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Capital and labor income Pareto exponents across time and space

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Capital and Labor Income Pareto Exponents across Time and Space

Tjeerd de Vries* Alexis Akira Toda†

May 28, 2020

Abstract

We estimate capital and labor income Pareto exponents across 348 country-year observations that span 51 countries over half a century. We document two stylized facts: (i) capital income is more unequally distributed than labor income; namely, the capital exponent (1–3) is smaller than labor (2–5), and (ii) capital and labor exponents are nearly uncorrelated. To explain these findings, we build an incomplete market model with job ladders and capital income risk that gives rise to a capital income Pareto exponent smaller than but nearly unrelated to the labor exponent. Our results suggest the importance of distinguishing income and wealth inequality.

Keywords: income fluctuation problem, inequality, power law.

JEL codes: C46, D15, D31, D52.

1 Introduction

The purpose of this paper is to estimate and document the Pareto exponents for capital and labor income separately for as many countries and years as possible. We say that a positive random variable X obeys a power law with Pareto exponent $\alpha > 0$ if the tail probability decays like a power function: $P(X > x) \sim x^{-\alpha}$ for large x . In the context of the income distribution, the Pareto exponent characterizes the tail heaviness of high incomes and hence top tail inequality. Our study is motivated by the following two observations. First, we are not aware of a comprehensive study that documents the capital and labor income Pareto exponents separately for many countries and years, despite their importance. Second, the Pareto exponent has desirable properties relative to other popular inequality measures such as top income shares.

Consider the first point. Conceptually, capital and labor income are very different. While the former is the return for providing capital (wealth), the latter is the return for providing labor services, and there is no particular reason to expect a relation between the two. Although these two forms of income are conceptually distinct, it is often put together as just “income” and discussed in the context of inequality and related policies. If capital and labor income

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are quantitatively different, a policy design based on total income may be misleading. To give one example, consider the theory of optimal taxation (Saez, 2001), where the income Pareto exponent plays an important role. Saez and Stantcheva (2018) carefully distinguish capital and labor income and apply the theory of optimal taxation in the United States. They find that with an income elasticity of $e = 0.5$, the optimal top marginal tax rate is about 50% for labor and 60% for capital (see their Figure 5). This difference directly comes from the fact that capital and labor income Pareto exponents are distinct. Thus, distinguishing capital and labor income inequality is potentially important for policy designs.

Consider the second point. In the applied literature such as Piketty (2003) and Piketty and Saez (2003), top income shares (such as the top 1% income share) are more commonly reported than the income Pareto exponent, perhaps because top shares are summary statistics that can be computed without specifying functional forms or can be understood by non-experts without special knowledge of statistics. However, Atkinson (2005) documents methodological problems regarding the cross-country comparison of top income shares, citing the differences in tax units (e.g., individual or household) and legislation (e.g., whether social security benefits are taxable). One of the reasons such issues arise is because it is not always clear how to define the population and measure small units. For example, suppose one is interested in the top 1% share of population in cities or fresh water surface area in lakes. Clearly the resulting numbers highly depend on how we define cities and lakes (versus villages and ponds). Using the Pareto exponents significantly alleviates these definition and measurement issues because the Pareto distribution is scale invariant (see Jessen and Mikosch, 2006 for a summary).

In this paper, we use the harmonized *Luxembourg Income Study* database (hereafter LIS) to document the capital and labor income Pareto exponents across all available 348 country-year observations that span 51 countries over half a century. We document two empirical findings. First, we find that the capital income Pareto exponent is roughly in the range 1–3 and is smaller than the labor income Pareto exponent, which ranges between 2–5. This implies that capital income is more unequally distributed than labor income. This fact is unsurprising and well known for a specific country or year (see, for example, the Lorenz curve in Figure 1 of Saez and Stantcheva, 2018). However, we are not aware of a comprehensive study that systematically analyzes datasets from many countries and years, and therefore our finding suggests that capital income is generally more unequal than labor income. More specifically, we formally test the equality of capital and labor income Pareto exponents and the null is rejected in 85% of samples. In every single case of rejection, the capital exponent is smaller than the labor exponent. Second, we find that the capital income Pareto exponent is nearly unrelated to the labor exponent. In particular, the correlation between the two exponents across countries is close to zero.

To explain our empirical findings, we build a simple incomplete market model with job ladders and capital income risk. In the model, agents get randomly promoted to the next job ladder. Because individual income follows a random growth process, we obtain a Pareto-tailed labor income distribution. The agents also save assets and face idiosyncratic investment risk, which generates a Pareto-tailed wealth (hence capital income) distribution. Because the capital income Pareto exponent is mainly determined by the asset return distribution, while

the labor income Pareto exponent is mainly determined by the income growth distribution, the relation between the two is weak. Furthermore, we analytically characterize the capital and labor income Pareto exponents and show that the former tends to be smaller than the latter for common parametrization. Our results suggest the importance of distinguishing income and wealth inequality.

Related literature The power law behavior of income was first recognized by Pareto (1895, 1896, 1897), who used tabulation data of tax returns in many European countries. More recent research that employs micro data include Reed (2001) for U.S., Reed (2003) for U.S., Canada, Sri Lanka, and Bohemia, Nirei and Souma (2007) for Japan, Toda (2011, 2012) for U.S., and Ibragimov and Ibragimov (2018) for Russia. Bandourian et al. (2002) estimate eleven parametric distributions (some of which exhibit Pareto tails) using 82 household labor income datasets from *Luxembourg Income Study* (LIS) as we do, though they neither focus on the Pareto exponent nor consider capital income. Atkinson and Piketty (2010, Table 13A.23) document income Pareto exponents across countries and years estimated from top income share data based on tax returns. However, their estimates are based on total income, and since (as we document in Section 3.3) the capital income Pareto exponents tend to be smaller than labor exponents, their estimates are best understood as capital income (hence wealth) Pareto exponents. See Gabaix (2009) for an introduction to power law.

2 Data

In this section we describe the dataset that we use and discuss its limitations.

2.1 The LIS database

We use the data from the *Luxembourg Income Study* (LIS), which is a large, harmonized database of micro-level income data that covers about 50 countries worldwide and many years since the late 1960s. In many countries, the data derive from government surveys (for example, the U.S. data is based on the *Current Population Survey*). The LIS data are available at both individual and household level. We focus on the household labor and capital income because (i) it is reasonable to assume that economic decisions such as financial planning are made at the household level, and (ii) incomes among couples are likely correlated due to assortative matching in the marriage market (Siow, 2015), which invalidates statistical estimation.¹ The LIS defines *labor income* as “cash payments and value of goods and services received from dependent employment, as well as profits/losses and value of goods from self-employment, including own consumption”. *Capital income* is defined as “cash payments from property and capital (including financial and non-financial assets), including interest and dividends, rental income and royalties, and other capital income from investment in self-employment activity”. Together these two categories make up total *factor income*. See the LIS 2019 USER GUIDE² for a detailed summary on how these data are retrieved and calculated.

¹In our data, we find an average correlation of 0.22 between labor income of husband and wife, which underpins the conjectured dependency.

²<https://www.lisdatacenter.org/wp-content/uploads/files/data-lis-guide.pdf>

2.2 Data limitations

Our analysis draws upon datasets from many different countries that are harmonized into a common framework by LIS. However, many details about the collection of data in the different countries are omitted. For example, we find evidence of top-coding in some countries and years, as the largest income order statistic is equal to the second largest.³ Top-coding induces an upward bias in the estimation of the Pareto exponent. This issue is not necessarily resolved if, instead, one relies on administrative tax income data, for similar biases arise such as rich households trying to understate their taxable income (Atkinson et al., 2011). Burkhauser et al. (2012) detail a method that can be used to overcome the bias due to top-coding, however at the end of their paper they show that the results are robust even if estimates are based on the top-coded series. For these reasons we treat the datasets as not being top-coded in our analysis.

Another limitation of the LIS database is that it is based on government surveys and the measurement error may be larger compared to administrative data based on tax returns. The fact that the income distribution in administrative data is often reported as tabulations, not micro data, causes no problem for estimating Pareto exponents, as Toda and Wang (2020) provide an efficient estimation method for such data. In fact, Atkinson and Piketty (2010, Table 13A.23) document income Pareto exponents across countries and years estimated from top income share data. However, their table is based on total income, and since (as we document in Section 3.3) the capital income Pareto exponents tend to be smaller than labor exponents, the estimates in Atkinson and Piketty (2010) are best understood as capital income (hence wealth) Pareto exponents. Since we are not aware of a comprehensive income database that distinguishes capital and labor income, we decided to use the LIS database.

3 Pareto exponents across countries and years

In this section we estimate the capital and labor income Pareto exponents for all countries and years that are available in the LIS database.

3.1 Estimation method

For each country and year, we suppose that the (capital or labor) income observations $\{X_n\}_{n=1}^N$ are independent and identically distributed (i.i.d.) with cumulative distribution function (CDF) $F(x) = P(X_n \leq x)$. The assumption that the upper tail of income obeys a power law with Pareto exponent $\alpha > 0$ translates into the regular variation condition

$$1 - F(x) = x^{-\alpha} \ell(x) \tag{3.1}$$

³Among all 348 country-year observations, the first and second order statistics are equal in 2 cases for labor income and 7 cases for capital income. Therefore we conjecture that the top-coding issue is not severe.

for some slowly varying function ℓ .⁴ We are interested in estimating the Pareto exponent α for each country and year. For this purpose, we employ the [Hill \(1975\)](#) (maximum likelihood) estimator

$$\frac{1}{\hat{\alpha}(k)} := \frac{1}{k} \sum_{n=1}^k \log \left(\frac{X_{n:N}}{X_{k:N}} \right). \quad (3.2)$$

Here $X_{n:N}$ denotes the n -th largest order statistic from the sample $\{X_n\}_{n=1}^N$ and $k \in \{1, \dots, N\}$ denotes the number of tail observations used to estimate the Pareto exponent.⁵

When the population distribution is known to be exactly Pareto (so ℓ in (3.1) is zero below the minimum size $x_{\min}$ and constant above this threshold), it is well known that the Hill estimator for the full sample ($k = N$) is consistent, asymptotically normal, and asymptotically efficient because it is a maximum likelihood estimator. In practice, the CDF is not exactly Pareto and the researcher needs to select an appropriate value of k . For instance, if $F(x)$ satisfies

$$1 - F(x) = Cx^{-\alpha}(1 + Dx^{-\beta} + o(x^{-\beta}))$$

with some $\beta > 0$, then [Hall \(1982\)](#) shows that choosing $k = o(N^{2\beta/(2\beta+\alpha)})$ together with $k \rightarrow \infty$ as $N \rightarrow \infty$ is sufficient for consistency and asymptotic normality (see also [Embrechts et al., 2013](#)). Notice that this choice puts a bound on the growth rate of k .

Despite these asymptotic results, it is notoriously difficult to pick k optimally in finite samples ([Hall, 1990](#); [Resnick and Stărică, 1997](#); [Danielsson et al., 2001](#)). In practice, researchers often plot the Hill estimator (3.2) over a range of k to find a flat region or plot the log rank $\log 1, \dots, \log N$ against the log size $\log X_{1:N}, \dots, \log X_{N:N}$ to find a region that exhibits a straight line pattern and choose a size threshold to run the log-rank regression.⁶⁷ Unfortunately, this graphical approach is not feasible in our setting because [LIS](#) does not allow researchers to download the micro data for confidentiality concerns (researchers are required to submit their execution files to conduct statistical analyses) and there is little scope for exploratory graphical data analysis. To overcome this issue, we apply the recent work by [Danielsson et al. \(2016\)](#), who propose a quantile-based heuristic to choose k in finite samples, which seems to work well as illustrated by their extensive simulation studies. Their approach can

⁴A function $\ell : (0, \infty) \rightarrow \mathbb{R}$ is said to be *slowly varying (at infinity)* if it is nonzero for sufficiently large x and $\lim_{x \rightarrow \infty} \ell(tx)/\ell(x) = 1$ for each $t > 0$. See [Bingham et al. \(1987\)](#) for a comprehensive treatment of the theory of regular variation.

⁵We use the Hill estimator because it is most natural and popular. [Gomes and Guillou \(2015\)](#) review 13 commonly used estimators. [Fedotenkov \(2018\)](#) review more than 100.

⁶[Gabaix and Ibragimov \(2011\)](#) study the asymptotic behavior of log rank regression and show that the standard error is larger by a factor of $\sqrt{2}$ than the Hill estimator. However, they do not discuss how to select the threshold. In their empirical application, they consider the size distribution of population in U.S. metropolitan statistical areas, which are already far into the tail and hence the threshold selection is less of an issue. [Ibragimov and Ibragimov \(2018\)](#) apply the same methodology to Russian household income data and consider the top 5% and 10% thresholds.

⁷An alternative approach is to estimate a parametric distribution F that admits a Pareto upper tail by maximum likelihood using the entire sample. The double Pareto-lognormal distribution proposed by [Reed \(2003\)](#) and [Reed and Jorgensen \(2004\)](#) often performs best. See [Toda \(2012\)](#) for a horse race across several parametric distributions in the context of U.S. labor income.

be explained as follows. The theoretical top j/N -quantile can be estimated by $X_{j:N}$. On the other hand, assuming a Pareto distribution in the upper tail, we can also estimate the quantile by $(k/j)^{1/\hat{\alpha}(k)} X_{k:N}$, where k is the number of tail observations used to estimate the Pareto exponent. Hence, we would want the Kolmogorov-Smirnov distance (in the quantile domain)

$$\max_{j=1,\dots,k} \left| X_{j:N} - (k/j)^{1/\hat{\alpha}(k)} X_{k:N} \right| \quad (3.3)$$

to be small. [Danielsson et al. \(2016\)](#) propose to choose k according to the min-max criterion

$$k^* = \arg \min_{k=k_{\min},\dots,k_{\max}} \max_{j=1,\dots,k} \left| X_{j:N} - (k/j)^{1/\hat{\alpha}(k)} X_{k:N} \right|, \quad (3.4)$$

where $k_{\min}$ and $k_{\max}$ are thresholds set by the researcher to bound the minimum and maximum value of acceptable k . Below, we adopt this methodology to select the optimal k , where we set the bounds $k_{\min} = \lfloor 0.01N \rfloor$ and $k_{\max} = \lfloor 0.1N \rfloor$ so that we force the estimation sample to include at least the top 1% but no more than the top 10% observations. As a robustness check, we also consider setting $k^* = \lfloor 0.05N \rfloor$ (top 5% observations).

3.2 Capital and labor income Pareto exponents

We apply the automated estimation procedure described in Section 3.1 to estimate the capital and labor income Pareto exponents for all countries and years available in the [LIS](#) database. The database spans 51 countries across the years 1967–2016, with a total of 348 country-year observations. The point estimates of the capital and labor income Pareto exponents for each country and year as well as their standard errors can be found in Table 2 in Appendix B. For visibility, Figure 1 shows histograms and scatter plots of the capital and labor income Pareto exponents. The left and right panels show the results for the automatic procedure for threshold selection and the top 5% threshold, respectively.

The top panels of Figure 1 show the histograms of the capital and labor income Pareto exponents pooled across all available countries and years. The capital and labor income Pareto exponents are generally in the range 1–3 and 2–5, respectively. This suggests that (i) capital income is generally more unequally distributed than labor income, but (ii) there is significant heterogeneity in both capital and labor income inequality across countries and years. The bottom panels of Figure 1 show the scatter plots of the Pareto exponents together with the 45° degree line. We find that the correlation between the two exponents is weak. Furthermore, for the vast majority of countries and years, the capital income Pareto exponent is smaller than the labor exponent, again suggesting that capital income is more unequal than labor income.

How do the Pareto exponents evolve over time? Because countries appear only sporadically in the [LIS](#) database, we do as follows. First, we consider only the countries that appear in ten or more years in the database. There are eleven such countries, namely: Australia, Canada, Chile, Germany, Israel, Italy, Mexico, Spain, Taiwan, United Kingdom, and United States. Second, for each of these countries, we linearly interpolate the capital and labor Pareto exponents for all years from the available years. (We extrapolate by constants outside the range using the first and last observation year.) Finally, for each

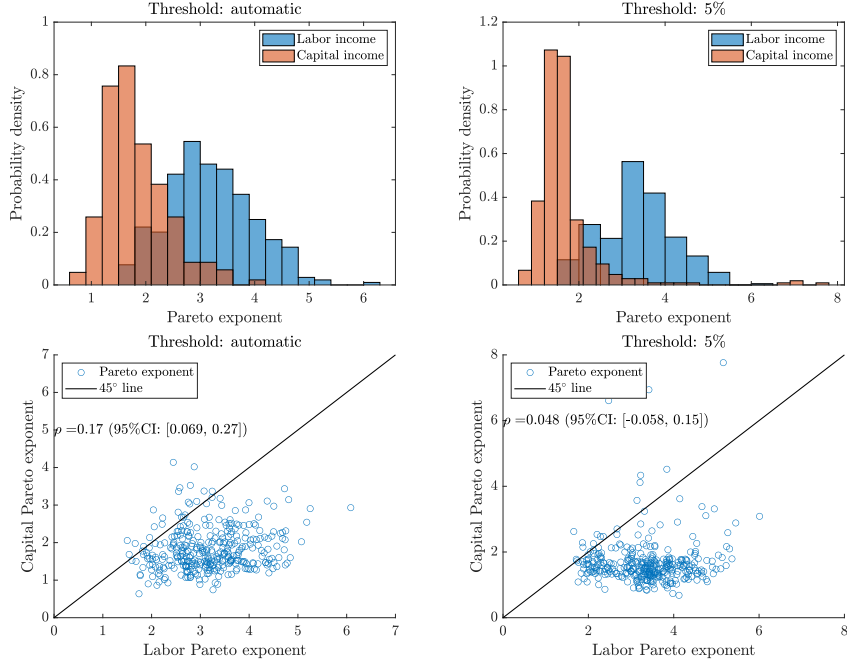


Figure 1: Histogram and scatter plot of capital and labor income Pareto exponents.

year we compute the median Pareto exponent across countries. Figure 2 shows the evolution of capital and labor income Pareto exponents over half a century (1967–2016). We observe that (i) the labor income Pareto exponent decreased from about 3.5 to around 3 over the decade of 1990–2000, but (ii) the capital income Pareto exponent has been stable at slightly below 2. Again, capital income appears to be more unequally distributed than the labor income.

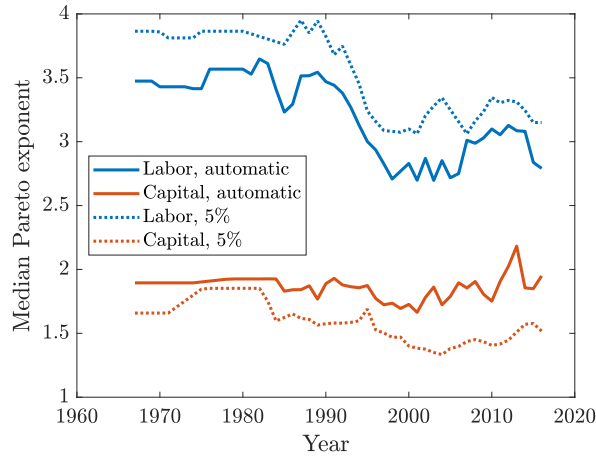


Figure 2: Time evolution of capital and labor income Pareto exponents.

3.3 Testing equality of capital and labor Pareto exponents

We now formally test whether the capital and labor Pareto exponents are equal. In particular our test is

$$H_0 : \alpha_{\text{lab}} = \alpha_{\text{cap}} \quad \text{against} \quad H_1 : \alpha_{\text{lab}} \neq \alpha_{\text{cap}},$$

where $\alpha_{\text{cap}}, \alpha_{\text{lab}}$ denote the capital and labor income Pareto exponents. Testing the null hypothesis H_0 is complicated by the fact that there is dependency between labor income $\{X_{\text{lab},n}\}_{n=1}^N$ and capital income $\{X_{\text{cap},n}\}_{n=1}^N$, because individuals who are rich (receive high labor income) tend to be wealthy and receive high capital income. Therefore, we apply the test recently developed by [Hoga \(2018\)](#), which allows for weak dependence in the data. The test is based upon the inverse of the Hill estimator [\(3.2\)](#), which we denote by $\hat{\gamma} := 1/\hat{\alpha}$. The test statistic is defined by

$$T_N = \frac{(\hat{\gamma}_{\text{lab}}(1) - \hat{\gamma}_{\text{cap}}(1))^2}{\int_{t_0}^1 t^2 [(\hat{\gamma}_{\text{lab}}(t) - \hat{\gamma}_{\text{cap}}(t)) - (\hat{\gamma}_{\text{lab}}(1) - \hat{\gamma}_{\text{cap}}(1))]^2 dt}, \quad (3.5)$$

where $t_0 \in (0, 1)$ is a tuning parameter and $\hat{\gamma}(t)$ is the inverse Hill estimator

$$\hat{\gamma}(t) := \frac{1}{[kt]} \sum_{n=1}^{[kt]} \log \left(\frac{X_{n:[Nt]}}{X_{[kt]:[Nt]}} \right). \quad (3.6)$$

Using the Hill estimator based on the subsample with only $[Nt]$ observations leads to self-normalization of the test statistic T_N and renders a test that is asymptotically pivotal. The limiting distribution is

$$T_N \xrightarrow{d} \frac{W(1)^2}{\int_{t_0}^1 [W(t) - tW(1)]^2 dt}, \quad (3.7)$$

where $W(t)$ is a standard Brownian motion. Note that since the test statistic [\(3.5\)](#) can be computed using only the Hill estimator and conducting numerical integration, there is no need to estimate the (potentially difficult) tail covariance. The tuning parameter t_0 affects the size of the test: high values of t_0 make the integral in [\(3.5\)](#) based on too few differences of $\hat{\gamma}$, whereas low values of t_0 yield volatile $\hat{\gamma}$ in [\(3.6\)](#) when t is close to t_0 . Both of these effects may cause size distortions. Therefore we set $t_0 = 0.2$ following the recommendation of [Hoga \(2018\)](#), who finds that this choice leads to favorable size properties. According to Table I of [Hoga \(2018\)](#), the 95 percentile of [\(3.7\)](#) for $t_0 = 0.2$ is 55.44, which we use as the critical value for testing H_0 at 5% significance level.

One issue with the test statistic [\(3.5\)](#) is that it requires the same number of tail observations k for both cross-sections of capital and labor income. Hence the automated selection procedure discussed in Section [3.1](#) is not applicable because in general we have $k_{\text{lab}}^* \neq k_{\text{cap}}^*$. Furthermore, these two numbers can be substantially different because our observations of capital income contain many zeros (many households do not hold liquid financial wealth and hence have no capital income). The resulting test may thus not be robust since the optimal k_{lab}^* is wildly different from k_{cap}^* . To overcome this issue, we only test the equality of Pareto exponents for countries that have more than 1,000 positive capital income observations and set $k = \lfloor 0.05N \rfloor$, where N is the number of positive

capital income observations. This sample selection results in 245 country-year observations out of 348. Table 3 in Appendix B shows the test results. Figure 3 shows the histogram of the logarithm of the test statistic (3.5).

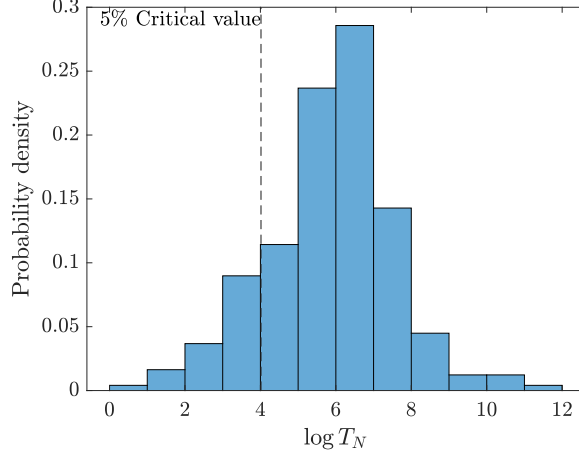


Figure 3: Histogram of logarithm of test statistic T_N in (3.5).

We reject the null hypothesis $H_0 : \alpha_{\text{lab}} = \alpha_{\text{cap}}$ in 208 country-year observations out of 245 (84.9%) that meet our sample selection criterion. In every single case of rejection, we have $\hat{\alpha}_{\text{lab}} > \hat{\alpha}_{\text{cap}}$, and therefore we formally confirm the observation in Section 3.2 that capital income is more unequally distributed than labor income.

4 A simple model of capital and labor Pareto exponents

Our empirical analysis in Section 3 suggests that (i) the capital income Pareto exponent is smaller than the labor one (i.e., capital income is more unequally distributed than labor income), and (ii) the correlation between capital and labor income Pareto exponents is weak. To explain these empirical findings, we present a simple dynamic model of consumption and savings.

4.1 Income fluctuation problem

Time is discrete and denoted by $t = 0, 1, 2, \dots$. Let a_t be the financial wealth of a typical agent at the beginning of period t including current income. The agent chooses consumption $c_t \geq 0$ and saves the remaining wealth $a_t - c_t$. The period utility function is u , the discount factor is $\beta > 0$, and the gross return on wealth and non-financial income in period t are denoted by $R_t, Y_t > 0$. Thus

the agent solves

$$\begin{aligned} \text{maximize} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \end{aligned} \quad (4.1a)$$

$$\text{subject to} \quad a_{t+1} = R_{t+1}(a_t - c_t) + Y_{t+1}, \quad (4.1b)$$

$$0 \leq c_t \leq a_t, \quad (4.1c)$$

where the initial wealth $a_0 = a > 0$ is given, (4.1b) is the budget constraint, and (4.1c) implies that the agent cannot borrow (which is without loss of generality according to the discussion in [Chamberlain and Wilson, 2000](#)). Throughout the rest of the paper we maintain the following assumptions.

Assumption 1 (CRRA utility). *The utility function exhibits constant relative risk aversion (CRRA) with coefficient $\gamma > 0$, so $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ if $\gamma \neq 1$ and $u(c) = \log c$ if $\gamma = 1$.*

Assumption 2 (I.i.d. shocks). *Let $G_{t+1} := Y_{t+1}/Y_t$ be the income growth. The sequence $\{R_{t+1}, G_{t+1}\}_{t=0}^{\infty}$ is independent and identically distributed (i.i.d.).*

These assumptions are similar to [Carroll \(2020\)](#), except that we allow for stochastic returns on savings. Note that the asset return R_{t+1} and income growth G_{t+1} are potentially mutually dependent. Due to the i.i.d. assumption 2, the state variables of the income fluctuation problem (4.1) are financial wealth $a_t > 0$ and current income $Y_t > 0$. Exploiting homotheticity (Assumption 1), we can reduce the number of state variables to just one, namely the wealth-income ratio (normalized wealth) $\tilde{a}_t := a_t/Y_t$. To see this, letting $\tilde{c}_t := c_t/Y_t$ be the consumption-income ratio (normalized consumption), dividing the borrowing constraint (4.1c) by Y_t , we obtain $0 \leq \tilde{c}_t \leq \tilde{a}_t$. Similarly, dividing the budget constraint (4.1b) by Y_{t+1} , we obtain

$$\begin{aligned} \tilde{a}_{t+1} &= a_{t+1}/Y_{t+1} = (R_{t+1}Y_t/Y_{t+1})(a_t/Y_t - c_t/Y_t) + 1 \\ &= (R_{t+1}/G_{t+1})(\tilde{a}_t - \tilde{c}_t) + 1 \\ &= \tilde{R}_{t+1}(\tilde{a}_t - \tilde{c}_t) + 1, \end{aligned} \quad (4.2)$$

where $\tilde{R}_{t+1} := R_{t+1}/G_{t+1}$ is the asset return relative to income growth. As for the utility function, since

$$c_t = Y_t \tilde{c}_t = Y_0 \left(\prod_{s=1}^t G_s \right) \tilde{c}_t,$$

(here we interpret $\prod_{s=1}^0 \bullet = 1$) assuming $Y_0 = 1$ (which is without loss of generality) and $\gamma \neq 1$, it follows from (4.1a) that

$$\begin{aligned} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) &= \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{s=1}^t \beta G_s^{1-\gamma} \right) \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} \\ &= \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{s=1}^t \tilde{\beta}_s \right) \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma}, \end{aligned} \quad (4.3)$$

where $\tilde{\beta}_t := \beta G_t^{1-\gamma}$. The discussion for $\gamma = 1$ is similar. Therefore the problem reduces to an income fluctuation problem with CRRA utility, random discount factors $\{\tilde{\beta}_t\}_{t=1}^{\infty}$, stochastic returns $\{\tilde{R}_t\}_{t=1}^{\infty}$ on wealth, and constant income ($\tilde{Y}_t \equiv 1$). The general theory of income fluctuation problems with stochastic discounting, returns, and income in a Markovian setting was developed by [Ma et al. \(2020\)](#). Therefore we immediately obtain the following result.

Proposition 1. *Suppose Assumptions 1, 2 hold and*

$$\beta \mathbb{E} G^{1-\gamma} < 1 \quad \text{and} \quad \beta \mathbb{E} R G^{-\gamma} < 1. \quad (4.4)$$

Then the income fluctuation problem (4.1) has a unique solution. The consumption function can be expressed as

$$c(a, Y) = Y \tilde{c}(a/Y),$$

where $\tilde{c} : (0, \infty) \rightarrow (0, \infty)$ is the consumption function of the detrended problem (maximizing (4.3) subject to (4.2)), which can be computed by policy function iteration.⁸

Proof. Applying Theorem 2.2 of [Ma et al. \(2020\)](#) to the i.i.d. case, a sufficient condition for the existence of a solution to the detrended problem is $\mathbb{E} \tilde{\beta} < 1$ and $\mathbb{E} \tilde{\beta} \tilde{R} < 1$, which is equivalent to (4.4). $\square$

4.2 Tail behavior of income and wealth

We now characterize the tail behavior of income and wealth in the context of the income fluctuation problem in Section 4.1.

To make the model stationary, suppose that agents survive to the next period with probability $v \in (0, 1)$ (perpetual youth model as in [Yaari, 1965](#)). Whenever agents die, they are replaced by newborn agents. For simplicity, assume that the discount factor β in (4.1a) already accounts for survival probability and that there is no market for life insurance (allowing for life insurance only changes R to R/v and is thus mathematically equivalent after reparametrization). Without loss of generality, suppose that newborn agents start with income $Y_0 = 1$. Then the income of a randomly selected agent is Y_T , where T is a geometric random variable with mean $\frac{1}{1-v}$. By the assumption on income growth, the log income of a randomly selected agent

$$\log Y_T = \log(Y_T/Y_0) = \sum_{t=1}^T \log G_t$$

is a geometric sum of i.i.d. random variables, for which we can characterize the tail behavior as follows.

Proposition 2 (Income Pareto exponent). *Suppose that $\mathbb{P}(G > 1) > 0$ and $1 < v \mathbb{E} G^z < \infty$ for some $z > 0$. Then the cross-sectional income distribution has a Pareto upper tail with exponent α_Y , which is the unique positive solution to*

$$v \mathbb{E} G^z = 1. \quad (4.5)$$

⁸See [Li and Stachurski \(2014\)](#) and [Ma et al. \(2020\)](#) for details on policy function iteration.

Proof. See [Beare and Toda \(2017, Theorem 3.4\)](#). $\square$

Characterizing the tail behavior of wealth is more difficult. We first note that the normalized consumption function $\tilde{c}$ in Proposition 1 is asymptotically linear.

Proposition 3 (Asymptotic linearity). *Let everything be as in Proposition 1. Then*

$$\lim_{a \rightarrow \infty} \frac{\tilde{c}(a)}{a} = \begin{cases} 1 - (\mathbb{E} \beta R^{1-\gamma})^{1/\gamma} & \text{if } \mathbb{E} \beta R^{1-\gamma} < 1, \\ 0 & \text{otherwise.} \end{cases} \quad (4.6)$$

Proof. The asymptotic linearity of $\tilde{c}$ follows from [Ma and Toda \(2020, Theorem 2.2\)](#). Noting that

$$\mathbb{E} \tilde{\beta} \tilde{R}^{1-\gamma} = \mathbb{E} \beta G^{1-\gamma} (R/G)^{1-\gamma} = \mathbb{E} \beta R^{1-\gamma},$$

the limit (4.6) follows from their Example 2.2. $\square$

Using Proposition 3 and setting $\rho = \min \{(\mathbb{E} \beta R^{1-\gamma})^{1/\gamma}, 1\}$, for high enough asset level, the detrended budget constraint (4.2) becomes approximately

$$\tilde{a}_{t+1} \approx \rho \tilde{R}_{t+1} \tilde{a}_t + 1,$$

which is a random growth process. Under specific assumptions, [Ma et al. \(2020, Theorem 3.3\)](#) prove that the upper tail of the stationary distribution of normalized wealth $\tilde{a}_t$ has a Pareto lower bound. Although a sharp characterization of the tail behavior is a difficult open problem, we proceed heuristically following the discussion of [Toda \(2019\)](#) and conjecture that the [Beare and Toda \(2017\)](#) formula is applicable in this setting. Hence we obtain the following result.

Claim. *Let $\rho = \min \{(\mathbb{E} \beta R^{1-\gamma})^{1/\gamma}, 1\}$ and $H = \rho \tilde{R}$. Suppose that $\mathbb{P}(H > 1) > 0$ and $1 < v \mathbb{E} H^z < \infty$ for some $z > 0$. Then the cross-sectional normalized wealth distribution has a Pareto upper tail with exponent $\tilde{\alpha}$, which is the unique positive solution of*

$$v \mathbb{E} H^z = 1. \quad (4.7)$$

The Pareto exponent for wealth and capital income is then $\alpha = \min \{\tilde{\alpha}, \alpha_Y\}$.

Proof. By accounting, capital income (excluding capital loss) is

$$Y_{\text{cap}} := \max \{R - 1, 0\} (a - c(a)) = \max \{R - 1, 0\} Y(\tilde{a} - \tilde{c}(\tilde{a})).$$

Using Proposition 3, this quantity is approximately equal to $\rho \max \{R - 1, 0\} Y \tilde{a}$. The claim $\alpha = \min \{\tilde{\alpha}, \alpha_Y\}$ then follows because asset return R is thin-tailed under the assumption $v \mathbb{E} H^z < \infty$ and $a_t = Y_t \tilde{a}_t$ is the product of two (potentially dependent) random variables with Pareto upper tails, which inherits the smallest Pareto exponent by the result in [Jessen and Mikosch \(2006\)](#). $\square$

Remark. The normalized wealth Pareto exponent $\tilde{\alpha}$, which solves (4.7), generally depends on both asset return R and income growth G because $H = \rho \tilde{R} = \rho R/G$. This does not contradict the result of [Stachurski and Toda \(2019\)](#) because income is stationary in their setting, whereas by Assumption 2 income exhibits random growth in our model.

We discuss an analytically solvable example to build intuition.

Example 1. Let $\Delta > 0$ be the length of time of one period and the discount factor be $\beta = e^{-\delta\Delta}$, where $\delta > 0$ is the discount rate. Suppose income grows at a constant rate $g > 0$, so $G = e^{g\Delta}$. Suppose asset return is risk-free, so $R = e^{r\Delta}$ with $r > 0$. Finally, let the survival probability be $v = e^{-\eta\Delta}$, where η is the death rate. Then (4.5) becomes

$$1 = e^{-\eta\Delta} e^{zg\Delta} \iff z = \eta/g,$$

so the income Pareto exponent is $\alpha_Y = \eta/g$. (This is the classical result of [Wold and Whittle \(1957\)](#) in discrete-time.) Suppose in addition that $-\eta + r(1 - \gamma) < 0$ so that $\beta R^{1-\gamma} < 1$. Since

$$H = (E \beta R^{1-\gamma})^{1/\gamma} \tilde{R} = (\beta R)^{1/\gamma} / G = e^{(\frac{r-\eta}{\gamma} - g)\Delta},$$

solving (4.7) the normalized wealth Pareto exponent is

$$\tilde{\alpha} = \frac{\eta\gamma}{r - \eta - g\gamma}$$

assuming $r - \eta - g\gamma > 0$. Therefore

$$\tilde{\alpha} < \alpha_Y \iff \frac{\eta\gamma}{r - \eta - g\gamma} < \frac{\eta}{g} \iff r > \eta + 2g\gamma,$$

so the wealth (hence capital income) Pareto exponent is smaller than the labor income Pareto exponent if the return on wealth r is sufficiently large. In summary, we obtain the following result: suppose $-\eta + r(1 - \gamma) < 0$ and let $\alpha_{\text{cap}}, \alpha_{\text{lab}}$ be the capital and labor income Pareto exponents. Then

$$\begin{cases} \alpha_{\text{cap}} = \alpha_{\text{lab}} = \frac{\eta}{g} & \text{if } r \leq \eta + 2g\gamma, \\ \alpha_{\text{cap}} = \frac{\eta\gamma}{r - \eta - g\gamma} < \frac{\eta}{g} = \alpha_{\text{lab}} & \text{if } r > \eta + 2g\gamma. \end{cases} \quad (4.8)$$

Note that the labor income Pareto exponent $\alpha_{\text{lab}} = \eta/g$ is highly sensitive to the income growth rate g . However, provided that $r > \eta + 2g\gamma$, the capital income Pareto exponent α_{cap} is not very sensitive to the value of g because the denominator is $r - \eta - g\gamma$. This example is consistent with our result in [Section 3.2](#) that the capital Pareto exponent is smaller than the labor Pareto exponent but the two values are only weakly related.

4.3 Numerical example

We further examine the tail behavior of income and wealth using a numerical example of the income fluctuation problem (4.1). Suppose that asset return is i.i.d. lognormal, so $\log R \sim N((\mu - \sigma^2/2)\Delta, \sigma^2\Delta)$, where $\Delta > 0$ is the length of one period, μ is the expected return, and σ is volatility. Suppose every period the agent is “promoted” with some probability, so the income growth rate is

$$G_{t+1} = Y_{t+1}/Y_t = \begin{cases} 1 & \text{with probability } 1 - p, \\ e^g & \text{with probability } p, \end{cases}$$

where $p \in (0, 1)$ is the promotion probability and g is the log income growth rate conditional on promotion. We parametrize the promotion probability as

$p = 1 - e^{-\Delta/L}$, where L is the expected length of time until a promotion. Using (4.5), the labor income Pareto exponent is determined such that

$$1 = v E G^{\alpha_Y} = v(1 - p + p e^{g\alpha_Y}) \iff \alpha_Y = \frac{1}{g} \log \frac{1 - v + vp}{vp}. \quad (4.9)$$

We set the parameter values as in Table 1. One unit of time corresponds to a year and one period is a quarter, so $\Delta = 1/4$. The preference parameters (discount rate and risk aversion) are standard. The death rate of $\eta = 0.025$ implies an average (economically active) age of $1/\eta = 40$ years. The expected return and volatility roughly correspond to the stock market. We set the labor income Pareto exponent to $\alpha_Y = 3$, which is roughly the median value in Figure 1. Using the survival probability $v = e^{-\eta\Delta}$ and (4.9), the implied value of income growth upon promotion is $g = 0.0403$. The wealth Pareto exponent determined by (4.7) is then $\alpha = 1.201$.

Table 1: Parameter values

Parameter	Symbol	Value
Length of one period	Δ	1/4
Discount rate	δ	0.04
Relative risk aversion	γ	2
Death rate	η	0.025
Expected return	μ	0.07
Volatility	σ	0.15
Expected time to promotion	L	5
Labor income Pareto exponent	α_Y	3

To numerically solve the income fluctuation problem (4.1), we discretize the log asset return $\log R$ using a 7-point Gauss-Hermite quadrature and apply policy function iteration (see Appendix A). After solving the individual problem, we apply the Pareto extrapolation algorithm developed in Gouin-Bonenfant and Toda (2018) to accurately compute the stationary (normalized) wealth distribution. Finally, we also simulate an economy with 10^4 agents. Figure 4 shows the results.

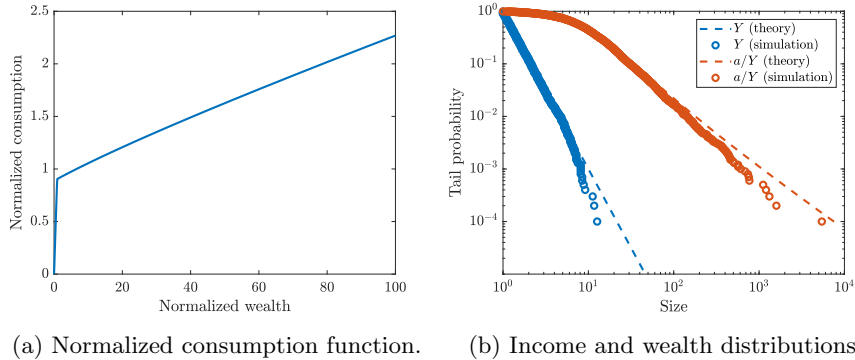


Figure 4: Solution to income fluctuation problem.

Figure 4a shows the normalized consumption function $\tilde{c}(\tilde{a})$ in the range $\tilde{a} \in [0, 100]$. Consistent with Proposition 3, the consumption function is roughly linear for high asset level. Figure 4b shows the size distributions of income Y normalized wealth $\tilde{a} = a/Y$ in a log-log plot, both from the theoretical model and the simulation. The fact that the tail probability $P(X > x)$ exhibits a straight line pattern in a log-log plot suggests that the size distributions have Pareto upper tails, consistent with theory. Furthermore, the slope for income is steeper than that of normalized wealth, so wealth (hence capital income) is more unequally distributed than labor income.

Finally, Figure 5 shows the income and wealth Pareto exponents when we change the income growth rate g in the range $g \in [0.02, 0.1]$, fixing other parameters. Because the income Pareto exponent is inversely proportional to income growth by (4.9), the labor income Pareto exponent is highly sensitive to income growth. On the other hand, the wealth (capital income) Pareto exponent does not depend much on income growth by the same intuition as in Example 1. Thus our model is consistent with our empirical findings in Section 3.2 that capital and labor income Pareto exponents are only weakly related.

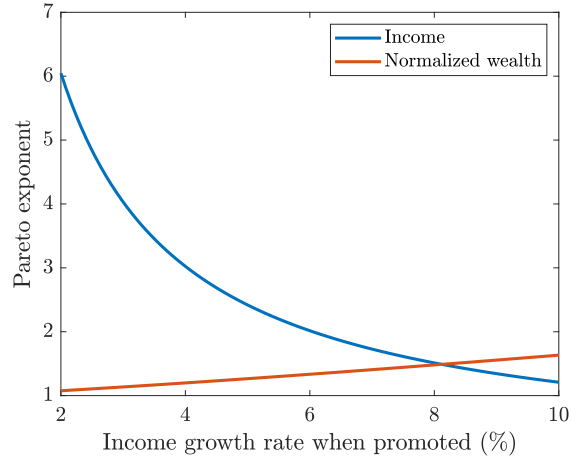


Figure 5: Dependence of income and wealth Pareto exponents on g .

4.4 Determinants of inequality

This section examines the relation between our estimated Pareto exponents α for capital and labor income and explanatory variables based on our model in Section 4.2.

Data We obtain regressors from two main sources: *Penn World Table*⁹ and *World Bank*.¹⁰ We construct the GDP growth rate series (g) based real GDP per capita expressed in current U.S. dollars. In addition, we use the number of deaths per 1,000 population as a proxy for the death rate (η). Finally, we use the internal rate of return (r) from the Penn World Table as a proxy for the return

⁹<https://www.rug.nl/ggdc/productivity/pwt/>

¹⁰<https://data.worldbank.org/>

on capital. Since our panel consisting of estimated α is highly unbalanced, we average all covariates over a five-year rolling window to smooth their effect over time.

Results We use a fixed effect panel regression approach to estimate our model, which allows the fixed effects to be correlated with the explanatory variables. This is important in our context, since there might be underlying forces specific to a country (e.g., culture) that correlate with economic growth, mortality rate, and other factors (see also [Acemoglu et al. \(2008\)](#) for a similar context). The regressions for labor and capital income are reported in (4.10a) and (4.10b), respectively. Standard errors (in parentheses) are heteroskedasticity-robust and clustered by country to account for possible dependency of the error terms within countries over time.

$$\hat{\alpha}_{\text{lab},it} = \underset{(0.746)}{2.475} + \underset{(1.182)}{1.623}g_{it} - \underset{(3.251)}{6.275}r_{it} + \underset{(0.067)}{0.112}\eta_{it} + \varepsilon_{it}, \quad (4.10a)$$

$$\hat{\alpha}_{\text{cap},it} = \underset{(0.894)}{1.514} + \underset{(0.768)}{0.949}g_{it} - \underset{(2.280)}{0.892}r_{it} + \underset{(0.071)}{0.028}\eta_{it} + \varepsilon_{it}. \quad (4.10b)$$

Since we set the average fixed effect to zero, the intercept can be interpreted as the mean Pareto exponent. Hence, the panel regression confirms our earlier finding that capital income is more unequal than labor income. Only the r and η coefficients are significant at 10% level and other coefficients (except the intercept) are insignificant. The fact that the labor income Pareto exponent α_{lab} is positively related with the death rate η in (4.10a) is consistent with the model, see for example (4.8). Although the g coefficient in (4.10a) is insignificant, it has the correct sign. The fact that α_{lab} is negatively related to the return r in (4.10a) is not predicted by theory.

Overall, the panel regression result is uninformative about the determinants of inequality. Although the coefficients have the correct signs, most of them are insignificant possibly due to the limited sample size. In addition, these regressions document only correlation and abstract from endogeneity problems. We view our results as predictive in nature and leave the causal inference for future research.

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A Solving the income fluctuation problem

In this appendix we discuss how to solve the detrended income fluctuation problem. After detrending, the problem becomes

$$\begin{aligned} & \text{maximize} && E_0 \sum_{t=0}^{\infty} \left(\prod_{s=1}^t \beta_s \right) \frac{c_t^{1-\gamma}}{1-\gamma} \\ & \text{subject to} && a_{t+1} = R_{t+1}(a_t - c_t) + 1, \\ & && 0 \leq c_t \leq a_t, \end{aligned}$$

where $\{R_t, \beta_t\}_{t=1}^{\infty}$ is i.i.d. (though R_t and β_t are generally correlated.) According to [Ma et al. \(2020\)](#), the Euler equation is

$$c_t^{-\gamma} = \max \{ E_t \beta_{t+1} R_{t+1} c_{t+1}^{-\gamma}, a_t^{-\gamma} \}. \quad (\text{A.1})$$

Let $c(a)$ be the consumption function. Taking the $-1/\gamma$ -th power of (A.1), we obtain

$$c(a) = \min \left\{ [E \beta R c(a')^{-\gamma}]^{-1/\gamma}, a \right\}, \quad (\text{A.2})$$

where $a' = R(a - c(a)) + 1$. Therefore we can compute the consumption function using the following variant of the policy function iteration algorithm:

1. Initialize the consumption function $c(a)$. For example, we can set $c(a) = \min \{a, 1 + ma\}$, where $m = \max \{1 - (E \beta R^{1-\gamma})^{1/\gamma}, 0\}$ is the theoretical asymptotic marginal propensity to consume according to (4.6).
2. Update $c(a)$ by the right-hand side of (A.2), where $a' = R(a - c(a)) + 1$.
3. Iterate the above step until $c(a)$ converges.

While the above algorithm has no guarantee to converge unlike the “true” policy function iteration algorithm discussed in [Ma et al. \(2020\)](#), it has the advantage of avoiding root-finding and hence it is fast.

In Section 4, we use this algorithm on a 100-point exponential grid for normalized wealth $\tilde{a}$ that spans $[0, 10^4]$, with a median grid point of 10. The details on the exponential grid are discussed in [Gouin-Bonenfant and Toda \(2018\)](#).

B Tables

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	k	N	$\hat{\alpha}$	s.e.	k	N
Australia	1981	5.25	0.24	463	10568	2.9	0.3	96	9488
Australia	1985	3.23	0.41	61	5492	1.79	0.2	81	5232
Australia	1989	3.7	0.12	1034	10629	1.98	0.19	111	10973
Australia	1995	2.92	0.29	99	4574	1.82	0.15	153	3364
Australia	2001	3.62	0.53	46	4510	1.52	0.13	132	3105
Australia	2003	2.66	0.29	86	6741	1.86	0.17	122	4473
Australia	2004	3.49	0.13	739	7893	1.72	0.14	162	6145
Australia	2008	2.99	0.15	411	6666	1.41	0.12	141	4749
Australia	2010	3.58	0.31	137	10949	1.44	0.08	296	9030
Australia	2014	2.79	0.23	146	9788	1.29	0.07	365	9569
Austria	1994	3.46	0.25	197	2183	1.52	0.13	131	1693
Austria	1997	3.55	0.41	74	1998	1.84	0.14	173	1787
Austria	2000	3.08	0.33	88	1584	3.37	0.72	22	1547
Austria	2004	3.57	0.31	134	3749	1.73	0.26	45	4064
Austria	2007	3.3	0.2	269	4101	1.51	0.22	48	4334
Austria	2010	3.61	0.35	106	4323	1.54	0.19	65	4752
Austria	2013	3.25	0.17	378	4022	1.38	0.18	60	4471
Belgium	1985	4.81	0.7	47	4357	3.14	0.64	24	397
Belgium	1988	3.39	0.64	28	2518	1.58	0.37	18	332
Belgium	1992	4.69	0.33	202	2581	2.11	0.36	35	349
Belgium	1995	2.93	0.43	46	1790	3.1	0.42	54	893
Belgium	1997	4	0.24	288	2906	3.06	0.44	48	4112
Belgium	2000	1.93	0.5	15	1431	0.9	0.32	8	715
Brazil	2006	2.77	0.08	1122	99559	2.16	0.2	119	9594
Brazil	2009	2.35	0.05	2045	100002	1.4	0.04	1081	19059
Brazil	2011	2.68	0.09	947	89066	1.93	0.14	198	19291
Brazil	2013	2.74	0.09	955	91657	1.77	0.1	340	21092
Canada	1971	4	0.11	1239	20386	2.48	0.22	129	9970
Canada	1975	4.13	0.22	344	21750	2.47	0.21	134	10717
Canada	1981	5.18	0.41	158	12331	2.54	0.28	85	8342
Canada	1987	4.5	0.48	89	8667	2.29	0.21	118	5190
Canada	1991	3.59	0.29	154	15257	2.51	0.17	223	9847
Canada	1994	3.56	0.21	283	28155	2.31	0.19	154	14077
Canada	1997	3.75	0.08	2385	25018	1.53	0.05	1117	11273
Canada	1998	2.71	0.17	241	23969	1.74	0.15	143	13486
Canada	2000	2.89	0.19	225	22298	1.53	0.06	624	13264
Canada	2004	2.85	0.18	239	21594	2.05	0.13	244	11861
Canada	2007	2.45	0.15	260	20918	1.96	0.16	150	12533
Canada	2010	3.32	0.08	1866	19500	2.3	0.19	149	10341
Canada	2013	4.1	0.29	194	18069	2.5	0.25	102	9741
Chile	1990	2.41	0.12	380	22755	1.39	0.14	100	1242
Chile	1992	2.39	0.12	369	31934	1.86	0.31	37	1678
Chile	1994	1.81	0.03	3985	39851	1.78	0.13	199	2071

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	k	N	$\hat{\alpha}$	s.e.	k	N
Chile	1996	2.71	0.15	323	30177	1.35	0.21	40	3437
Chile	1998	1.96	0.04	2317	42903	1.4	0.09	254	3335
Chile	2000	2.17	0.07	856	57716	1.27	0.12	115	4432
Chile	2003	2.08	0.05	1463	60055	1.35	0.08	274	4637
Chile	2006	2.46	0.09	815	65111	1.32	0.06	493	5374
Chile	2009	2.73	0.11	642	59668	2.29	0.34	45	3619
Chile	2011	2.82	0.11	669	50887	1.91	0.27	49	3312
Chile	2013	2.59	0.11	583	57079	2.2	0.29	56	3635
Chile	2015	2.73	0.09	927	69790	1.51	0.1	214	5314
China	2002	3.58	0.26	186	16745	1.61	0.11	198	2156
China	2013	2.78	0.07	1588	15878	1.61	0.06	825	10098
Colombia	2004	1.91	0.07	779	8255	1.64	0.17	99	1196
Colombia	2007	1.92	0.05	1357	14298	1.57	0.15	103	1961
Colombia	2010	2.26	0.08	824	14705	1.3	0.09	204	2037
Colombia	2013	2.83	0.22	161	13986	1.38	0.11	172	1983
Czech Rep.	1992	2.94	0.23	168	13287	1.13	0.2	33	444
Czech Rep.	1996	3.05	0.19	270	23587	1.79	0.27	43	3077
Czech Rep.	2002	3.17	0.14	502	6082	2.01	0.35	33	973
Czech Rep.	2004	2.38	0.42	32	3125	1.52	0.51	9	729
Czech Rep.	2007	3.33	0.12	823	8254	1.27	0.17	55	1735
Czech Rep.	2010	3.64	0.17	463	6551	2.29	0.5	21	1422
Czech Rep.	2013	3.18	0.13	597	5996	1.34	0.17	66	1198
Denmark	1987	4.18	0.29	214	9199	1.44	0.05	852	11575
Denmark	1992	3.75	0.38	97	9164	1.3	0.05	818	11853
Denmark	1995	4.41	0.06	5719	57306	1.19	0.02	4704	70200
Denmark	2000	4.11	0.05	5879	59549	1.3	0.03	2092	69714
Denmark	2004	3.46	0.14	624	59824	1.4	0.03	1853	67886
Denmark	2007	3.19	0.13	614	61255	1.26	0.02	2575	78755
Denmark	2010	2.6	0.11	596	59488	1.13	0.02	3900	69287
Denmark	2013	2.97	0.12	614	60880	1.29	0.03	2012	68088
Dominican Rep.	2007	2.9	0.31	85	7471	1.18	0.27	19	544
Egypt	2012	1.82	0.09	380	10095	1.34	0.17	65	727
Estonia	2000	3.26	0.26	162	5196	0.74	0.37	4	206
Estonia	2004	3.72	0.36	104	3116	2.85	0.82	12	238
Estonia	2007	3.91	0.24	271	4201	1.49	0.31	23	2250
Estonia	2010	4.51	0.41	122	4241	1.47	0.32	21	1948
Estonia	2013	4.37	0.58	57	4951	1.4	0.32	19	1450
Finland	1987	3.94	0.12	1120	11225	1.44	0.06	627	7906
Finland	1991	4.41	0.36	150	10380	1.55	0.12	174	8332
Finland	1995	4.02	0.42	92	7867	1.28	0.19	47	4236
Finland	2000	2.7	0.28	91	8977	1.07	0.06	314	5923
Finland	2004	3.92	0.39	100	9368	1.31	0.11	154	8061
Finland	2007	3.61	0.15	570	8664	1.46	0.15	93	8502
Finland	2010	3.64	0.13	739	7537	1.16	0.09	152	7571
Finland	2013	3.73	0.15	601	8899	1.25	0.08	257	9145

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	k	N	$\hat{\alpha}$	s.e.	k	N
France	1978	2.53	0.24	113	7664	1.63	0.17	91	1426
France	1984	4.72	0.51	84	7875	1.81	0.19	90	1974
France	1989	3.64	0.37	95	5942	1.55	0.14	125	1677
France	1994	2.99	0.11	738	7880	1.92	0.15	162	8734
France	2000	3.88	0.44	79	7112	1.48	0.06	556	8530
France	2005	3.24	0.15	495	7987	1.75	0.14	147	8330
France	2010	2.55	0.23	119	11820	2.3	0.21	124	11660
Germany	1973	3.43	0.09	1470	37614	2.23	0.03	4459	44891
Germany	1978	2.52	0.13	392	35731	2.3	0.03	4348	44654
Germany	1981	3.51	0.73	23	2024	2.54	0.73	12	163
Germany	1983	3.79	0.2	377	32288	2.47	0.12	458	40130
Germany	1984	2.62	0.4	43	4240	1.79	0.27	45	4259
Germany	1987	3.63	0.57	40	3863	1.52	0.21	52	4050
Germany	1989	2.68	0.43	39	3733	1.54	0.18	74	3940
Germany	1991	4.08	0.18	508	5448	1.46	0.14	117	5789
Germany	1994	3.35	0.43	62	5384	1.55	0.14	118	5758
Germany	1995	3.96	0.17	514	5275	1.3	0.07	361	5802
Germany	1998	4.27	0.18	551	5531	1.64	0.16	102	6136
Germany	2000	3.42	0.36	89	8784	1.75	0.13	177	9829
Germany	2001	2.7	0.28	95	9273	1.38	0.08	281	10174
Germany	2002	3.37	0.11	882	8850	1.34	0.06	436	9798
Germany	2003	2.79	0.29	91	8600	1.07	0.03	935	9875
Germany	2004	3.38	0.12	825	8300	1.1	0.04	917	9544
Germany	2005	2.74	0.29	90	8869	1.24	0.05	516	10338
Germany	2006	3.15	0.11	827	8276	1.47	0.1	238	9562
Germany	2007	3.14	0.35	79	7778	1.17	0.05	608	9055
Germany	2008	2.61	0.28	89	7295	1.51	0.15	103	8497
Germany	2009	3.35	0.21	258	10611	1.58	0.09	288	11102
Germany	2010	3.16	0.09	1239	12512	1.54	0.08	343	12913
Germany	2011	2.83	0.25	124	12291	1.5	0.11	192	12664
Germany	2012	2.92	0.25	140	13912	1.41	0.08	312	13051
Germany	2013	2.7	0.24	130	12199	1.38	0.07	437	11824
Germany	2014	3.22	0.29	124	12193	1.51	0.13	144	11176
Germany	2015	2.54	0.24	114	11000	1.49	0.14	110	10625
Georgia	2010	2.62	0.15	305	4867	1.5	0.75	4	170
Georgia	2013	2.61	0.18	217	2512	2.68	1.2	5	79
Georgia	2016	2.88	0.22	178	2390	1.24	0.44	8	81
Greece	1995	3.13	0.27	138	3375	3.04	0.51	36	1033
Greece	2000	3.46	0.29	146	2580	2.12	0.29	55	724
Greece	2004	3.47	0.54	41	3987	1.99	0.34	34	1006
Greece	2007	2.65	0.31	75	4533	2.68	0.65	17	1253
Greece	2010	2.74	0.24	128	3858	2.54	0.57	20	1170
Greece	2013	2.75	0.14	383	5015	3.08	0.6	26	1517
Guatemala	2006	1.89	0.14	173	13448	1.74	0.31	32	668
Guatemala	2011	1.66	0.15	127	12622	1.98	0.31	41	438

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	k	N	$\hat{\alpha}$	s.e.	k	N
Guatemala	2014	1.84	0.17	113	11203	1.9	0.35	29	392
Hungary	1991	3.1	0.69	20	1693	1.03	0.46	5	56
Hungary	1994	2.52	0.21	144	1532	2.18	1.54	2	85
Hungary	1999	3.48	0.48	53	1531	1.68	0.25	46	666
Hungary	2005	2.72	0.23	138	1384	1.63	0.54	9	94
Hungary	2007	2.87	0.25	133	1328	4.02	2.84	2	68
Hungary	2009	3.61	0.44	66	1250	2.61	1.31	4	41
Hungary	2012	2.73	0.4	46	1343	2.29	1.32	3	49
Hungary	2015	3.29	0.26	156	1809	1.25	0.47	7	125
Iceland	2004	4.49	0.52	75	2632	1.31	0.21	39	1836
Iceland	2007	3.73	0.25	216	2583	1.44	0.21	46	2112
Iceland	2010	4.15	0.54	59	2675	1.27	0.08	241	2994
India	2004	2.18	0.04	3783	39832	1.94	0.14	184	1874
India	2011	2.16	0.05	1818	39958	1.36	0.13	104	2011
Ireland	1987	2.53	0.46	30	2482	1.61	0.18	76	1998
Ireland	1994	2.17	0.34	40	2426	1.13	0.11	105	1113
Ireland	1995	1.96	0.42	22	2146	1.58	0.28	32	1025
Ireland	1996	1.76	0.38	22	2021	1.15	0.12	91	1022
Ireland	2000	3.19	0.34	90	1893	2.46	0.43	33	1017
Ireland	2004	2.22	0.34	43	3728	1.56	0.28	31	857
Ireland	2007	2.24	0.39	33	3137	1.34	0.21	40	1510
Ireland	2010	3.11	0.2	250	2511	2.46	0.78	10	766
Israel	1979	3.41	0.53	42	1941	1.24	0.27	21	365
Israel	1986	3.09	0.16	353	3942	1.5	0.39	15	420
Israel	1992	3.38	0.18	349	4057	2.61	1.07	6	449
Israel	1997	2.83	0.16	324	4072	1.44	0.21	48	485
Israel	2001	2.33	0.35	45	4382	0.93	0.21	19	556
Israel	2005	3.14	0.15	452	4789	1.26	0.15	71	1475
Israel	2007	4.34	0.61	50	4682	1.47	0.24	39	970
Israel	2010	2.28	0.33	48	4746	1.07	0.32	11	980
Israel	2012	3.13	0.12	678	6854	1.41	0.25	33	1305
Israel	2014	4.61	0.55	71	6607	1.6	0.19	71	1330
Israel	2016	4.23	0.5	71	6968	2.11	0.2	112	1215
Italy	1986	2.59	0.33	62	6016	1.93	0.19	99	3474
Italy	1987	3.64	0.26	198	6045	2.52	0.3	69	6844
Italy	1989	3.55	0.38	85	6154	1.77	0.18	102	5865
Italy	1991	3.39	0.44	59	5827	2.01	0.24	69	6587
Italy	1993	3.46	0.16	464	5444	1.86	0.16	132	6724
Italy	1995	2.8	0.3	88	5487	2.27	0.26	75	6811
Italy	1998	2.71	0.36	57	4876	1.92	0.24	63	6191
Italy	2000	2.83	0.2	203	5279	1.98	0.24	66	6516
Italy	2004	2.07	0.17	146	4913	1.98	0.21	87	6641
Italy	2008	3.13	0.2	248	4778	1.81	0.22	70	6552
Italy	2010	3.1	0.16	374	4719	1.59	0.14	124	6490
Italy	2014	3.08	0.25	157	4401	1.75	0.13	182	6748

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	k	N	$\hat{\alpha}$	s.e.	k	N
Côte d'Ivoire	2002	2.02	0.19	115	9811	1.66	0.31	29	618
Côte d'Ivoire	2008	1.74	0.07	584	11594	0.64	0.23	8	621
Côte d'Ivoire	2015	1.77	0.05	1127	11293	1.12	0.26	19	664
Japan	2008	3.25	0.2	270	3256	2.02	0.49	17	598
Lithuania	2010	4.74	0.36	169	3640	1.4	0.27	28	564
Lithuania	2013	4.64	0.71	43	3785	2.25	0.85	7	523
Luxembourg	1985	5.07	0.46	120	1552	2.02	0.46	19	199
Luxembourg	1991	4.78	0.48	101	1535	1.45	0.21	49	716
Luxembourg	1994	4.37	0.5	76	1362	3.11	0.6	27	531
Luxembourg	1997	4.57	0.35	169	1906	1.91	0.58	11	779
Luxembourg	2000	3.84	0.66	34	1840	1.85	0.41	20	892
Luxembourg	2004	3.44	0.21	271	2753	2.99	0.43	49	887
Luxembourg	2007	2.78	0.16	302	3028	2.49	0.41	37	2066
Luxembourg	2010	3.18	0.28	129	4111	2.68	0.45	36	3494
Luxembourg	2013	3.2	0.49	43	2828	1.42	0.1	215	2534
Mexico	1984	2.27	0.31	54	4461	1.99	0.45	20	297
Mexico	1989	2.14	0.08	709	10876	1.17	0.15	60	602
Mexico	1992	1.95	0.06	986	9959	1.86	0.33	31	414
Mexico	1994	2.04	0.06	1139	12068	2.87	1.17	6	450
Mexico	1996	2.02	0.06	1311	13128	1.42	0.22	40	505
Mexico	1998	2.05	0.07	834	10092	2.42	0.86	8	393
Mexico	2000	2.51	0.23	115	9262	1.73	0.45	15	348
Mexico	2002	3.24	0.21	239	15905	3.2	1.01	10	587
Mexico	2004	2.15	0.05	1989	20928	0.95	0.27	12	1055
Mexico	2008	2.08	0.04	2622	26414	1.61	0.23	49	1635
Mexico	2010	2.64	0.12	471	24218	1.75	0.39	20	1323
Mexico	2012	2.54	0.17	219	8032	2.81	0.64	19	433
Netherlands	1983	4.51	0.8	32	3090	2.97	0.9	11	182
Netherlands	1987	3.92	0.73	29	2792	2.12	0.5	18	231
Netherlands	1990	3.13	0.54	34	2971	1.78	0.33	30	2570
Netherlands	1993	3.62	0.56	42	3565	1.5	0.16	92	2423
Netherlands	1999	3.41	0.6	32	3061	2.02	0.45	20	1942
Netherlands	2004	3.17	0.29	116	7166	1.58	0.13	142	8171
Netherlands	2007	2.98	0.26	135	8011	1.01	0.04	544	9136
Netherlands	2010	3.27	0.27	145	8001	1.4	0.07	395	9539
Netherlands	2013	3.1	0.31	101	7808	1.33	0.06	547	9486
Norway	1979	1.5	0.05	781	8276	2.06	0.17	141	6048
Norway	1986	4.75	0.23	418	4254	2.31	0.31	57	2588
Norway	1991	3.8	0.2	359	7361	1.8	0.1	346	7448
Norway	1995	3.17	0.28	131	8027	1.18	0.11	124	9800
Norway	2000	2.74	0.25	116	11474	1.08	0.08	161	12804
Norway	2004	3.3	0.28	141	10947	0.93	0.06	251	12979
Norway	2007	3.1	0.08	1701	169708	1.37	0.03	2156	215363
Norway	2010	3.36	0.05	4809	177229	0.89	0.01	16510	225584
Norway	2013	2.99	0.05	3693	183146	0.89	0.01	22423	233192

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	k	N	$\hat{\alpha}$	s.e.	k	N
Panama	2007	2.73	0.23	146	11693	1.21	0.16	56	566
Panama	2010	2.23	0.09	619	11855	2.66	1	7	525
Panama	2013	2.95	0.24	145	10577	2.16	0.46	22	337
Paraguay	2000	1.68	0.06	689	7685	1.5	0.21	51	585
Paraguay	2004	1.83	0.07	709	7434	1.68	0.34	25	343
Paraguay	2007	1.6	0.15	119	4528	1.52	0.3	26	263
Paraguay	2010	1.54	0.2	62	4697	1.69	0.47	13	261
Paraguay	2013	2.1	0.17	152	5073	1.55	0.47	11	318
Paraguay	2016	2.02	0.12	298	9470	1.56	0.24	43	468
Peru	2004	2.51	0.16	256	17289	1.66	0.12	177	2193
Peru	2007	3	0.19	250	20464	1.48	0.1	242	2571
Peru	2010	2.62	0.19	199	19720	1.88	0.24	61	3137
Peru	2013	3.3	0.2	282	27670	2.12	0.24	80	3827
Poland	1986	3.57	0.26	186	9213	NaN	NaN	NaN	NaN
Poland	1992	3.57	0.34	111	5944	1.14	0.2	34	456
Poland	1995	2.26	0.15	226	21091	1.63	0.34	23	230
Poland	1999	3.09	0.06	2418	24216	2.62	0.5	27	284
Poland	2004	2.72	0.06	2216	23557	1.36	0.28	24	305
Poland	2007	2.65	0.16	287	27872	1.39	0.24	33	340
Poland	2010	2.86	0.05	2761	27640	1.72	0.38	21	366
Poland	2013	2.75	0.05	2643	26475	1.49	0.26	33	422
Poland	2016	2.93	0.07	1905	25983	2.39	0.47	26	383
Romania	1995	2.95	0.15	372	31519	1.5	0.33	21	325
Romania	1997	3.05	0.17	322	32122	1.74	0.29	36	374
Russia	2000	2.11	0.13	253	2631	0.86	0.5	3	55
Russia	2004	3.88	0.43	83	2705	1.5	1.06	2	66
Russia	2007	4.03	0.61	44	2986	2.07	1.19	3	81
Russia	2010	3.41	0.23	218	5025	3.18	1.06	9	107
Russia	2011	4.19	0.3	191	8697	2.68	0.63	18	513
Russia	2013	3.93	0.15	726	38297	2.87	0.37	61	4360
Russia	2014	4.32	0.16	735	38433	2.15	0.17	153	4412
Russia	2015	4.31	0.12	1272	50779	1.73	0.1	301	5752
Russia	2016	4.73	0.12	1554	134781	3.44	0.17	400	15716
Serbia	2006	3.53	0.33	113	3491	2.46	1.74	2	108
Serbia	2010	2.76	0.45	38	3270	2.19	0.77	8	107
Serbia	2013	1.96	0.34	33	3201	2.13	0.71	9	86
Serbia	2016	3.24	0.2	276	4566	3.37	0.87	15	146
Slovakia	1992	3.28	0.31	115	11439	1.31	0.31	18	177
Slovakia	1996	4.09	0.11	1318	13176	1.44	0.22	44	709
Slovakia	2004	2.59	0.28	86	3987	1.64	0.47	12	319
Slovakia	2007	3.89	0.53	54	4538	2.46	0.68	13	653
Slovakia	2010	3.49	0.18	393	4372	1.71	0.36	23	1024
Slovakia	2013	2.47	0.36	48	4527	2.17	0.53	17	1289
Slovenia	1997	4.84	0.37	174	2378	1.6	0.31	27	376
Slovenia	1999	4.34	0.23	342	3492	1.47	0.26	32	601

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	k	N	$\hat{\alpha}$	s.e.	k	N
Slovenia	2004	4.47	0.25	326	3275	2.17	0.4	29	521
Slovenia	2007	4.47	0.27	267	3219	1.58	0.23	48	501
Slovenia	2010	4.68	0.26	323	3468	2.27	0.8	8	493
Slovenia	2012	4.57	0.28	263	3252	1.36	0.11	150	1841
South Africa	2008	2.42	0.33	54	4866	2.15	0.62	12	198
South Africa	2010	2.55	0.26	98	3859	3.36	2.38	2	93
South Africa	2012	2.02	0.11	328	5112	1.09	0.45	6	154
Rep. of Korea	2006	3.71	0.1	1438	14390	2.37	0.32	55	4353
Rep. of Korea	2008	3.62	0.1	1196	12542	2.23	0.34	42	3616
Rep. of Korea	2010	3.65	0.11	1196	12276	1.94	0.27	50	3003
Rep. of Korea	2012	3.74	0.11	1112	11911	1.66	0.15	119	3329
Spain	1980	3.57	0.08	1859	19651	1.64	0.15	123	1470
Spain	1985	3.37	0.23	216	2707	1.7	0.37	21	344
Spain	1990	3.76	0.09	1591	16178	2.05	0.23	77	4036
Spain	1995	3.34	0.33	101	4088	1.62	0.12	171	2249
Spain	2000	3.15	0.2	245	3379	1.6	0.2	63	4206
Spain	2004	3.78	0.13	803	9175	1.74	0.23	58	4433
Spain	2007	4.74	0.43	119	9701	2.08	0.26	64	5327
Spain	2010	6.08	0.49	152	8822	2.93	0.5	35	3043
Spain	2013	4.22	0.33	165	8823	2.32	0.28	68	6681
Spain	2016	3.61	0.24	223	10165	2	0.2	102	7451
Sweden	1967	3.45	0.35	95	4912	1.51	0.15	99	988
Sweden	1975	3.77	0.14	709	9535	1.67	0.18	82	7845
Sweden	1981	4.15	0.15	726	8635	2.08	0.09	582	6092
Sweden	1987	4.39	0.16	752	8658	1.45	0.16	81	7953
Sweden	1992	4.25	0.16	694	10824	1.79	0.05	1072	11215
Sweden	1995	3.78	0.11	1130	11655	1.53	0.05	959	12993
Sweden	2000	2.79	0.27	108	10319	1.08	0.03	1041	11273
Sweden	2005	2.26	0.2	127	11951	1.02	0.03	1161	12120
Switzerland	1982	2	0.16	165	5570	1.27	0.07	330	4233
Switzerland	1992	3.21	0.31	110	4486	1.45	0.11	179	4846
Switzerland	2002	2.72	0.38	50	3140	1.24	0.08	245	2646
Switzerland	2000	2.64	0.47	32	3074	1.7	0.2	75	2578
Switzerland	2004	3.99	0.5	65	2697	1.3	0.09	203	2648
Switzerland	2007	2.22	0.23	93	5424	2.49	0.31	63	5057
Switzerland	2010	2.42	0.31	59	5759	1.83	0.22	71	6749
Switzerland	2013	3.6	0.3	140	5097	2.39	0.29	69	6190
Taiwan	1981	3.86	0.31	160	14997	1.89	0.18	107	10635
Taiwan	1986	3.23	0.26	160	15911	1.98	0.15	165	12864
Taiwan	1991	3.84	0.1	1546	15756	1.5	0.06	577	14978
Taiwan	1995	4.07	0.11	1302	13619	1.86	0.09	431	14184
Taiwan	1997	4.39	0.22	385	12572	2.11	0.14	222	13311
Taiwan	2000	3.74	0.11	1213	12301	2.49	0.21	135	13396
Taiwan	2005	4.08	0.16	689	11903	2.1	0.13	246	13127
Taiwan	2007	3.93	0.28	192	12042	2.26	0.19	137	13290

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	k	N	$\hat{\alpha}$	s.e.	k	N
Taiwan	2010	3.74	0.13	840	12748	2.17	0.17	159	14287
Taiwan	2013	3.68	0.13	841	13300	1.65	0.07	510	15615
Taiwan	2016	3.59	0.1	1353	13648	1.95	0.12	250	16449
United Kingdom	1969	3.47	0.45	59	5669	1.33	0.14	90	4102
United Kingdom	1974	3.05	0.42	53	5181	1.84	0.25	53	3547
United Kingdom	1979	4.39	0.2	485	4937	1.82	0.2	82	4280
United Kingdom	1986	4.02	0.19	438	4515	1.96	0.2	96	4942
United Kingdom	1991	2.76	0.34	66	4445	1.37	0.06	480	5174
United Kingdom	1994	2.51	0.18	203	15751	1.86	0.12	232	18563
United Kingdom	1995	3	0.38	63	4091	1.87	0.27	47	4584
United Kingdom	1999	2.44	0.15	250	15200	1.93	0.15	173	17223
United Kingdom	2004	2.23	0.17	171	17026	1.68	0.12	190	18531
United Kingdom	2007	3.01	0.22	180	15272	2.01	0.15	173	16371
United Kingdom	2010	2.98	0.24	156	15237	1.71	0.16	121	9839
United Kingdom	2013	3.2	0.25	170	12258	2.18	0.25	78	7714
United Kingdom	2016	2.66	0.22	145	11768	1.5	0.11	177	7340
United States	1974	3.92	0.17	514	9366	1.89	0.25	59	5558
United States	1979	4.18	0.18	564	52783	1.93	0.08	527	44749
United States	1986	3.5	0.07	2304	45660	2.02	0.1	401	38274
United States	1991	3.57	0.17	466	46199	2.16	0.1	433	38875
United States	1994	2.63	0.04	3614	44261	2.1	0.11	387	37355
United States	1997	2.62	0.04	3805	39812	2.32	0.13	313	30913
United States	2000	2.59	0.03	6343	64575	2.43	0.11	482	46961
United States	2004	2.65	0.04	3590	62334	2.45	0.12	434	42354
United States	2007	2.79	0.04	4879	62005	3.52	0.17	439	39315
United States	2010	2.78	0.04	5213	59937	3.03	0.16	378	37388
United States	2013	2.58	0.05	3060	40721	3.46	0.19	333	24494
United States	2016	2.44	0.04	3657	54727	4.14	0.17	565	45401
Uruguay	2004	2.46	0.11	467	13671	2.01	0.3	46	1241
Uruguay	2007	2.89	0.12	591	37097	1.45	0.07	402	4236
Uruguay	2010	2.93	0.1	778	35428	1.87	0.23	65	4085
Uruguay	2013	2.69	0.05	3498	35264	2.74	0.44	38	3497
Uruguay	2016	3.13	0.16	402	33484	1.5	0.09	313	3632
Vietnam	2011	2.8	0.28	101	9282	1.13	0.25	20	1362
Vietnam	2013	2.8	0.11	706	9282	1.66	0.17	98	1384

Note: $\hat{\alpha}$: point estimate of Pareto exponent; s.e.: standard error (calculated using the inverse Fisher information); k : the number of order statistics used for the Hill-estimator; N : the number of positive labor (resp. capital) income observations.

Table 3: Hypothesis testing of $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$.

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	N	Reject H_0 ?
Australia	1981	5.26	0.24	2.1	0.1	9488	Yes
Australia	1985	4.22	0.26	1.4	0.09	5232	Yes
Australia	1989	3.95	0.17	1.46	0.06	10629	Yes

Table 3: Hypothesis testing of $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$.

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	N	Reject H_0 ?
Australia	1995	2.91	0.22	1.7	0.13	3364	Yes
Australia	2001	3.24	0.26	1.45	0.12	3105	Yes
Australia	2003	3.25	0.22	1.51	0.1	4473	Yes
Australia	2004	3.38	0.19	1.41	0.08	6145	Yes
Australia	2008	3.3	0.21	1.27	0.08	4749	Yes
Australia	2010	3.31	0.16	1.29	0.06	9093	Yes
Australia	2014	3.14	0.14	1.19	0.05	9616	Yes
Austria	1994	3.12	0.34	1.59	0.17	1695	Yes
Austria	1997	3.55	0.37	1.78	0.19	1800	No
Austria	2000	3.1	0.35	2.1	0.24	1584	No
Austria	2004	3.34	0.24	0.99	0.07	3750	Yes
Austria	2007	3.23	0.23	1.17	0.08	4101	Yes
Austria	2010	3.51	0.24	1.05	0.07	4323	Yes
Austria	2013	3.36	0.24	1.12	0.08	4022	Yes
Belgium	1997	4.25	0.35	1.82	0.15	2906	Yes
Brazil	2006	3.32	0.21	1.53	0.1	4804	No
Brazil	2009	3.19	0.21	1.98	0.13	4724	No
Brazil	2011	2.89	0.24	1.7	0.14	2897	No
Brazil	2013	2.94	0.25	1.48	0.12	2869	Yes
Canada	1971	3.56	0.16	1.66	0.07	9970	Yes
Canada	1975	4.02	0.17	1.84	0.08	10717	Yes
Canada	1981	4.59	0.22	2.01	0.1	8342	Yes
Canada	1987	4.78	0.3	1.88	0.12	5190	Yes
Canada	1991	4.34	0.2	2	0.09	9847	Yes
Canada	1994	4.07	0.15	1.75	0.07	14077	Yes
Canada	1997	4.01	0.17	1.79	0.08	11273	Yes
Canada	1998	3.22	0.12	1.46	0.06	13486	Yes
Canada	2000	3.17	0.12	1.47	0.06	13264	Yes
Canada	2004	3.15	0.13	1.56	0.06	11861	Yes
Canada	2007	2.85	0.11	1.49	0.06	12533	Yes
Canada	2010	3.06	0.13	1.4	0.06	10341	Yes
Canada	2013	3.93	0.18	1.56	0.07	9741	Yes
Chile	1990	2.24	0.28	1.62	0.21	1242	No
Chile	1992	3.42	0.37	1.76	0.19	1678	Yes
Chile	1994	2.61	0.26	1.91	0.19	2071	No
Chile	1996	2.71	0.21	1.52	0.12	3437	Yes
Chile	1998	2.43	0.19	1.64	0.13	3335	No
Chile	2000	2.28	0.15	1.38	0.09	4432	Yes
Chile	2003	2.28	0.15	1.45	0.1	4637	No
Chile	2006	2.31	0.14	1.38	0.08	5374	Yes
Chile	2009	3.02	0.22	1.48	0.11	3619	Yes
Chile	2011	3.48	0.27	1.82	0.14	3312	Yes
Chile	2013	2.87	0.21	1.76	0.13	3635	No
Chile	2015	3.27	0.2	1.71	0.11	5314	Yes
China	2002	4.24	0.41	1.72	0.17	2167	Yes
China	2013	3.1	0.14	2.06	0.09	10098	No

Table 3: Hypothesis testing of $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$.

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	N	Reject H_0 ?
Colombia	2004	2.37	0.28	1.97	0.23	1411	No
Colombia	2007	2.02	0.06	1.7	0.05	23816	Yes
Colombia	2010	2.51	0.07	1.63	0.05	24975	No
Colombia	2013	2.55	0.07	1.74	0.05	25674	Yes
Czech Rep	1996	3	0.24	1.14	0.09	3077	Yes
Czech Rep	2007	3.62	0.39	1.11	0.12	1735	Yes
Czech Rep	2010	4.09	0.49	1.23	0.15	1422	Yes
Czech Rep	2013	3.12	0.4	1.54	0.2	1198	No
Denmark	1987	4.46	0.21	1.64	0.08	9199	Yes
Denmark	1992	4.57	0.21	1.46	0.07	9164	Yes
Denmark	1995	4.68	0.09	1.26	0.02	57306	Yes
Denmark	2000	4.26	0.08	1.19	0.02	59549	Yes
Denmark	2004	4.03	0.07	1.17	0.02	59824	Yes
Denmark	2007	3.82	0.07	1.17	0.02	61255	Yes
Denmark	2010	3.37	0.06	1.2	0.02	59488	Yes
Denmark	2013	3.47	0.06	1.13	0.02	60880	Yes
Estonia	2007	4.53	0.43	0.67	0.06	2250	Yes
Estonia	2010	4.68	0.48	0.91	0.09	1948	Yes
Estonia	2013	3.65	0.42	1.16	0.13	1549	Yes
Finland	1987	4.35	0.22	1.45	0.07	7906	Yes
Finland	1991	4.48	0.22	1.43	0.07	8332	Yes
Finland	1995	4.5	0.31	1.25	0.09	4236	Yes
Finland	2000	3.53	0.21	1.09	0.06	5923	Yes
Finland	2004	3.91	0.19	1.02	0.05	8061	Yes
Finland	2007	3.82	0.19	1.01	0.05	8502	Yes
Finland	2010	3.91	0.2	1.05	0.05	7537	Yes
Finland	2013	3.9	0.19	1.04	0.05	8899	Yes
France	1978	2.4	0.28	1.75	0.21	1426	No
France	1984	1.52	0.15	1.73	0.17	1974	No
France	1989	3.43	0.37	1.65	0.18	1677	Yes
France	1994	3.13	0.16	1.68	0.08	7880	Yes
France	2000	3.51	0.19	1.72	0.09	7112	Yes
France	2005	3.33	0.17	1.53	0.08	7987	Yes
France	2010	3.02	0.12	1.49	0.06	11660	Yes
Germany	1973	3.56	0.08	2.13	0.05	37614	No
Germany	1978	3.36	0.08	2.18	0.05	35731	No
Germany	1983	4.54	0.11	2.23	0.06	32288	Yes
Germany	1984	3.96	0.27	1.03	0.07	4240	Yes
Germany	1987	4.23	0.3	1.17	0.08	3863	Yes
Germany	1989	3.29	0.24	1.18	0.09	3733	Yes
Germany	1991	4.77	0.29	1.1	0.07	5448	Yes
Germany	1994	4.54	0.28	1.32	0.08	5384	Yes
Germany	1995	4.65	0.29	1.33	0.08	5275	Yes
Germany	1998	4.56	0.27	1.31	0.08	5531	Yes
Germany	2000	3.91	0.19	1.51	0.07	8784	Yes
Germany	2001	3.46	0.16	1.31	0.06	9273	Yes

Table 3: Hypothesis testing of $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$.

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	N	Reject H_0 ?
Germany	2002	3.57	0.17	1.35	0.06	8850	Yes
Germany	2003	3.64	0.18	1.28	0.06	8600	Yes
Germany	2004	3.67	0.18	1.29	0.06	8300	Yes
Germany	2005	3.29	0.16	1.27	0.06	8869	Yes
Germany	2006	3.28	0.16	1.32	0.06	8276	Yes
Germany	2007	3.44	0.17	1.35	0.07	7778	Yes
Germany	2008	3.22	0.17	1.29	0.07	7295	Yes
Germany	2009	3.43	0.15	1.37	0.06	10611	Yes
Germany	2010	3.44	0.14	1.29	0.05	12512	Yes
Germany	2011	3.3	0.13	1.28	0.05	12291	Yes
Germany	2012	3.43	0.13	1.31	0.05	13051	Yes
Germany	2013	3.4	0.14	1.32	0.05	11824	Yes
Germany	2014	3.78	0.16	1.3	0.06	11176	Yes
Germany	2015	3.09	0.13	1.31	0.06	10625	Yes
Greece	1995	3.7	0.51	2.83	0.39	1033	No
Greece	2004	3.64	0.52	2.12	0.3	1006	No
Greece	2007	2.81	0.35	1.97	0.25	1253	No
Greece	2010	3.01	0.39	2.09	0.27	1170	No
Greece	2013	2.68	0.31	2.22	0.25	1517	No
Iceland	2004	4.71	0.49	1.15	0.12	1840	Yes
Iceland	2007	4.4	0.43	1.16	0.11	2113	Yes
Iceland	2010	4.51	0.39	1.13	0.1	2675	Yes
India	2004	2.85	0.29	2.47	0.25	1876	No
India	2011	2.21	0.22	1.43	0.14	2027	No
Ireland	1987	3.41	0.34	1.29	0.13	1998	Yes
Ireland	1994	2.54	0.34	1.26	0.17	1114	Yes
Ireland	1995	2.31	0.32	1.3	0.18	1026	Yes
Ireland	1996	2.68	0.38	1.56	0.22	1023	No
Ireland	2000	3.44	0.41	1.7	0.2	1371	Yes
Ireland	2007	2.64	0.3	1.49	0.17	1510	Yes
Israel	2005	3.67	0.43	1.25	0.15	1475	Yes
Israel	2012	3.49	0.43	1.3	0.16	1305	Yes
Israel	2014	4.76	0.59	1.69	0.21	1330	Yes
Israel	2016	4.23	0.54	2.14	0.27	1215	Yes
Italy	1986	3.17	0.24	1.85	0.14	3474	Yes
Italy	1987	3.51	0.2	1.66	0.1	6045	Yes
Italy	1989	3.95	0.23	1.56	0.09	5865	Yes
Italy	1991	4.43	0.26	1.64	0.1	5827	Yes
Italy	1993	3.76	0.23	1.52	0.09	5444	Yes
Italy	1995	3.24	0.2	1.62	0.1	5487	Yes
Italy	1998	3.37	0.22	1.46	0.09	4876	Yes
Italy	2000	3.05	0.19	1.33	0.08	5280	Yes
Italy	2004	2.39	0.15	1.52	0.1	4913	Yes
Italy	2008	3.13	0.2	1.44	0.09	4778	Yes
Italy	2010	3.35	0.22	1.34	0.09	4719	Yes
Italy	2014	2.75	0.19	1.61	0.11	4402	Yes

Table 3: Hypothesis testing of $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$.

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	N	Reject H_0 ?
Luxembourg	2007	3.26	0.32	1.61	0.16	2085	Yes
Luxembourg	2010	3.43	0.26	1.71	0.13	3494	Yes
Luxembourg	2013	3.68	0.33	1.59	0.14	2534	No
Mexico	2004	2.58	0.35	1.58	0.22	1055	Yes
Mexico	2008	2.21	0.24	1.43	0.16	1635	Yes
Mexico	2010	2.64	0.32	1.64	0.2	1323	Yes
Netherlands	1990	3.75	0.33	1.38	0.12	2570	Yes
Netherlands	1993	4.16	0.38	1.23	0.11	2423	Yes
Netherlands	1999	4.21	0.37	1.27	0.11	2563	Yes
Netherlands	2004	3.41	0.18	1.36	0.07	7166	Yes
Netherlands	2007	3.17	0.16	0.98	0.05	8011	Yes
Netherlands	2010	3.63	0.18	1.37	0.07	8001	Yes
Netherlands	2013	3.62	0.18	1.3	0.07	7808	Yes
Norway	1979	1.8	0.1	1.71	0.1	6048	No
Norway	1986	5.11	0.45	1.89	0.17	2588	Yes
Norway	1991	3.83	0.2	1.76	0.09	7361	Yes
Norway	1995	3.45	0.17	1.14	0.06	8027	Yes
Norway	2000	3.18	0.13	0.88	0.04	11474	Yes
Norway	2004	3.98	0.17	0.77	0.03	10947	Yes
Norway	2007	3.74	0.04	1.11	0.01	169708	Yes
Norway	2010	3.55	0.04	0.86	0.01	177229	Yes
Norway	2013	3.35	0.04	0.83	0.01	183146	Yes
Peru	2004	2.5	0.23	1.75	0.16	2463	Yes
Peru	2007	3.05	0.26	1.69	0.15	2667	Yes
Peru	2010	2.78	0.22	1.66	0.13	3239	Yes
Peru	2013	3.72	0.27	1.68	0.12	3930	Yes
Russia	2013	4.25	0.29	1.67	0.11	4360	Yes
Russia	2014	5.03	0.34	1.79	0.12	4412	Yes
Russia	2015	4.89	0.29	1.65	0.1	5752	No
Russia	2016	5.19	0.19	1.91	0.07	15716	No
Slovakia	2010	3.49	0.49	0.96	0.13	1024	Yes
Slovakia	2013	2.73	0.34	0.84	0.1	1289	Yes
Slovenia	2012	6.11	0.64	1.43	0.15	1841	Yes
South Korea	2006	4.67	0.32	1.69	0.11	4353	No
South Korea	2008	4.2	0.31	1.7	0.13	3616	No
South Korea	2010	4.38	0.36	1.62	0.13	3003	No
South Korea	2012	3.79	0.29	1.58	0.12	3329	No
Spain	1980	4.21	0.49	2.07	0.24	1503	No
Spain	1990	3.84	0.27	1.58	0.11	4036	Yes
Spain	1995	3.46	0.33	1.57	0.15	2250	Yes
Spain	2000	3.19	0.24	1.09	0.08	3549	Yes
Spain	2004	4.22	0.28	1.32	0.09	4433	Yes
Spain	2007	4.3	0.26	1.41	0.09	5327	Yes
Spain	2010	6.08	0.49	1.79	0.15	3043	Yes
Spain	2013	3.59	0.2	1.69	0.09	6681	Yes
Spain	2016	3.44	0.18	1.51	0.08	7451	Yes

Table 3: Hypothesis testing of $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$.

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	N	Reject H_0 ?
Sweden	1975	3.93	0.2	1.54	0.08	7845	Yes
Sweden	1981	4.56	0.26	2.47	0.14	6092	Yes
Sweden	1987	4.71	0.24	1.77	0.09	7953	Yes
Sweden	1992	4.49	0.19	1.98	0.08	10824	Yes
Sweden	1995	3.97	0.16	1.63	0.07	11655	Yes
Sweden	2000	3.6	0.16	1.26	0.06	10319	Yes
Sweden	2005	3.15	0.13	1.07	0.04	11951	Yes
Switzerland	1982	2.05	0.14	1.34	0.09	4233	Yes
Switzerland	1992	3.55	0.24	1.3	0.09	4486	Yes
Switzerland	2002	3.29	0.29	1.4	0.12	2646	Yes
Switzerland	2000	3.53	0.31	1.56	0.14	2578	Yes
Switzerland	2004	4.41	0.38	1.38	0.12	2648	Yes
Switzerland	2007	2.74	0.17	1.22	0.08	5057	Yes
Switzerland	2010	2.92	0.17	1.14	0.07	5759	Yes
Switzerland	2013	3.45	0.22	1.4	0.09	5097	Yes
Taiwan	1981	4.5	0.19	1.53	0.07	10635	Yes
Taiwan	1986	3.88	0.15	1.34	0.05	12864	Yes
Taiwan	1991	4.11	0.15	1.41	0.05	14978	Yes
Taiwan	1995	4.29	0.16	1.77	0.07	13619	Yes
Taiwan	1997	4.27	0.17	1.83	0.07	12572	Yes
Taiwan	2000	4.19	0.17	1.76	0.07	12301	Yes
Taiwan	2005	4.17	0.17	1.8	0.07	11903	Yes
Taiwan	2007	3.82	0.16	1.75	0.07	12042	Yes
Taiwan	2010	3.85	0.15	1.74	0.07	12748	Yes
Taiwan	2013	3.85	0.15	1.55	0.06	13300	Yes
Taiwan	2016	3.88	0.15	1.66	0.06	13648	Yes
UK	1969	3.73	0.26	1.11	0.08	4102	Yes
UK	1974	3.56	0.27	1.35	0.1	3547	Yes
UK	1979	5.03	0.34	1.34	0.09	4280	Yes
UK	1986	4.48	0.3	1.79	0.12	4515	Yes
UK	1991	3.56	0.24	1.69	0.11	4445	Yes
UK	1994	3	0.11	1.42	0.05	15751	Yes
UK	1995	3.21	0.22	1.48	0.1	4091	Yes
UK	1999	2.89	0.1	1.4	0.05	15200	Yes
UK	2004	2.7	0.09	1.27	0.04	17314	Yes
UK	2007	2.87	0.1	1.34	0.05	15272	Yes
UK	2010	2.57	0.12	1.34	0.06	9839	Yes
UK	2013	2.79	0.14	1.5	0.08	7715	Yes
UK	2016	2.49	0.13	1.32	0.07	7340	Yes
US	1974	3.86	0.23	1.51	0.09	5558	Yes
US	1979	3.7	0.08	1.46	0.03	44749	Yes
US	1986	3.36	0.08	1.64	0.04	38274	Yes
US	1991	3.45	0.08	1.54	0.04	38875	Yes
US	1994	2.43	0.06	1.45	0.03	37355	Yes
US	1997	2.29	0.06	1.47	0.04	30913	Yes
US	2000	2.19	0.05	1.4	0.03	46961	Yes

Table 3: Hypothesis testing of $\hat{\alpha}_{lab} = \hat{\alpha}_{cap}$.

Country	Year	$\hat{\alpha}_{lab}$	s.e.	$\hat{\alpha}_{cap}$	s.e.	N	Reject H_0 ?
US	2004	2.39	0.05	1.33	0.03	42354	Yes
US	2007	2.34	0.05	1.49	0.03	39315	Yes
US	2010	2.56	0.06	1.41	0.03	37388	Yes
US	2013	2.4	0.07	1.4	0.04	24494	Yes
US	2016	2.29	0.05	1.42	0.03	45401	Yes
Uruguay	2004	2.53	0.32	1.81	0.23	1241	No
Uruguay	2007	3.26	0.22	1.53	0.11	4236	Yes
Uruguay	2010	3.1	0.22	1.63	0.11	4085	Yes
Uruguay	2013	3.69	0.28	1.8	0.14	3497	Yes
Uruguay	2016	3.53	0.26	1.69	0.13	3632	Yes
Vietnam	2011	3.26	0.4	1.23	0.15	1362	Yes
Vietnam	2013	2.61	0.31	1.64	0.2	1384	No

Note: $\hat{\alpha}_{lab}$ (resp. $\hat{\alpha}_{cap}$): point estimate of Pareto exponent for labor (resp. capital) income; s.e.: standard error (calculated using the inverse Fisher information); N : the number of positive capital income observations; the last column denotes if H_0 is rejected.