 \cdot open + \beta_6 \cdot dum + \varepsilon,$$

where ttm and mon are the time to maturity and the absolute moneyness of the option.[3] Squared terms ttm^2 and mon^2 are included to account for

[2]In Section 5 we further comment on the cases when a 200-strike option attracts less trading than its 100-strike neighbor.

[3]Absolute option moneyness is defined as $mon = |absK/F - 1|$ where K is the options strike and F is the futures price of the index.

possible non-linear effects. The regressor *open* is the open interest from the previous trading day. Finally, the dummy variable *dum* takes on value one if the corresponding option is a 200-strike contract and zero otherwise. From the Tables 1 and 2 is clear that there is a significant difference between the trading in out-of-the money and in-the-money options. Therefore, we run separate regressions for out-of-the-money and in-the-money contracts. Additionally, we distinguish between the four maturity classes. Tables 5 and 6 present the linear regression results. Note that the regressions R^2's are higher for the out-of-the-money contracts than for in-the-money contracts. Additionally, they decrease with the maturity class. The coefficients on the regressors *ttm* and *mon* are as to be expected: the trading volume decreases with moneyness and time to maturity. Furthermore, the trading volume is positively related to the previous days open interest. The dummy variable *dum* which indicates if the corresponding option is a 200-strike contract is always significant for out-of-the money options.

The regression results show that the trading on out-of-the-money options can be much better explained than the trading on in-the-money options indicated by the higher values of R^2. Additionally, the in-the-money contracts are much less traded than their out-of-the-money counterparts. This is not surprising since one of the main reason for trading on options is that they provide leverage and the degree of leverage decreases with moneyness. The degree of which the 200-strike contracts are more traded than 100-strike contracts also seem to depend on moneyness. For out of-the-money options the dummy coefficient β_6 is always positive and significant, and also greater than the corresponding coefficient for the in-the-money options. On the other hand, the dummy coefficient for in-the-money options is most of the time not significant, and for the call options in the first maturity class is even negative and significant at the 10% percent level. Overall, it seems that there is a difference in the options trading patterns depending whether they are in- or out-of-the money. The trading on in-the-money options is less intensive and noisier and the concentration of trading on the 200-strike contracts is much less pronounced and most of the time insignificant. We further take this observations into account when we discuss the formulation and estimation of the econometric model for the distribution of trading activity in Sections 5 and 6.

4 Factors that Determine the Degree of Substitution Between Options with Neighboring Strike Prices

Our hypothesis is that when the degree of substitution between the neighboring 200- and 100-strike options is high the trading shifts to the more attractive 200-strike contract. As in Koch and Lazarov(2003), we consider the following six factors that determine the degree of substitution between neighboring options:

Level of the DAX Index

When the level of the DAX index goes up, the 100 point difference between two neighboring 200-strike and 100-strike options relative to the index level decreases and therefore becomes less economically significant. The smaller the relative distance between two options is, the greater the degree of substitution between them. Hence, the 200-strike options should attract more trading volume when the index is high. Similar argument is used by Harris (1991) to justify the relation between the price clustering and the level of stock prices in the US equity market.

Time to Maturity and Volatility of Index Returns

Many investors in options markets are directional traders who pursue buy and hold strategies, i.e. they close their positions near maturity or exercise options. These traders are interested in forecasting the index level at or near maturity. The accuracy with which traders can predict the final index level decreases with increasing time to maturity and/or volatility. If investors' predictions are less precise, then a fixed difference of 100 points between strike prices of neighboring 200-strike and 100-strike contracts will be less important to them, and correspondingly the degree of substitution will increase. This argument is analogous to that of Ball,Torus, and Tschoegl (1985) for the case of price clustering in the gold futures market. The authors argue that traders choose their desired price grid depending on the degree of how accurately they can forecast the future gold price. Harris (1991) applies the same argument in analyzing the price clustering in the US equity market.

Options' deltas

Delta, or the first derivative of the option Black-Scholes price with respect

to the underlying, is the most important and most often used risk measure. Many market participants, especially market makers, try to keep the overall delta of their portfolios close to zero. For that kind of investors, two options with similar delta values are close substitutes.
A natural way to account for the impact of the option's delta on the degree of substitution is to compute the absolute delta difference of each pair of neighboring options. However, for the sake of simplicity we use one measure for the difference in deltas for all pairs of neighboring 200-strike and 100-strike options with the same time to maturity. Visual inspection of the data shows that the delta of the DAX index options is approximately a linear function of the strike price after accounting for the volatility smile. That is, the absolute difference in deltas of neighboring 200- and 100-strike options is relatively stable across all pairs of options. Taking into account this observation, we compute the first derivative of the at-the-money delta with respect to the strike price and use this value to summarize the absolute delta differences across all option pairs. In principle, in our model it is possible to account for the delta differences in each individual pair of neighboring 200- and 100-strike contracts. However this leads to unnecessary complications in the econometric estimation, while little is gained in terms of precision.

Options' Moneyness

In options markets trading tends to concentrate near to the at-the-money point and it decreases for options further in- or out-of-the-money. It can be expected that the attractiveness of 200-strike options increases with the absolute moneyness, since traders strive to coordinate trades on the more attractive 200-strike contract in order to generate liquidity.

Options' Open Interest

Open interest reflects the potential liquidity of an option. When two neighboring 200-strike and 100-strike options do not differ much into the previous factors then the traders will prefer the one with the higher open interest, which in most of cases the 200-strike contract. Note that sometimes a 100-strike option may have significantly higher open interest then its 200-strike neighbor, which could lead to higher trading on it than on the 200-strike contract.

5 An Econometric Model for the Distribution of the Trading Activity across Strike Prices

In this section we give description of the model that is used to characterize how the trading is distributed across options with the same time to maturity. Any such model should take into account that options' trading volumes are interdependent. Intuitively, the trading on the at-the-money option is related to the trading of its two neighboring in- and out-the-money options. The trading on these two contracts is at the same time related to the trading on their neighbors and so on. The approach of comparing only the trading on individual pairs of options with next to each other strike prices could potentiality lead to biases since it does not take into account the impact of the neighboring contracts.

We propose a way to simultaneously model the distribution of trading across all options belonging to the same maturity class. The starting point is the observation that only a discrete set of strike prices are available and the market participants who would potentially like to trade a continuum of options are faced with the decision of choosing among a limited number of contracts. To state these ideas formally, we assume a trading day and a particular maturity class. For simplicity, we assume that there exists a listed 200-strike option which is exactly at-the-money with strike price s_0^2. Additionally, we assume that there exists a continuous band of n alternating 100- and 200-strike out-of-the-money options which spread around the at-the-money point s_0^2. Denote the strike prices of these contracts by $s_{-1}^1, s_{-1}^2, s_{-2}^1, s_{-2}^2, ..., s_{-n}^1, s_{-n}^2$ where $s_i^1, i = 1, ...n$, are 100-strike options and $s_i^2, i = 1, ...n$, are 200-strike options, respectively. All neighboring options are 100 points apart. Similarly, we assume that there exists a continuous band of n alternating 100- and 200-strike in-the-money options with strike prices $s_1^1, s_1^2, s_2^1, s_2^2, ..., s_n^1, s_n^2$. The total number of the considered options is $4n + 1$.

First, we deal only with the in-the-the money contracts. We assume that in each of the intervals determined by the strikes prices of the pairs of neighboring options $[s_i^1; s_i^2], i \in 1, n$ and $\left[s_i^2; s_{i+1}^1\right], i \in 0, n-1$, the traders would like to execute m_{2i} and m_{2i+1} trades[4], respectively on options with strike prices that fall in these intervals. Market participants are faced with the decision to choose between options with strike prices s_i^1 and s_i^2 in the first case, and

[4]Note that in our case m_{2i} and m_{2i+1} represent the number of transactions. One can use the total volume (the total number of the traded contracts) instead. We prefer the number of transactions since it is much less noisier than the total volume. Another alternative is discussed at the end of the section.

between options with strike prices s_i^2 and s_{i+1}^1 in the second case.
To model how traders form their decisions we assume that there exist functions $s_{2i}(\cdot) \in [0;1], i = 1,..,n$, such that in the interval $[s_i^1; s_i^2]$, $s_{2i}(\cdot) \cdot m_{2i}$ of the m_{2i} trades are executed on the contract with strike s_i^2 and the remaining $(1 - s_{2i}(\cdot)) \cdot m_{2i}$ trades are executed on the contract with strike s_i^1. Similarly, we assume that here exist functions $s_{2i+1}(\cdot) \in [0;1], i = 0,..,n-1$ such that in the interval $\left[s_i^2; s_{i+1}^1\right]$, $s_{2i+1}(\cdot) \cdot m_{2i+1}$ of the m_{2i+1} trades are executed on the contract with strike s_i^2 and the remaining $(1 - s_{2i+1}(\cdot)) \cdot m_{2i+1}$ trades are executed on the contract with strike s_{i+1}^1. The functions $s_{2i}(\cdot)$ and $s_{2i+1}(\cdot)$ depend on the degree of substitution between the two options in the corresponding pair. Accordingly, the parameters that enter in these functions account for the impact of the factors discussed in Section 3. Our assumption is that when the degree of substitution is high, then the 200-strike options attract higher volume and in our setting the functions $s_{2i}(\cdot)$ and $s_{2i+1}(\cdot)$ will take on higher values. If $s_{2i+1}(\cdot) \geq \frac{1}{2}$ and/or $s_{2i}(\cdot) \geq \frac{1}{2}$, then the corresponding 200-strike contract attracts greater volume. Note that we do not restrict $s_{2i}(\cdot)$ and $s_{2i+1}(\cdot)$ to have values always greater than $\frac{1}{2}$. Sometimes the 100-strike option may have much greater open interest than its 200-strike neighbor and correspondingly attract greater trading. Since we later specify these functions to depend on the ratio of the open interests of the two corresponding contracts this could drive the value of $s_{2i}(\cdot)$ or $s_{2i+1}(\cdot)$ below $\frac{1}{2}$. Finally, it is assumed that there is no demand for options with strike prices greater than s_n^2. [5]
From the last assumption, the trading volume of the 200-strike option with strike price s_n^2 can be readily computed as:

$$v_n^2 = s_{2n}(\cdot)\, m_{2n}. \tag{1}$$

The trading volumes of the intermediate 200-strike in-the money options are given by

$$v_i^2 = s_{2i}(\cdot)\, m_{2i} + s_{2i+1}(\cdot)\, m_{2i+1}, i = 1,..,n-1, \tag{2}$$

i.e. the option s_i^2 attracts $s_{2i}(\cdot)\, m_{2i}$ trades from the interval $[s_i^1; s_i^2]$ and $s_{2i+1}(\cdot)\, m_{2i+1}$ trades from the interval $\left[s_i^2; s_{i+1}^1\right]$. Similarly, the trading volumes of intermediate 100-strike options are given by

[5] Although this assumption can be relaxed, there is no need to do that, since for the value of n which we consider ($n = 6$), the liquidity of the option with strike price s_n^2 is negligible.

$$v_i^1 = (1 - s_{2i-1}(\cdot))\, m_{2i-1} + (1 - s_{2i}(\cdot))\, m_{2i}, i = 1, ..., n, \tag{3}$$

i.e. the option s_{i-1}^2 attracts $s_{2i-1}(\cdot)\, m_{2i-1}$ trades from the interval $\left[s_{i-1}^2; s_i^1\right]$ and $s_{2i}(\cdot)\, m_{2i}$ trades from the interval $[s_i^1; s_i^2]$. By substituting $i = n$ into (3), it follows that

$$v_n^1 = (1 - s_{2n-1}(\cdot))\, m_{2n-1} + (1 - s_{2n}(\cdot))\, m_{2n}, \tag{4}$$

and by combining (1) and (4) we get:

$$m_{2n-1} = \frac{1}{(1 - s_{2n-1}(\cdot))} v_n^1 - \frac{1}{(1 - s_{2n-1}(\cdot))} \cdot \frac{(1 - s_{2n}(\cdot))}{s_{2n}(\cdot)} v_n^2. \tag{5}$$

We can continue going backwards using the relationship

$$v_{n-1}^2 = s_{2n-2}(\cdot)\, m_{2n-2} + s_{2n-1}(\cdot)\, m_{2n-1}, \tag{6}$$

which is derived from expression (2) by setting $i = n - 1$. Using (5) we get:

$$\begin{aligned} m_{2n-2} = {} & \frac{1}{s_{2n-2}(\cdot)} v_{n-1}^2 - \frac{1}{s_{2n-2}(\cdot)} \cdot \frac{s_{2n-1}(\cdot)}{(1 - s_{2n-1}(\cdot))} v_n^1 \\ & + \frac{1}{s_{2n-2}(\cdot)} \cdot \frac{s_{2n-1}(\cdot)}{(1 - s_{2n-1}(\cdot))} \cdot \frac{(1 - s_{2n}(\cdot))}{s_{2n}(\cdot)} v_n^2. \end{aligned} \tag{7}$$

Continuing in a similar fashion and alternating between expressions (2) and (3) it follows that:

$$\begin{aligned} m_1 = {} & \frac{1}{1 - s_1(\cdot)} \cdot \\ & \left(\sum_{j=1}^{n} v_j^1 \prod_{k=2}^{2j-1} \left(\frac{s_{k(\cdot)}}{1 - s_{k(\cdot)}} \right)^{(-1)^{k-1}} - \sum_{j=1}^{n} v_j^2 \prod_{k=2}^{2j} \left(\frac{s_{k(\cdot)}}{1 - s_{k(\cdot)}} \right)^{(-1)^{k-1}} \right). \end{aligned} \tag{8}$$

We make similar assumptions for out-of-the-money options and in an analogous way the following equation is derived as:

$$m_{-1} = \frac{1}{1 - s_{-1}(\cdot)} \cdot$$
$$\left(\sum_{j=1}^{n} v_{-j}^{1} \prod_{k=2}^{2j-1} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}} - \sum_{j=1}^{n} v_{-j}^{2} \prod_{k=2}^{2j} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}} \right). \quad (9)$$

The trading volume of the at-the-money option is given by $v_0^2 = s_{-1}(\cdot)\, m_{-1} + s_1(\cdot)\, m_1$. Combining the expressions (8) and (9), yields:

$$v_0^2 = \sum_{j=1}^{n} v_j^1 \prod_{k=1}^{2j-1} \left(\frac{s_{k(\cdot)}}{1 - s_{k(\cdot)}} \right)^{(-1)^{k-1}} - \sum_{j=1}^{n} v_j^2 \prod_{k=1}^{2j} \left(\frac{s_{k(\cdot)}}{1 - s_{k(\cdot)}} \right)^{(-1)^{k-1}} \quad (10)$$
$$+ \sum_{j=1}^{n} v_{-j}^{1} \prod_{k=1}^{2j-1} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}} - \sum_{j=1}^{n} v_{-j}^{2} \prod_{k=1}^{2j} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}}.$$

After rearranging the terms we get:

$$\sum_{j=2}^{n} v_{-j}^{2} \prod_{k=1}^{2j} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}} + v_0^2 + \sum_{j=2}^{n} v_j^2 \prod_{k=1}^{2j} \left(\frac{s_{k(\cdot)}}{1 - s_{k(\cdot)}} \right)^{(-1)^{k-1}} =$$
$$\sum_{j=1}^{n} v_{-j}^{1} \prod_{k=1}^{2j-1} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}} + \sum_{j=1}^{n} v_j^1 \prod_{k=1}^{2j-1} \left(\frac{s_{k(\cdot)}}{1 - s_{k(\cdot)}} \right)^{(-1)^{k-1}}. \quad (11)$$

The last expression links the degree of substitution between 200- and 100-strike options and their trading volumes. The left side is a weighted sum of the trading volumes of the 200-strike options and the right side is a weighted sum of the 100-strike options. Intuitively, the weights on the 100-strike options should be greater than the weights on the 200-strike options in order for the two sums to be equal. In the next section we discuss how to specify the functions $s_{\pm k(\cdot)}$ in way that allows the estimation of an equation of the type (11) from the data.

From the summary statistics for the number of trades on options with different moneyness and maturity, it is clear that out-of-the money contracts are

much more traded than in-the-money contracts for both call and put options. This is not surprising since one of the main reason for trading on options is that they provide leverage and the degree of leverage decreases with moneyness. We can reasonably expect that there will be some differences in the distribution of the trading activity across the out-of-the-money and in-the-money contracts. Our model can be modified to account for such differences by separately modelling the trading in these two classes of options by making the additional assumption that $s_1(\cdot)\, m_1 = s_{-1}(\cdot)\, m_{-1}$. The last expression simply means that the number trades that are attracted to the at-the-money 200-strike option from the in-the-money 100-strike option next to it equals the number trades that are attracted to the at-the-money 200-strike option from the out-the-money 100-strike option next to it. This seems a reasonable assumption, since intuitively around the at-the-money point the impact of factors that determine the distribution of option trading such as time to maturity, moneyness and open interest is minimized. Taking this into account, we can separate the modelling of the distribution of trading activity of in- and out-of-the-money options. The expression $v_0^2 = s_{-1}(\cdot)\, m_{-1} + s_1(\cdot)\, m_1$ can be rewritten as:

$$v_0^2 = 2 \cdot s_1(\cdot)\, m_1. \tag{12}$$

By substituting (8) into (12) it follows that:

$$\frac{1}{2}v_0^2 + \sum_{j=1}^{n} v_j^2 \prod_{k=1}^{2j} \left(\frac{s_{k(\cdot)}}{1 - s_{k(\cdot)}} \right)^{(-1)^{k-1}} = \sum_{j=1}^{n} v_j^1 \prod_{k=1}^{2j-1} \left(\frac{s_{k(\cdot)}}{1 - s_{k(\cdot)}} \right)^{(-1)^{k-1}}. \tag{13}$$

Similarly, from the assumption $v_0^2 = 2 \cdot s_{-1}(\cdot)\, m_{-1}$, we can derive the following expression for out-of-the money options:

$$\frac{1}{2}v_0^2 + \sum_{j=1}^{n} v_{-j}^2 \prod_{k=1}^{2j} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}} = \sum_{j=1}^{n} v_{-j}^1 \prod_{k=1}^{2j-1} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}}. \tag{14}$$

5.1 Remarks

In the DAX index options market besides 200- and 100-strike options there exist 50-strike options with strike prices mixed in between the strike prices of 200- and 100-strike options. In our analysis so far we have ignored this fact. Although the trading on 50-strike options is negligible compared to the trading on 200- and 100-strike options it still can have an impact on any econometric inference based on (13), (14) or (11). If a 50-strike contract is present between two neighboring 200-strike and 100-strike options, then it could attract trading volume, which otherwise would go to either the 200-strike or to the 100-strike option. However, we argue that this is not crucial for the way the distribution of trading activity between 200- and 100-strike options is modeled. For example, let consider a pair of 200-strike and 100-strike options which has a 50-strike option in between. We assume that traders want to execute m^* trades on these three contracts in a two-stage process. First, depending on the degree of substitution, part of the trades go to the 50-strike option and the remaining trades are executed on the 200-strike or on the 100-strike option. As before, let a function $s^*(\cdot) \in [0;1]$ which increases with the degree of substitution determines that $(1 - s^*(\cdot)) \cdot m^*$ of these m^* trades are executed on the 50-strike contract. In the second step, the remaining $m = s^*(\cdot) m^*$ trades are distributed between the 200-strike and the 100-strike option, respectively. Now, as in our model, we assume that there exists a function $s(\cdot) \in [0;1]$ depending on the degree of substitution which determines that $s(\cdot) \cdot m$ of these m trades are executed on the 200-strike contract and the remaining $(1 - s(\cdot)) \cdot m$ trades are executed on the 100-strike contract. The function $s(\cdot)$ and the variable m are analogous to the functions $s_i(\cdot)$ and the variables m_i, respectively. In that way modelling the distribution of trades between 100-strike and 200-strike options can be considered separately from modelling the distribution of the trades between 50-strike and 200/100-strike options. Correspondingly, the derivation of the expressions (13), (14) and (11) is unaffected by the existence of the 50-strike options.

The summary statistics in Section 3 show that sometimes for a pair of neighboring 200- and 100-strike options, the 100-strike contract has higher average trading volume than the 200-strike contract. In all such cases the 200-strike option is further from the at-the-money point than the 100-strike option. Our model allows such cases to happen even though the corresponding 200-strike contract colud be more attractive than its neighbor. For example, consider two neighboring in-the-money contracts s_i^1 and s_i^2. In our setting the trading volumes of these two contracts are given by:

$$v_i^2 = s_{2i}(\cdot)\, m_{2i} + s_{2i+1}(\cdot)\, m_{2i+1}$$
$$v_i^1 = (1 - s_{2i-1}(\cdot))\, m_{2i-1} + (1 - s_{2i}(\cdot))\, m_{2i}.$$

Since the demand for options decreases with the absolute moneyness we can expect that $m_{2i-1} > m_{2i} > m_{2i+1}$. The difference $v_i^2 - v_i^1$ equals:

$$v_i^2 - v_i^1 = (2 \cdot s_{2i}(\cdot) - 1)\, m_{2i} + s_{2i+1}(\cdot)\, m_{2i+1} - (1 - s_{2i-1}(\cdot))\, m_{2i-1}.$$

Even though the 200-strike contract s_i^2 might be more attractive than its 100-strike neighbor s_i^1 (i.e. $s_{2i}(\cdot) \geq \frac{1}{2}$), it is still possible that $v_i^2 - v_i^1 < 0$ if the value of m_{2i-1} is sufficiently larger than that of m_{2i} and m_{2i+1}.
Another important point is which measures for the trading activity v_i^j, $i = -n, ..., n$, $j = 1, 2$ to use. Popular choices are the total volume (the total number of all traded contracts) and the number of transactions. These two measures are widely used in the research for markets in primary securities such as equities, bonds and so on. However, options are artificial financial instruments and they allow for constructing measures of trading activity which are better suited for our purposes.
Ideally, in our model the variables v_i^j should reflect the trading generated by market participants which seek to take *new* short or long positions. The number of transactions as well as the total volume reflect in addition the trading by market participants which exit their short or long positions for various reasons such as portfolio rebalancing, profit taking or terminating a losing position. This kind of trading is unrelated to the degree of substitution between options. In principle, we can infer the number of the newly introduced positions, either short or long and use this measure for the variables v_i^j . This can be done in the following way. At the beginning of each trading day the total number of short and long positions on a particular option is given by two times the open interest from the previous trading day $2 \cdot p_{-1}$. Remember that for each open option contract there exist one market participants that holds a long position in this contract and another one that holds a short position. During the day there are four different trades that are executed: trades to close an existing short position, trades to close an existing long position, trades to open a new short position, and trades to open a new long position (by trade her we assume the act of taking or exiting a *single* short/long position). Let denote the number of such trades as cs, cl, ns and nl, respectively. Note that $2 \cdot vol = cs + cl + ns + nl$, where vol is the total number of traded contracts during the day. The value of

$ns + nl$ gives the number of the newly introduced short and long positions. During the trading day each closed short or long position decreases the value of two times the open interest from the previous day $2 \cdot p_{-1}$ by one, and each new short or long position increases the value of $2 \cdot p_{-1}$ by one. That is, two times the open interest at the end of the trading day $2 \cdot p$ is given by $2 \cdot p = 2 \cdot p_{-1} - cs - cl + ns + nl$. From expression $2 \cdot vol = cs + cl + ns + nl$ and the last expression it follows that $ns + nl = p + vol - p_{-1}$. Thus, the number of the newly introduced short/long positions on a given trading day equals the open interest plus the volume on that trading day minus the previous day open interest.
Unfortunately, this measure cannot be computed for the DAX index options due to the way in which the exchange calculates the open interest. According to the EUREX rules at the end of the trading day only the open and closed positions of market-makers are matched, while the positions of the other market participants are left unmatched in the calculation of the open interest. This makes the reported open interest a distorted measure of the real one. The calculation of the number of the newly introduced positions in the above way results in negative numbers in a lot of cases. We use the number of trades as a measure of the trading activity instead which is less volatile than the total volume.

6 Estimation of the Model

In this section we discuss how to estimate an equation of type (14). In practice, we restrict our attention to a model for the distribution of trading activity among out-of-the-money options specified by (14). The full model (11) for the trading across all options is not estimated, because of the significantly lower volume on in-the-money options, which could introduce bias.
First, it should be noted that an exact relationship between the trading volumes on the 200- and 100-strike options of the type (14) can never hold in practice. To introduce a noise factor in (14) we assume that the trading volume on the options is observed with noise: $v_i^{*k} = v_i^k + \varepsilon_i^k$, where the error terms ε_i^k have zero mean value and additionally are uncorrelated with the substitution factors discussed in Section 4. This noise in observed trading volumes could be due to various reasons such as portfolio rebalancing, profit taking or loss cutting.
Second, given the relatively complicated general form of the expression (14) convenient but economically meaningful specifications for the function $s_{-k(\cdot)}$ must be specified which allow for parsimonious econometric estimation. We introduce such specifications in the following way. Set:

$$\gamma_{-k}(\cdot) = \log \frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}}. \tag{15}$$

The function $\log \frac{x}{1-x}$ is a monotonically increasing transformation of the interval $(0; 1)$ into the interval $(-\infty, \infty)$. Instead of working with the functions $s_{-k(\cdot)}$ we will be using $\gamma_{-k}(\cdot)$. Since the new functions are monotone transformations of $s_{-k(\cdot)}$, the qualitative impact of the functions' parameters is preserved. It follows that:

$$\prod_{k=1}^{m} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}} = \exp\left(\sum_{k=1}^{m} (-1)^{k-1} \cdot \gamma_{-k}(\cdot) \right). \tag{16}$$

We set $\gamma_{-k}(\cdot)$ to be linear functions of parameters that account for the factors which determine the degree of substitution between options discussed in Section 4. More specifically, we assume that:

$$\gamma_{-k}(\cdot) = \alpha + \beta_1 ttm + \beta_2 sd + \beta_3 \cdot \frac{1}{Index} + \beta_4 \cdot delta + \beta_5 \cdot open_k + \beta_6 \cdot mon_k, \tag{17}$$

where ttm is the time to maturity. The impact of market volatility on the degree of substitution is represented by the GARCH(1,1) volatility estimate sd. To account for the effect of the index level, the inverse of the index level $\frac{1}{Index}$ is used. As discussed in Section 4, we include the parameter $delta$, which is the derivative with respect to the strike price of the at-the-money delta. The parameter $open_k$ equals $\log\left((1 + o^2) / (1 + o^1)\right)$ where o^2 is the open interest from the previous trading day of the 200-strike contract in the corresponding option pair, and o^1 is the open interest from the previous trading day of the neighboring 100-strike contract. This parameter reflects the irregular distribution of the trading on the two options caused by the open interest. One is added to o^1 and o^2 to account for the cases when one of them is zero. Finally, the parameter mon_k equals the moneyness of the option in the corresponding option pair which is further way from the at-the-money point. Note that all coefficient are expected to be positive, with the exception of β_4.

Given the assumed functional form of $\gamma_{-k}(\cdot)$, the expression (14) can be significantly simplified. Set

$$\gamma(\cdot) = \alpha + \beta_1 ttm + \beta_2 sd + \beta_3 \cdot \frac{1}{Index} + \beta_4 \cdot delta, \tag{18}$$

and then the following conclusion is satisfied:

$$\sum_{k=1}^{m} (-1)^{k-1} \cdot \gamma_{-k}(\cdot) = \tag{19}$$

$$\sum_{k=1}^{m} (-1)^{k-1} \cdot \gamma + \sum_{k=1}^{m} (-1)^{k-1} \cdot \beta_5 \cdot open_k + \sum_{k=1}^{m} (-1)^{k-1} \cdot \beta_6 \cdot mon_k.$$

It is obvious that:

$$\sum_{k=1}^{m} (-1)^{k-1} \cdot \gamma(\cdot) = \begin{cases} 0, & \text{if } m \text{ is even} \\ \\ \gamma(\cdot) & \text{if } m \text{ is odd.} \end{cases} \tag{20}$$

The parameter that accounts for the impact of the open interest $open_k$ is defined as:

$$open_k = \begin{cases} \log\left(\left(open\left(s^2_{-j}\right)+1\right) / \left(open\left(s^1_{-j}\right)+1\right)\right), & \text{if } k = 2j \\ \\ \log\left(\left(open\left(s^2_{-(j-1)}\right)+1\right) / \left(open\left(s^1_{-j}\right)+1\right)\right), & \text{if } k = 2j-1. \end{cases} \tag{21}$$

After a straightforward calculation, the sum $\sum_{k=1}^{m} (-1)^{k-1} \cdot open_k$ reduces to:

$$\sum_{k=1}^{m} (-1)^{k-1} \cdot open_k =$$

$$= \begin{cases} \log\left(open\left(\left(s^2_0\right)+1\right) / \left(open\left(s^2_{-j}\right)+1\right)\right), & \text{if } m = 2j \\ \\ \log\left(open\left(\left(s^2_0\right)+1\right) / \left(open\left(s^1_{-j}\right)+1\right)\right), & \text{if } m = 2j-1. \end{cases} \tag{22}$$

The parameter that accounts for the impact of moneyness is given by $mon_k = k \cdot \frac{100}{Index}$ and it follows that:

$$\sum_{k=1}^{m} (-1)^{k-1} \cdot mon_k = \begin{cases} -j \cdot \frac{100}{Index}, & \text{if } m = 2j \\ +j \cdot \frac{100}{Index}, & \text{if } m = 2j-1. \end{cases} \tag{23}$$

Now the sum (16) reduces to

$$\prod_{k=1}^{m} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}} = \exp\left(\sum_{k=1}^{m} (-1)^{k-1} \cdot \gamma_{-k}(\cdot) \right) = e^{\log\left(open\left(\left(s_0^2\right)+1\right)/\left(open\left(s_{-j}^2\right)+1\right)\right) - j \cdot \frac{100}{Index}}, \tag{24}$$

when $m = 2j$, and it reduces to:

$$\prod_{k=1}^{m} \left(\frac{s_{-k(\cdot)}}{1 - s_{-k(\cdot)}} \right)^{(-1)^{k-1}} = \exp\left(\sum_{k=1}^{m} (-1)^{k-1} \cdot \gamma_{-k}(\cdot) \right) = e^{\gamma(\cdot) + \log\left(\left(open\left(s_0^2\right)+1\right)/\left(open\left(s_{-j}^1\right)+1\right)\right) + j \cdot \frac{100}{Index}}, \tag{25}$$

when $m = 2j - 1$. After substituting (24) and (25) into (14), it follows that:

$$\frac{1}{2} v_0^2 + \sum_{j=1}^{n} v_{-j}^2 e^{\beta_5 \cdot \log\left(\left(open\left(s_0^2\right)+1\right)/\left(open\left(s_{-j}^2\right)+1\right)\right) - \beta_6 \cdot \left(j \frac{100}{Index}\right)} = \tag{26}$$

$$\exp(\gamma(\cdot)) \left(\sum_{j=1}^{n} v_{-j}^1 e^{\beta_5 \cdot \log\left(\left(open\left(s_0^2\right)+1\right)/\left(open\left(s_{-j}^1\right)+1\right)\right) + \beta_6 \left(j \cdot \frac{100}{Index}\right)} \right).$$

Finally, by dividing both sides of (26) by $e^{\beta_5 \cdot \log\left(open\left(s_0^2+1\right)\right)}$ we get:

$$\frac{1}{2}v_0^2 e^{-\beta_5 \cdot \log\left(open\left(s_0^2\right)+1\right)} + \sum_{j=1}^{n} v_{-j}^2 e^{-\beta_5 \cdot \log\left(open\left(s_{-j}^2\right)+1\right) - \beta_6\left(j \cdot \frac{100}{Index}\right)} =$$

$$\exp\left(\gamma\left(\cdot\right)\right) \cdot \sum_{j=1}^{n} v_{-j}^1 e^{-\beta_5 \cdot \log\left(open\left(s_{-j}^1\right)+1\right) + \beta_6 \cdot \left(j \cdot \frac{100}{Index}\right)}. \tag{27}$$

The last expression has a clear intuitive interpretation. The impact of the time to maturity, volatility, index level and the absolute difference options' deltas is the same for all contracts, which is reflected by the exponent of a linear combination of these factors in front of the weighted sum of 100-strike options in (27). Each option is weighted by an exponent of its moneyness, such that 200-strike options have decreasing weights with moneyness, and 100-strike options have increasing weights with moneyness. Finally, the higher the open interest, the greater the future trading which is reflected by the negative sign in fron of β_5 for both 200- and 100-strike options. Since the open interest is lower for 100-strike contracts, the weights on 100-strike contracts given by the open interest will be higher than the corresponding weights on 200-strike contracts.

To estimate the model we employ the data used to produce the preliminary statistics in Section 3. That is, the data spans 908 trading days in the period between 4 January 1999 and 31 July 2002. We consider only the first four maturity classes where most of the liquidity is concentrated. Additionally, options with time to maturity less than eight days are discarded form the sample since it is not possible to get reliable estimates for the implied volatilities and correspondingly, options' deltas for very short-term to maturity contracts. This leaves us with a total of 3304 observation. The equation (27) is estimated with the Generalized Method of Moments. The set of instruments spans the variables ttm, sd, $\frac{1}{Index}$ and $delta$. Additionally, the trading volumes v_{-j}^2, $j = 1, ..., 6$, and open interests $open\left(s_{-j}^2\right)$, $j = 1, ..., 6$, of the 200-strike options are included. The corresponding figures for the 100-strike contracts are not used since they are highly correlated with their 200-strike counterparts and their inclusion could lead to over-identification.

The estimation results are reported in Table 7. First, note that all significant coefficients have the expected signs. For call options all coefficients are significant with the exception of the moneyness coefficient. On the other hand, for put options the coefficients for the time to maturity, volatility and delta are not significant, while the coefficient for the moneyness is significant. These results support the hypothesized relationship between the degree of substitution and the distribution of trading activity. All significant

variables seem to be also economically significant with the exception of the delta variable for the call options. This impact of the delta difference can be seen from the range of change of the variable $\beta_4 \cdot delta$ in (18) given by the interval $[-0.139; -0.019]$. For comparison, the variables $\beta_1 ttm$, $\beta_2 sd$, and $\beta_3 \cdot \frac{1}{Index}$ take values in the intervals $[0.032; 0.744]$, $[0.498; 1.835]$ and $[-0.451; -1.762\ ]$, respectively
is interesting to evaluate the differences in trading activity across options pairs with different moneyness and maturity. To give such statistics, we compute the average values of the functions $s_{-k}, k = \{1, ..., 12\}$ which determine what percentage of the trades in the corresponding option pair goes to the 200-strike option. Remember that we consider seven 200-strike and six 100-strike contracts, which account for exactly 12 option pairs and substitution functions $s_{-k}, k = \{1, ..., 12\}$, associated with them. The values of s_{-k} are calculated from the values of γ_{-k} using the relation

$$s_{-k} = \frac{\exp(\gamma_{-k})}{1 + \exp(\gamma_{-k})}, \tag{28}$$

which is derived from (15). Additionally, we distinguish between the four maturity classes. The corresponding statistics are reported in Table 8. First, note that the clustering of trading on the 200-strike contract increases with maturity for both call and put options, such that for put options the speed of the increase is greater. In the first three maturity classes, the clustering is most of the time greater for the call options than for the put options, and that situation reverses in the fourth maturity class.
Remember that the moneyness coefficient is not significant in the case of the call options, and in Table 8 we can see that a lot of times for two neighboring option pairs, the clustering in the more out-of-the-money pair is smaller than the clustering in its neighbor. However, a more careful look reveals that in *all* such cases the nearest to the at-the-money point option pair has odd number, i.e. the first pair consists of options with strikes $\left[s^2_{-i}; s^1_{-(i+1)}\right]$ for some $i \in 0, .., 5$, and the second pair consists of the options with the strikes $\left[s^1_{-(i+1)}; s^2_{-(i+1)}\right]$. Note that in the first case the strike $s^1_{-(i+1)}$ is farther out-of-the money than the strike s^2_{-i}, and in the second case it is closer to the at-the-money point than the strike $s^2_{-(i+1)}$. This subtle moneyness effect could drive the clustering in the first option pair higher than that in the second pair, although the first option pair is nearer to the at-the-money point than the second one. The importance of this effect can be further seen in the generally higher degree of clustering in option pairs with odd numbers for both call and put options.

The put options show a steady increase of clustering in options pairs with moneyness, with two exceptions for the 12th option pair in the first and the second maturity class. The results in Table 8 show that the 100-strike contract in the first option pair attracts on average more trades than its 200-strike neighbor for the first three maturity classes. As we noted before, the average clustering in put options is generally smaller than that in call options. Additionally, it seems that clustering disappears for very near to the at-the-money point put options. Finally, the statistics indicate that for options pairs with higher moneyness and maturity, there is a significant shift of trading to the more attractive 200-strike contract for both call and put options. Further research is needed to evaluate the importance of this fact for market design issues.

7 Conclusion

In this paper we propose and estimate an econometric model for the cross-sectional distribution of trading activity in the DAX index options market. The starting point of the model is the assumption that market participants who would like to trade a continuum of contracts have to choose between a limited number of listed options. The observed irregularity in trading of the DAX index options gives us intuition how to model the trading decisions of the market participants. Namely, in this market trading is higher for contracts ending on 000, 200, 400, 600 and 800 (the class of 200-strike options) than for contracts with strike prices ending on 100, 300, 500, 700 and 900 (the class of 100-strike options). We propose the hypothesis that when traders have to choose between two neighboring 200-strike and 100-strike options, they prefer the 200-strike contract if the degree of substitution between these two options is high. This allows us to derive an equation which links the trading and the degree of substitution between these two classes of options. We use a set of variables which account for the options' substitution, and show that with this particular set of factors, the equation can be significantly simplified. The newly derived equation has convenient analytical properties, a clear intuitive interpretation and also can be readily estimated from the data.

The estimation results confirm the hypothesized relation between the degree of option substitution and the concentration of trading on the 200-strike contracts. In that way we confirm the conclusions of Koch and Lazarov (2003), who study this concentration of trading using linear regressions. Additionally, we obtain some statistics for the percentage of the trades that are at-

tracted to the 200-strike options across different maturities and strike prices. Our results indicate that for options with higher maturities and moneyness, there is a significant shift of trades to the 200-strike contracts. Further research is needed to evaluate the importance of this fact for market design issues.
To our knowledge no such model has been proposed in the literature. Note that our framework is quite flexible and can be applied to model the distribution of trading in markets, where there are many assets which are close substitutes such as the LIBOR futures market. Finally, the phenomenon of irregular trading is observed in many other options markets. It would be interesting to see how our methodology fares in these cases. This is best left for future research.

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Table 1
Call Options Summary Statistics for the Trading Volume

	Option Moneyness												
Maturity Class 1	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	1.75	4.80	12.14	29.76	78.56	170.03	181.1	39.91	10.59	5.20	2.97	2.00	1.34
100-strikes	0.88	3.42	10.89	34.70	96.67	182.70	-----	79.41	12.78	3.97	1.66	0.90	0.57

Maturity Class 2	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	2.86	7.39	16.44	31.89	53.27	57.69	41.66	12.48	3.71	1.85	0.96	0.64	0.38
100-strikes	0.94	3.53	11.55	26.53	40.11	38.15	-----	17.76	3.91	1.27	0.60	0.26	0.12

Maturity Class 3	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	2.09	3.98	9.34	14.22	15.3	13.88	10.79	4.68	1.92	1.14	0.63	0.45	0.32
100-strikes	0.83	1.79	4.21	7.46	8.53	7.40	-----	4.64	1.08	0.46	0.22	0.15	0.10

Maturity Class 4	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	6.03	8.71	11.28	13.67	13.07	11.8	9.46	4.35	2.05	1.11	0.88	0.50	0.50
100-strikes	1.35	2.50	3.87	4.68	5.40	5.38	-----	3.27	0.93	0.51	0.26	0.16	0.08

Table 2
Put Options Summary Statistics for the Trading Volume

	Option Moneyness												
Maturity Class 1	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	4.37	9.87	20.23	37.11	65.91	119.83	200.18	92.72	21.19	8.42	4.51	2.75	4.37
100-strikes	3.03	6.03	12.15	28.20	60.93	125.57	-----	141.86	32.58	9.35	4.04	1.77	3.03

Maturity Class 2	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	8.53	15.19	24.75	35.38	44.96	51.73	50.43	22.35	7.81	3.52	1.88	1.2	8.53
100-strikes	3.30	6.21	13.57	22.95	29.76	34.55	-----	28.35	9.06	3.38	1.55	0.83	3.3

Maturity Class 3	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	4.84	7.58	11.04	13.76	15.25	15.82	13.49	6.55	2.6	1.42	0.82	0.55	0.30
100-strikes	1.42	2.23	3.99	6.10	7.43	8.05	-----	6.19	2.23	1.00	0.52	0.36	0.24

Maturity Class 4	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	5.81	7.41	8.15	9.79	11.51	12.68	11.66	5.96	2.82	1.85	1.43	1.1	5.81
100-strikes	1.04	1.85	3.11	3.98	5.17	5.82	-----	4.52	1.94	0.96	0.54	0.37	1.04

Table 3
Call Options Summary Statistics for the Open Interest

	Option Moneyness												
Maturity Class 1	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	7350	10213	12828	15955	19056	19511	15226	9531	6147	3995	2985	1970	1498
100-strikes	3384	4973	7219	10565	13451	13116	-----	8158	4401	2877	1566	833	618

Maturity Class 2	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	6449	8066	10395	12576	14030	13773	10545	7475	4720	3023	2275	1396	1132
100-strikes	1787	2557	4637	7105	8332	6849	-----	4668	2813	1774	984	618	311

Maturity Class 3	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	5399	6327	7944	8355	8267	7737	6611	5162	3768	2747	1995	1673	1412
100-strikes	1456	2131	2783	3386	3585	3348	-----	2614	1545	858	719	511	227

Maturity Class 4	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	14071	16942	17679	18267	16687	15141	13655	10805	7697	5454	4047	3494	2113
100-strikes	2634	3713	4043	4953	5426	5476	-----	4097	2832	1961	1059	611	319

Table 4
Put Options Summary Statistics for the Open Interest

	Option Moneyness												
Maturity Class 1	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	8116	11837	15876	18744	20447	21181	19208	14038	9706	6713	5212	4018	8116
100-strikes	3355	5335	7478	10000	11738	12609	-----	11511	7892	4893	3137	1726	3355

Maturity Class 2	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	6555	9823	12377	14273	15502	14896	13146	9710	6746	4675	3440	2668	6555
100-strikes	1995	3317	5535	7261	7830	7635	-----	5975	4210	2775	1844	1147	1995

Maturity Class 3	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	5211	6470	7117	8833	9918	9469	8141	6649	5271	4214	3455	2756	5211
100-strikes	1194	1883	2744	3241	3591	3288	-----	3039	2338	1916	1504	1312	1194

Maturity Class 4	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
200-strikes	11372	13436	15225	16651	18224	17994	16452	13981	11208	8938	7564	5992	11372
100-strikes	1813	3066	4238	5528	5674	5351	-----	4615	4301	3410	2382	1767	1813

Table 5
Linear Regressions for the Trading Volume of the 200- vs 100-strike Call Options

In-the-money Options

	Maturity Class			
Variable	**1**	**2**	**3**	**4**
c	37.024 (3.234)***	28.624 (3.289)***	3.846 (3.195)	6.129 (0.777)***
ttm	-0.646 (0.313)**	-0.780 (0.143)***	-0.013 (0.087)	-0.065 (0.011)***
ttm²	0.009 (0.008)	0.007 (0.002)***	-4.22E-05 (5.88E-04)	2.13E-04 (4.12E-05)***
mon	-251.234 (10.874)***	-74.842 (3.173)***	-22.557 (1.218)***	-11.544 (0.624)***
mon²	403.726 (19.068)***	117.949 (5.710)***	33.250 (2.047)***	16.052 (0.919)***
open	0.002 (1.52E-04)***	7.62E-04 (6.32E-05)***	3.06E-04 (2.63E-05)***	1.70E-04 (1.23E-05)***
dum	-0.837 (0.447)*	0.174 (0.146)	0.407 (0.069)***	0.335 (1.23E-05)***
R²	**0.35**	**0.29**	**0.21**	**0.21**

Out-of-the-money Options

	Maturity Class			
Variable	**1**	**2**	**3**	**4**
c	121.533 (7.613)***	131.962 (11.344)***	57.566 (9.948)***	26.677 (3.353)***
ttm	0.340 (0.787)	-3.491 (0.499)***	-1.178 (0.271)***	-0.261 (0.048)***
ttm²	-0.004 (0.020)	0.030 (0.005)***	0.007 (0.001)***	8.07E-04 (1.69E-04)***
mon	-701.818 (17.723)***	-207.809 (5.413)***	-51.730 (1.614)***	-27.805 (0.934)***
mon²	717.141 (27.425)***	175.090 (6.795)***	33.633 (1.759)***	14.825 (0.821)***
open	0.002 (1.52E-04)***	7.70E-04 (4.34E-05)***	4.09E-04 (2.10E-05)***	2.24E-04 (1.16E-05)***
dum	3.983 (0.963)***	3.540 (0.442)***	2.165 (0.222)***	2.356 (0.162)***
R²	**0.48**	**0.40**	**0.26**	**0.22**

Table 6
Linear Regressions for the Trading Volume of the 200- vs 100-strike Put Options

In-the-money Options

	Maturity Class			
Variable	**1**	**2**	**3**	**4**
c	33.080 (3.631)***	54.605 (5.278)***	27.701 (4.050)***	18.790 (1.933)***
ttm	-0.514 (0.3815)	-1.814 (0.231)***	-0.640 (0.109)***	-0.238 (0.027)***
ttm²	0.010 (0.010)	0.017 (0.002)***	0.004 (7.23E-04)***	7.86E-04 (9.24E-05)***
mon	-180.780 (8.361)***	-58.918 (2.851)***	-17.050 (1.003)***	-7.275 (0.642)***
mon²	202.732 (11.450)***	57.837 (3.576)***	14.862 (1.158)***	5.663 (0.630)***
open	0.001 (8.35E-05)***	6.83E-04 (4.94E-05)***	3.22E-04 (2.15E-05)***	2.32E-04 (1.50E-05)***
dum	-0.712 (0.459)	-0.0257 (0.196)	0.215 (0.084)**	0.138 (0.063)**
R²	**0.28**	**0.25**	**0.23**	**0.20**

Out-of-the Money Options

	Maturity Class			
Variable	**1**	**2**	**3**	**4**
c	144.547 (5.748)***	117.120 (9.254)***	48.428 (10.853)***	22.890 (3.353)***
ttm	-0.490 (0.582)	-2.968 (0.408)***	-0.923 (0.292)***	-0.218 (0.048)***
ttm²	0.015 (0.015)	0.026 (0.005)***	0.005 (0.002)***	6.59E-04 (1.71E-04)***
mon	-962.026 (17.400)***	-229.452 (7.457)***	-45.870 (2.928)***	-31.127 (1.866)***
mon²	1328.134 (31.206)***	256.626 (12.320)***	32.329 (4.650)***	26.684 (0.821)***
open	2.830E-04 (6.79E-05)***	6.86E-04 (3.95E-05)***	5.35E-04 (3.22E-05)***	2.25E-04 (1.37E-05)***
dum	13.979 (0.851)***	7.290 (0.401)***	3.712 (0.214)***	2.873 (0.182)***
R²	**0.61**	**0.41**	**0.27**	**0.21**

Table 7

GMM Estimation of the Econometric Model for the Distribution of Trading Activity

Call Options

Variable	c	ttm	sd	1/Index	delta	open	mon
Coefficient[1]	1.742 (0.198)***	0.004 (0.001)***	3.109 (0.395)***	-14206.994 (1678.114)***	-0.014 (0.006)**	0.391 (0.074)***	1.858 (1.921)

number of observations: 3304

Put Options

Variable	c	ttm	sd	1/Index	delta	open	mon
Coefficient[1]	0.570 (0.273)**	-1.548e-05 (9.468e-04)	0.971 (0.610)	-5972.016 (1888.939)***	1.7902e-04 (4.821e-03)	0.305 (0.081)***	6.494 (1.993)***

number of observations: 3304

[1] All standard errors are adjusted for heteroscedasticity and autocorrelation using the Newey-West (1987) procedure

Table 8

Average Distrisbution of Trading Activity across Neighboring Options with Different Moneyness and Maturity[1]

Call Options

Maturity Class	Option Pair											
	1	2	3	4	5	6	7	8	9	10	11	12
1	0.53	0.54	0.57	0.55	0.62	0.58	0.67	0.60	0.69	0.62	0.70	0.62
2	0.58	0.59	0.62	0.61	0.67	0.63	0.72	0.65	0.72	0.65	0.71	0.63
3	0.64	0.64	0.68	0.68	0.73	0.70	0.75	0.66	0.71	0.65	0.65	0.66
4	0.73	0.74	0.76	0.76	0.79	0.79	0.81	0.81	0.83	0.82	0.84	0.82

Put Options

Maturity Class	Option Pair											
	1	2	3	4	5	6	7	8	9	10	11	12
1	0.43	0.51	0.54	0.59	0.64	0.67	0.74	0.76	0.81	0.82	0.87	0.86
2	0.44	0.53	0.54	0.60	0.64	0.68	0.74	0.76	0.82	0.82	0.88	0.87
3	0.45	0.56	0.57	0.61	0.67	0.70	0.75	0.77	0.83	0.82	0.88	0.87
4	0.52	0.59	0.60	0.64	0.68	0.71	0.75	0.77	0.82	0.84	0.88	0.90

[1] The numbers in the cells indicate what percentage of the trades are attracted to the 200-strike contract in the corresponding option pair.