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# BONN ECON DISCUSSION PAPERS

Discussion Paper 16/2002

**Regulation under Financial Constraints**

by

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# Regulation under Financial Constraints\*

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## Abstract

This article studies the problem of regulating a monopolist with unknown marginal cost. The originality of the paper is to consider that the regulator faces a cash-in-advance constraint. The introduction of such a constraint not only reduces the amount of public good provided but also limits the instruments available to the regulator. The wealth constraint could change the optimal regulatory contract from a two-part tariff, where the quantities produced depend on the firm's cost, to a fixed fee where the firm produces the same quantity whatever its cost.

**JEL codes:** D82 H42 L51

**Keywords:** Regulation, asymmetric information, monopolist

## 1 Introduction

This article analyzes the following problem: a public authority faces a monopolistic producer with unknown cost.<sup>1</sup> The firm provides an essential facility (bridge, road, infrastructure or sewer system) to a bunch of consumers. The authority regulates the term under which the producer delivers the service to consumers.

Under full-information, the regulator requires that the firm produces the quantity of public good that maximizes the consumer's surplus net of the firm's production cost. In the case where the regulator ignores the firm's production cost, it designs a regulatory contract that gives incentives for the firm to reveal its private information. When the firm's cost could take two values, the optimal (second-best) regulatory contract has the following features: first, the efficient firm (the low cost one) produces the same amount of public good than under full-information but receives a higher payment (information rent). Second, the high cost firm produces less than under full-information and makes a zero profit.

This paper adds macroeconomic constraints to the standard problem of regulation under asymmetric information. It is assumed that the public authority has only limited funds at its

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<sup>1</sup>The framework is similar to Baron and Myerson (1982) except that the firm has no fixed cost.

disposal. For example, if the authority has limited possibilities of raising taxes. This constraint limits the possibilities for the government to finance the infrastructure. This article describes the optimal regulatory contract when the authority has limits in the amount it could transfer to the firm.

Financial constraints are particularly relevant for the case of financing infrastructures in developing countries. The World development reports (World Bank, 2000) stresses the role of local collectivities and local governments as a vector of development. These collectivities are responsible for building the most attractive infrastructure for their citizens and the investors. The benefits of these infrastructures being established, the question is how to finance them ?

Local governments can finance their new responsibilities in several ways. Development fees, connection charges, and local tax revenue can all generate funds that can be used for investment. While such resources can make a significant contribution to investment financing, particularly in slow-growing cities, they may not be enough to finance all infrastructure investments at the peak of the urban transition. In this case debt financing may be required and can make financial sense. Roads, schools, and pipelines have long useful lives, and debt spreads over their lifetimes. But what options do local governments have for borrowing ? The experience of industrial countries suggests two: municipal bonds and municipal funds. (World development report pages 132-133)

But, the conditions for raising external funds are not always satisfied:

In many developing countries, few of these conditions exist. Long histories of macroeconomics instability make long term financial commitment extremely risky. Information on potential borrowers is unreliable. The legal framework needed to provide investors with recourse in case of default is underdeveloped and often untested. Municipal governments in these countries are viewed -often correctly- as particularly unattractive borrowers because they lack the autonomy to raise revenues or reduce spending, particularly on personnel. Moreover, local governments often have no credible political commitment to long-term financial obligations. Under these conditions, even if long-term private capital is available, local governments generally can borrow only at very high rate of interest, if at all. (World development report page 133)

From this discussion, it seems that a public authorities in developing countries face a cash-in-advance constraint that limits their possibilities to finance infrastructure.<sup>2</sup> Even with a limited budget at disposal, some infrastructure building could not be delayed and will be financed even if funds are scarce.

In these circumstances, there is under-provision of the public good (or lower quality infrastructure). In the second best-problem, for incentive purpose, the quantity produced by the high cost firm is reduced to lower the information rent paid to the low cost firm. When, in addition, the authority is wealth constrained, the transfer paid to the low cost firm should be lowered. To limit the transfer, the regulator lowers the quantity produced by the efficient firm to reduce its compensation but also lowers the quantity produced by the inefficient firm to lower the information rent. These 'third best' distortions in the quantities, that come on top of the

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<sup>2</sup> A similar problem arose in Europe with the EMU. The Maastricht treaty forced the European governments (including local one) to cut down their public spending.

traditional 'second best' distortions, are necessary to fulfill the wealth constraint and are unequally distributed among the quantities produced by each type of firm. For the regulator, there is a trade-off between the efficiency cost of reducing the production of the low cost firm and the efficiency cost of reducing the information rent.

These distortions may lead to the collapse of the incentive system. It may be impossible for the regulator to design a separating contract where the firm produces different quantities according to its cost. It will be the case when the efficiency cost of reducing the information rent is relatively high. In that case, the separating contract is such that the low cost firm produces less than the high cost one, and is therefore not feasible. When the regulator is constrained, bunching is a non trivial issue.

The wealth constraint not only reduces the amount of public good provided but also limits the instruments available to the regulator. The wealth constraint could change the optimal regulatory contract from a two-part tariff, where the quantities produced depend on the firm's cost, to a fixed fee where the firm produces the same quantity whatever its cost. There are two inefficiencies associated with cash-in-advance constraint. The quantity of the public good provided to consumers is reduced and the regulator uses less efficient regulatory instruments.<sup>3</sup>

There are several recent papers that study incentive problems when the better informed party has limited wealth. Che and Gale (1998) study auctions with bidders that have private information about their willingness to pay and their ability to pay. In this model with two dimensional asymmetric information, the first-price auctions yield higher expected revenue than second-price auctions. Lewis and Sappington (2000) consider privatizations to wealth-constrained operators. As the most efficient operator is not necessarily the one who has the largest resources, privatization through an auction is not ex-post efficient. To have an ex-post efficient privatization, should design another procedure. Wealth constraints then modify the privatization process. To select an efficient operator, the government limits the operator's stake in the privatized firm. With limited stake, only the most efficient operators agree to operate the privatized company. Wealth constraints change the optimal privatization mechanism and imply that the government keeps shares in the company.

In the same line we show that the optimal regulation scheme changes when the regulator (the non informed party) faces financial constraints. A financially constrained regulator is sometimes forced to use different and less efficient regulatory instruments.

## 2 Model

The model is a simple model of adverse selection: the public authority (principal) contracts with a monopolistic private firm (agent) for the provision of a public utility. The agent is responsible for the production and the principal finances the production with transfers. At the time of contracting, the principal does not know the cost conditions under which the firm can produce. The firm produces with a constant return to scale technology. The cost function of the firm is  $\theta q$ , where  $\theta$  is a constant marginal cost and  $q$  is the quantity produced.<sup>4</sup> The marginal cost is private information to the firm. The principal only knows that  $\theta \in \Theta \equiv \{\theta_1, \theta_2\}$  with  $\theta_1 < \theta_2$

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<sup>3</sup>Price cap regulation would be another potential application. Under a price cap, the regulated firm cannot charge more than a given amount to consumers. It would be interesting to know if the imposition of a price cap limits the possibilities of price discrimination of the firm (see Armstrong and Vickers (1991) for the case of perfect information.)

<sup>4</sup>Equivalently,  $q$  could be interpreted as the quality of the good produced.

and the probabilities  $v_1$  and  $v_2 = 1 - v_1$  of agent being  $\theta_1$  and  $\theta_2$ . We call  $\Delta\theta = \theta_2 - \theta_1$ .

The profit of the firm is the difference between the transfer received from the principal ( $T$ ) and the cost of producing a quantity  $q$ :

$$U^A = T - \theta q$$

The firm accepts to produce if it realizes a non-negative profit (individual rationality constraint).

When the agent produces a quantity  $q$ , the principal collects a surplus  $S(q)$ . We assume that  $S' \geq 0$ ,  $S'(0) = +\infty$ ,  $S'' < 0$  and  $S''' > 0$ . The assumptions on  $S$  ensure that it is optimal to have the agent producing whatever its cost.

The utility of the principal is the difference between the surplus and the transfer:

$$U^P = S(q) - T$$

The regulator offers a contract specifying the transfer  $T$  and the quantity  $q$ . We call  $T_1, q_1$ , the transfer paid to the type  $\theta_1$  agent when he produces  $q_1$  and similarly,  $T_2, q_2$ , the transfer and production of type  $\theta_2$  agent.

The cash in advance constraint limits the transfer: it cannot exceed an upper limit denoted  $\bar{T}$ .

## 3 Results

### 3.1 Second best equilibrium

Without the cash-in-advance constraint, the objective of the principal is to maximize its expected utility, subject to incentive compatible and participation constraints. The incentive constraints ensure that the agent of type  $\theta_1$  selects the contract  $(T_1, q_1)$  rather than the contract  $(T_2, q_2)$ , and similarly for the agent of type  $\theta_2$ .

Program [P1]

$$\max_{q_1, q_2, T_1, T_2} v_1(S(q_1) - T_1) + v_2(S(q_2) - T_2)$$

subject to:

$$T_1 - \theta_1 q_1 \geq T_2 - \theta_1 q_2 \quad (IC_1)$$

$$T_2 - \theta_2 q_2 \geq T_1 - \theta_2 q_1 \quad (IC_2)$$

$$T_1 - \theta_1 q_1 \geq 0 \quad (IR_1)$$

$$T_2 - \theta_2 q_2 \geq 0 \quad (IR_2)$$

The two relevant constraints of this problem are  $IC_1$  and  $IR_2$ .

**Proposition 1** *The solution to the problem [P1] is given by:*

$$q_1^{SB} = S'^{-1}(\theta_1) \quad (1)$$

$$q_2^{SB} = S'^{-1}\left(\theta_2 + \frac{v_1}{v_2} \Delta\theta\right) \quad (2)$$

$$T_1 = \theta_1 q_1^{SB} + \Delta\theta q_2^{SB} \quad (3)$$

$$T_2 = \theta_2 q_2^{SB} \quad (4)$$

This solution is standard. The low cost firm produces the quantity of the good that maximizes the total surplus and makes a positive profit (information rent). This information rent is necessary to fulfill the incentive constraint. The inefficient firm makes a zero-profit and produces less than the quantity that maximizes the total surplus. The quantity  $q_2^{SB}$  is reduced compared to the first best in order to lower the profit made by the low cost firm (rent extraction-efficiency trade-off).

### 3.2 Wealth-constrained equilibria

We now introduce the wealth constraint. The constraint implies that the principal cannot transfer the agent more than  $\bar{T}$ . We said that the constraint is *relevant* if the maximal transfer  $\bar{T}$  is smaller than the highest transfer paid by the principal in the second best equilibrium.<sup>5</sup> When the principal is wealth-constrained, its optimization program becomes:

Program [P2]:

$$\max_{q_1, q_2, T_1, T_2} v_1(S(q_1) - T_1) + v_2(S(q_2) - T_2)$$

subject to:  $(IC_1)$ ,  $(IC_2)$ ,  $(IR_1)$ ,  $(IR_2)$  and

$$T_1, T_2 \leq \bar{T} \tag{WC}$$

**Lemma 1** *When  $\bar{T} \leq \theta_1 q_1^{SB} + \Delta \theta q_2^{SB}$ , the transfer paid to the low cost firm equals  $\bar{T}$ .*

**Proof.** If  $\bar{T} < T_1^{SB}$ , the solution of [P1] cannot be replicated in [P2]. Then, at least one of the transfers in [P2] is given by the constraint (WC). A necessary condition for implementation is:  $q_1 \geq q_2$  and  $T_1 \geq T_2$ . Then the constraint (WC) binds (at least) for  $T_1$ . ■

Using the relevant constraints  $IC_1$ ,  $IR_2$  and the result of lemma 1, the program [P2] can be rewritten as:

Program[P3]:

$$\max_{q_1, q_2} v_1(S(q_1) - \bar{T}) + v_2(S(q_2) - \theta_2 q_2)$$

s.t.

$$\bar{T} = \theta_1 q_1 + \Delta \theta q_2 \tag{μ_1}$$

$$T_2 = q_2 \theta_2 \leq \bar{T} \tag{μ_2}$$

Where we indicate the Lagrange multipliers in brackets. The second constraint is equivalent to  $q_2 \leq q_1$ . The solution of this optimization program is given proposition 2.

**Proposition 2** *(i) If  $\theta_1 \geq v_1 \theta_2$  and  $\bar{T} \leq \bar{T}^* = \theta_2 S'^{-1}(\frac{\theta_2 \theta_1 v_2}{\theta_1 - v_1 \theta_2})$ , the equilibrium is a pooling equilibrium:*

$$q_1 = q_2 = \frac{\bar{T}}{\theta_2} \tag{5}$$

$$T_1 = T_2 = \bar{T} \tag{6}$$

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<sup>5</sup>The constraint is relevant if:  $\bar{T} \leq \theta_1 q_1^{SB} + \Delta \theta q_2^{SB}$ .

(ii) otherwise, the equilibrium is a separating equilibrium characterized by the following first order conditions:

$$q_1^{WC} = S'^{-1}\left(\frac{\mu_1}{v_1}\theta_1\right) \quad (7)$$

$$q_2^{WC} = S'^{-1}\left(\theta_2 + \frac{\mu_1}{v_2}\Delta\theta\right) \quad (8)$$

$$\bar{T} - \theta_1 q_1^{WC} - \Delta\theta q_2^{WC} = 0 \quad (9)$$

And the transfers are given by the constraints WC and IR<sub>2</sub>.

$$T_1 = \bar{T} \quad (10)$$

$$T_2 = \theta_2 q_2^{WC} \quad (11)$$

(iii) In the separating equilibrium, the value of  $\mu_1$  is a decreasing and convex function of  $\bar{T}$ , with  $\lim_{\bar{T} \rightarrow 0} \mu_1 = +\infty$  and  $\lim_{\bar{T} \rightarrow T_1^{SB}} \mu_1 = v_1$

The proof of proposition 2 is relegated to an appendix. Figure 1 illustrates the quantities produced when the wealth constraint is relevant.

INSERT FIGURE 1 HERE

Consider first the separating equilibrium. The transfer paid to the agent of type  $\theta_1$  could be decomposed into (i) a compensation for its production cost ( $\theta_1 q_1$ ) and (ii) an informational rent ( $\Delta\theta q_2$ ) which is necessary to satisfy the incentive constraint. If this transfer is fixed to  $\bar{T}$ , the principal has to reduce both the direct compensation and the informational rent i.e. reduces the quantity produced by both types of firm to fulfill the wealth constraint. On the top of the traditional second best trade-off between efficiency and rent extraction that leads to distortions in  $q_2$ , the wealth constraint implies a third best distortion in quantities  $q_1$  and  $q_2$ .

If we call  $\mu'_1 = \mu_1 - v_1$ , we can rewrite (7) and (8) in order to isolate the second and third best distortions:

$$q_1^{WC} = S'^{-1}\left(\theta_1 + \frac{\mu'_1}{v_1}\theta_1\right) \quad (12)$$

$$q_2^{WC} = S'^{-1}\left(\theta_2 + \frac{v_1}{v_2}\Delta\theta + \frac{\mu'_1}{v_2}\Delta\theta\right) \quad (13)$$

In these two expressions, the last terms on the right hand member measure the distortions imposed to keep the transfer  $T_1$  equals to  $\bar{T}$ . These distortions depend on the highest possible transfer  $\bar{T}$ . As part (iii) of proposition 2 shows,  $\mu'_1$  increases when the wealth constraint becomes more severe. The lower  $\bar{T}$  is, the larger are the distortions in quantities. But these distortions are unequally distributed among  $q_1$  and  $q_2$ . The wealth constrained regulator selects the most efficient way to reduce the transfer paid. For that, it compares the efficiency cost of reducing the information rent (reducing  $q_2$ ) and the efficiency cost of reducing the production cost (reducing  $q_1$ ). The efficiency cost is measured by the reduction in the expected surplus. If  $v_1\Delta\theta$  is larger (resp. lower) than  $\theta_1$ , a given reduction in  $q_2$  reduces more (resp. less) the transfer than a similar reduction in  $q_1$ . Consequently,  $q_2$  will be relatively more (resp. less) reduced than  $q_1$ .

The addition of a third best distortion in  $q_1$  and  $q_2$  may lead to the collapse of the incentive system. It will be the case if the quantities  $q_1^{WC}$  and  $q_2^{WC}$  do not satisfy the necessary condition

for implementation, namely keeping  $q_1$  greater than  $q_2$ .<sup>6</sup> If  $\mu_1$  is such that  $q_1^{WC} < q_2^{WC}$ , the only feasible mechanism is a pooling mechanism where the firm produces the same quantity whatever its cost. If  $q_1$  is more distorted than  $q_2$ <sup>7</sup>, there is a level of  $\mu_1'$  and a corresponding level of  $\bar{T}$  (called  $\bar{T}^*$ ) such that the value of  $q_1$  given by (7) is smaller than the value of  $q_2$  given by (8). Therefore, for  $\bar{T} \leq \bar{T}^*$ , the only feasible mechanism is a pooling mechanism.

Given that the equilibrium is not always separating, the implications in term of regulatory policy are drawn in the following corollary:

**Corollary 1** *The optimal regulation policy for a wealth-constrained principal is a fixed fee  $\bar{T}$  for quantity  $q = \frac{\bar{T}}{\theta_2}$  when the separating equilibrium does not exist and, otherwise, a two part tariff where the firm receives  $T_2 = \theta_2 q_2^{WC}$  for the quantity  $q_2^{WC}$  and a bonus  $\bar{T} - T_2$  if the firm produces the additional quantities  $q_1^{WC} - q_2^{WC}$ .*

The wealth constraint has two consequences on the regulatory scheme designed by the principal. First, there is under provision of the public good (or lower quality infrastructures) compared to the second best equilibrium. Second, in the case where the separating equilibrium does not exist, the regulation policy is less efficient. In this case, the principal uses a low-powered regulation scheme rather than a two-parts tariff. Regulation with fixed fee implies additional welfare losses due to the use of a less efficient regulatory instrument. The wealth constraint not only reduces the quantities of the good but also the number of instruments available to the regulator.

## 4 Conclusion

In this paper, we have shown that when the government is constrained on the level of transfer he can make to the firms, there is under-provision of public facilities. Moreover pooling contracts where both types of firm produce the same quantities and receive the same transfer could be the optimal contract. In the standard regulation problem, and given that the model satisfies some regularity conditions, pooling contracts are ruled out. When financial constraints are introduced, pooling contracts become an issue. Wealth constraints may lead to low-powered incentive scheme where the regulator pays a constant fee for a fixed quantity.

A wealth-constrained regulator changes the regulatory contract. Along the same lines, Lewis and Sappington (2000) show that facing wealth-constrained operators, a public authority will modify the privatization process. In both cases (regulation and privatization), the presence of wealth-constrained actors modifies the outcome of the process but also the instruments used.

## A Proof of proposition 2

The first order conditions of [P3] are:

$$S'(q_1) = \frac{\mu_1 \theta_1}{v_1} \tag{14}$$

$$S'(q_2) = \theta_2 + \frac{\mu_1}{v_2} \Delta\theta + \frac{\mu_2 \theta_2}{v_2} \tag{15}$$

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<sup>6</sup>If  $q_2 < q_1$ , it is impossible to satisfy the two incentive constraints at the same time. This is not an issue in the second best problem, because only the action of the inefficient firm ( $\theta_2$ ) is distorted.

<sup>7</sup>This is the case when  $\theta_1 > v_1 \theta_2$ .

$$\bar{T} - \theta_1 q_1 - \Delta\theta q_2 = 0 \quad (16)$$

$$\mu_2(\bar{T} - \theta_2 q_2) = 0 \quad (17)$$

We know by lemma 1 that  $\mu_1 > 0$  if the wealth constraint is relevant. There are two possible solutions to this system of equation: a separating solution when  $\mu_2 = 0$  and a pooling solution when  $\mu_2$  is positive.

If  $\mu_2 > 0$ , (17) becomes  $\bar{T} = \theta_2 q_2$ , then  $q_2 = \frac{\bar{T}}{\theta_2}$ . Replacing this value in (16), we have  $q_1 = q_2 = \frac{\bar{T}}{\theta_2}$ .

If  $\mu_2 = 0$ , the separating solution is given by:

$$S'(q_1) = \frac{\mu_1}{v_1} \theta_1 \quad (18)$$

$$S'(q_2) = \theta_2 + \frac{\mu_1}{v_2} \Delta\theta \quad (19)$$

$$\bar{T} - \theta_1 q_1 - \Delta\theta q_2 = 0 \quad (20)$$

To know which solution applies, we check when  $\mu_2$  is positive. As long as  $q_2$  is smaller than  $q_1$ , the transfer  $T_2$  is smaller than  $\bar{T}$ . Therefore, the second wealth constraint is slack when  $q_2 \leq q_1$ . This corresponds to the following condition:

$$\frac{\mu_1}{v_1} \theta_1 \geq \theta_2 + \frac{\mu_1}{v_2} \quad (21)$$

where the value of  $\mu_1$  is given by (20). Take the limit case where (21) is satisfied with equality, and solve for  $\mu_1$  we have:  $\mu_1 = \frac{v_2 \theta_2 \theta_1}{\theta_1 - v_1 \theta_2}$ . As long as the actual  $\mu_1$  is smaller than this value (call it  $\mu_1^*$ ),  $q_2$  is smaller than  $q_1$ .

$\mu_1^*$  is negative if  $\theta_1 \leq v_1 \theta_2$ . In this case, whatever  $\bar{T}$ ,  $q_2$  is smaller than  $q_1$ , except in the limit case where  $\bar{T}$  is null where both quantities are equal to zero.

If  $\theta_1 > v_1 \theta_2$ , we have to find the value of  $\bar{T}$  that generates value of  $\mu_1$  equals to  $\mu_1^*$ . To do this, we solve (18), (19) and (20) for  $\bar{T}$  when  $\mu = \mu_1^*$ . This gives a value  $\bar{T}^* = \theta_2 S'^{-1}(\frac{\theta_2 \theta_1 v_2}{\theta_1 - v_1 \theta_2})$ .

Now we can show that  $\mu_1$  increases when  $\bar{T}$  decreases:

(i) From the first order conditions we have:  $\bar{T} = \theta_1 S'^{-1}(\frac{\mu_1}{v_1} \theta_1) + \Delta\theta S'^{-1}(\theta_2 + \frac{\mu_1}{v_2} \Delta\theta)$ . Call the right hand side  $G(\mu_1)$ . Then  $\mu_1 = G^{-1}(\bar{T})$ . Given our assumptions on  $S$ ,  $G$  is increasing and concave, because  $S'^{-1}$  is increasing and concave. Then  $G^{-1}$  is decreasing and convex. (ii) At the limit when  $\bar{T}$  goes to  $T_1^{SB}$ , the problem is identical to the problem [P1] and therefore the solution is identical. i.e.  $\mu_1 = v_1$ . When  $\bar{T}$  goes to zero, the right hand side of the (9) must go to zero. Given that  $S'(0) = +\infty$ , we have that  $G^{-1}(0) = +\infty$ .

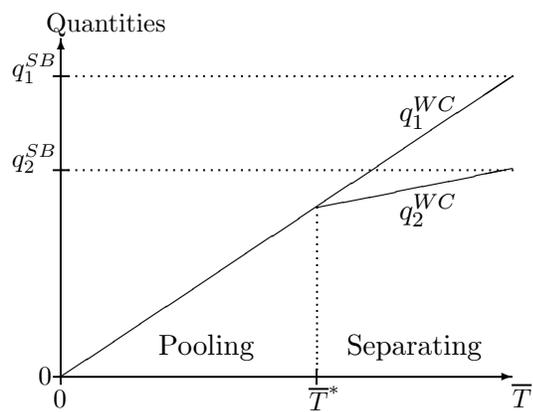
Hence, when  $\bar{T} \leq \bar{T}^*$  and  $\theta_1 > v_1 \theta_2$ ,  $\mu_2$  is positive and the solution is the pooling equilibrium. When  $\bar{T} \geq \bar{T}^*$ ,  $\mu_2$  is null and the solution is the separating equilibrium.

The second order conditions of [P3] are always satisfied thanks to the concavity of the problem.

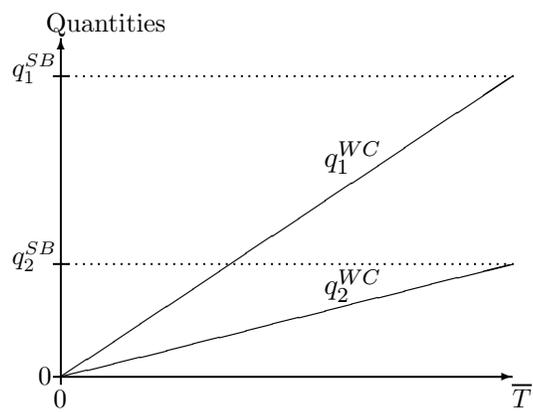
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Case 1:  $\theta_1 \geq v_1\theta_2$



Case 2:  $\theta_1 \leq v_1\theta_2$

Figure 1: Quantities produced when the principal is wealth constrained