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## Informing the public about a pandemic

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# Informing the Public about a Pandemic

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This paper explores how governments may efficiently inform the public about an epidemic to induce compliance with their confinement measures. Using an information design framework, we find that governments have an incentive to either downplay or exaggerate the severity of the epidemic if the government heavily prioritizes the economy over population health or vice versa. Importantly, we find that the level of economic inequality in the population has an effect on these distortions. The more unequal the disease’s economic impact on the population is, the less the government exaggerates and the more it downplays the severity of the epidemic. When the government weighs the economy and population health sufficiently equally, however, the government should always be fully transparent about the severity of the epidemic.

*Key words:* Public Health, Epidemic Control, Information Design, Strategic Behavior

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## 1 Introduction

Lockdowns and confinement measures constitute effective governmental interventions for slowing the spread of an epidemic (Anderson et al. 2020, Fowler et al. 2020). These measures enable the deployment of more time-consuming medical responses and may even bring the spread of the virus to a halt (Ferguson et al. 2020, Ji et al. 2020). The efficacy of these restrictions, however, relies heavily on individual compliance (Wright et al. 2020), which political leaders and governmental agencies influence by disseminating information on the epidemic’s severity (Webster et al. 2020). However, several factors greatly complicate the proper choice of a communication strategy to influence the public compliance.

First, implementing confinement measures requires making trade-offs between health benefits and the economic costs that social distancing brings about (Hargreaves Heap et al. 2020, IMF 2020). Depending on whether health or the economy is of higher priority, a political leader might highlight or downplay the severity of the epidemic. For instance, political leaders in the United States appear to have belittled the risks associated with

the COVID-19 epidemic (Paz 2020), while several European leaders have insisted on its severity (Bennhold and Eddy 2020).

These messages affect the population’s perception of the risk associated with social interactions. However, compliance with confinement measures also depends on the socio-economic status of individuals (Atchison et al. 2020, Van Rooij et al. 2020). In deciding to remain isolated, individuals weigh the benefits to their health against the income they might forgo (Wright et al. 2020, Gitmez et al. 2020). Thus, part of the population might have preferences that do not fully align with the government’s priorities. The more heterogeneous a population is, the more individual preferences may differ (Bottan et al. 2020).

Furthermore, the choice of an individual to remain isolated (or not) creates externalities for the rest of the population. The more individuals isolate themselves, the less exposed the population is, which decreases the expected health risk for everyone (Bethune and Korinek 2020). This, however, decreases the incentive to remain confined and thus gives rise to strategic interactions within the population.

This paper explores how policymakers may efficiently inform the public about the severity of an epidemic to induce public compliance with their confinement measures. To address this question, we develop an information design framework, which accounts for the health-economy trade-off faced by both governments and individuals, the socioeconomic heterogeneity within the population and the externalities that social distancing brings about.

Our analysis reveals that governments should always fully disclose their information about the epidemic when they weigh the economy and population health equally. Consistent with the existing literature (and several pieces of anecdotal evidence - see Alizamir et al. 2020 or Paz 2020 for instance), we also find that governments sometimes have a rationale to misrepresent information on the epidemic. If the government’s priorities are heavily biased toward the economy or the population’s health, they may indeed downplay or exaggerate the severity of the epidemic, respectively. These distortions occur when the public incentives are overall aligned with the government’s priorities.

More specifically, consider a government that is heavily biased toward protecting the population’s health over minimizing economic losses. When the symptoms of the disease are generally mild, individuals do not have strong incentives to isolate themselves, which does not fully align with the government’s objective. Nevertheless, we find in our setup that the government always fully discloses its information about the severity of the epidemic.

In contrast, when symptoms are serious, individuals have a strong incentive to remain confined, which does align with the government’s priorities. However, we find that the government sometimes *exaggerates* the severity of the epidemic. In fact, when symptoms are extremely serious, the government does not disclose any information. We obtain similar results when the government is heavily biased toward minimizing economic costs. In this case, the government may *downplay* the severity of the epidemic when the symptoms are mild, which, again, does not align with the government’s priorities.

Further, we find that the level of economic inequality in the population has an effect on these misrepresentations. The more heterogeneous the disease’s economic impact is, the less the government exaggerates and the more it downplays the severity of the epidemic. In other words, the government of a population with significant levels of economic disparity has a greater tendency to downplay the severity of the epidemic than the government of a more equal society.

The general information design framework we consider in this paper was introduced by Kamenica and Gentzkow (2011), with growing applications in operations research and management science (see Candogan 2020, and the references therein). In particular, this approach has been especially fruitful for studying broad public health questions such as ours (see Shi 2013 and Alizamir et al. 2020 for instance). Our findings also complement the rapidly growing literature on global health and disaster management that has emerged since the COVID-19 outbreak, which analyzes different approaches to limit the spread of the disease (see, e.g., Birge et al. 2020, Housni et al. 2020 and Ramdas et al. 2020). In contrast to this literature, our study explores the role of information and public behavior in containing the epidemic and is related to agency problems and strategic behaviors in healthcare systems (see Adida et al. 2017, Zorc et al. 2017 for instance).

## 2 Information Design Framework

We next develop an information design model in which a government (the sender) seeks to induce a certain level of social distancing in its population (the receivers), which is experiencing the spread of an infectious disease. The sender informs the public about the epidemic’s severity in a way that may or may not reflect its proprietary information. Given the sender’s message, each receiver decides whether to avoid social interactions. To make this choice, individuals need to weigh the economic losses that social isolation brings about

against the risk of being infected. Each individual choice affects the probability of infection in the rest of the population, giving rise to a game within the population.

### Individual choices to social distance

Each individual in the population chooses  $\alpha \in \{0, 1\}$ , such that the individual remains isolated when  $\alpha = 1$  or engages in social interactions otherwise.<sup>1</sup> To make this choice, an individual needs to assess the risk of being infected when engaging in social interactions.

We assume that an infected individual incurs an expected *healthcare cost*  $\kappa > 0$  due to the illness.<sup>2</sup> The health risk then corresponds to the perceived probability of being infected times cost  $\kappa$ . The perceived probability of infection for an individual engaging in social interactions is the product of the perceived infectiousness of the disease, denoted by  $\mu$ , and the size of the socially active population, denoted by  $P$  (which yields  $\mu P$ ).

Perceived infectiousness  $\mu$  represents the subjective probability of becoming infected via contact with one individual in the public.<sup>3</sup> This probability is influenced by the information that the government disseminates about the epidemic. Population size  $P$  is the size of the population that engages in social interactions and is the equilibrium outcome of individual decisions. Thus, the product  $\mu P$  reflects the homogeneous mixing assumption typical of epidemic models (e.g., Anderson et al. 1992). Taken together, these values indicate that an individual faces a health risk equal to  $(1 - \alpha)\mu P\kappa$  given choice  $\alpha \in \{0, 1\}$ .

Self-isolation, however, is economically costly, as individuals may forgo their source of income. We denote  $c$  as the *economic cost* that a confined individual incurs. We further consider a continuum of population heterogeneity in this economic cost. Specifically, we normalize the population size to 1 and assume that  $c$  follows a uniform distribution with mean  $\theta$ , standard deviation  $\sigma$  and nonnegative support  $\mathcal{C}$ , i.e.,  $\mathcal{C} = [\theta - \sigma\sqrt{3}, \theta + \sigma\sqrt{3}]$  with  $\theta \geq \sigma\sqrt{3} > 0$ . In particular, the larger  $\sigma$  is, the more unequal the economic impact of the epidemic is.

Thus, given her perception of infectiousness  $\mu$  and the socially active population size  $P$ , an individual decides whether to self-isolate,  $\alpha \in \{0, 1\}$ , to minimize her total expected economic and health costs  $\alpha c + (1 - \alpha)\mu P$ .

<sup>1</sup> We restrict the individual decision to a binary choice for clarity, but our model easily accounts for settings where individuals decide on the proportion of time they remain confined (in which case  $\alpha$  takes on real values with  $\alpha \in [0, 1]$ ).

<sup>2</sup> Cost  $\kappa$  captures the symptoms' seriousness and may include additional economic costs due to hospitalization.

<sup>3</sup> More precisely,  $\mu$  corresponds to the product of (i) the (subjective) probability of disease transmission through contact between the individual and an infectious subject and (ii) the (perceived) proportion of infected individuals in the population.

## Information structure and public information policies

Each individual is privately informed about her economic cost  $c$  but is uninformed about the severity of the epidemic. We refer to the infectiousness of the disease as  $\omega$ ,<sup>4</sup> which is a binary random variable  $\omega \in \{\omega_\ell, \omega_h\}$  taking a high value  $\omega_h \in (0, 1)$  (representing a *highly severe* epidemic) with prior probability  $\rho^\circ \in (0, 1)$  and a low value  $\omega_\ell \in (0, \omega_h)$  (representing a *less severe* epidemic) with the complement probability. Thus, the prior belief about the epidemic's infectiousness is given by  $\mu^\circ = \rho^\circ \omega_h + (1 - \rho^\circ) \omega_\ell$ . We assume that  $\omega_\ell \kappa \geq \inf \mathcal{C}$ , i.e., the health risk in the absence of any self-isolation is not too small.<sup>5</sup>

In contrast, the government observes the actual the realization of  $\omega \in \{\omega_\ell, \omega_h\}$ , and can further influence individual behavior by strategically disclosing this information. Formally, before  $\omega$  realizes, the government commits to a public information policy  $\Gamma = (\pi, \mathcal{M})$ , which specifies (i) the space of all possible messages  $\mathcal{M}$  that can be sent to the population and (ii) the probability  $\pi(m | \omega)$  of sending each message  $m \in \mathcal{M}$  given each realization of  $\omega \in \{\omega_\ell, \omega_h\}$ . Consistent with the literature, probability  $\pi$  can be thought of as an abstraction of the intensity with which messages are sent (see, e.g., Alizamir et al. 2020).<sup>6</sup>

We do not restrict the search for the optimal policy, but we show in Section 4 that the optimal information policy is of the following binary type. Consider the binary message space  $\mathcal{M}_B = \{m_\ell, m_h\}$  with  $m_h \neq m_\ell$ , where  $m_\ell$  and  $m_h$  represent “low-severity” and “high-severity” alerts, respectively. An information policy is then given by two probabilities  $\pi(m_i | \omega_i) \in [0, 1]$  for  $i \in \{\ell, h\}$ , with  $\pi(m_j | \omega_i) = 1 - \pi(m_i | \omega_i)$  for  $j \neq i$  and  $i, j \in \{\ell, h\}$ . This class of policies includes the following two extreme policies as special cases.

*Full disclosure.* An information policy fully discloses the severity of the epidemic if  $\pi(m_i | \omega_i) = 1$  for  $i \in \{\ell, h\}$ . Under this policy, message  $m_i$  precisely reveals the government's private information  $\omega_i$ .

*No disclosure.* An information policy discloses no information if  $\pi(m_i | \omega_i) = \pi(m_i | \omega_j)$  for all  $i, j \in \{\ell, h\}$ , i.e., the probability of sending  $m_i$  is independent of  $\omega$ . In this case, the message carries no inferential information about the government's private information.

<sup>4</sup> Specifically,  $\omega$  corresponds to the product of (i) the probability of disease transmission through contact between the individual and an infectious subject and (ii) the proportion of infected individuals in the population, which we assume to be known by the government.

<sup>5</sup> This condition ensures that a positive proportion of the population engages in social distancing at equilibrium, which simplifies the analysis. Note also that any positive  $\kappa$  satisfies this condition when  $\inf \mathcal{C} = 0$ .

<sup>6</sup> The more frequently and intensively the government communicates its messages, the more likely they will be received, i.e., the closer the value of  $\pi(m | \omega)$  is to one.

There is a plethora of information policies between these two polar cases, which reveal some information about the government's private information, albeit with distortion. Of particular interest are the following two types of information distortions.

*Downplay.* An information policy is said to *downplay* the severity of the epidemic if  $\pi(m_\ell | \omega_\ell) = 1$  and  $\pi(m_\ell | \omega_h)$  is positive. In other words, the government may claim with positive probability that the epidemic is not severe (i.e., it sends message  $m_\ell$ ), even though its information indicates otherwise (i.e., it observes  $\omega_h$ ). The higher the probability  $\pi(m_\ell | \omega_h)$ , the more the government downplays the severity.

*Exaggerate.* An information policy is said to *exaggerate* if  $\pi(m_h | \omega_h) = 1$  and  $\pi(m_h | \omega_\ell)$  is positive. That is, the government may claim with positive probability that the epidemic is severe (i.e., it sends message  $m_h$ ) even though its information indicates otherwise (i.e., it observes  $\omega_\ell$ ). The higher the probability  $\pi(m_h | \omega_\ell)$ , the more the government exaggerates.

## Equilibrium

Given the government's information policy  $\Gamma$ , the population makes Bayesian inferences about the government's knowledge of  $\omega \in \{\omega_h, \omega_\ell\}$  upon receiving message  $m$ . That is, each individual updates her prior  $\mu^\circ$ , and we denote the corresponding posterior as  $\mu_m \triangleq \mathbb{E}[\omega | m, \Gamma] \in [\omega_\ell, \omega_h]$  (see online Appendix B).

Given posterior  $\mu_m$ , an individual with economic cost  $c$  determines  $\alpha$  to minimize her total expected economic and healthcare costs, i.e.,

$$a(c, \mu_m) \in \arg \min_{\alpha \in \{0,1\}} \alpha c + (1 - \alpha) \mu_m P_{a, \mu_m} \kappa, \quad (1)$$

where  $a(c, \mu_m)$  is the individual optimal choice, and

$$P_{a, \mu_m} \triangleq \mathbb{E}_c [1 - a(c, \mu_m)] \quad (2)$$

is the size of the socially active population.<sup>7</sup>

The probability of an individual becoming infected,  $(1 - \alpha) \mu_m P_{a, \mu_m}$ , depends on  $P_{a, \mu_m}$ , which captures the *externalities* that engaging in social interactions creates. As fewer people self-isolate (as  $P_{a, \mu_m}$  increases), a socially active individual is more likely to become infected, raising her incentives to self-isolate, giving rise to a game among individuals. A strategy profile  $a : \mathcal{C} \times [0, 1] \rightarrow \{0, 1\}$  (which maps the economic cost  $c$  and perceived

<sup>7</sup> In this paper,  $\mathbb{E}_x[\cdot]$  represents the expectation with respect to the random variable  $x$ .



infectiousness  $\mu_m$  to the isolation choice) forms a Bayesian Nash equilibrium (BNE) if it satisfies (1) -(2).<sup>8</sup> We denote the set of BNEs induced by information policy  $\Gamma$  as  $\mathcal{B}(\Gamma)$ .

### Government's problem

When managing the epidemic, the government needs to balance its population's health with the economy. The government assigns different levels of priority over these conflicting objectives, which may reflect different long- and short-term political goals. Specifically, we refer to  $C_h$  and  $C_e$  as the population's total expected health costs and economic costs, respectively, given the government's information policy and the population's corresponding response. The government balances these costs by assigning weights  $\lambda \in [0, 1]$  and  $1 - \lambda$  so that its objective function,  $C_\lambda$ , is equal to

$$C_\lambda(\Gamma, a) = \lambda C_e(\Gamma, a) + (1 - \lambda) C_h(\Gamma, a), \quad \text{for } a \in \mathcal{B}(\Gamma), \quad (3)$$

where

$$C_e(\Gamma, a) = \mathbb{E}_{c,m} [a(c, \mu_m)c \mid \Gamma], \quad \text{for } a \in \mathcal{B}(\Gamma), \quad (4)$$

is the population's expected economic cost, and

$$C_h(\Gamma, a) = \mathbb{E}_{c,m} [(1 - a(c, \mu_m))\mu_m P_{a,\mu_m} \kappa \mid \Gamma], \quad \text{for } a \in \mathcal{B}(\Gamma). \quad (5)$$

is the population's expected health cost. The government considers the total health and economic costs across the whole population, which the expectation over  $c$  in the expressions of  $C_e$  and  $C_h$  captures. Furthermore, the government may send its message with different levels of intensity as reflected by probability  $\pi$ , hence the expectation over  $m$ .

The government is more biased toward the economy when  $\lambda > 1/2$  and toward the population's health when  $\lambda < 1/2$ . At the extremes, the government's priority lies solely in reducing either economic costs  $C_e$  (when  $\lambda = 1$ ) or health costs  $C_h$  (when  $\lambda = 0$ ).

The government's problem is then to design an information policy that minimizes the total expected costs. Although we show in Section 3 that the optimal population equilibrium is unique, an information policy may induce multiple BNEs, and we denote as  $\mathcal{B}(\Gamma)$  the set of BNEs induced by policy  $\Gamma$ . As is standard in the information design literature (e.g., Kamenica and Gentzkow 2011), the sender optimizes over  $\Gamma$  by focusing on

<sup>8</sup> This is consistent with the equilibrium definition of nonatomic games (Schmeidler 1973, Mas-Colell 1984).

the equilibrium in  $\mathcal{B}(\Gamma)$ , which minimizes the sender's objective function (also referred to as the sender-preferred equilibrium). Thus, the government chooses  $(\Gamma, a)$  with  $a \in \mathcal{B}(\Gamma)$  to minimize the total expected cost, namely,

$$C^* \triangleq \min_{\Gamma, a \in \mathcal{B}(\Gamma)} C_\lambda(\Gamma, a) \quad (6)$$

In the following, we refer to  $(\Gamma^*, a^*)$  as the corresponding optimal policy and equilibrium.

### 3 Social Distancing Equilibrium

We next show that for any perceived infectiousness  $\mu$ , a unique social distancing equilibrium exists in the population. This equilibrium is characterized by a threshold in the individual's economic cost. (We provide the closed-form expressions for this and the following results as well as all the proofs in the online appendix.)

**PROPOSITION 1 (Equilibrium Characterization).** *Given  $\mu \in [\omega_\ell, \omega_h]$ , a unique BNE exists, which is given by threshold  $c^*(\mu)$  such that*

$$a^*(c, \mu) = \begin{cases} 1 & c \leq c^*(\mu), \\ 0 & c \geq c^*(\mu), \end{cases} \quad (7)$$

and the size of the socially active population in equilibrium is

$$P_{a^*, \mu} = \frac{c^*(\mu)}{\mu\kappa}. \quad (8)$$

Furthermore, threshold  $c^*(\mu)$  increases concavely and size  $P_{a^*, \mu}$  decreases in  $\mu$ .

In other words, the equilibrium behavior in the population is uniquely characterized by a single threshold  $c^*$  in the economic cost of isolation, such that only individuals who suffer an economic cost less than this threshold stay in confinement. The rest of the population, i.e., individuals with higher economic costs, engages in social interactions.

Proposition 1 also sheds light on the role of the externalities that individual choices create in the population. Indeed, an individual whose economic cost is exactly at threshold  $c^*$  is indifferent between avoiding and engaging in social interactions, i.e.,  $c^*(\mu) = \mu\kappa P_{a^*, \mu}$  per (8). Thus, two countervailing forces shape the behavior of  $c^*$  as a function of  $\mu$ . On the one hand, the higher the perceived infectiousness  $\mu$  is, the higher the value of  $c^*$  for a fixed size  $P_{a^*, \mu}$ , i.e., the more individuals have a great enough incentive to isolate themselves. On

the other hand, a higher  $\mu$  lowers the equilibrium size of the socially active population  $P_{a^*,\mu}$  because fewer individuals engage in social interactions. The net effect is that  $c^*$  increases in  $\mu$ , albeit at a diminishing rate. In other words, because of the externalities that individual choices create (via size  $P_{a^*,\mu}$ ), the additional number of confined individuals decreases when the epidemic appears to be more severe.

Proposition 1 enables reformulating the government's problem in terms of threshold  $c^*$ . Specifically, for any given perceived infectiousness  $\mu$ , the population's economic and healthcare costs as well as the government's total cost can be written as

$$K_e(c^*(\mu)) \triangleq \mathbb{E}_c [a^*(c, \mu)c] = \mathbb{E}_c [c \mathbb{1}\{c \leq c^*(\mu)\}], \quad (9)$$

$$K_h(c^*(\mu)) \triangleq \mathbb{E}_c [(1 - a^*(c, \mu))\mu P_{a^*,\mu}\kappa] = c^*(\mu) \mathbb{E}_c [\mathbb{1}\{c \geq c^*(\mu)\}] \quad \text{and} \quad (10)$$

$$K_\lambda(c^*(\mu)) \triangleq \lambda K_e(c^*(\mu)) + (1 - \lambda)K_h(c^*(\mu)). \quad (11)$$

In particular,  $C_e = \mathbb{E}_m [K_e(c^*(\mu_m)) | \Gamma]$  and  $C_h = \mathbb{E}_m [K_h(c^*(\mu_m)) | \Gamma]$ . The government's problem becomes then,

$$C^* = \min_{\Gamma} \mathbb{E}_m [K_\lambda(c^*(\mu_m)) | \Gamma] \quad (12)$$

The optimal policy that solves (12) is fully determined by the lower convex envelope of total cost  $K_\lambda(c^*(\cdot))$  (see Kamenica and Gentzkow 2011 for why this is the case and online Appendix C for additional intuition regarding our setup). However, cost  $K_\lambda(\cdot)$  is in general neither concave nor convex, and threshold  $c^*(\cdot)$  is nonlinear due to negative externalities within the population (per Proposition 1), which makes the analysis highly nontrivial.

## 4 Optimal Information Policy

We are now ready to present the main findings of our analysis.

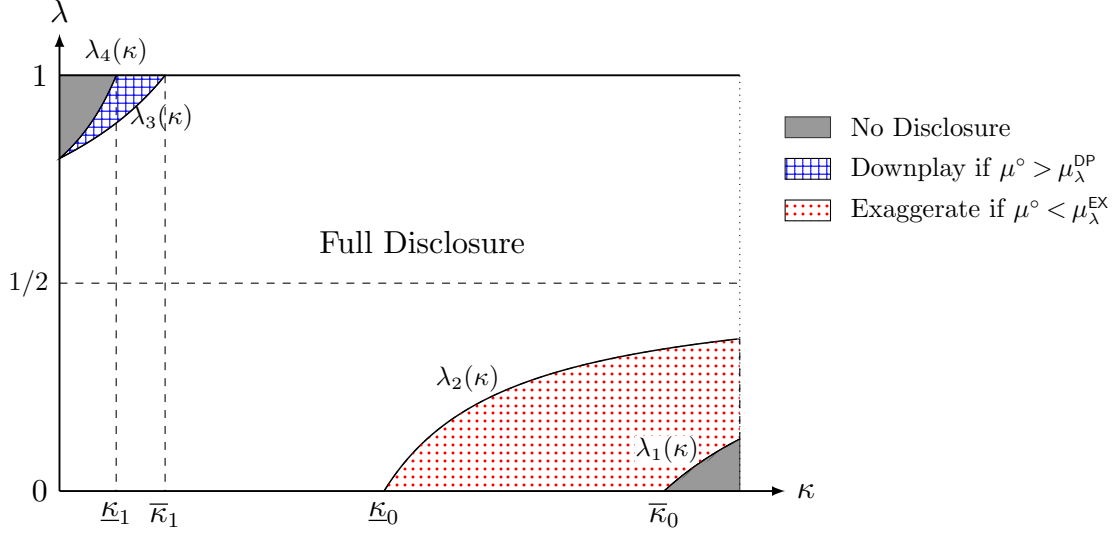
### **Governments may misinform the public.**

The next result determines when the government fully discloses its private information, when it exaggerates or downplays the severity of the epidemic, and when it refrains from disclosing any information (where these different information policies are as formally defined in Section 2).

**THEOREM 1 (Optimal Information Policy).** *Given  $\sigma > 0$  and  $\kappa > 0$ , the optimal information policy that solves (12) is characterized by four thresholds  $\lambda_1, \lambda_2, \lambda_3$ , and  $\lambda_4$  in  $\lambda$ , with  $\lambda_1 \leq \lambda_2 \leq 1/2 \leq \lambda_3 \leq \lambda_4$ , and two thresholds  $\mu_\lambda^{\text{EX}}$  and  $\mu_\lambda^{\text{DP}}$  in  $\mu^\circ$ , such that if*

- $\lambda \leq \lambda_1$  or  $\lambda \geq \lambda_4$ , no disclosure is optimal.
- $\lambda_2 \leq \lambda \leq \lambda_3$ , full disclosure is optimal.
- $\lambda_1 < \lambda < \lambda_2$ , exaggerating is optimal if  $\mu^\circ < \mu_\lambda^{\text{EX}}$ , and no disclosure is optimal otherwise.
- $\lambda_3 < \lambda < \lambda_4$ , downplaying is optimal if  $\mu^\circ > \mu_\lambda^{\text{DP}}$ , and no disclosure is optimal otherwise.

Furthermore, the thresholds  $\lambda_1, \lambda_2, \lambda_3$  and  $\lambda_4$  are all nondecreasing in  $\kappa$ , with  $\lambda_1(\bar{\kappa}_0) = \lambda_2(\underline{\kappa}_0) = 0$  and  $\lambda_3(\bar{\kappa}_1) = \lambda_4(\underline{\kappa}_1) = 1$  for positive cutoffs  $\bar{\kappa}_0 > \underline{\kappa}_0 > \bar{\kappa}_1 > \underline{\kappa}_1$ .



**Figure 1** Optimal policy in  $(\kappa, \lambda)$ -space for  $\omega_\ell = 0.3$ ,  $\omega_h = 0.8$ ,  $\theta = 10$  and  $\sigma = 10/\sqrt{3}$ .

Theorem 1 reveals that governments sometimes inform the public in a way that is not consistent with their own information about the epidemic. The incentives to do so depend on both the government's priorities ( $\lambda$ ) and the health cost that individuals are facing ( $\kappa$ ). Figure 1 depicts the distortions characterized by the theorem as a function of  $\lambda$  and  $\kappa$ .

When the government weighs the economy and population health sufficiently equally, i.e., when  $\lambda$  is sufficiently close to  $1/2$ , the government fully discloses its information about the severity of the epidemic. In fact, the government is always fully transparent when  $\lambda \in [\lambda_2, \lambda_3]$  (with  $\lambda_2 \leq 1/2 \leq \lambda_3$ ). However, if the government has more polarized preferences, i.e.,  $\lambda$  is sufficiently far from  $1/2$ , the government may misrepresent its information. This happens when health costs also take on extreme values (the dotted and hatched areas in Figure 1). Specifically, when  $\lambda$  is small and  $\kappa$  large, the government exaggerates the epidemic's severity if the population's perception of the risk is low ( $\mu^\circ < \mu_\lambda^{\text{EX}}$ ). And when  $\lambda$  is large and  $\kappa$  small, the government downplays the epidemic's severity if the population's

perception of the risk is high ( $\mu^\circ > \mu_\lambda^{\text{DP}}$ ). If both  $\lambda$  and  $\kappa$  take on very extreme values, the government prefers not to disclose any information (the shaded areas in Figure 1).

These distortions are perhaps best illustrated by the limit cases in which the government is concerned solely with either the economy ( $\lambda = 1$ ) or with population health ( $\lambda = 0$ ).

**COROLLARY 1 (Economy-biased Government).** *If the government's objective is solely to minimize economic costs ( $\lambda = 1$ ), it is optimal to*

- *fully disclose if  $\kappa \geq \bar{\kappa}_1$ ,*
- *downplay if  $\underline{\kappa}_1 < \kappa < \bar{\kappa}_1$  and  $\mu^\circ > \mu_1^{\text{DP}}$ , and*
- *not disclose any information otherwise.*

Downplaying the severity of the epidemic should always promote public behaviors that favor the economy, and a government that focuses solely on this objective may be prone to systematically misrepresenting its information in this way. However, Corollary 1 indicates that such a government refrains from misinforming the public when health costs are sufficiently high ( $\kappa \geq \bar{\kappa}_1$ ). This is despite the fact that high health costs incentivize individuals to remain confined, undermining the government's economy-biased objective. In contrast, the government may downplay the severity of the epidemic when health costs are small, which is precisely when individuals have stronger incentives to participate in the economy. When the health costs are very small, the government does not disclose any information.

Opposite results hold when the government focuses solely on health.

**COROLLARY 2 (Healthcare-biased Government).** *If the government's objective is solely to minimize healthcare costs ( $\lambda = 0$ ), it is optimal to*

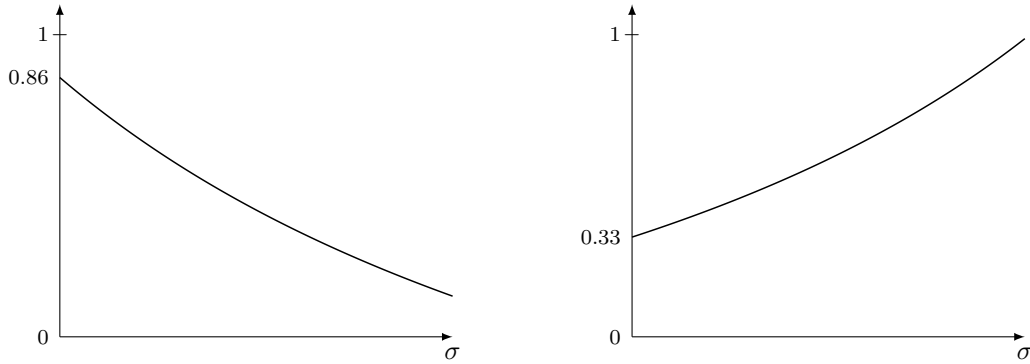
- *fully disclose if  $\kappa \leq \underline{\kappa}_0$ ,*
- *exaggerate if  $\underline{\kappa}_0 < \kappa < \bar{\kappa}_0$  and  $\mu^\circ < \mu_0^{\text{EX}}$ , and*
- *not disclose any information otherwise.*

Thus, when the government focuses solely on health, full disclosure is optimal when the health costs are small enough, which in fact incentivizes more individuals to engage in social interactions. The government may exaggerate the epidemic's severity precisely when the health costs are large, giving the population has strong incentives to remain confined.

### Governments exaggerate less but downplay more as economic inequalities increase.

This paper's main finding is a link between the unequal economic impact of the epidemic and the misinformation identified in Theorem 1. In our setup, the unequal impact of the epidemic is captured by  $\sigma$ , the effect of which is characterized next.

**THEOREM 2 (Effect of economic inequality).** *Thresholds  $\lambda_1, \lambda_2, \lambda_3$  and  $\lambda_4$  are non-increasing in  $\sigma$ . Further, the optimal exaggeration probability  $\pi^*(m_h | \omega_\ell)$  is decreasing and the optimal downplaying probability  $\pi^*(m_\ell | \omega_h)$  is increasing in  $\sigma$ .*



(a)  $\pi^*(m_h | \omega_\ell)$  for  $\lambda = 0$  and  $\kappa = 75 \in [\underline{\kappa}_0, \bar{\kappa}_0]$       (b)  $\pi^*(m_\ell | \omega_h)$  for  $\lambda = 1$  and  $\kappa = 16 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

**Figure 2** Optimal distortion probabilities as a function of  $\sigma$  for  $\omega_\ell = 0.3$ ,  $\omega_h = 0.8$  and  $\theta = 10$ .

In other words, the more unequal the economic impact of an epidemic is, the less the government may exaggerate, but the more it may downplay the epidemic's severity. First, the parametric regions in which the government may exaggerate and downplay (as depicted in Figure 1) shrink and expand, respectively, as heterogeneity  $\sigma$  increases. Second, when these misrepresentations occur, the probabilities of exaggerating and downplaying decrease and increase, respectively. Figure 2 illustrates this point for the limit cases. Figure 2a and Figure 2b depict the probabilities of exaggerating and downplaying the severity of the epidemic for  $\lambda = 0$  and  $\lambda = 1$ , respectively, as heterogeneity  $\sigma$  increases.

## 5 Discussion

Informing a population about an epidemic is challenging because different individuals trade off between their wealth and health differently, which further creates externalities within the rest of the population. The more unequal the epidemic's economic impact is in the population, the more heterogeneous the public reaction to governmental messages can be.

To tackle this problem, we have followed an analytic approach based on an information design framework. This approach yields three main findings: i) governments should always fully disclose their information about the epidemic when they weigh the economy and population health sufficiently equally, ii) but governments have an incentive to misrepresent their information about the epidemic when they have polarized objectives that are aligned with the overall public incentives, and iii) the more unequal the epidemic's economic impact is in the population, the less the government exaggerates but the more it is tempted to downplay the severity of the epidemic. The latter result is especially relevant given the rise of economic inequality in many societies (Piketty 2013).

Formally establishing these results requires several simplifications. In particular, we assume that the economic impact of the epidemic is uniformly distributed within the population. This assumption is not fundamental to our results, and our insights hold overall, at least numerically, with a beta distribution. In addition, all individuals in our setup expect similar levels of symptom severity from infection. Nevertheless, our main insights should hold as long as the economic costs of remaining isolated are not too strongly correlated with the severity of the symptoms. Furthermore, we assume that the expected severity of symptoms is public information. We can easily account for situations where this information is private to the government, in which case all our results continue to hold.<sup>9</sup>

Empirical and survey-based studies (e.g., Webster et al. 2020, Sabat et al. 2020) have recently established the paramount importance of the government's role in providing information about the COVID-19 outbreak, and called for policymakers to adopt effective communication strategies to contain the epidemic. What constitutes such strategies, however, is difficult to study empirically, especially when accounting for different governmental priorities and population characteristics, and largely remains an open question. Nonetheless, our analytical approach allows uncovering important mechanisms to consider when inducing public compliance.

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<sup>9</sup> Technically, this only requires redefining the government's private information as  $\omega\kappa$ , instead of  $\omega$ .

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## A Proofs of Results

Before providing proofs, we first provide some definitions used in the statements of the results in the paper and the proofs. First, let  $\gamma \triangleq \sigma 2\sqrt{3}$  (which also corresponds to  $\gamma = \sup \mathcal{C} - \inf \mathcal{C}$ ).

The closed-form expression of the equilibrium threshold  $c^*(\mu)$  is

$$c^*(\mu) = \frac{\mu\kappa(\theta + \sigma\sqrt{3})}{\sigma 2\sqrt{3} + \mu\kappa}.$$

The thresholds over  $\mu$  are given by

$$\mu_\lambda^{\text{DP}} \triangleq \rho_\lambda^{\text{DP}}(\omega_h - \omega_\ell) + \omega_\ell \quad (13)$$

$$\mu_\lambda^{\text{EX}} \triangleq \rho_\lambda^{\text{EX}}(\omega_h - \omega_\ell) + \omega_\ell \quad (14)$$

where

$$\rho_\lambda^{\text{DP}} \triangleq \frac{\gamma^2(5\lambda - 4) - 2\kappa^2\omega_\ell\omega_h(2\lambda - 1) - \gamma\kappa(2(1 - \lambda)\omega_h + \lambda\omega_\ell)}{\kappa(\omega_h - \omega_\ell)(2\kappa\omega_h(2\lambda - 1) + \gamma\lambda)} \quad (15)$$

$$\rho_\lambda^{\text{EX}} \triangleq \frac{\left(2\kappa\omega_\ell(1 - 2\lambda) - \gamma(4 - 5\lambda)\right)(\kappa\omega_\ell + \gamma)}{\kappa(\omega_h - \omega_\ell)(2\kappa\omega_\ell(2\lambda - 1) + \gamma\lambda)}. \quad (16)$$

Then, the optimal downplaying and exaggerating probabilities are in order respectively.

To highlight the difference between those terms, we use subscripts.

$$\pi_{\text{DP}}^*(m_h | \omega_h) = \frac{\omega_h - \omega_\ell}{\mu^\circ - \omega_\ell} \left( \frac{\mu^\circ - \mu_\lambda^{\text{DP}}}{\omega_h - \mu_\lambda^{\text{DP}}} \right) \text{ and } \pi_{\text{DP}}^*(m_\ell | \omega_\ell) = 1. \quad (17)$$

$$\pi_{\text{EX}}^*(m_h | \omega_h) = 1 \text{ and } \pi_{\text{EX}}^*(m_\ell | \omega_\ell) = \frac{(\mu_\lambda^{\text{EX}} - \mu^\circ)(\omega_h - \omega_\ell)}{(\omega_h - \mu^\circ)(\mu_\lambda^{\text{EX}} - \omega_\ell)}. \quad (18)$$

Furthermore, we define for a given  $\gamma$  (and hence  $\sigma$ ),

$$\lambda_1(\kappa) \triangleq \max \left( -\epsilon, G_\ell^{-1} \left( \frac{\kappa}{\gamma} \right) \right) \quad (19)$$

$$\lambda_2(\kappa) \triangleq \max \left( -\epsilon, G_2^{-1} \left( \frac{\kappa}{\gamma} \right) \right) \quad (20)$$

$$\lambda_3(\kappa) \triangleq \min \left( 1 + \epsilon, G_1^{-1} \left( \frac{\kappa}{\gamma} \right) \right) \quad (21)$$

$$\lambda_4(\kappa) \triangleq \min \left( 1 + \epsilon, G_h^{-1} \left( \frac{\kappa}{\gamma} \right) \right) \quad (22)$$

for  $\epsilon > 0$  and  $Y^{-1}(\cdot)$  represents the inverse of a function  $Y$ . Note here that we project functions  $\lambda_i$ ,  $i = 1, \dots, 4$  in order to extend their domains to positive real numbers because  $\kappa/\gamma \geq 0$ . Since  $\lambda \in [0, 1]$ , the extended parts ( $1 + \epsilon$  and  $-\epsilon$ ) do not affect our analysis.

$$G_h(\lambda) \triangleq \frac{1 - \frac{5\lambda}{4}}{\omega_h(\frac{1}{2} - \lambda)} \text{ and } G_\ell(\lambda) \triangleq \frac{1 - \frac{5\lambda}{4}}{\omega_\ell(\frac{1}{2} - \lambda)} \text{ for } \lambda \in (-\infty, 1/2) \cup (1/2, \infty), \quad (23)$$

$$G_1(\lambda) \triangleq \sqrt{\frac{5\lambda - 4}{2(2\lambda - 1)\omega_h\omega_\ell} + \left(\frac{2(1-\lambda)\omega_h + \lambda\omega_\ell}{4(2\lambda - 1)\omega_h\omega_\ell}\right)^2} - \frac{2(1-\lambda)\omega_h + \lambda\omega_\ell}{4(2\lambda - 1)\omega_h\omega_\ell} \text{ for } \lambda \in (4/5, \infty), \quad (24)$$

$$G_2(\lambda) \triangleq \sqrt{\frac{4 - 5\lambda}{2(1 - 2\lambda)\omega_h\omega_\ell} + \left(\frac{\lambda\omega_h + 2(1-\lambda)\omega_\ell}{4(1 - 2\lambda)\omega_h\omega_\ell}\right)^2} + \frac{\lambda\omega_h + 2(1-\lambda)\omega_\ell}{4(1 - 2\lambda)\omega_h\omega_\ell} \text{ for } \lambda \in (-\infty, 1/2). \quad (25)$$

We also define

$$\begin{aligned} \bar{\kappa}_0 &\triangleq \gamma G_\ell(0) \\ \underline{\kappa}_0 &\triangleq \gamma G_2(0) \\ \bar{\kappa}_1 &\triangleq \gamma G_1(1) \\ \underline{\kappa}_1 &\triangleq \gamma G_h(1). \end{aligned} \quad (26)$$

In the proofs of results, we denote the first and second derivatives of an arbitrary function  $Y(x)$  with respect to  $x$  by  $Y'(x)$  and, respectively  $Y''(x)$ .

*Proof of Proposition 1.* Fix  $\mu$ . It is straightforward to see from (1) and (2), the equilibrium action  $a^*(c, \mu)$  in (7) of an individual with cost  $c$  is the unique solution to

$$\min_{\alpha \in \{0,1\}} \alpha c + (1 - \alpha)\mu P_{a^*, \mu} \kappa. \quad (27)$$

for  $c^*(\mu) = \mu P_{a^*, \mu} \kappa$ . We next show that  $P_{a^*, \mu}$  is uniquely pinned down by  $a^*(c, \mu)$  for  $c^*(\mu) = \mu P_{a^*, \mu} \kappa$  and (2). Let  $F(\cdot)$  denote the c.d.f. of the uniform distribution with support  $\mathcal{C}$ .

$$P_{a^*, \mu} = 1 - F(\mu P_{a^*, \mu} \kappa). \quad (28)$$

Let  $h(x) \triangleq x - 1 + F(\mu x \kappa)$  for all  $x \in [0, 1]$ . The function  $h$  is continuous and strictly increasing. Furthermore,  $h(0) = -1 < 0$  since  $\theta - \gamma/2 \geq 0$  and  $h(1) = F(\mu \kappa) \geq 0$ . Thus, we conclude that  $P_{a^*, \mu}$  is the unique point such that  $P_{a^*, \mu} = h^{-1}(0)$ . Finally, solving the unique  $P_{a^*, \mu}$  of  $P_{a^*, \mu} = h^{-1}(0)$  for a uniform distribution over  $[\theta - \gamma/2, \theta + \gamma/2]$  leads to  $P_{a^*, \mu} = (\theta + \sigma\sqrt{3}) / (2\sigma\sqrt{3} + \mu\kappa)$ . Using the fact that  $c^*(\mu) = \mu P_{a^*, \mu} \kappa$ , we obtain  $c^*(\mu) = \frac{\mu\kappa(\theta + \sigma\sqrt{3})}{\sigma 2\sqrt{3} + \mu\kappa}$ . It is straightforward to show the monotonicity and concavity properties by taking first and second derivatives of  $P_{a^*, \mu}$  and  $c^*(\mu)$  with respect to  $\mu$ . Q.E.D.

*Proof of Theorem 1.* Before proving Theorem 1, we first provide four lemmas and some new notation. We delegate the proofs of those lemmas to the end. For simplicity, we define  $\tilde{K}_i(\mu) \triangleq K_i(c^*(\mu))$  for  $i \in \{e, h\}$  and so  $\tilde{K}_\lambda(\mu) \triangleq K_\lambda(c^*(\mu))$  for  $\lambda \in [0, 1]$ .

LEMMA 1. *Functions  $G_h, G_\ell, G_1$  and  $G_2$  defined respectively in (23)-(25) are strictly increasing. Furthermore,  $G_1(\lambda) > G_h(\lambda)$ ,  $G_\ell(\lambda) > G_1(\lambda)$  for  $\lambda \in (4/5, 1]$ , and  $G_\ell(\lambda) > G_2(\lambda)$ ,  $G_2(\lambda) > G_h(\lambda)$  for  $\lambda \in [0, 1/2)$ .*

LEMMA 2. *Consider  $\lambda_i$ , for  $i = 1, \dots, 4$  defined in (19)-(22), then  $\lambda_1 \leq \lambda_2 \leq 1/2 \leq 4/5 \leq \lambda_3 \leq \lambda_4$ . Further, thresholds  $\lambda_1, \lambda_2, \lambda_3$  and  $\lambda_4$  are all nondecreasing in  $\kappa$  with  $\lambda_1(\bar{\kappa}_0) = \lambda_2(\underline{\kappa}_0) = 0$  and  $\lambda_3(\bar{\kappa}_1) = \lambda_4(\underline{\kappa}_1) = 1$  for positive cutoffs  $\bar{\kappa}_0 > \underline{\kappa}_0 > \bar{\kappa}_1 > \underline{\kappa}_1$  defined in (26).*

LEMMA 3. *We have the following.*

- A. *The function  $\tilde{K}_\lambda(\mu)$  is concave if either one of the following conditions holds.*
1.  $\lambda \in [1/2, 4/5]$ .
  2.  $\lambda \in (4/5, 1] \wedge \kappa/\gamma \geq G_\ell(\lambda)$ .
  3.  $\lambda \in [0, 1/2) \wedge G_h(\lambda) \geq \kappa/\gamma$ .
- B. *The function  $\tilde{K}_\lambda(\mu)$  is convex if either one of the following conditions holds.*
1.  $\lambda \in [0, 1/2) \wedge \kappa/\gamma \geq G_\ell(\lambda)$ ,
  2.  $\lambda \in (4/5, 1] \wedge G_h(\lambda) \geq \kappa/\gamma$ .
- C. *If  $\lambda \in (4/5, 1] \wedge G_\ell(\lambda) > \kappa/\gamma \geq G_1(\lambda)$ , then  $\tilde{K}_\lambda(\mu)$  is first convex and then concave with inflection point  $\mu_{in} \in (\omega_\ell, \omega_h)$  and it satisfies  $\tilde{K}'_\lambda(\omega_\ell)(\omega_h - \omega_\ell) \geq \tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)$ .*
- D. *If  $\lambda \in [0, 1/2) \wedge G_2(\lambda) \geq \kappa/\gamma > G_h(\lambda)$ , then  $\tilde{K}_\lambda(\mu)$  is first concave and then convex with inflection point  $\tilde{\mu}_{in} \in (\omega_\ell, \omega_h)$  and it satisfies  $\tilde{K}_\lambda(\omega_h) - \tilde{K}'_\lambda(\omega_h)(\omega_h - \omega_\ell) \geq \tilde{K}_\lambda(\omega_\ell)$ .*
- E. *If  $\lambda \in (4/5, 1] \wedge G_1(\lambda) > \kappa/\gamma > G_h(\lambda)$ , then  $\tilde{K}_\lambda(\mu)$  is first convex and then concave with inflection point  $\mu_{in} \in (\omega_\ell, \omega_h)$  such that  $\mu_\lambda^{DP} \leq \mu_{in}$ . Moreover,  $\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\mu_\lambda^{DP}) = \tilde{K}'(\mu_\lambda^{DP})(\omega_h - \mu_\lambda^{DP})$ .*
- F. *If  $\lambda \in [0, 1/2) \wedge G_\ell(\lambda) > \kappa/\gamma > G_2(\lambda)$ , then  $\tilde{K}_\lambda(\mu)$  is first concave and then convex with inflection point  $\tilde{\mu}_{in} \in (\omega_\ell, \omega_h)$  such that  $\mu_\lambda^{EX} \geq \tilde{\mu}_{in}$ . Moreover,  $\tilde{K}_\lambda(\mu_\lambda^{EX}) - \tilde{K}_\lambda(\omega_\ell) = \tilde{K}'_\lambda(\mu_\lambda^{EX})(\mu_\lambda^{EX} - \omega_\ell)$ .*

We further define the following conditions.

$$\text{FD}_1 : \lambda \in [1/2, 4/5] \tag{29}$$

$$\text{FD}_2 : \lambda \in (4/5, 1] \wedge \kappa/\gamma \geq G_1(\lambda) \tag{30}$$

$$\text{FD}_3 : \lambda \in [0, 1/2) \wedge G_2(\lambda) \geq \kappa/\gamma \quad (31)$$

$$\text{ND}_1 : \lambda \in [0, 1/2) \wedge \kappa/\gamma \geq G_\ell(\lambda) \quad (32)$$

$$\text{ND}_2 : \lambda \in (4/5, 1] \wedge G_h(\lambda) \geq \kappa/\gamma \quad (33)$$

$$\text{DP} : \lambda \in (4/5, 1] \wedge G_1(\lambda) > \kappa/\gamma > G_h(\lambda) \quad (34)$$

$$\text{EX} : \lambda \in [0, 1/2) \wedge G_\ell(\lambda) > \kappa/\gamma > G_2(\lambda) \quad (35)$$

LEMMA 4. *The lower convex envelope  $k_\lambda(\mu)$  of  $\tilde{K}_\lambda(\mu)$  is*

$$k_\lambda(\mu) \triangleq \begin{cases} \tilde{K}_\lambda(\omega_\ell) + \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell} [\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)] & \text{if } \text{FD}_1 \vee \text{FD}_2 \vee \text{FD}_3, \\ \tilde{K}_\lambda(\mu) & \text{if } \text{ND}_1 \vee \text{ND}_2, \\ \tilde{K}_\lambda(\mu) 1_{\{\mu \leq \mu_\lambda^{\text{DP}}\}} + \left[ \tilde{K}_\lambda(\mu_\lambda^{\text{DP}}) + (\mu - \mu_\lambda^{\text{DP}}) \frac{\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\mu_\lambda^{\text{DP}})}{(\omega_h - \mu_\lambda^{\text{DP}})} \right] 1_{\{\mu > \mu_\lambda^{\text{DP}}\}} & \text{if } \text{DP}, \\ \tilde{K}_\lambda(\mu) 1_{\{\mu \geq \mu_\lambda^{\text{EX}}\}} + \left[ \tilde{K}_\lambda(\omega_\ell) + (\mu - \omega_\ell) \frac{\tilde{K}_\lambda(\mu_\lambda^{\text{EX}}) - \tilde{K}_\lambda(\omega_\ell)}{(\mu_\lambda^{\text{EX}} - \omega_\ell)} \right] 1_{\{\mu < \mu_\lambda^{\text{EX}}\}} & \text{if } \text{EX}. \end{cases} \quad (36)$$

where  $1_{\{\cdot\}}$  represents the indicator function.

Combining these lemmas, we are now ready to prove the theorem.

Following Corollary 2 of Kamenica and Gentzkow (2011), we know that the optimal cost corresponding to any prior mean  $\mu$  is given by  $k_\lambda(\mu)$  because  $k_\lambda(\mu)$  is the lower convex envelope of  $\tilde{K}_\lambda(\mu)$  (see Lemma 4). We first provide a set of conditions, and characterize the optimal information disclosure policy for each bullet point.

- If  $\text{FD}_1$  or  $\text{FD}_2$  or  $\text{FD}_3$ , full disclosure is optimal. This item follows because we have  $k_\lambda(\lambda) = \tilde{K}_\lambda(\omega_\ell) + (\mu - \omega_\ell) [\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)] / (\omega_h - \omega_\ell)$  so the optimal value can be achieved by the perfect information policy which induces posterior means  $\omega_h$  and  $\omega_\ell$ . Thus, the binary message space  $\mathcal{M}_B = \{m_\ell, m_h\}$  with  $\pi^*(m_i | \omega_i) = 1$  for  $i \in \{\ell, h\}$  achieves the optimal cost and hence constitute the optimal policy.

Next, we show that if  $\lambda_2 \leq \lambda \leq \lambda_3$ , then  $\text{FD}_1$  or  $\text{FD}_2$  or  $\text{FD}_3$  and hence full disclosure is optimal.

- When  $4/5 < \lambda \leq \lambda_3$ , ( $4/5 \leq \lambda_3$ , see Lemma 2), we have  $\lambda \leq 1$  and  $\lambda \leq G_1^{-1}(\kappa/\gamma)$  from  $\lambda \leq \lambda_3$ . Thus, it follows that  $\lambda \in (4/5, 1]$  and  $\kappa/\gamma \geq G_1(\lambda)$  and hence  $\text{FD}_2$ .
- When  $1/2 \leq \lambda \leq 4/5$ , we directly obtain  $\text{FD}_1$ .
- When  $\lambda_2 \leq \lambda < 1/2$ , ( $\lambda_2 \leq 1/2$ , see Lemma 2), we have  $\lambda \geq 0$  and  $\lambda \geq G_2^{-1}(\kappa/\gamma)$  from  $\lambda \geq \lambda_2$ . Thus, it follows that  $\lambda \in [0, 1/2)$  and  $G_2(\lambda) \geq \kappa/\gamma$  and hence  $\text{FD}_3$ .

- If  $\text{ND}_1$  or  $\text{ND}_2$ , no disclosure is optimal. The lower convex envelope  $k_\lambda(\mu)$  is equal to the function  $\tilde{K}_\lambda(\mu)$ . Thus, no disclosure policy is the uniquely optimal. Note that  $\lambda \leq \lambda_1$  implies  $\text{ND}_1$ , and  $\lambda \geq \lambda_4$  implies  $\text{ND}_2$  from the definitions of  $\lambda_1$  and  $\lambda_4$ . Hence, we prove the second bullet point of the theorem.
- If  $\text{DP}$  and  $\mu^\circ > \mu_\lambda^{\text{DP}}$ , it is optimal to downplay the risk. The optimal cost can be achieved by no disclosure policy if  $\mu^\circ \leq \mu_\lambda^{\text{DP}}$  because  $k_\lambda(\mu) = \tilde{K}_\lambda(\mu)$  in that case. Otherwise, the optimal cost can be achieved by inducing posterior means  $\omega_h$  with probability  $(\mu^\circ - \mu_\lambda^{\text{DP}})/(\omega_h - \mu_\lambda^{\text{DP}})$  and  $\mu_\lambda^{\text{DP}}$  with the remaining probability. Specifically, the following probability distribution provided in (17) with a binary message space  $\mathcal{M}_B$  achieves this posterior distribution. We next show that  $\lambda_3 < \lambda < \lambda_4$  implies  $\text{DP}$ . Since,  $\lambda_3 \geq 4/5$ , we obtain that  $\lambda \in (4/5, 1]$ . Definitions of  $\lambda_3$  and  $\lambda_4$  (see (19)-(22)) imply  $G_1(\lambda) > \kappa/\gamma > G_h(\lambda)$ .
- If  $\text{EX}$  and  $\mu^\circ < \mu_\lambda^{\text{EX}}$ , it is optimal to exaggerate the risk. The optimal cost can be achieved by no disclosure policy if  $\mu^\circ \geq \mu_\lambda^{\text{EX}}$ . Otherwise, the optimal cost can be achieved by inducing posterior means  $\omega_\ell$  with probability  $(\mu_\lambda^{\text{EX}} - \mu^\circ)/(\mu_\lambda^{\text{EX}} - \omega_\ell)$  and  $\mu_\lambda^{\text{EX}}$  with the remaining probability. Specifically, the following probability distribution provided in (18) with a binary message space  $\mathcal{M}_B$  achieves this posterior distribution. We conclude the proof by showing that  $\lambda_1 < \lambda < \lambda_2$  implies  $\text{EX}$ . Since  $\lambda_2 \leq 1/2$ , we obtain that  $\lambda \in [0, 1/2)$ . Definitions of  $\lambda_3$  and  $\lambda_4$  (see (19)-(22))  $G_2(\lambda) < \kappa/\gamma < G_\ell(\lambda)$ .

*Proof of Lemma 1.* To prove this result, we first provide the derivatives of  $G_\ell$  and  $G_h$  with respect to  $\lambda$ .

$$G'_h(\lambda) = \frac{3}{2\omega_h(1-2\lambda)^2} \text{ and } G'_\ell(\lambda) = \frac{3}{2\omega_\ell(1-2\lambda)^2} \quad (37)$$

Since these terms above are positive, the claim follows. Now consider,  $G_2$ . The terms  $\frac{4-5\lambda}{2(1-2\lambda)\omega_h\omega_\ell}$  and  $\frac{\lambda\omega_h+2(1-\lambda)\omega_\ell}{4(1-2\lambda)\omega_h\omega_\ell}$  are increasing functions of  $\lambda$ . Hence,  $G_2$  is also increasing. In order to prove,  $G_1$  is increasing we define the following notation for simplicity. Let  $A_1(\lambda) \triangleq \frac{5\lambda-4}{2(2\lambda-1)\omega_h\omega_\ell}$ ,  $A_2(\lambda) \triangleq \frac{2(1-\lambda)\omega_h+\lambda\omega_\ell}{4(2\lambda-1)\omega_h\omega_\ell}$ , i.e.,  $G_1(\lambda) = \sqrt{A_1(\lambda) + (A_2(\lambda))^2} - A_2(\lambda)$ . Taking derivative of  $A_1$  and  $A_2$ , it can be shown that  $A_1$  is increasing and  $A_2$  is decreasing. Furthermore, it follows that

$$G'_1(\lambda) = \frac{A'_1(\lambda)}{2\sqrt{A_1(\lambda) + (A_2(\lambda))^2}} - A'_2(\lambda) \left[ 1 - \frac{1}{\sqrt{\frac{A_1(\lambda)}{(A_2(\lambda))^2} + 1}} \right]. \quad (38)$$

This expression reveals that  $G'_1(\lambda) > 0$  because  $A'_1(\lambda)$  is positive, and  $A'_2(\lambda)$  is negative.

Next, we prove  $G_1(\lambda) > G_h(\lambda)$  and  $G_\ell(\lambda) \geq G_1(\lambda)$  for  $\lambda \in (4/5, 1]$ . Note that we can write  $G_h(\lambda) = A_1(\lambda)\omega_\ell$  and  $G_\ell(\lambda) = A_1(\lambda)\omega_h$ . First, we consider  $G_1(\lambda) \geq G_h(\lambda)$ , we need to show that

$$\sqrt{A_1(\lambda) + [A_2(\lambda)]^2} > A_1(\lambda)\omega_\ell + A_2(\lambda)$$

Using simple algebraic operations, we equivalently represent the inequality above as

$$1 > \frac{(1-\lambda)\omega_h + (3\lambda-2)\omega_\ell}{\omega_h(2\lambda-1)} \iff \omega_h > \omega_\ell.$$

For the last inequality we use the fact that  $\lambda \in (4/5, 1]$  so  $(3\lambda-2) > 0$ . Now, we consider  $G_\ell(\lambda) > G_1(\lambda)$ , we need to show that

$$A_1(\lambda)\omega_h + A_2(\lambda) > \sqrt{A_1(\lambda) + [A_2(\lambda)]^2}$$

Algebraic operations yield that the above inequality can be written as follows:

$$\frac{(3\omega_h + \omega_\ell)\lambda - 2\omega_h}{4\lambda\omega_\ell - 2\omega_\ell} > 1 \iff \omega_h > \omega_\ell$$

Here, we use the fact that  $\lambda \in (4/5, 1]$  so  $(3\lambda-2) > 0$ .

We next prove  $G_\ell(\lambda) > G_2(\lambda)$ ,  $G_2(\lambda) > G_h(\lambda)$  for  $\lambda \in [0, 1/2)$ . For simplicity we define  $B_1(\lambda) = G_\ell(\lambda)/\omega_h$  and  $B_2(\lambda) \triangleq \frac{\lambda\omega_h + 2(1-\lambda)\omega_\ell}{4(1-2\lambda)\omega_h\omega_\ell}$ . Using this notation, we first need to show that

$$\omega_h B_1(\lambda) > \sqrt{B_1(\lambda) + [B_2(\lambda)]^2} + B_2(\lambda)$$

Simplifying this inequality we obtain that

$$\frac{((3\omega_h - \omega_\ell)\lambda - 2\omega_h + \omega_\ell)}{(2\lambda - 1)\omega_\ell} > 1 \iff \omega_h > \omega_\ell.$$

To obtain the last inequality, we use the fact that  $\lambda \in [0, 1/2)$  to show  $2 - 3\lambda > 0$ . Thus,  $G_\ell(\lambda) > G_2(\lambda)$  follows.

Finally,  $G_2(\lambda) > G_h(\lambda)$  which can equivalently be represented as follows:

$$\sqrt{B_1(\lambda) + [B_2(\lambda)]^2} + B_2(\lambda) > \omega_\ell B_1(\lambda)$$

Using straightforward algebraic operations, we get the equivalent inequality.

$$1 > \frac{\omega_\ell(2-3\lambda) - \lambda\omega_h}{2\omega_h(1-2\lambda)} \iff \omega_h > \omega_\ell$$

As in the previous steps, we use the fact that  $\lambda \in [0, 1/2)$ .

Q.E.D.

*Proof of Lemma 2* Since we know  $G_h, G_\ell, G_1$  and  $G_2$  (see (23)-(25)) are strictly increasing in their domains from Lemma 1, their inverse are also strictly increasing (see, Binmore 1982, p. 111). Since  $\kappa/\gamma$  is strictly increasing in  $\kappa$ , thresholds  $\lambda_i$  are all nondecreasing in  $\kappa$ .

Because  $G_1(4/5) = G_h(4/5) = 0$ , it follows that  $\lambda_3 \geq 4/5$  and  $\lambda_4 \geq 4/5$ . Furthermore,  $G_1(\lambda) > G_h(\lambda)$  for  $\lambda \in (4/5, 1]$  (see Lemma 1). Therefore,  $\lambda_4 \geq \lambda_3 \geq 4/5$ . Note that  $\lim_{\lambda \rightarrow 1/2} G_\ell(\lambda) = \lim_{\lambda \rightarrow 1/2} G_2(\lambda) = \infty$ , and  $G_\ell$  and  $G_2$  are increasing so we get  $1/2 \geq \lambda_2$  and  $1/2 \geq \lambda_1$ . Since,  $G_\ell(\lambda) > G_2(\lambda)$  for  $\lambda \in [0, 1/2)$  it follows that  $1/2 \geq \lambda_2 \geq \lambda_1$ .

Using the definitions of  $G_h, G_\ell, G_1$  and  $G_2$ , we compute  $\bar{\kappa}_0 = \gamma G_\ell(0) = 2\gamma/\omega_\ell$ ,  $\underline{\kappa}_0 = \gamma G_2(0) = \gamma(\sqrt{2/(\omega_h\omega_\ell)} + 1/\omega_h - (1/\omega_h)^2)$ ,  $\bar{\kappa}_1 = \gamma G_1(1) = \gamma(\sqrt{1/(2\omega_h\omega_\ell)} + (1/(4\omega_h))^2 - 1/(4\omega_h))$  and  $\underline{\kappa}_1 = \gamma G_h(1) = \gamma/(2\omega_h)$ . Then, we obtain  $\bar{\kappa}_0 > \underline{\kappa}_0 > \bar{\kappa}_1 > \underline{\kappa}_1$ .

Finally, the definitions of  $\bar{\kappa}_i, \underline{\kappa}_i$  for  $i \in \{0, 1\}$  imply that  $\lambda_1(\bar{\kappa}_0) = \lambda_2(\underline{\kappa}_0) = 0$  and  $\lambda_3(\bar{\kappa}_1) = \lambda_4(\underline{\kappa}_1) = 1$ . Q.E.D.

*Proof of Lemma 3.* Using the derived expression of  $c^*(\mu)$  from Proposition 1 and the definition of  $\tilde{K}_\lambda(\mu)$ , we obtain that

$$\tilde{K}_\lambda(\mu) = \frac{(3\lambda - 2)}{2\gamma} \left( \frac{\mu\kappa(\theta + \gamma/2)}{\gamma + \mu\kappa} \right)^2 + (1 - \lambda) \left( \frac{\mu\kappa(\theta + \gamma/2)}{\gamma + \mu\kappa} \right) \frac{\theta + \gamma/2}{\gamma} - \frac{\lambda(\theta - \gamma/2)^2}{2\gamma}. \quad (39)$$

The second derivative of  $\tilde{K}_\lambda$  with respect to  $\mu$  is given by

$$\tilde{K}_\lambda''(\mu) = - \frac{4(\theta + \gamma/2)^2 \kappa^2 \gamma \left[ \frac{\kappa}{\gamma} \mu \left( \lambda - \frac{1}{2} \right) + \left( 1 - \frac{5\lambda}{4} \right) \right]}{(\kappa\mu + \gamma)^4}. \quad (40)$$

*Part A.* Note that  $\tilde{K}_\lambda''(\mu)$  in (40) is negative when  $\lambda \in [1/2, 4/5]$ . Thus, the first item follows. Recalling the definition of  $G_h$  and  $G_\ell$  from (23), one can show that the second and third items imply negative  $\tilde{K}_\lambda''(\mu)$  because  $\frac{\kappa}{\gamma} \mu \left( \lambda - \frac{1}{2} \right) + \left( 1 - \frac{5\lambda}{4} \right)$  is positive under those conditions.

*Part B.* To prove this part, we again use (40). Simple algebra yields that  $\frac{\kappa}{\gamma} \mu \left( \lambda - \frac{1}{2} \right) + \left( 1 - \frac{5\lambda}{4} \right)$  is negative and hence  $\tilde{K}_\lambda''(\mu)$  is positive when the first and second conditions hold.

*Part C.* First note that  $G_1(\lambda) > G_h(\lambda)$  (see Lemma 1). Thus, we know the parameter regime is such that neither the second item of Part A nor the second item of Part B holds. This implies there exists an inflection point  $\mu_{in}$  such that  $\tilde{K}_\lambda(\mu)$  is first concave and then



convex because  $\tilde{K}''_\lambda(\mu)$  is first negative and then positive. We next derive  $\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell) - \tilde{K}'_\lambda(\omega_\ell)(\omega_h - \omega_\ell)$  and show that it is nonpositive when  $\lambda \in (4/5, 1] \wedge G_\ell(\lambda) > \kappa/\gamma \geq G_1(\lambda)$ .

$$\frac{2[(\omega_h - \omega_\ell)(\theta + \gamma/2)\kappa\gamma]^2(\lambda - \frac{1}{2}) \left[ \omega_\ell\omega_h \left(\frac{\kappa}{\gamma}\right)^2 + \frac{(2(1-\lambda)\omega_h + \lambda\omega_\ell)\kappa}{2(2\lambda-1)}\frac{\kappa}{\gamma} - \frac{(5\lambda-4)}{2(2\lambda-1)} \right]}{(\kappa\omega_h + \gamma)^2(\kappa\omega_\ell + \gamma)^3} \quad (41)$$

In order to prove  $\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell) - \tilde{K}'_\lambda(\omega_\ell)(\omega_h - \omega_\ell) \leq 0$ , it is sufficient to show the following inequality holds.

$$\left(\frac{\kappa}{\gamma}\right)^2 + \frac{(2(1-\lambda)\omega_h + \lambda\omega_\ell)\kappa}{2(2\lambda-1)\omega_h\omega_\ell}\frac{\kappa}{\gamma} - \frac{(5\lambda-4)}{2(2\lambda-1)\omega_h\omega_\ell} \geq 0 \quad (42)$$

We can rewrite the left-hand side of the above inequality as

$$\left(\frac{\kappa}{\gamma}\right)^2 + \frac{(2(1-\lambda)\omega_h + \lambda\omega_\ell)\kappa}{2(2\lambda-1)\omega_h\omega_\ell}\frac{\kappa}{\gamma} - \frac{(5\lambda-4)}{2(2\lambda-1)\omega_h\omega_\ell} \quad (43)$$

$$= \left(\frac{\kappa}{\gamma} + \frac{(2(1-\lambda)\omega_h + \lambda\omega_\ell)}{4(2\lambda-1)\omega_h\omega_\ell}\right)^2 - \left(\frac{(2(1-\lambda)\omega_h + \lambda\omega_\ell)}{4(2\lambda-1)\omega_h\omega_\ell}\right)^2 - \frac{(5\lambda-4)}{2(2\lambda-1)\omega_h\omega_\ell}. \quad (44)$$

This expression above is nonnegative whenever  $\kappa/\gamma \geq G_1(\lambda)$  and we conclude the proof of this part.

*Part D.* Note that  $G_\ell(\lambda) > G_2(\lambda)$  (see Lemma 1) so the parameter regime in this part is such that neither the third item of Part A nor the first item of Part B holds. Thus, there exists an inflection point  $\tilde{\mu}_{in} \in (\omega_\ell, \omega_h)$  such that  $\tilde{K}_\lambda(\mu)$  is first convex and then concave because  $\tilde{K}''_\lambda(\mu)$  is first positive and then negative. Next, we show that  $\tilde{K}_\lambda(\omega_h) - \tilde{K}'_\lambda(\omega_h)(\omega_h - \omega_\ell) - \tilde{K}_\lambda(\omega_\ell)$  is nonnegative when  $\lambda \in [0, 1/2) \wedge G_2(\lambda) \geq \kappa/\gamma > G_h(\lambda)$ . The expression is given by

$$\frac{2[(\omega_h - \omega_\ell)(\theta + \gamma/2)\kappa\gamma]^2(\frac{1}{2} - \lambda) \left[ \omega_\ell\omega_h \left(\frac{\kappa}{\gamma}\right)^2 + \frac{(2(1-\lambda)\omega_\ell + \omega_h\lambda)\kappa}{2(1-2\lambda)}\frac{\kappa}{\gamma} - \frac{(5\lambda-4)}{2(1-2\lambda)} \right]}{(\kappa\omega_h + \gamma)^3(\kappa\omega_\ell + \gamma)^2}.$$

To show  $\tilde{K}_\lambda(\omega_h) - \tilde{K}'_\lambda(\omega_h)(\omega_h - \omega_\ell) - \tilde{K}_\lambda(\omega_\ell) \geq 0$ , we can directly focus on the following inequality.

$$\left(\frac{\kappa}{\gamma}\right)^2 + \frac{(2(1-\lambda)\omega_\ell + \omega_h\lambda)\kappa}{2(1-2\lambda)\omega_h\omega_\ell}\frac{\kappa}{\gamma} - \frac{(5\lambda-4)}{2(1-2\lambda)\omega_h\omega_\ell} \leq 0 \quad (45)$$

Rewriting this expression, we obtain the following.

$$\left(\frac{\kappa}{\gamma}\right)^2 + \frac{(2(1-\lambda)\omega_\ell + \omega_h\lambda)\kappa}{2(1-2\lambda)\omega_h\omega_\ell}\frac{\kappa}{\gamma} - \frac{(5\lambda-4)}{2(1-2\lambda)\omega_h\omega_\ell} \quad (46)$$

$$= \left(\frac{\kappa}{\gamma} + \frac{(2(1-\lambda)\omega_\ell + \omega_h\lambda)}{4(1-2\lambda)\omega_h\omega_\ell}\right)^2 - \left(\frac{(2(1-\lambda)\omega_\ell + \omega_h\lambda)}{4(1-2\lambda)\omega_h\omega_\ell}\right)^2 - \frac{(5\lambda-4)}{2(1-2\lambda)\omega_h\omega_\ell} \leq 0 \quad (47)$$

The last inequality holds because  $\kappa/\gamma \leq G_2(\lambda)$ .

*Part E.* Since  $G_\ell(\lambda) > G_1(\lambda)$  (see Lemma 1), there exists an inflection point  $\mu_{in}$  as in Part C. Differently from Part C, this time  $G_1(\lambda) > \kappa/\gamma$  so  $\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell) - \tilde{K}'_\lambda(\omega_\ell)(\omega_h - \omega_\ell) > 0$ . Moreover, we know  $\tilde{K}_\lambda(\mu) + \tilde{K}'_\lambda(\mu)(\omega_h - \mu)$  is an increasing function of  $\mu \in (\omega_\ell, \mu_{in})$  due to convexity and  $\tilde{K}_\lambda(\mu_{in}) + \tilde{K}'_\lambda(\mu_{in})(\omega_h - \mu_{in}) > \tilde{K}_\lambda(\omega_h)$  due to concavity for  $\mu \in (\mu_{in}, \omega_h)$ . Thus,  $\tilde{K}_\lambda(\mu) + \tilde{K}'_\lambda(\mu)(\omega_h - \mu)$  crosses  $\tilde{K}_\lambda(\omega_h)$  for some  $\mu = \mu_\lambda^{\text{DP}}$  such that  $\mu_\lambda^{\text{DP}} \leq \mu_{in}$ .

*Part F.* Note that  $G_2(\lambda) > G_h(\lambda)$  (see Lemma 1) therefore there exists an inflection point  $\tilde{\mu}_{in}$  as in Part D. Unlike Part D,  $\kappa/\gamma > G_2(\lambda)$  thus  $\tilde{K}_\lambda(\omega_h) - \tilde{K}'_\lambda(\omega_h)(\omega_h - \omega_\ell) - \tilde{K}_\lambda(\omega_\ell) < 0$ . Due to concavity in  $(\omega_\ell, \tilde{\mu}_{in})$ , we know  $\tilde{K}_\lambda(\omega_\ell) < \tilde{K}_\lambda(\tilde{\mu}_{in}) - \tilde{K}'_\lambda(\tilde{\mu}_{in})(\tilde{\mu}_{in} - \omega_\ell)$ . Besides,  $\tilde{K}_\lambda(\mu) - \tilde{K}'_\lambda(\mu)(\mu - \omega_\ell)$  is a decreasing function of  $\mu \in (\tilde{\mu}_{in}, \omega_h]$  due to convexity after the inflection point  $\tilde{\mu}_{in}$ . Thus, there must be a point  $\mu_\lambda^{\text{EX}}$  in  $(\tilde{\mu}_{in}, \omega_h)$  such that  $\tilde{K}_\lambda(\mu_\lambda^{\text{EX}}) - \tilde{K}'_\lambda(\mu_\lambda^{\text{EX}})(\mu_\lambda^{\text{EX}} - \omega_\ell) = \tilde{K}_\lambda(\omega_\ell)$ . Q.E.D.

*Proof of Lemma 4.* In order to check if the proposed function  $k_\lambda$  is the lower convex envelope of  $\tilde{K}_\lambda$  or not, we use the verification approach provided in Oberman (2007). In particular, we show that the function  $k_\lambda$  satisfies (Ob) of Oberman (2007) equation for  $\tilde{K}_\lambda$ . This equation is

$$\max\{k_\lambda(\mu) - \tilde{K}_\lambda(\mu), -k''_\lambda(\mu)\} = 0. \quad (\text{Ob})$$

- $\text{FD}_1 \vee \text{FD}_2 \vee \text{FD}_3$ : We need to show  $\tilde{K}_\lambda(\mu) \geq v(\mu)$  because  $k''_\lambda(\mu) = 0$ . Since  $k_\lambda(\mu)$  is in fact the line connecting  $\tilde{K}_\lambda(\omega_\ell)$  and  $\tilde{K}_\lambda(\omega_h)$  in this case, the result immediately follows when  $\tilde{K}_\lambda$  is concave. Note that Part A of Lemma 3 shows that  $\tilde{K}_\lambda$  is concave when  $\text{FD}_1$ . For  $\text{FD}_2$  and  $\text{FD}_3$ , we need to complement that the same part of the lemma by considering  $\lambda \in (4/5, 1] \wedge G_\ell(\lambda) > \kappa/\gamma \geq G_1(\lambda)$  and  $\lambda \in [0, 1/2) \wedge G_2(\lambda) \geq \kappa/\gamma > G_h(\lambda)$ , respectively.

Assume that  $\lambda \in (4/5, 1] \wedge G_\ell(\lambda) > \kappa/\gamma \geq G_1(\lambda)$ . For  $\mu \leq \mu_{in}$ , Taylor's theorem implies that  $\tilde{K}_\lambda(\omega_\ell) + \tilde{K}'_\lambda(\omega_\ell)(\mu - \omega_\ell) < \tilde{K}_\lambda(\mu)$  because  $\tilde{K}_\lambda$  is convex in  $[\omega_\ell, \mu_{in}]$ . Using Part C of Lemma 3 to replace  $\tilde{K}'_\lambda(\omega_\ell)$  with its lower bound, we obtain that  $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell} [\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$ . When  $\mu \geq \mu_{in}$ , we know  $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\mu_{in})x + \tilde{K}_\lambda(\omega_h)(1 - x)$  for any  $x \in (0, 1)$  due to concavity. Furthermore, we know that  $\tilde{K}_\lambda(\mu_{in}) \geq \tilde{K}_\lambda(\omega_\ell) + \frac{\mu_{in} - \omega_\ell}{\omega_h - \omega_\ell} [\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$  from the previous analysis. Thus, we obtain that

$$\tilde{K}_\lambda(\mu) > \left(1 - \frac{\mu_{in} - \omega_\ell}{\omega_h - \omega_\ell}\right) x \tilde{K}_\lambda(\omega_\ell) + \left(x \frac{\mu_{in} - \omega_\ell}{\omega_h - \omega_\ell} + 1 - x\right) \tilde{K}_\lambda(\omega_h)$$

Here, we can set  $\left(1 - \frac{\mu_{in} - \omega_\ell}{\omega_h - \omega_\ell}\right) x = \left(1 - \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell}\right)$  because  $0 \leq \frac{1 - \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell}}{1 - \frac{\mu_{in} - \omega_\ell}{\omega_h - \omega_\ell}} \leq 1$  since  $\mu \geq \mu_{in}$ . Thus, we obtain that  $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell} [\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$  for  $\mu \geq \mu_{in}$ , too.

Next, assume that  $\lambda \in [0, 1/2) \wedge G_2(\lambda) \geq \kappa/\gamma > G_h(\lambda)$ . Taylor's theorem implies that  $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_h) + \tilde{K}'_\lambda(\omega_h)(\mu - \omega_h)$  for  $\mu > \tilde{\mu}_{in}$  due to convexity. Since  $\mu - \omega_h$  is negative, we replace  $\tilde{K}'_\lambda(\omega_h)$  with its upper bound obtained in Part D of Lemma 3. Thus, it follows that  $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell} [\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$  for  $\mu > \tilde{\mu}_{in}$ . When  $\mu \leq \tilde{\mu}_{in}$ , we know that  $\tilde{K}_\lambda(\mu) \geq \tilde{K}_\lambda(\tilde{\mu}_{in})x + (1 - x)\tilde{K}_\lambda(\omega_\ell)$  for any  $x \in [0, 1]$  because  $\tilde{K}_\lambda$  is concave in  $[\omega_\ell, \tilde{\mu}_{in}]$ . Moreover, we know that  $\tilde{K}_\lambda(\tilde{\mu}_{in}) \geq \tilde{K}_\lambda(\omega_\ell) + \frac{\tilde{\mu}_{in} - \omega_\ell}{\omega_h - \omega_\ell} [\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$  from the previous analysis. Combining these observations, we obtain

$$\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell)x + \frac{\tilde{\mu}_{in} - \omega_\ell}{\omega_h - \omega_\ell} [\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]x + (1 - x)\tilde{K}_\lambda(\omega_\ell) \quad (48)$$

$$= \tilde{K}_\lambda(\omega_\ell) + x \frac{\tilde{\mu}_{in} - \omega_\ell}{\omega_h - \omega_\ell} [\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)] \quad (49)$$

Here, we can set  $x = \left( \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell} \right) / \left( \frac{\tilde{\mu}_{in} - \omega_\ell}{\omega_h - \omega_\ell} \right)$  because  $\mu \leq \tilde{\mu}_{in}$ , and obtain that  $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell} [\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$ . This completes the proof of this item of the lemma.

- ND<sub>1</sub> ∨ ND<sub>2</sub>: Part B of Lemma 3 shows that  $\tilde{K}_\lambda$  is convex, thus its lower convex envelope is equal to  $\tilde{K}_\lambda(\mu)$  itself (see Boyd and Vandenberghe 2004, pp. 94).
- DP: We consider two cases separately. First, assume that  $\mu \leq \mu_\lambda^{\text{DP}}$ , then  $k_\lambda(\mu) = \tilde{K}_\lambda(\mu)$  and  $\tilde{K}_\lambda(\mu)$  is convex because  $\mu_\lambda^{\text{DP}} \leq \mu_{in}$  (see Part E of Lemma 3). Next, assume that  $\mu \geq \mu_\lambda^{\text{DP}}$ , then  $k''_\lambda(\mu) = 0$  by its definition. Thus, we need to show that  $k_\lambda(\mu) - \tilde{K}_\lambda(\mu) \leq 0$  for  $\mu \geq \mu_\lambda^{\text{DP}}$  to prove that  $k_\lambda(\mu)$  satisfies (Ob). Taylor's theorem implies that  $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\mu_\lambda^{\text{DP}}) + (\mu - \mu_\lambda^{\text{DP}})\tilde{K}'_\lambda(\mu_\lambda^{\text{DP}})$  for  $\mu_{in} \geq \mu$  due to convexity. From Part E of Lemma 3, we also know  $k_\lambda(\mu) = \tilde{K}_\lambda(\mu_\lambda^{\text{DP}}) + (\mu - \mu_\lambda^{\text{DP}})\tilde{K}'_\lambda(\mu_\lambda^{\text{DP}})$ . Thus,  $k_\lambda(\mu) - \tilde{K}_\lambda(\mu) \leq 0$  for  $\mu_{in} \geq \mu \geq \mu_\lambda^{\text{DP}}$ .

Next, we consider  $\mu \geq \mu_{in}$ . When we look at the boundary  $\mu = \mu_{in}$ , the former observation implies  $\tilde{K}_\lambda(\mu_{in}) \geq k_\lambda(\mu_{in})$ . Because  $\tilde{K}_\lambda$  is concave when  $\mu \geq \mu_{in}$ , it follows that  $\tilde{K}_\lambda(\mu) \geq \tilde{K}_\lambda(\mu_{in}) + (\mu - \mu_{in})[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\mu_{in})]/(\omega_h - \mu_{in})$  which constitutes the line connecting  $\tilde{K}_\lambda(\mu_{in})$  to  $\tilde{K}_\lambda(\omega_h)$ . Because  $k_\lambda(\mu)$  is in fact the line connecting  $k_\lambda(\mu_{in})$  to  $k_\lambda(\omega_h) = \tilde{K}_\lambda(\omega_h)$  by definition, it follows that  $\tilde{K}_\lambda(\mu) \geq k_\lambda(\mu)$  for  $\mu \geq \mu_{in}$ , too.

Combining these, we conclude that  $k_\lambda(\mu)$  satisfies (Ob).

- EX: We consider two cases separately. If  $\mu \geq \mu_\lambda^{\text{EX}}$ , then  $k_\lambda(\mu) = \tilde{K}_\lambda(\mu)$  and  $\tilde{K}_\lambda(\mu)$  is convex from Part F of Lemma 3 because  $\mu_\lambda^{\text{EX}} \geq \tilde{\mu}_{in}$ . We next consider  $\mu_\lambda^{\text{EX}} \geq \mu$ . By its definition,  $k''_\lambda(\mu) = 0$  when  $\mu_\lambda^{\text{EX}} \geq \mu$ . Thus, we need to show that  $k_\lambda(\mu) - \tilde{K}_\lambda(\mu) \leq 0$  for  $\mu_\lambda^{\text{EX}} \geq \mu$  to prove that  $k_\lambda(\mu)$  satisfies (Ob).

When  $\mu_\lambda^{\text{EX}} \geq \mu \geq \tilde{\mu}_{in}$ , Taylor's theorem implies that  $\tilde{K}_\lambda(\mu) \geq \tilde{K}_\lambda(\mu_\lambda^{\text{EX}}) + (\mu - \mu_\lambda^{\text{EX}})\tilde{K}'_\lambda(\mu_\lambda^{\text{EX}})$  because  $\tilde{K}_\lambda(\mu)$  is convex for  $\mu \geq \tilde{\mu}_{in}$ . Furthermore, we know  $k_\lambda(\mu) = \tilde{K}_\lambda(\mu_\lambda^{\text{EX}}) + (\mu - \mu_\lambda^{\text{EX}})\tilde{K}'_\lambda(\mu_\lambda^{\text{EX}})$  because  $\tilde{K}_\lambda(\mu_\lambda^{\text{EX}}) - \tilde{K}_\lambda(\omega_\ell) = \tilde{K}'_\lambda(\mu_\lambda^{\text{EX}})(\mu_\lambda^{\text{EX}} - \omega_\ell)$  (see Part F of Lemma 3). Thus,  $k_\lambda(\mu) \leq \tilde{K}_\lambda(\mu)$  for  $\mu_\lambda^{\text{EX}} \geq \mu \geq \tilde{\mu}_{in}$ .

Next, we consider  $\tilde{\mu}_{in} \geq \mu$ . By definition  $k_\lambda(\omega_\ell) = \tilde{K}_\lambda(\omega_\ell)$ , and  $k_\lambda(\mu)$  is the line connecting  $k_\lambda(\omega_\ell)$  and  $k_\lambda(\tilde{\mu}_{in})$  for  $\tilde{\mu}_{in} \geq \mu$ . We also know  $\tilde{K}_\lambda(\mu)$  is above the line connecting  $\tilde{K}_\lambda(\omega_\ell)$  and  $\tilde{K}_\lambda(\tilde{\mu}_{in})$  because  $\tilde{K}_\lambda$  is concave for  $\tilde{\mu}_{in} \geq \mu$ . Hence, we need to show  $k_\lambda(\tilde{\mu}_{in}) \leq \tilde{K}_\lambda(\tilde{\mu}_{in})$  to complete the proof. The previous observation also implies that  $k_\lambda(\tilde{\mu}_{in}) \leq \tilde{K}_\lambda(\tilde{\mu}_{in})$  so the claim follows. Q.E.D.

*Proof of Theorem 2.* We first prove the monotonicity of downplaying and exaggerating probabilities in  $\gamma$ . Because  $\gamma = 2\sigma\sqrt{3}$ , we also obtain the same monotonicity behavior in  $\sigma$ .

Note that the optimal downplaying probability, see (17), is increasing in  $\mu_\lambda^{\text{DP}}$  when  $\lambda_1 < \lambda < \lambda_2$ , and exaggerating probability, see (18), is decreasing  $\mu_\lambda^{\text{EX}}$  when  $\lambda_3 < \lambda < \lambda_4$ . Thus, we focus on the derivatives of  $\mu_\lambda^{\text{DP}}$  and  $\mu_\lambda^{\text{EX}}$  with respect to  $\gamma$ .

The derivative of  $\mu_\lambda^{\text{DP}}$  with respect to  $\gamma$  is given by

$$\frac{\gamma^2(5\lambda - 4) \left[ -8(1 - \lambda) \frac{(\lambda - \frac{1}{2}) \kappa^2 \omega_h^2}{(5\lambda - 4) \gamma^2} + 8 \left( \lambda - \frac{1}{2} \right) \frac{\kappa \omega_h}{\gamma} + \lambda \right]}{16 \left( \kappa \omega_h \left( \lambda - \frac{1}{2} \right) + \frac{\gamma \lambda}{4} \right)^2 \kappa}. \quad (50)$$

Since  $\lambda$  is inside  $(4/5, 1]$  when downplaying, the derivative is positive if and only if the term inside square brackets is positive. Thus, we focus on that term.

$$-8(1 - \lambda) \frac{(\lambda - \frac{1}{2}) \kappa^2 \omega_h^2}{(5\lambda - 4) \gamma^2} + 8 \left( \lambda - \frac{1}{2} \right) \frac{\kappa \omega_h}{\gamma} + \lambda = 8 \left( \lambda - \frac{1}{2} \right) \frac{\kappa \omega_h}{\gamma} \left( 1 - \frac{(1 - \lambda) \kappa \omega_h}{(5\lambda - 4) \gamma} \right) + \lambda \quad (51)$$

Next, we show that  $\left( 1 - \frac{(1 - \lambda) \kappa \omega_h}{(5\lambda - 4) \gamma} \right)$  is positive because the remaining terms are positive. To do so, we use the upper bound  $G_1(\lambda)$  on  $\kappa/\gamma$ . In particular, we prove that  $\frac{5\lambda - 4}{1 - \lambda} \geq \omega_h G_1(\lambda)$ .

First note that

$$\begin{aligned} & \frac{5\lambda - 4}{1 - \lambda} - \omega_h G_1(\lambda) \\ &= \frac{5\lambda - 4}{1 - \lambda} - \sqrt{\frac{(5\lambda - 4)\omega_h}{2(2\lambda - 1)\omega_\ell} + \left( \frac{2(1 - \lambda)\omega_h + \lambda\omega_\ell}{4(2\lambda - 1)\omega_\ell} \right)^2} + \frac{2(1 - \lambda)\omega_h + \lambda\omega_\ell}{4(2\lambda - 1)\omega_\ell}. \end{aligned} \quad (52)$$

In the following, we show that the sum of positive terms above is larger than the negative term.

$$\left[ \frac{5\lambda - 4}{1 - \lambda} + \frac{2(1 - \lambda)\omega_h + \lambda\omega_\ell}{4(2\lambda - 1)\omega_\ell} \right]^2 - \frac{(5\lambda - 4)\omega_h}{2(2\lambda - 1)\omega_\ell} - \left( \frac{2(1 - \lambda)\omega_h + \lambda\omega_\ell}{4(2\lambda - 1)\omega_\ell} \right)^2 \quad (53)$$

$$= \frac{(5\lambda - 4)\omega_h}{2(2\lambda - 1)\omega_\ell} \left\{ \frac{2(2\lambda - 1)\omega_\ell}{(1 - \lambda)\omega_h} \left[ \frac{5\lambda - 4}{1 - \lambda} + \frac{2(1 - \lambda)\omega_h + \lambda\omega_\ell}{2(2\lambda - 1)\omega_\ell} \right] - 1 \right\} \quad (54)$$

$$= \frac{(5\lambda - 4)\omega_h}{2(2\lambda - 1)\omega_\ell} \left\{ \frac{2(2\lambda - 1)\omega_\ell(5\lambda - 4)}{\omega_h(1 - \lambda)^2} + \frac{\lambda\omega_\ell}{\omega_h(1 - \lambda)} + 1 \right\} > 0 \quad (55)$$

The last inequality follows because all terms are positive. Combining these, we conclude that  $\mu_\lambda^{\text{DP}}$  is increasing in  $\gamma$ .

The derivative of  $\mu_\lambda^{\text{EX}}$  with respect to  $\gamma$  is given by

$$\frac{\gamma^2(4 - 5\lambda) \left[ 8(1 - \lambda) \frac{\kappa^2\omega_\ell^2 \left(\frac{1}{2} - \lambda\right)}{\gamma^2(4 - 5\lambda)} + 8 \frac{\kappa\omega_\ell}{\gamma} \left(\frac{1}{2} - \lambda\right) - \lambda \right]}{16 \left( \kappa\omega_\ell \left(\lambda - \frac{1}{2}\right) + \frac{\gamma\lambda}{4} \right)^2 \kappa}. \quad (56)$$

Recall that  $\lambda$  is inside  $[0, 1/2)$  when exaggerating, so the derivative is positive if and only if the term inside square brackets is positive. Hence, we focus on that term.

$$8(1 - \lambda) \frac{\kappa^2\omega_\ell^2 \left(\frac{1}{2} - \lambda\right)}{\gamma^2(4 - 5\lambda)} + 8 \frac{\kappa\omega_\ell}{\gamma} \left(\frac{1}{2} - \lambda\right) - \lambda = 8 \frac{\kappa\omega_\ell}{\gamma} \left(\frac{1}{2} - \lambda\right) \left( \frac{\kappa\omega_\ell(1 - \lambda)}{\gamma(4 - 5\lambda)} + 1 \right) - \lambda \quad (57)$$

Here, note that  $\left( \frac{\kappa\omega_\ell(1 - \lambda)}{\gamma(4 - 5\lambda)} + 1 \right)$  is larger than 1, thus we next show that  $8 \frac{\kappa\omega_\ell}{\gamma} (1/2 - \lambda) \geq \lambda$  to complete the proof. To do so, we use the lower bound on  $\kappa/\gamma$ .

$$\frac{\kappa}{\gamma} 8\omega_\ell(1/2 - \lambda) \geq G_2(\lambda) 8\omega_\ell(1/2 - \lambda) = \lambda + \frac{2(1 - \lambda)\omega_\ell}{\omega_h} \geq \lambda. \quad (58)$$

Thus, we conclude that  $\mu_\lambda^{\text{EX}}$  is increasing in  $\gamma$ .

The remaining part of the theorem follows because functions  $G_2$  and  $G_\ell$  on  $[0, 1/2)$  and  $G_1, G_h$  on  $(4/5, 1]$  are increasing functions and  $\kappa/\gamma$  is a decreasing function of  $\sigma$ . Figure 3 provides an illustration for this result. Q.E.D.

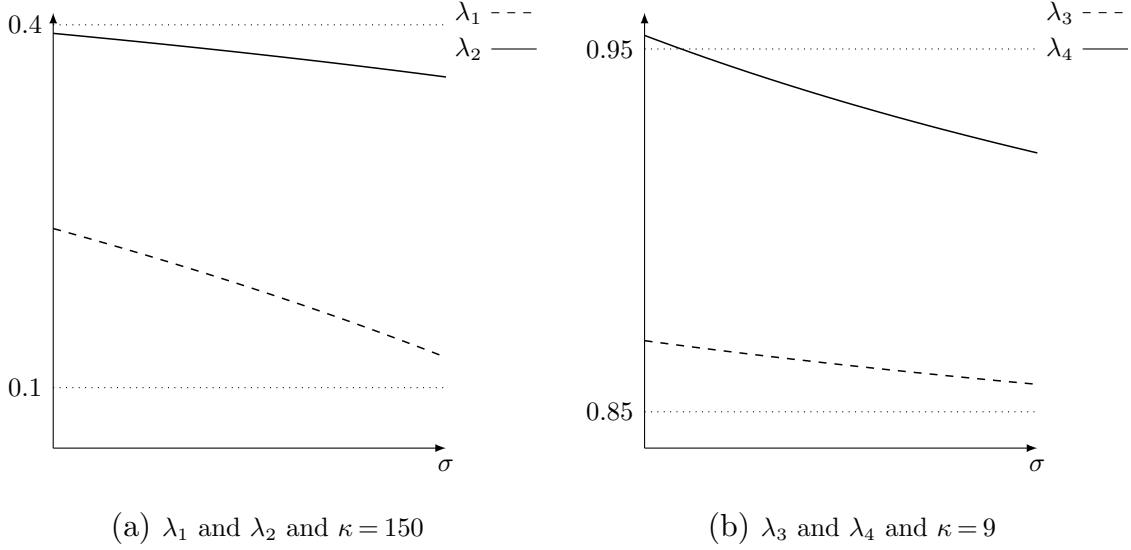
## B Belief Update using Bayes' Rule

The prior belief of the population is  $\rho^\circ$ , and suppose that the sender commits to an information disclosure policy  $\Gamma = (\pi, \mathcal{M})$ . After receiving message  $m$ , receivers update their beliefs from  $\rho^\circ$  to  $\rho_m$  according Bayes' rule as follows.

$$\rho_m \triangleq \mathbb{P}(\omega = \omega_h | m) = \frac{\pi(m | \omega_h)\rho^\circ}{\pi(m | \omega_h)\rho^\circ + \pi(m | \omega_\ell)(1 - \rho^\circ)}. \quad (59)$$

Accordingly, the posterior mean corresponding to the posterior belief is

$$\mu_m \triangleq \rho_m\omega_h + (1 - \rho_m)\omega_\ell. \quad (60)$$



**Figure 3** Threshold functions  $\lambda_i$ ,  $i = 1, \dots, 4$  as a function of  $\sigma$  for  $\omega_\ell = 0.3$ ,  $\omega_h = 0.8$  and  $\theta = 10$ .

## C Technical Intuition of Our Results

In this section of the appendix, we provide further intuition regarding Theorem 1. To do so, we introduce the notation  $c^*(\mu\kappa)$  such that  $c^*(\mu\kappa) = c^*(\mu)$  in Proposition 1 with some abuse of notation.

As discussed in Section 3, the shape of  $K_\lambda(c^*(\mu\kappa))$  is jointly determined by that of  $K_i(\cdot)$  ( $i \in \{e, h\}$ ) and that of the threshold  $c^*(\mu\kappa)$ , which we recall is an increasing concave function of  $\mu$  due to negative externalities among the receivers (as implied by Proposition 1). Therefore, we next explore the shape of  $K_i(\cdot)$  ( $i \in \{e, h\}$ ).

**PROPOSITION 2.** *For any  $x \in \mathcal{C}$ , the function  $K_e(x)$  is increasing convex, whereas the function  $K_h(x)$  is concave and unimodal with the peak point at  $x = (\theta + \sigma\sqrt{3})/2$ .*

*Proof of Proposition 2.* We first derive the expressions for  $K_i(x)$  for  $i \in \{e, h\}$  for uniform distribution with mean  $\theta$  and support  $\mathcal{C}$ . Note that we assume  $x \in \mathcal{C}$ .

$$K_e(x) = \mathbb{E}_c[1_{\{c \leq x\}}c] = \int_{\theta - \gamma/2}^x \frac{x}{\gamma} dx = \frac{1}{2\gamma} \left[ (x)^2 - \left( \theta - \frac{\gamma}{2} \right)^2 \right] \quad (61)$$

$$K_h(x) = x(1 - F(x)) = x \left( 1 - \frac{x - \theta + \gamma/2}{\gamma} \right) \quad (62)$$

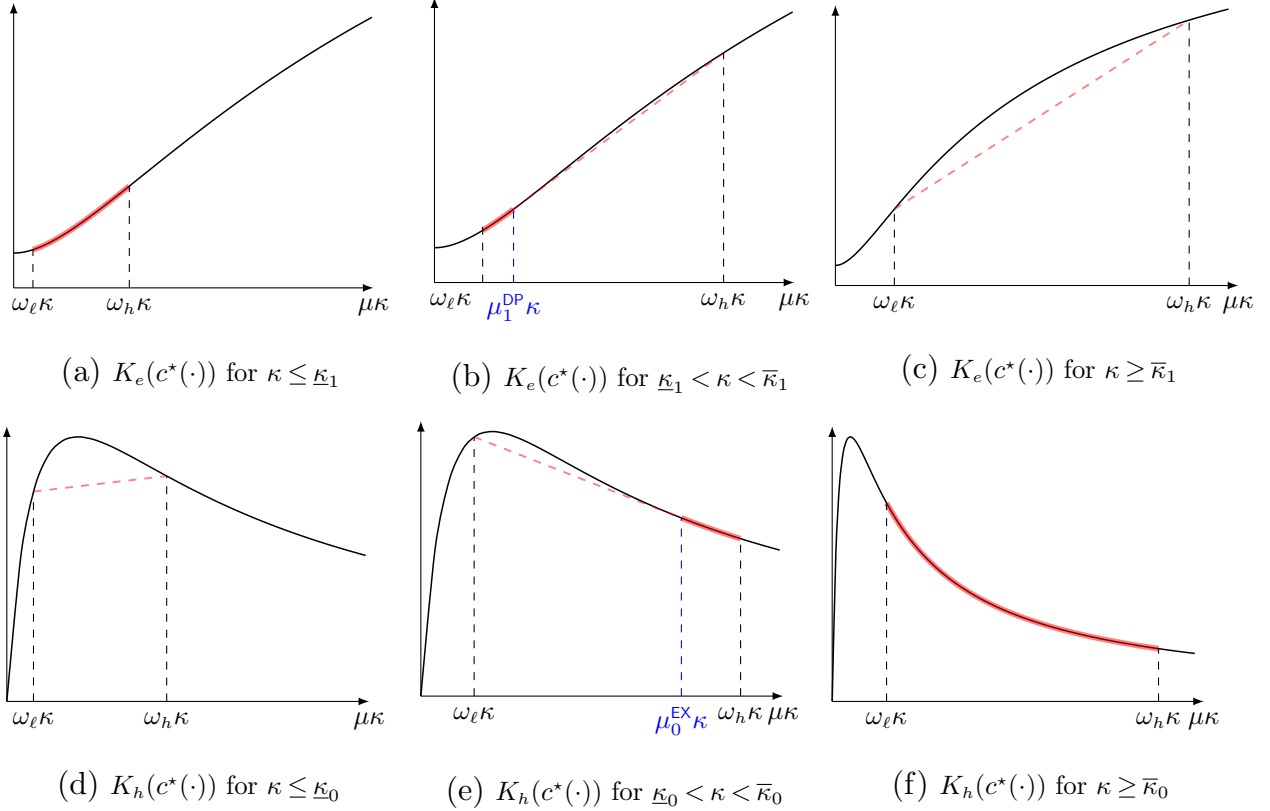
Using these, we get  $K'_e(x) = x/\gamma$  and  $K''_e(x) = 1/\gamma$  and prove the first bullet point. Furthermore, it follows that  $K'_h(x) = [(\theta + \gamma/2) - 2x]/\gamma$  and  $K''_h(x) = -2/\gamma$ . Because the second derivative is negative, it follows that  $K_h(x)$  is concave. Since  $K'_h(x) \geq 0$  when  $x \leq (\theta + \gamma/2)/2$ , and  $K'_h(x) \leq 0$  when  $x \geq (\theta + \gamma/2)/2$ , the result follows. Q.E.D.

Thus, as equilibrium threshold  $c^*$  increases, more individuals incurring increasingly higher economic cost remain in confinement. Hence, the overall economic cost increases at an increasing rate (i.e.,  $K_e(\cdot)$  is increasing convex). In contrast, total healthcare cost  $K_h(\cdot)$  first increases as  $c^*$  increases, because the expected healthcare cost per individual is given by  $\mu\kappa P_{a^*,\mu} = c^*$ . However, as threshold  $c^*$  keeps increasing, fewer individuals choose to engage in social interaction, eventually lowering the total healthcare cost (i.e.,  $K_h(\cdot)$  then decreases).

As the optimal policy is obtained through the lower convex envelope of  $K_\lambda(c^*(\mu\kappa))$  as a function of  $\mu$ , the nature of the optimal information policy characterized above is fundamentally driven by the second-order behavior of function  $K_\lambda(c^*(\cdot))$ . Specifically, as illustrated by Figure 4,  $K_e(c_\gamma^*(\cdot))$  is first convex and then concave, whereas  $K_h(c_\gamma^*(\cdot))$  is the opposite, i.e, first concave and then convex. The domain of these functions are  $[\omega_\ell\kappa, \omega_h\kappa]$  because  $\mu \in [\omega_\ell, \omega_h]$ . In effect,  $\kappa$  acts to control the active domain of function  $K_\lambda(c^*(\cdot))$ , over which the lower convex envelope will be constructed.

Take the economy-biased government ( $\lambda = 1$ ) as an example. When the healthcare cost is low ( $\kappa \leq \underline{\kappa}_1$ ), function  $K_e(c^*(\cdot))$  is entirely convex over  $[\omega_\ell\kappa, \omega_h\kappa]$  and hence its lower convex envelope is itself (see Figure 4a), suggesting that the optimal martingale split of  $\mu^\circ$  is simply not to split and hence no disclosure is optimal. When the healthcare cost moves into the intermediate range ( $\underline{\kappa}_1 < \kappa < \bar{\kappa}_1$ ), the inflection point of function  $K_\lambda(c^*(\cdot))$  falls within  $[\omega_\ell\kappa, \omega_h\kappa]$  and hence its lower convex envelope consists of itself over  $[\omega_\ell\kappa, \mu_1^{\text{DP}}\kappa]$  and the straight line connecting  $(\mu_1^{\text{DP}}\kappa, K_e(c^*(\mu_1^{\text{DP}}\kappa)))$  and  $(\omega_h\kappa, K_e(c^*(\omega_h\kappa)))$  (see Figure 4b), where the straight line and function  $K_e(c^*(\cdot))$  are tangent with each other at  $(\mu_1^{\text{DP}}\kappa, K_e(c^*(\mu_1^{\text{DP}}\kappa)))$  (see (13) for the closed-form expression of  $\mu_1^{\text{DP}}$ ). Thus, as in the previous case, no disclosure is optimal if  $\mu^\circ \leq \mu_1^{\text{DP}}$ ; otherwise, the optimal martingale split of  $\mu^\circ$  would induce posterior beliefs at  $\mu_1^{\text{DP}}$  and  $\omega_h$ , which can be implemented by downplaying (see (17) in for exact expression of  $\pi^*(m_\ell | \omega_h)$ ). When the healthcare cost is sufficiently large ( $\kappa \geq \bar{\kappa}_1$ ), function  $K_e(c^*(\cdot))$  is entirely concave over  $[\omega_\ell\kappa, \omega_h\kappa]$  and hence its lower convex envelope is the straight line connecting  $(\omega_\ell\kappa, K_e(c_\gamma^*(\omega_\ell\kappa)))$  and  $(\omega_h\kappa, K_e(c_\gamma^*(\omega_h\kappa)))$  (see Figure 4c), suggesting that the optimal martingale split of  $\mu^\circ$  would induce posterior beliefs at  $\omega_\ell$  and  $\omega_h$  and hence full disclosure is optimal.

Similarly, the healthcare-biased government's optimal policy can be constructed from the lower convex envelope of  $K_h(c^*(\cdot))$  (see Figures 4d-4f), albeit in the direction opposite



**Figure 4** Convexification of  $K_\lambda(c^*(\mu\kappa))$  as a function of  $\mu$  for  $\lambda \in \{1, 0\}$ ,  $\omega_\ell = 0.15$ ,  $\omega_h = 0.9$ ,  $\gamma = 20$  and  $\theta = 10$ . Solid red lines represent that  $K_\lambda(c^*(\mu\kappa))$  coincides with its lower convex envelope, whereas dashed red lines represent  $K_\lambda(c^*(\mu\kappa))$  is strictly above its lower convex envelope.

to that of the economy-biased policy. This is because  $K_h(c^*(\cdot))$  demonstrates the second-order behavior in contrast to that of  $K_e(c^*(\cdot))$ . Therefore, when the government needs to make a trade off between the economic and healthcare costs by minimizing  $K_\lambda(c^*(\mu\kappa))$  for some  $\lambda \in [0, 1]$ , the opposing behaviors of  $K_e(c^*(\cdot))$  and  $K_h(c^*(\cdot))$  may offset each other and subsequently damp down the information distortion in the optimal policy.



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