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# The Impact of Inflation Targeting: Testing the Good Luck Hypothesis

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# The Impact of Inflation Targeting: Testing the Good Luck Hypothesis

#### Abstract

Starting in the mid 1980s, the level and volatility of inflation decreased across industrial countries. The inflation stabilization can be explained by a shift in monetary policy or by a lucky period of low volatility in business cycle shocks. To test the "good luck hypothesis", we examine the inflation experience of Canada, one of the earliest and most successful adopters of an inflation targeting monetary policy. We Kalman-filter the historical structural shocks consistent with an estimated DSGE model. The estimated shocks are used to build counterfactual histories. The good luck hypothesis can explain only a minor portion of the change in the path and volatility of inflation after the shift in policy. Most of inflation and output stabilization is accounted by the impact on expectations. Unconditionally, the inflation targeting policy does not improve on the previous policy in terms of inflation volatility, but supports a more favorable trade-off, reducing substantially output volatility.

#### Resume

Fra midten af 1980'erne faldt niveauet og volatiliteten i inflationen på tværs af industrilandene. Stabiliseringen i inflationen kan forklares enten ved et skift i pengepolitikken eller ved en heldig periode med lav volatilitet i stødene til konjunkturerne - "lykkehypotesen". For at teste lykkehypotesen undersøger vi inflationserfaringen i Canada, et af de tidligste og mest succesfulde lande til at vedtage en inflationsmålrettet pengepolitik. Vi Kalman-filtrerer de historiske strukturelle stød, der er i overensstemmelse med en estimeret DSGEmodel. De estimerede stød bruges til at opstille kontrafaktiske historier. Lykkehypothesen kan kun forklare en mindre del af ændringen i inflationsstien og -volatiliteten efter ændringen i pengepolitikken. Størstedelen af stabiliseringen i inflation og output kan forklares af effekten af forventninger. Den inflationsmålrettede pengepolitik forbedrer ikke ubetinget den tidligere politik med hensyn til inflationsvolatilitet, men understøtter en mere gunstig afvejning, hvilket reducerer outputvolatiliteten betydeligt.

#### Key words

Business cycle shocks; Kalman ⊠filter; Credibility; Inflation targeting.

#### **JEL classification**

E42; E52; E58.

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The authors alone are responsible for any remaining errors.

# The Impact of Inflation Targeting: Testing the Good Luck Hypothesis

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October 5, 2019

#### Abstract

Starting in the mid 1980s, the level and volatility of inflation decreased across industrial countries. The inflation stabilization can be explained by a shift in monetary policy or by a lucky period of low volatility in business cycle shocks. To test the 'good luck hypothesis', we examine the inflation experience of Canada, one of the earliest and most successful adopters of an inflation targeting monetary policy. We Kalman-filter the historical structural shocks consistent with an estimated DSGE model. The estimated shocks are used to build counterfactual histories. The good luck hypothesis can explain only a minor portion of the change in the path and volatility of inflation after the shift in policy. Most of inflation targeting policy does not improve on the previous policy in terms of inflation volatility, but supports a more favorable trade-off, reducing substantially output volatility.

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### 1 Introduction

Over the Great Moderation period, between the mid 1980s and mid 2000s, many industrial countries experienced a marked decline in the level and volatility of inflation, interest rates, and long-term inflation expectations. This period of low and stable inflation can be attributed either to a change in the propagation of shocks through the economy – the most prominent explanation being a shift in the management of monetary policy – or to a reduction in the exogenous volatility of business cycle shocks.

We examine the inflation performance of Canada, an early and successful adopter of an inflation targeting monetary policy since February 1991, and ask whether it can be explained by the 'good luck hypothesis'. Accepting the good luck hypothesis means that conditional on the exogenous shocks that hit the Canadian economy since 1991 the inflation time series would not have been significantly different under an alternative monetary policy. We focus on the impact of inflation targeting in reducing inflation and output volatility in Canada. In particular, the standard deviation of quarterly inflation more than halved from 0.004 over the period 1971-1990 to 0.0018 over the period 1992-2000, and the standard deviation of output fell from 0.0161 over the period 1971-1990 to 0.0104 over the period 1992-2000 (see also Longworth, 2002, Murray, 2006). Our estimates show that the good luck hypothesis can be rejected. A very large share of the impact of inflation targeting on the behavior of inflation was caused by the change in the private sector's beliefs for monetary policy. The estimated model predicts that unconditionally the change in the volatility of shocks would have only a minor impact on the volatility of inflation, and a much smaller role than what the conditional counterfactual attributes to the shift in policy. Finally, inflation stabilization did not result in larger output volatility.

The experience of inflation targeting countries is especially suitable to assess whether good luck or good policy can account for the observed change in inflation behavior across industrial countries, since the monetary authority explicitly announced – and committed to – the inflation targeting policy.

We investigate the impact of the shift in monetary policy by building counterfactual histories of the Canadian economy conditional on alternative assumptions for monetary policy, shocks, expectations. The shocks are Kalman-filtered from data on nine aggregate variables, and rest on a Bayesian estimation of an open-economy DSGE model with staggered wage and price adjustment. Our approach encompasses a number of alternative modeling hypotheses (from flexible to sticky prices and wages, from backward to forward looking price-adjustments, from wage-indexation to no indexation at all, from full pass-through to pricing to market for import prices, from none to very large price sensitivity of imports and export demand, from standard to habit-persistent preferences) to let the data select an appropriate description of the observables. The model is estimated over the floating-rate regime period (1971-1990) and over the inflation targeting period (1992-2018). The sample covers the Great Recession period, and uses estimated shadow interest rates to account for the zero-lower bound of interest rates.<sup>1</sup>

We innovate relative to the literature by adopting two alternative ways to build counterfactuals so as to evaluate how much of the economy dynamics in the inflation targeting period is accounted for by the change in the path of the policy instrument (a shift in the *actual policy*) and by the announcement of an inflation target that is credible and affects expectations (a shift in the *perceived policy*).<sup>2</sup>

Section 2 presents the model, with the most standard derivations confined to Appendix 5.1. Section 3 presents the estimation and the counterfactual experiments. Section 4 concludes.

# 2 The General Equilibrium Model

The Canadian economy is modeled as a small open economy with nominal price and wage rigidities (see Galí and Monacelli, 2005, Kollmann, 2001). The domestic (H) sector utilizes labor to produce a consumption-good basket that is both consumed by domestic households and exported to the foreign (F) sector, in exchange for a foreign-produced consumption good.

A fraction of households and domestic firms set respectively nominal wages according to the Erceg, Henderson and Levin (1999) staggered contracts mechanism and prices as in Calvo (1983). The remaining fraction is assumed to follow a rule of thumb, so that price and wage setting is partially

<sup>&</sup>lt;sup>1</sup>Dib et al. (2008) provide an estimation of an open economy DSGE model for Canada, over the period 1981-2007.

 $<sup>^{2}</sup>$ The approach to build conditional counterfactuals with model-consistent expectations we adopt has been used by a large number of authors, including Arias, Hansen and Ohanian (2007), Benati and Mumtaz (2008), Justiniano and Primiceri (2008), King and Rebelo (1998), Rotemberg and Woodford (1998), Stock and Watson (2003), and Chari, Kehoe and McGrattan (2007).

backward-looking. Households' preferences have a habit-persistence specification. These three features improve the performance of sticky-price models, whose failure to generate plausible degrees of output and inflation persistence and to match the empirical correlation between real wages and output is well known (Fuhrer, 2000). Finally, the model allows for short-term incomplete pass-through from the foreign to the domestic price of imported goods.

#### 2.1 Households

Assume a continuum of infinitely lived households, indexed by  $j \in [0, 1]$ . Preferences are described by the instantaneous utility function

$$U_{t}^{j} = \left\{ \log(C_{t}^{j} - bC_{t-1}^{j})D_{t} - \frac{\ell N_{t}^{j^{1+\eta}}}{1+\eta} + \nu\left(\frac{M_{t}^{j}}{P_{t}}\right) \right\},\$$

where  $M_t/P_t$  is real money balances,  $N_t$  is the amount of labor service supplied, and  $D_t$  is an exogenous demand shock that distorts the labor-leisure decision. When b > 0, preferences are characterized by habit persistence. State-contingent claims ensure consumption levels are identical across households who supply different amounts of labor services.

The consumption aggregate,  $C_t^j$ , combines a basket of differentiated home-produced,  $C_{t,H}$ , and foreign-produced,  $C_{t,F}$ , goods:

$$C_t^j = \left[ (1 - \gamma)^{\frac{1}{\rho}} (C_{H,t}^j)^{\frac{\rho-1}{\rho}} + \gamma^{\frac{1}{\rho}} (C_{F,t}^j)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}},$$
(1)

where  $0 \leq \gamma \leq 1$  is the share of the foreign good and  $\rho > 0$  is the elasticity of substitution between domestic and foreign goods. The domestic good  $(C_{t,H})$  and the foreign good  $(C_{t,F})$  are themselves Dixit-Stiglitz aggregates, each defined over a continuum of differentiated goods indexed by  $i \in [0, 1]$ with an elasticity of substitution  $\vartheta > 1$ . Households allocate their expenditure optimally across goods.  $P_t$ ,  $P_{H,t}$  and  $P_{F,t}$  indicate the price indices for the aggregate, domestic, and foreign good consumption basket.

Households maximize the expected discounted utility flow,  $U^j = \mathbb{E}_0 \{ \sum_{t=0}^{\infty} \beta^t U_t^j(C_t^j, N_t^j, \frac{M_t^j}{P_t}, D_t) \},\$ 

subject to eq. (1) and the budget constraint

$$P_t C_t^j + M_t^j + e_t v_t^* B_t^{*j} + \overrightarrow{v}_t \overrightarrow{B}_t^j \le W_t^j N_t^j + M_{t-1}^j + e_t B_{t-1}^{*j} + B_{t-1}^j + \Pi_t^j - \tau_t,$$
(2)

where  $e_t$  is the nominal exchange rate,  $v_t^*$  is the price of a zero-coupon risk-free bond priced in foreign currency,  $B_t^*$  is the amount of foreign asset purchased,  $W_t$  is the wage rate,  $\Pi^j$  is the share of profit from the monopolistic firms rebated to the household, and  $\tau_t$  is a lump-sum government tax. Each element of the row vector  $\vec{v}_t$  represents the price of an asset that will pay one unit of currency in a particular state of nature in period t + 1. The corresponding element of  $\vec{B}_t$  represents the quantity of such claims purchased by the household.  $B_{t-1}$  indicates the value of the household portfolio of claims against domestic residents given the current state of nature.

Firms regard each household j's labor supply,  $N_t^j$ , as an imperfect substitute for the labor offered by other households. A CES labor aggregator,  $N_t = \left(\int_0^1 N_t^{j\frac{\phi}{\phi-1}} dj\right)^{\frac{\phi-1}{\phi}}$ , combines households' labor services in the same proportions as firms would optimally choose. The wage index,  $W_t = \left(\int_0^1 W_t^{j^{1-\phi}} dj\right)^{\frac{1}{1-\phi}}$ , gives the least expenditure that buys a unit of the labor index. Firms' optimal demand for each type of labor j, conditional on the total demand for labor services  $N_t$ , give household j's downward sloping demand function for  $N^j$ :

$$N_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\phi} N_t.$$
(3)

Households set the nominal wage  $W^j$  in contracts which can be renegotiated with probability  $(1-\theta_w) \in (0,1]$ . Of the households resetting the wage contract, a fraction  $(1 - \omega_w) \in (0,1]$  updates the wage optimally, while a fraction  $\omega_w$  follows a backward-looking rule of thumb. As in the staggered wage adjustment model of Erceg et al. (1999), any household j optimally resetting the wage at time t maximizes its utility functional with respect to the nominal wage  $\tilde{W}_t^j$  subject to the sequence of budget constraints (eq. 2) and the labor demand function (eq. 3) at time t + s. We assume the rule of thumb adopted by a fraction  $\omega_w$  of the wage-resetting households takes into account the average nominal

wage, the consumer price inflation rate and the average contract duration  $\frac{1}{1-\theta_w}$ , as in Rabanal (2001). Backward-looking households update the wage to the average level prevalent across all contracts at time t - 1, adjusted for the current inflation rate  $\pi_t$ :

$$W_t^{RT^j} = W_{t-1}(1+\pi_t)^{\frac{1}{1-\theta_w}}.$$
(4)

For  $\omega_w \to 0$ , the model converges to the Erceg et al. (1999) wage-updating mechanism. This hybrid model implies aggregate nominal wage inflation depends explicitly on current and expected consumer price inflation through the indexing rule (4). In the estimation, we allow for the elasticity of substitution across labor services  $\phi$  to be time-varying. This results in an additive shock in the linearized wage-setting equation. Since variations in  $\phi$  generate variations in the steady-state wage-markup, the literature usually labels this shock as the wage-markup shock  $\mu_t^w$  (Smets and Wouters, 2003, 2007). It can also be interpreted as a shock observationally equivalent to a time-varying labor wedge or as fluctuations in the bargaining power of workers.

#### 2.2 Firms

The *H* production sector is made up of a continuum of firms indexed by  $i \in [0, 1]$ . Domestic firms produce goods by combining labor services supplied by households. The firm producing good *i* employs a CRS technology:  $Y_{H,t}(i) = A_t N_t(i)$ , where  $A_t$  is an aggregate productivity shock. The firms' cost minimization problem implies that when inputs quantities are chosen optimally the real marginal cost,  $MC_t$ , is independent of the scale of production. We adopt the hybrid pricing model in Galí and Gertler (1999). As in the time-dependent Calvo (1983) pricing model, in every period *t* firms adjust their prices with probability  $(1 - \theta_p) \in (0, 1]$ . A fraction  $(1 - \omega_p) \in (0, 1]$  of the price resetting firms update the price optimally, while a fraction  $\omega_p$  follows a backward-looking rule of thumb. The problem of the firm optimally setting the price at time *t* consists of choosing  $P_{H,t}(i)$  to maximize

$$\mathbb{E}_{t}\left\{\sum_{s=0}^{\infty}(\theta_{p}\beta)^{s}\Lambda_{t,t+s}\left[\frac{P_{H,t}(i)}{P_{H,t+s}}Y_{H,t+s}(i)-\frac{MC_{t+s}^{N}}{P_{H,t+s}}Y_{H,t+s}(i)\right]\right\}$$
(5)

subject to

$$Y_{H,t+s}(i) = \left[\frac{P_{H,t}(i)}{P_{H,t+s}}\right]^{-\vartheta} C_{t+s}^W,$$
(6)

$$MC_{t+s}^{N} = P_{H,t+s}MC_{t+s} = \frac{W_{t+s}}{MPL_{t+s}},$$
(7)

where  $MC^N$  is the nominal marginal cost and MPL is the marginal product of labor. In (6),  $Y_{H,t+s}(i)$ is the demand function for firm *i* output at time t + s, conditional on the price set *s* periods in advance at time *t*,  $P_{H,t}(i)$ . Market clearing ensures that  $Y_{H,t}(i) = C_t^W(i) \equiv C_{H,t}(i) + C_{H,t}^*(i)$ , where  $C_{H,t}^*(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\vartheta} C_{H,t}^*$  is foreign demand for good *i*,  $C_{H,t}^* = \gamma^* S_t^{\rho^*} C_t^*$  is foreign demand for domestic exports, and  $C_t^*$  is the exogenously given aggregate foreign demand. Aggregate world demand is defined as  $C_t^W \equiv C_{H,t} + C_{H,t}^*$ . The stochastic discount factor between period *t* and period t + s is  $\beta^s \Lambda_{t,t+s}$ . Backward-looking firms update their price to the average level set in the most recent round of price adjustment,  $\overline{P}_{t-1}$ , adjusted for the lagged domestically-produced goods inflation rate  $\pi_{H,t-1}$ :

$$P_{H,t}^{RT}(i) = \overline{P}_{H,t-1}(1 + \pi_{H,t-1}).$$

Conditional on the shocks vector at time t, the rule of thumb price  $P_{H,t+k}^{RT}(i)$  converges to the optimal price as  $k \to \infty$ . This hybrid model ensures that current inflation is determined partly by lagged and partly by expected inflation. In the estimation, we allow for the elasticity of substitution across individual goods,  $\vartheta$ , to be time-varying. This introduces a price-markup shock  $\mu_t^{\pi}$  in the linearized equation relating current, past, and future inflation  $\pi_H$  to marginal costs, which has proven important to explain business cycle dynamics in several estimated new Keynesian medium-scale models (Smets and Wouters, 2003, 2007).

#### 2.3 Import Sector

We model incomplete pass-through of imported goods prices by assuming that the foreign-produced good F is purchased by a continuum of monopolistically competitive firms in the import sector as

an input for production. Each firm z can costlessly differentiate the imported good  $X_F$  to produce a consumption good  $C_F(z)$  using the production technology  $Y_F(z) = X_F(z)$ , where  $X_F(z)$  denotes the amount of input imported by firm z. The nominal marginal cost of producing one unit of output is defined as  $MC_{F,t}^N(z) = e_t P_{F,t}^*$ , where  $P_{F,t}^*$  is the foreign-currency price of  $X_F$ . The domestic-currency price,  $P_F(z)$ , is set following the Calvo (1983) pricing model with a probability of price re-optimization equal to  $(1 - \theta_f) \in (0, 1]$ . The producer faces an aggregate demand schedule given by

$$Y_{F,t+s}(z) = \left[\frac{P_{F,t}(z)}{P_{F,t+s}}\right]^{-\vartheta} C_{F,t+s},$$

where market clearing implies that  $Y_{F,t}(z) = C_{F,t}(z)$ . This production structure generates deviations from the law of one price in the short run, while asymptotically the pass-through from the price of the imported good to the price of the consumption basket F is complete.

#### 2.4 Government Sector and Aggregate Shocks

The government budget is balanced in every period. Monetary policy is set by an interest rate rule. To ease interpretation, we discuss the rule in its log-linear approximation,

$$i_t^{tar} = \pi_t^{tar} + \omega_\pi \mathbb{E}_t \left\{ (\pi_{t+1} - \pi_{t+1}^{tar}) \right\} + \omega_y y_{H,t} + \omega_e \Delta e_t, \tag{8}$$

where  $\omega_{\pi} > 1$  is the feedback coefficient to deviations of the expected gross consumer price inflation rate from its target value  $\pi_t^{tar}$ ,  $\omega_y \in \mathbb{R}$  is the feedback coefficient to deviations of domestic output from its steady-state value, and  $\omega_e \in \mathbb{R}$  is the feedback coefficient to the exchange rate log-change  $\Delta e_t$  between time t - 1 and t. The interest rate target  $i_t^{tar}$  responds to changes in the nominal exchange rate, to accommodate the possibility that the central bank smooths the volatility of the foreign value of its currency. The policy rule, as is often the case in medium-scale monetary DSGE models estimated over long samples (Smets and Wouter, 2003), allows for the possibility of exogenous shifts in the long-run target inflation rate  $\pi_t^{tar}$ . The rule can be interpreted as prescribing that, ceteris paribus, an increase in the ex-post target real interest rate ( $i_t^{tar} - \pi_t^{tar}$ ) only occurs if the inflation rate is expected to rise above the target inflation rate, while an expected exogenous increase in  $\pi_{t+1}^{tar}$  would lead to a fall in  $(i_t^{tar} - \pi_t^{tar})$ . Finally, we assume the policymaker adjusts the interest rate only gradually to the target rate  $i_t^{tar}$ , so that  $i_t = (1 - \chi)i_t^{tar} + \chi i_t + \varepsilon_{i,t}$ , where  $\chi \in [0, 1)$  is the degree of smoothing and the exogenous shock  $\varepsilon_{i,t}$  represents non-systematic movements in the monetary policy instrument.

The logarithm of the exogenous demand shock,  $D_t$ , the technology shock,  $A_t$ , aggregate foreign demand,  $C_t^*$ , the price and wage markup shocks,  $\mu_t^{\pi}$  and  $\mu_t^w$ , the inflation target shock,  $\pi_t^{tar}$ , the world interest rate,  $\tilde{\imath}_t^*$ , and imports' price inflation,  $P_{F,t}^*/P_{F,t-1}^*$ , follow a first order autoregressive stochastic process, with stochastic innovations  $\varepsilon_{j,t} \sim N(0, \sigma_j^2)$ . The exogenous policy shock,  $\varepsilon_{i,t}$ , is assumed to have no serial correlation. Market clearing conditions and aggregate equilibrium conditions are presented in Appendix 5.1.

# 3 The Impact of Inflation Targeting

#### 3.1 Methodology

Write the linearized DSGE model equilibrium law of motion as

$$\xi_{t+1} = F\xi_t + v_{t+1}, \tag{9}$$

$$q_t = H'\xi_t, \tag{10}$$

where  $q_t$  is a vector of observable variables,  $\xi_t = [\xi_t^1 \ \xi_t^2]'$  is a vector of endogenous  $\xi_t^1$  and exogenous  $\xi_t^2$ state variables, and the only non-zero element of  $v_{t+1}$  is a multivariate Gaussian stochastic process  $\varepsilon_t$ . We build a counterfactual history  $[\overline{q}]_{t=1}^T$  by simulating the model in eqs. (9) and (10) conditional on a counterfactual law of motion,  $\overline{F}^{11}$ ,  $\overline{F}^{12}$ , and  $\overline{H}'$ , and on the estimate  $[\xi_{t|T}^2]_{t=1}^T$ :

$$\overline{\xi}_{t+1} \equiv \begin{bmatrix} \overline{\xi}_{t+1}^1 \\ \overline{\xi}_{t+1}^2 \end{bmatrix} = \begin{bmatrix} \overline{F}^{11} & \overline{F}^{12} \\ 0 & I \end{bmatrix} \begin{bmatrix} \overline{\xi}_t^1 \\ \xi_{t+1|T}^2 + \widetilde{\xi}_{t+1}^2 \end{bmatrix},$$
(11)

$$\overline{q}_t = \overline{H}'_t \overline{\xi}_t. \tag{12}$$

For  $\tilde{\xi}_t^2 = 0 \ \forall t$ , the system in eqs. (11) and (12) simulates the economy dynamics conditional on the historical exogenous shocks. In some instances, it is useful for some of the  $\tilde{\xi}_t^2$  components to be nonzero in order to build counterfactual histories conditional on alternative shocks series.

#### 3.1.1 Data

We estimate the parameters and shocks of the model through a Bayesian procedure, and evaluate the likelihood using the Kalman filter. Details of the estimation procedure are provided in Appendix 5.3.

The model is estimated on two separate samples, at a quarterly frequency. The 'inflation targeting' sample runs from 1992Q1 to 2018Q4. Inflation targeting was formally adopted in the beginning of 1991, and the first target was set to reduce consumer price inflation in the 1-3 pct. range by the end of 1995. Before the announcement of specific inflation targets, the Bank of Canada had embarked in a three-year campaign to promote price stability as the long-term objective of monetary policy, though it made little headway against the momentum in inflation expectations that had built up. In the fourth quarter of 1990, inflation was still at 4.2 pct. The 'pre-inflation targeting' sample runs from 1971Q1 (following the eight-year period ending in 1970 when Canada pegged its exchange rate to the US dollar) to 1990Q4.

We estimate the model using nine observable variables: (1) Domestic output,  $Y_{H,t}$ , is measured by real gross domestic product;<sup>3</sup> (2) domestic consumer price inflation,  $\pi_t \equiv \log(P_t) - \log(P_{t-1})$ , is measured by the log-change in the consumer price index excluding food, energy, and the effect of indirect taxes; (3) the domestic interest rate,  $i_t$ , is measured by the Bank of Canada target for the overnight rate; (4) domestic terms of trade,  $S_t = \frac{P_{F,t}}{P_{H,t}}$ , are measured by the ratio of the Laspeyres index for import and export prices; (5) the domestic nominal exchange rate depreciation,  $\frac{e_t}{e_{t-1}}$ , is measured by the log-change in the Canadian/US dollar exchange rate; (6) aggregate foreign demand,  $C_t^*$ , is measured by the real output of the US nonfarm business sector; (7) the foreign interest rate,  $\tilde{i}_t^*$ , measured by the quarterly US effective federal funds rate; (8) domestic wage inflation,  $\xi_t \equiv \log(W_t) - \log(W_{t-1})$ , is the

 $<sup>^{3}</sup>$ The HP-filtered series of this measure of output correlates strongly with the multivariate output gap computed by the Bank of Canada since 1981Q1. The results in the paper – both from the estimation and the simulated counterfactuals – are robust to using alternative definitions of output.

log-change in the average hourly earnings of manufacturing workers; and (9) domestic employment,  $N_t$ , is measured as total employment multiplied by the average weekly hours in the manufacturing sector. The series measuring  $Y_{H,t}$ ,  $S_t$ ,  $C_t^*$ , and  $N_t$  are logged and detrended using a Hodrick-Prescott filter. All variables, with exception of the two interest rates, have been seasonally adjusted.

We replace the Bank of Canada target for the overnight rate with the shadow bank target rate from MacDonald and Popiel (2017) in the period 2009Q2-2010Q1, when this rate was at its zerolower bound. Likewise, we replace the effective federal funds rate with the shadow effective federal funds rate from Wu and Xia (2016) in the period 2009Q1-2015Q4. These substitutions are necessary to prevent, in particular, our monetary policy coefficients from being misidentified, as a result of the observed policy rate not correctly measuring how accommodative monetary policy was at the zerolower bound. By contrast, the shadow rate – using information on short-term forward rates and the historical relationship between these rates and the policy rate – captures the policy rate which would have materialized in the absence of the zero-lower bound.<sup>4</sup>

Figure 1 plots the time series for the interest rates used in the estimation. Appendix 5.2 provides details on the data sources and plots of all variables used in the estimation.

#### 3.1.2 Parameterization and Prior Distribution

A subset of the parameters are parameterized using information complementary to the estimation sample. The parameters  $\eta$  and  $\phi$  are not separately identified. To estimate  $\eta$ , the inverse of the steadystate labor supply elasticity, the value of the steady-state wage markup,  $\phi/(\phi - 1)$ , is assumed equal to 10%, implying  $\phi = 11$ . We use the model's steady-state restrictions to set the value of some additional parameters. The quarterly discount factor,  $\beta$ , is set to 0.99, which implies a steady-state real world interest rate of 4%. The foreign good share,  $\gamma$ , is equal to the steady-state ratio between imports and domestic output, and is set to 0.29, the average Canadian import/output ratio over 1971-2018. Finally,

<sup>&</sup>lt;sup>4</sup>The shadow rate is estimated using a nonlinear term structure model. This model posits the existence of an unobserved shadow interest rate that is linear in Gaussian factors, with the observed policy rate being the maximum of the shadow rate and zero. The results in the paper – both from the estimation and the simulated counterfactuals – are robust to not using the shadow rates in the zero-lower-bound periods.

the elasticity of substitution between domestic goods and between foreign goods,  $\vartheta$ , is set equal to 6, so that the markup in a flexible-price steady state is 20% (Galí and Monacelli, 2005).

Table 1 reports the prior distributions of the estimated parameters. A detailed description of these distributions and comparisons with the existing literature is contained in Appendix 5.3.

We assume that the covariance matrix,  $\Sigma$ , of the shock innovations is diagonal, except for the submatrix describing the covariances for the foreign shocks,  $C_t^*$ ,  $\tilde{\imath}_t^*$ , and  $P_{F,t}^*/P_{F,t-1}^*$ . The non-zero correlation across these shocks allows the model to generate richer dynamics for the foreign sector, resembling the correlations we would expect from a small structural model of the foreign economy.

#### 3.1.3 Posterior Distribution

Table 1 reports the posterior distributions from the two estimations of the model.<sup>5</sup> The two posterior distributions report marked differences in the estimation of the monetary policy feedback coefficients. While the interest rate smoothing coefficient ( $\chi$ ) and the inflation coefficient ( $\omega_{\pi}$ ) are of similar order of magnitude, the output coefficient ( $\omega_y$ ) more than doubles in size in the IT period. The exchange rate coefficient ( $\omega_e$ ) is much higher in the period 1971-1990 than in the period 1992-2018. This is consistent with the Bank of Canada at various times in the 1970s and 1980s being concerned about the value of the Canadian dollar against the US dollar, thus taking action to smooth exchange rate fluctuations (Powell, 2005). The weight on this policy objective was – according to our estimates – set to zero with the implementation of the inflation target policy, putting relatively more emphasis on inflation stabilization. Overall, it is hard to interpret how the response of monetary policy changed across the two samples, since the variables to which the policy rule responds are correlated. To answer this question, we will use the counterfactuals and volatility estimates for macroeconomic outcomes in the next subsection.

Most the of remaining structural parameters do not differ substantially across the two estimation samples. However, two exceptions to this are the backward-looking price and wage parameters, which

<sup>&</sup>lt;sup>5</sup>Estimation diagnostics, posterior distributions for estimated parameters, and estimated historical shock series are available upon request.

are, respectively, higher and lower in the 1992-2018 sample than in the 1971-1991 sample. The higher share of backward-looking price adjustments in the late sample (0.21 vs. 0.40) possibly reflects that price inflation was more stable in the later period than in the early one, causing backward lookingness to be a better description of price adjustments. Conversely, the lower share of backward-looking wage adjustments in the later sample (0.34 vs. 0.19) could reflect less need for automatic wage indexation, again in the context of a more stable evolution in prices in this period. Taking statistical uncertainty into consideration, these point estimates are not significantly different from the point estimates that, e.g., Smets and Wouters (2007) and Galí, Smets, and Wouters (2012) have found for the post-war US economy. Finally, habit formation in consumption is lower in the later sample than in the early one. This might result from nominal and real interest rates having been more stable lately, reducing the need for habits to explain a given degree of consumption stability.

The estimates of inertia in price and wage setting are in the range of 0.37-0.85, with import prices always being less rigid than domestic prices. These estimates imply average price and wage durations of 1.6-6.7 quarters. Such degrees of rigidity are commensurate with what Dib et al. (2008) find for Canada during 1981-2007. Furthermore, the point estimates are again well in line with the estimates obtained by Smets and Wouters (2007) and Galí, Smets, and Wouters (2012).

We next evaluate the role of nominal rigidities in the performance of the model. Table 2 computes the posterior odds ratio comparing the benchmark model to a restricted version where domestic wages and prices are reset optimally in every period (i.e.,  $\omega_p = \omega_w = \theta_p = \theta_w = 0$ ). The restrictions are massively rejected, implying that nominal rigidities play a statistically significant role in fitting the data. Finally, we test separately for the significance of the nominal rigidity in the foreign good import sector. This test is meant to discriminate between the producer and consumer currency-pricing hypothesis for the imported good. A value of  $\theta_f = 0$  implies instantaneous, complete pass-though of foreign price movements. This restriction can also be rejected.

The standard deviations of the demand innovation (0.2517 and 0.2055) are large compared to the other disturbances. Formally, this is a result of the shock entering the model as a first difference, due to habit formation.<sup>6</sup> Large standard deviations for demand innovations are not unusual in models with nominal rigidities. For instance, Del Negro et al. (2006) estimate a demand shock standard deviation of 0.4054 within a large-scale New Keynesian model of the US economy. Hall's (1997) empirical decomposition shows that a very large share of the volatility in US labor hours can be attributed to a preference shift between market and non-market activities, and is consistent with the empirical evidence in Eichenbaum, Hansen, and Singleton (1988) on the co-movements of real wages, consumption, and work effort.

Overall, we see that the Canadian economy has transitioned from mainly being a cost-push driven economy in 1971-1990 to predominantly being a demand driven economy in 1992-2018. Price and wage markup shocks are considerably more persistent in the early sample than in the later one, giving them more relevance early on. This contrasts demand and foreign interest rate shocks which are more persistent in the late sample, giving them more relevance here. Furthermore, we see that the monetary policy objective has stabilized in the late sample, in that the inflation target shock is more persistent here. These interpretations are consistent with the actual shock realizations identified at the posterior mode. According to these realizations, adverse technology and price markup shocks impact the economy around the Oil Crises in the 1970s, and adverse demand shocks impact the economy around the Great Recession in the 2000s. Finally, the inflation target shock increases in the 1970s and falls in the 1980s.

Summing up, Canada experienced two structural shifts around 1991: a new monetary policy and a change in the type of shocks affecting the economy. These two changes eventually led to the two candidate explanations for the decline in the level and volatility of inflation highlighted in the introduction, namely inflation targeting monetary policy and the 'good luck hypothesis'. In the following, we will in a number of counterfactual simulations compare these competing explanations.

<sup>&</sup>lt;sup>6</sup>The demand shock enters the model as  $D_t - b\beta \mathbb{E}_t \{D_{t+1}\}$  via the expression for the marginal utility of consumption.

#### 3.2 The Good Luck Hypothesis and the Impact of Inflation Targeting

Using the two estimates of the model for the pre-inflation targeting (pre-IT) and the inflation targeting (IT) period, we build several counterfactuals to assess the impact of the change in the policy rule, and of the change in the stochastic shocks hitting the economy across the two subperiods.<sup>7</sup> Table 3 provides a summary of the changes in standard deviation and levels for inflation and detrended output. Figures 2-6 build conditional counterfactual paths in the 1992-2018 period.

The first thought experiment compares the historical annual consumer price inflation  $\pi_t$  and output  $y_{H,t}$  with the counterfactual paths  $\overline{\pi}_t, \overline{y}_t$  under the hypothesis that inflation targeting had not been adopted in 1991 (Figure 2). It assumes that  $\overline{\xi}_t^2 = \xi_{t|T}^2$  and  $\overline{F}^{11}, \overline{F}^{12}, \overline{H}'$  are computed conditional on the estimated pre-inflation targeting policy rule. We assume identical long-term inflation goals under the two regimes.

Figure 2.a shows that a large portion of the major swings in inflation over the period would be amplified under the pre-IT policy. After 2012, inflation would have been nearly always below its historical level, and further below the 2% inflation target than the historical inflation level. The Bank of Canada success in reducing inflation in the first half of the 1990s was associated by some critics with a high cost in terms of unemployment (Fortin, 1996). But it is by no means clear that the unemployment rate increase from 8% to over 11% between 1990 and 1993 could be avoided using a different monetary policy (Mishkin and Posen, 1997). In the same period, world oil markets created inflationary pressures, while low commodity prices harmed Canadian exports. The output counterfactual (Figure 2.b) shows that the output slowdown in the initial years of IT implementation cannot be ascribed to the inflation targeting policy. The pre-IT policy would have resulted in a much larger slowdown. On the contrary, the pre-IT policy would have lead to a substantial overheating of the economy in the expansion of the 2000s, and in additional volatility in the Great Recession period.

Macroeconomic volatility outcomes under various scenarios are reported in Table 3. The table shows that over 1992-2018, inflation volatility would have doubled, and detrended output volatility

<sup>&</sup>lt;sup>7</sup>The simulations are based on smoothed shock estimates,  $\xi_{t|T}^2$ , which we Kalman-filtered out from the model parameterized to the posterior mode.

would have more than doubled, under the pre-IT policy. The shift in policy did not affect the average level of inflation.

Obviously the inflation behavior results are also dependent on the parameter estimates. The estimates for  $\omega_p$  in the price-adjustment equation imply inflation has some inertia, and could potentially limit the sensitivity of inflation to the policy rule. Table 3 shows that estimating the model under the constraint that the share of rule-of-thumb firms is equal to zero has a negligible impact on the results. Finally, it is legitimate to ask whether, given the Kalman-filtered shocks vector  $\xi_{t|T}^2$ , the DSGE model under some alternative parameterization is able to generate a counterfactual where the pre-IT policy would have reduced inflation. This is indeed the case. Table 3 includes the result from the counterfactual experiment conducted in the estimated economy where we set  $\omega_p = 0$ , without reestimating the whole model. In this economy, the pre-IT policy would have *lowered* inflation volatility by 14%.

The estimation results established that both the policy rule, and the volatility of business cycles shocks, changed across the two estimation subperiods. The second thought experiment asks whether the change in the shocks volatility between the pre-IT and the IT period was relevant for the behavior of output and inflation. We address this question by computing the *unconditional* standard deviation of  $\pi_t$  and  $y_{H,t}$  in the IT-period estimated model, assuming the shocks process is described by the covariance matrix  $\hat{Q}$  estimated over the pre-IT period, while monetary policy follows its historical IT rule. Table 3 shows that the volatility of inflation and output would increase respectively by 15% and 32%. These results suggests that we should expect a limited impact from the change in business cycle shocks on the inflation and output behavior under the IT regime, while conditionally on the shocks the change to the IT policy halved (more than halved) inflation (output) volatility.

The third thought experiment examines whether we should expect the impressive performance of the shift to the IT policy to be repeated in the future. Should the IT policy be expected to perform better than the pre-IT policy, given the estimated IT-period model parameters and covariance matrix  $\hat{Q}$ ? Table 3 reports that unconditionally, the pre-IT policy *reduces* volatility of inflation by 15%, albeit at the cost of increasing the volatility of output by 72%. Notice that alternative covariance matrices Q could easily yield different results. The estimation of the model in the pre-IT period provides one such matrix. In our fourth thought experiment, we compare the relative performance of the IT and pre-IT rule policies keeping the model parameters (with exception of the monetary policy coefficients) equal to the ones estimated in the IT period, but changing the covariance matrix  $\hat{Q}$  to the one estimated in the pre-IT period. First, we consider the effects of just changing the standard deviations of the monetary policy innovation and inflation target innovation. In this way, both the deterministic and stochastic components of monetary policy are at the pre-IT values, while the remaining model is the IT one. Doing so does not change the unconditional volatilities considerably. Inflation volatility now falls by 7%, and output volatility increases by 75%. If we instead change the entire covariance matrix  $\hat{Q}$ , the results are qualitatively similar, but quantitatively larger, to the previous experiment. Unconditionally, the pre-IT policy now *reduces* volatility of inflation by 8%, albeit at the cost of increasing the volatility of output by 146%.

Finally, we can ask whether the IT policy would have improved the performance of the Canadian economy, conditional on the estimated series of shocks for the 1971-1990 period. The fifth thought experiment estimates the counterfactual behavior of output and inflation. The large increase in inflation in the 1975-1983 period would have been higher by an average of two percentage points. In the 1988-1990 period, instead, inflation would have been on average substantially lower, and output substantially higher. Table 3 reports that the conditional standard deviation of inflation under the counterfactual IT policy would increase by 57%, with inflation increasing on average from 6.27% to 7.27% over the period 1971-1990. In summary, the IT policy performed well over the periods in which it was implemented, but would *not* have contributed to reducing inflation volatility if applied in the 1971-1990 period.

## 3.3 Sources of Macroeconomic Stabilization in the Inflation Targeting Period

In this section, we examine more closely how the changes in the monetary policy rule and in the shocks' volatility estimated to have happened between the 1971-1990 and 1992-2018 period affected macroeconomic performance in the IT period.

We have established using the estimated model that unconditionally the pre-IT policy delivers

a slight (if changing the monetary policy behavior with the pre-IT policy rule) or moderate (if changing only the monetary policy coefficients with the pre-IT coefficients) decrease in inflation volatility  $\sigma_{\pi}$ , and a very large rise in output volatility  $\sigma_y$ . Table 4 investigates which change in the behavior of the monetary authority generates the unconditional volatility results for  $\pi_t$  and  $y_{H,t}$ . A counterfactual policymaker using the pre-IT period policy rule feedback coefficients  $\omega_e$  (measuring the response to exchange rate log-changes) and  $\omega_y$  (measuring the output response) in the estimated economy for the IT sample, but switching to the IT-period coefficient  $\omega_{\pi}$  (measuring the response to expected inflation) would only marginally affect the counterfactual volatility of output and inflation, since the estimates for  $\omega_{\pi}^{IT}$  and  $\omega_{\pi}^{pre-IT}$  are very close. The second line of the table shows that the increase in  $\sigma_y$  observed under the counterfactual policy is owed in large part to the lower response to output adopted in the pre-IT period. It turns out that the slight reduction in  $\sigma_{\pi}$  achieved under the counterfactual policy is entirely dependent on the pre-IT policy reacting to changes in the nominal exchange rate. Setting the parameter  $\omega_e$  to the value estimated in the IT period – virtually nil – would lead the counterfactual policymaker to an increase in inflation volatility of around 9% relative to the one under the estimated IT policy.

We measure the role played by the change in the volatility of the shocks in the IT period by building conditional counterfactuals under alternative assumptions on the shocks. Figure 3 displays the counterfactual path of inflation, output, and nominal interest rate under the assumption that the size of the innovations in the 1992-2018 sample is rescaled so that the volatility of the shock state pertaining to a given series of innovations is equal to the volatility of the same shock state in the estimation using the 1971-1990 sample. Under this scenario, output peaks and troughs would have been more extreme, especially in the 2000s, when deviations from trend would have peaked at values occasionally over 100% higher. The impact on the volatility of inflation is larger, returning a standard deviation several times higher than the historical one over virtually any subsample.

It is relevant to ask whether any single shock had a large impact on the estimated increase in conditional volatility computed when replaying the IT-period economy with the size of the shocks estimated from the earlier period. We address this question in Table 5 by comparing the counterfactual conditional volatility for  $\pi_t$  and  $y_{H,t}$  under alternative assumptions as to the volatility of which shock changed between the pre-IT and the IT period. The first line of Table 5 reports the ratio of the 1992-2018 volatility for  $\pi_t$  and  $y_{H,t}$  relative to its values in the 1971-1990 period. We then ask: by how much would the volatility in the IT period increase, if the historical innovations had been rescaled to match the pre-IT conditional volatility? Comparing the first two lines of the table shows that inflation volatility in the IT period was only 21% of inflation volatility in the pre-IT period, while rescaling the innovations would imply that inflation volatility in the IT period would increase to 65% of its pre-IT period value. This implies that scaling the shocks to their pre-IT period level increases  $\sigma_{\pi}$  and  $\sigma_y$  by respectively 219% and 96%. Table 5 reports that the price and wage markup shocks are responsible for most of the increase in  $\sigma_{\pi}$  that the 1971-1990 shocks would have brought about. Overall, the 1971-1990 markups and demand shock account for about half of the increase in  $\sigma_y$ .

We found that the shift to inflation targeting *does* generate a large decrease in the conditional inflation volatility. We then ask what drives this result, given that unconditionally the estimates show that the inflation behavior is similar regardless of the monetary regime.

Consider that the monetary policy rule for each subperiod is estimated jointly with all other parameters of the model, and is therefore returning the most likely estimate of the policy coefficients conditional on the model specification and the estimated values of all other coefficients. Thus, different estimated models could return a more limited or more extreme estimate in the shift of the monetary policy after the introduction of inflation targeting. In our estimate, it turns out that the pre-IT policy larger relative weight on exchange rate stabilization results *unconditionally* also in substantial inflation stabilization – at a cost of a large volatility in output. The difference between the inflation volatility in the unconditional and conditional counterfactual economy depends on the specific draw of shocks that is estimated over the 1992-2018 sample. When comparing the unconditional variance decomposition to the share of inflation variance explained in-sample by each shock over the IT period, we find that price and wage markup shocks account for about 50% of  $\sigma_{\pi}$  in the sample, against an unconditional variance share of only 30.6%. On the contrary, unconditionally the demand shock accounts for 25% of  $\sigma_{\pi}$ , while in sample the same shock explains only 16.5% of inflation volatility. Finally, the wedge between the unconditional and conditional counterfactual inflation is also driven by the shocks' correlation. The correlation across domestic shocks, and between domestic and foreign shocks, is zero unconditionally, but in sample can be non-zero. For example, we obtain that the correlation between the price markup shock and the domestic demand shock is 0.67, and the correlation between the latter shock and the foreign inflation shock is -0.27.

#### 3.4 The Impact of Policy Shocks

Did surprise deviations from the IT policy have an impact on macroeconomic variables in the 1992-2018 period? Figure 4 builds a counterfactual path by simulating an economy conditional on the inflation targeting policy but setting to zero the monetary policy disturbance. The counterfactual assumes  $\overline{F}^{11} = F^{11}$ ,  $\overline{F}^{12} = F^{12}$ , and  $\overline{H}' = H'$ . We set the  $j^{th}$  component of the vector  $\tilde{\xi}_t^2$ , corresponding to the policy shock  $\varepsilon_{i,t}$ , equal to  $-\xi_{t|T}^{2j}$ , so that  $\overline{\xi}_t^{2j} = 0 \forall t$ . The vector composed of all the elements of  $\overline{\xi}_t^2$ except the  $j^{th}$  one,  $\overline{\xi}_t^{2^{-j}}$ , is unchanged and equal to  $\xi_{t|T}^{2^{-j}}$ .

The impact of policy shocks on inflation is limited over the IT period. In a minority of the sample, policy shocks account for a difference with the historical path of the order of a quarter of a percentage point, and in many periods the difference close to zero. Note that this outcome occurs despite Figure 4 showing policy shocks having been at times non-negligible, such as in the 2010-2018 period. Unexpected movements in monetary policy played a more important role in output dynamics. By lowering the interest rate level relative to the counterfactual path, they contributed over the 2015-2018 period to raise output by about one-half of a percentage point. The counterfactuals show that the non-systematic component of monetary policy did not carry any weight for output movements in the 1992-1994 period, soon after the shift to the IT monetary policy, while contributed to reduce output volatility in the second half of the 1990s.

#### 3.5 The Impact of Credibility

Central banks adopting inflation targeting frameworks consider the achieved macroeconomic stability, at least in part, a result of the increased credibility of the policy commitment to inflation stabilization. Some authors (see Kuttner and Posen, 1999) argue instead that the adoption of inflation targeting simply indicates the central bank's shift towards a greater conservatism with respect to the inflation goal, with increased credibility and transparency playing a minor role for the policy's outcome.

We evaluate the role of the inflation targeting credibility in Canada's inflation performance since 1991 by building a history under the assumption that the shift in monetary policy occurred but was not believed by the private sector (Figure 5). It assumes  $\overline{\xi}_t^2 = \xi_{t|T}^2$ . The matrices  $\overline{F}^{11}$ ,  $\overline{F}^{12}$ , and  $\overline{H}'$  are built under the hypothesis that the monetary authority adjusts interest rates according to the estimated post-1991 inflation targeting policy ( $\overline{F}^{11} = F^{11}$ ,  $\overline{F}^{12} = F^{12}$ , and  $\overline{H}' = H'$ ), while the private sector expectation  $\widetilde{E}_t^{ps}[s_{t+1}]$  of any variable  $s_t$  is conditioned on the belief the central bank adopts the policy rule estimated for the earlier period. Thus, while the *actual policy* changes relative to the pre-inflation targeting period, the *perceived policy* used to form the private sector's expectation is unchanged. Effectively, the private sector interprets as non-systematic movements in the interest rate  $\varepsilon_{i,t}^{ps}$  what truly is the sum of a policy shock and the distance between the interest rate implied by the true and believed policy rule:

$$i_{t} - \left\{ (1 - \chi^{ps}) \left[ \pi^{tar}_{t} + \omega^{ps}_{\pi} \widetilde{\mathbb{E}}^{ps}_{t} (\pi_{t+1} - \pi^{tar}_{t+1}) + \omega^{ps}_{y} y_{H,t} + \omega^{ps}_{e} \Delta e_{t} \right] + \chi^{ps} i_{t-1} \right\} = \varepsilon^{ps}_{i,t}$$

$$\varepsilon_{i,t}^{ps} = \varepsilon_{i,t} + (\chi^{cb} - \chi^{ps})i_{t-1} + \left[ (1 - \chi^{cb})\omega_y^{cb} - (1 - \chi^{ps})\omega_y^{ps} \right] y_{H,t} \\
+ \left[ (1 - \chi^{cb})\omega_\pi^{cb} \mathbb{E}_t^{cb} \left\{ (\pi_{t+1} - \pi_{t+1}^{tar}) \right\} - (1 - \chi^{ps})\omega_\pi^{ps} \widetilde{\mathbb{E}}_t^{ps} \left\{ (\pi_{t+1} - \pi_{t+1}^{tar}) \right\} \right] \\
+ \left[ (1 - \chi^{cb})\omega_e^{cb} - (1 - \chi^{ps})\omega_e^{ps} \right] \Delta e_t,$$

where the index *ps* indicates the private sector believes, the index *cb* indicates the true value of a parameter in the central bank policy rule, and we assume the central bank's expectation of inflation is model-consistent:  $\mathbb{E}_t^{cb}{\{\pi_{t+1}\}} = \mathbb{E}_t{\{\pi_{t+1}\}}$ . Appendix 5.4 describes the equilibrium concept adopted to solve the DSGE model when the private sector holds incorrect believes.

Figure 5 shows that despite the monetary authority increased aggressiveness towards inflation, a non-credible policy would achieve an inflation path as volatile as - and in many periods, close to - the one under the assumption inflation targeting had not been introduced at all (shown in Figure 5), that is, under a less inflation-averse, but fully credible policy. Over the 2011-2017 period – a time when inflation undershoot the target – lack of credibility would have returned a substantially *lower* level of inflation. By and large, the experiment shows that whatever change in inflation dynamics occurred over the inflation targeting period must be attributed to the change in the *perceived policy*: the private sector adjusting its behavior given the belief that the central bank's stance against inflation has become more aggressive.

To measure the gain from the expectation channel, we build an additional counterfactual path assuming the monetary authority still follows the inflation targeting policy without being believed by the private sector – but at each time t adjusts the nominal interest rate to bring inflation to its historical level (Figure 6). The counterfactual assumes the same matrices,  $\overline{F}^{11}$ ,  $\overline{F}^{12}$ , and  $\overline{H}'$ , as in the previous exercise while  $\tilde{\xi}_t^{2^j}$  is computed to ensure  $\overline{\pi}_t = \pi_t$ . Appendix 5.5 provides the recursion to build the appropriate  $\overline{\xi}_t^{2^j}$ . The counterfactual history describes an economy where the central bank achieves the given historical inflation path by actually changing the path of the interest rate through surprise policy interventions, rather than relying on its commitment to a policy rule to adjust the interest rate in response to the state of the economy.

Figure 6.b shows the implied path of output. Overall replicating the inflation performance without credibility would have required a massive increase in output volatility. After 2012, when inflation undershoots the 2% target, a non-credible central bank would have needed a large fall in the policy rate to raise inflation to the historical path, resulting in massive increase in output. Between 1996 and 2002, instead, the nominal interest rate has to be kept at a higher level to achieve the historical inflation path. Correspondingly, counterfactual output decreases persistently and dramatically. The mechanism at work can be summarized as follows. When faced with inflationary shocks the monetary authority raises interest rates and forces the economy into a severe recession to be able to achieve the inflation target path. Firms' expect policy to be more accommodative towards inflation than it really is, and the incorrect believes would lead firms to set persistently higher prices, ceteris paribus. The central bank reacts with unexpected increases in the nominal interest rate, that translate in real interest rate increases, to curb demand and the firms' inflationary behavior. Because only a minority of the prices are updated in every period, incorrect believes are very persistent in the economy, and result in a prolonged recession.

## 4 Conclusions

This paper considers the inflation performance of an early and successful adopter of an inflation target monetary policy, Canada, to investigate whether the volatility and behavior of key macroeconomic variable can be explained by good luck – a favorable combination of business cycle shocks coincident with the adoption of inflation targeting – or by the shift in the management of monetary policy.

We estimate a DSGE model for the Canadian economy over the 1971-1990 period, and over the inflation targeting sample running from 1992 to 2018. The main conclusions we reach building several alternative counterfactuals are as follows.

The shift in monetary policy was highly relevant for the behavior of inflation after 1991. A counterfactual history conditional on the estimated shocks and using the pre-1991 policy would have doubled inflation volatility. Significantly, the reduction in inflation volatility did not come at the expense of output volatility, which was also reduced, compared to the counterfactual history.

To assess the impact of the change in shocks volatility between the two subsamples, and after the introduction of IT, we compare the unconditional volatility of inflation and output under the pre-IT and IT period estimated shocks' covariance matrix. Under the pre-IT shocks' covariance, the volatility of inflation would increase in the IT period by 15%, supporting the hypothesis that good luck played a smaller role than good policy in lowering inflation volatility. While we do not know what particular shocks' draw would have occurred in the IT-period given the pre-IT shocks covariance, an accounting exercise where we scale shocks over 1992-2018 to be consistent with the covariance matrix of shocks over 1971-1990 shows that conditional inflation volatility would have been much higher, suggesting a relevant role also for changes in business cycle shocks.

The counterfactuals also show that inflation targeting affected the behavior of inflation for the largest part through the impact on expectations. Monetary policy shocks are estimated to have nonnegligible variance, yet they contributed very little to inflation stabilization. Moreover, a monetary policy that did not affect private sector expectations, but nevertheless stabilized inflation at its historical level would have lead to a massive increase in output volatility. This result supports the claim that changes in policy regime can dramatically affect the economy dynamics by altering private agents' decision-making, and the empirical observation that inflation targeting in Canada managed to steer inflation expectations.

Finally, the analysis shows that the estimated IT policy is not necessarily 'good for all seasons'. In the 1971-1990 sample, it would have failed to reduce inflation volatility. Surprisingly, it returns an unconditional volatility in the estimated model for the IT period slightly *higher* for inflation, compared to the pre-IT policy. However, the results suggest that the IT regime shifted monetary policy towards a more favorable trade-off, by delivering unconditionally a very large reduction in output volatility, even if performing slightly worse in terms of inflation volatility.

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## 5 Appendix - for online publication only

#### 5.1 Market Clearing and Aggregate Equilibrium Conditions for the DSGE model

The household's intratemporal optimality condition for consumption allocation yields the equation for domestic demand of the F and H good, resulting in  $\frac{C_{F,t}}{C_{H,t}} = \frac{\gamma}{1-\gamma} (S_t)^{-\rho}$ , where  $S_t = P_{F,t}/P_{H,t}$ . Foreign consumption demand for the home-produced good is ultimately exogenous in a small open economy model. We assume it is elastic to the price charged by domestic producers. Foreign households' demand for F goods is assumed symmetric the domestic households', and equal to  $C_{H,t}^* = \gamma^* [\frac{P_{H,t}^*}{P_t^*}]^{-\rho^*} C_t^*$ . Using Purchasing Power Parity, and under the assumption that the rest of the world behaves like a closed economy, so that  $P_t^* = P_{F,t}^*$ , it holds that

$$C_{H,t}^* = \gamma^* \left[ \frac{P_{H,t}}{\mathbb{E}_t \{ P_{F,t}^* \}} \right]^{-\rho^*} C_t^* = \gamma^* S_t^{\rho^*} C_t^*.$$

Euler equations for domestic and foreign risk-free bonds can be combined to give

$$0 = \mathbb{E}_{t} \left\{ MUC_{t+1} \frac{P_{t}}{P_{t+1}} \left[ \frac{e_{t+1}}{e_{t}} (1+i_{t}^{*}) - (1+i_{t}) \right] \right\}$$
(13)  
$$MUC_{t} = \mathbb{E}_{t} \left\{ \frac{D_{t}}{C_{t} - bC_{t-1}} - \beta b \frac{D_{t+1}}{C_{t+1} - bC_{t}} \right\},$$

where  $MUC_t$  is the marginal utility of consumption,  $(1 + i_t) = v_t^{-1}$  is the gross nominal interest rate and  $(1 + i_t^*) = v_t^{*^{-1}}$  is the interest rate paid by domestic residents to borrow on the international capital market.

Market clearing in the domestic economy requires that

$$Y_{H,t} = \int_0^1 A_t N_t(i) di = A_t N_t = \int_0^1 \left[ \frac{P_{H,t}(i)}{P_{H,t}} \right]^{-\vartheta} (C_{H,t} + C_{H,t}^*) di$$

or  $Y_{H,t} = (C_{H,t} + C_{H,t}^*)s_t$ , where  $s_t = \int_0^1 \left[\frac{P_{H,t}(i)}{P_{H,t}}\right]^{-\vartheta} di$ . Since all firms face identical marginal costs, any firm belonging to the fraction  $(1 - \theta_p)$  resetting the price at t chooses the same new optimal price:

 $\tilde{P}_{H,t}(i) = \tilde{P}_{H,t}$  and  $P_{H,t}^{RT}(i) = P_{H,t}^{RT}$ . The solution to the optimal pricing problem is given by

$$\tilde{P}_{H,t}(i)\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (\theta_p \beta)^s \Lambda_{t,t+s} \left[ \frac{\tilde{P}_{H,t}(i)}{P_{H,t+s}} \right]^{1-\vartheta} C_{t+s}^W \right\} = \frac{\vartheta}{\vartheta - 1} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (\theta_p \beta)^s \Lambda_{t,t+s} M C_{t+s}^N \left[ \frac{\tilde{P}_{H,t}(i)}{P_{H,t+s}} \right]^{1-\vartheta} C_{t+s}^W \right\}$$
(14)

The consumer price index evolves according to

$$P_{H,t} = \left[\theta_p P_{H,t-1}^{1-\vartheta} + (1-\theta_p)\overline{P}_{H,t}^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}},$$
  
$$\overline{P}_{H,t} = \left[\omega_p P_{H,t}^{RT^{1-\vartheta}} + (1-\omega_p)\widetilde{P}_{H,t}^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}.$$

Each household purchases an equal amount  $C_{H,t}^{j}(i)$  so that  $C_{H,t}^{j}(i) = C_{H,t}(i)$ . Since the marginal rate of substitution between consumption and labor will be equal across all households who can reset the wage at a given time, the wage setting equation (15) implies that all household contracting a new wage at t will choose the same new optimal wage, so that  $\tilde{W}_{t}^{j} = \tilde{W}_{t}$ . Let  $MUN_{t+s}^{j} = \frac{\partial U_{t+s}^{j}}{\partial N_{t+s}^{j}}$ . The first order condition for household j is

$$\mathbb{E}_{t}\left\{\sum_{s=0}^{\infty}(\theta_{w}^{s}\beta^{s})\left[\frac{\phi-1}{\phi}MUC_{t+s}^{j}\frac{\tilde{W}_{t}^{j}}{P_{t+s}}\right]N_{t+s}^{j}\right\} = -\mathbb{E}_{t}\left\{\sum_{s=0}^{\infty}(\theta_{w}^{s}\beta^{s})\left[MUN_{t+s}^{j}\right]N_{t+s}^{j}\right\}.$$
 (15)

Using the wage index definition and  $W_t^{RT^j} = W_t^{RT}$ , the aggregate wage index evolves according to

$$W_t = \left[\theta_w W_{t-1}^{1-\phi} + (1-\theta_w)(1-\omega_w)\tilde{W}_t^{1-\phi} + (1-\theta_w)\omega_w W_t^{RT^{1-\phi}}\right]^{\frac{1}{1-\phi}}$$

Aggregating over firms in the importing sector results in the aggregate foreign-good price index  $P_{F,t}$ equal to  $\left[\theta_f P_{F,t-1}^{1-\vartheta} + (1-\theta_p) \widetilde{P}_{F,t}^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}$ , where  $\widetilde{P}_{F,t}$  is the price chosen by re-optimizing firms. The law of motion for the net foreign asset stock  $B_t^*$  can be derived using the households' budget constraints and the market clearing conditions. To ensure stationarity we assume that  $i^*$  is given by the exogenous world interest rate  $\tilde{i}^*$  plus a premium increasing in the real value of the country's stock of foreign debt:  $(1+i_t^*) = (1+\tilde{i}_t^*)g(-\tilde{B}_t)$ , where  $\tilde{B}_t = \frac{e_t B_t^*}{P_{H,t}}$  and  $g(\cdot)$  is a positive, increasing function.

#### 5.2 Data Sources and Plots

We measure nine theoretical variables of the model by the following time series:

- **Domestic output**: Real gross domestic product. Statistics Canada: "Gross domestic product, expenditure-based, Canada, quarterly" (36-10-0104-01).
- Domestic inflation: Log-change in the consumer price index excluding food, energy, and the effect of indirect taxes. Statistics Canada: Consumer Price Index, monthly, not seasonally adjusted (18-10-0004-01) for 1971-1983 and "Consumer Price Index (CPI) statistics, alternative measures, unadjusted and seasonally adjusted" (10-10-0106-01) for 1984-2018.
- **Domestic interest rate**: Quarterly Bank of Canada target for the overnight rate. OECD Main Economic Indicators.
- Domestic terms of trade: Ratio of the Laspeyres index for import and export prices. IMF International Financial Statistics for 1971-1996. Statistics Canada: "International merchandise trade, by commodity, price and volume indexes, quarterly" (12-10-0125-01) for 1997-2018.
- Domestic nominal exchange rate depreciation: Log-change in the Canadian/US dollar exchange rate. Exchange Rates, National Currency Per US Dollar, Period Average, Rate, International Financial Statistics.
- Foreign output: Real output of the US nonfarm business sector. FRED database: OUTNFB.
- Foreign interest rate: Quarterly US effective federal funds rate. FRED database: FEDFUNDS.
- **Domestic wage inflation**: Log-change in the average hourly earnings of manufacturing workers. OECD Main Economic Indicators.
- **Domestic employment**: Total employment in persons multiplied by the average weekly hours in the manufacturing sector. OECD Main Economic Indicators.

[FIGURES A.1-A.2 HERE]

#### 5.3 Estimation Procedure and Prior Distributions

We estimate the model by Bayesian maximum likelihood, such as reviewed in An and Schorfheide (2007). The details of the solution and estimation procedures are laid out in Adjemian et al. (2019).

Our estimation procedure encompasses the following steps. First, the mode of the posterior distribution is obtained by maximizing the posterior kernel with respect to a parameter vector. Second, after having obtained the posterior mode, the posterior distribution is simulated with point of origin at the mode, using the random-walk Metropolis-Hasting algorithm. This algorithm is a Monte-Carlo Markov chain rejection sampling method. The idea of the algorithm is to generate a number of Markov chains of parameter realizations, such that the entire domain of the parameter space is explored. We simulate two parallel Markov chains, each containing 200,000 realizations with the 100,000 initial realizations being discarded. The probability distribution from which each proposal parameter vector is generated within an iteration is called the 'jumping distribution'. We set the variance scale factor of the jumping distribution to 0.30. The resulting acceptance rates are 30.4 and 31.8 for the period 1971Q1-1990Q4 and 36.6 and 36.8 for the period 1992Q1-2018Q4.

The posterior kernel is defined as the sum of the log-likelihood function and the log-prior distribution. The prior distribution is motivated below. The likelihood function is formed by writing the model up on a linear state-space representation. Then, a Kalman filter is used to retrieve the innovations which, for a given parameterization of the model, explain the observed data. When maximizing the posterior kernel, it is the joint sum of the squared innovations that is minimized conditional on the prior distribution.

We now motive our prior distribution. The distributions of the individual parameters are reported in Table 1. The prior mean of the elasticity of the marginal disutility of labor supply ( $\varphi = 5.00$ ) implies a real wage elasticity of labor supply of  $\frac{1}{5}$ , consistent with the micro-estimates in MaCurdy (1981) and Altonji (1986). The prior means of the elasticity of substitution between domestic and foreign goods ( $\rho = 2.23$ ) and the elasticity of foreign demand with respect to the terms of trade ( $\rho_1 = 2.23$ ) are set to the estimate obtained in an earlier version of the paper, where the substitution elasticity was estimated by classical maximum likelihood. Such substitution elasticities above unity have also been found in other empirical studies (Whalley, 1984). Finally, the parameters in the monetary policy reaction function are identical to the prior means that Smets and Wouters (2007) and Galí, Smets, and Wouters (2012) use, with the response of monetary policy to the exchange rate being identical to the response to output ( $\omega_y = \omega_e = 0.125$ ). The prior means of the remaining estimated parameters are similar to the prior means of the corresponding parameters in, e.g., Smets and Wouters (2007) and Galí, Smets, and Wouters (2012).

#### 5.4 DSGE Model Solution Method under Private Sector Incorrect Believes

In the following it is useful to refer to the the standard VAR representation used in the DSGE literature, adopting the nomenclature where  $x_t$  is the vector of endogenous state variables,  $z_t$  is the vector of exogenous state variables, and  $y_t$  is the vector of control variables. To relate the VAR representation to the state-space representation, (9) and (10), let  $\xi_t^1 = x_{t-1}$ ,  $\xi_t^2 = z_t$ , and  $q_t = y_t$ . Then

$$Y_{t} = \Gamma_{1}Y_{t-1} + \Gamma_{2}z_{t}, \qquad (16)$$

$$z_{t} = Nz_{t-1} + \varepsilon_{t},$$

$$Y_{t} = \begin{bmatrix} x_{t} \\ y_{t} \end{bmatrix}, \quad \Gamma_{1} = \begin{bmatrix} F^{11} & 0 \\ H'^{1} & 0 \end{bmatrix}, \quad \Gamma_{2} = \begin{bmatrix} F^{12} \\ H'^{2} \end{bmatrix}, \quad N = [F^{22}],$$

where the vector  $Y'_t = [x_t, y_t]$  has dimension  $1 \times n + r$ . Assume that the vector  $z_t$  has the dimension m = n + r, therefore the number n + r of observable variables is equal to the number of shocks, and the matrix  $\Gamma_2$  is square (see Ravenna, 2007, for details on the non-square case). Since

$$z_t = \Gamma_2^{-1} Y_t - \Gamma_2^{-1} \Gamma_1 Y_{t-1} = N[\Gamma_2^{-1} Y_{t-1} - \Gamma_2^{-1} \Gamma_1 Y_{t-2}] + \varepsilon_t,$$

we obtain a VAR(2) representation for eq. (16),

$$Y_{t} = (\Gamma_{1} + \Gamma_{2}N\Gamma_{2}^{-1})Y_{t-1} - (\Gamma_{2}N\Gamma_{2}^{-1}\Gamma_{1})Y_{t-2} + \Gamma_{2}\varepsilon_{t}$$
  
$$= \overline{\Gamma}_{1}Y_{t-1} + \overline{\Gamma}_{2}Y_{t-2} + \eta_{t}, \qquad (17)$$

Typically DSGE models have a larger number of observable variables than unobservable shocks, that is m < n + r. In this case, a non-singular VAR representation for the observable variables can still be obtained by including only an *m*-dimensional subset of the vector  $Y_t$ . In general, for the DSGE equilibrium to have a finite-order VAR representation,  $Y_t$  must include all the elements of the vector  $x_t$  (see Ravenna, 2007). For N = [0], eq. (17) simplifies to a VAR(1).

Assume a DSGE model is described by the system of stochastic difference equations:

$$0 = A\mathbb{E}_t\{Y_{t+1}\} + BY_t + CY_{t-1} + Dz_t, \tag{18}$$

$$z_t = N z_{t-1} + \varepsilon_t. \tag{19}$$

Define the rational expectation equilibrium for  $Y_t$  under the monetary policy rule  $L_a$  as

$$Y_t = \Gamma_{a1} Y_{t-1} + \Gamma_{a2} z_t,$$

where  $z_t = N z_{t-1} + \varepsilon_t$ . If the private sector expects the central bank to behave according to the policy rule  $L_b$ , expectations are consistent with the rational expectation equilibrium for  $Y_t$  defined by

$$Y_t = \Gamma_{b1} Y_{t-1} + \Gamma_{b2} z_t. \tag{20}$$

Let  $\tilde{\mathbb{E}}_t^L$  indicate the expectation of a variable under the belief that the central bank follows the policy rule L. The structural model (18) for  $Y_t$  can then be written as

$$0 = A\tilde{\mathbb{E}}_{t}^{b}\{Y_{t+1}\} + BY_{t} + CY_{t-1} + Dz_{t}$$
  
=  $A[\Gamma_{b1}Y_{t} + \Gamma_{b2}Nz_{t}] + BY_{t} + CY_{t-1} + Dz_{t}.$  (21)

The model (21) can be solved for  $Y_t$ . For any policy  $L_a \neq L_b$ , the solution for (21) will be different from the reduced-form law of motion (20). If the central bank's policy  $L_a$  is specified in terms of expected inflation, the DSGE model under incorrect private sector believes is given by

$$0 = A[\Gamma_{b1}Y_t + \Gamma_{b2}Nz_t] + A^{cb}\mathbb{E}_t\{Y_{t+1}\} + BY_t + CY_{t-1} + Dz_t.$$
(22)

The expectation term appears because the central bank's instrument reacts to the rational expectation inflation forecast.  $A^{cb}$  includes the coefficients on the central bank's expected values of  $Y_{t+1}$  in the policy rule. Solving the model (22) for the rational expectations equilibrium yields the law of motion for  $Y_t$ :

$$Y_t = \Gamma_{c1} Y_{t-1} + \Gamma_{c2} z_t,$$

which assumes the central bank knows the private sector incorrect believes about the policy regime L, and takes them into account when formulating its own inflation forecast.

#### 5.5 Building Policy Shock Series to Achieve Historical Inflation Path

Let  $q_t^i$  define the  $i^{th}$  row of the vector  $q_t$  and  $q_t^{-i}$  the vector including all the rows of  $q_t$  except the  $i^{th}$  one, which we assume is the row corresponding to the inflation variable. Eqs. (11) and (12) imply that for any counterfactual path  $\xi_t^{1\prime}$  and Kalman-filtered estimate  $\xi_{tT}^2$ :

$$\overline{\xi}_{t}^{1} = \overline{F}^{11}\overline{\xi}_{t-1}^{1} + \overline{F}^{12}\xi_{t-1|T}^{2},$$

$$q_{t}^{i} = \overline{H}^{\prime 1i}\overline{\xi}_{t}^{1} + \overline{H}^{\prime 2i}\xi_{t|T}^{2} = \overline{H}^{\prime 1i}\overline{F}^{11}\overline{\xi}_{t-1}^{1} + \overline{H}^{\prime 1i}\overline{F}^{12}\xi_{t-1|T}^{2} + \overline{H}^{\prime 2i}\xi_{t|T}^{2}$$

$$= \overline{H}^{\prime 1i}\overline{F}^{11}\overline{\xi}_{t-1}^{1} + \overline{H}^{\prime 1i}\overline{F}^{12}\xi_{t-1|T}^{2} + \overline{H}^{\prime 2i^{-j}}\xi_{t|T}^{2-j} + \overline{H}^{\prime 2i^{j}}\xi_{t|T}^{2j},$$
(23)

where  $\overline{H}^{\prime 1i}$  ( $\overline{H}^{\prime 2i}$ ) is the *i*<sup>th</sup> row of the matrix  $\overline{H}^{\prime 1}$  ( $\overline{H}^{\prime 2}$ ),  $\overline{H}^{\prime 2i^{j}}$  is the *j*<sup>th</sup> column of the matrix  $\overline{H}^{\prime 2i}$ ,  $\overline{H}^{\prime 2i^{-j}}$  is a matrix including all the columns of  $\overline{H}^{\prime 2i}$  except the *j*<sup>th</sup>, which we assume is the column corresponding to the policy shock variable  $\xi_{t|T}^{2j}$ , and  $\xi_{t|T}^{2^{-j}}$  is the vector composed of all the elements of  $\xi_{t|T}^{2}$  except the *j*<sup>th</sup> one. By setting the variable  $\overline{q}_{t}^{i}$  equal to its historical smoothed estimate,  $q_{t|T}^{i}$ , eq. (23) can be solved recursively for the shock  $\hat{\xi}_t^{2^j} = \hat{\varepsilon}_{i,t}$ :

$$\widehat{\xi}_t^{2^j} = \left(\overline{H}^{\prime 2i^j}\right)^{-1} \left[ q_{t|T}^i \overline{H}^{\prime 1i} - \overline{F}^{11} \overline{\xi}_{t-1}^1 - \overline{H}^{\prime 1i} \overline{F}^{12} \xi_{t-1|T}^2 - \overline{H}^{\prime 2i^{-j}} \xi_{t|T}^{2^{-j}} \right].$$

Since it is assumed that for the inflation equation  $w_t^i = 0 \forall t$ , it follows that  $q_{t|T}^i = q_t^{o^i}$  and  $\hat{\xi}_t^{2^j}$  is the value of the policy shock ensuring that inflation equals its historical value in the counterfactual history where the dynamics of the economy is described by the matrices  $\overline{F}^{11}$ ,  $\overline{F}^{12}$ , and  $\overline{H}'$ .

	Prior Distribution		Posterior Distributions						
				197	71Q1-199	0Q4	199	2Q1-201	8Q4
	Type	Mean	S.D.	Mode	5  pct.	95 pct.	Mode	5  pct.	95 pct.
Structural Paramete	ers								
$\eta$	Ν	5.00	0.50	5.09	4.17	5.83	5.42	4.62	6.19
ho	Ν	2.23	0.15	2.04	1.81	2.31	2.09	1.84	2.33
$ ho_1$	Ν	2.23	0.15	1.96	1.74	2.23	2.04	1.78	2.29
$\omega_p$	В	0.50	0.10	0.21	0.11	0.30	0.40	0.29	0.49
$\omega_w$	В	0.50	0.10	0.34	0.24	0.48	0.19	0.11	0.28
$ heta_p$	В	0.67	0.05	0.84	0.75	0.90	0.85	0.80	0.89
$\hat{ heta_w}$	В	0.67	0.05	0.63	0.54	0.70	0.68	0.58	0.76
$ heta_f$	В	0.67	0.05	0.47	0.42	0.52	0.37	0.34	0.42
b	В	0.70	0.10	0.90	0.85	0.95	0.62	0.58	0.72
$\chi$	В	0.75	0.05	0.74	0.69	0.78	0.78	0.74	0.84
$\omega_{\pi}$	Ν	1.50	0.50	2.17	1.75	2.70	1.96	1.64	2.37
$\omega_y$	Ν	0.125	0.10	0.12	0.10	0.24	0.30	0.21	0.40
$\omega_e$	Ν	0.125	0.10	0.55	0.45	0.68	0.01	-0.02	0.06
AR(1) Parameters for	or Shoc	k Proce	sses						
Demand	В	0.50	0.20	0.52	0.32	0.65	0.94	0.90	0.95
Inflation target	В	0.50	0.20	0.91	0.88	0.97	0.98	0.96	0.99
Technology	В	0.50	0.20	0.49	0.34	0.65	0.42	0.28	0.55
Price markup	В	0.50	0.20	0.87	0.01	0.17	0.06	0.01	0.15
Wage markup	В	0.50	0.20	0.43	0.31	0.60	0.12	0.03	0.22
Foreign demand	В	0.50	0.20	0.82	0.74	0.89	0.85	0.78	0.91
Foreign interest rate	В	0.50	0.20	0.78	0.74	0.84	0.94	0.92	0.95
Foreign inflation	В	0.50	0.20	0.11	0.03	0.19	0.05	0.02	0.08
Standard Deviations	of Inn	ovations							
Demand	IG	0.001	0.01	0.2517	0.1405	0.3983	0.2055	0.1767	0.2603
Mon. policy inn.	IG	0.001	0.01	0.0038	0.0034	0.0045	0.0017	0.0015	0.0020
Inflation target	IG	0.001	0.01	0.0037	0.0029	0.0053	0.0033	0.0026	0.0042
Technology	IG	0.001	0.01	0.0075	0.0067	0.0087	0.0091	0.0082	0.0102
Price markup	IG	0.001	0.01	0.0014	0.0028	0.0038	0.0071	0.0063	0.0081
Wage markup	IG	0.001	0.01	0.0047	0.0035	0.0058	0.0111	0.0096	0.0128
Foreign demand	IG	0.001	0.01	0.0121	0.0108	0.0140	0.0065	0.0059	0.0074
Foreign interest rate	IG	0.001	0.01	0.0034	0.0030	0.0040	0.0011	0.0010	0.0013
Foreign inflation	IG	0.001	0.01	0.0202	0.0173	0.0240	0.0381	0.0344	0.0439
Correlations between Foreign Innovations									
Demand and int. rate	N	0.00	0.20	0.23	0.06	0.38	0.25	0.12	0.39
Demand and inflation	Ν	0.00	0.20	-0.19	-0.34	-0.03	0.03	-0.12	0.16
Int. rate and inflation	Ν	0.00	0.20	-0.08	-0.24	0.07	0.07	-0.07	0.20
				1					

Table 1: PRIOR DISTRIBUTION AND POSTERIOR DISTRIBUTIONS

Distributions: N: Normal. B: Beta. IG: Inverse-Gamma.

*Note:* Parameter and shock process estimates for the DSGE model. The model is estimated on two samples covering 1971Q1-1990Q4 and 1992Q1-2018Q4. "Mode", "5 pct.", and "95 pct." refer to the mode, 5 percentiles, and 95 percentile of the posterior distributions.

	Marginal D	Pata Density	Posterior Odds Ratio	
	Baseline	Restricted		
Flexible domestic producer prices and wages				
1971-1990	2378.96	2289.70	$\exp(89.26)$	
1992-2018	3232.91	2767.21	$\exp(465.70)$	
Flexible import price	S			
1971-1990	2372.21	2360.17	$\exp(12.04)$	
1992-2018	3232.80	3230.82	$\exp(1.98)$	

Table 2: MODEL PERFORMANCE TESTS

*Note:* Comparison of baseline model against a model with flexible domestic price and wage setting and against a model with flexible import prices. Marginal data densities and posterior odds ratios for the DSGE model estimated over 1971-1990 and 1992-2018. The absolute value of the marginal data densities are computed at the respective posterior modes.

Rat	Ratio: Counterfactual Relative to Historical Value				
An	nual Inflation	Output			
(a) Conditional on the Historical Shock Realizations in 1992-2018					
Baseline Model					
Standard deviation	2.00	2.64			
Level	1.02				
Constrained Model: No Rule-of-Thumb Firms ( $\omega_p = 0$ ), Model Reestimated					
Standard deviation	2.00	2.76			
Level	1.03				
Constrained Model: No Rule-of-Thumb Firms ( $\omega_p = 0$ ), Model Not Reestimated					
Standard deviation	0.86	2.60			
Inflation	1.02				
(b) Unconditional Distribution in 1992-2018					
Exogenous Shock Processes as in 19	71-1990				
Standard deviation	1.15	1.32			
Monetary Policy Coefficients as in 2	1971-1990				
Standard deviation	0.85	1.72			
Monetary Policy Rule as in 1971-1990					
Standard deviation	0.93	1.75			
Exogenous Shock Processes and Monetary Policy Rule as in 1971-1990					
Standard deviation	0.92	2.46			
(c) Conditional on the Historical Shock Realizations in 1971-1990					
Baseline Model					
Standard deviation	1.57	0.68			
<b>T</b> 1					

#### Table 3: COMPARING MONETARY POLICIES

Note: All cells report the standard deviation or average level of annual inflation or output in a counterfactual simulation relative to the historical data. The DSGE model is parameterized to the posterior mode of the 1992-2018 sample in panel (a)-(b) and to the posterior mode of the 1971-1990 sample in panel (c). "Monetary Policy Coefficients as in 1971-1990" assumes only changing the monetary policy coefficients,  $\chi$ ,  $\omega_{\pi}$ ,  $\omega_{y}$ , and  $\omega_{e}$ , while "Monetary Policy Rule as in 1971-1990" additionally assumes changing the standard deviations of the monetary policy innovation and inflation target innovation. In the baseline of panel (a), average inflation increases from 1.58% to 1.62%. In panel (c), average inflation increases from 6.27% to 7.27%.

Ratio of Unconditional Standard Deviations: Counterfactual Relative to Historical Value			
Annual Inflation	Output		
Monetary Policy Coefficients as in 1971-1990			
0.85	1.72		
Monetary Policy Coefficients as in 1971-1990: $\omega_y$	, at 1992-2018 Value		
0.89	1.20		
Monetary Policy Coefficients as in 1971-1990: $\omega_e$	at 1992-2018 Value		
1.09	1.45		

*Note:* Counterfactual under pre-inflation targeting policy vs. historical policy. The DSGE model is parameterized to the posterior mode of the 1992-2018 sample. The experiments involve changing the monetary policy coefficients,  $\chi$ ,  $\omega_{\pi}$ ,  $\omega_{y}$ , and  $\omega_{e}$ .

Table 5: The Role of Shock Volatility: Volatility as in 1	971-1990
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Ratio of Conditional Standard Deviations: Counterfactual Relative to Historical Value				
Annual Inflation	Output			
Historical Ratios (i.e., 1992-2018 vs. 1971-1990)				
0.21	0.65			
Shock Volatility as in 1971-1990				
0.67	1.28			
Shock Volatility as in 1971-1990: Except for the Price Markup Shock				
0.40	1.27			
Shock Volatility as in 1971-1990: Except for t	he Price and Wage Markup Shocks			
0.29	1.07			
Shock Volatility as in 1971-1990: Except for t	he Price, Wage Markup, and Demand Shocks			
0.24	0.94			

*Note:* Counterfactual under shock volatility as in 1971-1990 vs. historical series. The DSGE model is parameterized to the posterior mode of the 1992-2018 sample and simulated across this period. Each series of innovations is rescaled so that the volatility of the shock state pertaining to a given series of innovations is equal to the volatility of the same shock state in the estimation using the 1971-1990 sample.



Figure 1: POLICY INTEREST RATES

*Note:* The interest rates are averages at a quarterly rate. Shadow rates measure the estimated level of the policy rate consistent with the overall monetary policy stance in the absence of the zero-lower bound. Shadow rates are computed by MacDonald and Popiel (2017) for Canada and Wu and Xia (2016) for the U.S. The shaded areas indicate the zero-lower-bound periods, where the policy target rate deviates from the shadow rate.



Figure 2: Comparing Monetary Policies

*Note:* Counterfactual under pre-inflation targeting policy vs. historical series. The DSGE model is parameterized to the posterior mode of the 1992-2018 sample and simulated across this period. The counterfactual path is obtained by simulating the model with monetary policy coefficients parameterized to the posterior mode of the 1971-1990 sample.



Figure 3: The Role of Shock Volatility: Volatility as in 1971-1990

*Note:* Counterfactual under shock volatility as in 1971-1990 vs. historical series. The DSGE model is parameterized to the posterior mode of the 1992-2018 sample and simulated across this period. Each series of innovations is rescaled so that the volatility of the shock state pertaining to a given series of innovations is equal to the volatility of the same shock state in the estimation using the 1971-1990 sample. The shaded area indicates the zero-lower-bound period in Canada.



*Note:* Counterfactual under no monetary policy innovations vs. historical series. The DSGE model is parameterized to the posterior mode of the 1992-2018 sample and simulated across this period. The monetary policy innovations have been set to zero in the counterfactual simulation. The shaded area indicates the zero-lower-bound period in Canada.



Figure 5: NO CREDIBILITY: INFLATION TARGETING IMPLEMENTED AS EXO. INNOVATIONS

*Note:* Counterfactual under no credibility vs. historical series. The DSGE model is parameterized to the posterior mode of the 1992-2018 sample and simulated across this period. The counterfactual path is obtained by parameterizing the monetary policy coefficients to the posterior mode of the 1971-1990 sample and recursively simulating a single-period monetary policy innovation which ensures that the domestic interest rate is equal to the historical overnight target rate for Canada.



Figure 6: NO CREDIBILITY: TARGET INFLATION TO ITS HISTORICAL LEVEL

*Note:* Counterfactual under no credibility vs. historical series. The DSGE model is parameterized to the posterior mode of the 1992-2018 sample and simulated across this period. The counterfactual path is obtained by parameterizing the monetary policy coefficients to the posterior mode of the 1971-1990 sample and recursively simulating a single-period monetary policy innovation which ensures that the domestic inflation is equal to the historical consumer price inflation in Canada. The shaded area indicates the zero-lower-bound period in Canada.





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