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$$\begin{aligned} \frac{n!}{m!} \binom{n}{m} p^m (1-p)^{n-m} &= p \sum_{\ell=0}^{n-1} \frac{\ell+1}{n} \frac{(n-1)!}{(n-1-\ell)! \ell!} p^\ell (1-p)^{n-1-\ell} \\ &= p \frac{n-1}{n} \sum_{\ell=0}^{n-1} \left[ \frac{\ell}{n-1} + \frac{1}{n-1} \right] \frac{(n-1)!}{(n-1-\ell)! \ell!} p^\ell (1-p)^{n-1-\ell} = p^2 \frac{n-1}{n} + \end{aligned}$$

$$\frac{\ell!}{(n-1)!} p^{m-1} (1-p)^{n-m} = p \sum_{\ell=0}^{n-1} \frac{\ell+1}{n} \frac{(n-1)!}{(n-1-\ell)! \ell!} p^\ell (1-p)^{n-1-\ell} = p \frac{n-1}{n} \sum_{\ell=0}^{n-1} \left[ \frac{\ell}{n-1} + \frac{1}{n-1} \right] \frac{(n-1)!}{(n-1-\ell)! \ell!} p^\ell (1-p)^{n-1-\ell} = p^2 \frac{n-1}{n} +$$

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# Czech BEERs with PEERs: Tackling the Uncertainty

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## Abstract:

The alternative specifications of the behavioural equilibrium exchange rate models (the BEERs) and their permanent counterparts (the PEERs) often deliver diverse estimates of the equilibrium exchange rate. In the case of the Czech koruna against the euro exchange rate, the discrepancy among the estimated BEERs often exceeds 10 %, and it had been wide before the introduction of the exchange rate commitment in November 2013 as well. Thus, the BEERs do not provide reliable guidance about the equilibrium exchange rate and the size of the equilibrium. To tackle the model uncertainty, we propose to learn about the BEERs and PEERs from the model combination to retain the information of all individual models. From a policy perspective, our results provide weak support to claims that the Czech koruna had been slightly overvalued before the introduction of the commitment in 2013. However, the koruna became increasingly undervalued since the mid of 2015, and this timing overlaps with the need to support the exchange rate commitment by the exchange rate interventions.

**JEL:** F31, O24, E58, C52

**Keywords:** Equilibrium exchange rate, BEER, PEER, exchange rate commitment, model averaging

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# I. Introduction

The knowledge of the equilibrium exchange rate is essential for monetary policy in small open economies. The exchange rate misalignments can translate into inflationary or deflationary pressures in the economy. Moreover, even under inflation targeting to the central banks might resort to exchange rate policy as an additional instrument of monetary policy. Last but not least, countries considering to join the Eurozone might wish fix to avoid permanent fixation of their currencies at overvalued or undervalued exchange rates.

However, the equilibrium exchange rate is unobserved, and its estimation is far from being straightforward despite the availability of well-established approaches. Driver and Westaway (2005) emphasize in their survey that the alternative approaches differ according to their respective time dimension. For example, the behavioural equilibrium exchange rate, so-called BEER model, assumes short-term deviations of the exchange rate. Other models, such as the fundamental equilibrium exchange rate model (FEER), allow for more persistent deviations. Nevertheless, the different time dimension of alternative models is not the only complication. More strikingly, many implementations of models of the same class lead to markedly different results, due to different choice of variables and other identifying assumptions employed in estimations. In this case, the theory does not guide the selection of the best model, and the size or even a sign of the exchange rate misalignment remains uncertain.

In this paper, we provide estimates of the equilibrium exchange rate of the Czech koruna against the euro and the respective misalignments.<sup>1</sup> These estimates are not readily available for recent years. The Czech National Bank does not publish them, and the projections of the exchange rate against the euro consistent with the inflation forecasts were not published until 2018.<sup>2</sup> Also, the available estimates of the equilibrium exchange rate by the Peterson Institute of International Economics (PIIE), and by the Centre d'études prospectives et d'informations internationales (CEPII) provide even different signs of the koruna misalignments.

Furthermore, the estimations of the past misalignments are essential for an assessment of the exchange rate commitment introduced by the Czech National Bank in November 2013. With interest rates already constrained by the zero lower bound, and while facing deflationary outlook, the central bank decided for a 5% devaluation and committed to keeping the exchange rate above 27 CZK/EUR as long as the inflation does not return to the inflation target of 2 %. The resort to exchange rate policy was inspired by Lars Svensson's proposal to escape the liquidity trap (Svensson, 2003). However, the core of

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<sup>1</sup> The CZK/EUR exchange rate is the far most dominant exchange rate for the Czech exporters, since the share of exports to the EU accounted for 83.6 % of total exports in 2017. Hence, we do not have to resort to panel regressions, that would have been more relevant for countries with higher diversification across export partners. See Adler and Grisse (2014) for an illustration of a panel approach.

<sup>2</sup> Nevertheless, the quantitative projections of the USD/EUR exchange rate have been published by the Czech National Bank regularly, within the official Inflation Reports. The first Inflation Report with the CZK/EUR projection is the 2018/1 issue.

his proposal was real undervaluation<sup>3</sup>, not the initial depreciation, and these estimates of the exchange rate misalignments achieved by the commitment are scarce and rather inconsistent<sup>4</sup>. Our paper fills this gap.

We focus on the estimation of the behavioural equilibrium exchange rate models, the BEERs, that are the least demanding in terms of the assumptions of long-term sustainable values of current accounts and other variables. These sustainable values usually cannot be estimated within the model of the equilibrium exchange rate itself and are subject to somewhat arbitrary choices. In principle, the BEER models regress the real exchange rate on a set of its short-term and medium-term determinants, and they were applied on the bilateral exchange rate CZK/EUR already, notably by Babetski and Égert (2005) and Komárek and Motl (2012). Both models differ in specifications and depart from the benchmark set of variables suggested in Clark and MacDonald (1998) as well.

The main novelty of our paper is, however, that we focus on the inherent model uncertainty associated with the variable selection for the BEER models, which is usually left behind in previous studies. We estimate seven distinct BEER models, inspired by the previous literature, and we present the dispersion in the resulting equilibrium exchange rates. Then, we combine the predictions via averaging over the estimates of the individual models, to obtain the final estimate of the equilibrium exchange rate. To estimate the long-run relationships, we employ the ARDL model with bounds testing of cointegration (Pesaran, Shin, and Smith, 2001). This methodology is well-suited for models with a mix of  $I(1)$  and  $I(0)$  variables.

Our sample covers the period of the exchange rate commitment when the exchange rate was deliberately undervalued. By definition, the BEER models fit the actual real exchange rate so that they might underestimate the actual level of misalignment. To alleviate this bias, we consider two alternatives to the full-sample estimates. First, we use static forecasts of models estimated on the data up to the start of the commitment in November 2013. Second, following Daude, Yeyati, and Nagengast (2016), we include the interventions of the central bank needed to maintain the commitment explicitly in the BEER model, as an additional determinant of the equilibrium exchange rate.

After estimating the BEER models, we estimate the long-term equilibrium exchange rate with the help of the permanent equilibrium exchange rates (PEER). Following Égert (2004), we construct the PEER model by replacing the regressors in the cointegrating relationships by their long-term components. These long-term components are constructed via the HP-filter, as recommended by Égert (2004). To address the

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<sup>3</sup> Note that Svensson proposed to complement the exchange rate policy by price-level targeting. However, the Czech National Bank deliberately did not follow this advice. The discussion, along with the assessment whether the price-level targeting would make a difference, is summarized in Baxa and Šestořád (2019).

<sup>4</sup> To our knowledge, the only exceptions include Komárek (2017), who presents updated results originally appearing in Komárek and Motl (2012), the previously mentioned estimates of the Peterson Institute of International Economics, and of the Centre d'études prospectives et d'informations internationales (CEPII).

end-sample bias of the HP-filter, we use the one-sided filter and Hamilton's (2018) filter as sensitivity checks.

Our results confirm the wide discrepancy among the individual BEER models that easily exceeds 10% in several periods. On the other hand, the differences among the three alternative strategies of tackling the exchange rate commitment are smaller and do not exceed 5%, when the averages across models are considered. We also argue that it is hard to provide sound justification for the rejection of some specifications based on economic intuition. Hence, the model averaging appears as a reasonable option of summarizing the knowledge of multiple models.

From a policy perspective, our results imply that there is some indication of moderate overvaluation of the Czech koruna before the introduction of the exchange rate commitment, both with respect to the behavioural and permanent equilibrium exchange rates. However, the depreciation did not cause short-term misalignments in 2014. This observation is consistent with a limited response of the headline inflation to the introduction of the commitment, discussed in CNB(2017), and more extensively in Baxa and Sestorad (2019). Short-term misalignments of the koruna emerged since the mid of 2015 and converged to the long-term misalignments at 5 % in 2016. Interestingly, the timing of the increase of the short-term misalignments overlaps with the need to support the exchange rate commitment by the exchange rate intervention by the Czech National Bank. Hence, the speculations betting on an appreciation of the koruna reflected the fundamentals, at least to some extent.

The remainder of this paper is organized as follows. We review the theory of the equilibrium exchange rates in section two. Then, we present the estimation methodology and the models included in our paper. Section four contains a data description. The results and the sensitivity analysis follow, and the final section summarizes the main findings and provides some policy conclusions.

## II. A Theory of the equilibrium exchange rates

A classic survey of the literature and the theory of the equilibrium exchange rates are provided by Driver and Westaway (2005). They present various alternative approaches to the identification and estimation of the equilibrium exchange rates in a unified framework. Importantly, Driver and Westaway stress the role of time horizon over which the equilibrium shall be achieved. While in a short-term, the equilibrium exchange rate shall be consistent with the current values of fundamental determinants of the exchange rate, the medium and the long-term equilibrium exchange rate depends on steady states of the respective fundamental variables.

Following Driver and Westaway (2005), and Clark and MacDonald (1997), we can describe the dynamics of the exchange rate using the following equation:

$$q_t = \beta Z_t + \theta T_t + \varepsilon_t \quad (1)$$

where the  $q_t$  is the real exchange rate at time  $t$ ,  $Z_t$  is a vector of fundamentals that are assumed to affect the exchange rate over the medium and the long term, and  $T_t$  is a

vector of transitory factors. Among transitory factors we can list both variables that are expected to have solely a short-term effect on the exchange rate, or the short-term fluctuations of the fundamentals represented, for example, by their first differences.

The short-term equilibrium is then defined as the value of the exchange rate as if there were no unexpected shocks, hence:

$$q_t^{ST} = \beta Z_t \quad (2)$$

The equation (2) is a general representation of a family of the so called behavioral equilibrium exchange rate (BEER) models.<sup>5</sup>

The BEER models use econometric methods to establish a behavioral link between the real exchange rate and the relevant fundamental variables (Clark and MacDonald, 1998). It is implicitly assumed that the exchange rate is on average in equilibrium on the estimation sample so that the exchange rate misalignments are stationary. Hence, the list of variables  $Z_t$  in the equations (1) and (2) includes a vector of fundamentals that affect the exchange rate over the long run, and over the business cycle as well.

Although it might not be apparent at the first look, the BEER models account for the expectations of the future developments of the underlying fundamentals. Their selection exploits the uncovered interest rate parity. Accordingly, the current interest rate differentials mirror the expected changes in the spot exchange rate, and the risk premium. The trick is then to approximate the expectations about the future real exchange rate by the observed variables, and to find a suitable proxy for the risk premium.<sup>6</sup> The open economy macroeconomics provides guidance for a selection of the

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<sup>5</sup> An alternative measure of the short-term equilibrium would include the transitory factors  $T_t$  as well. Here, we follow Clark and MacDonald (1998) and we keep them aside. Nevertheless, we relax the assumption that some of the determinants of the exchange rate have only transitory effect in our sensitivity analysis.

<sup>6</sup> Note that the selection of BEER-variables exploits the uncovered interest rate parity, through which the spot exchange rate reflects the underlying fundamentals:

$$E_t [\Delta s_{t+k}] = -(i_t - i_t^*) + \lambda_t \quad (i)$$

where the left hand side represents the expected change of the spot exchange rate over a period  $k$ , the  $i_t$  and  $i_t^*$  are the domestic and the foreign nominal interest rate, and the  $\lambda_t$  is the time-varying risk premium.

It is supposed that the increasing risk premium shall cause nominal and real depreciation as well. Clark and MacDonald approximate the risk premium by the relative government debt, i.e. a ratio of the domestic government debt/GDP ratio to the foreign debt/GDP ratio, since they increase a risk of monetization of government debt and a risk of default.

The equation (i) can be restated in the real terms by subtracting the expected inflation differential,  $E_t [\Delta p_{t+k}] - E_t [\Delta p_{f,t+k}]$  from the spot exchange rate and from the interest rate differential. Hence, the real exchange rate  $q_t$  becomes:

$$q_t = E_t [q_{t+k}] + (r_t - r_t^*) + \lambda_t \quad (ii)$$

In (ii) the  $r_t$  is the ex-ante real interest rate. The sign at the risk premium is now negative, since a presence of a risk leads to depreciation.

For the BEER estimation based on the reduced form, the expected real exchange rate is

variables, however the BEER model is not structural, and so the variable selection remains a bit arbitrary.

Additionally, the long-term equilibrium exchange rate is defined as:

$$q_t^{LT} = \beta \bar{Z}_t \quad (3)$$

where the equilibrium exchange rate is determined by the sustainable or long-term values of the fundamentals  $\bar{Z}_t$ . The family of the fundamental equilibrium exchange rates (FEER) models replaces the  $\bar{Z}_t$  by values consistent with sustainable internal and external balances, and those sustainable levels are usually calculated separately, for example following the approach by Chinn and Prasad (2003). Then, the FEER shall be consistent with a stock-flow equilibrium at which changes to asset stocks (as a percentage of GDP) remain zero (Driver and Westaway, 2005). The FEER models were originated by Williamson (1983), the recent contributions include Cline (2008), Phillips, et al. (2013), and Cline (2015).

On the other hand, the statistical approaches such as the permanent equilibrium exchange rates (PEER), consider the long-term filtered series as a suitable approximation of sustainable levels. The methodology of the PEER models is very similar to the BEER models, and sometimes, BEER models are considered as a starting point for the PEER models. In particular, Égert (2004) replaces the variables in the BEER model by their HP-filtered values of fundamentals. Others, i.e. Costa (2005), follow Gonzalo and Granger (1995) to estimate the common long-memory component from the fundamentals. However, Égert (2004) or Driver and Westaway (2005) find both methods as being equivalent, with no preference for any of them.

Recently, Chen and MacDonald (2015) develop an unobserved component of the PEER model where both temporary and permanent component are estimated simultaneously within a framework of a state space model.<sup>7</sup>

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approximated by a regressed of the current real exchange rates on observed values of economic fundamentals.

<sup>7</sup> The other concepts of the medium-term and long-term models of the equilibrium exchange rate include for example the desired equilibrium exchange rate model (DEER), natural real exchange rate model (NATREX) and others. See the survey by Driver and Westaway (2005) for more detailed discussion of these models. Also, the DSGE models are sometimes used for prediction of the long-term component of the equilibrium exchange rate, see Ca'Zorzi, Kolasa, and Rubaszek (2017) for a recent example.

Over the very long term, the equilibrium exchange rate is often associated with the purchasing power parity, and this concept is used quite often in the growth literature to assess the effects of exchange rate misalignments on economic growth, perhaps the most prominent example includes Rodrik (2008).



### III. Methodology

#### III.1 Behavioral equilibrium exchange rate models

For the estimation of the equilibrium exchange rates of the Czech Republic, we rely on the BEER and the PEER models. In contrast to the alternatives, such as the fundamental exchange rate (FEER) models, the BEER and PEER models have minimum requirements on assumptions on the long-run equilibrium values of underlying fundamentals, such as of the equilibrium growth rates, or the sustainable current accounts. Instead, the BEER models estimate the long-term relationship among the fundamentals directly, as indicated in equations (1) and (2).

The estimation of the equilibrium exchange rate of the Czech koruna is not new. Shortly after 2000, the equilibrium exchange rate of the Czech koruna was estimated quite frequently, with the help of various methods. The EU accession motivated the interest, with the expectations of the euro adoption in the future. This literature is summarized in Égert et al., (2006), and by Babetski and Égert (2005) who provide their BEER estimates as well. However, since then, the interest faded away, and it has revived slightly after the Great Recession. Most recently, Komárek and Motl (2012) estimated both the BEER and the FEER model of the Czech koruna, on a sample up to 2011.

Importantly, although there were numerous attempts to estimate the BEER models in the spirit of Clark and MacDonald (1998), these models frequently used different combinations of variables to approximate the expected exchange rate. The models included the net foreign assets and productivity differential, but apparently, there is no consensus about the right set of variables for the BEER model.

Hence, while our primary goal is to learn about the equilibrium exchange rate, we need to take deal with this model uncertainty. Because the previous studies differed in numerous aspects, and not just in the variable selection, we use the existing literature as a guidance which variables can form an appropriate BEER model, rather than replicating them.<sup>8</sup> Overall, we employ seven alternative specifications, and their survey is provided in Table 1.

#### III.2 Impact of the exchange rate commitment

We used three alternative approaches to account for the potential effect of exchange rate commitment on the equilibrium exchange rate. Along with estimates on the full sample, we estimate the models on a sample up to November 2013 (the start of the exchange rate interventions), and since then, the equilibrium exchange rate is obtained as a prediction while knowing the evolution of other variables. Third, following Daude,

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<sup>8</sup> The previous studies employed different estimation methodologies, alternative data series represented the the variables suggested by the theoretical model, and obviously the sample was different as well. Also, some of the earlier studies used variables in levels rather than in logs (such as Komárek and Motl, 2012).

Yeyati, and Nagengast (2016), we include exchange rate interventions that were needed to establish and to maintain the exchange rate commitment explicitly in equation (1).<sup>9</sup>

In the spirit of Driver and Westaway (2005), we assume only a short-term effect of the interventions on the exchange rate in our baseline. Thus, the long-term equilibrium exchange rate is supposed to remain unaffected.<sup>10</sup> By this approach, we treat potential omitted variable bias caused by the omission of exchange rate interventions in equation (1). We relax this assumption in the sensitivity analysis when the interventions are included among the long-run variables as well.

**Table 1: The BEER models**

	1	2	3	4	5	6	7
Net foreign assets ( <i>nfa</i> )	X	X	X	X	X	X	X
Productivity differential ( <i>dprod</i> )	X	X				X	X
Relative export prices ( <i>tnt</i> )			X	X	X	X	X
Terms of trade ( <i>tot</i> )			X	X	X	X	X
Government debt (relative)				o			o
Government debt (log)					o		
Government expenditure (ratio)						o	
Interest rate differential				o	o		o
Net exports (ratio to GDP)	X						
Investments (ratio to GDP)	X						

Note: X = long-run effect; o = transitory effect; model 1 - Komárek and Motl (2012), model 2 - Babelski and Égert (2005), model 3 - Clark and MacDonald (1998) with no transitory variables, model 4 - Clark and MacDonald (1998) full specification, model 5 - Clark and MacDonald (1998) full specification but with log of government debt, model 6 - Grifoli et al. (2015) but with productivity differential rather than real GDP, model 7 - Chen and MacDonald (2015) extended for the net foreign assets and the relative export prices. The dependent variable is always the real exchange rate, variables are in logs.

<sup>9</sup> The Czech National Bank makes the data on exchange rate interventions public so their explicit inclusion is feasible and we do not have to resort to a necessity to proxy this variable for example by the difference between growth in reserves and money aggregate M2 used by Daude et al. (2016) or by the so called relative money gap, the difference between growth of M2 and growth of output which are used by Grifoli et al. (2015).

<sup>10</sup> Driver and Westaway (2005) argue: "Monetary policy will not be able to influence the medium and long-run values of fundamentals; however, it may well be able to influence the transition path back to equilibrium."

### III.3 Model averaging

Overall, we estimated seven distinct BEER models, all with three different treatments of the exchange rate interventions, that is 21 models in total. To learn about the equilibrium exchange rate from this wide range of models with potentially conflicting implications, we resorted to averaging of the estimated equilibrium exchange rates.

The idea that the combination of forecasts or models can bring better insight than the individual models has been increasingly popular. An overview of methods and applications is provided in Weiss, Raviv, and Roetzer (2016).

We opted for a simple averaging against the other methods that help to address the model uncertainty, such as the Bayesian model averaging, because these alternative models give higher posterior probability to models with higher goodness of fit. However, in our context, we do not have any reliable a priori information that models with better fit are better approximations of the equilibrium exchange rate in comparison to models that predict higher and more persistent exchange rate misalignments. Also, a combination of forecasts using simple averaging is known as a surprisingly simple and powerful benchmark, even in comparison with more sophisticated methods (Genre, Kenny, Meyler, and Timmermann, 2013).

### III.4 Estimation method

All models were estimated using the autoregressive distributed lags (ARDL) model. It enables estimation of the long-run relationship among non-stationary and potentially cointegrated variables. To test for cointegration, and implicitly for the existence of a stable and long-run relationship between the real exchange rate and the fundamentals, we use the bounds testing procedure developed for the ARDL models (Pesaran, Shin, and Smith, 2001). As it shows up, the existence of the long-term relationship is supported by the bounds testing in all models under consideration.

We use the "restricted constant" structure of the deterministic terms that allows to include constant both in ARDL and in the cointegrating equation as well. Although the inclusion of constant in the equation of the equilibrium exchange rate is not consistent with the theory, it is in line with most of the literature, including Clark and MacDonald, 1998).

The number of lags is set according to the Schwarz information criterion (SIC), mainly to avoid overfitting. However, when significant autocorrelation in residuals appeared, the Akaike information criterion is used.<sup>11</sup>

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<sup>11</sup> While the estimates of the respective BEERs were affected by the utilisation of the AIC instead of the SIC, the impact on the average BEER was rather limited. However, since the bounds testing procedure assumes uncorrelated residuals, we preferred to keep well-specified models, rather than to insist on consistent model selection rules.

### III.5 Permanent equilibrium exchange rate models

Along with the BEER models, we estimate their long-run counterparts, the Permanent equilibrium exchange rates (PEER). We depart from the estimated coefficients of the long-run relationship among the fundamentals of all of the BEER models. Then, following Égert (2004), we replace the fundamental variables by their filtered values. As with the BEER models, we average across individual PEER models to obtain the final estimate of the PEER model.

In our baseline, we opt for the conventional HP filter, with the smoothing parameter set to 1600. However, and because of the growing awareness of utilisation of the HP filter due to its end-sample bias, we use two alternative estimates of the long-run components as additional sensitivity checks. The first option is the one-sided HP filter, and the second one is the regression filter proposed by Hamilton (2018) in which the variable at  $t+h$  is regressed on the four values as of the date  $t$ .<sup>12</sup>

None of these two methods exploits the information from future observations, so the trend is not revised backwards when a new observation arrives. Hence, the results from these filters mimic the information available to policymakers in times of their decisions better than the HP filter.

## IV. Data

Our dataset starts in the first quarter of 2001 and ends in the second quarter of 2017. The initial point was determined by data availability. In particular, the net foreign assets of the banking sector are not available before 2001 on a quarterly frequency. We opted for the quarterly against the monthly frequency because the data from national accounts, i.e. the labour productivity and fiscal data, are unavailable at this higher frequency, and we wanted to avoid interpolation. The primary sources for our data were the Czech National Bank database (ARAD), the Czech Statistical Office, and Eurostat.

The dependent variable is always the logarithm of the real bilateral CZK/EUR exchange, which is the log of the nominal exchange rate deflated by the industrial producer price index.

Clark and MacDonald (1998) constructed the BEER models around three main fundamentals determining the long-run equilibrium exchange rate: the net foreign assets (NFA), the relative export prices (TNT) that serve as a control for the Balassa-Samuelson effect, and the terms of trade. The net foreign assets are represented by the log of the ratio of the net foreign assets of the banking sector on the GDP, which is a standard proxy for the net foreign assets in the literature. The relative export prices, TNT, is a ratio of the domestic prices to the domestic export prices, in relative terms to the same ratio of the Euro area. Also, the terms of trade are expressed in relative terms, i.e., as a log of the ratio of the domestic and the Euro area terms of trade.

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<sup>12</sup> Hamilton (2018) proposes  $h=8$ , so that the regression is  $y_t = a_1 y_{t-8} + a_2 y_{t-9} + a_3 y_{t-10} + a_4 y_{t-11} + u_t$ .

Following Babetski and Égert (2005) and Komárek and Motl (2012), we control for the differential in labour productivity as well. The narrowing gap between labour productivity in the Czech Republic and the Euro area reflects the real convergence and fosters the trend appreciation of the koruna. Until recently, the productivity differential used to appear mainly in models of the equilibrium exchange rate of emerging countries. However, the importance of productivity differentials as a determinant of the real exchange rate even for advanced countries has been recently corroborated by Bordo et al. (2017). It appears in the permanent equilibrium exchange rate model of the EUR/USD exchange rate by Chen and MacDonald (2015) as well.

Finally, we have extended the baseline BEER models for the exchange rate interventions, to account for their effect on the exchange rate. The Czech National Bank makes the information about the scale of the exchange rate interventions publicly available so that no indirect approximation is required.

The list of all variables and their respective transformations can be found in Appendix 1, and the plot of all variables in Appendix 2.

## V. Results

### V.1. Average BEERs with PEERs

Our main results, the estimates of the average BEER models, along with the average PEER models, are presented in Figure 1.<sup>13</sup> Broadly, the average equilibrium exchange rates signalize that the Czech koruna had been persistently undervalued before the Great Recession, approximately by 5% till 2006, and below 2% till 2008. Then, over the most volatile years 2008 and 2009, the exchange rate mostly followed the equilibrium exchange rate.

According to the average BEER model, the misalignments (Figure 2) were low even in the aftermath of the Great Recession. Since then, the discrepancy among individual BEER models gradually increased, and while some BEERs suggested moderate overvaluation of the koruna before the introduction of the exchange rate commitment in late 2013, other models implied the opposite. Hence, in times when the knowledge of the actual equilibrium exchange rate was needed at most for a design of the exchange rate policy, the BEERs failed to provide consistent results.

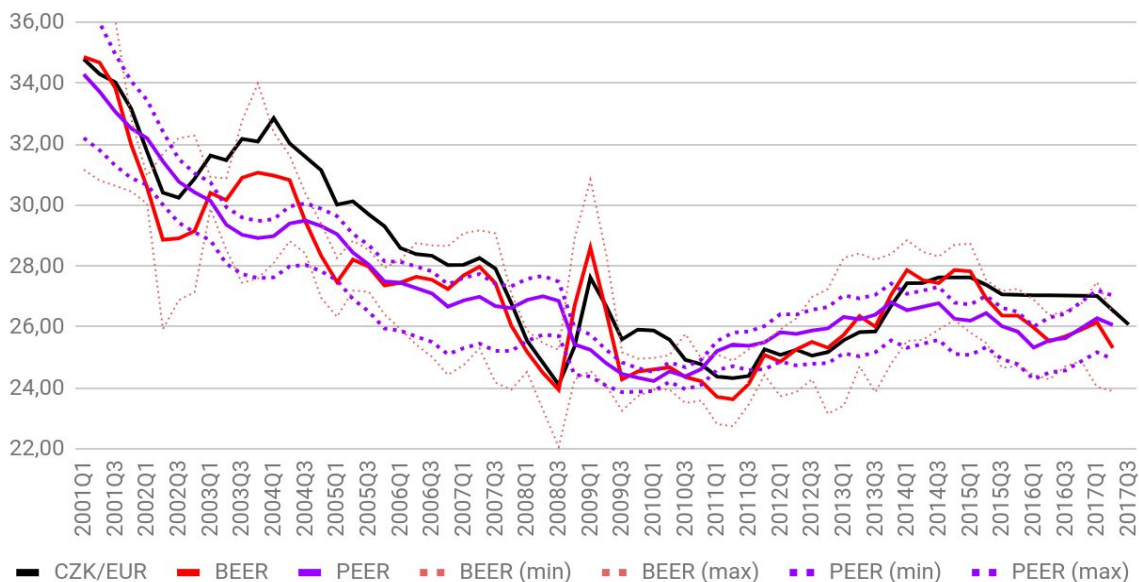
Our estimates imply that the equilibrium exchange rate depreciated with the introduction of the exchange rate commitment as well. Hence, the short-term misalignments remained rather low in 2014, due to the evolution of the underlying fundamental variables. On the other hand, the commitment lead to a long-term misalignment vis-a-vis the PEER model.

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<sup>13</sup> The estimated equilibrium exchange rates, misalignments, and all estimation outputs are available on-line at <https://docs.google.com/spreadsheets/d/1sJ-paemsjUAm5v4i159Uhg25Bjx7Bno-UKCBBSJIU0E/edit?usp=sharing>.

# Figure 1: Equilibrium exchange rate - CZK/EUR

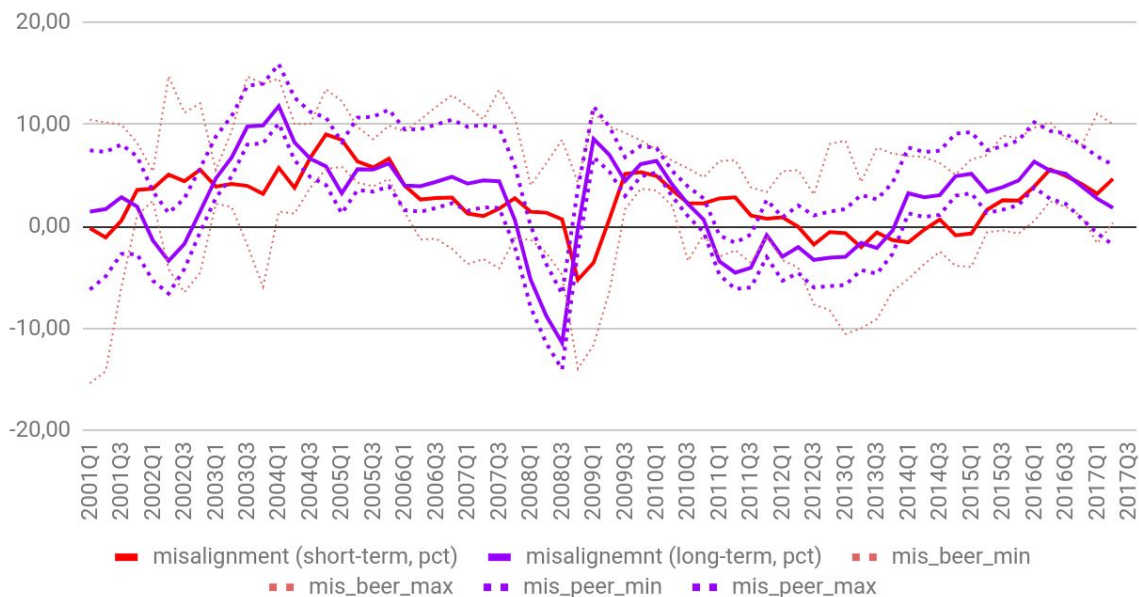
Average BEER and PEER



Note: The dotted lines show the minimum and the maximum BEER (PEER) estimated for a given point of time across all models. Hence, the dotted lines should not be associated with any individual BEER or PEER models.

# Figure 2: Exchange rate misalignments

Short-term and long-term misalignments, %



Note: The dotted lines show the minimum and the maximum BEER (PEER) misalignment estimated for a given point of time across all models. Hence, the dotted lines should not be associated with any individual BEER or PEER misalignments.

The average BEER model started to depart from the actual exchange rate in 2015 and approached to the long-term equilibrium approximated by the PEER model. Then, in 2016 and 2017, the koruna was misaligned by 5% to the equilibrium exchange rate. These misalignments imply that the rising pressure of the market on the Czech National Bank to remove the commitment was not purely speculative, and it has reflected the underlying fundamentals, at least to some extent. The central bank was forced to support the commitment by interventions that first occurred in the third quarter of 2015, and lasted until the commitment has been removed.

However, it should be noted that the misalignments during the commitment were less persistent and not as sizeable as those in the pre-crisis period without an active exchange rate policy.

In comparison to the other literature, our average BEER model leads to higher and more persistent deviations than the BEER-based misalignments presented by Komárek and Motl (2012).<sup>14</sup> However, our results are broadly consistent with their FEER-based estimates. Qualitatively, our PEER results are consistent with the estimates of the FEER model of the Peterson Institute of International Economics, but opposite to those of the Centre d'études prospectives et d'informations internationales.<sup>15</sup>

## V.2. Individual BEERs and PEERs

We can turn our attention to the individual BEER and PEER models now. The resulting trajectories all individual models are shown in Figures 3a and 3b; along with the respective time-averages, and with the actual exchange rate.

The subplots of Figures 3a and 3b report the BEER and PEER models for the three different treatments of the period of the commitment: the full sample estimates, predictions based on the estimates on the sample up to 2013 Q3, and the models with interventions explicitly included among explanatory variables.

These plots demonstrate a wide discrepancy among the individual BEERs that reaches up to 4 CZK/EUR in several periods, especially in 2002/2004, 2006/2008, and in 2012/2013. For the last two years of the commitment (in 2015–2017), all models imply undervaluation of koruna vis a vis the equilibrium exchange rate, although the amplitude of the deviations remains model-specific.

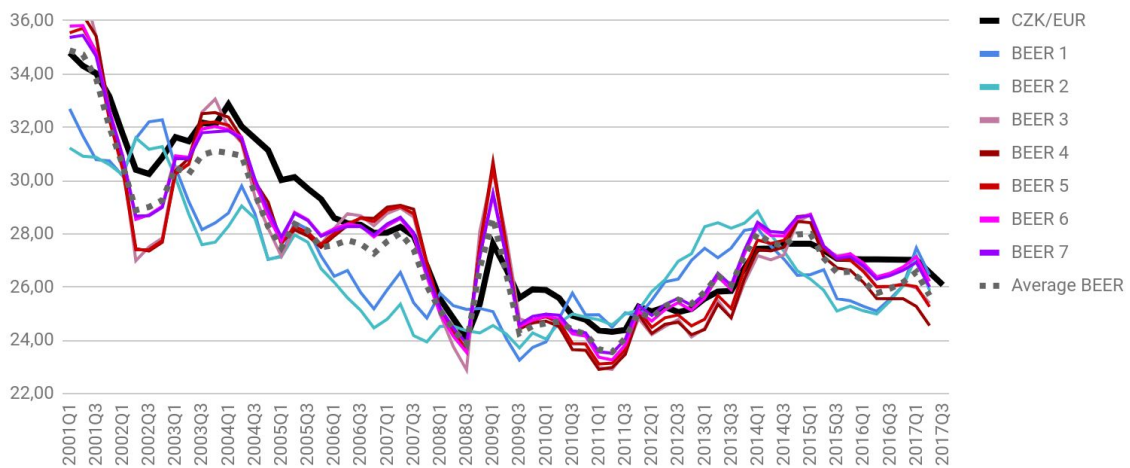
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<sup>14</sup> We suspect the difference might be driven by different estimation methodology rather than by some differences in specifications of time series entering the model, but Komárek and Motl do not specify the estimation method of their BEER model.

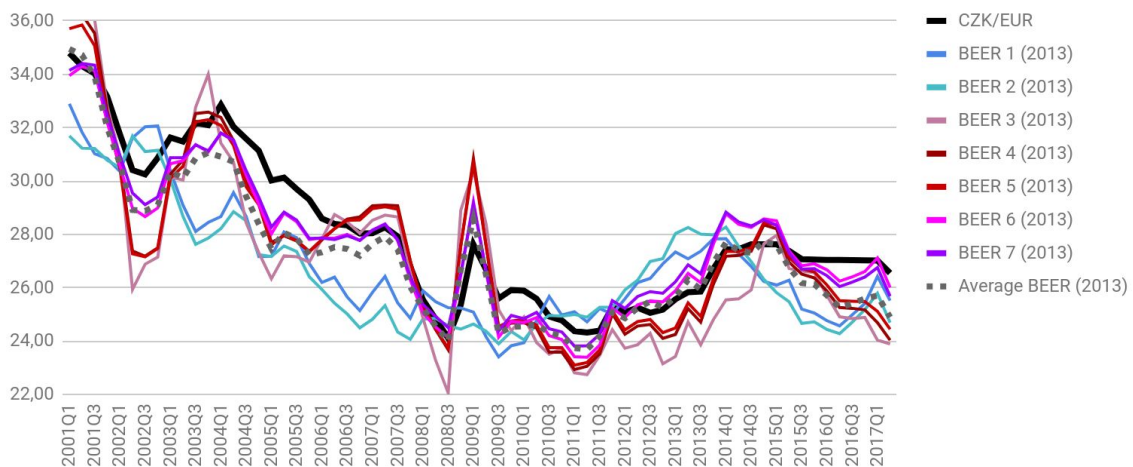
<sup>15</sup> In our paper, we focus on the nominal equilibrium exchange rate and nominal misalignments. However, the exchange rate commitment is supposed to work via real misalignments mainly (Svensson, 2003). In the case of the recent decade, the differences between the real and the nominal exchange rates are rather minor, because of low and stable inflation. The real misalignments are reported in Appendix 4.

**Figure 3a: Individual BEER models**

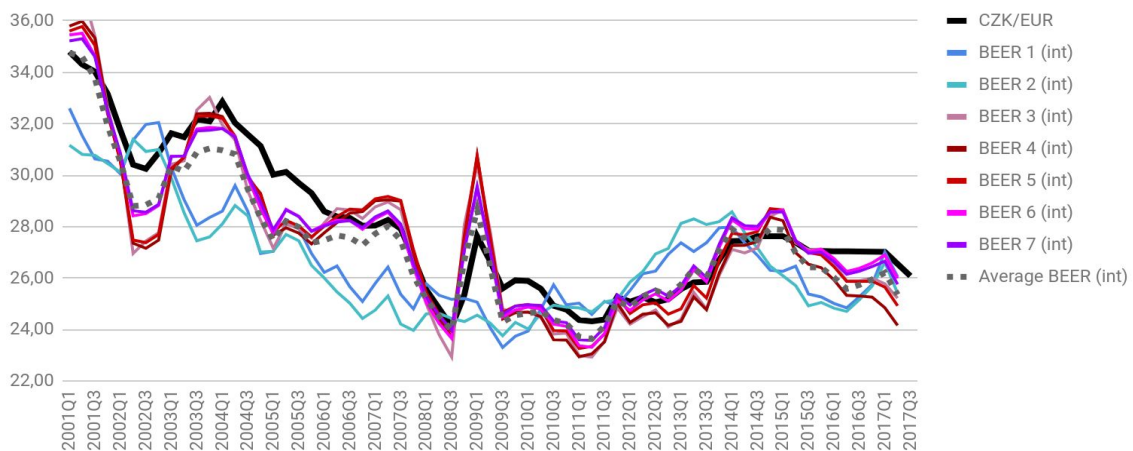
**BEERs based on full sample**



**BEERs based on 2013 sample**



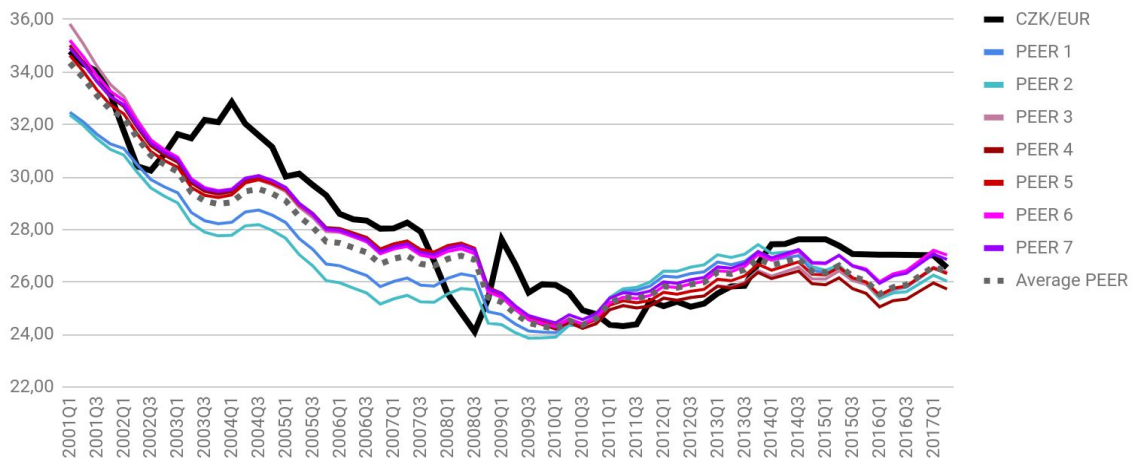
**BEERs based on models with interventions**



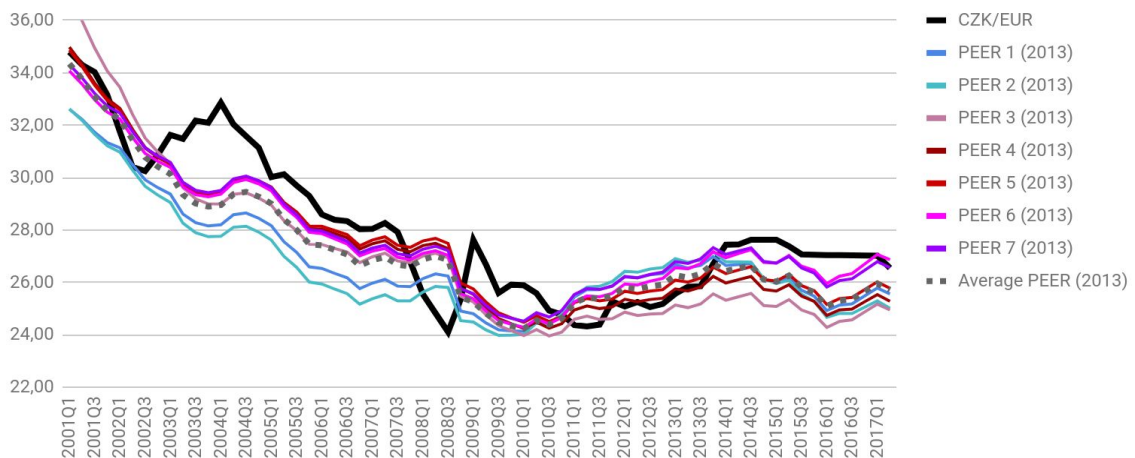


**Figure 3b: Individual PEER models**

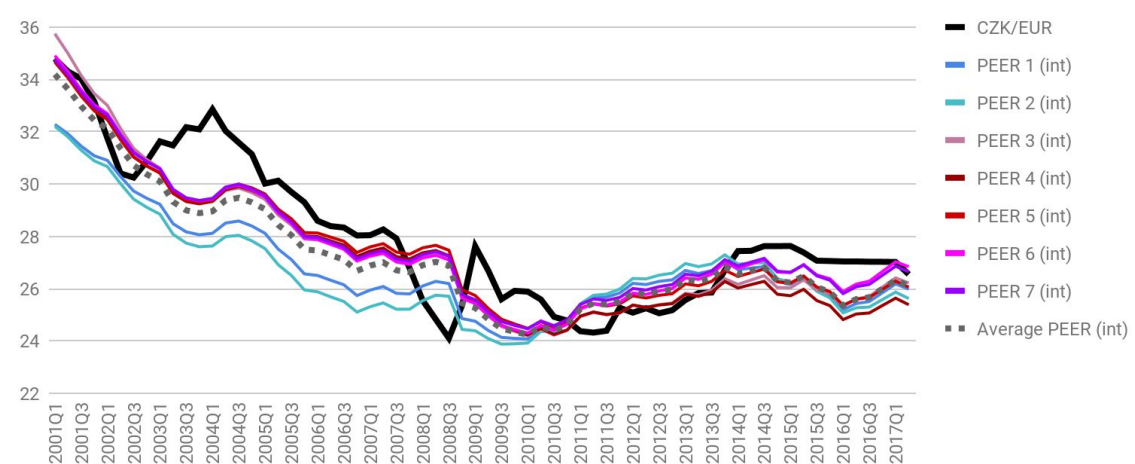
PEERs based on full sample



PEERs based on 2013 sample



PEERs based on models with interventions



As expected, the PEER models smooth out most of the short-term fluctuations, and they are also less dissimilar than the BEER models. Hence, the policy lessons from PEER models are more consistent across individual models as well. In particular, the models confirm that the actual exchange rate had been undervalued before the Great Recession, but slightly overvalued before the adoption of the exchange rate commitment in November 2013. To what extent are these implications driven by choice of the two-sided Hodrick-Prescott filter will be subject of the sensitivity analysis.

All the subplots of Figures 3a and 3b reveal consistent patterns for all three alternative treatments of the commitment, which is confirmed by the respective averages depicted in Figure 4. Moreover, due to averaging, the dispersion among these alternative sets of models arises at the end of the sample only, and it is way smaller than the discrepancy among individual models with the same treatment of the interventions.

When the exchange rate commitment was abandoned in the second quarter of 2017, the discrepancy among individual BEER models reached 2 CZK/EUR, but the difference among the time-averages across different treatments of the commitment is only 0.8 CZK EUR. In any case, all BEER models imply that the koruna was undervalued at the end of the commitment.

Among the seven models we have considered, we can identify three groups of models that move together and differ from models from the other groups. The first group consists of models 1 and 2 in which the trends in relative productivity are approximated by the productivity differential only, and they do not control for changes in relative export prices. Their model specification follows Komárek and Motl (2012) and Babecky and Égert (2005). These models suggest the most substantial and most persistent undervaluation of the koruna in 2002-2007 and imply the largest overvaluation of the koruna before the introduction of the exchange rate commitment. At the same time, these models predict the largest misalignments during in 2015/2016.

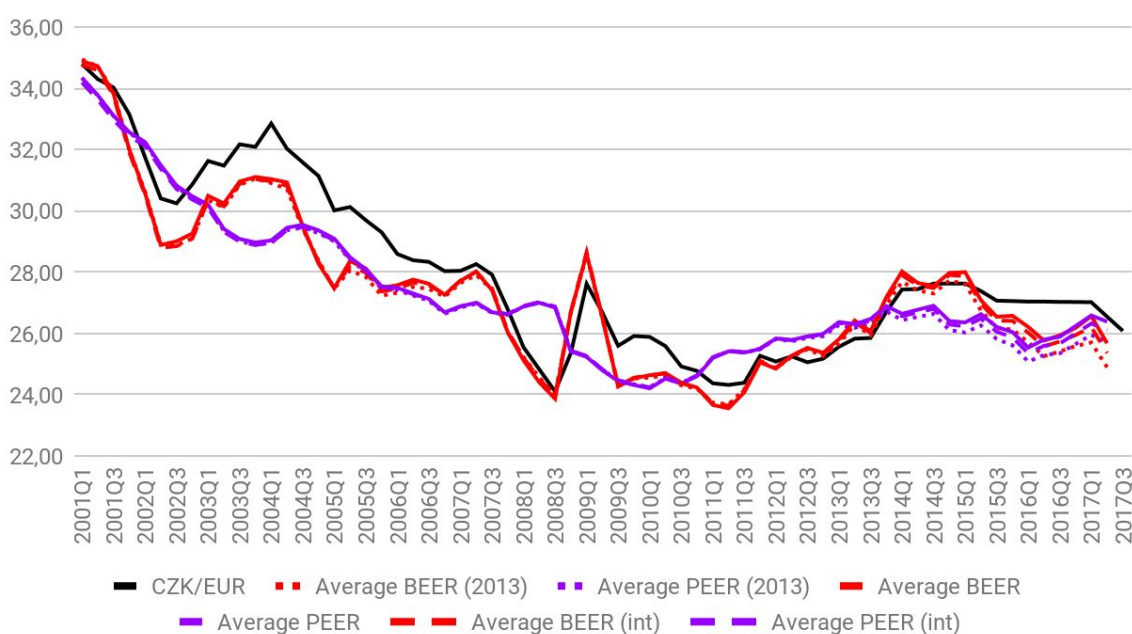
The other models that control for the change in the relative export prices (TNT\_PPIS) are more volatile. They follow the developments of the actual exchange rate during the Great Recession (2008/2009), and in the initial period of the commitment as well. The prominent role of the relative export prices in the dynamics of the fitted equilibrium exchange rates should not be surprising because this variable is directly affected by a change in the actual exchange rate. When depreciation occurs, the export prices expressed in the domestic currency increase, and, all other variables being constant, the TNT\_PPIS decreases.

The last two models (6 and 7) that include the differential in labour productivity and the relative export prices simultaneously follow the average BEER most closely. Furthermore, their respective PEER counterparts imply almost perfect alignment of the koruna to the equilibrium at the end of the commitment in 2017Q3. Nevertheless, the other PEER models imply larger undervaluation, of 1 CZK/EUR at least.

All in all, the results demonstrate that the uncertainty from the alternative variable selection is larger than the uncertainty about the proper treatment of the exchange rate commitment.

Figure 4: Different treatments of the commitment

BEER and PEER models



Note: Average\_2013 denotes BEER and PEER estimates on a sample ending in 3Q 2013. Average\_full shows average BEERs and PEERs estimated on full sample. Average\_fullint represent the averages across models with interventions among the explanatory variables.

### V.3. Coefficient estimates

Naturally, the discrepancies among the estimated BEERs and PEERs reflect the differences in their estimated coefficients. The cointegrating equations that define the equilibrium exchange rates for the full sample estimates are presented in Table 2, and the remaining samples are in Appendix 3.

It can be seen that besides the specification, the main differences are in the coefficients at the net foreign assets with even different signs across models. However, both signs appear in the empirical literature for the Central and Eastern European countries, as documented by Égert (2004). Intuitively, an increase in the net foreign assets shall increase domestic income and prices, so that the real exchange rate shall appreciate, and the sign shall be negative. On the other hand, decreasing net foreign assets can mirror capital inflow which, in the context of the EU New Member States, helped to increase the domestic labour productivity and to boost the competitiveness, and so a decrease of the net foreign assets can bring appreciation as well. Hence, both signs are plausible, and the differences in estimated coefficients do not lead to rejection of any of the models for providing counterintuitive results<sup>16</sup>. The differences in the coefficients at

<sup>16</sup> Égert (2004) in section 6.2.2 provides a detailed survey of that time literature, and he shows that the proportion of both signs of the coefficients at the net foreign assets is almost the same. Also, he reminds that the NATREX models consider the net foreign assets as endogenous: "(Within the model...) if investment rises in the open sector, capital inflows reflected in a decline in

the net foreign assets might also reflect collinearity between the net foreign assets and other determinants of the equilibrium exchange rate models in the regressions because these determinants are the drivers of the net foreign assets as well<sup>17</sup>.

The coefficients at the relative export prices (TNT\_PPIS) and the terms of trade (TOT\_t) are both negative and in line with the intuition across all models, although the coefficients at the terms of trade are often insignificant. Hence, an increase in both variables implies appreciation (as in Clark and McDonald, 1998).

In all models, we rejected the null hypothesis of no long-run relationship using the F-bounds test, at least at 5%. Hence, the long-run relationship has an equilibrium nature, and the misalignments are mean-reverting. The Q-stats confirm the null of no remaining autocorrelation in residuals, and so the results of the F-bounds test are valid.

Overall, the estimation results do not provide reasons to reject any of the models under scrutiny due to apparent misspecification, or a failure to meet the specification tests. Indeed, model averaging becomes a natural approach to retain information from all the individual models.

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*net foreign assets causes the real exchange rate to appreciate in the medium-run. In the long run, when investment starts working in the open sector, the trade balance ameliorates, resulting an increase in net foreign assets, leads to the appreciation of the real exchange rate in the second phase."* When looking at our results, one might wonder whether the sign of the net foreign assets does depend on model specification, as it is positive or insignificant, when the productivity differential is included. However, this conclusion cannot be generalized. Komárek and Motl (2012) end up with a negative sign of NFA even with differential of labour productivity included the model.

<sup>17</sup> Collinearity is stressed as the main reason for the ambiguity of the results by Chen and MacDonald (2015).

**Table 2: Coefficient estimates, BEER models (full sample)**

	<b>m01</b>	<b>m02</b>	<b>m03</b>	<b>m04</b>	<b>m05</b>	<b>m06</b>	<b>m07</b>
Method	ARDL, BIC	ARDL, BIC	ARDL, BIC	ARDL, BIC	ARDL, BIC	ARDL, AIC	ARDL, BIC
Dependent variable	LOG_RER	LOG_RER	LOG_RER	LOG_RER	LOG_RER	LOG_RER	LOG_RER
Net foreign assets	0,330 (0,077**)	0,275 (0,077**)	-0,139 (0,081*)	-0,169 (0,075**)	-0,122 (0,058**)	-0,004 (0,044)	-0,018 (0,032)
Productivity differential	-1,412 (0,341**)	-1,768 (0,145**)				-0,327 (0,148**)	-0,350 (0,093**)
TNT (relative exp prices)			-2,778 (0,163**)	-2,816 (0,231**)	-2,669 (0,185**)	-2,232 (0,155**)	-2,172 (0,112**)
Terms of trade			-2,087 (1,015**)	-0,892 (0,546)	-0,966 (0,542*)	-1,136 (0,501**)	-0,871 (0,340**)
Net exports	-0,100 (0,131)						
Investment	0,214 (0,198)						
Constant	1,606 (0,894**)	2,221 (0,257**)	3,559 (0,269**)	3,651 (0,235**)	3,507 (0,183**)	3,128 (0,143**)	3,179 (0,104**)
<b>F-Bounds test</b>							
F-statistics	6,149	8,072	6,968	11,449	10,912	7,315	12,043
k	4	2	3	3	3	4	4
5% lower bound	2,750	3,285	2,976	2,976	2,976	2,750	2,750
5% upper bound	3,755	4,070	3,896	3,896	3,896	3,755	3,755
<b>AR component in residuals (12 lags)</b>							
Q-stat	7,865	6,022	12,731	5,104	4,288	15,972	6,525
p-value	0,796	0,915	0,389	0,954	0,978	0,193	0,887

Note: The table reports the long-run coefficients of the cointegrating relations that define the long-run relationships. Standard errors in parentheses. \* - significant at 10%, \*\* - significant at 5%. The F-statistics always exceeds the upper bound, and the Q statistics does not indicate significant autocorrelation. Critical values for the F-Bounds test are for 65 observations.

## VI. Sensitivity analysis

Besides estimating the average BEER over multiple models and across different treatments of the period of the commitment, two additional sensitivity checks are provided. First, we include additional explanatory variables within the cointegrating equation, and second, we apply different trend and cycle decompositions for the PEER models.

### VI.1 All variables within the cointegrating vector

Models 8-11 contain additional explanatory variables that were considered as short-term determinants of the exchange rate: The relative government debt, the debt ratio, the government expenditure ratio, the interest rate differential, and the log of interventions. These variables are supposed not to have any impact on the equilibrium exchange rate, and they were excluded from the cointegrating equation. However, the choice which variable is supposed to have a long-term or short-term effect is at least to some extent arbitrary. Also, the deviations of some of these variables from means are persistent, and the also the vast extent of the exchange rate interventions to defend the exchange rate commitment in 2015-2017 might have long-lasting effects on the exchange rate as well.

Hence, we re-estimated the relevant models with all variables considered as determinants of the equilibrium exchange rate. All lags were selected according to the Schwarz information criterion, or the Akaike information criterion when the residuals of the smaller models were autocorrelated.

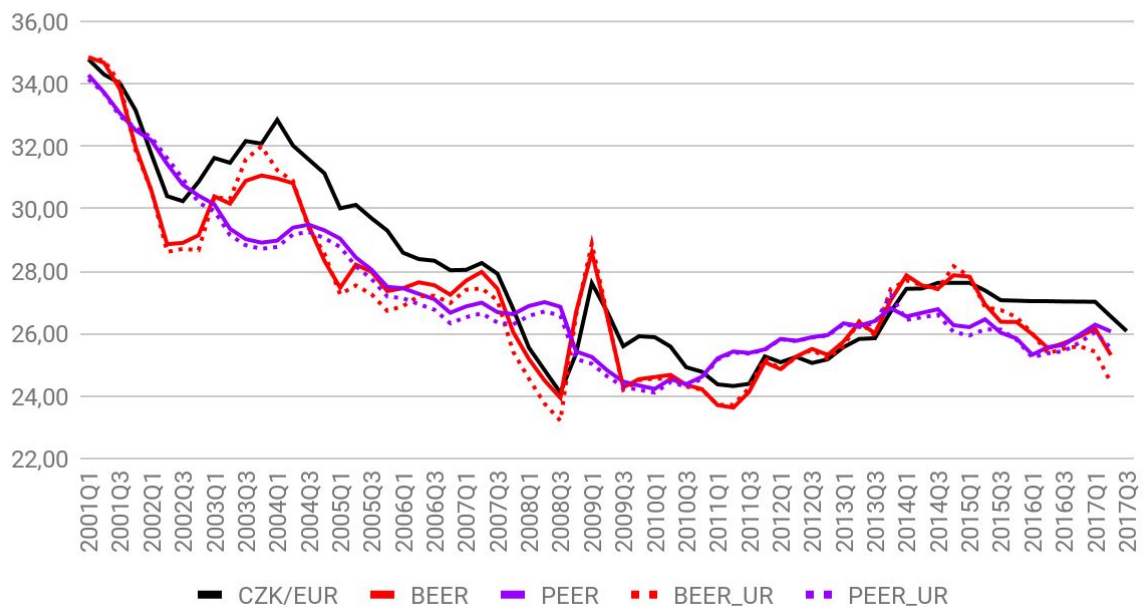
The resulting individual BEERs are more volatile than those with a restricted set of variables included within the cointegrating vectors; this holds especially for the models estimated on the sample till 2013 (Appendix 5). However, when looking at the average BEER, PEER, and their respective misalignments, the results generally confirm our previous findings (Figures 5 and 6), although the unrestricted models predict more substantial undervaluation at the end of the commitment.<sup>18</sup>

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<sup>18</sup> Larger undervaluation appears across all three treatments of the exchange rate commitment.

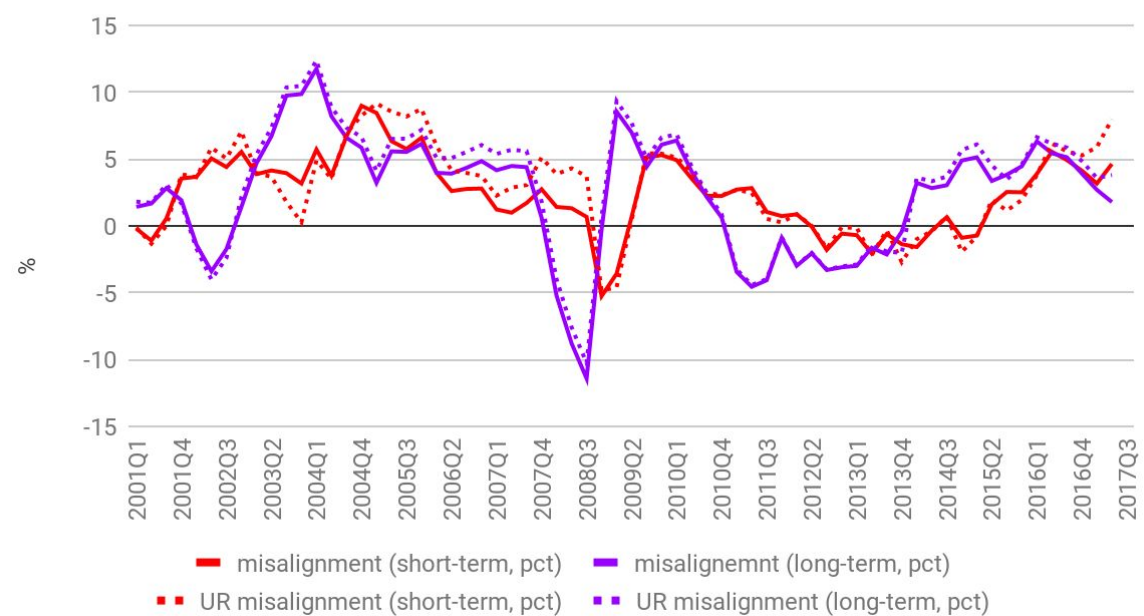
# Figure 5: Sensitivity analysis - Unrestricted ARDL

Sensitivity analysis (Unrestricted ARDL models - UR)



# Figure 6: Sensitivity analysis - Exchange rate misalignments

Short-term and Long-term; Sensitivity analysis (Unrestricted ARDL models - UR)





## VI.2 Alternative trend/cycle decomposition for PEERs

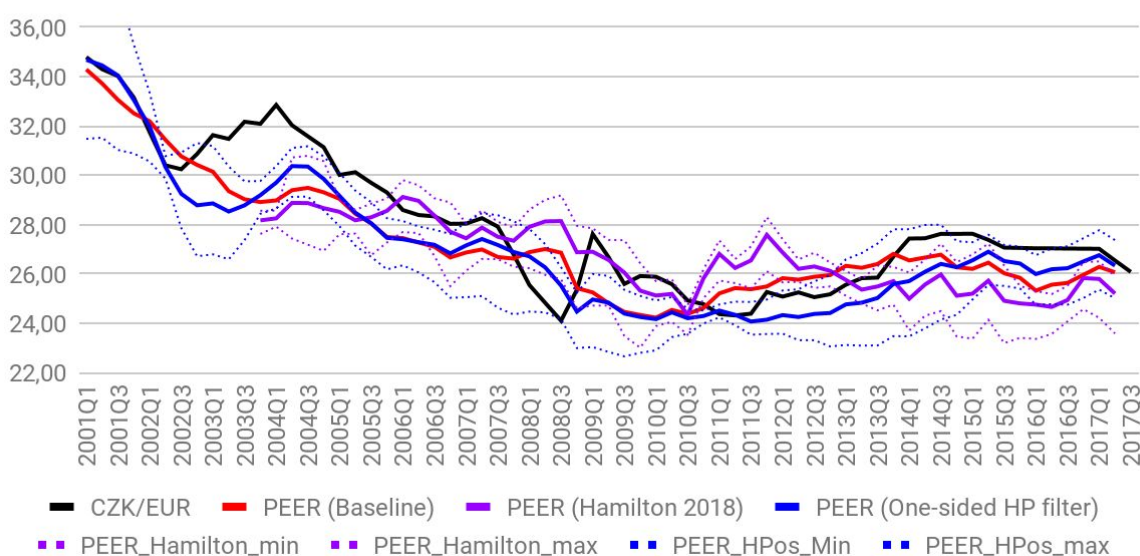
The main drawback of the Hodrick-Prescott filter is its end-sample bias so that the filtered trend is being revised backwards when the new data arrives. Around the business cycle turning points, these revisions can be quite substantial. Hence, we use two alternative trend-cycle decompositions for the estimation of the PEER models, the one-sided HP-filter, and the 2-year ahead predictions proposed by Hamilton (2018).

These alternative PEER models are based on one-sided filters. Thus their values are not affected by future observations. Thus, they are more closely related to what the policymakers could have observed in times of their policy-decisions, although we still abstract from potential time-variation in coefficients.<sup>19</sup>

Several insights appear. First of all, even with these alternative filters, the undervaluation of the koruna in times of the exchange rate commitment remains corroborated. The most significant undervaluation is predicted by the Hamilton's filter. However, these alternative trends provide a contradictory assessment about the exchange rate misalignment before 2013, when the decision about the exchange rate commitment has been made. Hence, our results point to the difficulties facing by the Czech National Bank to set the appropriate value of the exchange rate floor when facing the inherent uncertainty about the real value of the equilibrium exchange rate.

**Figure 7: Alternative trends in PEERs**

HP filter (baseline), One-sided HP filter, Hamilton (2018) filter



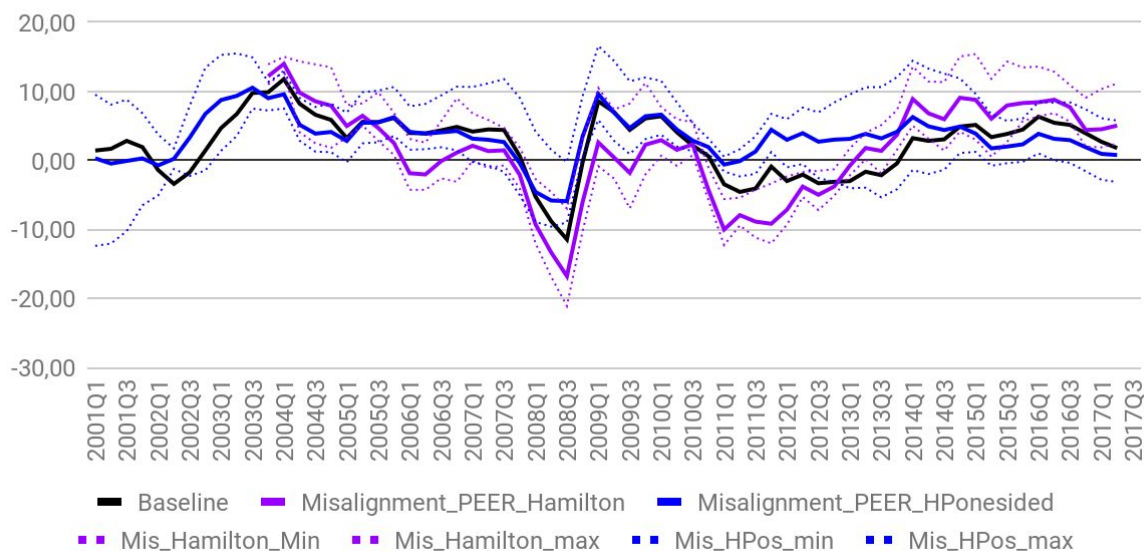
Note: The dotted lines show the minimum and the maximum BEER (PEER) estimated for a given point of time across all models. Hence, the dotted lines should not be associated with any individual BEER or PEER models.

<sup>19</sup> The potential time-variation is indirectly addressed by the estimations of all BEER models on full sample and on a sample till 2013.



**Figure 8: Alternative trends in PEER-Misalignments**

HP filter (baseline), One-sided HP filter, Hamilton (2018) filter



Note: The dotted lines show the minimum and the maximum BEER (PEER) misalignment estimated for a given point of time across all models. Hence, the dotted lines should not be associated with any individual BEER or PEER misalignments.

## VII. Conclusions and policy implications

We have estimated several behavioural equilibrium exchange rate models (the BEERs) of the Czech koruna against the euro, and their long-run counterparts, the permanent equilibrium exchange rate models (the PEERs). We focused on the development of the equilibrium exchange rate in the years of the exchange rate commitment, adopted by the Czech National Bank as a response to deflationary outlooks and the policy rate already constrained by the zero lower bound.

Unlike other papers, we tackled the model uncertainty associated with the behavioural equilibrium exchange rates explicitly. We have considered seven alternative BEER specifications, that appeared in the previous literature, and three different approaches to treating the times of the exchange rate commitment, thus 21 models in total.

These differences in specifications did matter for the resulting estimates of the equilibrium exchange rate. Although none of the models was rejected due to misspecification, the discrepancy among the individual BEERs exceeded 10 % in many parts of the sample. Thus, the exchange rate misalignments derived from the individual BEER and PEER models suffer by substantial model uncertainty. Interestingly, the primary source of uncertainty was the choice of variables for the BEER model and not the way how the exchange rate commitment was treated.

To deal with the model uncertainty, we have proposed to learn about the equilibrium exchange rate from model combination achieved via averaging across all estimated

BEER and PEER models. By doing so, we incorporated the information provided by all of the models, and the minimal and maximal estimates of the BEERs and PEERs provide information about the uncertainty surrounding the estimates.

Our results imply that the size and sign of the exchange rate misalignments became uncertain after the Great Recession. Still, there is some weak indication that the koruna had been slightly overvalued just before the introduction of the exchange rate commitment in November 2013. Then, the adoption of the exchange rate commitment lead to a misalignment vis-a-vis the permanent exchange rate, but the koruna remained aligned to the short-term equilibrium represented by the average BEER. Although the koruna appeared aligned with the equilibrium in 2014, we do not argue that the koruna would have depreciated even without the intervention of the Bank, because the past misalignments were quite persistent.

In any case, our results suggest that the Czech National Bank faced considerable uncertainty in 2013, namely when deciding about the actual value of the exchange rate floor to be defended in the upcoming years. Although officially the Czech National Bank employs own large-scale DSGE models for forecasting, several CNB's economists published papers with the reduced-form estimates of the equilibrium exchange rate of the Czech Republic in the past. Hence it is quite likely that they did similar internal calculations in 2013 as well. The uncertainty behind the Bank's decision could have been a reason for somewhat inconsistent communication of the exchange rate commitment to the public, and it also justifies the occasional notes that the devaluation should have been more profound.

On the other hand, the uncertainty about the sign of the misalignment disappeared in 2015. We have provided robust evidence that the koruna became undervalued since then, and the exchange rate misalignments were gradually increasing. Interestingly, the timing of the emergence of misalignments overlaps with the start of exchange rate interventions of the Czech National Bank intended to defend the exchange rate commitment. Based on our estimations, the Czech koruna had been undervalued by 5% in 2016 and 2017, before the exchange rate commitment was finally abandoned.

All in all, we have demonstrated that even the exchange rate commitment policy remains more art than science, no matter that its mechanisms have been investigated in numerous structural models already. However, in these models, the policy is supposed to work through the exchange rate misalignment which implies that the main instrument of the monetary policy is, in fact, an unobserved variable, beyond the direct control of the central bank. To what extent the central bank shall communicate these difficulties, and the uncertainty about its own policy to the public remains to be learned.

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# Appendix

## Appendix 1: Variable definitions and data sources

Exchange rate: Nominal CZK/EUR exchange rate, monthly average, Czech National Bank.  
Deflator for the real exchange rate: PPI in industry, quarterly, 2005=100, Czech Statistical Office.

Net foreign assets ratio: Net foreign assets of the banking sector, divided by the annual nominal GDP; EUR, mil.; IMF IFS, net foreign assets, depository institutions (equivalent to Czech National Bank, ARAD database, series net foreign assets in SDDS Plus, Financial sector, Analytical accounts of monetary and financial institutions).

Labour productivity: Eurostat, Labour productivity and unit labour costs [namq\_10\_lp\_ulc], seasonally adjusted using TRAMO/SEAS. Productivity differential calculated as the log difference between the labour productivity in the Czech Republic and in the Euro area.

Relative export prices:  $TNT = \ln \left( \frac{PPI_{domestic}(CZ)}{PPI_{export}(CZ)} / \frac{PPI_{domestic}(EA)}{PPI_{export}(EA)} \right)$ .

PPI: 2010=100, Eurostat, Producer prices in industry, domestic market - quarterly data [sts\_inppd\_q] and Producer prices in industry, non domestic market - quarterly data [sts\_inppnd\_q].

Terms of trade:  $TOT = \ln \left( \frac{P_{export}(CZ)}{P_{import}(CZ)} / \frac{P_{export}(EA)}{P_{import}(EA)} \right)$ ;

Price levels of export and import: Eurostat, GDP and main components (output, expenditure and income) [namq\_10\_gdp]

Relative stock of government debt. Following Clark-MacDonald (1998) it express the time-varying risk-premium, they suggest using the *gnfl* series from the OECD. However, the OECD data are at annual frequency only. Hence, we used equivalent Eurostat data for the net foreign assets of the public sector, i.e. *nfa\_g\_eu15\_pct* and *nfa\_g\_cz\_pct* (Eurostat, Quarterly financial accounts for general government [gov\_10q\_ggfa])

Interventions, EUR, mil., available via the Czech National Bank ([https://www.cnb.cz/cs/financni\\_trhy/devizovy\\_trh/DEVOP\\_EUR.HTML](https://www.cnb.cz/cs/financni_trhy/devizovy_trh/DEVOP_EUR.HTML)); spot trades. Quarterly data: sum over months. Spot trades of 2004 to 2012 were not intended to affect the exchange rate, so the value of interventions restricted to zero for this period. Interventions enter the model in logs, i.e.,  $\log(\text{interventions} + 1)$ , so that the variable is zero in times of no interventions even in logs.

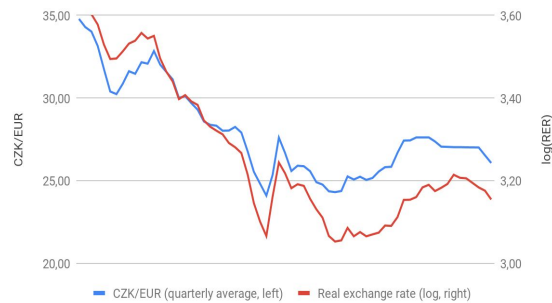
Interest rate differential: Real interest rate differential vis-a-vis the Euro area. Real rates: annual nominal bond rate - change in HICP from previous year.

Bond rates: %, Eurostat, Long term government bond yields - Maastricht definition (average). Monthly data converted to quarterly by averaging. Euro area (EA11-2000, EA12-2006, EA13-2007, EA15-2008, EA16-2010, EA17-2013, EA18-2014, EA19).

## Appendix 2: Plot of the main variables

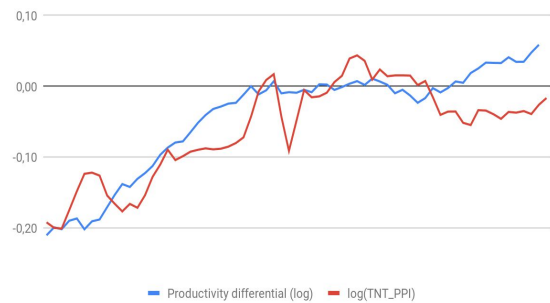
### Nominal and Real exchange rate (log)

Real exchange rate deflated by PPI of industrial goods



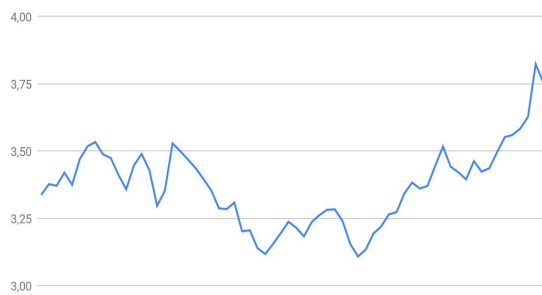
### Productivity differential and the Balassa-Samuelson Effect

Relative labour productivity and relative domestic to export prices (tnt\_ppi)



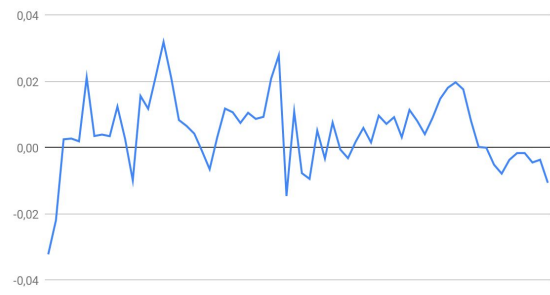
### Net foreign assets

Net foreign assets of the banking sector (% of GDP, log)



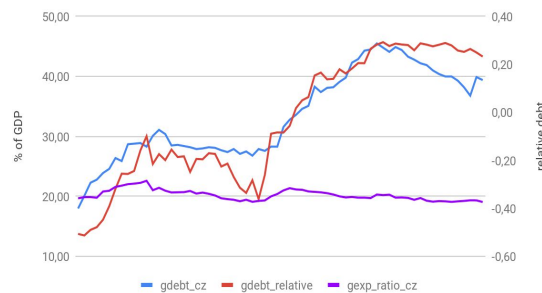
### Terms of trade

Relative, CZ vis-a-vis EU



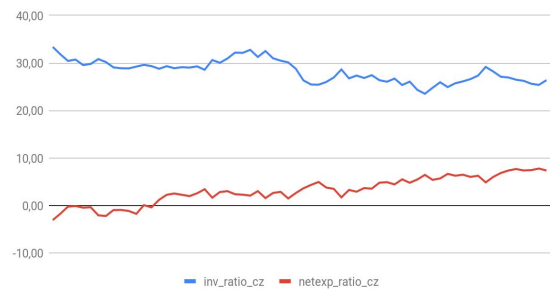
### Fiscal sector

Government debt and expenditure ratios, and the relative stock of government debt (right)



### Gross investments and net export

% of GDP



### Exchange rate interventions

Log(Interventions + 1), + implies interventions against CZK



## Appendix 3.1: Estimated BEER models (long-run coefficients)

Cointegrating equations	2013 Sample							Full sample							Full sample with interventions variable						
Model	m01	m02	m03	m04	m05	m06	m07	m01	m02	m03	m04	m05	m06	m07	m01	m02	m03	m04	m05	m06	m07
Net foreign assets	0,263	0,191	-0,282	-0,210	-0,184	0,024	0,009	0,330	0,275	-0,139	-0,169	-0,122	-0,004	-0,018	0,302	0,241	-0,150	-0,196	-0,156	-0,019	-0,035
	0,135*	0,141**	0,315	0,147	0,139	0,045	0,043	0,077**	0,077**	0,081*	0,075**	0,058**	0,044	0,032	0,096**	0,102**	0,097	0,084*	0,069**	0,049	0,037
Productivity differential	-1,514	-1,868				-0,388	-0,545	-1,412	-1,768				-0,327	-0,350	-1,405	-1,776				-0,307	-0,350
	0,390**	0,219**				0,116**	0,090**	0,341**	0,145**				0,148**	0,093**	0,340**	0,102**				0,150**	0,094**
TNT (relative exp prices)			-3,016	-2,862	-2,753	-2,062	-1,895			-2,778	-2,816	-2,669	-2,232	-2,172			-2,786	-2,823	-2,694	-2,240	-2,173
			0,488**	0,314**	0,281**	0,105**	0,117**			0,163**	0,231**	0,185**	0,155**	0,112**			0,168**	0,233**	0,190**	0,155**	0,112**
Terms of trade			-3,429	-0,796	-0,879	-0,141	-0,033			-2,087	-0,892	-0,966	-1,136	-0,871			-2,034	-0,741	-0,823	-0,988	-0,774
			2,225	0,756	0,757	0,314	0,274			1,015**	0,546	0,542*	0,501**	0,340**			1,042*	0,562	0,553	0,528**	0,351**
Net exports	-0,090							-0,100							-0,108						
	0,141							0,131							0,132						
Investment	0,200							0,214							0,195						
	0,200							0,198							0,202						
Constant	1,840	2,497	4,016	3,783	3,702	3,035	3,092	1,606	2,221	3,559	3,651	3,507	3,128	3,179	1,776	2,332	3,596	3,738	3,613	3,176	3,232
	0,999*	0,458**	1,013**	0,469**	0,445**	0,145**	0,137**	0,894**	0,257**	0,269**	0,235**	0,183**	0,143**	0,104**	0,971*	0,335**	0,316**	0,264**	0,220**	0,160**	0,120**
<b>F-Bounds test</b>																					
F-statistics	4,572	5,926	5,699	8,906	8,576	5,033	12,572	6,149	8,072	6,968	11,449	10,912	7,315	12,043	6,091	8,016	6,673	11,501	11,109	7,282	12,155
k	4	2	3	3	3	4	4	4	2	3	3	3	4	4	4	2	3	3	3	4	4
5% lower bound	2,763	3,368	3,048	3,048	3,048	2,763	2,763	2,750	3,285	2,976	2,976	2,976	2,750	2,750	2,750	3,285	2,976	2,976	2,976	2,750	2,750
5% upper bound	3,813	4,178	4,002	4,002	4,002	3,813	3,813	3,755	4,070	3,896	3,896	3,896	3,755	3,755	3,755	4,070	3,896	3,896	3,896	3,755	3,755
Observations	50	50	50	50	50	50	50	65	65	65	65	65	65	65	65	65	65	65	65	65	65
<b>AR component in residuals (12 lags)</b>																					
Q-stat	9,488	7,883	17,588	5,691	5,346	19,171	18,513	7,865	6,022	12,731	5,104	4,288	15,972	6,525	7,790	6,317	12,525	5,573	4,941	14,938	7,254
p-value	0,661	0,794	0,129	0,931	0,945	0,084	0,101	0,796	0,915	0,389	0,954	0,978	0,193	0,887	0,801	0,899	0,404	0,936	0,960	0,245	0,840



## Appendix 4: Real misalignments

**Figure A4: Real exchange rate misalignment (%)**

Short-term and long-term misalignments



## Appendix 5: BEERs with all variables in cointegrating equations

Figure A5.1 BEERs, based on full sample

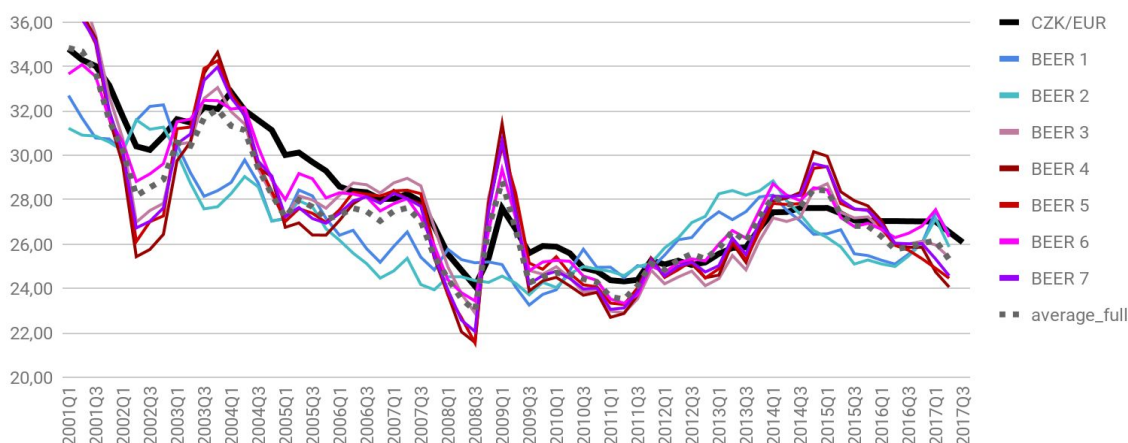


Figure A5.2 BEERs based on 2013 sample

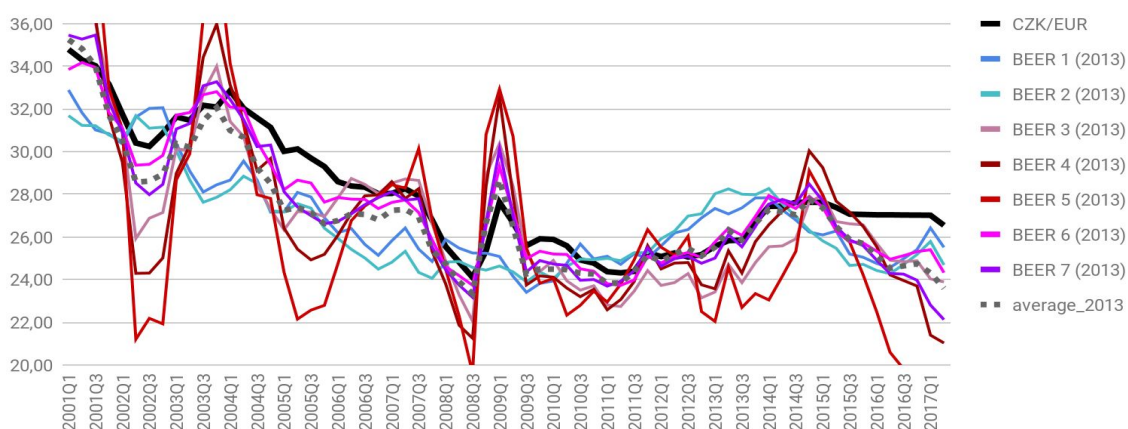
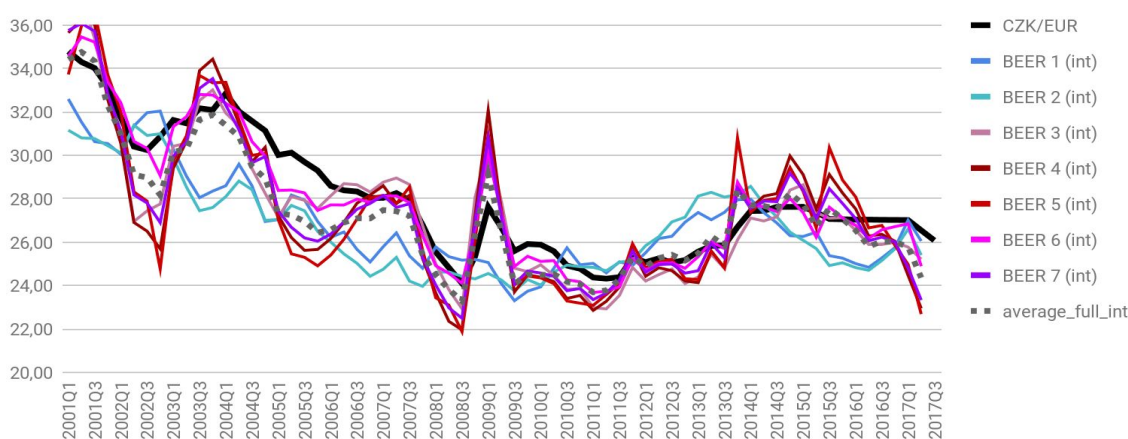


Figure A5.3 BEERs based on models with interventions



Note: Model 5 has been excluded from the average in Figure A4.2 as an apparent outlier.

## Supplementary online materials

The data are available at:

[https://docs.google.com/spreadsheets/d/1nwkbz2RLETN0FazCbgpZinsVK\\_SarhXLW-EE2gkn74s/edit?usp=sharing](https://docs.google.com/spreadsheets/d/1nwkbz2RLETN0FazCbgpZinsVK_SarhXLW-EE2gkn74s/edit?usp=sharing)

The estimated equilibrium exchange rates, misalignments, and all outputs are available on-line at <https://docs.google.com/spreadsheets/d/1sJ-paemsjUAm5v4i159Uhg25Bjx7Bno-UKCBBSJIU0E/edit?usp=sharing>.

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