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## Will an optimal deposit insurance always increase financial stability?

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# BONN ECON DISCUSSION PAPERS

Discussion Paper 28/2002

**Will an optimal deposit insurance always increase  
financial stability?**

by

**Mathias Drehmann**

October 2002



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# Will an optimal deposit insurance always increase ...nancial stability?

## Second Version

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## Abstract

In this paper we show that deposit insurance can increase the probability of systemic banking crisis, even though it is optimally designed and its premium is risk related. This is driven by the possibility of contagious bank runs. We prove that contagion only occurs if the correlation between the portfolios of banks is high enough. Without deposit insurance contagious bank runs can impose such great losses on banks, that banks choose less correlated portfolios to avoid contagion altogether. Optimal deposit insurance eliminates this incentive and thus the correlation of portfolios and with it the probability of systemic banking crisis can increase.

Keywords: Bank runs, contagion, systemic risk, investment of banks, deposit insurance.

JEL-Classification: G21, G28.

# 1 Introduction

If there is such a thing as a consensus amongst academics and practitioners there might be one concerning the effects of deposit insurance. Laurence H. Mayer, a member of the Board of Governors of the US Federal Reserve System, summed up it nicely: "On the one hand, there are benefits from the contribution of deposit insurance to overall financial stability. On the other hand, deposit insurance imposes costs from the inducement of risk taking and the misallocation of resources" and "because of reduced market discipline and moral hazard, there is an intensified need for government supervision..." (Mayer 2001 p.1)<sup>1</sup> Given that the benefits - the enhancement of financial stability- seem to be obvious, the debate and policy action focused in recent years on the costs and ways to reduce them. The core idea of many policy proposals to eliminate moral hazard is to impose risk-related capital requirements or deposit insurance premia. The new Basel II accord is the most prominent initiative in that direction.

This paper wants to take a different approach. Assuming a world where such banking supervision reforms have been successfully implemented, we pose the question does such deposit insurance unambiguously increase financial stability. At first glance the answer seems obvious. A deposit insurance scheme which is perfectly risk-related eliminates all sun-spot and contagious bank runs. This increases financial stability. However increasing financial stability by eliminating contagious bank runs has an effect on the investment behaviour of banks. Without such deposit insurance banks have an incentive to differentiate themselves from other banks to protect against possible contagious runs. This incentive is eliminated by deposit insurance. This leads to greater similarities between banks, an increase in the occurrence of systemic crisis and a decline in welfare.

A contagious bank run takes place if depositors in one bank observe a failure of another bank and start a run on their bank. whether such a run starts depends on the informational content of the failure of the first bank to depositors in other banks. If the investment portfolio of all banks are completely independent then depositors in banks learn nothing about the probability that their bank fails from a failure of another bank.<sup>2</sup>

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<sup>1</sup>Testimony to the Subcommittee of Financial Institutions and Consumer Credit of the Committee on Financial Services, House of Representatives, 26th July 2001.

<sup>2</sup>This argument assumes that there are no cross-holdings between banks. This assumption will be

Thus there is no reason to alter withdrawal behaviour and no run will take place. Highly correlated portfolios on the other hand imply that a failure of one bank is a very good signal that my bank will also soon be bust. Given a "first come first serve" rule it is then optimal for any depositor to try to withdraw money as quickly as possible and a run will take place. Contagious bank runs impose expected losses on banks as they might be hit by a run, which is unwarranted because the bank is fundamentally solvent. Banks might want to avoid these losses by investing in less correlated portfolios and so reducing the likelihood of contagious bank runs. Deposit insurance removes this incentive because it eliminates the threat of unwarranted contagious bank runs. This changes the investment behaviour of banks and can imply that financial stability and welfare declines.

The historical example of the Australian free banking system in the 19th century gives more intuition for the results of our model.<sup>3</sup> The 1880s saw a property market boom in Australia. It came to an halt towards the end of the 1880s and property prices started to decline. As many banks were heavily exposed to the property market, severe strains in the financial system were observed and several small banks failed in the following years. In January 1893 the first big Australian bank, the Federal Bank, became bankrupt. Once it became clear that the government and no other bank would rescue the Federal Bank, withdrawals from other institutions rose to panic levels. Consequently several banks failed in the following months. However, one could also observe a flight to quality. In particular three big banks, the Australasia, the Union and the Bank of New South Wales, received substantial deposit inflows. It is interesting to note, that these three banks pulled out of the property market before the downturn in the end of the 1880s.

This highlights several key aspects of our theoretical work. In line with other empirical findings (e.g. Calomiris and Gorton (1991) or Benston and Kaufman(1996)) it seems that sun-spot type bank runs did not play a role and depositors did not run randomly on banks. In contrast the inflows of deposits to the Australasian, the Union and the Bank of New South Wales indicate that depositors were able to discriminate between banks according to their portfolios and contagious runs were driven by informational spillovers. This is

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kept throughout the paper.

<sup>3</sup>The following overview is taken from K. Dowd (1992), who gives an extensive discussion on the Australian free banking era.

exactly what our model predicts. We can also rationalise that some banks dropped out and others continued to invest in the property market in the end of the 1880s. A reduction of the exposure of some banks to the property market lowered the correlation between the portfolios of all banks and thus had an indirect effect on the expected profits of all banks. The reduction might have been enough such that it was optimal for the remaining banks to keep up their investments. The effects of an optimal deposit insurance are a straightforward result of this insight.

Beside this historical example there is econometric support that deposit insurance decreases financial stability. Demirgüç-Kunt and Detragiache (2000) find in a panel of over 60 countries that an explicit deposit insurance increases the probability of banking crisis. They attribute their empirical findings to the well known problems of moral hazard. The insight of our paper however put these findings into a different light. It well may be that not the design of the deposit insurance but shifts in the investment behaviour of banks are the driving force in the increase in the occurrence of banking crisis.

The paper is organised as follows. Section 2 discusses the related literature, Section 3 sets up the model; Section 4 looks at conditions for bank runs. Their implications on the investment behaviour of banks are analysed in Section 5. In Section 6 we consider welfare and show the effects of existing safety nets to prevent contagion. In Section 7 we conclude the paper.

## 2 Related literature

Our paper is closely related to the theoretical literature on bank runs. An important building block is the seminal paper by Diamond and Dybvig (1983) (hereafter D&D). The general idea of D&D is that banks act as insurance for unexpected liquidity needs. This implies the optimality of demand deposits. However given unobservable liquidity needs and costs a bank has to incur if it liquidates its investments early, sun-spot type bank runs are an equilibrium. Here it is individually optimal for each depositor to run on a fundamentally sound bank as everyone else is running as well. In contrast to D&D we look at different types of bank runs, which do not rest on the assumption of multiple equilibria. Bank runs in our model are only driven by information. Either depositors

know that their bank is fundamentally bankrupt, or they can observe a failure of another bank and start a run. The latter type of runs distinguishes the paper from early papers on information driven bank runs (e.g. Jacklin and Bhattacharya (1988) or Chari and Jagannathan (1988)). In a recent paper Goldstein and Pauzner (2000) extend the D&D set up to a global games framework. Here depositors receive noisy information about the true payoff of the investment. This leads to a unique equilibrium, where bank runs are sometimes efficient and sometimes not. Again, their focus is on a bank run on an individual bank and not on a series of bank runs on several banks.

The literature showed two main channels of contagion: the interbank market and informational spillovers.<sup>4</sup> We focus on the latter one and discuss the implications of an interbank market in the conclusion. Contagion driven by information was first formalised by Chen (1999). His paper takes the general set up of D&D and extends it to an economy with several banks which all invest in different projects. The realisation of the good or bad project outcome in each bank is independent of the realisation of projects in other banks. However the ex ante probability of the good outcome depends on the unknown "general state" of the banking sector (which can be low or high). In period 1 informed depositors receive perfect information on the true outcome of the investment of their bank. They will start a run when the bank is fundamentally bankrupt. Information contagion can occur, as information revelation is not simultaneous in all banks. Informed depositors in a fraction of banks receive information on their bank before depositors in other banks are informed about their bank. Depositors in later banks can observe the number of bank failures of earlier banks. The more banks failed the more likely that the state is low. Given that at least a certain number of banks failed, the probability that the state is low is so high that the expected payoff for depositors of running on their bank is higher than waiting. We use a simplified version of Chen's model with only two banks and information revelation to all depositors. The innovation of our paper is that we drop the assumption of an exogenous good or bad state and model the distribution of returns continuously. In our model banks have several risky investment possibilities, which imply different correlations between the banks portfolios. This allows banks to

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<sup>4</sup>For a discussion of contagion via the interbank market see for example Allen and Gale (2000) and Rochet and Tirole (1996).

have some influence on the correlation, which can be so low that contagious bank runs are not observed in equilibrium. We also look at deposit insurance from another angle. Chen derives the optimal insurance contract such that depositors only run on a bank once it is truly bankrupt. This eliminates all the moral hazard problems in his model. We assume that deposit insurance premia are risk related and that deposit insurance does not bail out any insolvent bank. We then show even such an optimal scheme can decrease welfare and increase the fragility of the financial system.

Our main result is also in the spirit of Calomiris and Kahn (1991). They show in their paper that demandable debt plays an important role to discipline bankers from fraudulent behaviour. Demandable debt in our framework allows the possibility of contagious bank runs. They constitute a potential threat to banks and hence can discipline banks from investing in high risk portfolios.

Our paper also touches the literature of the effects of charter value on the investment behaviour of banks. It is well known (e.g. Herring and Vankudre (1987) or Keeley (1990)) that a higher charter value of banks decreases risk taking. An increase in risk on the one hand increases current profits but on the other hand it increases the probability of failure and thus the probability of a loss of the continuation value. This sets an upper bound on the risk taking by banks. In our set up one can observe a similar effect. There is however an additional effect as the asset choice has not only an effect on the probability of failure but also on the probability of getting hit by a contagious bank run. This effect can strengthen the standard argument such that banks choose less risky investments because they also imply less correlation with other banks.

### 3 The model

The model follows the structure of Chen (1999) and thus draws heavily on D&D. In contrast to Chen we look at the simplest possible case where contagious bank runs can be observed: a three period model with two banks and depositors, who consume early or late. In a nutshell, the timing of the model is as follows:

After receiving one unit of funds from depositors both banks simultaneously take their investment decision in  $T_0$ .  $T_1$  is structured into three sub-periods  $T_{11}$  to  $T_{13}$ : At  $T_{11}$

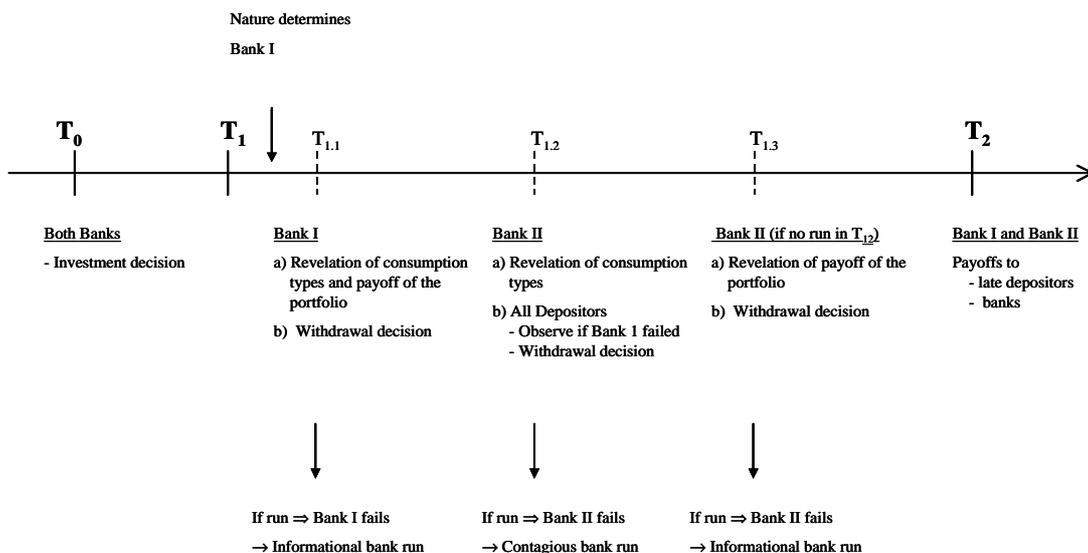


Figure 1: The timing of the game

there is one bank - Bank I - where information about the true payoff of the investment is revealed to its depositors. Given this information depositors decide, whether to withdraw or not. If everyone withdraws a run takes place and Bank I fails. Depositors in Bank II can observe this. In  $T_{1.2}$  they also take a withdrawal decision and a run might take place before information about the true payoff of the portfolio of Bank II is revealed to depositors in  $T_{1.3}$ . After the information revelation depositors can again decide whether to withdraw or not.  $T_2$  is the payoff stage; if banks did not fail in  $T_1$ ; remaining depositors get their money and banks receive their profits - if any.

Conceptually we can distinguish two types of bank runs in our model. Bank runs in period  $T_{1.1}$  or  $T_{1.3}$  are started by information on the true payoff of the investment. Hence we call them informational bank runs. Bank runs in  $T_{1.2}$  are also driven by information. The informational quality is however different. Runs in  $T_{1.2}$  start by observing the failure of Bank I, which is an imprecise signal if Bank II is solvent or not. We call these bank runs contagious runs. Figure 1 sums up the timing of the game.

Before we want to examine the issue of informational and contagious bank runs more formally, we further clarify the set up of the model.

There is a unit mass of depositors in each bank, who may be as in D&D of two types.

Early depositors consume in  $T_1$  and late depositors in  $T_2$ . In period 0 depositors do not know their type and after revelation in  $T_{11}$  (if they are depositors of Bank I) and  $T_{12}$  (if depositors of Bank II) it is private information to them. In each population of depositors there is a fraction  $\alpha \in [0, 1[$  of early and a fraction  $(1 - \alpha)$  of late depositors. This is common knowledge: Late depositors can withdraw in  $T_1$  and store the money with a storage technology till  $T_2$ : This storage technology gives a certain return of 1 per unit invested.

The utility of early (late) depositors only depends on the consumption  $C_1(C_2)$  in  $T_1(T_2)$  and is given by  $U(C_1)(U(C_2))$ . No specific shape of the utility function has to be assumed, except that  $U' > 0$ : For mathematical convenience we also normalise  $U$  so that  $U(0) = 0$ . The shape of the utility function - and thus whether depositors are risk averse or not - is only important to determine the interest rate on deposits (see Appendix 9.1).

Note, that in contrast to Chen we do not assume that there is a fraction of informed and uninformed depositors in each bank. This keeps the model more tractable. But in the Appendix 9.3 we show that our model is easily extended to the case where only a fraction of depositors receives information.

Even though there is one bank where information is revealed earlier both bank are ex ante identical. Each bank is equally likely to be Bank I or Bank II and must make its investment decision before this is determined.

Both banks are risk neutral and profit maximisers. For simplicity we assume that there are no cross holdings between banks<sup>5</sup> and both banks face a binary decision problem in  $T_0$ : they can either invest all their funds in a risky portfolio  $Z_{ia}$  or a risky portfolio  $Z_{ib}$  with  $i \in \{I, II\}$  indicating the investment of Bank I or II.<sup>6</sup> The underlying distributions of  $Z_{ia}[Z_{Ia}]$  is identical to the underlying distribution of  $Z_{ib}[Z_{IIb}]$ . We assume that  $Z_{Ij}$  and  $Z_{IIk}$  ( $j, k \in \{a, b\}$ ) are joint normally distributed. The exact specification is discussed in section 5. It is important in our model is that payoffs of the portfolios are not independent across banks. We assume that different investment choices of banks imply different correlations  $\rho_{I,II} > 0$ . The investment choices and thus the correlation

<sup>5</sup>A short discussion on the implications of cross holdings between banks is included in the last section.

<sup>6</sup>That we index banks with I, II is a slight abuse of notation, as in  $T_0$  it is not decided yet which bank becomes I or II. Introducing another index would, however, complicate the notation without giving any additional insight.

are commonly known.<sup>7</sup>

Portfolio  $Z_{ia}$  as well as  $Z_{ib}$  realise payoffs in  $T_2$ : Investments can be partially or fully liquidated at  $T_1$ : We assume throughout the paper that one unit invested can be liquidated for one unit in  $T_1$ . As losses of fire sales in banking crises are important in reality, we discuss a modified model in the Appendix 9.4, in which assets can be liquidated at only a fraction of their value in  $T_1$ . This does not change the qualitative results. A bank will be declared bankrupt in  $T_2$  if the payoff of its portfolio is less than its remaining liabilities to depositors. If the bank has to declare bankruptcy or is subject to a run the bank will earn zero because of limited liability.<sup>8</sup> In case of bankruptcy in  $T_2$  the remaining assets are distributed equally among the remaining depositors.

Demand deposit contracts offered by banks have the standard form: depositors can withdraw their money without notice on a first come first serve basis (either at period  $T_1$  or  $T_2$ ) and receive an interest rate of  $r$  ( $r_1$  in  $T_1$  and  $r_2$  in  $T_2$ ): If more depositors want to withdraw in  $T_1$  than the bank is able to pay out, the order of withdrawals is determined randomly with equal probability of being able to withdraw. As the focus of the paper is not risk sharing we assume that  $r_1$  and  $r_2$  are exogenously given and the same for both banks with  $r_2 > r_1 > 1$ .<sup>9</sup> The interest rates are chosen in such a way that they satisfy the participation constraint of depositors. For this to be possible we need to assume that unconditional expected payoffs of all portfolios are such that no runs are observed before any information is revealed to depositors. This is the case if  $r_1 < 1$  and  $pE[U(\frac{(1-p)r_1 Z_i}{(1-p)} + jZ_i \cdot z) + (1-p)U(r_2)] > U(r_1)$  for  $i \in \{a, b\}$ ;  $p = p(Z_i \cdot z)$  and  $z = \frac{(1-p)r_1}{(1-p)r_1}$ : The first condition has to hold as otherwise the bank is not able to pay out all early depositors even if it liquidates all its assets. The second condition is discussed in section 4.1. In general this condition also implies that there are no contagious bank runs on bank II in  $T_{12}$  in case there was no informational bank run on bank I in  $T_{11}$ : However there are peculiar distributions where it is not the case. These are ruled out by

<sup>7</sup>It is only necessary to assume that at least one late depositor in each bank knows the correlations between the portfolios of banks. Assuming that all do makes it easier to solve for the equilibrium.

<sup>8</sup>The reader should be aware that we distinguish between "bankrupt" and "failing". As a convention we use "bankrupt" only if the payoff of the investment in  $T_2$  is not enough to satisfy all the claims by depositors. On the other hand we use "failing" only when we talk about  $T_1$  and a bank going bust in that period. Without deposit insurance a failure in  $T_1$  is always driven by bank runs which may or may not be driven by depositors knowing that the bank will be bankrupt in  $T_2$ :

<sup>9</sup>A short discussion on the derivation of  $r_1$  and  $r_2$  is included in the Appendix 9.1.

assumption. The exact technical condition for this is discussed below.

The focus of this paper is on bank runs driven by information, we do not want to consider sun-spot type bank runs. To exclude them we make the assumption that in case of multiple equilibria players always coordinate on the no-run equilibrium. As we are solving for Nash equilibria in pure strategies, only two equilibria in the D&D set up exist; either all late depositors wait till  $T_2$  or everyone runs at  $T_1$ : Given that our set up is based on D&D the same kind of multiplicity of equilibria can be observed in  $T_{11}$ ;  $T_{12}$  and  $T_{13}$ : As there are only two equilibria in pure strategies the above assumption is equivalent to assuming that in case of multiple equilibria players coordinate on the Pareto superior equilibrium.

We feel that we can make this assumption on several grounds. Most important, it seems that sun-spot type bank runs are empirically not a relevant phenomena (see for example Calomiris and Gorton (1991) or Benston and Kaufman(1996)). Furthermore the assumption is most crucial in the analysis of bank runs in period  $T_{11}$ : As will be seen in the next subsection, bank runs in period  $T_{11}$  will only occur if the bank is fundamentally bankrupt. A bank run on Bank I acts as a well defined signal for depositors in Bank II, whether their bank is bankrupt or not. Dropping the above assumption would make the signal more fuzzy and one would have to introduce beliefs about the probability that the bank run in  $T_{11}$  was driven by fundamental information to solve the model. However allowing for an interbank market in such an environment would imply that Bank I could always raise the necessary liquidity in the interbank market in case of a sun-spot type bank run as - given full information revelation - it is common knowledge that the bank is fundamentally solvent. Hence sun-spot type bank runs would not start from the onset in  $T_{11}$ .

In the following we will solve our model for Nash equilibria in pure strategies. In case of depositors we also only look at symmetric equilibria. The more formal reader should keep in mind that we actually solve for a perfect Bayesian Nash equilibrium in pure strategies, where depositors have beliefs over the probability that their bank is bankrupt. However, full information revelation ...xes beliefs in  $T_{11}$  and  $T_{13}$ : It will also become apparent that beliefs in  $T_{12}$  are always on the equilibrium path and thus determined by Bayes rule. Furthermore there are no strategic interactions between depositors in Bank I and Bank

II. This set up combined with the exclusion of sun-spot type bank runs allows us to solve the game backwards. Introducing beliefs would only complicate the notation. In the appendix we explicitly model beliefs as this is necessary if some depositors do and others do not receive information in  $T_{11}$  and  $T_{13}$ . The general results will be similar to the full information case, though some threshold conditions change.

Before solving the model, we introduce some convenient notation;  $R$  [NR] indicates the case that there was a [no] run on Bank I ; if we allow for [exclude] contagious bank runs in our model we use  $C$  [NC] as an index;  $j, k \in \{1, 2\}$  indicate that Bank I chose portfolio  $Z_{1j}$  and Bank II chose  $Z_{11k}$ .

## 4 Bank runs

The driving forces of bank runs are always late depositors. Given that early depositors only have a positive utility if they consume in  $T_1$  they try to withdraw their money as soon as possible. Thus we see withdrawals of at least  $\alpha$  (i.e. all the early) depositors in Bank I at  $T_{11}$  and in Bank II at  $T_{12}$ : With this behaviour in mind we solve for the optimal behaviour of late depositors.

### 4.1 Informational bank runs

It will later become apparent that Bank II only exists in  $T_{13}$  if no late depositor withdraws at  $T_{12}$ :

Taking this as given, the utility of a late depositor when he withdraws at  $T_2$  and no other late depositor withdraws at  $T_{13}$  is

$$U_2 = \min \left\{ U \left( \frac{(1 - \alpha)r_1 z_{11}}{(1 - \alpha)} \right); U(r_2) \right\} \quad (1)$$

At  $T_2$  late depositors receive  $r_2$  if the bank is solvent and  $\frac{(1 - \alpha)r_1 z_{11}}{(1 - \alpha)}$  otherwise. The denominator of the latter expression is the payoff of the portfolio  $z_{11}$  after the bank liquidated  $\alpha r_1$  assets to pay out early depositors in  $T_{12}$ . According to our bankruptcy rule the remaining assets are equally distributed among the  $(1 - \alpha)$  late depositors in  $T_2$ . Note that with our information structure the true realisation of  $z_{11}$  is revealed to

depositors in  $T_{13}$ : If

$$U_2 \cdot U(r_1) \tag{2}$$

it is optimal for an individual depositor to deviate from the no-withdrawal equilibrium and withdraw  $r_1$  with certainty in  $T_{13}$  and store it with the storage technology till  $T_2$ : If, however, it is optimal for one late depositor to deviate from the non-withdrawal equilibrium it is optimal for all late depositors. Thus a bank run will take place if  $\min(\frac{(1-i_s)r_1 z_{11}}{(1-i_s)}; r_2) < r_1$  or since  $r_2 > r_1$  if

$$z_{11} \cdot \frac{(1-i_s)r_1}{(1-i_s)r_1} := z$$

That bank runs are uniquely determined by  $z_{11}$  being small or larger than  $z$  depends on our assumption that depositors coordinate on the Pareto superior equilibrium in case of multiple equilibria. There are also equilibria where there is a full or partial run, even though  $z_{11} > z$ . They are however Pareto inferior as in these equilibria no one consumes more than  $r_1$  and some consume even zero, in contrast to the equilibrium where every late depositor leaves the money in the bank and is able to consume at least  $r_1$  in  $T_2$ : An informational bank run is however the unique equilibrium once  $z < z$ :

A similar argument holds for Bank I in  $T_{11}$ . If all late depositors in Bank I do not withdraw in  $T_{11}$ , then their payoff in  $T_2$  is determined by equation (1) where  $z_{11}$  is replaced by Bank I's payoff  $z_1$ . Again  $z_1$  is known by depositors of Bank I in  $T_{11}$ : As above this implies that a bank run on Bank I would take place if  $z_1 < z$ :

Equation (2) gives also the intuition why we needed to assume for the existence of banks that  $pE \cdot U \frac{(1-i_s)r_1 z_i}{(1-i_s)} - jZ_i < \frac{(1-i_s)r_1}{(1-i_s)} + (1-i_p)U(r_2) > U(r_1)$ : The left hand side of this expression represents the expected value of  $U_2$  at  $T_{11}$  or  $T_{13}$  given no information. If this would be greater than  $U(r_1)$  late depositors would always start a run in  $T_1$  and banks would not exist.

The results so far are not unexpected. Given full information revelation late depositors know if the bank is bankrupt or not and if so how much money they would get in  $T_2$ .  $r_1$  however is the amount of money a late depositor can secure himself by pretending to be an early one. Given that he can store the money till  $T_2$  it is obvious that a late depositor withdraws at  $T_1$  where this gives him a higher return than waiting.

The above result is also equivalent to the observation that a bank is only subject to an informational bank run in  $T_{11}$  or  $T_{13}$  if it will be bankrupt in  $T_2$ : It is never the case that a bank run forces a bank into failure even though it could pay out depositors in  $T_2$ .<sup>10</sup>

The results of this section are summed up in Proposition 1:

**Proposition 1** An informational bank run is only observed in  $T_{11}(T_{13})$  if the bank is bankrupt and the truly realised return  $z_1(z_{11})$  is less than or equal to  $z := \frac{(1-i)r_1}{(1-i)r_1}$ :

## 4.2 Contagious bank runs

A bank run on Bank I is observable to depositors in the second bank at  $T_{12}$ . Hence depositors know if  $z_1 \cdot z$  or not. As both portfolios are correlated this contains some information on the probability that  $z_{11} \cdot z$  and an informational bank run happening in  $T_{13}$ . Let  $p = p(z_{11} \cdot z)$  denote the unconditional probability that there will be an informational bank run in  $T_{13}$  and  $p_R = p(z_{11} \cdot z | R) = p(z_{11} \cdot z | z_1 \cdot z)$  the conditional probability that  $z_{11} \cdot z$  given a run on Bank I. Note that this conditional probability depends on the investment of Bank II as well as Bank I as these determine the correlation of portfolios.

In  $T_{12}$  the bank has to liquidate at least  $i r_1$  assets to satisfy the demand of early depositors. Given this the expected utility in  $T_{12}$  of late depositors  $U_{I,R}$  given a run on Bank I and no late depositor withdraws at  $T_{12}$  is

$$U_{I,R} = p_R \frac{(1-i)r_1}{(1-i)r_1} U(r_1) + (1-i-p_R) E(U_{2,NR}) \quad (3)$$

The first term on the right hand side is the expected utility given  $z_{11} \cdot z$  in  $T_{13}$ . In that case an informational bank run occurs in  $T_{13}$ : With the first come first serve rule depositors are able to withdraw  $r_1$  as long as the bank can liquidate some assets. This is the case for the first  $\frac{(1-i)r_1}{(1-i)r_1}$  in the queue. Given random ordering the probability to be among the first lucky depositor is also  $\frac{(1-i)r_1}{(1-i)r_1}$ . When depositors have a place later in the queue they are unable to withdraw anything yielding a utility of  $U(0) = 0$ .

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<sup>10</sup>This must be the case as the run-threshold implies that the bank is bankrupt ( $z < \frac{(1-i)r_2}{(1-i)r_1}$ ).

The second term of equation (3) is the expected utility given  $z_{11} > z$  in  $T_{13}$  and all late depositors wait till  $T_2$ :

$$E(U_{2;NR}) = U(r_2) \int_{z_{11}}^{\infty} f(Z_{11}|z_{11} > z; R) + \int_{z_{11}}^{\infty} U\left(\frac{(1+r_2)z_{11}}{(1+r_1)}\right) f(Z_{11}|z_{11} > z; R) \quad (4)$$

where  $f(z|z_{11} > z; R)$  is the conditional distribution of the payoff of the investment of Bank II given there is no run in  $T_{13}$  but there was a run on Bank I in  $T_{11}$ . Here depositors either get  $r_2$  or  $\frac{(1+r_1)z_{11}}{(1+r_2)} > r_1$ , if Bank II is bankrupt at  $T_2$  but it is still better for depositors to wait.

It is optimal for a late depositor to deviate from the non-withdrawal equilibrium and withdraw  $r_1$  in  $T_{12}$  if

$$U(r_1) > U_{1;R} \quad (5)$$

Solving equation (5) for  $p_R$  leads to the following Proposition:

**Proposition 2** Given a run on Bank I, a contagious bank run on Bank II is observed in  $T_{12}$  if the conditional probability  $p_R$  that the payoff of the investment  $z_{11}$  is less than or equal to  $z$  is so high that

$$p_R > \frac{E(U_{2;NR}) - U(r_1)}{E(U_{2;NR}) - \frac{(1+r_1)z_{11}}{(1+r_2)} U(r_1)} =: \bar{p} \quad (6)$$

Whether Proposition 2 applies and a contagious bank run is observed in equilibrium depends on the correlation of the banks portfolios. For high correlations between the portfolio of Bank I and Bank II the conditional probability of an informational bank run at  $T_{13}$  goes to 1 as for  $\rho_{11} \rightarrow 1$   $p_R = p(z_{11} \leq z|z_{11} > z) \rightarrow 1$ . It is also easy to see that  $\bar{p} < 1$  independent of the correlation as  $r_1 > 1$ : Furthermore  $\bar{p}$  will change with a change in the correlation as well. Given a run on bank I a higher correlation implies a greater possibility that even though  $z_{11} > z$  the bank is bankrupt and depositors earn less than  $r_2$ : Hence  $E(U_{2;NR})$  declines and with it  $\bar{p}$ : Proposition 2 and the joint normality of  $Z_{1j}$  and  $Z_{1k}$  ( $j, k = 2, \dots, n$ ) implies that a contagious bank run on Bank II only takes place

if  $\frac{1}{2} \rho_{1,1}$  is greater than a correlation threshold  $\frac{1}{2}$  and there was an informational bank run on Bank I. <sup>11</sup>

Given the previous discussion we can reformulate Proposition 2:

**Corrolary 1** A contagious bank run on Bank II is only observed in  $T_{12}$  if the correlation between both portfolios  $\frac{1}{2} \rho_{1,1}$  is greater than a threshold  $\frac{1}{2}$  and there was an informational run on Bank I in  $T_{11}$ .

This result is again intuitive. Contagious banks runs are driven by observing a bank run on the Bank I. If portfolios are independent this information has no value for depositors in the Bank II and thus should not alter the behaviour of depositors. The higher the correlation however the greater the informational content of a bank run on the first bank for depositors of the second. When both banks choose the same portfolio, perfect correlation implies that a bank run on the first is a perfect signal that the second bank will fail as well. Thus there must exists a correlation threshold for which running on the bank is optimal rather then waiting till the true information is revealed.

We started our discussion of informational bank runs with the assumption that Bank II only exists in  $T_{13}$  if no late depositor withdraws his funds in  $T_{12}$ : This is the outcome of Proposition 2, as either all the late depositors withdraw in  $T_{12}$  or none. The uniqueness of this equilibrium also rests on our assumption that in case of multiple equilibria players coordinate on the Pareto superior equilibrium.

## 5 The investment decision of banks

The investment decision of banks is very simple as they have to decide between 2 portfolios  $Z_i \in \{Z_{ia}; Z_{ib}\} \quad i \in \{I; II\}$ : Banks will choose the portfolio, which maximises their expected profits. Looking at a world where contagion is excluded by assumption the expected

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<sup>11</sup>Comparing equation 6 with the assumption that  $pE \sum_{i=1}^3 U \left( \frac{(1-p)r_1 Z_i}{(1-p)r_1} \right) + (1-p)U(r_2) > U(r_1)$  one can see that under most distributions the assumption is enough such that no contagious bank runs on Bank II are observed in  $T_{12}$  in case there was no informational bank run on Bank I in  $T_{11}$ . Otherwise we assume that  $p_{NR} \cdot \frac{E_{NR}(U_{2;NR})_i U(r_1)}{E_{NR}(U_{2;NR})_i \left( \frac{(1-p)r_1}{(1-p)r_1} \right) U(r_1)}$  where  $E_{NR}$  is as equation 4 but given no run on Bank I:

profit for a bank of investing in  $Z_{ij}$ <sup>12</sup> is

$$E(\pi_{z_i})_{NC} = E[\max((1 - \alpha)r_1 Z_i - (1 - \alpha)r_2; 0)] \quad (7)$$

$$= E(\pi_{z_i}) \quad (8)$$

where the subscript NC indicates the no-contagion case. Given limited liability banks can never earn less than 0. In case the bank does not go bankrupt, it keeps the remaining payoff of the investment  $Z_i$  after liquidating  $\alpha r_1$  in  $T_1$  to satisfy the demand of early depositors and paying out  $r_2$  to  $(1 - \alpha)$  late depositors: Expected profits without contagion are also the same as expected profits in a world where there are no informational runs. This is a result of Proposition I which showed that information runs only occur if the bank is bankrupt and hence earns already 0 because of limited liability.

To see the implications of contagion on the investment decision, it is helpful to rewrite expected profits to match the timing of our model. At  $T_0$  each bank has the probability of  $\frac{1}{2}$  of becoming the first or the second bank. Expected profits in the no contagion case can be therefore also expressed as

$$E(\pi_z)_{NC} = \frac{1}{2}E(\pi_{z_i}) + \frac{1}{2}[(1 - \phi_i)E(\pi_{z_{i,j}}|NR) + \phi_i E(\pi_{z_{i,j}}|R_{z_i})] \quad (9)$$

The first term in equation (9) is the expected profit of Bank I. The second term is the sum of the expected profits of being Bank II given there was no run on Bank I ( $E(\pi_{i,j}|NR)$ ) and given there was a run ( $E(\pi_{i,j}|R)$ ) with  $\phi_i = p(z_i < z)$  the probability that there is a run on Bank I in  $T_{11}$ .

The key insight of the last section was, that contagious bank runs always occur if there is a run on Bank I in  $T_0$  and the correlation  $\rho_{z_i, z}$  is greater than  $\frac{1}{2}$ : An informational bank run on Bank I is however not a precise signal that the payoff of Bank II's investment is also lower than  $z$ . This implies that there are situations where there is a contagious bank run on Bank II in  $T_{12}$ , even though the bank is fundamentally sound and no informational bank run would happen in  $T_{13}$ . Given that depositors in Bank II always run on their

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<sup>12</sup>To keep the notation simple we drop the index  $j$  to indicate whether bank  $i$  invested in  $Z_{ia}$  or  $Z_{ib}$ : We only use it when it is necessary to clarify an argument.

bank if  $\frac{1}{2} > \frac{1}{2}$  and there was a run on Bank I, expected profits are lower if contagion is possible.

$$E(z)_C = E(z)_{NC} + q_{I;II} E(z_{II} | R) \quad (10)$$

where

$$q_{I;II} = \begin{cases} 0 & \text{if } \frac{1}{2} < \frac{1}{2} \\ \frac{1}{2} q_I = \frac{1}{2} p(z_I > z) & \text{if } \frac{1}{2} > \frac{1}{2} \end{cases} \quad (11)$$

$q_{I;II}$  is the probability that the bank is second and subject to a contagious bank run. Given Corollary 1 we know that a contagious run takes only place if  $\frac{1}{2} > \frac{1}{2}$  and there is an informational run on Bank I in T; which is the case with probability  $p(z_I > z)$ :

Before we can see the implication of contagion on the investment behaviour of banks, we need to look more detailed into the investment possibilities. As mentioned we assume the banks face a binary decision problem. They can either invest all their funds in a risky portfolio  $Z_{ia}$  or a risky portfolio  $Z_{ib}$  with  $i \in \{I, II\}$  indicating the investment of Bank I or II. The underlying distributions of  $Z_{ia}[Z_{IIa}]$  is identical to the underlying distribution of  $Z_{ib}[Z_{IIb}]$ . For all our results the exact distribution is not important. We only need that different investment choices imply different correlations between portfolios of banks. By convention the correlation  $\frac{1}{2}_{i;II}$  is highest if both banks invest in portfolio  $Z_a$  and lowest if both invest in  $Z_b$ , thus  $\frac{1}{2}_{aa} > \frac{1}{2}_{ab} > \frac{1}{2}_{bb}$ . We want to focus on the case where

$$\frac{1}{2}_{aa} > \frac{1}{2} > \frac{1}{2}_{ab} > \frac{1}{2}_{bb}$$

We also limit the analysis to the case where the "high" correlation asset is more risky but that is more profitable for a bank to invest in it because of limited liability.

$$E(z_A) > E(z_B) \quad (12)$$

with

$$\sigma_a > \sigma_b \text{ and } \frac{1}{4}_a > \frac{1}{4}_b \quad (13)$$

We made these assumption as they generate the most puzzling results especially in the welfare analysis. However we discuss the robustness of our results later in the paper. One can see already that given Corollary 1 distributions where  $\frac{1}{2} > \frac{1}{2}_{aa}$  or  $\frac{1}{2}_{bb} > \frac{1}{2}$  are of no

great interest. In both cases the occurrence of contagious bank runs are irrelevant for the investment decision of banks. For the first case contagious bank runs on Bank II would never in contrast to the latter where they would always be observed if an informational bank run took place on Bank I.

Even though the exact distributions are not crucial for the results it is helpful to set up an example to see how investment choices by banks influence the correlation and to give intuition to results later in the paper. Assume there are three underlying assets:  $X$ ;  $Y_I$  and  $Y_{II}$ . Both banks can invest in asset  $X$ , but  $Y_i$  is specific to Bank  $I$  ( $I$ ;  $II$ ). With

$$X \sim N(\mu_x, \sigma_x^2)$$

and

$$Y_i \sim N(\mu_y, \sigma_y^2)$$

and

$$\mu_x < \mu_y \text{ and } \sigma_x > \sigma_y$$

$X$ ;  $Y_I$  and  $Y_{II}$  are all independently distributed from each other.

The decision problem of banks is, which fraction  $a$  or  $b$  to invest in the common asset  $X$ . Without loss of generality we assume  $0 < b < a < 1$ : Thus  $Z_{ai}$  and  $Z_{bi}$  are

$$\begin{aligned} Z_{ai} &= aX + (1 - a)Y_i \\ &\sim N(\mu_a, \sigma_a^2) \end{aligned}$$

and

$$\begin{aligned} Z_{bi} &= bX + (1 - b)Y_i \\ &\sim N(\mu_b, \sigma_b^2) \end{aligned}$$

This implies  $\sigma_a < \sigma_b$  and  $\beta_a > \beta_b$  and correlations of portfolio  $Z_i$  with  $Z_{ii}$ <sup>13</sup>

$$\rho_{aa} > \rho_{ab} = \rho_{ba} > \rho_{bb}$$

For  $\sigma_a$  large and  $\sigma_b$  small enough it is always the case that  $\rho_{aa} > \frac{1}{2} > \rho_{ab}$ : One can then find values for  $\sigma_x, \sigma_y, \beta_x$  and  $\beta_y$  which satisfy condition (12) and (13):

Such a set up can be motivated along two lines. The first is that one can essentially divide the risk banks take on in two orthogonal components; macroeconomic or common risk and idiosyncratic risk. Every investment that each bank undertakes has both components of risk, although to a different degree. An investment in a stock market index contains much more macroeconomic risk than a credit to a local plumber, who only borrows at his local bank. This gives the bank some leverage on the risk-mix it wants to have. The other motivation behind this set up is more in line with the introductory example of the Australian banking crises. This banking crisis as many others was driven by investments in the property market, which later collapsed. In our model the property market would be represented by  $X$ , whereas all the other investments of banks are denoted by  $Y_i$ : What is important for us is that in both interpretations the bank is able to choose the risks it wants to take and thus is also able to influence the correlation with the portfolios of other banks.

We can now easily see the effects of contagion on the investment decision of banks. Without contagion both banks invest in the portfolio  $Z_a$  as  $E(r_a) > E(r_b)$ : However we know from the previous discussion that contagion lowers expected profits. Looking at equation (10) the expected losses of contagion ( $q_{i,j} E(r_{Z_{ij}})$ ) might be so high, that at least one bank invests in the low risk portfolio  $Z_b$ : This can lower the correlation between both banks sufficiently so that the informational content of a failure of Bank I is not high enough to make it optimal for late depositors in Bank II to run on their bank in  $T_{12}$ : Hence contagious bank runs will not be observed in equilibrium.

This intuition is easily formalised. The condition  $\rho_{aa} > \frac{1}{2} > \rho_{ab}$  is equivalent to  $q_{aa} > 0$  and  $q_{ba}, q_{ab}, q_{bb} = 0$ : The investment game has then the form as in Table 1.

As long as the potential losses of contagion  $q_{aa} E(r_{Z_a})$  are greater than the benefits

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<sup>13</sup>  $\rho_{aa} = \frac{a^2 \beta_x^2}{\beta_a^2} > \frac{1}{2} > \rho_{ab} = \rho_{ba} = \frac{ab \beta_x^2}{\beta_a \beta_b} > \rho_{bb} = \frac{b^2 \beta_x^2}{\beta_b^2}$

Table 1: Payoff table if  $\frac{1}{2}q_{aa} > \frac{1}{2}$  and  $\frac{1}{2}q_{ab}; \frac{1}{2}q_{ba}; \frac{1}{2}q_{bb} < \frac{1}{2}$

Bank I Bank II	$Z_{Ia}$	$Z_{Ib}$
$Z_{IIa}$	$E(\cdot a) + q_{aa}E(\cdot a)R_a$ $E(\cdot a) + q_{aa}E(\cdot a)R_a$	$E(\cdot b)$ $E(\cdot a)$
$Z_{IIb}$	$E(\cdot a)$ $E(\cdot b)$	$E(\cdot b)$ $E(\cdot b)$

$E(\cdot|a) > E(\cdot|b)$  of investing in  $Z_a$  relative to  $Z_b$   $aa$  can not be an equilibrium. But if at least one bank invests in  $Z_b$ ; then the correlation between both banks is low enough that contagion does not occur in equilibrium. Given  $E(\cdot|a) > E(\cdot|b)$   $bb$  can also not be an equilibrium and only one bank invests in  $Z_a$  whereas the other in  $Z_b$ :

This result is summed up in Proposition 3

**Proposition 3** If  $q_{aa}E(\cdot|a)R_a > E(\cdot|a) + E(\cdot|b)$  contagious bank runs are not observed in equilibrium; one bank invests in the low return portfolio  $Z_b$  and the other in the high return portfolio  $Z_a$ .

For Proposition 3 to hold the assumption that  $\frac{1}{2}q_{aa} > \frac{1}{2} > \frac{1}{2}q_{ab}$  and the condition  $q_{aa}E(\cdot|a)R_a > E(\cdot|a) + E(\cdot|b)$  are crucial. Put into words they simply say that it must be possible to avoid contagion and the gains of doing so must be high enough.

The question clearly is how often can one observe such severe losses that banks want to avoid contagion, and they are able to do so. Even with our simple distributional example it is hard to specify exact conditions for the underlying distributions for which Proposition 3 applies or not. Arguing intuitively with our example the share  $b$  of the common investment in  $Z_b$  should not be too high as otherwise contagion is also observed in the  $ab$  or  $bb$  equilibrium. Furthermore the share  $a$  of common investment in  $Z_a$  should not be too high since as  $a \rightarrow 1; \frac{1}{2}q_{aa} \rightarrow 1$  and  $q_{aa} > 0$ ; but  $q_{aa}E(\cdot|a)R_a \rightarrow 0$ . If the correlation is very high, then the informational content of a failure is very high and depositors will always run on the second bank. It is then rarely the case that a contagious bank run hits a bank which is fundamentally sound so the possibility of contagious bank runs does not harm banks much. Furthermore  $Z_a$  should not be much more profitable than the expected losses as otherwise the benefits of investing in  $Z_a$  outweigh the expected losses of contagion.

## 5.1 The effects of charter value

The discussion in the previous section might suggest that Proposition 3 does not have much bite. Banks would only avoid contagious bank runs if  $b$  is "small",  $a$  is "medium" range and portfolios  $Z_a$  and  $Z_b$  are relatively similar. However the intuition has much greater application if one includes charter value of banks in the analysis. It was already discussed in the Introduction that a higher charter value decreases risk taking. In general it is optimal for banks to choose the asset with the highest risk in a one shot game (ceteris paribus) because of the option like nature of the profit function. In a multi period setting there exists a continuation value after the first period - the charter value. This decreases risk taking for the first period as high risk taking leads not only to a high probability of bankruptcy but also to a high probability of the loss of the charter value. This sets an upper bound on the risk taking by banks. In our set up one can observe a similar effect. There is however an additional effect as the asset choice not only has an effect on the probability of bankruptcy and the loss of the continuation value but also on the probability of getting hit by a contagious bank run.

With the introduction of charter value, banks in our set up do not only want to maximise expected profits in  $T_0$  but net present value of the bank including the charter value. Assume for simplicity that the charter value  $K$  is independent of the investment behaviour of banks. The net present value of banks in  $T_0$  in the no contagion case is then

$$NPV_{NC} = E(\pi)_{NC} + p_i^0 K$$

$p_i^0 = p(z_i > \frac{(1+r_2)r_2}{(1+r_1)})$  is the probability that the bank is not bankrupt. In the contagion case the net present value changes to

$$NPV_C = NPV_{NC} + q_{i;11} [E(\pi_{i;11}R) + p_{i;11}^0 K]$$

where  $q_{i;11}$  is still

$$q_{i;11} = \begin{cases} 8 < 0 & \text{if } \frac{1}{2}z_{i;11} < \frac{1}{2} \\ \frac{1}{2}q_i = \frac{1}{2}p(z_i > z) & \text{if } \frac{1}{2}z_{i;11} > \frac{1}{2} \end{cases}$$

If  $\frac{1}{2} > \frac{1}{2} > \frac{1}{2}$  the condition for Proposition 3 to hold changes to:

$$E(\cdot|_a) - E(\cdot|_b) + (p_a^0 - p_b^0)K \cdot q_{aa}[E(\cdot|_a)R_a + p_{ajR}^0K] \quad (14)$$

If  $E(\cdot|_a) - E(\cdot|_b) + (p_a^0 - p_b^0)K < 0$  we see the standard effect of charter value. Even though  $E(\cdot|_a) - E(\cdot|_b) > 0$  it can be the case that the NPV of investing in  $Z_b$  is higher than of  $Z_a$  as for the latter the likelihood of losing the charter value  $(1 - p^0)$  is much greater. New in our analysis is the additional effect on the right side. This strengthens the standard effects when the potential current gains of investing in  $Z_a$  are big enough to offset the possible losses of the charter value so that  $E(\cdot|_a) - E(\cdot|_b) + (p_b^0 - p_a^0)K > 0$ : In that case the potential losses of a unwarranted contagious run, i.e. the loss of profits for that period plus the loss of the continuation value, can act as another deterrent of investing in the high risk and high correlation asset  $Z_a$ :

Bearing in mind that charter value strengthens Proposition 3 we return in the following section to the model without charter value.

## 6 Welfare and the effects of an optimal deposit insurance

In this section we look on welfare implications of contagion as well as the effects of deposit insurance (DI) on welfare. As it is a well known fact that mispriced deposit insurance and forbearance induces moral hazard and leads to excessive risk taking, we want to exclude this by assumption. We rather assume that the DI mimics the behaviour of depositors in the non-deposit insurance case, i.e. the DI premium is fixed before the investment decision and just replicates the demanded interest rates. This also means that the DI does not bail out any banks, which are truly insolvent. The only effects of the introduction of such a scheme in our model is the elimination of contagious bank runs<sup>14</sup>. We call this DI the optimal DI.

Given that we did not specify utility functions for depositors, measuring welfare as

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<sup>14</sup>Informational bank runs would also not be observed in equilibrium. However, whenever they occur the bank is truly insolvent. Hence the optimal DI would liquidate the bank in these cases.

the unweighted sum of expected utilities of all agents is not possible. If we look at the unweighted sum of expected payoffs to all parties as a measure then the first best in our model is clear: both banks should invest in portfolio  $Z_b$  ( $^1_{Z_b} > ^1_{Z_a}$ ). The welfare argument for  $Z_b$  is strengthened if depositors are risk averse, because they face all the downward risk and the probability of bankruptcy is greater for  $Z_a$ :

In our model however  $bb$  can not be an equilibrium and the first best can not be achieved. The realised equilibrium depends on whether Proposition 3 applies or not. If expected losses of contagious bank runs are not high enough then both banks invest in  $Z_a$ . In the case where Proposition 3 holds the threat of contagion implies that welfare is higher because one bank invests in  $Z_b$  instead of  $Z_a$  getting closer to the first best.

Taking the contagion case as a benchmark the effects on welfare of an optimal DI are clear. In situations where Proposition 3 does not apply welfare is raised. The investment equilibrium remains  $aa$  but the DI eliminates all unwarranted bank runs; this increases expected total payoffs and hence increases welfare. If however Proposition 3 holds welfare is lowered by the introduction of DI. This must be so, as the DI eliminates the threat of contagious runs. Hence there is no reason for one bank to shift its investment to  $Z_b$  and  $aa$  is the equilibrium instead of  $ab$ . The DI has also no beneficial effect in this situation as no unwarranted contagious runs would be observed in equilibrium anyhow.

Such a simple welfare analysis might not be warranted. As was shown in the introduction there is a mayor concern for banking stability and for the prevention of systemic crises. We talk of a systemic crisis in our model when both banks fail in  $T_1$ : In reality systemic banking crises impose externalities on the rest of the economy. These could be for example via a bank lending channel as in Bernanke (1983). For a full blown welfare analysis one could model these externalities by assuming some loss  $L$  in case of one bank failure and in case of a systemic crisis a loss which is greater than  $2L$ .<sup>15</sup> Given that this is somewhat arbitrary let us focus on the probability that systemic banking crises occur.

Financial stability is affected in two ways by DI. If Proposition 3 does not hold DI decreases the probability of systemic banking crises as it eliminates the occurrence of runs on fundamentally sound banks. On the other hand, given the previous discussion,

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<sup>15</sup>See for example Matutes and Vives (1998) for such an approach.

correlation of portfolios increases with DI if Proposition 3 applies as the equilibrium in the investment game changes from ab to aa. This also has an effect on the probability of systemic crises  $s_{I;II}$  which without DI is

$$\begin{aligned} s_{ab} &= p(Z_{aI} \cdot z \setminus Z_{bII} \cdot z) \\ &= p(Z_{aI} \cdot z) p(Z_{bII} \cdot z | Z_{aI} \cdot z) \end{aligned}$$

With DI, both banks invest in  $Z_a$  and the probability of a systemic crisis is  $s_{aa}$

$$\begin{aligned} s_{aa} &= p(Z_{aI} \cdot z \setminus Z_{aII} \cdot z) \\ &= p(Z_{aI} \cdot z) p(Z_{aII} \cdot z | Z_{aI} \cdot z) \end{aligned}$$

One can show that

$$s_{ab} < s_{aa} \tag{15}$$

as  $p(Z_{bII} \cdot z | Z_{aI} \cdot z) < p(Z_{aII} \cdot z | Z_{aI} \cdot z)$  if  $\frac{p}{(1 - \frac{1}{2} \frac{z_b^2}{z_a^2})} > \frac{p}{(1 - \frac{1}{2} \frac{z_a^2}{z_a^2})} \hat{A}$  where  $\hat{A} < 1$ : The exact definition of  $\hat{A}$  and further details are referred to the Appendix 9.2.

Hence in cases where Proposition 3 holds and  $\frac{p}{(1 - \frac{1}{2} \frac{z_b^2}{z_a^2})} > \frac{p}{(1 - \frac{1}{2} \frac{z_a^2}{z_a^2})} \hat{A}$ , the DI decreases welfare not only by inducing banks to invest in the high risk asset but also by increasing the joint probability of a systemic banking crisis implying an increase in cost of banking failures.

These results are summed up in the following Proposition.

**Proposition 4** Optimal deposit insurance decreases welfare, whenever Proposition 3 applies. It also increases the probability of systemic banking crises if  $\frac{p}{(1 - \frac{1}{2} \frac{z_b^2}{z_a^2})} > \frac{p}{(1 - \frac{1}{2} \frac{z_a^2}{z_a^2})} \hat{A}$ .

The reader might ask, if the described deposit insurance is truly optimal as it has negative welfare implications in situations where Proposition 3 applies. The question is clearly why the DI can not act like depositors and liquidate banks when contagious bank runs would normally be observed. The driving force for contagious bank runs are negative payoff externalities of withdrawals by late depositors in  $T_{13}$ . As there is a "first come first serve" rule and depositors can withdraw  $r_1 > 1$  there is the possibility that a

late depositor is so far back in the queue in an informational bank run in  $T_{13}$ , that he is unable to withdraw anything because the bank is already illiquid. If the chance of an informational bank run is too high withdrawing  $r_1$  in  $T_{12}$  yields the higher expected payoff. Not so if there is only one late depositor. Here no negative payoff externality exists and the utility of a single late depositor of withdrawing at  $T_{12}$  is the same as of withdrawing at  $T_{13}$  when the bank is bankrupt.<sup>16</sup> As there is always a chance that the bank is not bankrupt a single late depositor would always want to wait till  $T_{13}$  before he withdraws in  $T_1$ . As the DI is equivalent to a single depositor the argument implies that the DI always wants to wait till  $T_{13}$  before it liquidates the second bank. Hence even if the DI would announce ex ante that it liquidates banks whenever a contagious bank run would occur, it can not credibly commit to do so.

Consider two alternative forms of DI which at first glance thought to improve on the above deposit insurance scheme: a) a reflection of cross-correlations in deposit insurance rates b) insuring only uninformed depositors. Given the previous discussion a successful prevention of risk shifting behaviour depends on the establishment of a credible threat to liquidate banks in case they are second and a run on the first banks occurs. This is clearly not the case if the DI demands a premium which reflects cross-correlations in some form. The DI can still not commit to force the bank to liquidate all its assets in  $T_{12}$ . As banks choose their investments after the DI rate is fixed, including the charges for higher correlation, they will always choose the portfolio, which achieves the highest expected profits,  $Z_a$ : Hence welfare is lowered with DI when Proposition 3 holds.

We call deposit insurance which only insures uninformed depositors a partial DI.<sup>17</sup> In the Appendix 9.3 we show that Proposition 1-3 do not change substantially if only a subset of depositors is informed about the true payoff of the banks portfolio in  $T_2$ : Proposition 4 continues to hold if all informed and uninformed depositors are covered by the DI. To determine welfare implications of a partial DI the question is whether early and informed late depositors are able to force the bank to liquidate all its assets in  $T_{12}$

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<sup>16</sup>In both cases the utility is  $U(1 - r_1)$ : Substituting this into equation (6) implies that  $p_R$  must be greater than 1 for a contagious bank run to happen.

<sup>17</sup>In reality there is a identification problem who is informed and who is not. However existing partial DIs differentiate according to size as it is generally assumed that large scale institutional investors have the capacity to collect information in contrast to small depositors.

and if claims of informed are more or less junior than of uninformed ones. The detailed welfare analysis is also referred to the Appendix 9.3 but it is interesting to note that in our model a partial DI has either no welfare effect, or if there is one, the partial DI performs worse than a DI which covers all depositors (the full DI). Negative welfare implications are driven by a shift in the investment behaviour of banks. If such a shift takes place under a partial DI, then it would also take place under a full DI, implying the same negative effects. However if the partial DI is welfare improving than a full DI would be as well but with higher welfare gains. Because the full DI prevents the liquidation of all assets when there is an unwarranted contagious run in contrast to a partial DI which only prevents the liquidation of assets to pay out uninformed depositors. From a pure welfare perspective this might be a tentative argument for forbearance of informed depositors.<sup>18</sup>

The reader might ask how robust our welfare results are to three crucial assumption of no liquidation costs,  $\frac{1}{2}_{aa} > \frac{1}{2} > \frac{1}{2}_{ab}$  and that the high risk asset is more profitable. We discuss each of this assumptions in turn.

In reality fire sales of assets in a banking crisis have a huge impact on welfare. We do not take this into account, as banks in our analysis could always liquidate the long term asset for a return of one in  $T_1$ : In the Appendix 9.4 we adopt our model so that banks can only sell the long term asset in  $T_1$  for a price which is below the current expected value of the asset. Introducing liquidation costs does not alter much of the previous results except that depending on liquidation costs banks want to invest  $\frac{1}{2}r_1$  in the riskless asset. Thresholds that induce runs change slightly but Proposition 3 still applies. From an ex ante perspective  $Z_b$  continues to be the first best with liquidation costs. Even more so as from a welfare perspective it is also important to prevent early liquidation and investments in  $Z_a$  have a higher probability of contagious runs. Early liquidation because of runs can be prevented by a DI as it can credibly commit not to sell any assets in  $T_1$  and wait until  $T_2$  to receive the payoff of the investments.<sup>19</sup> This raises welfare. But with a DI both banks will invest in  $Z_a$ . Again if the condition in Proposition 3 does

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<sup>18</sup>If the partial DI has no effect on welfare and the probability of systematic banking, which is the case if all early and informed depositors can force the bank into illiquidity, then the question should be asked, why it was implemented in the first place.

<sup>19</sup>The optimal DI in this case does not liquidate the bank in  $T_1$  but takes over the management to prevent any gamble for resurrection.

not hold then DI is clearly welfare improving as it not only prevents bank runs on solvent institutions but also any liquidation in  $T_1$ : If the condition in Proposition 3 is satisfied then the effects are ambiguous as welfare depends on the comparison of expected welfare gains from avoiding early liquidation and losses from investments in  $Z_a$  instead of  $Z_b$ :

The welfare analysis is simple if  $\frac{1}{2} > \frac{1}{2}_{aa}$  or  $\frac{1}{2} < \frac{1}{2}_{bb}$ : As mentioned earlier the possibility of contagious bank runs does not have an influence on the investment decision of banks and  $aa$  is the investment equilibrium. Hence the introduction of a DI has a beneficial effect as it eliminates unwarranted contagious bank runs. If however  $\frac{1}{2}_{ab} > \frac{1}{2} > \frac{1}{2}_{bb}$  it is possible that both banks do not want to invest in the high correlation asset. However conditions are much more stringent than in Proposition 3.<sup>20</sup> The negative welfare implications are also driven by our assumptions that the high correlation asset is more risky. If  $Z_b$  is the more profitable asset than a DI would either have no effect (if contagious bank runs would not be observed in equilibrium anyhow) or a positive one because of its elimination of unwarranted contagious bank runs. The analysis is not that clear cut if the more profitable high correlation asset has a higher mean ( $\mu_a > \mu_b$ ) and a lower variance ( $\sigma_a < \sigma_b$ ): In that case welfare is always increasing even if Proposition 3 applies, because  $aa$  is the first best. However it still may be that the introduction of a DI increases the probability of a systemic banking crisis due to an increase in the correlation.<sup>21</sup> If there are externalities of banking crises this can imply a reduction in overall welfare.

## 7 Conclusion

This paper highlighted several aspects of contagious bank runs driven by informational externalities. In the first part we showed that contagious bank runs are only observed if the conditional probability of a failure given a run on another bank is higher than a certain threshold. Leaving the reasons for an interbank market aside interbank deposits can strengthen this argument. Assume that there are some interbank deposits or banks agree

<sup>20</sup>If  $bb$  is an equilibrium then  $aa$  is usually an equilibrium as well. This implies that  $E(\mu_a) > E(\mu_b) > q_{aa}E(\mu_a|R_a) > q_{ab}E(\mu_b|R_a)$  ( $aa$  equilibrium condition) and  $E(\mu_a) > E(\mu_b) > q_{ba}E(\mu_a|R_b) > q_{bb}E(\mu_b|R_b)$  ( $bb$  equilibrium condition). That both banks choose  $Z_b$  the  $bb$  equilibrium must be Pareto superior  $E(\mu_a) > E(\mu_b) > q_{aa}E(\mu_a|R_a)$ . As  $q_{aa} = q_{ab}$  and  $E(\mu_a|R_a) > E(\mu_a|R_b)$  the last condition is the hardest to satisfy.

<sup>21</sup>The condition for this is further on  $\frac{\sigma_b}{(1 - \frac{1}{2}_{ab})} > \frac{\sigma_a}{(1 - \frac{1}{2}_{aa})}$ .

credit lines in case of unexpected liquidity needs. If Bank I fails due to an informational bank run, it will not be able to repay its interbank deposits. Such a failure not only contains information on the likelihood that Bank II will fail as well but it also implies that the second bank lost some money with certainty. This increases the likelihood that late depositors start a contagious bank run and Proposition 2 and 3 will be more easily satisfied.<sup>22</sup>

The important insight of this paper was that even an optimally designed DI can be a double-edged sword. In situations where we would observe contagion in equilibrium, it has a beneficial effect as it eliminates the possibility that depositors run on banks which are actually fundamentally sound. However possible contagious runs can act as a discipline device for banks to prevent investments in highly correlated portfolios. This threat is eliminated by DI implying that even if all banking supervision reforms have been successfully implemented and the deposit insurance premium is perfectly risk-related, deposit insurance can have negative effects on welfare and systemic risk. It is clear that this result is not only limited to an optimal DI but also to any other safety net like a lender of last resort for example. If the lender of last resort is designed in the spirit of Bagehot and does not lend to truly insolvent banks it also only eliminates contagious bank runs on solvent banks. Hence it has the same effects on the investment behaviour of banks, as the ones described in this paper.

Applying the results of this paper to Basel II, it is clear that it may fall short of its expected impact. Systemic banking crises may still be observed in the future. Basel II focuses on risk taking by individual bank and tries to eliminate moral hazard implied by mispriced deposit insurance. This is clearly important and increasing financial stability. However we showed that correlation of assets across banks is at the core of systemic risk. Basel II has little to say about these issues. But even if it would address it we also argued that it will be very hard to implement a regulatory system without some negative welfare effects as long as banks have a chance to change their investments after the regulatory

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<sup>22</sup>In general the effects of an interbank market on financial stability are less clear cut. In our model there are only efficient bank runs in  $T_{11}$ : Thus no other bank would support bank I in case it is subject to a run in  $T_1$ : Inefficient bank runs only occur in  $T_{12}$ ; when the first bank went under already and is thus not able to support the second bank. As discussed previously interbank market would have a role if we drop the assumption of no-sun spot type bank runs as banks support each other in case of inefficient runs increasing financial stability.

charges are fixed.<sup>23</sup>

The general question is how often something similar to Proposition 3 applies in the real world where investment opportunities change. However this can not be addressed in a theoretical model. Future empirical research is necessary to clarify this.<sup>24</sup> However this paper highlighted an effect which had previously not been remarked on and may be important. Reconsidering our example of the Australian banking system in the introduction of this thesis. It is not hard to claim that our assumptions were satisfied in this situation. Clearly the expected profits as well as the risk was higher for investments in the property market than for other ventures. Furthermore one can argue that the inflows of deposits to some banks show that the observed bank runs were informational rather than contagious bank runs, implying that Proposition 3 as well as the equivalent to  $\frac{1}{2}_{aa} > \frac{1}{2} > \frac{1}{2}_{bb}$  was satisfied. The hypothetical effects of a DI in this case are therefore that all banks would have invested in the property market and welfare losses of the crisis would have been higher. As property market booms or other speculative investments are often at the heart of a banking crises it might well be that even an optimal deposit insurance decreases welfare and increases systemic risk more often than not.

## 8 Bibliography

Allen, F. and D. Gale (2000), Financial Contagion, *Journal of Political Economy* 108(1)

Benston, G. and G. Kaufman (1996), The Appropriate Role of Bank Regulation, *The Economic Journal* 106 (2)

Bernanke, B. S. (1983), Non-Monetary Effects of Financial Crisis in the Propagation of the Great Depression, *American Economic Review* 73(3)

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<sup>23</sup>Clearly if investments of banks are known when the DI premium is fixed than the premium can perfectly reflect the risk and correlation of the investments undertaken. This can be done ex ante. In our model one could construct charges which limit the charter value proportional to the risk. However in times of crisis when the moral hazard problem is most imminent banks have little charter value and such a scheme has no bite.

<sup>24</sup>It might very well be that in normal times correlation between banks portfolios are lower than the critical correlation. The correlation might only reach the critical level in times of speculative booms. It is then only interesting to know if potential losses of contagion are high enough to deter banks to invest in highly correlated assets without a DI, which will be hard to measure.

Billingston, P. (1995), *Probability and Measure*, A Wiley Interscience Publication, John Wiley & Sons, New York

Calomiris, C. and G. Gorton (1991), *The Origins of Banking Panics: Models, Facts and Bank Regulation*, in *Financial Markets and Financial Crisis*, edited by G. Hubbard, Chicago, University of Chicago Press

Chari, V. and R. Jagannathan (1988), *Banking Panics, Information and Rational Expectations Equilibrium*, *Journal of Finance* 18 (3)

Chen, F. (1999), *Banking Panics: The Role of the First Come, First Serve Rule and Information Externalities*, *Journal of Political Economy* 107(5)

Demirgüç-Kunt, A. and E. Detragiache (2000), *Does Deposit Insurance Increase Banking System Stability?*, IMF Workingpaper WP/00/03

Diamond, D. and P. Dybvig (1983), *Bank Runs, Deposit Insurance and Liquidity*, *Journal of Political Economy* 91(3)

Dowd, K. (1992), *The Experience of Free Banking*, Routledge, London and New York

Freixas, X. and J. C. Rochet (1999), *Microeconomics of Banking*, The MIT Press, Cambridge, London

Goldstein, I. and A. Pauzner (2000), *Demand Deposit Contracts and the Probability of Bank Runs*, [www.tau.ac.il/~pauzner](http://www.tau.ac.il/~pauzner)

Herring, R. and P. Vankudre (1987), *Growth Opportunities and Risk Taking by Financial Intermediaries*, *Journal of Finance* 47 (3)

Jacklin, C. and S. Bhattacharya (1988), *Distinguishing Panics and Information-Based Bank Runs: Welfare and Policy Implications*, *Journal of Political Economy* 96 (3)

Keeley, M. (1990), *Deposit Insurance, Risk and Market Power in Banking*, *American Economic Review* 80 (5)

Matutes, C. and X. Vives (2000) *Imperfect Competition, Risk Taking and Regulation in Banking*, *European Economic Review* 44 (1)

Mayer, L. H.(2001), *Testimony to the Subcommittee of Financial Institutions and Consumer Credit of the Committee on Financial Services, House of Representatives, 26th July 2001*, BIS Review 68,2001, [www.bis.org](http://www.bis.org)

Rochet, J. C. and J. Tirole (1996), *Interbank Lending and systemic Risk*, *Journal of Money, Credit and Banking* 28(4)

## 9 Appendix

### 9.1 Interest rates

Profit maximisation of banks imply that contracts satisfy the participation constraint of depositors. As depositors can use the storage technology in  $T_0$  the participation constraint is

$$U(1) \geq \frac{1}{2}U^{\text{early};I} + \frac{1}{2}U^{\text{early};II} + (1 - \frac{1}{2}) \left( \frac{1}{2}U^{\text{late};I} + \frac{1}{2}U^{\text{late};II} \right) \quad (16)$$

where I/II stand for being in the Bank I or Bank II. For  $p_j = p(z_j \cdot z)$

$$U^{\text{early};b1} = p_j^\circ U(r_1) + (1 - p_j)U(r_1)$$

$$U^{\text{late};b1} = p_j^\circ U(r_1) + (1 - p_j)E(U_{2;NR})$$

where  $\circ = \frac{1}{r_1}$  is the probability that one is able to withdraw if there is a bank run. (For  $E(U_{2;NR})$  see equation (4) in the main text).

If  $p_R < \bar{p}$  no contagious bank runs are observed and late depositors wait till  $T_{13}$ : Thus

$$U^{\text{early};b2} = U(r_1)$$

and

$$U^{\text{late};b2} = p_j^{\circ 0} U(r_1) + (1 - p_j)E(U_{2;NR})$$

where  $\circ 0 = \frac{(1 - \frac{1}{r_1})}{(1 - \frac{1}{r_1})}$  is the probability that a late depositors is able to withdraw, given that all late depositors are running and all early depositors withdrew already

If contagious bank runs are observed in equilibrium ( $p_R \geq \bar{p}$ )

$$U^{\text{early};b2} = \phi^\circ U(r_1) + (1 - \phi)U(r_1)$$

where  $\phi = p(z_1 \cdot z)$  the probability that a run occurs and everyone is able withdraw  $r_1$

with probability  $\phi$ :

$$U^{\text{late;b2}} = \phi U(r_1) + (1 - \phi)[p_{NR} U(r_1) + (1 - p_{NR})E(U_{2;NR})]$$

Profit maximising banks will offer an interest rate such that the participation constraint (16) is satisfied with equality. It is obvious that it has to be that  $r_2 \geq r_1$ : If  $r_1 < r_2$  late depositors would always pretend to be early ones going to the bank at  $T_1$  and storing it till  $T_2$ .  $r_1$  also implicitly determines the conditions for an informational and contagious bank run.

## 9.2 Conditions that DI decreases financial stability

The condition that an optimal DI increases the probability of a systemic banking crisis were given in the text by equation (15)

$$S_{ab} < S_{aa} \quad (17)$$

$$p(Z_{b|I} \cdot z|Z_{a|I} \cdot z) < p(Z_{a|I} \cdot z|Z_{a|I} \cdot z) \quad (18)$$

For equation (17) to hold it must be the case that

$$\frac{z_i - 1_{bjR_a}}{\sigma_{bjR_a}^2} < \frac{z_i - 1_{ajR_a}}{\sigma_{ajR_a}^2} \quad (19)$$

With the assumptions of a normal distribution (see Billingsley (1995)) we see that

$$\begin{aligned} 1_{ajR_a} &= \int_{i=1}^z \left(1 - \frac{\sigma_{aa}}{\sigma_a^2}\right) 1_a + \frac{\sigma_{aa}}{\sigma_a^2} Z_{Ia} f(Z_{Ia}|Z_{Ia} \cdot z) \\ &= 1_a + \frac{\sigma_{aa}}{\sigma_a^2} \left(1_a - E(Z_{Ia}|Z_{Ia} \cdot z)\right) \end{aligned}$$

$$\sigma_{ajR_a}^2 = \sigma_a^2 \left(1 - \frac{\sigma_{aa}^2}{\sigma_a^2}\right)$$

and

$$\begin{aligned} 1_{bjR_a} &= \int_{i=1}^z \left(1 - \frac{\sigma_{ab}}{\sigma_a^2}\right) 1_a + \frac{\sigma_{ab}}{\sigma_a^2} Z_{Ia} f(Z_{Ia}|Z_{Ia} \cdot z) \\ &= 1_b + \frac{\sigma_{ab}}{\sigma_a^2} \left(1_a - E(Z_{Ia}|Z_{Ia} \cdot z)\right) \end{aligned}$$

$$\text{Var}(z_{11}|z_1, z) = \frac{1}{4} \frac{1}{b^2} (1 - \frac{1}{2} \frac{a^2}{ab})$$

This implies that

$$z_{11} < z_{12} < z_{13}$$

Hence an easy condition for equation (17) to be satisfied is

$$\frac{q}{\frac{1}{4} \frac{1}{b^2} (1 - \frac{1}{2} \frac{a^2}{ab})} > \frac{p}{\frac{1}{4} \frac{1}{a^2} (1 - \frac{1}{2} \frac{a^2}{aa})} \hat{A}$$

where

$$\hat{A} = \frac{z_{12} < z_{13}}{z_{11} < z_{12}} < 1$$

$\hat{A} < 1$  as  $\frac{1}{b} > \frac{1}{a}$  and  $\frac{1}{4} \frac{1}{ab} < \frac{1}{4} \frac{1}{aa}$ :

### 9.3 The model with informed and uninformed depositors

In the main text we assumed that all depositors in Bank I /Bank II see the true payoff of the investment of their bank in  $T_{11}/T_{13}$ : Here we drop this assumption. Instead there are informed and (1 -  $\mu$ ) uninformed depositors in each group of early and late depositors. Only informed depositors see the true payoff of the investment in  $T_{11}$  or  $T_{13}$ . Each depositors has two actions: withdrawal (W) or not withdrawal (N). Furthermore depositors have a belief  $\mu$  about the probability that  $z_1 = z = \frac{(1-\mu)r_1}{(1-\mu)r_1}$ : We only write  $\mu$  explicitly in  $T_{11}$  or  $T_{13}$  for uninformed depositors as informed ones know  $z_1 = z_{11}$  in equilibrium. In  $T_{12}$  beliefs for all depositors are on the equilibrium path and the same as in the main text and therefore not explicitly formulated. To simplify the maths, we assume that informed are always in front of uninformed and that there is enough money such that every informed depositors can withdraw. We also keep the assumption that depositors coordinate on the Pareto superior equilibrium.

We solve for the perfect Bayesian Nash equilibrium in the game in  $T_{13}$ :

Proposition 5 Informational bank runs are observed in  $T_{13}$  if

$$z_{11} \cdot z$$

Or:

There is a Perfect Bayesian Nash equilibrium (separating) in the game in  $T_{13}$  with:

- a) Late informed depositors play W if  $z_{11} \cdot z$  and N if  $z_{11} > z$
- b) Late uninformed depositors have the belief  $\mu = 1$  and play W if the take up rate is greater than  $\bar{z}$ . If the take up rate is equal to  $\bar{z}$  they have a belief  $\mu = 0$  and play N.

Proof. If there are no withdrawals by late depositors in  $T_{13}$  the payoff to (all) late depositors at  $T_2$  is

$$U_2^0 = U \left[ \mu \frac{(1 - \bar{z} + r_1)z_{11}}{(1 - \bar{z})} ; r_2 \right] \quad (21)$$

For  $z_{11} > z$  )  $U_2 > U(r_1)$  and for  $z_{11} \cdot z$  )  $U_2 < U(r_1)$ :

The payoff of late uninformed depositor if all informed play W and all uninformed play N with the belief  $\mu = 1$  is

$$\begin{aligned} U_{WNj\mu} &= E \left[ U \left( \frac{(1 - \bar{z} + (1 - \bar{z})^-)r_1 z_{11}}{(1 - \bar{z})(1 - \bar{z})^-} ; j\mu = 1 \right) \right] \\ &= \int_{i=1}^z r_1 U \left( \frac{(1 - \bar{z} + (1 - \bar{z})^-)r_1 z_{11}}{(1 - \bar{z})(1 - \bar{z})^-} ; f(z_{11}|z_{11} \cdot z) \right) \\ &< U(r_1) \end{aligned} \quad (22)$$

where  $(\bar{z} + (1 - \bar{z})^-)r_1$  is the amount the bank had to pay out to satisfy the demand of early and late informed depositors.  $(1 - \bar{z})(1 - \bar{z})^-$  are the remaining late uninformed depositors. The inequality in equation (22) is satisfied as

$$\begin{aligned} \frac{(1 - \bar{z} + (1 - \bar{z})^-)r_1 E(z_{11}|z_{11} \cdot z)}{(1 - \bar{z})(1 - \bar{z})^-} &< \frac{(1 - \bar{z} + r_1)E(z_{11}|z_{11} \cdot z)}{(1 - \bar{z})} \\ &< r_1 \end{aligned}$$

This implies that it is optimal for one uninformed depositor to withdraw, given that all informed play W and all other uninformed N and the belief  $\mu = 1$ : Hence all uninformed play W.

Now look at the payoff of late uninformed depositor if all informed play N and all uninformed play N with the belief  $\mu = 0$ :

$$\begin{aligned}
 E(U_{2N} | \mu = 0) &= E \left[ U \left( \min \left( \frac{(1-i)z + r_1}{(1-i)z}, r_2 \right) \right) \right] \\
 &= U(r_2) f(Z_{11} | Z_{11} > z) \\
 &\quad + \int_z^{(1-i)z + r_1} U \left( \frac{(1-i)z + r_1}{(1-i)z} \right) f(Z_{11} | Z_{11} > z) \\
 &> U(r_1)
 \end{aligned}$$

This implies that it is optimal to wait till  $T_2$  if no informed depositor withdraws. Given the belief system and the equilibrium strategies it is optimal for informed to play W if  $z < z^*$  and N if  $z > z^*$ : Hence Proposition 5 is a perfect Bayesian Nash equilibrium. ■

This is the unique equilibrium in the class of symmetric equilibria, where agents coordinate on the Pareto superior equilibrium, as long as one restricts the off-equilibrium beliefs in case of pooling to  $\mu_{off} = p(Z_{11} | z \text{ history})$ . If this is the case it is never optimal for an informed depositor to play W if  $z > z^*$  as all late uninformed play N if they see that all informed play N.

The intuition behind Proposition 5 is simple: Late informed depositors will only withdraw money when the bank is truly bankrupt. Knowing that it is optimal for late uninformed depositors to wait till  $T_2$  if no more than  $s$  people withdraw in Bank II.

The equilibrium solution for  $T_{11}$  follows the same pattern and is thus not repeated. The condition for a contagious bank run is also in line with section 3.2 except that runs are determined by late uninformed depositors. Informed depositors would always prefer to wait as they have an informational advantage in  $T_{13}$  and can retrieve  $r_1$  with near certainty. Uninformed depositors are therefore the driving force of contagious bank runs.  $U_{I;R}^{uninf}$  for uninformed depositors becomes

$$U_{I;R}^{uninf} = p_R \frac{(1-i)(s + (1-i)z^-)r_1}{(1-i)z(1-i)z^-} U(r_1) + (1-i)p_R E(U_{2;NR})$$

leading to a different run condition  $\bar{p}^{\text{uninf}} < \bar{p}$ : Hence contagious runs will be more often observed. This is intuitive as uninformed depositors have a higher incentive to run because their chances of being able to withdraw  $r_1$  in  $T_{13}$  are lower because we assumed that informed depositors are in the front of the line. With the above argument Proposition 2 and Proposition 3 are easily reformulated.

### 9.3.1 A partial deposit insurance

In case there is a partial DI which only covers uninformed depositors the question is whether early and informed late depositors are able to force the bank to liquidate all its assets in  $T_{12}$  and if claims of informed are more or less junior than of uninformed ones.

If claims of insured late depositors are more junior it is not the case anymore that uninformed late depositors start a run. This time  $U_{I;R}^{\text{inf}}$  determines whether informed depositors run or not

$$U_{I;R}^{\text{inf}} = p_R \frac{1 - i_s + (1 - i_s)(1 - i_s^-)r_1}{(1 - i_s^-)r_1} U(r_1) + (1 - i_p) E(U_{2;NR})$$

$U_{I;R}^{\text{inf}} < U_{I;R}^{\text{uninf}}$  for  $\bar{c} < \frac{1}{2}$  which is sensible to assume. Hence  $\bar{p}^{\text{inf}} < \bar{p}^{\text{uninf}} < \bar{p}$  and late informed depositors start to withdraw already when the correlation of the portfolios is lower than previously.

As discussed in the main text, a DI can not commit to liquidate the bank in  $T_{12}$ : Whether the DI has an effect on the investment behaviour of banks depends on the question if informed depositors are a credible threat or not. If all early and informed depositors can force the bank into liquidation by running in  $T_{12}$  then contagious bank runs will be observed and Proposition 3 will continue to hold even with a partial DI. As a partial DI neither changes investment incentives nor eliminates unwarranted bank runs by informed depositors welfare does not change and the probability of systemic banking crises remains the same.

However if claims are junior and  $1 > (i_s + (1 - i_s^-)r_1)$  such that the second bank is able to pay out early and informed late depositors in case there is a run on Bank I and

$p_R > p^{inf}$  welfare effects are not clear cut anymore. Prospects in this case are

$$E(z)_C^{inf} = E(z)_{NC} + q_{1,1}(1 - \alpha)^{-r_1} E(Z_{11} | R) \quad (23)$$

with  $\frac{1}{2}_{aa} > \frac{1}{2}^{uninf} > \frac{1}{2}_{bb}$  it can still be the case that a similar proposition to Proposition 3 applies even with the partial DI where the sufficient condition is  $E(z)_a < E(z) < q_{1,1}(1 - \alpha)^{-r_1} E(Z_{11} | R)$ . As  $(1 - \alpha)^{-r_1} < 1$  by definition this is much harder to satisfy. Especially if there is only a small fraction of informed depositors then the partial DI implies that the investment equilibrium is always  $aa$ . However informed depositors will still start contagious bank runs, which implies that in case such a run would be unwarranted the DI has a lower welfare benefit as a full DI.

Welfare also declines in cases where Proposition 3 applies and claims of informed depositors are more senior. Here informed depositors have never an incentive to start contagious bank run. As uninformed do not have this incentive either no contagious bank runs are observed with a partial DI and  $aa$  becomes the investment equilibrium. A partial DI with senior claims for informed depositors has therefore the same welfare effects as a full DI.<sup>25</sup>

#### 9.4 The model with an asset market and liquidation costs

In this paragraph we look at the model, when there is an active asset market and banks can only liquidate the long term asset in  $T_1$  on this market. Given our informational set up, prices in this market would reflect public beliefs of the expected value of the asset at each interval in  $T_1$ . This makes the calculation some more complex and several equilibria emerge without giving more insights. To avoid this, we want to change the informational structure such that information on the true payoff is not only observable to depositors in each bank but to everyone in the market. However we still assume that there are liquidation costs so that the bank can only retrieve a constant fraction  $\frac{1}{3} < 1$  of the value of the asset if it is liquidated in  $T_1$ . Given the importance of relationship banking and

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<sup>25</sup>It can also be possible that claims of informed and uninformed have the same seniority. However to analyse this case one has to assume a specific allocation rule of assets to informed depositors in case of a failure. The spirit of such a discussion would be the same as in this subsection and is therefore omitted.

that our portfolios always include bank specific assets, liquidation cost are not far fetched even though the true value of the asset is known.

In this set up banks sometimes want to invest a fraction  $\alpha r_1$  in the storage technology. Assume first that they do. This implies that banks do only have to liquidate assets, if there is a bank run. Starting the analysis in  $T_{13}$  given there was no contagious bank run in  $T_{12}$  the utility of late depositors in case no other late depositor runs in  $T_{13}$  is now

$$U_2^0 = \min \left\{ U \left( \frac{(1 - \alpha r_1) z_{11}}{(1 - \alpha)} \right); U(r_2) \right\}$$

$U_2^0$  is the same  $U_2$  in the general section (compare with equation (1)). This is not surprising as the investment in the save asset gives the same return as liquidating the asset for a return of one. Following the same argumentation as before informational bank runs are observed in  $T_{11}$  and  $T_{13}$  if  $z_1 \cdot z$ :

The expected utility for late depositors in  $T_{12}$  given no late depositor withdraws and there was an informational run on Bank I is now

$$U_{I;R}^0 = p_R \frac{(1 - \alpha r_1) E(z_{11} | z_{11} \cdot z; R)}{(1 - \alpha)^3 r_1} U(r_1) + (1 - p_R) E(U_{2;NR})$$

where  $E(U_{2;NR})$  is given in equation (4). In comparison with  $U_{I;R}$  (compare with equation (3)) we see that only the probability of being able to withdraw changes from the base set up. Depositors at  $T_{12}$  know that if there is an informational bank run in  $T_{13}$ , the bank has to liquidate all it assets which yields an expected return of  $\frac{1}{3} E(z_{11} | z_{11} \cdot z; R)$  implying that a depositor has the probability of  $\frac{(1 - \alpha r_1) E(z_{11} | z_{11} \cdot z; R)}{(1 - \alpha)^3 r_1}$  to be able to withdraw  $r_1$ : The new  $U_{I;R}^0$  leads to the run threshold  $p^0$ : A contagious bank run will therefore be observed if

$$p_R \geq \frac{E(U_{2;NR}) + U(r_1)}{E(U_{2;NR}) + \frac{(1 - \alpha r_1)}{(1 - \alpha)^3 r_1} E(z_{11} | z_{11} \cdot z; R) U(r_1)} = p^0$$

If  $\frac{1}{3} E(z_{11} | z_{11} \cdot z; R) < 1$  then  $p^0 < p$  implying the correlation threshold  $\frac{1}{2}^0$  is smaller than  $\frac{1}{2}$  in the main text. But the spirit of Proposition 2 and Corollary 1 remains. It is also

easy to show that Proposition 3 is unchanged as expected profits are exactly the same.

If the bank does not do an investment in the risk free asset it has to liquidate at least  $\frac{\alpha r_1}{z_1}$  of its portfolio in  $T_{11}$  or  $T_{12}$  to pay out  $\alpha r_1$  early depositors. At  $T_0$  the bank therefore expects to receive  $\frac{E_{T_1}(zjNB)}{\alpha}$  for each unit liquidated. This changes the expected profits in the non-contagion case to

$$E(\pi|z)_{NC}^0 = E[\max(Z - \alpha r_1; (1 - \alpha)r_2; 0)] \quad (24)$$

A comparison of equation (24) with profits in case the bank invests in the riskless asset (equation (9)) shows that the bank prefers to invest in the riskless asset if  $\alpha E(zjNB) < 1$ .<sup>26</sup> The run conditions for informational and contagious bank runs will change as well if the bank does not invest in the riskless asset and there are transaction cost. However showing them explicitly does not change the spirit of Proposition 1-3 and hence it is omitted for brevity.

Most welfare aspects are discussed in the main section. However there is also a welfare issues concerning the investment in the safe asset. To maximise expected payoffs to all parties a fraction  $\alpha r_1$  should be invested in the safe asset when  $\alpha E(z) < 1$ : But there are values of  $\alpha$  for which  $\alpha E(z) < 1 < \alpha E(zjNB)$  and banks continue to invest everything in the risky assets, which is not optimal from a welfare perspective. However DI or contagion has no influence on this investment decision. Welfare can only be raised if regulators demand a mandatory investment of  $\alpha r_1$  in the safe asset.

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<sup>26</sup>Given this argument combined with the fact that banks will always be bankrupt once a bank runs starts it is also clear that it is optimal to invest exactly  $\alpha r_1$  in the storage technology.