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**BEYOND CONNECTEDNESS: A COVARIANCE
DECOMPOSITION BASED NETWORK RISK MODEL**

Umut Akovalı

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Beyond Connectedness: A Covariance Decomposition based Network Risk Model

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February 24, 2020

Abstract

This study extends the Diebold-Yilmaz Connectedness Index (DYCI) methodology and, based on forecast error covariance decompositions, derives a network risk model for a portfolio of assets. As a normalized measure of the sum of variance contributions, system-wide connectedness averages out the information embedded in the covariance matrix in aggregating pairwise directional measures. This actually does matter, especially when there are large differences in asset variances. As a first step towards deriving the network risk model, the portfolio covariance matrix is decomposed to obtain the network-driven component of the portfolio variance using covariance decompositions. A second step shows that a common factor model can be estimated to obtain both the variance and covariance decompositions. In a third step, using quantile regressions, the proposed network risk model is estimated for different shock sizes. It is shown, in contrast to the DYCI model, the dynamic quantile estimation of the network risk model can differentiate even small shocks at both tails. This result is obtained because the network risk model makes full use of information embedded in the covariance matrix. Estimation results show that in two recent episodes of financial market turmoil, the proposed network risk model captures the responses to systemic events better than the system-wide index.

Key Words: Connectedness; Covariance decomposition; Factor models, Idiosyncratic risk; Portfolio risk; Quantile regressions; Systemic risk; Vector Autoregressions; Variance decomposition.

JEL codes: C32, G21

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1 Introduction

In a series of papers, Diebold and Yilmaz (2012, 2014) develop the connectedness index (DYCI) methodology to analyze how shocks are transmitted among a set of financial variables. They propose several connectedness measures that range from the aggregate level (system-wide index) to the granular level (pairwise directional measures) and are obtained via forecast error variance decompositions from a vector autoregression. The system-wide connectedness index aggregates the pairwise directional measures on the basis of *average* “cross”-variance decomposition shares. The index, which indicates the strength of the overall connectedness in the system, can be used to identify changes in financial networks when the primary focus is providing descriptive results in order to monitor overall shock transmissions across financial assets.

Although connectedness arises also through the disturbance covariance matrix (Demirer et al., 2018), system-wide connectedness fails to account for the asymmetry in asset variances as a result of normalizing the rows of variance decomposition matrix. This raises an important issue: when assets are not homogeneous in terms of their risk exposures, a central asset may carry less risk exposure, or sometimes a riskier asset may be less central in the network. In these cases, systemic risk can be poorly approximated if one utilizes only the normalized measure of connectedness. On the other hand, forecast error variances by definition are the sum of squared impulse responses, and so system-wide connectedness practically takes the row-means of cross-variance decomposition shares. This is indeed a simple and often effective way to provide descriptive results for overall connectedness in the system. However, as an outer product of impulse response functions, covariances are also critically important from a systemic risk perspective.

In this study, I propose an approximate covariance decomposition methodology that builds on Joo (2000) and Goto and Volkanov (2002) in order to improve the standard system-wide connectedness measure. Incorporating covariance decompositions into a connectedness framework paves the way for a more general method to aggregate pairwise directional measures. Assuming that the financial system is a portfolio of assets, network-driven portfolio variance is calculated via covariance decompositions. Instead of taking the row means of variance decomposition shares, I use the ratio of network-driven portfolio variance to total portfolio variance to measure overall network effect. Noticeably, the share of network-driven portfolio variance has the same meaning with standard system-wide connectedness measure. Moreover, it distinguishes differences among asset variances, and unlike the conventional system-wide connectedness measure, covariances are allowed to have an impact. Furthermore, network-driven variance can easily be converted into a network-driven risk measure that quantifies the impact of network connectivity on portfolio risk in nominal terms. For this reason, I refer to this framework as a “network risk model”.

Using daily or higher frequency stock returns, the standard assumptions of vector autoregressions, such as those employed by Diebold and Yilmaz (2014), can be used to estimate the network risk model. As suggested by Claeys and Vašíček (2014), it is also possible to take the factor structure into account when considering the impact of common shocks on variables. In

the spirit of network analysis, this requires a further effort to interpret the factor structure in a network model. A rich literature examines the network interpretation of factor structure in neural network models (Kumar et al. (2012), Ravi and Pramodh (2008), O’Farrell et al. (2005)), bayesian graphical models (Ahelegbey, 2016) and psychometrics (Borsboom and Cramer (2013), Epskamp et al. (2016)). I also demonstrate a further result that common factors can be analyzed in the spirit of variance decomposition networks. Incorporating the factor structure to the network risk model yields an explanation similar to the well-known “*robust yet fragile*” phenomenon (Gai and Kapadia (2010), Gai (2013)). Increasing the number of assets in the portfolio enables the diversification of idiosyncratic risks, while at the same time, overall portfolio risk approaches to the undiversifiable (or systematic) risk component. In addition, larger portfolios are more likely to display higher network risks for two reasons: a) a large number of assets increase the number of possible linkages among assets, b) adding more assets boosts the pre-existing network relations, allowing further feedbacks.

It is also possible to estimate the network risk model for different shock sizes by relying on approximate quantile regressions. Ando et al. (2018) apply the quantile regression model to the DYCI framework to obtain connectedness measures for different shock sizes. Using a static factor model to remove cross-sectional correlations, they apply quantile regressions on an equation-by-equation basis and find that financial networks get denser at both tails of the shock distribution. However, the dynamic estimation of the *quantile connectedness* at both tails fails to distinguish good or bad market conditions, showing very little change in system-wide connectedness through time. Unlike the quantile estimation of the connectedness model, the proposed network risk model can distinguish shocks at both tails because it also takes into account the full information available at the covariance matrix.

This paper is organized as follows. Section 2 discusses the connectedness framework in order to provide a basis for further discussion of the network risk model. Next, Section 3 discovers the effect of financial networks on the covariance matrix by exploiting a simple payoff function associated with network effects. Section 4 proposes the approximate covariance decompositions and introduces the network risk model. It also incorporates common shocks into the network risk model and provides a network interpretation in the spirit of variance decomposition based networks. Section 5 presents a detailed exposition of the empirical estimation of the network risk model. Section 6 reports the network risk model estimation results and compares them to the results obtained from the standard DYCI estimation. Finally, Section 7 concludes.

2 DYCI and Variance Decompositions

Diebold and Yilmaz (2014) develop a methodology to obtain a connectedness measure (Diebold-Yilmaz Connectedness Index – DYCI) through variance decompositions using approximating VAR models. By virtue of variance decompositions, connectedness is defined as the share of the forecast error variance of a variable due to shocks arising from other variables. In the N -variable vector autoregression model, the connectedness measure c_{ij}^H is defined as the ij -th element of the H -step forecast error variance decomposition matrix. It is calculated as the proportion of

i -th variable's H -step forecast error variance due to shocks in variable j , where $i \neq j$.

	x_1	x_2	\dots	x_N	From Others
x_1	c_{11}^H	c_{12}^H	\dots	c_{1N}^H	$\sum_{j=1}^N c_{1j}^H, j \neq 1$
x_2	c_{21}^H	c_{22}^H	\dots	c_{2N}^H	$\sum_{j=1}^N c_{2j}^H, j \neq 2$
\vdots	\vdots	\vdots	\ddots	\vdots	\vdots
x_N	c_{N1}^H	c_{N2}^H	\dots	c_{NN}^H	$\sum_{j=1}^N c_{Nj}^H, j \neq N$
To Others	$\sum_{i=1}^N c_{i1}^H$ $i \neq 1$	$\sum_{i=1}^N c_{i2}^H$ $i \neq 2$	\dots	$\sum_{i=1}^N c_{iN}^H$ $i \neq N$	$\frac{1}{N} \sum_{i,j=1}^N c_{ij}^H$ $i \neq j$

Table 1: The Connectedness Table

Table 1, the connectedness table, is generated from the forecast error variance decomposition matrix. The off-diagonal elements are the measures of pairwise directional connectedness. The last column, called “*From Others*” or simply *from-connectedness*, is defined as the row-sum of the off-diagonal elements. The last row, called “*To Others*” or *to-connectedness*, is the column-sum of the off-diagonal elements. The diagonal elements of the connectedness table (c_{ii}^H for $i = 1, 2, \dots, N$) are called *own connectedness*, indicates the share of the i -th variance due to shocks that hit the i -th variable's itself. Noticeably, own connectedness, or the *self loop* in network terminology, is kept out of the “*To – connectedness*” and “*From – connectedness*”, which are the directional measures of connectedness. The bottom-right element is the *system-wide connectedness* or “connectedness index”; it summarizes the overall connectedness in the system.

Connectedness is just one of the proposed methodologies to analyze financial networks. Other methodologies proposed in literature include the correlation networks of (Barigozzi et al., 2018) and (Brownlees et al., 2018), and the Granger-causality networks of Billio et al. (2011). However, the connectedness methodology has certain advantages: the estimated edges are defined bilaterally with possibly different weights owing to the variance decompositions argued in Diebold and Yilmaz (2014). The method provides rich information that explains sophisticated network relationships compared to undirected, unweighted network structures. It also flexibly defines asymmetric linkages among assets, which is highly relevant to the financial networks.

3 Financial Networks and Portfolio Risk

This section explores the close links between the covariance matrix and financial networks. Benoit et al. (2017) utilizes a standard asset pricing model to review the features of systemic risk models in the literature. They assume N financial institutions indexed by i , each of which has a risk exposure x_i . They denote the share of systematic component on total risk exposure as α_i ; the remaining share $1 - \alpha_i$ is reserved for the idiosyncratic component. The systematic risk exposure of asset i is defined as $y_i^S = \alpha_i x_i$; $y_i^I = (1 - \alpha_i)x_i$ shows the idiosyncratic risk

exposure. The benchmark payoff function of a financial institution, $\hat{\pi}_i$, is the weighted sum of $\rho^S + \epsilon^S$ and $\rho^I + \epsilon^I$ where ρ^S and ρ^I are constants and ϵ^S and ϵ^I are *iid* random shocks.

$$\hat{\pi}_i = (\rho^S + \epsilon^S) \times y_i^S + (\rho^I + \epsilon^I) \times y_i^I \quad (1)$$

Equation (1) defines the financial gains/losses as a function of the systematic and idiosyncratic shocks and exposures. Noticeably, $\hat{\pi}_i(y_i^S, y_i^I, \epsilon^S, \epsilon^I)$ allows simultaneous financial losses only through the systematic component. Benoit et al. (2017) argue that the model fails to approximate systemic risk because it does not take into account simultaneous losses generated through asset linkages. They modify the $\hat{\pi}_i(\cdot)$, assuming that the direct exposures among assets are collected in an $N \times N$ adjacency matrix B and redefine the benchmark payoff function with network effects as $\pi_i(y_i^S, y_i^I, B, \epsilon^S, \epsilon^I)$. Finally, they argue that systemic event in systemic risk literature is a joint statement about the $\pi_i(\cdot)$ in general.

To uncover the impact of asset linkages on the covariance matrix, I specify the payoff function with network effects borrowing the $\pi_i(\cdot)$ from (Benoit et al., 2017). Assuming an N -asset portfolio \mathcal{P} , I define the function $f_i(B, \pi_j)$ that connects the payoff π_i to the payoff of others in the portfolio π_j where $j \in \mathcal{P} \setminus \{i\}$. The function $f_i(B, \pi_j)$ represents the network driven payoff. For the sake of simplicity, I assume that the functional forms for asset payoffs are the same for all assets in the portfolio. Then, the benchmark payoff of i -th asset is defined as

$$\pi_i = \pi(\pi_i^S, \pi_i^I, f_i(B, \pi_j)). \quad (2)$$

a) Network Effects in the Static Model

To isolate the effects of asset linkages on asset variance, I specify a simple payoff function, $\pi_i^*(\cdot)$ with no common shocks. Then, assuming that there are three or more assets, I draw two assets indexed by i and k where $i \neq k$. Assuming the payoff function π_i^* is additively separable and $f_i(B, \pi_j)$ is linear, I organize the payoff of asset i as

$$\pi_i^* = \pi_i^I + \sum_j b_{ij} \pi_j^I$$

wherein b_{ij} is the ij -th element of the adjacency matrix B that connects asset payoffs to each other. For independent asset-specific shocks, the variance of π_i^* is found as

$$\sigma_i^{*2} = \sigma_i^2 + \sum_j b_{ij}^2 \sigma_j^2.$$

where σ_i^2 is the variance of the asset-specific shock π_i^I . The first term on the right-hand side corresponds to the asset-specific variance contribution of i -th asset to its own variance; the latter term is the variance contributions of others in the portfolio. For the values of $b_{ij} \neq 0$, the asset variance σ_i^{*2} is directly affected by the others' asset-specific shocks. Therefore, the

presence of network effects makes the asset payoffs move together simultaneously even when the systematic shocks are absent.

Similarly, the impact of direct asset linkages (even there is no systematic shocks) becomes evident in covariances that happen to be essential for measuring portfolio risk. Given the adjacency matrix and the distribution of asset-specific shocks, one can also measure the covariance among assets in terms of asset-specific variances and bilateral linkages. The covariance among asset i and k is calculated by taking the cross-multiplication of payoffs for $j \in \mathcal{P} \setminus \{i, k\}$

$$\begin{aligned}\pi_i^* &= \pi_i^I + b_{ik}\pi_k^I + \sum_j b_{ij}\pi_j^I \\ \pi_k^* &= \pi_k^I + b_{ki}\pi_i^I + \sum_j b_{kj}\pi_j^I\end{aligned}$$

Therefore, $Cov(\pi_i^*, \pi_k^*)$ is

$$Cov(\pi_i^*, \pi_k^*) = (b_{ik}\sigma_i^2 + b_{ki}\sigma_k^2) + \sum_j b_{ij}b_{kj}\sigma_j^2$$

The first part of the covariance equation associates the asset-specific variances and connectiveness for i and k . This is the pairwise contributions of i -th and k -th assets to their own covariance. The latter term captures the contributions of other assets to $Cov(\pi_i^*, \pi_k^*)$.

Noticeably, network driven variance and covariances are derived as a function of the adjacency and covariance matrices. Without a loss of generality, the simple model above considers only the first-order network effects; it assumes that asset-specific shocks instantly transmitted to the others and that the equilibrium solution is then reached. Given that evidence of the presence of network effects on portfolio covariance matrix has already been provided, one can argue that higher order network effects may trigger a further propagation of asset-specific shocks. Further iterations are required to arrive at a static equilibrium solution; the complexity of the variance and covariance equations increases because the model includes higher order network effects for large portfolios.

b) Detection of Network Effects through Spatial Regressions

Billio et al. (2016), who takes a spatial regression approach, offers a variation of the traditional asset pricing model that includes the contemporaneous and/or lagged links across assets. Their work shows that incorporating network effects into the asset pricing model introduces two new components to the conventional (i) systematic and (ii) idiosyncratic components. These components are (iii) the impact that network connectivity among asset returns has on the systematic component and (iv) the impact that network connectivity among asset returns has on the idiosyncratic component. The model assumes that the network structure W is given and that the factor structure captures the effect of common shocks. They utilize the following spatial regression model:

$$\hat{\Pi}_t = \rho W \hat{\Pi}_t + \Lambda F_t + \eta_t \quad \eta_t \sim N(0, \Omega) \quad (3)$$

where the coefficient ρ accounts for the impact of other assets that are weighted by the $N \times N$ proximity matrix W (the adjacency matrix in network terminology, or the spatial weight matrix in spatial regression terminology); $\hat{\Pi}_t$ is the vector of excess returns, Λ is a $N \times M$ matrix of factor loadings, where M is the number of factors; F_t is M -dimensional vector of common factors; Ω is the diagonal covariance matrix of error terms η_t . The diagonal elements of W are assumed to be zero; therefore, self-loops are not allowed. Furthermore, the W is symmetric for *undirected* networks, and so one can introduce asymmetry in order to deal with directed network structures. Equation (3) is equivalently written as

$$\begin{aligned} (I - \rho W) \hat{\Pi}_t &= \Lambda F_t + \eta_t \\ \hat{\Pi}_t &= \Lambda^* F_t + \eta_t^* \end{aligned} \quad (4)$$

where $\Lambda^* = (I - \rho W)^{-1} \Lambda$ and $\eta_t^* = (I - \rho W)^{-1} \eta_t$. Using the property $(I - \rho W)^{-1} = I + \rho W + \rho^2 W^2 + \rho^3 W^3 + \dots$ following LeSage and Pace (2009), Billio et al. (2016) rewrite the Equation (4) as below

$$\hat{\Pi}_t = \Lambda F_t + \sum_{j=1}^{\infty} \rho^j W^j \Lambda F_t + \eta_t + \sum_{j=1}^{\infty} \rho^j W^j \eta_t. \quad (5)$$

There are four components of the excess returns $\hat{\Pi}_t$ in equation (5). ΛF_t and η_t are the well-known systematic and idiosyncratic components, and they are independent of the network exposures. The network exposure to the common factor is captured by $\sum_{j=1}^{\infty} \rho^j W^j \Lambda F_t$; the network exposure to the idiosyncratic shocks is defined as $\sum_{j=1}^{\infty} \rho^j W^j \eta_t$. Noticeably, the model allows for higher order network effects on systematic and idiosyncratic components. Billio et al. (2016) argue that contemporaneous relations among asset returns can help to deal with correlated idiosyncratic shocks in the estimation of the model. They also note that the network effect pushes the diversification benefit down because the idiosyncratic risks that are amplified by the network connections generates an additional non-diversifiable risk component. This approach, which can be used to decompose portfolio risk, requires the estimation of the network structure W , which Billio et al. (2016) do not address.

4 The Network Risk Model

Static and spatial models are useful for exploring the effects of financial networks on the portfolio covariance matrix. In practice, we observe asset payoffs and estimate the covariance matrix Σ_π . Unfortunately, without prior knowledge on the asset linkages, it is not always feasible to measure the effect of financial networks in static or spatial model, as discussed earlier. To have a proper portfolio risk decomposition with network effects, one also needs to estimate the adjacency matrix (denoted by network parameters $\{b_{ij}\}$ or W in previous section). In

this section, I introduce the network risk model and propose a methodology to deliver network driven covariances. The estimation of the network risk model, which utilizes the standard assumptions of vector autoregressive model, allows us to estimate the adjacency and covariance matrix together. To obtain the network driven covariances, I propose approximate covariance decompositions using the adjacency matrix. Below, I briefly share the details of the network risk model. Additional details are provided in Appendix A.

4.1 Forecast Errors and Covariance Decompositions

Consider an N -dimensional covariance stationary data-generating process with orthogonal shocks w_t :

$$y_t = \Theta(L)w_t, \quad \Theta(L) = \Theta_0 + \Theta_1 + \Theta_2 + \dots, \quad E(w_t w_t') = I \quad (6)$$

with Θ_i for $i = 1, 2, 3, \dots$ are each $N \times N$ coefficient matrices, and $\Theta(L)$ is the lag-polynomial. The error of the optimal h -step forecast is the difference between the actual value y_{t+h} and the optimal h -step forecast $y_t(h)$:

$$y_{t+h} - y_t(h) = \sum_{i=0}^{h-1} \Theta_i w_{t+h-i}.$$

Because $w_{n,t}$'s are assumed to be uncorrelated and have unit variances, the mean-squared error (MSE) of $y_{j,t+h}$ is

$$E(y_{j,t+h} - y_{j,t}(h))^2 = \sum_{n=1}^N (\theta_{jn,0}^2 + \dots + \theta_{jn,h-1}^2) \quad (7)$$

This analysis is often called *innovation accounting*, which accumulates the effect of individual innovations to the variance of a particular variable. (Lütkepohl, 2005). To provide variance decompositions, Equation (7) can arbitrarily be rewritten as

$$E(y_{j,t+h} - y_{j,t}(h))^2 = \sum_{i=0}^{h-1} \theta_{jj,i}^2 + \sum_{i=0}^{h-1} \sum_{n=1; j \neq n}^N (\theta_{jn,i}^2) \quad (8)$$

The first term on the right-hand side (RHS) corresponds to the contribution of innovations in variable j , while the second term stands for the contribution of innovations in variables *other than* j to forecast the error variance of the h -step forecast of variable j . Variance decomposition shares are obtained by dividing RHS to $MSE[y_{j,t}(h)]$

$$\underbrace{\frac{\sum_{i=0}^{h-1} \theta_{jj,i}^2}{MSE[y_{j,t}(h)]}}_{\text{Own Contribution}} + \underbrace{\frac{\sum_{i=0}^{h-1} \sum_{n=1; j \neq n}^N (\theta_{jn,i}^2)}{MSE[y_{j,t}(h)]}}_{\text{Contribution of Others}} \quad (9)$$

4.1.1 Covariance Decompositions

Joo (2000) and Goto and Volkanov (2002) propose covariance decompositions as a “natural” extension of variance decomposition analysis; instead of accumulating only the squared shocks, they also consider the accumulation of the cross-multiplied shocks that can be defined as forecast error covariances. Noticeably, variance decompositions provide an $N \times N$ matrix that indicates the contribution of each N variables to N different individual asset variances, while covariance decomposition gives an $N \times N \times N$ dimensional array that provides the contribution of N variables to N^2 variances and covariances. However, extending the variance decompositions to covariance decompositions is not feasible in most cases. To see this, recall Equation (8) in matrix form, which gives contribution of innovations in variable n to h -step forecast error variance of variable j ,

$$\sum_{i=0}^{h-1} (e'_j \Theta_i e_n)^2$$

where e_j is the j -th column of the $N \times N$ identity matrix, I_N . To obtain covariances among m and j , rather than taking squares of $(e'_j \Theta_i e_n)$ terms, multiply the $(e'_m \Theta_i e_n)$ and $(e'_j \Theta_i e_n)$ terms.

$$\sum_{i=0}^{h-1} (e'_m \Theta_i e_n)(e'_j \Theta_i e_n) \quad (10)$$

Consider the special case, $m = j$, which is simply equal to variance decomposition. For this reason, it is convenient to regard covariance decomposition as a generalization of variance decompositions. Equation (10) shows the contribution of innovations in variable n to the h -step forecast error covariance of variable m and j . Now, regarding the MSE as covariance matrix, the mj -th and jm -th components of MSE matrix are given as

$$\sigma_{mj} = \sigma_{jm} = \sum_{i=0}^{h-1} \sum_{n=1}^N \theta_{mn,i} \theta_{jn,i} = \sum_{i=0}^{h-1} \sum_{n=1}^N \theta_{jn,i} \theta_{mn,i} \quad (11)$$

Analogous to variance decompositions, both Joo (2000) and Goto and Volkanov (2002) define covariance decomposition as

$$\omega_{mj,n,h} = \frac{\sum_{i=0}^{h-1} (e'_m \Theta_i e_n)(e'_j \Theta_i e_n)}{MSE_{mj,t}(h)} \quad (12)$$

However, this approach has two important drawbacks. First, if any pair of $\theta_{jn,i}$ and $\theta_{mn,i}$ has opposite signs, their interactions will be negative. Positive and negative terms in $MSE_{mj,t}(h)$ cancel out each other; then it is no longer an appropriate denominator that $\omega \in [0, 1]$ is not guaranteed, or covariance decomposition may be undefined since denominator can be zero. Second, since we are interested in covariances (instead of shares), it may be preferable to obtain covariances directly instead of having the ω s. This eliminates the first drawback, but it fails to satisfy one of the basic assumptions of covariance matrix: *positive semi-definiteness*. To see

this, modify the Equation (11) and write it explicitly as

$$\sum_{i=0}^{h-1} (\theta_{jj,i} \theta_{mj,i} + \theta_{mm,i} \theta_{jm,i}) + \sum_{i=0}^{h-1} \sum_{\substack{n=1 \\ j \neq n \\ m \neq n}}^N (\theta_{jn,i} \theta_{mn,i}). \quad (13)$$

This equation fails to guarantee the positive semi-definiteness of the covariance matrix because it arbitrarily splits the variance-covariance matrix into two pieces, both of which are not necessarily positive semi-definite. It should be noted that the symmetry property of the covariance matrix is satisfied in Equation (13).

4.1.2 Approximate Covariance Decomposition via Variance Decomposition

Apparently, the “natural” extension of variance decompositions fails to provide a feasible solution to covariance decompositions. Instead, I propose an approximate covariance decomposition that relies on the information from variance decompositions. To isolate the network-driven variance and covariances from the covariance matrix, I follow a two-step procedure that satisfies the symmetry and positive semi-definiteness properties of covariance matrix. The procedure relies on basic variance and covariance properties,

$$Var(aX + b) = a^2 Var(X)$$

$$Cov(aX + b, cY + d) = ac Cov(X, Y)$$

Because variance decomposition delivers a proportion of the h -step forecast error variance of variable j , accounted for by w_{nt} innovations, we easily apply backward-engineering to the variance equation to procure a^2 and c^2 . Recall Equation (9) such as

$$\underbrace{\frac{\sum_{i=0}^{h-1} \theta_{jj,i}^2}{MSE[y_{j,t}(h)]}}_{\text{Own Contribution}} + \underbrace{\frac{\sum_{i=0}^{h-1} \sum_{n=1; j \neq n}^N (\theta_{jn,i}^2)}{MSE[y_{j,t}(h)]}}_{\text{Contribution of Others}}$$

The second term represents the contribution of innovations in variables **other than** j to the forecast error variance of the h -step forecast of variable j . Let us call that term $d_{o,j}^2$

$$\underbrace{\frac{\sum_{i=0}^{h-1} \sum_{n=1; j \neq n}^N (\theta_{jn,i}^2)}{MSE[y_{j,t}(h)]}}_{\text{Contribution of Others}} = d_{o,j}^2$$

Because variance decompositions provide the portion of variance that is contributed by others, a^2 or c^2 can be replaced with $d_{o,j}^2$ in the variance equation. In matrix form, the procedure is as follows. I form a diagonal matrix of the “contribution of others”, D_o^2 , which locates the share of the contribution of others to the variance of a particular variable.

$$D_o^2 = \begin{bmatrix} d_{o,1}^2 & 0 & \dots & 0 \\ 0 & d_{o,2}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & d_{o,N}^2 \end{bmatrix}$$

Owing to the fact that $d_{o,j}^2 \geq 0$ for all j and it is a diagonal matrix, I can take the square root of D_o^2 . In order to ensure the positive semi-definiteness of the resulting covariance matrix, the $d_{o,i}$ s are assumed to be greater than or equal to zero, $d_{o,i} \geq 0$. In order to weight the covariance matrix in the next step, I pre- and post-multiply the covariance matrix with the squared matrices.

$$\begin{bmatrix} d_{o,1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & d_{o,N} \end{bmatrix} \begin{bmatrix} \sigma_{11} & \dots & \sigma_{1K} \\ \vdots & \ddots & \vdots \\ \sigma_{K1} & \dots & \sigma_{KK} \end{bmatrix} \begin{bmatrix} d_{o,1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & d_{o,K} \end{bmatrix}$$

Then the positive semi-definite *network-driven* covariance matrix is

$$\Sigma_{nd} = \begin{bmatrix} \overbrace{\sigma_{11}d_{o,1}^2}^{a^2Var(X)} & \overbrace{\sigma_{12}d_{o,1}d_{o,2}}^{acCov(X,Y)} & \dots & \sigma_{1K}d_{o,1}d_{o,K} \\ \sigma_{21}d_{o,2}d_{o,1} & \sigma_{22}d_{o,2}^2 & \dots & \sigma_{2K}d_{o,2}d_{o,K} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{K1}d_{o,K}d_{o,1} & \sigma_{K2}d_{o,K}d_{o,2} & \dots & \sigma_{KK}d_{o,K}^2 \end{bmatrix} \quad (14)$$

After obtaining the network-driven covariance matrix, I subtract it from the portfolio covariance matrix. The residual part is denoted as the *asset specific* covariance matrix which can be diagonal or not. Noticeably, the correlations among variables are equal in the network driven covariance matrix and the portfolio covariance matrix because approximate covariance decomposition keeps the underlying correlation structure constant. It scales only the asset volatilities using basic variance and covariance properties. Clearly, this assumption leads to the argument that the network driven covariances play a primary role on determining the correlation structure, and asset-specific covariances are treated as residual variation. This is similar to factor models in which the covariance among assets is explained only through the systematic component. The network risk model provides an additional component to the portfolio risk problem: after accounting for the systematic shocks, network driven covariances identifies the correlation structure when the covariance matrix is non-diagonal.

The network risk model presented above extends the connectedness framework, incorporating the adjacency and covariance matrices for a portfolio of assets. Conventionally, the connectedness framework addresses the network implications for a portfolio of assets, while the network risk model (achieved via approximate covariance decompositions) provides a rich environment for analyzing the portfolio risk implications employing the covariance matrix and network structure together.

As discussed earlier, the connectedness index by definition is the average of the off-diagonal variance decomposition shares. This is introduced as a summary measure for the financial connectedness among variables. The network risk model generalizes the connectedness index taking a portfolio variance approach and proposes the share of the network driven variance as a more general approximation to systemic risk. This is the proportion of portfolio variance weighted by the adjacency matrix; it also accounts for the covariances among assets such as

$$\text{The Share of Network-Driven Variance} = \frac{w' D_o \Sigma D_o w}{w' \Sigma w}$$

where w is the vector of portfolio weights. The following example emphasizes the difference among connectedness index and the share of network variance using the simple two assets (equally-weighted) portfolio. I borrow the notation from Equation (14) and explicitly write connectedness index and the share of the network driven variance as

$$\begin{aligned} \text{Connectedness Index} &= \frac{d_{o,1}^2 + d_{o,2}^2}{2} \\ \text{The Share of the Network-Driven Variance} &= \frac{\sigma_{11}d_{o,1}^2 + \sigma_{22}d_{o,2}^2 + 2\sigma_{12}d_{o,1}d_{o,2}}{\sigma_{11} + \sigma_{22} + 2\sigma_{12}} \end{aligned}$$

Noticeably, the connectedness index is a special case of the share of the network-driven variance for a portfolio; they are equal when $\sigma_{11} = \sigma_{22}$ and $\sigma_{12} = 0$. Also, approximate covariance decompositions introduce non-linearity in connectedness as the multiplication of the square-root of the *from-connectedness* appears in the network driven variance. From the perspective of portfolio risk, further diversification is achieved when we take into account the covariances in the connectedness framework ($\sigma_{12} \neq 0$); now the covariances can be positive or negative, implying that there is a difference between the connectedness index and the share of network driven variances.

Furthermore, the network risk model goes a step further, quantifying the network-driven and asset-specific risks following the estimation of covariance matrix and the network structure of the portfolio. The network-driven portfolio risk and the asset-specific portfolio risk are just the square root of the network driven variance and asset specific covariance matrices as follows

$$\begin{aligned} Risk_{nd} &= \sqrt{w' D_o \Sigma D_o w} \\ Risk_{as} &= \sqrt{w' (\Sigma - \Sigma_{nd}) w} \end{aligned}$$

4.1.3 The Network Risk Model: A Numerical Exercise

Armed with the approximate FECVD, I undertake a numerical exercise to show the properties of the network risk model. The first task is to investigate the direct impact of connectedness on network-driven and asset-specific risks. To isolate the impact of connectedness, I assume an equicorrelation matrix and equal standard deviations for each node in the system. I also assume equal connectedness weights for each node; the only thing I allow to change is the expected system-wide connectedness that measures the expected value of off-diagonal elements

of adjacency matrix. Figure 1 (a) shows the decomposed portfolio risk changes with the system-wide connectedness and reveals the trade-off between the network-driven risk and asset-specific risk that, as expected, are inversely related to each other. When connectedness is high, the network-driven risk takes a higher share from the portfolio risk, which undermines the share of asset specific risk, causing it to be lower. Figure 1 (b) also takes into account the portfolio variance; as expected, if portfolio variance is high, then the network-driven risks goes up because the network risk model rescales the network effect with portfolio variance.

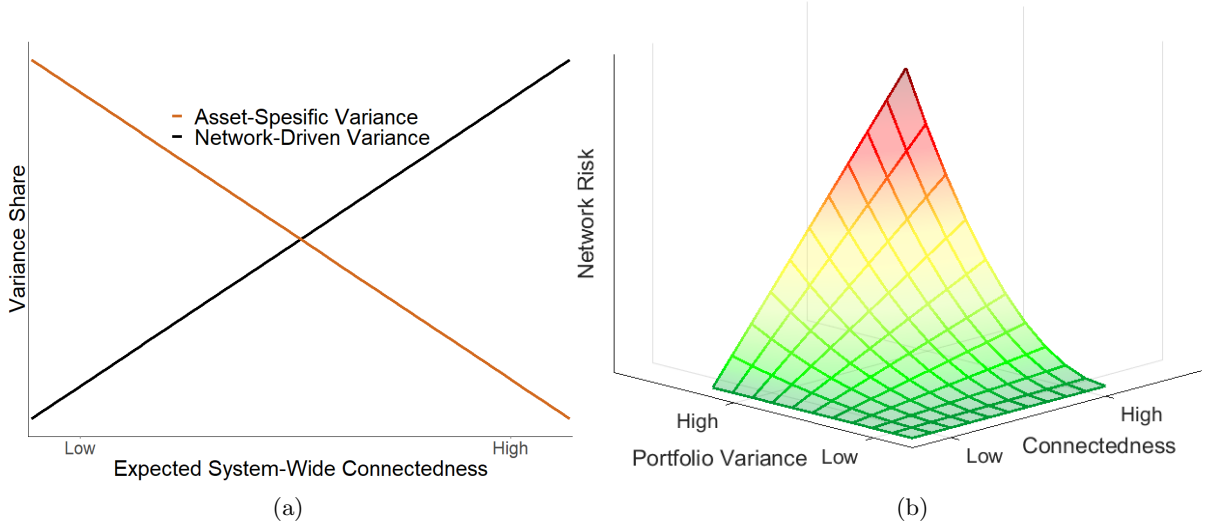


Figure 1: The Network Risk Model

As stated earlier, the system-wide connectedness index does not distinguish the effect of large differences among asset variances. This becomes an issue when the risk exposures of assets are not homogeneous because a central asset may carry less risk exposure or in some cases a riskier asset may be less central in the network. In these cases, systemic risk can be poorly approximated if one utilizes only the normalized measure of connectedness. However, the share of network variance by construction exploits the full information embedded in the covariance matrix and it is expected to provide a better approximation to systemic risk in these cases. Below, I provide a numerical exercise that uses the two asset example in Equation (15) to understand how the share of network-driven variance differentiates when there are large differences among asset variances. To allow asset variances to be different, I assume that assets may have different volatilities. I define the variable *risk spread* as a measure of the difference in volatilities among arbitrarily assigned *High* and *Low*-connected assets. When risk spread is zero, then volatilities of both assets are assumed to be one.¹ Figure 2 provides the result of the exercise.

¹One may argue correctly that the simulation result is highly dependent on the minimum and maximum values of risk spread parameter. However, this only changes the speed of convergence and the shape of the curve, the intuition is still valid. In that regard, to make the plot more beautiful, I define risk spread as a grid of $[-5, 5]$ with step-size 0.1. I fixed the variance of low (high) connected asset to one when risk spread is positive (negative).

The result points out to several conclusions. First, the connectedness index and the share of network-driven variance are equal when two conditions are met: the asset variances are equal and the covariance is equal to zero. This confirms the analytic result of Equation (15), which indicates that under certain conditions the share of network driven variance is equal to connectedness. Second, I focus on the green line, which allows only asset variances to vary without any further action on covariances. In this case, the share of network driven variance is larger when the high connected asset is more volatile. The figure also shows that the share of network driven variance is bounded by $[c_{LowCon}, c_{HighCon}]$, which represents directional pairwise connectedness measures for *Low* and *High*-connected assets. Third, the black and blue lines introduce the effect of covariances. A positive correlation means weaker diversification of total portfolio risk with respect to network-driven risks; that is, the connectedness index and the share of network driven variance converge. If assets are negatively correlated, a small difference among asset variances can lead to a large difference in both measures.

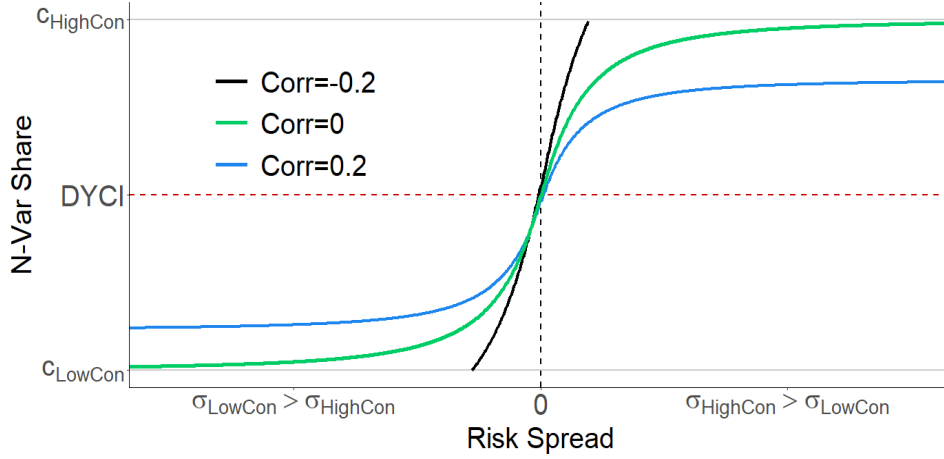


Figure 2: Connectedness Index and The Share of Network Driven Variance

The connectedness framework draws a complete, weighted, and directed network, and within the framework it is possible to evaluate the distributional properties of different network structures. To test the network risk model with different network structures I design a numerical exercise. First, I consider the sparsity of the network. The sparsity, by definition, is trivial in terms of variance decomposition-based networks because the network structure is complete. However, I introduce sparsity to the complete variance decomposition-based network via connectedness weights because connectedness weights indicate the strength of linkages among financial assets.

I prefer not to drop any of the nodes and linkages in order to achieve sparsity; instead, I allow the directional “to-connectedness” measures to vary keeping the expected system-wide connectedness constant. Hence, the presence of to-connectedness weights that are almost equal in all assets of a portfolio signifies that the network structure allows more shock transmission. Otherwise, a small number of assets can have extremely high directional to-connectedness weights while a large number of assets receive low to-connectedness shares. This apparently implies a sparse network structure wherein shocks to low to-connectedness variables are hardly

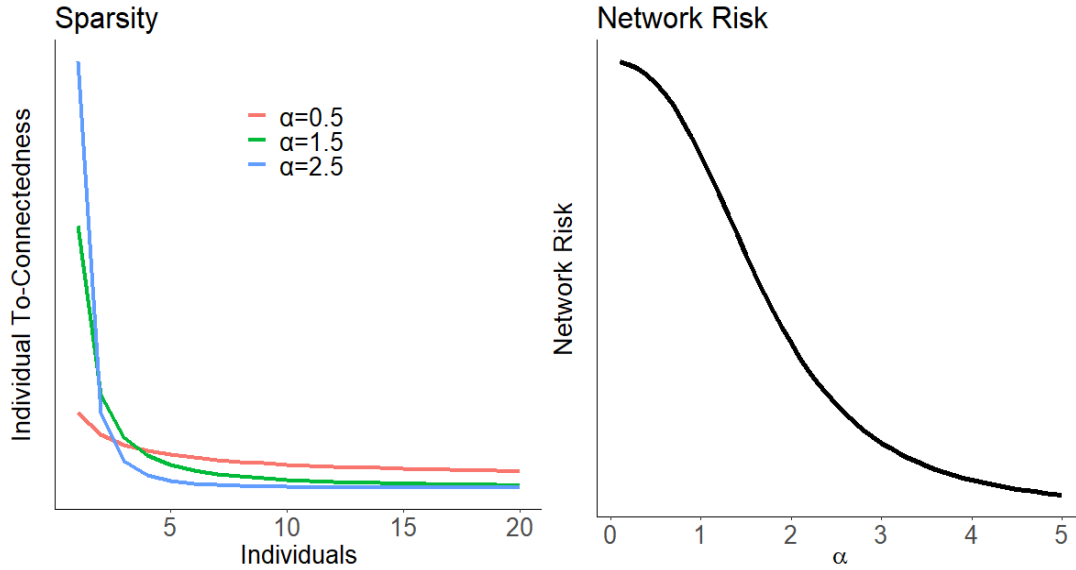


Figure 3: Sparsity in Networks

transmitted to others in the portfolio. To introduce this form of sparsity, I make use of power distribution, which is routinely used in network studies (Gabaix (2009), Jackson (2010)). In general, a power distribution with a scale parameter α is used to measure the degree distribution of networks. I use it to generate heterogeneous to-connectedness weights for different nodes. Consequently, I do not need to remove the weak linkages. The left panel of Figure 3 shows that the parameter α determines the shape of the to-connectedness distribution. Where $\alpha \rightarrow 0$, connectedness weights tends to be equal and the network is denser. In contrast, $\alpha \rightarrow \infty$ implies sparser networks in which the to-connectedness weights for many of the nodes shrink towards zero; then, there are only a few assets which have remarkable to-connectedness weights. Assuming that all nodes are exposed to equal shocks, and because sparse networks implies weaker propagation of shocks, the network-driven risk is expected to be lower. To achieve comparable results, I normalize the connectedness weights by the sum of system-wide connectedness. The right panel of Figure 3 shows the network risk results for different α values. As α increases, confirming the prior expectation, network risk goes down. Moreover, the power function imposes sparsity in a non-linear fashion as α gets bigger; the network-driven risk, too, goes down non-linearly.

The network risk model is also able to capture different network structures and explore their effects on the portfolio risk. Here I examine two well-known network structures, *star* and *tree* networks to be examined². Star networks (Figure 4(a)), have only a single central node and $N - 1$ peripheral nodes are connected to the center node with $N - 1$ paths. The central node is a hub, and so shocks to the central node is transmitted to all peripheral nodes. Figure 4(b) illustrates the numerical network risk result as a function of connectedness. The blue line indicates the benchmark network-driven portfolio risk with one standard deviation shocks. The red (green) line provides the network-driven portfolio risk when shock size is doubled for the

²See Hojman and Szeidl (2008), Jackson (2010) for detailed information.

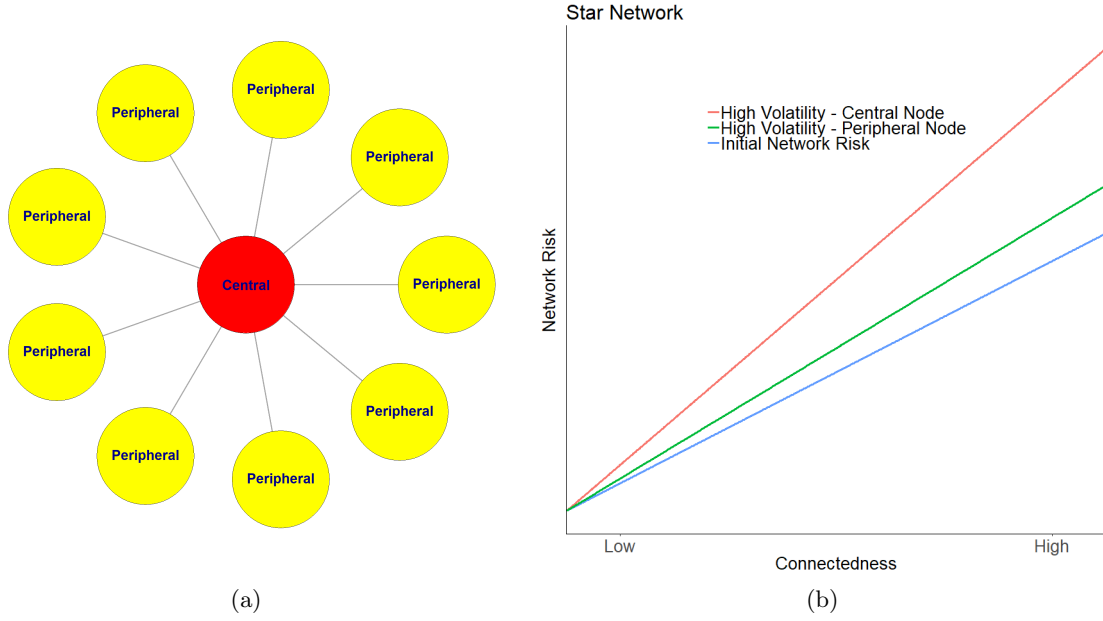


Figure 4: Network Risk in Star Network

central (a peripheral) node. That result shows that increasing the shock size in the central node surges the network driven portfolio risk more than in the peripheral node.

A tree network (Figure 5(a)) is a network with no cycles. I define a tree network with a central node that has four children (child 1), all of whom (child 1) have two children (child 2). Here, the central node remains the most *important* in terms of portfolio risk; however, Child 1 is also crucial in terms of portfolio risk because it is *locally* central. Figure 5(b) shows a similar numerical result; here large shocks to central node increase the network-driven portfolio risk more than Child 1 does, Child 1 impacts more than Child 2.

4.2 Connectedness and Factor Structure

In Section 2, I discuss in detail the properties of the Diebold and Yilmaz (2014) connectedness framework. Notably, the standard DYCI framework does not account for the factor structure explicitly. As shown by Claeys and Vaříček (2014), who employ a factor augmented VAR model, if variables are exposed to common shocks, these shocks amplify the connectedness among assets. When the factor structure is not explicitly identified, the connectedness impact of the common shocks are allocated to pairwise directional connectedness measures. The connectedness framework with/without factor structure is useful, when the primary focus of the study is providing a descriptive analysis of overall connectedness in the financial markets. However, when network-driven risk is being measured, the model should explicitly account for common shocks.

The network interpretation of observed/latent factor models is examined intensively in discussion of neural network models (Kumar et al. (2012), Ravi and Pramodh (2008), O'Farrell et al. (2005)) and bayesian graphical models (Ahelegbey, 2016) and in the psychometrics liter-

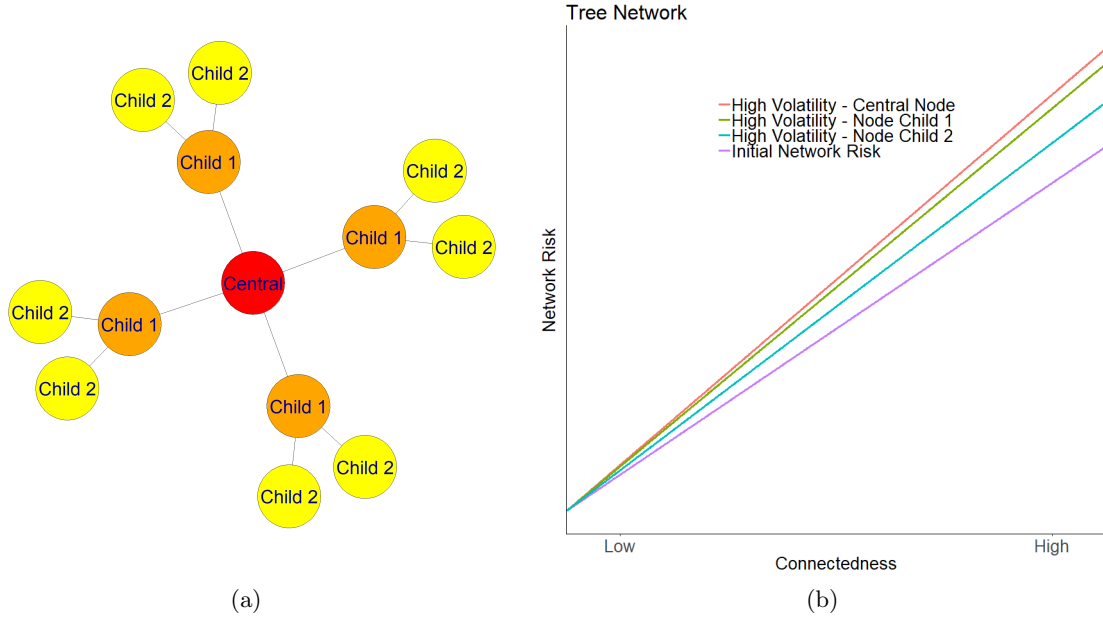


Figure 5: Network Risk in Tree Network

ature (Borsboom and Cramer (2013), Epskamp et al. (2016)). Below, I provide a brief introduction to the network interpretation of the factor structure in variance decomposition-based networks and their use in the network risk model.

I start with a simple factor model to discover the network properties of common effects assuming that assets are conditionally independent given common factors. In literature, variance decompositions (or volatility decompositions (Campbell et al., 2001)) is a standard tool used to analyze the components of the asset and portfolio risk. In the simplest form, the common shocks are regarded as the *common cause* of dependence among asset returns. Then, it is convenient to explore the network properties using causal graphical models that date back to the *conjunctive forks* of Reichenbach (1956). Cartwright (1988) defines the conjunctive fork as a causal diagram in which common causes screen off joint effects; that is, the correlation among variables disappears given the common causes. In this manner, the factor structure can be drawn as a graph with N paths (where N is the number of assets) and the relationships between asset returns can be interpreted as *forks*. The variance decompositions can be used to provide a distance metric that indicates the strength of the unilateral linkage from common factors to asset returns.

The single factor model yields a special case in which the correlation coefficient among single common factor and asset returns is directly related to the variance decompositions. To show this property I write a single factor model in regression form such that

$$R_{i,t} = \alpha_i + \beta_i F_t + \epsilon_{i,t}$$

Equivalently, the regression equation can be set to go through the origin

$$\hat{R}_{i,t} = \beta_i \hat{F}_t + \epsilon_i \quad (15)$$

where $\hat{R}_{i,t} = R_{i,t} - \mu_{R_i}$ and $\hat{F}_t = F_t - \mu_F$. Then, I write down β_i explicitly as $Cov(\hat{R}_i, \hat{F})/Var(\hat{F})$,

$$\hat{R}_{i,t} = \frac{\hat{\sigma}_{iF}}{\hat{\sigma}_F^2} \hat{R}_{F_t} + \epsilon_i \quad (16)$$

where $\hat{\sigma}_{iF}$ is the covariance among the asset return i and the single factor. The variance decomposition for asset i is derived as

$$\hat{\sigma}_i^2 = \rho_{iF}^2 \hat{\sigma}_i^2 + \sigma_{\epsilon_i}^2 \text{ or alternatively} \quad (17)$$

$$\hat{\sigma}_{\epsilon_i}^2 = (1 - \rho_{iF}^2) \hat{\sigma}_i^2 \quad (18)$$

where ρ_{iF} is the correlation coefficient between asset returns $\tilde{R}_{i,t}$ and \tilde{F}_t . Because the mean adjustment in Equation 15 does not change the covariances and correlations, we can replace hats and tildes. The above equation shows the share of variance that can be accounted for common and idiosyncratic shocks; the share of common shocks is equal to ρ_{iF}^2 while the share of idiosyncratic shock is $(1 - \rho_{iF}^2)$. The geometric interpretation of the correlation coefficient among variables (or effects) is widely studied in learning methods, clustering, and network literature considering its similarity to the distance metrics (Berthold and Höppner, 2016). Jaskowiak et al. (2013) formulates the inverse relation between correlation and distance. In this manner, the correlation coefficient can be represented as a measure of the asset's distance metric to the common factor when the common factor is hypothetically located at the origin.

Furthermore, it is possible to extend this argument to the factor loadings $\beta_i = \rho_{iF} \frac{\hat{\sigma}_i}{\hat{\sigma}_F}$. From a network perspective, β_i can be interpreted as a volatility-adjusted distance metric. Observably, an asset i with $\beta_i = 1$ in a single factor model (i.e. capital asset pricing model) does not necessarily mean that the asset return i is following market returns because of its distance to the common factor (i.e. $\rho_{iF} = 1$). The ratio of asset and market volatilities can be high and it can recover the low correlation, pushing the market beta to be equal to 1. On the other hand, factor loading has a nice feature in single latent factor models, the loading of the single latent factor (the first principal component) is definitionally equivalent to the *eigenvector centrality* in network terminology, as argued in Billio et al. (2017).

Since Ross (1976), most factor models in finance have exploited multiple factors. The previous results on the network properties of single factor models can be easily generalized for orthogonal multiple factors; assets live in K (the number of factors) different causal networks. When common factors are allowed to be correlated, factor model is still appealing under the assumption of orthogonality of the common and idiosyncratic components. In this case, *component-wise* variance decompositions are available, not to obtain distance metrics, but to decompose the portfolio covariance matrix.

4.2.1 Latent and Residual Network Modeling

Epskamp et al. (2016) provide a simple network representation for latent variable models. In the first step, they discuss well-known *latent network modeling* (LNM) which relies on the local independence assumption (Lord et al. (1968), Lazarsfeld and Henry (1968)) and in which the observed indicators are independent given the latent factors. LNM explores a unique structure of network interactions among common latent variables (or factors). In the following step, and taking into account the possible violations of local independence assumption, Epskamp et al. (2016) allow some degree of correlation among observed indicators after accounting for the effect of latent factors. In proposing *residual network modeling* (RNM), they aim to capture pairwise linear effects among residuals terms. Accounting for residual interactions, they show that RNM performs better than standard models. Assuming that the factor model is correctly identified, it does not change the interpretation of latent factors because the factor structure is kept intact; that is, the model extracts further information about the interaction of variables, which remain after controlling for the effect of common factors.

Assuming the feedbacks from the variables (or observed indicators) to the factors are blocked, the connectedness among common factors (visually the top layer) forms a network structure that takes into account the interactions among common factors; the connectedness among variables (visually the bottom layer) forms another network structure. Causal relationship among factors and individual variables are also accounted for by unilateral linkages that run from top layer to bottom layer of the network. Similarly, the interpretation of the factor structure remains the same, and the residual network provides additional information and that allows us to estimate the network-driven risks.

Apparently, the LNM & RNM approach yields a natural component-wise decomposition of covariance matrix using both the factor loadings and the residual adjacency matrix. I adopt Epskamp et al. (2016) in order to provide a component-wise decomposition of the portfolio covariance matrix. Practically, LNM extracts the common component in the covariance matrix by estimating the factor model in the first step. Using residuals of the estimated factor model, I estimate vector autoregressions to estimate the connectedness matrix and covariances among residual terms. Applying approximate covariance decompositions (see Section 4.1.2) to the covariance matrix of the residuals, I provide further decomposition of the residual covariance matrix and I obtain network-driven covariances and asset-specific covariance matrix.

The argument on static factor models can be extended to dynamic factor models (DFM) by taking into account the time dynamics of the factor structure and their dynamic effects on the variables. As Stock and Watson (2005) have argued, if the model is identified, the impulse responses and variance decompositions are available; then it is possible to apply connectedness and network risk analyses enjoying very rich information about the dynamics of the factor structure and its implications. The estimation of the FAVAR model usually includes exclusion restrictions that blocks feedbacks from variables to common factors. Bai and Wang (2015) discusses details of the implications and identification of the DFM. However, providing a proper identification for a portfolio of asset is not usually straightforward, the connectedness analysis

and the network risk model in dynamic factor models are reserved for the future work.

4.2.2 Network Risk Model with Factor Structure

To estimate the network risk model, I utilize a multivariate version of a static factor model that allows dynamic interactions among variables. This model allows measurement of the network-driven risk together with the common and idiosyncratic components using the variance decompositions. Borrowing from Ando et al. (2018), I write the standard factor model for N assets as simultaneous regression equations in the dynamic form as follows

$$\pi_{i,t} = \sum_p \sum_j b_{ij,p} \pi_{j,t-p} + \varepsilon_{i,t} \quad (19)$$

$$\varepsilon_{i,t} = \lambda_i' f_t + \nu_{i,t} \quad \nu_{i,t} \sim N(0, \nu_{i,t}^2) \quad (20)$$

where K is the number of factors, λ_i is the $K \times 1$ vector of factor loadings, f_t is the K -dimensional vector of common factors, $p = 1, \dots, P$ is lag order, and $b_{ij,p}$ is the ij -th element of coefficient matrix B for lag order p . Notably, the idiosyncratic component π_i^I (see Equation 2) is replaced by an iid random shock $\nu_{i,t}$, while factors and loadings $\lambda_i' f_t$ address the common component π_i^S . Again, if there is nonzero b_{ij} for any i, j where $i \neq j$, then $\sum_p \sum_j b_{ij,p} \pi_{j,t-p}$ corresponds to the dynamic network effect in the payoff function.

Assuming static and observable exogenous factors, plugging 20 into 19 yields a VARX(p,0) representation of the factor model with lagged endogenous variables and static exogenous variables *common factors*.

$$\pi_{i,t} = \sum_p \sum_j b_{ij,p} \pi_{j,t-p} + \lambda_i' f_t + \nu_{i,t} \quad (21)$$

Equation (21) can be estimated using standard assumptions of vector autoregressions. Fortunately, variance decomposition in the standard toolbox of VARs is also available to provide a reasonable solution. As suggested in Diebold and Yilmaz (2014), the variance decompositions imply a normalized measure of network connectivity, while the decomposition of portfolio covariance matrix according to Equation (21) provides a reasonable means of capturing the network-driven risk component. Since common shocks are static and exogenous, VARX(p,0) representation allows RNM to estimate the adjacency matrix via variance decompositions. Moreover, approximate covariance decompositions are also available; thus, portfolio covariance matrix can be split into common, network-driven and asset-specific components. In this way, network-driven risk can be obtained by isolating the network driven covariances.

5 Estimation

The proposed network risk model employs vector autoregressions for asset returns in order to estimate the connectedness and covariance matrices for a portfolio of assets. I evaluate the model in two cases. First, exploiting the full-sample information, I apply network risk model

to S&P-100 stock market portfolio for 107 stocks ³. In this case, I emphasize the portfolio risk implications of the network risk model. Second, borrowing the global banking dataset from Demirer et al. (2018) and using rolling window analysis, I discover the systemic risk implications of the network risk model for a portfolio of international banks. Considering the heavy computational burden, I drop smaller and low-connected banks (see Demirer et al. (2018)) in the dynamic estimation and I employ 75 banks from 25 countries. The list of assets in both portfolios is provided in Appendix D. The stock market sample covers the period from the May 2012 to July 2019 and the number of daily observations is 1811. Again, the global banking portfolio dataset spans a longer period, from September 2003 to December 2013, and consists of 2676 daily asset returns.

Stock and Watson (2002) show that static/dynamic factors models can be consistently estimated using the factors obtained by the principal component analysis, when the number of variables and time series are both large. In that regard, when estimating the network risk model with common shocks, I make use of latent factors that are extracted through principal component analysis (PCA). Determining the number of static factors using the well-known *minimum average partial* method. Following Bai and Ng (2013), I identify the factor structure by employing the normalization restrictions $\mathbf{F}'\mathbf{F}/T$ is identity matrix and first block of factor loading matrix is lower diagonal.

The estimation of network risk model via approximate quantile regressions is discussed in Section 5.3, Wang and Wang (2016) warn that tail quantiles suffer from high variability when the sample size is small. I assume that the length of the time series is sufficient for a full-sample estimation of the stock market portfolio. However, the rolling window analysis of the global banking system requires sizeable windows in order to deal with small sample size limitations. Therefore, I select 1000-day overlapping rolling windows to estimate the time-dynamics of the network risk model during the sample period. As noted below in the discussion of results, this poses a drawback that the impact of large shocks are preserved in connectedness outcomes throughout rolling windows for those who include the observation from systemic event. Observably, the bankruptcy of Lehman Brothers had huge impact on the estimation results, the shock remained in the system for a long period. Therefore, I do not provide the full set of time-dynamics. Instead, I focus on two episodes of financial market turmoil: a) before and after the global financial crisis and b) the Fed's decision to taper QE in 2013. To deal with long rolling windows, network risk model can be estimated using high-frequency data (For an elegant application of vector autoregressions to intra-day data see Hotchkiss and Ronen (2002)). I reserve this issue for future work.

The estimated parameters in vector autoregressions are usually sensitive to the selection of the lag structure, as argued in Nicholson et al. (2014), and so are the variance decompositions. Several methods are available in standard toolbox of vector autoregressions to determine the optimal lag structure of the VAR model (Lütkepohl, 2005). Tiwari and Ludwig (2015) argue

³The dataset consider entry and exit of firms to the stock market index. I exclude the *new* stocks whose public trading started later after the beginning of the sample period. I obtain the daily stock market data from the Bloomberg terminal.

that the rolling window estimation raises a further concern about the interpretation of the variation in the lag structure during different periods. The statistical nature of the optimal lag structure models (as discussed by Tsay (2013)) makes them highly sensitive to the changes in the data; therefore, the shifts in lag structure for different rolling windows can lead to unidentified jumps in connectedness especially in extreme events. In these cases, fluctuations in observed outcomes turn out to be a mixture of effects from both the lag structure shifts and the shocks to the variables; consequently it is difficult to interpret the connectedness results. To minimize the impact of window-wise lag selection, I assume a fixed one-lag model which provides stable results, experimenting higher lag orders do not add much to the estimation results. After inspecting $\{5, 10, 15\}$ periods, I select a 10-period forecast horizon, but the results are not materially different.

5.1 Identification

A reduced form vector autoregression model can easily be estimated using standard estimation techniques. However, recovering uncorrelated structural shocks from usually correlated reduced form shocks requires a further effort to identify the source of contemporaneous correlation among residuals. To achieve proper impulse responses, additional restrictions should be considered to sort out the contemporaneous effects among variables. Since publication of the influential work of Sims (1980), a popular solution has been *orthogonalized impulse responses*. When the true "causal" ordering of the variables is known, orthogonalized impulse responses utilizes the cholesky decomposition of a positive semidefinite variance-covariance matrix (which yields a unique lower triangular matrix) to identify the vector autoregressions. In this case, variance decomposition analysis can be undertaken without prior knowledge on the structural relationship between the endogeneous variables, except that the causal ordering of variables must be known. Therefore, an important practical problem emerges when the ordering of variables is either not known or is not explicit, the network structure estimated via orthogonalized impulse responses also depends on the ordering of variables.

(Demiralp and Hoover, 2003) argue that there is no consensus about the statistical or empirical basis for the choice of the ordering problem; orthogonalized impulse responses are not unique in general, and this may lead to unclear/misleading estimation outcomes. Instead, Klößner and Wagner (2014) suggest an algorithm to search for all possible orderings that can be applied to the orthogonalized impulse responses. Thus, they provide upper and lower bounds to variance decompositions. However, their approach becomes costly when large VAR models are estimated, in which case the number of possible orderings explodes with the number of assets in the portfolio.

But rather, one can prefer to be more specific on identification problem via explicit assumptions on the "structure" of how variables behave and interact to recover uncorrelated shocks from the residuals. Structural VARs have attracted considerable attention because they enable researchers devise a behavioural interpretation of the model parameters using theory-driven explanations that transform the reduced-form model. Structural estimation models usually

requires setting up a number of structural equations that are founded on certain assumptions.

Researchers sometimes prefer to search alternative identification methods due to lack of sufficient and credible identifying assumptions, including an unreasonable ordering or inadequate theoretical knowledge. There are two crucial drawbacks to applying both the Choleski and structural approaches to the network-risk model. First, the aim of this study is to discover the network of financial assets. Apparently, it is difficult to devise a structural or reasonable "causal" model that provide an ordering of financial asset returns. Second, the number of financial assets in the portfolio sometimes is too large that it causes two problems: the assets cannot be uniquely ordered; and a structural model is not achievable (e.g. the estimation would require an unfeasibly large number of structural equations). Therefore, I consider an alternative strategy.

Let $\psi_j^o(n) = \Theta_h \mathbf{P} e_j$ be the orthogonalized impulse response of the model (Equation (6)), where \mathbf{P} is a lower triangular matrix obtained by Choleski decomposition of covariance matrix Σ . As discussed, the ordering of the variables matters because orthogonalized impulse responses requires a lower-triangular decomposition of the covariance matrix. Alternatively, Koop et al. (1996) develop ordering invariant impulse responses, *generalized impulse responses*. They assume that error terms are distributed as multivariate normal and that a shock hits only one equation at a time. Consequently, the ordering of the variables has no effect on the impulse-responses. Pesaran and Shin (1998) show that generalized impulse responses yield the same result as the orthogonalized impulse responses in the first equation under the multivariate normality assumption.⁴ This property can be exploited so that, in any linear vector autoregression model, the estimate of the model is repeated as each equation becomes the first equation in each loop; then apply the Cholesky decomposition is applied only for the first equation and store the orthogonalized impulse responses. This identification procedure explicitly allows each variable to cause all variables in the system to become the first chain of the "causal" model. Thus, we can see that the generalized impulse responses are far more agnostic than the orthogonalized impulse responses that one no longer needs to invest much to discover the ordering of variables. From the economics perspective, this can be seen as a drawback because it disregards the true causal chain (if such a chain exist); indeed, it can lead to unclear analysis outcomes. However, as it is true in some cases, when a true causal chain is not achievable and, therefore, a unique ordering is not deliverable, generalized impulse responses can provide a practical solution to the identification problem.

5.2 Shrinkage and Selection

Both the DYCI framework and covariance decomposition framework rely on the estimation of an approximating VAR model. However, because there are a high number of endogenous variables in the vector autoregression model, it is probably not effective to go ahead with the classical least squares estimation of the VAR model. To overcome the high dimensionality prob-

⁴For proof, see Pesaran and Shin (1998). The derivation and the details of the generalized impulse responses are discussed in Appendix B.

lem, constrained estimation methods are employed that achieve proper shrinkage and selection. Tibshirani (1996) defines least-absolute shrinkage and selection operator (LASSO), which is a regression method that applies both regularization and selection to solve a constrained convex optimization problem

$$\begin{aligned}\hat{\beta} &= \arg \min_{\beta} \left(\sum_{i=1}^N \left(y_t - \sum_i \beta_i x_{it} \right)^2 \right) \\ \text{subject to} \quad & \sum_{i=1}^K |\beta|^q \leq c\end{aligned}$$

Demirer et al. (2018) introduce the penalized estimation of connectedness that selects and shrinks the coefficients of the estimated VAR model. They argue that making the estimated VAR coefficient matrix sparse generally does not lead to incomplete networks owing to the nonlinear transformation of the coefficient matrix to obtain variance decompositions.

Selection and shrinkage handle the high dimensionality problem, but an interesting issue emerges when we think a factor model and residual network model together. For instance, consider Bernanke et al. (2005)'s FAVAR estimation that treats dynamic factors as variables and imposes no additional restrictions. Yet, one could also argue that the factor structure applies to each variable in the system and, thus, that the selection and shrinkage of the factor loadings are not appropriate. Because the Lasso estimator is flexible, one can decide to penalize particular variables (their coefficients) or not penalize some of them. Then, the optimization function can be written to take into account the non-penalized factor structure, such as

$$\hat{\beta}_{lasso} = \arg \min_{\beta} \left(\sum_{t=1}^T \left(y_t - \sum_i \gamma_i f_t + \sum_i \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^K |\beta_i| \right).$$

Noticeably, the only the penalized vector of coefficients is associated to variables; factors are not included in the constraint. Therefore, it ensures that common shocks to affect each of the variable in the system. In that regard, component-wise decomposition of the covariance matrix is properly defined.

5.3 Quantile VARs: A Value-at-Risk type of Network Risk Measure

Risk measurement techniques usually focus on the tails of the distribution in order to provide a statistical measure of expected loss with a given probability. Portfolio risk decomposition is supposed to consider the impacts of the different size of shocks, especially the larger ones that are at the tails of the distributions. This approach is also employed in the systemic risk literature, in tail dependence models (Balla et al., 2014), in copula methods, in the $\Delta CoVaR$ of Adrian and Brunnermeier (2016).

In ordinary least squares regressions, the estimation problem solves, by construction, the conditional mean estimator. The estimation result is generalized to the entire distribution when the distribution is assumed to be well-behaved. However, one may be interested in how different

shock size affect the network structure and covariances. Moreover, quantile regressions are robust to non-gaussian errors and outliers. Ando et al. (2018), instead of using conditional mean estimators, consider a methodology that estimates the connectedness via quantile regressions (Koenker and Bassett Jr, 1978), and to this end they utilize a factor structure to remove cross-sectional dependence. They show that idiosyncratic credit risk shocks are powerful in tails of the shock distribution, where the median shocks do not propagate strongly. This evidence is obviously useful for the network risk model.

The covariance decomposition is straightforward in this case because the connectedness model of Ando et al. (2018) estimates variance decompositions and the covariance matrix. In this manner, one can focus on the tails of the distribution, the *tail* network-driven portfolio risk is *conceptually* similar to a value-at-risk (VaR) type of risk measure accounted for the network effects. The details of the quantile regression approach are provided in Appendix C.

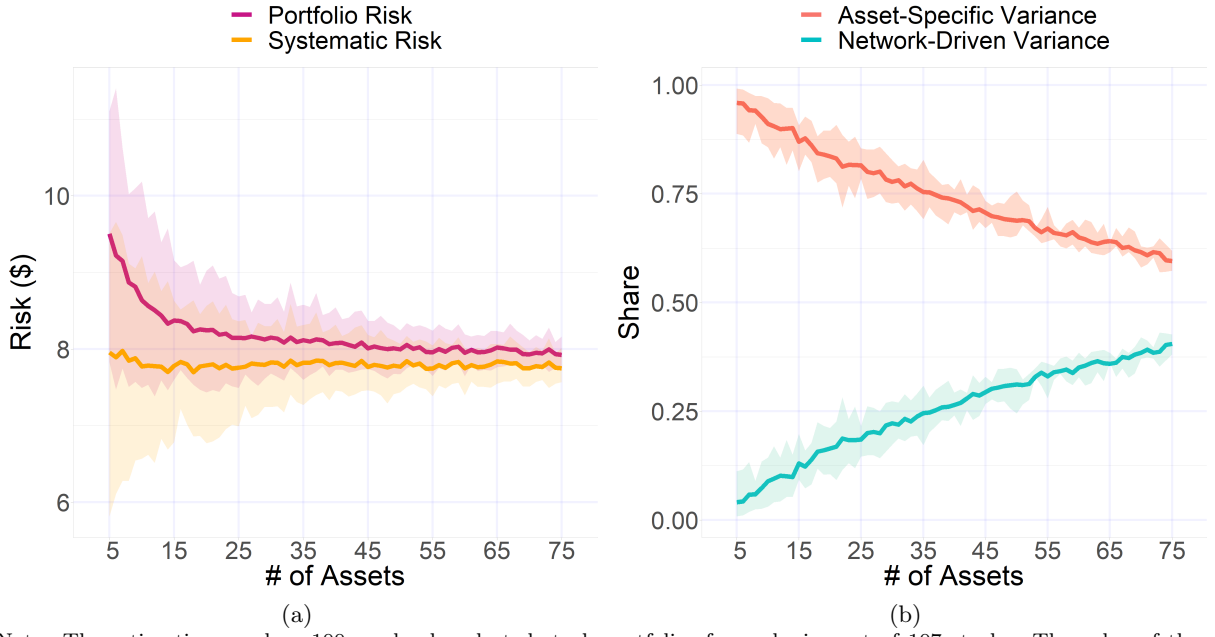
6 Empirical Results

The theoretical properties and implications of the network risk model were introduced in Section 4. As stated in that discussion, I evaluate the network risk model in two cases: a) the portfolio risk implications of the network risk model for a portfolio of 107 *S&P*-100 stocks; and b) the systemic risk implications of the network risk model for a portfolio of international banks.

Case 1) Stock Portfolio

In this case, I conduct a full-sample estimation in order to discover the portfolio risk implications of the network risk model. The details of the estimation are examined in Section 5. To start, I estimate the network risk model with latent factors that rely on conditional mean estimators. I obtain the portfolio risk measures for random portfolios, which allows the portfolio size to vary. This analysis emphasizes the role played by network effects in portfolio risk. I use a grid of [5, 75] to determine the number of assets in the portfolio. For each portfolio size, I generate 100 different random portfolios and apply the network risk model. Figure 6 indicates the estimation results. Panel (a) shows that the average portfolio risk for random portfolios decreases, which verify the diversification effect. As the number of assets increases, total portfolio risk gets closer to the common (or systematic) risk, thus diversifying the asset specific risks. However, the marginal diversification effect becomes smaller for more than 15 stocks, and it becomes very small for more than 45 stocks. The network risk model enables additional analysis of the portfolio risk by taking financial networks into account.

Figure 6(b) summarizes the results for network risk model which obtains the shares of asset-specific and network-driven variances. Larger portfolios are more likely to display high network-driven variance because a) added assets increase the number of possible linkages among assets and b) adding more assets enhances the existing network effect (feedbacks generated by the new assets). Not surprisingly, the share of the network-driven variance grows (in exchange for the asset-specific variance) as the number of assets in the portfolio increases. Figure 6



Note: The estimation employs 100 randomly selected stock portfolios for each size out of 107 stocks. The value of the portfolio is \$1000 and it is equally-weighted. The light areas indicates the 95% confidence intervals.

Figure 6: Portfolio Risk in Network Risk Model - Conditional Mean Estimation

yields results similar to those found in the well-known “*robust yet fragile*” phenomenon (Gai and Kapadia (2010), Gai (2013)). Increasing the number of assets in the portfolio leads to a diversification of asset-specific risks, while at the same time it prompts the overall portfolio risk approach to the undiversifiable (or systematic) risk component. In contrast, Vivier-Lirimont (2006) and Battiston et al. (2012) argue that as financial networks become denser, financial networks can become a destabilizing force that increase fragility by enabling the transmission and cascade of the financial shocks, especially in bad times. Figure 6 provides the basis for this discussion employing a portfolio risk approach.

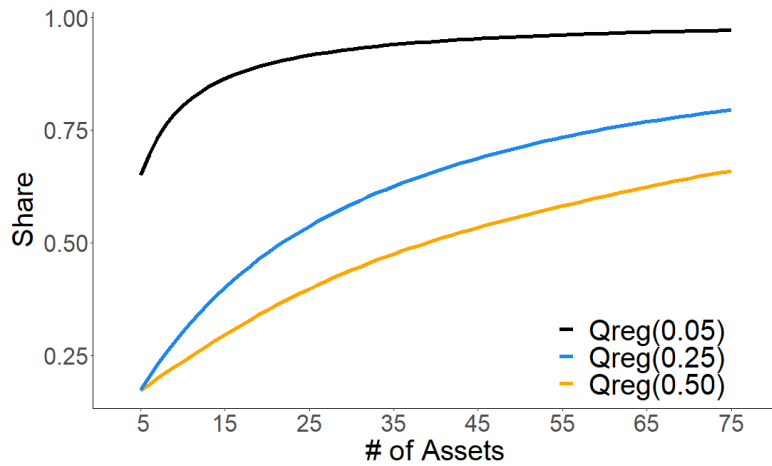


Figure 7: Network Driven Risk by Size and Quantile

The above result indicates that the share of network risk is lower with respect to asset-specific variance. To estimate the network and covariance matrices, it employs conditional mean estimator that accounts only for the average shocks. However, I also analyze the network effect at different quantiles, especially tail comovement and tail risks. To capture the tail behaviour of the network risk model, I use the quantile regression approach which leads to a risk measure analogous to the well-known *value-at-risk* at different quantiles. Figure 7 shows the share of network-driven risk for different portfolio sizes when $\tau = \{0.05, 0.25, 0.50\}$. Fulfilling the expectations, the share of the network effect increases with τ because connectedness and portfolio variance go up together towards the tails of the distribution. In the extreme quantiles, the share of network-driven variance converges immediately to an upper bound as portfolio size increases. This result verifies the Ando et al. (2018) conclusion that connectedness is higher at the tails of the distribution. Not surprisingly, the portfolio variance is also large at the tails, and the combination of both high connectedness and portfolio variance means the share of network-driven variance surges. This result is important for portfolio risk management because the relative increase in the network-driven risk at the tails (in exchange for the asset-specific risks) generates a significant non-diversifiable risk component.

Figure 8, which relies on the quantile estimation of network risk model by using the whole set of assets in the stock portfolio, provides richer information. First, Panel (a) compares the (standard system-wide) connectedness index and the share of network-driven variance at different quantiles. The results indicate an inverse bell-shaped curve for the connectedness index and the share of the network-driven variance along the quantiles; connectedness and portfolio variance go up towards the tails, and the conditional median estimator provides the minimum connectedness and the share of network driven variance. Second, the shapes of the curves are almost symmetrical; neither measure can tackle the asymmetry in the risk results at the rightmost and leftmost tails. Third, the share of network-driven variance towards median shocks is slightly higher than connectedness index. As financial network become denser at the tails, the differences disappears. This means that the upper and lower bounds (see Figure 2) narrow down, this implies the share of network-driven variances and system-wide connectedness measures get closer to each other.

Second, Panel (b) shows the network-driven and asset-specific risk measures in dollar terms rather than variance shares. Noticeably, this panel differs from the previous one in the respect that the risk results are not normalized; the panel quantifies the network-driven and asset-specific components of the portfolio risk. According to my estimation results, network-driven risk is almost equal to asset-specific risks around the median. Otherwise, network risk is higher at the tails, it distinguishes the asymmetric left and right tail shocks, which are highly relevant in both network studies Baruník et al. (2016) and risk measurement practices. Asset-specific risk is small at the tails which means that the portfolio risk (after accounting for the common shocks) is mainly dominated by the network effects. Third, Panel (c) compares the network-driven and common factor risks. As it is evident in Figure 6, I utilize the percentage changes from the median to recover the scale difference in common factor and network driven risk .

The plot draws a similar result for network driven risk. It also shows that the common factor risk grows more at the tails, however, network-driven risk grows faster than common factor (or systematic) risk component towards the extremes of the tail distribution.

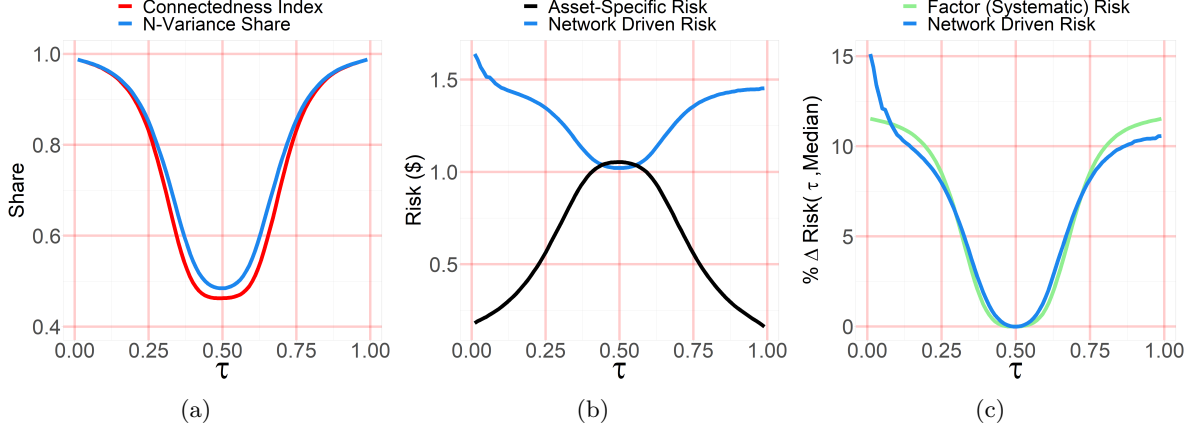


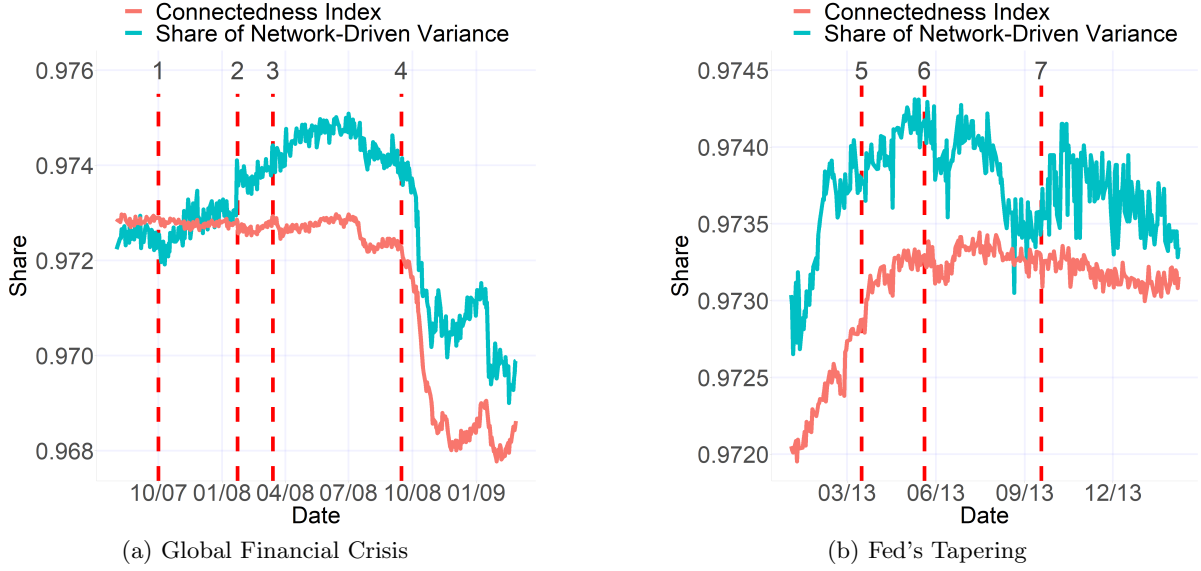
Figure 8: The Network Risk Model by Quantiles

Case 2) Global Banking Portfolio

Borrowing the global banking dataset from Demirer et al. (2018), I discover the systemic risk implications of the network risk model for a portfolio of international banks using rolling window analysis. The sample covers the period 2003 to 2014. As discussed above, the dynamic estimation of connectedness model is conducted using 1000-day overlapping rolling windows. Therefore, the first estimation outcome is provided by the third quarter of 2007, which coincides with the beginning of the mortgage crisis in U.S. markets. By selecting long rolling windows, I accept the risk of carrying the effect of large shocks in the system for a long time. Indeed, estimation results suffer from this issue, indicating that connectedness and network-driven risks are highly persistent during 2009-2012, or the period after the peak of the global financial crisis. In this respect, I divide the sample into two parts, which covers the a) global financial crisis and b) the Fed's tapering decision. The estimation results provide enough evidence to distinguish the contribution of the network risk model.

As argued in Glasserman and Young (2016), exposures to the U.S. subprime mortgage market have not been limited to the traditional mortgage lending; securitization of the mortgage debt has generated considerable uncertainty about the bank balance sheets. This has become evident when housing markets were shaken. When default rates on mortgage debt increased, the expected value of mortgage-backed securities and derivatives also were affected. Claessens et al. (2010) emphasize on the cross-border spillovers from the U.S. mortgage market to European banks; considering the U.S. originated mortgage-backed instruments were widely held by banks in other advanced economies. On the other hand, the Fed's tapering in recent decade has signalled the end of the era of *super-easy* monetary policy. In the years following the global financial crisis, major central banks pumped large amounts of money to fight the economic downturn. An unconventional monetary policy tool, large-scale asset purchases (LSAP) are

usually used by central banks to achieve domestic objectives. However, as Coeuré (2017) points out, LSAPs in a world of integrated financial markets are likely to have international effects. Bernanke (2017) and Khatiwada (2017) provide evidence of international spillovers from U.S. quantitative easing policies to other economies, especially emerging countries. In this respect, I confidently use these two recent episodes of the financial market turmoil as laboratories to test the implications of the network risk model for a pool of international banks. Figure 9-10 show the estimation results for the network risk model estimated for left tail shocks when $\tau = 0.05$.



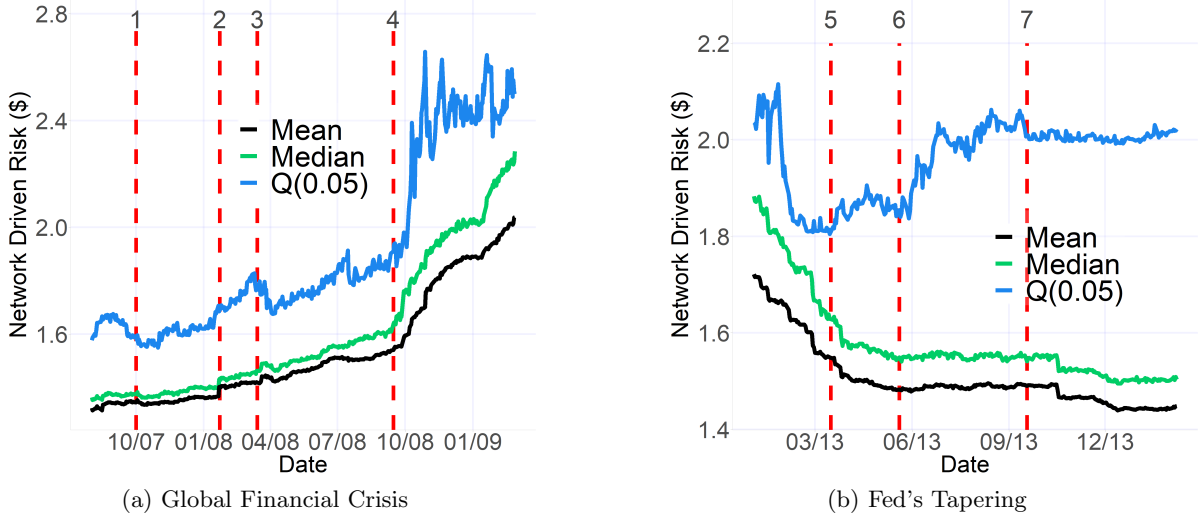
Note: Numbers on top of dashed lines indicates important dates: **1)** Major banks announces huge losses over subprime plunge **2)** Global stock market tumble on U.S. mortgage fears **3)** Collapse of Bear Stearns, **4)** Bankruptcy of Lehman Brothers, **5)** Peak of the Cyprus Crisis, **6)** Bernanke announces “tapering” **7)** Fed surprises markets postponing the start of tapering

Figure 9: Connectedness Index vs. The Share of Network Driven Risk in Important Dates

Figure 9 compares the connectedness index and the share of network-driven variance (N-var share). Panel (a) shows that connectedness among banks decreased after the collapse of Lehman Brothers which is usually referred as the peak of the global financial crisis. This result contradicts the well-known result that connectedness tends to increase under financial distress (e.g. Demirer et al. (2018)). Thus, it is important to observe, as noted above, that the network risk model also utilize the static latent factor model, as the details are discussed earlier. Therefore, the results shown in Figure 9 are the “de-factorised” connectedness measures, as defined by Claeys and Vašíček (2014), that analyzes the network effect after accounting for the common shocks. Then, it is possible to observe that connectedness and the share of network driven variance may go down when common shocks capture a greater share of covariation among asset returns during systemic events. In these cases, as an unscaled measure, network driven risk is still effective to verify the increasing network effects which is provided in Figure 10.

As claimed earlier, as an average measure of pairwise directional connectedness, the connectedness index draws a smoother pattern with respect to N-Var Share. Moreover, as evident in Figure (9), it fails to distinguish the impact of some important events. The equivalence of

connectedness and the N-Var share is broken in many cases. Usually this occurs when the N-Var share is pushed up, which indicates that high connected assets are, on average, the riskier ones. Clearly, the estimated N-Var share is more volatile than connectedness index because it utilizes richer information embedded in the covariance matrix.



Note: Numbers on top of dashed lines indicates important dates: **1)** Major banks announces huge losses over subprime plunge **2)** Global stock market tumble on U.S. mortgage fears **3)** Collapse of Bear Stearns, **4)** Bankruptcy of Lehman Brothers, **5)** Peak of the Cyprus Crisis, **6)** Bernanke announces “tapering” **7)** Fed surprises markets postponing the start of tapering

Figure 10: Network Driven Risk in Important Dates

In Figure (10), I provide the network risk outcome rather than indices. In contrast to Figure (9), it quantifies the network-driven risk in nominal terms. As an “unscaled” network measure, it does not rely on the proportions; instead, it calculates the network-driven portfolio risk. In that regard, either the connectedness index or the share of network-driven variance goes down because of normalization; as can be observed in Figure (10), network-driven risk goes up. I also include the network-driven risks for the conditional mean and conditional median estimators, which are clearly lower than tail risks. Furthermore, tail shocks are likely to generate a more responsive network risk measure because conditional covariances is also affected more at the tails.

7 Conclusion

Extending the DYCI methodology, and on the basis of forecast error covariance decompositions, this study derives the network risk model for a portfolio of assets. As a first step towards the derivation of the network risk model, I propose the approximate covariance decompositions to obtain network-driven portfolio variance. Unlike the “natural” covariance decompositions, approximate covariance decomposition ensures the positive semi-definiteness of the covariance matrix. Simulation and estimation results show that the network risk model can identify large

differences among asset variances when it aggregates the pairwise directional connectedness measures.

In addition, the proposed model uses static latent factors to take into account common shocks. Relying on the *latent* and *residual* network modeling of Epskamp et al. (2016), I show that a common factor model can be estimated to obtain both the variance and the covariance decompositions that enable the *component-wise* decomposition of portfolio variance. Incorporating the factor structure into the network risk model yields a property quite similar to the well-known “*robust yet fragile*” phenomenon: Although large portfolios are assumed to be well-diversified, they are more likely to display higher network driven risks.

Finally, the proposed network risk model is estimated for different shock sizes using quantile regressions. Results show that unlike the quantile estimation of the connectedness model, the network risk model can differentiate even small shocks at both tails because it also takes into account the information available at the covariance matrix.

There are many ways to improve the proposed network risk model. As noted above, the model does not consider the time dynamics of the common factors; it simply assumes static and exogenous common shocks. The dynamic factor models and hierarchical factor models of (Moench et al., 2013) can be used to describe the different aspects of a network risk model that utilizes different factor structures. Moreover, in our framework, the identification of the network risk model relies on the generalized variance decompositions of Koop et al. (1996). However recent studies (e.g. Bettendorf and Heinlein (2019), Scidá (2017)), favor data-driven identification methods that provide a statistical basis for the ordering problem and for the structural vector autoregressions. In any case, these new methods could improve connectedness analysis. Finally, while the network risk model penalizes only the coefficient matrix in high dimensional settings, sparse estimates of the covariance matrix could be obtained. This methodological issue is relevant to extension of the proposed network risk model because the derivation of the model depends critically on the covariance matrix and covariance decomposition.

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Appendices

A Covariance Decompositions

Previously, I discussed the network properties of asset pricing models with/without factors, the question of how portfolio risk decomposition is not answered yet. Obviously, the implications of variance decomposition based networks is limited in the analysis of portfolio risk, since the portfolio risk is more than asset return variances, the covariance among asset returns is crucial for the purpose of diversification and risk management.

In standard applications of factor models, the residuals of the factor model are assumed to be uncorrelated in multivariate analysis of asset returns, decomposing covariance matrix is pretty straightforward, asset return variance is a weighted sum of the systematic and idiosyncratic variances that the weights are closely related to the variable's closeness to the factor structure as discussed earlier. In this case, there is no need to decompose the covariances, since the error covariance matrix is assumed to be diagonal, the covariances are only generated by the systematic component. I have earlier discussed the covariance among asset returns can be different than zero when there exist direct linkages among assets, even if the factor structure is absent or is not able to capture the effect of network linkages on portfolio risks. Therefore, the covariances are not only provided by the systematic component, yet the linkages among assets also let the residual returns to be correlated.

For simplicity, I assume the factor structure is independent of the error terms, therefore I can estimate it in standard VAR framework. It could easily be extended to the exact factor models, extending it to the approximate factor models requires the adoption of further identification strategies in DFM, which is one of the directions for the future researches, except for FAVAR estimation of Bernanke et al. (2005) and Claeys and Vašíček (2014), the covariance decomposition framework is easily applicable in this case.

Data-Generating Process and Forecast Errors

Consider an N -dimensional covariance stationary data-generating process with orthogonal shocks w_t :

$$y_t = \Theta(L)w_t, \quad \Theta(L) = \Theta_0 + \Theta_1 + \Theta_2 + \dots, \quad E(w_t w_t') = I \quad (\text{A.1})$$

with Θ_i for $i = 1, 2, 3, \dots$ are each $N \times N$ coefficient matrices, $\Theta(L)$ is the lag-polynomial. The error of the optimal h -step forecast is the difference between actual value y_{t+h} and optimal h -step forecast $y_t(h)$:

$$y_{t+h} - y_t(h) = \sum_{i=0}^{h-1} \Theta_i w_{t+h-i}.$$

Explicitly, we can write optimal h -step forecast error of j -th component of y_t such as

$$y_{j,t+h} - y_{j,t}(h) = \sum_{i=0}^{h-1} (\theta_{j1,i} w_{1,t+h-i} + \cdots + \theta_{jK,i} w_{K,t+h-i})$$

where $\theta_{ji,i}$ are the elements of ji -th element of Θ_i . Equivalently, we can rewrite optimal h -step forecast error of j -th component of y_t by summing up the innovations over each variable n where $n = 1, 2, \dots, N$

$$y_{j,t+h} - y_{j,t}(h) = \sum_{n=1}^N (\theta_{jn,0} w_{n,t+h} + \cdots + \theta_{jn,h-1} w_{n,t+1}).$$

Since the $w_{n,t}$'s are assumed to be uncorrelated and have unit variances, the mean-squared error (MSE) of $y_{j,t+h}$ is

$$E(y_{j,t+h} - y_{j,t}(h))^2 = \sum_{n=1}^N (\theta_{jn,0}^2 + \cdots + \theta_{jn,h-1}^2) \quad (\text{A.2})$$

This analysis is often called *innovation accounting* which accumulates the effect of individual innovations to variance of a particular variable. (Lütkepohl, 2005)

Forecast Error Variance and Covariance Decompositions

Equation (A.2) can arbitrarily be rewritten as

$$E(y_{j,t+h} - y_{j,t}(h))^2 = \sum_{i=0}^{h-1} \theta_{jj,i}^2 + \sum_{i=0}^{h-1} \sum_{n=1; j \neq n}^N (\theta_{jn,i}^2) \quad (\text{A.3})$$

First term on the right-hand side (RHS) corresponds to the contribution of innovations in variable j and second term stands for the contribution of innovations in variables *other than* j to forecast error variance of the h -step forecast of variable j . Variance decompositions are obtained by dividing RHS to $MSE[y_{j,t}(h)]$

$$\underbrace{\frac{\sum_{i=0}^{h-1} \theta_{jj,i}^2}{MSE[y_{j,t}(h)]}}_{\text{Own Contribution}} + \underbrace{\frac{\sum_{i=0}^{h-1} \sum_{n=1; j \neq n}^N (\theta_{jn,i}^2)}{MSE[y_{j,t}(h)]}}_{\text{Contribution of Others}} \quad (\text{A.4})$$

We can also show individual contributions using Equation (A.3). The innovations in variable n contributes to h -step forecast error variance of variable j by

$$\theta_{jn,0}^2 + \cdots + \theta_{jn,h-1}^2 = \sum_{i=0}^{h-1} \theta_{jn,i}^2 \quad (\text{A.5})$$

We also rewrite the MSE of the j -th component as follows

$$MSE[y_{j,t}(h)] = \sum_{i=0}^{h-1} \sum_{n=1}^N \theta_{jn,i}^2 \quad (\text{A.6})$$

Dividing (A.5) to (A.6) gives

$$\omega_{jn,h} = \sum_{i=0}^{h-1} \theta_{jn,i}^2 / \sum_{i=0}^{h-1} \sum_{n=1}^N \theta_{jn,i}^2 \quad (\text{A.7})$$

is the proportion of the h -step forecast error variance of variable j , accounted for by w_{nt} innovations.

How to Decompose Covariances

Joo (2000) and Goto and Volkanov (2002) proposes covariance decompositions as a “natural” extension of variance decomposition analysis, instead of accumulating only the squared shocks, they also consider the accumulation of the cross-multiplied shocks which can be defined as forecast error covariances. Noticably, variance decompositions provides $N \times N$ matrix that indicates the contribution of each N variables to N different variances, covariance decomposition gives an $N \times N \times N$ dimensional array providing the contribution of N variable to N^2 variances and covariances. However, extending the variance decompositions to covariance decompositions is not feasible in most of the cases. To see this, recall Equation (A.5) in matrix form which gives contribution of innovations in variable n to h -step forecast error variance of variable j ,

$$\sum_{i=0}^{h-1} (e_j' \Theta_i e_n)^2$$

where e_j is the j -th column of $N \times N$ identity matrix, I_N . In order to obtain covariances among m and j , instead of taking squares of $(e_j' \Theta_i e_n)$ terms, multiply the $(e_m' \Theta_i e_n)$ and $(e_j' \Theta_i e_n)$ terms.

$$\sum_{i=0}^{h-1} (e_m' \Theta_i e_n)(e_j' \Theta_i e_n) \quad (\text{A.8})$$

Consider the special case, $m = j$. It is simply equal to variance decomposition. For this reason, it is convenient to think covariance decomposition as a generalization of variance decompositions. Equation (A.8) gives the contribution of innovations in variable n to h -step forecast error covariance of variable m and j . Now, think MSE as covariance matrix, the mj -th and jm -th component of MSE matrix is given as

$$\sigma_{mj} = \sigma_{jm} = \sum_{i=0}^{h-1} \sum_{n=1}^N \theta_{mn,i} \theta_{jn,i} = \sum_{i=0}^{h-1} \sum_{n=1}^N \theta_{jn,i} \theta_{mn,i} \quad (\text{A.9})$$

Analogous to variance decompositions, both Joo (2000) and Goto and Volkanov (2002) defines covariance decomposition as

$$\omega_{mj,n,h} = \frac{\sum_{i=0}^{h-1} (e_m' \Theta_i e_n)(e_j' \Theta_i e_n)}{MSE_{mj,t}(h)} \quad (\text{A.10})$$

However, there are two important drawbacks. First, if any pair of $\theta_{jn,i}$ and $\theta_{mn,i}$ has opposite

signs, their interactions will be negative. Positive and negative terms in $MSE_{m,j,t}(h)$ cancel out each other, then it is no longer an appropriate denominator that $\omega \in [0, 1]$ is not guaranteed. On the other hand, since we are interested in covariances, it may be preferred to obtain covariances directly instead of having the ω s. This eliminates first drawback for sure, however, it still fails to satisfy one of the basic assumptions of covariance matrix, *positive semi-definiteness*. To see this, modify the Equation (A.9) and write it explicitly such as

$$\sum_{i=0}^{h-1} (\theta_{jj,i} \theta_{mj,i} + \theta_{mm,i} \theta_{jm,i}) + \sum_{i=0}^{h-1} \sum_{\substack{n=1 \\ j \neq n \\ m \neq n}}^N (\theta_{jn,i} \theta_{mn,i}). \quad (\text{A.11})$$

It should be noted that symmetry property of the covariance matrix is satisfied in Equation (A.11). This equation fails to guarantee positive semi-definiteness of covariance matrix since it arbitrarily splits variance-covariance matrix into two pieces in which both are not needed to be positive semi-definite.

A.0.1 Approximate Covariance Decomposition via Variance Decomposition

Previously, I showed “natural” expansion of variance decompositions to decompose covariances fails to provide feasible solution to our problem. Instead, I propose an approximate covariance decomposition which relies on the information from variance decompositions. To isolate the network-driven variance and covariances from covariance matrix, I follow a two-step procedure which satisfies the symmetry and positive semi-definiteness properties. Procedure relies on basic variance and covariance properties,

$$\begin{aligned} \text{Var}(aX + b) &= a^2 \text{Var}(X) \\ \text{Cov}(aX + b, cY + d) &= ac \text{Cov}(X, Y) \end{aligned}$$

As variance decomposition delivers proportion of the h -step forecast error variance of variable j , accounted for by w_{nt} innovations, we easily apply backward-engineering to variance equation to procure a^2 and c^2 . Recall Equation (A.11) such as

$$\underbrace{\frac{\sum_{i=0}^{h-1} \theta_{jj,i}^2}{MSE[y_{j,t}(h)]}}_{\text{Own Contribution}} + \underbrace{\frac{\sum_{i=0}^{h-1} \sum_{n=1; j \neq n}^N (\theta_{jn,i}^2)}{MSE[y_{j,t}(h)]}}_{\text{Contribution of Others}}$$

Second term represents contribution of innovations in variables **other than** j to forecast error variance of the h -step forecast of variable j . Let us call that term $d_{o,j}^2$

$$\underbrace{\frac{\sum_{i=0}^{h-1} \sum_{n=1; j \neq n}^N (\theta_{jn,i}^2)}{MSE[y_{j,t}(h)]}}_{\text{Contribution of Others}} = d_{o,j}^2$$

Since variance decompositions provides the portion of variance contributed by others, a^2 or

c^2 can be replaced with $d_{o,j}^2$ in variance equation. In matrix form, we proceed as follows. We form a diagonal matrix of “contribution of others”, D_o^2 , which locates the share of contribution of others to variance of a particular variable.

$$D_o^2 = \begin{bmatrix} d_{o,1}^2 & 0 & \dots & 0 \\ 0 & d_{o,2}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & d_{o,N}^2 \end{bmatrix}$$

Owing to the fact that $d_{o,j}^2 \geq 0$ for all j and it is diagonal matrix, we can take square root of D_o^2 . In order to ensure the positive semi-definiteness of the resulting covariance matrix the $d_{o,i}$ s are assumed to be greater than or equal to zero, $d_{o,i} \geq 0$. In order to weight covariance matrix in the next step, we pre- and post-multiply the covariance matrix with the squared matrices.

$$\begin{bmatrix} d_{o,1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & d_{o,N} \end{bmatrix} \begin{bmatrix} \sigma_{11} & \dots & \sigma_{1K} \\ \vdots & \ddots & \vdots \\ \sigma_{K1} & \dots & \sigma_{KK} \end{bmatrix} \begin{bmatrix} d_{o,1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & d_{o,K} \end{bmatrix}$$

Then positive semi-definite *network-driven* covariance matrix is

$$\Sigma_{nd} = \begin{bmatrix} \overbrace{\sigma_{11}d_{o,1}^2}^{a^2Var(X)} & \overbrace{\sigma_{12}d_{o,1}d_{o,2}}^{acCov(X,Y)} & \dots & \sigma_{1K}d_{o,1}d_{o,K} \\ \sigma_{21}d_{o,2}d_{o,1} & \sigma_{22}d_{o,2}^2 & \dots & \sigma_{2K}d_{o,2}d_{o,K} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{K1}d_{o,K}d_{o,1} & \sigma_{K2}d_{o,K}d_{o,2} & \dots & \sigma_{KK}d_{o,K}^2 \end{bmatrix} \quad (\text{A.12})$$

B Generalized Variance Decompositions

Building on Koop et al. (1996), Pesaran and Shin (1998) proposes an agnostic approach to the identification problem which the estimated impulse responses are invariant to the order of variables. Recall the time series model in Equation A.1 with a covariance matrix Σ (not necessarily an identity matrix)

$$y_t = \sum_{l=0}^{\infty} \Theta_l w_{t-l}, \quad E(w_t w_t') = \Sigma$$

where Θ_l is $K \times K$ coefficient matrices, w_{t-l} is $K \times 1$ zero mean *iid* random shock with covariance matrix Σ . An impulse response function for y_t of the j -th shock is formally interpreted as

$$\begin{aligned} IRF(h, \delta_j, \Omega_{t-1}) &= E[y_{t+h}|w_{j,t} = \delta_j, w_{k \neq j,t} = 0, w_{t+1} = 0, \dots, w_{t+h} = 0, \Omega_{t-1}] \\ &\quad - E[y_{t+h}|w_t = 0, w_{t+1} = 0, \dots, w_{t+h} = 0, \Omega_{t-1}] \end{aligned}$$

where h is the forecast horizon, δ_j is the shock size hitting the j -th variable, Ω_{t-1} is the information set that is used to forecast y_t . Noticably, realization of y_{t+h} is dependent to both Ω_{t-1} and $\{w_t, \dots, w_{t+h}\}$. As Koop et al. (1996) called, *traditional impulse responses* measures the difference of expectations of two realizations of $y_{i,t+h}$. The first one is the realization of y_{t+h} when j -th variable is hit by a shock of size δ_j at time t and the rest of the shocks hitting other variables are assumed to be zero along time t and $t+h$. The second realization assumes no shocks hitting the system during $t, \dots, t+h$. In this setting, the impulse response function can be thought as an experiment to discover the dynamic effect of a shock hitting a particular variable at a particular time compared to the control, outcome of the baseline model assuming no shock hitting the system.

Identification problem takes place at this particular stage of the analysis. If reduced-form shocks are correlated, then any data generating process will fail to produce an outcome $w_{j,t} = \delta_j$ while $w_{k \neq j,t} = 0$, causing the experiment to be infeasible. As discussed above, from Choleksi-type of identification to the structural models relying on economic theory attempts to set this kind of experiment to find out the particular impact of a particular variable in the system. As a solution, Koop et al. (1996) defines the *generalized impulse response* function condition on the non-decreasing information set Ω_{t-1}

$$GIRF_{ij}(h, \delta_j, \Omega_{t-1}) = E[y_{t+h}|w_{j,t} = \delta_j, \Omega_{t-1}] - E[y_{t+h}|\Omega_{t-1}]$$

Assuming that w_t is distributed as multivariate normal, we obtain (see also Koop et al. (1996) and Pesaran and Shin (1998))

$$E(w_t|w_{jt} = \delta_j) = (\sigma_{1j}, \sigma_{2j}, \dots, \sigma_{Kj})' \sigma_{jj}^{-1} \delta_j = \Sigma e_j \sigma_{jj}^{-1} \delta_j$$

where e_j is a selection vector with *one* on the j -th entry and *zeros* elsewhere. Therefore, the generalized impulse response of the effect of a shock to the j -th equation at time t on y_{t+h} is

$$\left(\frac{\Theta_h \Sigma e_j}{\sqrt{\sigma_{jj}}} \right) \left(\frac{\delta_j}{\sqrt{\sigma_{jj}}} \right)$$

The remaining step is scaling the impulse response function defining a shock size equal to one standard deviation for j -th variable that eliminates the latter parenthesis from the last equation. We end up with the generalized impulse response function for variable j as follows

$$\psi_j^g(h) = \sigma_{jj}^{-\frac{1}{2}} \Theta_h \Sigma e_j$$

Now, we can discuss the differences between generalized and orthogonalized impulse re-

response functions. For comparison purposes, we can simply obtain orthogonalized impulse response, $\psi_j^o(n) = \Theta_h \mathbf{P} e_j$ where \mathbf{P} is an $m \times m$ lower triangular matrix obtained by Choleski decomposition of covariance matrix Σ . Apparently, orthogonalized impulse responses requires a lower-triangular decomposition of covariance matrix, the ordering of the variables matters. However, in generalized impulse responses, assuming error terms are distributed as multivariate normal, we define a shock hitting only one equation at a time, then the ordering does not change anything. On the other hand, Pesaran and Shin (1998) shows generalized impulse responses yields the same result with the orthogonalized impulse responses in the first equation under the multivariate normality assumption. (For proof, see Pesaran and Shin (1998)) This property can be exploited in a way that, for any linear vector autoregression model, estimate the model repeatedly as each equation becomes the first equation in each loop, then apply the Cholesky decomposition to select and store the orthogonalized impulse responses for the first equation. This identification procedure explicitly allow each variable to cause all variables in the system, to become the first chain of the “causal” model. In that regard, we can see the generalized impulse responses far more agnostic than the orthogonalized impulse responses, you do not spend your time to find out the ordering of variables. From the economics perspective, this may be seen as a drawback that disregarding the true causal chain, if there exists one, can lead to unclear analysis outcomes. However, as it is in some cases, when there is no true causal chain is achievable and therefore a unique ordering is non-deliverable, generalized impulse responses can provide a practical solution to the ordering problem.

One interesting issue arises when we proceed to generalized variance decompositions from generalized impulse responses. Koop et al. (1996) defines the ij -th element of h -step generalized variance decomposition matrix as

$$D_{ij}^g(h) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i' \Theta_h \Sigma e_j)^2}{\sum_{h=0}^{H-1} (e_i' \Theta_h \Sigma \Theta_h' e_j)}$$

where e_i' is similar to the e_j is a $K \times 1$ vector that i -th element is equal to one and else are zero. Since the shocks in generalized impulse responses are not necessarily orthogonal, row sums of D^g do not sum up to unity. This requires additional effort to rescale the rows to provide a convenient network representation. Diebold and Yilmaz (2014) normalizes the row sums to scale the variance decompositions. They prefer to use \tilde{D}^g as adjacency matrix where

$$\tilde{D}_{ij}^g = \frac{D_{ij}^g}{\sum_{j=1}^N D_{ij}^g}.$$

C Quantile VARs

Ando et al. (2018) introduces the studies the *quantile connectedness*, technically the quantile vector autoregressions for the estimation of connectedness in order to account for the different size of idiosyncratic shocks. They adopt equation-by-equation approach to estimate the quantile VARs of cross-sectionally correlated residuals proposed by Cecchetti and Li (2008) and Schöler (2014). They, similar to the Epskamp et al. (2016), focuses on the residual network after

isolating the impact of common shocks.

First, assuming a factor VAR(p) model

$$\mathbf{y}_t = \mu + \sum_{j=1}^p \Phi_j \mathbf{y}_{t-j} + \Lambda \mathbf{f}_t + \nu_t \quad (\text{A.13})$$

where \mathbf{f}_t is the k dimensional vector of common factors are observed or latent at time t , Λ is the $N \times k$ factor loading matrix, ν_t is the $N \times T$ error terms, Φ_j is the coefficient matrix and μ contains the intercept terms.

They rewrite the Equation A.13 in quantile factor model (QFVAR) form evaluated at τ th conditional quantile such as

$$\mathbf{y}_t = \mu_{(\tau)} + \sum_{j=1}^p \Phi_{j(\tau)} \mathbf{y}_{t-j} + \Lambda_{(\tau)} \mathbf{f}_t + \nu_{t(\tau)} \quad (\text{A.14})$$

Then, following Koenker and Xiao (2006) keeping the lag order as fixed across quantiles, they obtain the parameter estimates minimizing the function

$$\min_{\beta'_{i(\tau)}} \sum_{t=1}^T \xi_{\tau}(\mathbf{y}_{it} - \beta'_{i(\tau)} \mathbf{z}_t) \quad (\text{A.15})$$

where the $\beta'_{i(\tau)} \mathbf{z}_t$ is the compact form of right-hand side of the Equation A.14 with \mathbf{z}_t is an $(Np + k + 1) \times 1$ is the vector vector of regressors at time t , $\beta'_{i(\tau)}$ is the corresponding coefficient matrix evaluated at τ -th conditional quantile, $\xi_{\tau}(z)$ is the *check loss function* defined as $\xi_{\tau}(z) = z(\tau - 1_{[z \leq 0]})$ following Koenker and Hallock (2001). Once the parameters of the VAR model is done, the remaining procedure is similar to the usual Diebold and Yilmaz (2014), in order to extract the network structure of financial assets evaluated with different conditional quantiles. In order to see the estimation details, check Ando et al. (2018).

D Details of Dataset

Name	Country	Bank code	Name	Country	Bank code
National Australia Bank	Australia	nab.au	CIMB Group Holdings	Malaysia	cimb.my
Commonwealth Bank of Australia	Australia	cba.au	ING Groep	Netherlands	inga.nl
Australia and New Zealand Banking Group	Australia	anz.au	DNB ASA	Norway	dnb.no
Westpac Banking	Australia	wbc.au	Banco Comercial Portugues	Portugal	bcp.pr
Erste Group Bank	Austria	esb.at	Banco Espirito Santo	Portugal	bes.pr
KBC Groupe	Belgium	kbc.be	Sberbank Rossii	Russia	sber.ru
Dexia	Belgium	dexb.be	N/A	Singapore	d05.sg
Toronto-Dominion Bank	Canada	td.ca	United Overseas Bank	Singapore	u11.sg
Royal Bank of Canada	Canada	ry.ca	Banco Santander	Spain	san.es
Bank of Nova Scotia	Canada	bns.ca	Banco Bilbao Vizcaya Argentaria	Spain	bbva.es
Bank of Montreal	Canada	bmo.ca	Banco de Sabadell	Spain	sab.es
China Merchants Bank	China	cmb.cn	Norden Bank	Sweden	ndasek.se
Shanghai Pudong Development Bank	China	shgp.cn	Skandinaviska Enskilda Banken	Sweden	seba.se
China Minsheng Banking Corp	China	chinm6.cn	Svenska Handelsbanken	Sweden	shba.se
Ping An Bank	China	pab.cn	Swedbank	Sweden	sweda.se
Hua Xia Bank	China	hxb.cn	UBS	Switzerland	ubsn.ch
Danske Bank	Denmark	danke.dk	Credit Suisse Group	Switzerland	csgn.ch
Pohjola Bank	Finland	pohls.fi	HSBC Holdings	Switzerland	hsba.gb
Credit Agricole	France	aca.fr	Barclays	UK	barc.gb
BNP Paribas	France	bnp.fr	Royal Bank of Scotland Group	UK	rbs.gb
Societe Generale	France	gfe.fr	Lloyds Banking Group	UK	lloy.gb
Deutsche Bank	Germany	dbk.de	Standard Chartered	UK	stan.gb
Commerzbank	Germany	cbk.de	JPMorgan Chase	US	jpm.us
National Bank of Greece	Greece	ete.gr	Bank of America Corp	US	bac.us
Bank of Ireland	Ireland	bir.ie	Citigroup	US	c.us
Allied Irish Banks	Ireland	aib.ie	Wells Fargo	US	wfc.us
Unicredit	Italy	ucg.it	Goldman Sachs Group	US	gs.us
Intesa Sanpaolo	Italy	isp.it	Morgan Stanley	US	ms.us
Mitsubishi UFJ Financial Group	Japan	mtbh.to	Bank of New York Mellon	US	bk.us
Mizuho Financial Group	Japan	mzh.jp	U.S. BANCORP	US	usb.us
Sumitomo Mitsui Financial Group	Japan	sntm.jp	PNC Financial Services Group	US	pnc.us
Resona Holdings	Japan	rsnh.jp	Capital One Financial	US	cof.us
Nomura Holdings	Japan	nmrh.jp	BB T Corp	US	bbt.us
Sumitomo Mitsui Trust Holdings	Japan	sntm.jp	SunTrust Banks	US	sti.us
Woori Finance Holdings	Korea	wrfh.kr	American Express	US	exp.us
Shinhan Financial Group	Korea	shf.kr	Fifth Third Bancorp	US	fith.us
Industrial Bank of Korea	Korea	ibk.kr	Regions Financial	US	rf.us
Malayan Banking Berhad	Malaysia	may.my			

Table 2: Global Banking Portfolio - Banks

Company	Sector	Code	Company	Sector	Code
Microsoft Corp.	Information Technology	MSFT	3M Company	Industrials	MMM
Apple Inc.	Information Technology	AAPL	American Tower Corp.	Real Estate	AMT
Amazon.com Inc.	Consumer Discretionary	AMZN	QUALCOMM Inc.	Information Technology	QCOM
Berkshire Hathaway	Financials	BRK.B	Lockheed Martin Corp.	Industrials	LMT
Johnson & Johnson	Health Care	JNJ	Altria Group Inc	Consumer Staples	MO
JPMorgan Chase & Co.	Financials	JPM	Danaher Corp.	Health Care	DHR
Alphabet Inc Class C	Communication Services	GOOG	General Electric	Industrials	GE
Exxon Mobil Corp.	Energy	XOM	American Express Co	Financials	AXP
Visa Inc.	Information Technology	V	Gilead Sciences	Health Care	GILD
Procter & Gamble	Consumer Staples	PG	Lowe's Cos.	Consumer Discretionary	LOW
Bank of America Corp	Financials	BAC	Booking Holdings Inc	Consumer Discretionary	BKNG
Facebook, Inc.	Communication Services	FB	Mondelez International	Consumer Staples	MDLZ
The Walt Disney Company	Communication Services	DIS	U.S. Bancorp	Financials	USB
Mastercard Inc.	Information Technology	MA	Caterpillar Inc.	Industrials	CAT
AT&T Inc.	Communication Services	T	Bristol-Myers Squibb	Industrials	BMJ
Cisco Systems	Information Technology	CSCO	Anthem	Health Care	ANTM
Pfizer Inc.	Health Care	PFE	CME Group Inc.	Financials	CME
Chevron Corp.	Energy	CVX	CVS Health	Health Care	CVS
United Health Group Inc.	Health Care	UNH	Automatic Data Processing	Information Technology	ADP
Verizon Communications	Communication Services	VZ	United Parcel Service	Industrials	UPS
Home Depot	Consumer Discretionary	HD	Goldman Sachs Group	Financials	GS
Merck & Co.	Health Care	MRK	Fidelity National Information Services	Information Technology	FIS
Intel Corp.	Information Technology	INTC	Intuit Inc.	Information Technology	INTU
Coca-Cola Company	Consumer Staples	KO	Chubb Limited	Financials	CB
Comcast Corp.	Communication Services	CMCSA	Charter Communications	Communication Services	CHTR
Wells Fargo	Financials	WFC	Becton Dickinson	Health Care	BDX
Boeing Company	Industrials	BA	Stryker Corp.	Health Care	SYK
PepsiCo Inc.	Consumer Staples	PEP	Celgene Corp.	Health Care	CELG
Netflix Inc.	Communication Services	NFLX	Duke Energy	Utilities	DUK
Citigroup Inc.	Financials	C	TJX Companies Inc.	Consumer Discretionary	TJX
McDonald's Corp.	Consumer Discretionary	MCD	S&P Global, Inc.	Financials	SPGI
Walmart	Consumer Staples	WMT	Colgate-Palmolive	Consumer Staples	CL
Abbott Laboratories	Health Care	ABT	Northrop Grumman	Industrials	NOC
Adobe Systems Inc	Information Technology	ADBE	Dominion Energy	Utilities	D
Oracle Corp.	Information Technology	ORCL	Fiserv Inc	Information Technology	FISV
Medtronic plc	Health Care	MDT	Southern Co.	Utilities	SO
Honeywell Int'l Inc.	Industrials	HON	Crown Castle International Corp.	Real Estate	CCI
Philip Morris International	Consumer Staples	PM	Boston Scientific	Health Care	BSX
International Business Machines	Information Technology	IBM	ConocoPhillips	Energy	COP
Accenture plc	Information Technology	ACN	Intuitive Surgical Inc.	Health Care	ISRG
Union Pacific Corp	Industrials	UNP	CIGNA Corp.	Health Care	CI
Salesforce.com	Information Technology	CRM	PNC Financial Services	Financials	PNC
Thermo Fisher Scientific	Health Care	TMO	Target Corp.	Consumer Discretionary	TGT
Costco Wholesale Corp.	Consumer Staples	COST	Morgan Stanley	Financials	MS
Angen Inc.	Health Care	AMGN	Intercontinental Exchange	Financials	ICE
Nike	Consumer Discretionary	NKE	Prologis	Real Estate	PLD
Linde plc	Materials	LIN	Ecolab Inc.	Materials	ECL
Broadcom Inc.	Information Technology	AVGO	General Dynamics	Industrials	GD
Texas Instruments	Information Technology	TXN	Raytheon Co.	Industrials	RTN
United Technologies	Industrials	UTX	CSX Corp.	Industrials	CSX
Starbucks Corp.	Consumer Discretionary	SBUX	Marsh & McLennan	Financials	MMC
NextEra Energy	Utilities	NEE	DuPont de Nemours Inc	Materials	DD
Lilly (Eli) & Co.	Health Care	LLY	BlackRock	Financials	BLK
Nvidia Corporation	Information Technology	NVDA			

Table 3: Stock Market Portfolio - Companies