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Delegation of Authority as an Optimal (In)complete Contract

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Delegation of Authority as an Optimal (In)complete Contract

by

Andreas Roider

May 2003
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Delegation of Authority as an Optimal (In)complete Contract

Andreas Roider*

University of Bonn†

May 2003

Abstract

In a holdup framework, I provide conditions under which simple delegation of authority is a solution to the complete-contracting problem even though ex-post actions are ex-ante contractible, and unlimited transfer payments are feasible. In particular, delegation turns out to be optimal if the payoff functions of the parties satisfy certain separability and symmetry conditions, and the parties face an underinvestment problem. This result is extended to the case of potential overinvestment and to multi-dimensional effort provision. Besides providing a complete-contracting rationale for delegation, the findings contribute to the foundation of incomplete contracts and the property-rights theory of the firm.

Keywords: delegation, decentralization, authority, incomplete contracts, holdup, property rights.

JEL-Classification: D82, D23, L14, L22.

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1 Introduction

Delegation of authority is prevalent: frequently, managers are free to decide what to produce, or whom to hire. In procurement, contractors often have discretion with respect to their sourcing and subcontractors. Likewise, bureaucrats frequently decide on their own about public policy programs.

Why, however, might such a rather crude way of allocating decision rights be optimal? Indeed, if the Revelation Principle applies, any form of delegation is (weakly) dominated by a centralized structure in which agents send messages and actions are chosen from a prespecified menu. As a consequence, delegation has been mainly explained by imperfections in the contracting environment that render the Revelation Principle invalid: delegation/decentralization has been shown to be optimal when there are limits to communication or commitment, when it is costly to process information, when agents might collude, or when contracts can only be incomplete. In contrast, the present paper provides conditions under which simple delegation of authority is optimal even when such imperfections are absent: delegation turns out to be the solution to the complete-contracting problem of the parties.

I study a complete-contracting version of the holdup problem where ex-post actions are ex-ante contractible and “message games” are feasible. In the model, ex-ante investments have a direct positive effect on both parties. Such hybrid investments are common in employment contexts: frequently, a human capital investment by an agent will increase both his reservation utility and the surplus of the principal. In trading contexts, investments by a seller often not only reduce his costs, but also raise the quality of the good, thereby benefiting the buyer. Similarly, investments in physical capital are hybrid if both parties profit from increases in asset value. Two cases are considered: first, I consider the case that only one of the parties can invest. This might represent effort provision by an agent. Second, I study the case that both parties make transferable investments: for example, Hart (1995) argues that many investments in physical capital are transferable because frequently it does not matter which of the parties to a relationship actually invests. Similarly, in many marketing alliances or horizontal production joint ventures only the total amount invested matters.

The main findings of the paper are as follows. First, if the parties face an underinvestment problem, it is optimal to maximize investment incentives of one of the parties. I show that if the threatpoint payoffs satisfy certain separability and symmetry conditions, this is achieved by
simple delegation: an optimal contract specifies a fixed payment (e.g., a wage) and gives one of the parties authority to choose any action ex-post. In particular, simple delegation is optimal if threatpoint payoffs can be expressed as the product of an (idiosyncratic) decision-dependent factor and a common investment-dependent factor (possibly plus decision-independent terms). Second, this result can be extended to a multi-dimensional investment if only one of the parties may invest and investment components are strategic complements. And third, if simple delegation leads to overinvestment and some continuity requirements are met, simple delegation, however with restricted competencies, is optimal. That is, it is optimal to limit discretion to a subset of possible alternatives. Hence, the latter institution is similar to Tirole’s (1999) “authority with gatekeeping counterpower (or partial veto)” where a principal (the gatekeeper) defines competencies of a decision maker who subsequently is free to choose actions from the prespecified set.

Previous work showing that simple forms of delegation may do just as well as complicated message-dependent contracts has focused on settings of asymmetric information: such replication results have been derived in adverse selection models (see e.g., Melumad, Mookherjee and Reichelstein, 1992, 1995; McAfee and McMillan, 1995; Mookherjee and Reichelstein, 1997, 2001; and Baliga and Sjöström, 2001) or principal-multiagent problems with moral hazard (see e.g., Baliga and Sjöström, 1998). More closely related, Tirole (1999) provides a complete-contracting justification for the incomplete-contracting authority model of Aghion and Tirole (1997). However, the present model differs from Tirole (1999) in that he considers a moral hazard problem of information acquisition where the ex-post information structure, but not the ex-post actions are ex-ante verifiable.

Given its focus on the holdup problem and ex-ante investment incentives, the present paper also contributes to the literature on the foundations of incomplete contracts. On the one hand, this

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1In the model, I do not restrict attention to situations where one of the parties is superior to the other party: while in cases where there is no given hierarchical structure it would be more appropriate to speak of decentralization of authority, I will use the term “delegation” throughout the paper.

2See also Holmstrom (1984), Melumad and Shibano (1991) and Armstrong (1994) who study models where a principal can commit to a decision rule but not to monetary transfers, resulting in partial forms of delegation. While in the present model restricted competencies will serve to reduce incentives, Szalay (2000) shows that restricting feasible alternatives may lead to stronger incentives for information acquisition.

3Baliga and Sjöström (1998) also derive a replication result in the presence of collusion (see also Ihoh, 1993).

4For other incomplete-contracting approaches to delegation, see e.g., Hart and Moore (1999b) and Aghion, Dewatripont, and Rey (2002) who assume that control over actions, but not actions themselves, are contractible. See also Dessein (2002) who studies delegation as an alternative to (noisy) communication in the spirit of Crawford and Sobel (1982).
literature has identified circumstances under which the *complete absence of an ex-ante contract* is optimal (see e.g., Hart and Moore, 1999; Segal, 1999; and Che and Hausch, 1999). On the other hand, it has been shown that *simple contractual arrangements*, such as *non-contingent contracts* or *options contracts*, might be optimal either because they lead to the first-best, or because more complicated arrangements would not achieve a better outcome (see e.g., Nöldeke and Schmidt, 1995, 1998; Edlin and Reichelstein, 1996; Edlin and Hermalin, 2000; Schweizer, 2000; Lülkesmann, 2001; and Segal and Whinston, 2002). However, many typical holdup settings, such as employment relationships or joint ventures, are characterized by simple delegation/decentralization of authority. To the best of my knowledge, the present justification for this phenomenon is novel.5

Finally, the paper suggests that a mere allocation of property rights may suffice even when complete contracts are feasible. Maskin and Tirole (1999) have shown that even if message-dependent quantity contracts are possible, the parties may be unable to improve upon the simple allocation of ownership rights proposed by the property-rights theory (see e.g., Grossman and Hart, 1986; Hart, 1995; and Hart and Moore, 1990). However, Maskin and Tirole (1999) retain the assumption that ownership confers ex-ante non-contractible residual control rights on the owner. In contrast, the present paper provides conditions under which ownership suffices even when all ex-post actions are ex-ante contractible: suppose an asset is a critical resource in the sense that a party can take ex-post actions if and only if it has access to the asset (for example, the asset may be a machine necessary for production). In this case, simple delegation is equivalent to ownership of the asset by one of the parties.6 This complete-contracting justification of ownership is in the spirit of Tirole (1999, p. 771) who argues that “incomplete contracting is not a compulsory ingredient of a theory of institutions.”

The remainder of the paper is structured as follows. In Section 2 the model is introduced. Section 3 contains conditions under which simple delegation is the solution to the complete-contracting problem, followed by concluding remarks in Section 4.

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5If investments are hybrid, non-contingent contracts are in general not optimal (this is discussed in more detail below). Note that simple delegation is less complicated than option contracts because in the latter message-dependent payments have to be specified.

6In particular, simple ownership would be *strictly* optimal if there were (arbitrarily small) costs of writing more complex contracts.
2 The Model

I consider a standard holdup framework with two risk-neutral, symmetrically informed parties, \( j = 1, 2 \). In a first stage, the parties may invest in order to increase the attainable surplus. In a second stage, some ex-ante contractible actions \( a = (a^1, ..., a^I) \in A = A^1 \times ... \times A^I \subseteq \mathbb{R}^I, I \geq 1 \), have to be taken.

**Sequence of events** Figure 1 depicts the sequence of events. At *date 1*, the parties sign a (possibly message-dependent) contract \( C \). In addition to the actions, only transfer payments and messages sent between the parties are verifiable by a court.\(^7\) In line with the holdup-literature, all other variables, such as the ex-post state or payoffs of the parties, are assumed to be non-verifiable. Denote by \( t^j \in \mathbb{R} \) the ex-post transfer payment to party \( j \), where \( t^1 + t^2 = 0 \). The revelation principle allows to restrict attention to direct revelation mechanisms that induce truth-telling on and off the equilibrium path. Formally, a contract \( C \) is defined as a mapping \( [a, t^1, t^2] : \theta \rightarrow A \times \mathbb{R} \times \mathbb{R} \), where \( \theta \) denotes the ex-post state of the world (to be defined below).\(^8\) Note that in the following, the arguments of \( a(\cdot), t^1(\cdot), \) and \( t^2(\cdot) \) will sometimes be dropped for ease of exposition.

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Figure 1: The sequence of events

At *date 2*, the parties make investments \( i^j \in [0, \tilde{i}^j] \), \( j = 1, 2 \), that are observable, but not verifiable. Total investment is denoted by \( i \equiv i^1 + i^2 \). At *date 3*, uncertainty is resolved. That is, a random variable \( \epsilon \in \mathcal{E} \) is realized, where \( \mathcal{E} \) is a probability space. The ex-post state of the world is now known and denoted by \( \theta \equiv (i^1, i^2, \epsilon) \in \Theta \equiv [0, \tilde{i}^1] \times [0, \tilde{i}^2] \times \mathcal{E} \). At *date 4*, the parties send

\(^7\)Note that ex-ante transfer payments do not have any effect on investment incentives. Moreover, there will always exist ex-ante payments that induce participation by both parties.

\(^8\)Allowing for random mechanisms, which play a crucial role when parties are risk-averse, would not change the results qualitatively.
messages $\theta^j \in \Theta$, $j = 1, 2$. At date 5, they possibly renegotiate the initial contract, before actions are taken at date 6.

**Payoff functions and renegotiation** If renegotiations fail, the threatpoint payoffs of the parties depend on the initial contract $C$ and are given by $\pi^j(a, i, \epsilon) + t^j, j = 1, 2$, where $\pi^j$ is assumed to be twice continuously differentiable in $i$. As investments are assumed to be transferable, the threatpoint payoffs only depend on the total amount invested. Besides being interesting in their own right, transferable investments have some useful analytical properties that are discussed in more detail below. Two-sided non-transferable investments are discussed in the conclusion. As the parties have symmetric information, it is assumed that they always succeed in renegotiating the initial contract, and ex-post efficient actions are taken at date 6. Thereby, the parties create an ex-post surplus $\phi(\theta)$, where

$$\phi(i, \epsilon) \geq \max_{\alpha \in A} \left\{ \sum_j \pi^j(a, i, \epsilon) \right\} \text{ for all } i \text{ and } \epsilon. \quad (1)$$

Whenever the above inequality is strict, there are ex-ante non-contractible actions that only become contractible ex-post.\(^9\) I assume that $\phi(i, \epsilon)$ is non-negative and twice continuously differentiable in $i$, and that $\phi_i(i, \epsilon) > 0 \ \forall i, \epsilon. \quad (1)$

The resulting renegotiation surplus

$$\Delta(\theta^1, \theta^2, \theta) \equiv \phi(\theta) - \sum_j \pi^j(a(\theta^1, \theta^2), i, \epsilon) \geq 0, \quad (2)$$

is divided in Nash-bargaining with exogenously given bargaining powers $\beta^1(\epsilon)$ and $\beta^2(\epsilon)$, where $0 < \beta^1(\epsilon) < 1$ and $\beta^1(\epsilon) + \beta^2(\epsilon) = 1. \quad (1)$ As the parties anticipate the outcome of renegotiations, the post-renegotiation payoff of party $j$ consists of $\pi^j(a, i, \epsilon)$, the transfer payment $t^j$, and party $j$’s share of the renegotiation surplus:

$$\Pi^j(\theta^1, \theta^2, \theta) \equiv \pi^j(a(\theta^1, \theta^2), i, \epsilon) + \beta^j(\epsilon) \cdot \Delta(\theta^1, \theta^2, \theta) + t^j(\theta^1, \theta^2). \quad (3)$$

\(^9\)Note that none of the results depend on the presence of ex-ante non-contractible actions.
\(^10\)Throughout, subscripts denote partial derivatives.
\(^11\)While I allow for the possibility that the bargaining powers depend on the ex-post realization of uncertainty, the results do not depend on this feature in any way.
Define by \( \Pi^j (\theta) \equiv \Pi^j (\theta, \theta, \theta) \), \( j = 1, 2 \), the post-renegotiation payoffs given truthtelling of the parties.

**Optimization problem** The purpose of the initial contract is to influence threatpoint payoffs in a way such that the parties make investments that maximize the expected net surplus of the relationship. An optimal contract solves the following problem:

\[
\max_{a(\cdot), t^i(\cdot)} \left\{ E \left[ \phi (i, \epsilon) \right] - i \right\}, \tag{4}
\]

subject to:

\[
\Pi^1 (\theta) \geq \Pi^1 (\theta', \theta, \theta) \quad \text{and} \quad \Pi^2 (\theta) \geq \Pi^2 (\theta, \theta', \theta) \quad \forall \theta, \theta', \tag{5}
\]

\[
i^j \in \arg \max_i \left\{ E[\Pi^j (i + i^m, \epsilon)] - i \right\} \text{ for } j, m = 1, 2 \text{ and } j \neq m. \tag{6}
\]

Inequalities (5) ensures that truth-telling is a Nash-equilibrium on and off the equilibrium path. The search for optimal contracts is simplified by a useful property of investment equilibria:

**Lemma 1** Under any contract \( \mathcal{C} \), an equilibrium exists in which only one of the parties invests, and this equilibrium weakly dominates possible other equilibria in terms of the total amount invested.

**Proof.** Straightforward, and therefore omitted. □

Lemma 1 immediately follows from the transferability of investments. To illustrate the properties of investment equilibria further, suppose first that, for a given contract \( \mathcal{C} \), \( E[\Pi^j (i, \epsilon)] - i \) has a unique maximizer for all \( j = 1, 2 \). Then, generically, the investment equilibrium is unique. In the non-generic case, all investment equilibria lead to the same total amount invested. Second, only if for some \( j = 1, 2 \) the maximizer of the above expression is not unique, investment equilibria may differ with respect to the total amount invested.

Finally, an optimal contract achieves the first-best outcome if it induces total investments \( i^* \) defined by:

\[
i^* \in \arg \max_{i \in [0, i^1 + i^2]} \left\{ E[\phi (i, \epsilon)] - i \right\}, \tag{7}
\]
where \( i^* \) is assumed to be unique and strictly positive.

### 3 Delegation as Solution to the Complete-Contracting Problem

If investments are hybrid, non-contingent contracts provide in general insufficient investment incentives, and optimal contracts are necessarily message-dependent.\(^{12}\) That is, optimal contracts need to allow for some form of ex-post discretion over contract terms. To illustrate this, consider Segal and Whinston (2002) who provide general conditions under which non-contingent contracts are optimal.\(^{13}\) Their results are based on the assumption that actions that minimize (respectively maximize) investment returns of a party do not depend on the ex-post realization of uncertainty \( \varepsilon \), which in general will fail to hold when investments are hybrid. Consider the following example:

**Example 1** Suppose only party 1 invests, \( a \in [0, \bar{a}] \), \( \pi^j(\cdot) \) is differentiable in \( a \), \( \pi_1^1, \pi_2^1, \pi_1^2, \pi_2^2 > 0 \), and \( \beta^j(\varepsilon) \) is independent of \( \varepsilon \) for \( j = 1, 2 \). Hence, the investment is hybrid, and it depends on (i) the distribution of bargaining power, (ii) the technological impact of the investment on \( \pi^1 \) and \( \pi^2 \), and (iii) the realization of uncertainty whether, in a given ex-post state, \( \Pi_{ia}^1(a, i, \varepsilon) = \beta^2 \cdot \pi_1^1(a, i, \varepsilon) - \beta^1 \cdot \pi_2^2(a, i, \varepsilon) \) is positive or negative. In the former case, the investment is mainly selfish, and incentives are maximized by setting \( a = \bar{a} \). In the latter case, the investment is mainly cooperative, and incentives are maximized by setting \( a = 0 \). Hence, the incentive-maximizing action may depend on the realization \( (i, \varepsilon) \) of the ex-post state. In contrast, if the investment is not hybrid, but either purely selfish \( (\pi_1^1 > \pi_1^2 \equiv 0) \), or purely cooperative \( (\pi_2^2 > \pi_1^1 \equiv 0) \), Segal and Whinston’s (2002) condition is met. For more detailed discussions of these cases, see Edlin and Reichelstein (1996) and Che and Hausch (1999), respectively.

Hence, while in the present setting non-contingent contracts are in general too crude an instrument to provide optimal incentives, in the following, I present conditions under which optimal contracts take nevertheless an especially simple, empirically relevant form. First, I study the underinvestment case, and then turn to a setting where potential overinvestment is an issue.

\(^{12}\)Formally, a contract is called non-contingent if \( a(\theta^1, \theta^2) = a \) and \( t^1(\theta^1, \theta^2) = t \forall \theta^1, \theta^2 \in \Theta \).

\(^{13}\)For non-differentiable settings, see Schweizer (2000).
3.1 Underinvestment Case

If the parties face an underinvestment problem, it is optimal to maximize the total amount invested. Hence, given Lemma 1, it is optimal to maximize the investment incentives of one of the parties. Suppose that authority is delegated to party 1, say. In this case, party 1 chooses the actions that maximize its post-renegotiation payoff. In general, these actions will, however, fail to maximize the marginal return of an investment from an ex-ante perspective. However, ex-post and ex-ante incentives (with respect to the choice of action) are aligned if the following condition is met.

**Condition 1** The threatpoint payoffs of the parties can be expressed as

\[ \pi^j(a, i, \epsilon) \equiv \rho^j(a, \epsilon) : \nu(i, \epsilon) + \gamma^j(i, \epsilon) + \tau^j(\epsilon) \quad \text{for } j = 1, 2, \]

where \( \nu(\cdot) \) and \( \gamma^j(\cdot) \) are continuously differentiable in \( i, \rho^j(\cdot), \nu(\cdot) \geq 0, \) and \( \nu_i(\cdot) > 0. \)

Condition 1 requires that the threatpoint payoffs can be written as the product of an action-dependent factor \( \rho^j(a, \epsilon) \) and a common investment-dependent factor \( \nu(i, \epsilon) \), possibly plus action-independent terms. For instance, this condition would hold if payoff consequences for a principal and an agent would not only depend on ex-post actions of the agent, but also on the ex-ante investment of the agent in his human capital \( \nu(i, \epsilon) \). Alternatively, \( \nu(i, \epsilon) \) could represent the investment-dependent value of an asset. In this case, payoffs of the parties would additionally depend on how the asset is employed ex-post.\(^{14}\)

While Condition 1 ensures that, ex-post, a party prefers to pick the incentive-maximizing action, it remains to characterize under which circumstances potential overinvestment is not an issue. Suppose that authority is delegated to party \( j \) (i.e., party \( j \) is allowed to choose any \( a \in A \) ex-post), and the transfer payments are non-contingent. In this case, the preferred ex-post actions are

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\(^{14}\)To impose some structure on the payoff functions of the parties is not uncommon in the holdup literature. For example, Edlin and Reichelstein (1996, Condition A3) assume that threatpoint payoffs of the parties can be expressed as \( \pi(i', q, \epsilon) = \pi^2(i', \epsilon) \cdot q + \pi^1(q, \epsilon) + \pi^2(i', \epsilon) \) for \( j = 1, 2 \), where \( q \) denotes a variable trade quantity: similar to Condition 1 above, they require that the ex-post decision and the investment only interact multiplicatively but, in contrast to Condition 1, they consider a common decision term and an idiosyncratic investment term. They use this condition to show that a simple, non-contingent contract can induce efficient two-sided investments. Segal and Whinston (2002, Condition S) consider a generalized version of Edlin and Reichelstein’s (1996) condition that allows for hybrid investments. Finally, Segal and Whinston (2002, Condition A) consider a condition that states that investments can be aggregated into one dimension, and that the action-dependent parts of the post-renegotiation payoffs of the parties only depend on this investment aggregate.
given by

\[ a^j(\epsilon) \in \operatorname{arg\,max}_{a \in A} \{ \pi^j(a, i, \epsilon) + \beta^j(\epsilon) \cdot [\phi(\theta) - \sum_j \pi^j(a, i, \epsilon)] \}, \tag{8} \]

where \( j, m = 1, 2 \), and \( a^j(\epsilon) \) is assumed to exist for all \( j \) and \( \epsilon \). Given that authority is delegated to party \( j \), define by \( \bar{i}^j \) the resulting equilibrium total investment level.\(^{15}\) It is now possible to state the following result:

**Proposition 1** If Condition 1 holds and the parties face an underinvestment problem (i.e., if \( \max\{\bar{i}^1, \bar{i}^2\} \leq i^* \)), ex-ante delegation of authority over ex-post actions is optimal. That is, a contract is optimal that specifies a fixed payment and prescribes that one of the parties is free to choose any \( a \in A \) at the ex-post stage.

**Proof.** See Appendix A. \( \blacksquare \)

To illustrate Proposition 1, Appendix B provides a numerical example. Note that for Proposition 1 to hold, post-renegotiation payoffs are not required to be concave in investments, which will in general not be the case when investments are hybrid. Finally, a sufficient condition for \( \max\{\bar{i}^1, \bar{i}^2\} \leq i^* \) to hold is given by \( \rho^j(a^j(\epsilon), \epsilon) \cdot v_l(i, \epsilon) + \gamma^j_l(i, \epsilon) \leq \phi_l(i, \epsilon) \) for all \( i, \epsilon \) and \( j \).

**Extension: a multi-dimensional investment** In many settings an agent engages in various preparatory activities. Proposition 1 can be extended to such a multi-dimensional setting if only one of the parties, party 1 say, may provide effort. Under slight abuse of notation, suppose that party 1 chooses a \( n \)-dimensional effort vector \( \iota^1 \equiv (\iota^1, \ldots, \iota^n) \), \( n \geq 1 \), from a compact subset of \( \mathbb{R}^n \). Party 2 does not invest, i.e., \( \iota^2 \equiv 0 \). Define \( \iota^{-l} = (\iota^1, \ldots, \iota^{l-1}, \iota^{l+1}, \ldots, \iota^n) \) for \( l = 1, \ldots, n \). Redefine the ex-post state \( \theta \) and the first-best investment level \( i^* \) accordingly, and denote by \( \tilde{i} \) the equilibrium investment level when authority is delegated to party 1. To take account of the multi-dimensionality, Condition 1 has to be slightly adapted:

\(^{15}\)If \( \tilde{i}^2 \) is not unique, it is defined as the maximally possible equilibrium total amount invested.
Condition 2 The threatpoint payoffs of the parties can be expressed as

\[ \pi^j(a,i,e) \equiv \rho^j(a,e) \cdot \nu(i,e) + \tau^j(e) \text{ for } j = 1, 2, \]

where \( \nu(\cdot) \) is continuously differentiable in \( i \), \( \rho^j(\cdot), \nu(\cdot) \geq 0 \), and \( \nu, \rho > 0 \) for all \( l \in \{1, \ldots, n\} \).

Condition 2 is slightly stronger than Condition 1 in that it requires that there is no action-independent effect of investments on threatpoint payoffs.

Unfortunately, in contrast to the one-dimensional case, Condition 2 is not sufficient for delegation to be optimal. Suppose that Condition 2 holds, and that party 1 has authority over ex-post actions: for a given \( \iota^{-l} \), define \( \iota^l(\iota^{-l}) \) as the \( l \)th effort component party 1 will choose ex-ante. Formally:

\[ \iota^l(\iota^{-l}) \in \arg \max_{\iota^l} \{ E[\pi^j(a^{1}(\iota), i, e) + \beta^j(e) \cdot [\phi(\theta) - \sum_j \pi^j(a^{1}(\iota), i, e)] - \iota^l] \} \quad (9) \]

for \( l = 1, \ldots, n \).

Condition 3

(i) \( \phi(\cdot) \) is strictly concave in \( i \), \( \nu(\cdot) \) is concave in \( i \), and \( \nu_{\iota^{-l}m}, \phi_{\iota^{-l}m} > 0 \) for all \( l \neq m \), and

(ii) \( \forall e: \exists a^0 \in A \text{ such that } \rho^S(a^0, e) = \rho^B(a^0, e) = 0 \).

Condition 3 ensures that if authority is delegated to party 1, then the equilibrium investment vector \( \tilde{i} \) and \( \iota^l(\iota^{-l}) \) are unique for all \( l \), and investment components are strategic complements. Note that part (ii) of Condition 3 is rather mild: it only requires that there exist actions at which action-dependent parts of the threatpoint payoffs are zero. For example, in the bilateral trade models of Edlin and Reichelstein (1996) and Che and Hausch (1999), this is the case for a trade quantity equal to zero. It is now possible to establish the following result.

Proposition 2 Suppose that only party 1 provides (multi-dimensional) effort, Conditions 2 and 3 hold, and the parties face an underinvestment problem (i.e., \( \tilde{i} \leq i^* \)). Then ex-ante delegation of authority over ex-post actions is optimal. That is, a contract is optimal that specifies a fixed payment and prescribes that party 1 is free to choose any \( a \in A \) at the ex-post stage.

Proof. See Appendix C.
Figure 2 serves to illustrate Proposition 2. In the underinvestment settings of Propositions 1 and 2, simple delegation with unrestricted competencies turns out to be optimal because it induces maximal investment incentives for one of the parties. If potential overinvestment is an issue, delegation with restricted competencies may be optimal. This case is discussed in the next subsection.

Figure 2: Delegation and multi-dimensional effort provision. Suppose \( i^1 \equiv (i^1, i^2) \). In the setting of Proposition 2, the functions \( i^1(i^2) \) and \( i^2(i^1) \) are increasing, and \( \tilde{i} \) is unique. Hence, investment equilibria under any contract \( C \) are in the set \( \Omega \). That is, delegation, which leads to \( \tilde{i} \), is optimal.

### 3.2 Overinvestment is Possible

I now return to the initial setting in which both parties may make one-dimensional investments. Suppose that delegation with unrestricted competencies would lead to overinvestment. In this case, simple delegation with restricted competencies is optimal provided that two conditions are satisfied. The first condition is a somewhat stronger version of Condition 1.

**Condition 4** The threatpoint payoffs of the parties can be expressed as

\[
\pi^j(a, i, \epsilon) \equiv \rho^j(a, \epsilon) \cdot \nu(i, \epsilon) + \tau^j(\epsilon) \text{ for } j = 1, 2,
\]

where \( \rho^j(\cdot) \) is continuous in \( a \), \( \nu(\cdot) \) is continuously differentiable in \( i \), \( \rho^j(\cdot), \nu(\cdot) \geq 0 \), \( \nu_i(\cdot) > 0 \), and \( \nu_{ii}(\cdot) < 0 \).
Compared to Condition 1, Condition 4 additionally requires $\rho^j(\cdot)$ to be continuous in $a$, $\gamma^j(\cdot) \equiv 0$ and $\nu_{ii} < 0$. The second condition states that the ex-post surplus is concave, actions are continuous, and if no actions are taken, the action-dependent parts of the threatpoint payoffs are zero.

**Condition 5**

(i) $\phi(\cdot)$ is concave in $i$, and

(ii) the action space is given by $A = [0, \pi^1] \times ... \times [0, \pi^I]$, and $\rho^j((0, ..., 0), \epsilon) = 0$ for all $\epsilon$.

Even if only Condition 1 (see Subsection 3.1 above) is met, it holds true that a party’s investment incentives are reduced if its ex-post freedom of choice is restricted. However, if overinvestment is possible, Condition 1 is not sufficient for delegation to be optimal. On the one hand, one needs to ensure that, given delegation, there exists a truncated choice set such that $i^*$ satisfies the first-order condition of one of the parties. Given the continuity requirements in Conditions 4 and 5, this is the case. On the other hand, even if $\nu(\cdot)$ and $\phi(\cdot)$ are well-behaved, the post-renegotiation payoffs are in general not concave in investments. However, given that authority is delegated to party $j$, the post-renegotiation payoff of party $j$ turns out to be concave if $\gamma^1(\cdot) = \gamma^2(\cdot) \equiv 0$ holds and null-actions exist.

**Proposition 3** Suppose Conditions 4 and 5 hold. Then ex-ante delegation of authority over ex-post actions is optimal, but competencies need to be restricted. That is, a contract is optimal that specifies a fixed payment and prescribes that one of the parties is free to choose any $a \in A^C \subseteq A$ at the ex-post stage.

**Proof.** See Appendix D. ■

One simple way to optimally restrict competencies is to rule out extreme choices. That is, in the proof of the Proposition 3 it is shown that a choice set of the form $A^C = [0, \pi^1 - \omega^1] \times ... \times [0, \pi^I - \omega^I]$, where $\omega^1, ..., \omega^I \geq 0$, is optimal. Finally, while Condition 5 requires all actions to be continuous, this assumption is not necessary for a result of the above form to obtain. Appendix E provides an example to illustrate this point.

### 4 Conclusion

The present paper provides conditions under which simple delegation/decentralization of authority replicates the outcome of optimal message-dependent contracts. While such replication results have
previously been obtained in settings of asymmetric information, I consider a standard symmetric-information holdup framework but assume that all ex-post actions are ex-ante contractible. I argue that investments frequently benefit both parties, and show that if threatpoint payoffs satisfy certain separability and symmetry conditions, simple ex-ante delegation of authority over ex-post actions is the best way to tackle an underinvestment problem. This result can be extended to a multi-dimensional investment if investment components are strategic complements. If overinvestment is possible, I show that simple delegation, however with restricted competencies, is optimal if additional continuity requirements are met. To summarize, these results provide an (incentive-based) complete-contracting rationale for authority relationships: delegation/decentralization turns out to be optimal even though the Revelation Principle applies.

My results can be interpreted in two additional ways. First, the results contribute to the literature on the foundations of incomplete contracts. This literature aims to derive “incompleteness” of contracts endogenously, and so far has identified circumstances under which the lack of a contract or simple non-contingent/option contracts are optimal. Second, the property-rights theory of the firm argues that (ex-ante non-contractible) residual control rights lead to the relevance of ownership, and hence the relevance of the boundaries of the firm. In the spirit of Tirole (1999), Propositions 1 and 2 provide conditions under which an optimal contract consists of nothing more than an assignment of ownership of a critical resource even though all ex-post actions are contractible ex-ante.

Finally, an extension of the results to two-sided non-transferable investments would be interesting. Above I have provided conditions under which delegation maximizes the investment incentives of one of the parties. If investments are non-transferable, optimal contracts need to be more balanced (i.e., they need to provide investment incentives for both parties). However, if investments are non-transferable, but any action only affects the return of the investment of one of the parties, it should be possible to derive conditions under which divided authority is optimal where each of the parties has discretion over a subset of actions. A detailed analysis of this case awaits, however, future research.

16 For example, in a vertical relationship only production-oriented actions might affect the return of a cost-reducing investment by the seller, while only sales-oriented actions might affect the return of a marketing investment by the buyer.
Appendix

A Proof of Proposition 1

In a first step, I derive upper bounds for the investment returns of the parties. Note that the ex-post message game between the parties is constant-sum:

$$\Pi^1(\theta^1, \theta^2, \theta) + \Pi^2(\theta^1, \theta^2, \theta) = \phi(\theta) \forall \theta^1, \theta^2, \theta \in \Theta.$$  \hspace{1cm} (10)

This observation together with the truth-telling constraint (5) implies (see e.g., Maskin and Moore, 1999):

$$\Pi^1(\theta) - \Pi^1(\theta') \leq \Pi^1(\theta', \theta, \theta) - \Pi^1(\theta', 0, 0), \text{ and}$$

$$\Pi^2(\theta) - \Pi^2(\theta') \leq \Pi^2(\theta, \theta', \theta) - \Pi^2(\theta, \theta', 0),$$ \hspace{1cm} (11)

for arbitrary $\theta, \theta' \in \Theta$. For arbitrary $\theta = (i^1, i^2, \epsilon)$ and $\theta' = (i'^1, i'^2, \epsilon)$ where $i' = i^1 + i^2 > i^1 + i^2 = i$, the above inequalities together with Condition 1 imply

$$\frac{\delta \Pi^1(\theta)}{\delta \theta_i} \equiv \limsup_{i' \rightarrow i} \frac{\Pi^1(\theta) - \Pi^1(\theta')}{i' - i} \leq \beta^j(\epsilon) \cdot \phi_i(i, \epsilon)$$

$$+ \limsup_{i' \rightarrow i} \{\beta^m(\epsilon) \cdot \pi^j(a(\theta', \theta), i, \epsilon) - \beta^j(\epsilon) \cdot \pi^m(a(\theta', \theta), i, \epsilon)\}$$

$$= \beta^j(\epsilon) \cdot \phi_i(i, \epsilon) + \left[\beta^m(\epsilon) \cdot \gamma^j_i(i, \epsilon) - \beta^j(\epsilon) \cdot \gamma^m_i(i, \epsilon)\right]$$

$$+ \limsup_{i' \rightarrow i} \left\{\nu_i(i, \epsilon) \cdot [\beta^m(\epsilon) \cdot \rho^j(a(\theta', \theta), \epsilon) - \beta^j(\epsilon) \cdot \rho^m(a(\theta', \theta), \epsilon)]\right\},$$

$$\leq \beta^j(\epsilon) \cdot \phi_i(i, \epsilon) + \left[\beta^m(\epsilon) \cdot \gamma^j_i(i, \epsilon) - \beta^j(\epsilon) \cdot \gamma^m_i(i, \epsilon)\right]$$

$$+ \nu_i(i, \epsilon) \cdot [\beta^m(\epsilon) \cdot \rho^j(a^j(\epsilon), \epsilon) - \beta^j(\epsilon) \cdot \rho^m(a^j(\epsilon), \epsilon)],$$

for $j, m = 1, 2$ and $j \neq m$. The second inequality follows from (8). Hence, Lemma 1 and $\max\{i^S, i^B\} \leq i^*$ imply that it is optimal to maximize incentives of one of the parties. If the ex-ante contract specifies a fixed payment and that party $j$ is free to choose from set $A$ ex-post, then the post-renegotiation payoff of party $j$ as a function of $a$ is given by

$$\beta^j(\epsilon) \cdot \phi(i, \epsilon) + \beta^m(\epsilon) \cdot \pi^j(a, i, \epsilon) - \beta^j(\epsilon) \cdot \pi^m(a, i, \epsilon) + t^j$$

$$= \nu(i, \epsilon) \cdot \left[\beta^m(\epsilon) \cdot \rho^j(a, \epsilon) - \beta^j(\epsilon) \cdot \rho^m(a, \epsilon)\right] + [\text{terms independent of } a],$$ \hspace{1cm} (13)

for $j, m = 1, 2$ and $j \neq m$. Hence, for each realization of $\epsilon$, party $j$ chooses action $a^j(\epsilon)$, which maximizes its investment incentives from an ex-ante perspective.

B Illustration of Proposition 1

Suppose that

(i) $a = (a^1, a^2) \in A = \{0, 1\}^2$,

(ii) $\rho^j(a, \epsilon) = \epsilon \cdot a^1 + a^2$,

(iii) $\rho^m(a, \epsilon) = 2 \cdot a^1 + 0.25 \cdot \epsilon \cdot a^2$,

(iv) $\nu(i, \epsilon) = 0.5 \cdot \phi(i, \epsilon) = 2 \cdot \ln(1 + i)$,

(v) $\gamma^S(i, \epsilon) = \gamma^B(i, \epsilon) = 0$,

(vi) $\beta^1(\epsilon) = \beta^2(\epsilon) = 0.5$,
and (vii) $\epsilon$ is uniformly distributed on $[0, 5]$. In this case, the first-best investment level is given by $i^* = 3$. If authority is delegated to party 1, she will prefer actions $(a^1, a^2) = (1, 0)$ if $\epsilon \geq 4$, and $(a^1, a^2) = (0, 1)$ otherwise. This results in an investment level $i^1 = 2.3$. If authority is delegated to party 2, he will only invest $i^2 = 1.425$. Consequently, it is optimal to agree on a fixed transfer payment and to delegate authority to party 1.

C Proof of Proposition 2

First, consider delegation of authority where party 1 is free to choose from set $A$. Condition 3(ii) implies that
\[
\beta^2(\epsilon) \cdot \rho^1(a^1(\epsilon), \epsilon) - \beta^1(\epsilon) \cdot \rho^2(a^1(\epsilon), \epsilon) \geq 0 \quad \forall \epsilon.
\]

This observation together with Condition 3(ii) implies that (a) $i^l(\iota^{-l})$ is unique and non-decreasing in $\iota^{-l}$ for all $l$, and that (b) $\iota = (i_1^{l_1}, \iota^{-l_1})$ is unique where $i_l$ is implicitly defined by $i_l(\iota^{-l}) = \iota^l$ for all $l$. Second, similar to inequality (12) in the proof of Proposition 1, it follows from Condition 2 that under any arbitrary contract $C$ it holds that
\[
\frac{\delta \Pi^l}{\delta i^l}(\theta) \leq \beta^1(\epsilon) \cdot \phi(i, \epsilon) + \nu(i, \epsilon) \cdot [\beta^2(\epsilon) \cdot \rho^1(a^1(\epsilon), \epsilon) - \beta^1(\epsilon) \cdot \rho^2(a^1(\epsilon), \epsilon)] \quad \forall i, \epsilon.
\]

Hence, for given $\iota^{-l}$, the $i^l$ chosen when authority is delegated to party 1 is weakly larger than the $i^l$ chosen under any other contract $C$. Define $\Omega \equiv \{i^l \mid i^l \leq i^l(\iota^{-l}) \text{ for all } l\}$. Inequality (15) together with the fact that the functions $i^l(\cdot)$ are non-decreasing in their arguments implies that (a) any investment equilibrium $\hat{i}(C)$ under an arbitrary contract $C$ is in the set $\Omega$ (i.e., $\hat{i}(C) \in \Omega \forall C$), and that (b) $\hat{i}^l(C) \leq i^l \leq i^l(C)$ for all $l$. Hence, delegation of authority to party 1, which leads to $\hat{i}$, is optimal.

D Proof of Proposition 3

Case $\hat{i}^1, \hat{i}^2 < i^*$: the reasoning in the proof of Proposition 1 in Appendix A implies that unrestricted delegation of authority (i.e., $A^C = A$) is optimal.

Case $\max\{\hat{i}^1, \hat{i}^2\} \geq i^*$: define by $\hat{i}^{j,k}$ the investment level that party $j$ chooses if party $m \neq j$ does not invest and party $k$ has unrestricted authority. Formally:
\[
\hat{i}^{j,k} \in \arg \max_{i \in [0, \hat{\iota}]} \{E[\beta^j(\epsilon) \cdot \phi(i, \epsilon) + \nu(i, \epsilon) \cdot [\beta^m(\epsilon) \cdot \rho^j(a^k(\epsilon), \epsilon) - \beta^j(\epsilon) \cdot \rho^m(a^k(\epsilon), \epsilon)]\},
\]

where $j, k, m = 1, 2$ and $j \neq m$. Note that $\hat{i} = \max\{\hat{i}^{j,j}, \hat{i}^{m,j}\}$, and (by definition) $\hat{i}^{j,j} \geq \hat{i}^{m,m}$. Proposition 3 is proved for the case $\hat{i}^{1,1} \geq \max\{i^*, \hat{i}^{2,2}\}$. The proofs for the remaining cases are completely analogous, and therefore omitted. Define by $A(\omega) \equiv [0, \hat{\iota}^1 - \omega^1] \times ... \times [0, \hat{\iota}^l - \omega^l]$ a truncated action space, where $\omega^l \in [0, \hat{\iota}] \forall l \in \{1, ..., l\}$. In analogy to (8), define
\[
\check{\rho}^l(\epsilon, \omega) \equiv \max_{a \in A(\omega)} \{\beta^2(\epsilon) \cdot \rho^1(a, \epsilon) - \beta^1(\epsilon) \cdot \rho^2(a, \epsilon)\}.
\]

Conditions 4 and 5 ensure that a solution to (17) exists for all $\epsilon$ and $\omega$. Condition 5 implies
This observation together with Condition 5 and the concavity of $E[\phi(i, \epsilon)]$ implies

$$E[\beta^1(\epsilon) \cdot \phi_{it}(i, \epsilon) + \nu_{it}(i, \epsilon) \cdot \tilde{\rho}^1(\epsilon, \omega)] \leq 0 \forall i, \omega. \quad (18)$$

Hence, $\tilde{\rho}^1 \geq i^*$ together with (18) implies for $\omega^0 \equiv (0, \ldots, 0)$:

$$E[\nu_i(i^*, \epsilon) \cdot \tilde{\rho}^1(\epsilon, \omega^0)] \geq E[\beta^2(\epsilon) \cdot \phi_i(i^*, \epsilon)] > 0,$$

where the second inequality follows from $\beta^2(\cdot), \phi_i(\cdot) > 0$. Define $\mathcal{A} \equiv (a^1, \ldots, a^I)$. Condition 5 implies $\tilde{\rho}^1(\epsilon, \omega) = 0$, and hence $E[\nu_i(i^*, \epsilon) \cdot \tilde{\rho}^1(\epsilon, \omega)] = 0$. Moreover, as $\rho^1(\cdot)$ and $\rho^2(\cdot)$ are continuous in $a$, it follows from Berge’s theorem of the maximum (see e.g., De la Fuente, 2000, p. 301) that the value function $\tilde{\rho}^1(\epsilon, \omega)$ is continuous in $\omega$. Hence, the Intermediate-Value Theorem implies that there exists a $\omega^*$ such that:

$$E[\beta^1(\epsilon) \cdot \phi_i(i^*, \epsilon) + \nu_i(i^*, \epsilon) \cdot \tilde{\rho}^1(\epsilon, \omega^*)] = E[\phi_i(i^*, \epsilon)]. \quad (20)$$

Therefore, (18) and (20) imply that if party 1 is free to choose from set $A(\omega^*)$ ex-post, $i^1 = i^*$ is a best-response to $i^2 = 0$. It remains to be checked whether party 2 has an incentive to invest more than $i^*$. Note that, given (restricted) delegation of authority to party 1, the marginal investment return of party 2 is given by

$$E[\beta^2(\epsilon) \cdot \phi_i(i, \epsilon) - \nu_i(i, \epsilon) \cdot \tilde{\rho}^1(\epsilon, \omega^*)] \leq E[\beta^2(\epsilon) \cdot \phi_i(i, \epsilon)] \leq E[\phi_i(i, \epsilon)] \forall i,$$

where the first inequality follows from $\tilde{\rho}^1(\epsilon, \omega^*) \geq 0 \forall \epsilon$. Hence, party 2 does not want to invest more than $i^*$ which concludes the proof.

**E Potential Overinvestment and Discrete Actions: An Example**

Reconsider the example given in Appendix B, but suppose that

(i) $a = (a^1, a^2) \in A = [0, 1] \times \{0, 1\}$, and

(iv) $v(i, \epsilon) = 0.95 \cdot \phi(i, \epsilon) = 3.8 \cdot \ln(1 + i)$.

In this case, if party 1 is allowed to choose from set $A$, she invests $i = 3.47 > i^*$ (i.e., there is overinvestment). However, delegation of authority where party 1 is only free to choose from the restricted set $A^C = [0, 0.725] \times \{0, 1\}$ induces the first-best.
References


