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Institut für Volkswirtschaftslehre

Universität Augsburg

## Volkswirtschaftliche Diskussionsreihe

**Managing Credit Risk  
with Credit and Macro Derivatives**

**Udo Broll  
Gerhard Schweimayer  
Peter Welzel**

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# Managing Credit Risk with Credit and Macro Derivatives

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## Abstract

The industrial organization approach to the microeconomics of banking augmented by uncertainty and risk aversion is used to examine credit derivatives and macro derivatives as instruments to hedge credit risk for a large commercial bank. In a partial-analytic framework we distinguish between the probability of default and the loss given default, model different forms of derivatives, and derive hedge rules and strong and weak separation properties between deposit and loan decisions on the one hand and hedging decisions on the other. We also suggest how bank-specific macro derivatives could be designed from common macro indices which serve as underlyings of recently introduced financial products.

*Keywords:* banking, credit risk, systematic risk, credit derivative, macro derivative

*JEL classification:* G21

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# Managing Credit Risk with Credit and Macro Derivatives

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The industrial organization approach to the microeconomics of banking augmented by uncertainty and risk aversion is used to examine credit derivatives and macro derivatives as instruments to hedge credit risk for a large commercial bank. In a partial-analytic framework we distinguish between the probability of default and the loss given default, model different forms of derivatives, and derive hedge rules and strong and weak separation properties between deposit and loan decisions on the one hand and hedging decisions on the other. We also suggest how bank-specific macro derivatives could be designed from common macro indices which serve as underlyings of recently introduced financial products.

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## 1 Introduction

Credit risk can be considered the oldest and most significant form of risk faced by a commercial bank in its business of taking deposits and giving loans. Large scale borrower default carries the potential to force a bank into bankruptcy. Managing credit risk therefore has always been one of the prime challenges in running a bank. For decades this was mainly achieved through selecting and monitoring borrowers and through creating a well-diversified loan portfolio. More recently, new financial instruments and risk sharing markets have evolved (see Neal 1996, Bank for International Settlements 2001), and banks and specialized consulting firms alike put a lot of effort into developing sophisticated models (carrying fancy names such as Credit Metrics, Credit Monitor, CreditRisk+, or CreditSmartRisk) for measuring credit risk (for a survey on credit risk measurement see Altman and Saunders 1997). In particular, markets for credit derivatives virtually exploded during the 1990s. Data collected by the Bank for International Settlements which are probably broadest in coverage and have been corrected for double-counting indicate an increase from \$ 118 billion in mid-1998 to \$ 693 billion in mid-2001 (see Jeanneau 2002, p. 38).

The British Bankers' Association in its Credit Derivatives Report 2002 predicted an increase of about 400% from end of 2001 to end of 2004. In this London-dominated market so-called credit default swaps (CDS) are the most popular financial contracts traded, capturing nearly half of the market (British Bankers' Association 2002). Such a CDS is a unique contract based on a specific reference credit or a pool of credits between a purchaser and a seller of protection against losses from a credit event (default or other event). The buyer of protection removes credit exposure while retaining ownership of the asset and paying a premium or fee to the seller who in case of the credit event usually makes a cash payment to the seller. Following the International Swaps and Derivatives Association's (ISDA) conventions, credit events are typically defined as one or more of the following: bankruptcy, failure to pay, restructuring, repudiation, moratorium, obligation default, or obligation acceleration.

Against the background of this recent surge in the use of credit derivatives as financial instruments to transfer credit risk from lenders to third parties, we ask how the availability of such derivatives affects a bank's decisions concerning interest rates or loan and deposit volumes and what can be said about the optimal level of insurance against this type of risk. Before addressing these questions, we observe that it is by no means self-evident that credit risk can easily be transferred. The informational advantage of the risk seller from its existing relationship with the debtor creates a barrier to this kind of transaction. Furthermore, in the parlance of capital market theory "credit risk has both an idiosyncratic and a systematic component" (Loubergé and Schlesinger 2002, p. 1), i.e., there is risk of creditor default originating from the creditor herself and from factors unrelated to the creditor. Systematic risk of a loan may arise, for example, from the business cycle or from general political instability. According to Wilson (1998) only a small number of macroeconomic factors are sufficient to explain this type of risk which determines most part of all risk related to a loan contract. From the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT) we have an understanding that systematic risk can not be diversified away, but is tradeable, whereas unsystematic risk ought to be eliminated through diversification. Regional, sectoral or institutional constraints, however, may get in the way of this diversification of a bank's loan portfolio which will be far from perfect for, say, a bank focused on car or real estate financing, or a German savings or cooperative bank which by its statutes is confined to a local market. Furthermore, the information asymmetry mentioned above makes substituting risk diversification by selling risk difficult. Seen from this angle, a credit default swap looks like a rather crude instrument to reduce a bank's exposure to credit risk. Macro derivatives (sometimes also called economic derivatives) as an even more recent innovation in financial markets carry the potential to improve matters. They enable a bank to sell the systematic, tradeable part of credit risk while retaining the specific part which lies, due to informational aspects, in the core competence of the bank (for macro derivatives see Marshall et al. 1992, Topping 2001, Schweimayer

2003). Deutsche Bank and Goldman Sachs, for example, in October 2002 started to auction off derivatives on an U.S. retail sales index, a manufacturing index, and on the change in U.S. non-farm payrolls (The Wall Street Journal 2002).

In this paper we use the industrial economics approach to the microeconomics of banking to analyze the management of credit risk for the case of a large bank active in the deposit and loan markets. While this approach does not explicitly account for informational problems, our analysis of credit and macro derivatives which do not exactly offset the bank's credit risk nevertheless captures in a stylized way features of optimal bank behavior under asymmetric information. For example, the combination of uncertainty and asymmetric information suggests that the bank retain some of the risk in order to give an incentive for proper monitoring. This retention of risk will be included in all but one of our analyses. We follow the lead of Wong (1997) and supplement the industrial economics approach by risk aversion and uncertainty, more specifically credit risk in our case. As for the assumption of risk aversion and the need of active corporate risk management we refer our readers to the seminal work of Froot et al. (1993) and Froot and Stein (1998). Pausch and Welzel (2002) provide an application to the banking industry, showing that even a per se risk neutral bank exhibits risk averse behavior, if there is capital adequacy regulation. Our two main objectives are the following: First, we want to help close the gap between theoretical analysis and practice of credit derivatives as hedging instruments. We explicitly model a credit default swap as the main instrument traded and show how the design of the derivative affects its hedge properties and optimal bank decisions. Second, we split up credit risk into a systematic and a specific part and introduce the notion of a macro derivative used to hedge against systematic risk. Both the credit default swap and the macro derivative extend the scope of banking from a mere buy-and-hold strategy to active management of credit risk.

The plan of the paper is as follows: In section 2 we present the basic model which we use for an analysis of a credit default swap in section 3. In the section 4 we introduce a macro derivative as an alternative instrument to hedge against credit risk. Section 5 concludes.

## 2 The Model

In the framework of the industrial economics approach to the microeconomics of banking (see e.g. Freixas and Rochet 1997, chpt. 3) we consider a large bank in a one-period framework taking deposits  $D$  and giving loans  $L$ . By "large" we mean a bank with influence on interest rates both in the deposit and the loan market. Since we are not interested in the strategic interactions between banks, we focus on a monopolistic bank. As further motivation recall the estimations by Neven

and Rölller (1999) of a structural model of the type used below which indicate for seven European banking industries that the hypothesis of non-cooperative Nash competition can be rejected and actual competition is closer to the collusive, i.e., monopolistic, type. Let the interest rate on loans  $r_L(L)$  be negatively related to the loan volume  $L$ , i.e.,  $dr_L(L)/dL = r'_L < 0$ . In analogy to this assumption of normal demand behavior we also assume normal supply conditions implying a positive relationship between the deposit rate  $r_D(D)$  and the volume of deposits  $D$ , i.e.,  $dr_D(D)/dD = r'_D > 0$ . The bank also faces operational costs  $C(D, L)$  with strictly positive marginal costs  $C'_D$  and  $C'_L$ .

To our knowledge Wong (1997) was the first author to introduce uncertainty and risk aversion to this framework. We deviate from his analysis of credit risk by using an endogenously determined non-stochastic deposit rate and adding hedge instruments — a credit derivative or a macro derivative — to the model. A potential influence from credit risk through a rating of the bank on the cost of refinancing is neglected, as is a potential direct link between the bank's decision on a loan and the volume of deposits held by the loan applicant (for this kind of link see Chiappori et al. 1995).

Loans are subject to credit risk which is modelled by a random variable  $\tilde{\theta} \in [0, 1]$ . We want to emphasize the two components of credit risk: the default event  $\tilde{\delta} \in \{0, 1\}$  itself and the severity of default, the so-called loss given default (LGD)  $\tilde{\lambda} = (\tilde{\theta} \mid \tilde{\delta} = 1) \in (0, 1]$ . More precisely,

$$\tilde{\theta} = \tilde{\lambda} \cdot \tilde{\delta} \in [0, 1]. \quad (1)$$

Given a credit default, the bank loses a share  $\tilde{\lambda}$  of the payment  $(1 + r_L)L$  from its debtors due at the end of the period. Notice that we use as an implicit, but important, simplifying assumption that credit risk  $\tilde{\theta}$  is not affected by the level of interest rates  $r_L$  and  $r_D$  which result from the bank's decisions.

To keep the notation as simple as possible, we ignore a voluntary or mandatory holding of reserves on deposits, as we ignore regulatory capital requirements for loans.<sup>1</sup> The bank has a given equity capital  $K$ . The positive or negative balance  $M$  of capital available, deposits taken, and loans given is invested in or financed from an interbank market at a given deterministic interest rate  $r$ .<sup>2</sup> We abstract from a market risk in the bank's refinancing operations. This can be summarized in the balance sheet constraint

$$M + L = K + D. \quad (2)$$

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<sup>1</sup>For an analysis of capital requirements issues in the framework of the industrial economics approach with credit risk see Pausch and Welzel (2002).

<sup>2</sup>Think for example of banks being local monopolists meeting only in a competitive interbank market. Other interpretations for this given opportunity rate  $r$  found in the literature refer to a central bank giving and taking money at this rate or to international capital markets offering this rate.

Denoting the uncertain end-of-period profit by  $\tilde{\Pi}$ , the bank's profit function is given by

$$\tilde{\Pi} = (1 - \tilde{\theta})r_L(L)L - \tilde{\theta}L + rM - r_D(D)D - C(D, L). \quad (3)$$

Substituting for  $M$  from (2) and rearranging terms yields

$$\tilde{\Pi} = \left( (1 - \tilde{\theta})r_L(L) - r \right) L - \tilde{\theta}L + rK + (r - r_D(D))D - C(D, L), \quad (4)$$

where the first term includes the interest margin from the loan business corrected for credit default, the second is the loss of principal due to default, the third term is interest income from investing equity capital at the opportunity interest rate, the fourth includes the interest margin from the deposit business, and the fifth accounts for operating costs. The risk averse bank management maximizes a von Neumann-Morgenstern utility function  $U(\Pi)$  with  $U' > 0$  and  $U'' < 0$ .

### 3 Credit Derivatives

As noted in the introduction, credit derivatives have become increasingly popular to hedge against credit risk. As credit default swaps are by far the most widely used and liquid instrument, we will focus in this section on a stylized representation of such a CDS. We proceed in three steps: Initially a credit default swap is analyzed which pays the (ex ante) random amount  $\tilde{\lambda}$  per (marginal) contract in case of default, i.e., if  $\tilde{\delta} = 1$  holds at the end of the period. This is analyzed under the assumption of no basis risk which amounts to assuming a perfect negative correlation between credit risk and the credit default swap (3.1). In a second step we relax this assumption and examine the case of a CDS which carries so-called basis risk because its underlying is not perfectly correlated with the bank's credit risk (3.2). Finally, we examine the case of a CDS in which the random payment is replaced by a conditional but contractually pre-determined payment  $\lambda H$ , more specifically,  $\tilde{\lambda} \equiv \lambda$  (3.3). It will turn out that the ability of a bank to hedge its credit risk crucially depends on the specifics of the CDS at hand.

#### 3.1 Hedging without basis risk

Under a typical credit default swap the protection buyer pays a premium to the protection seller and receives the difference between face value and recovery value, if the credit event defined by the contract occurs (Prato 2002). In our first step, we model this in the following way: The bank as seller of credit risk and buyer of protection pays a given premium  $\bar{\theta}$  for one unit of credit risk. By selling a volume  $H$  the bank makes a deterministic payment  $\bar{\theta}H$  in exchange for a stochastic claim



$\tilde{\theta}H$  at the end of the period.<sup>3</sup> This stochastic claim offsets the loss due to credit default to an extent which is controlled by the decision variable  $H$ . Notice that this interpretation implicitly presumes rational bank behavior to lead to  $H > 0$  which will be shown to be true later. The hedge operation described contributes  $(\tilde{\theta} - \bar{\theta})H$  to the bank's profit. We therefore modify our profit definition (4) to arrive at

$$\tilde{\Pi} = \left( (1 - \tilde{\theta})r_L(L) - r \right) L - \tilde{\theta}L + rK + (r - r_D(D))D + (\tilde{\theta} - \bar{\theta})H - C(D, L). \quad (5)$$

The definition in (5) is a correct representation of the profit because credit default swaps by law do not enter the balance sheet and therefore do not affect the balance constraint (2). The decision problem of the bank management can then be described in terms of the expected utility maximization problem

$$\max_{D, L, H} \mathbb{E} \left[ U(\tilde{\Pi}) \right]. \quad (6)$$

Since the bank is modelled as a monopolist both in the deposit and the loan markets, quantity setting is equivalent to setting interest rates in these two markets, and we therefore formulate the whole problem in terms of quantities.

To simplify our exposition we suppress the optimal value arguments of the functions  $r_D$ ,  $r_L$  and  $C$  in the sequel. (6) leads to first-order necessary conditions with respect to  $D$ ,  $L$  and  $H$ :

$$\mathbb{E} \left[ U'(\tilde{\Pi}^*) \left( (r - r_D) - r'_D D^* - C'_D \right) \right] = 0 \quad (7)$$

$$\mathbb{E} \left[ U'(\tilde{\Pi}^*) \left( (1 - \tilde{\theta})r_L - r + (1 - \tilde{\theta})r'_L L^* - \tilde{\theta} - C'_L \right) \right] = 0 \quad (8)$$

$$\mathbb{E} \left[ U'(\tilde{\Pi}^*) (\tilde{\theta} - \bar{\theta}) \right] = 0 \quad (9)$$

Inspection of (7), (8) and (9) leads to the following

**Proposition 1** *Given a credit derivative with perfect negative correlation with the bank's exposure to credit risk, (a) the bank can separate its decision on credit risk management from its decisions on deposit and loan volumes, (b) the bank fully hedges its credit risk exposure, if the market for the credit default swap used as hedge instrument is unbiased.*

**Proof** (a) Substituting  $\mathbb{E}[U'(\cdot)\bar{\theta}]$  for  $\mathbb{E}[U'(\cdot)\tilde{\theta}]$  from (9) into (8), rearranging terms and dividing by  $\mathbb{E}[U'(\cdot)]$  in both (7) and (8) yields two deterministic equations in  $D$  and  $L$  which can be solved for the optimal values  $D^*$  and  $L^*$ :

$$r = r_D + r'_D D^* + C'_D \quad (10)$$

$$(1 - \bar{\theta})(r_L + r'_L L^*) - \bar{\theta} = r + C'_L \quad (11)$$

As the system (10)–(11) is totally independent from  $H^*$  and contains no stochastic term, the “production” decisions on deposit and loan volumes can be separated from

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<sup>3</sup>To keep the model as simple as possible we assume that the premium is paid at the end of the period, so there is not need for discounting.

the risk management decision on the hedging volume.

(b) Using the covariance decomposition  $\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$ , we can write (9) in the form

$$\frac{\text{Cov}[U'(\cdot), \tilde{\theta}]}{E[U'(\cdot)]} + E(\tilde{\theta}) = \bar{\theta}. \quad (12)$$

If the market for the CDS is unbiased, i.e.,  $E(\tilde{\lambda} \cdot \tilde{\delta}) = E(\tilde{\theta}) = \bar{\theta}$ , then (12) is equivalent to  $\text{Cov}[U'(\tilde{\Pi}^*), \tilde{\theta}] = 0$ , which can only be true for a deterministic bank profit  $\Pi^*$ . This in turn implies that the bank has no exposure to risk. From (5) we conclude that  $H^* = r_L L^* + L^*$ , i.e., the bank in the optimum completely hedges its risky position in the loan market, it uses a full hedge. q.e.d.

Part (a) of Proposition 1 is an example for the well-known (Fisher) separation property in the presence of a hedging instrument without basis risk which is familiar from analyses of optimal firm behavior under uncertainty. As a consequence the bank will choose the same volumes of deposits and loans under uncertainty as in the case of a deterministic default rate at level  $\bar{\theta} = E\tilde{\theta}$  (certainty equivalence). In contrast to a weaker notion of separation to be derived later in our paper, we call this strong separation. Our analysis includes as a special case the situation with a deterministic default rate  $E(\tilde{\theta})$ . First-order conditions for this case are given by (7) for  $D^*$  (after dividing by  $E(U(\cdot))$ ), and  $(1 - E(\tilde{\theta}))(r_L + r'_L L^*) - E(\tilde{\theta}) = r + C'_L$  for  $L^*$ . These are the familiar conditions of equality between marginal revenues and marginal costs in the deposit and loan markets. A stochastic default rate affects only the latter optimality condition by an additive term  $\text{Cov}[U'(\cdot), \tilde{\theta}] \cdot (r_L + r'_L L^* + 1) / E[U'(\cdot)]$  on the righthand side. Due to  $U''$  the sign of this covariance is positive. Closer inspection shows that under increasing marginal costs and in the absence of an instrument to hedge credit risk uncertainty about the default rate unambiguously implies a lower loan volume whereas the impact on the deposit volume depends on the cross derivative  $C''_{DL}$ , i.e., on economies or diseconomies of scope between taking deposits and giving loans. Returning briefly to our separation result we notice that the optimality conditions (10) and (11) contain the familiar Lerner Index (price minus marginal cost divided by price) formulation (cf. Freixas and Rochet 1997, p. 58)

$$\frac{r - r_D - C'_D}{r_D} = \frac{1}{\epsilon_D}, \quad (13)$$

$$\frac{(1 - \bar{\theta})r_L - \bar{\theta} - r - C'_L}{(1 - \bar{\theta})r_L} = \frac{1}{\epsilon_L}, \quad (14)$$

where  $\epsilon_D = (dD/dr_D)(r_D/D)$  and  $\epsilon_L = -(dL/dr_L)(r_L/L)$  are the elasticities of deposit supply and loan demand, respectively, and the term  $-\bar{\theta}$  on the lefthand side of (14) accounts for the marginal loss of principal in case of loan default.

The assumption of unbiasedness of the hedging market which we use in part (b) of Proposition 1 clearly is a simplifying one, defining a benchmark case of a derivatives market where buyers of risk are a large number of well-diversified investors.

Introducing a constant risk premium would lead to an optimal hedge below the full hedge, without substantially changing our analysis. The more challenging task of endogenizing the price of the credit derivative, i.e., explicitly including the derivatives market in the model, is beyond the scope of this paper. However, this is an issue which ought to be addressed in future research because market data shows that banks which we consider sellers of risk also play a major role on the buying side of the derivatives market.

### 3.2 Hedging with basis risk

In our introduction we mentioned already that a complete transfer of credit risk from the bank as initial lender to a third party is hard to imagine. We capture the notion of non-tradeable risk by introducing so-called basis risk into our model. The most important causes of basis risk discussed in the literature are differences in the maturities of the hedging instrument and the bank's risky position, and differences in the stochastic properties between the underlying of the hedging instrument and the risk the bank faces. In the case of credit risk the first problem could arise when the derivatives contract matures at an earlier date than the underlying loan contract. This, however, is not only beyond the scope of our one-period model, but should also be of relatively minor importance since credit derivatives are traded over the counter, enabling the contracting parties to adequately match maturity dates. We think that the second cause of basis risk is more relevant in our framework. Information asymmetries related to the individual loan contract create an unsystematic component of credit risk, put potential buyers of credit risk at an informational disadvantage and make a complete transfer of credit risk to a third party hard to imagine. This problem is augmented by the fact, that the unsystematic part of credit risk may be hard to diversify for institutional reasons mentioned earlier. In essence this means that a bank will not be able to find a hedging instrument perfectly offsetting the credit risk it currently holds. Our approach to model this in a simple and stylized way is now to add basis risk to our model.

Consider availability of a credit default swap at a given price as before. To model basis risk we introduce the following modification: The market uses no longer the share of non-performing loans  $\tilde{\theta}$ , but a share  $\tilde{g}$  as underlying of the derivatives contract.  $\tilde{g}$  can be interpreted as the portion of loans non-performing due to systematic risk. From this definition it is apparent that the two risks are not necessarily independent. We assume regression dependence between the two random variables (cf. Benninga et al. 1984), i.e.,

$$\tilde{\theta} = b + \beta \tilde{g} + \tilde{s}, \quad (15)$$

where  $b \geq 0$ ,  $\beta > 0$ , and  $\tilde{s}$  is a zero mean noise term stochastically independent from  $\tilde{g}$ . For each unit of the credit derivative sold the bank makes a deterministic payment  $\bar{g}$  in exchange for the stochastic amount  $\tilde{g}$ .

We assume unbiasedness of the derivatives market, i.e.,  $E(\tilde{g}) = \bar{g}$ , with  $\bar{g}$  denoting the market price of the underlying chosen by the contracting parties. This implies  $\tilde{\theta} = \beta\bar{g}$ , where we assume  $b = 0$  without loss of generality.

Suppressing arguments of  $r_D$  and  $r_L$ , the bank's profit can now be re-written as

$$\tilde{\Pi} = \left( (1 - \tilde{\theta})r_L - r \right) L - \tilde{\theta}L + (r - r_D)D + rK - C(D, L) + (\tilde{g} - \bar{g})H. \quad (16)$$

Maximization of expected utility yields (7) and (8) as in the case without basis risk. Condition (9) for the optimal hedge volume, however, is replaced by

$$E \left[ U'(\tilde{\Pi}^*)(\tilde{g} - \bar{g}) \right] = 0. \quad (17)$$

Inspection of the first-order conditions leads us to

**Proposition 2** (a) *In the presence of basis risk in an unbiased CDS market the bank hedges a portion  $\beta$  of its risky position  $r_L L^* + L^*$  (beta-hedge rule). (b) The usual separation property no longer exists. Instead, a weaker notion of separation holds. (c) In the absence of economies or diseconomies of scope, the optimal volume of deposits  $D^*$  can be determined as in the case of certainty.*

**Proof** (a) Unbiasedness of the derivatives market implies that (17) can be written as  $\text{Cov}[U'(\tilde{\Pi}^*), \tilde{g}] = 0$ . Replacing  $\tilde{\Pi}^*$  by (16) and using (15) yields

$$\text{Cov} \left[ U'(-\tilde{s}(r_L L^* + L^*) - \tilde{g}(\beta r_L L^* + \beta L^* - H^*) + \text{const}), \tilde{g} \right] = 0. \quad (18)$$

Due to the stochastic independence of  $\tilde{s}$  and  $\tilde{g}$  this can only be true, if

$$\beta = \frac{H^*}{r_L L^* + L^*}. \quad (19)$$

(b) Inserting (15) and the optimal hedge rule (19) into the first-order condition (8) for loans shows that  $L^*$  still depends on probabilities and risk preferences, even if  $D^*$  were known. This in turn implies from (7) that  $D^*$  also cannot be determined without knowledge of probabilities and risk preferences. More than market data is required to decide the optimal loan and deposit volumes, which prevents the traditional notion of separation of production and risk management. Notice, however, that the optimal hedge rule derived holds for any pair  $(D, L)$ . We can therefore imagine a bank choosing loan and deposit volumes arbitrarily (or on the basis of a simulation) and still minimizing its risk exposure by applying the beta-hedge. While the bank may find it impossible to determine the optimal values of  $D^*$  and  $L^*$  in the presence of basis risk, it can still separate its hedging decision from its production decisions. We call this a weak notion of separation.

(c) Inspection of (7) shows that for  $C''_{DL} = 0$  (neither economies nor diseconomies of scope)  $D^*$  can be determined on the basis of market data alone, i.e., without knowledge of probabilities, risk preferences, or the bank's hedging decision. q.e.d.

### 3.3 Hedging with a fixed pre-determined payment

We now turn to yet another situation where the bank cannot perfectly sell all its credit risk even if it wishes to do so. Consider the case of a fixed pre-determined payment of the insurance seller to the insurance buyer, i.e., the bank, in case of default. This situation, where a deterministic recovery value is included in the CDS contract, making the payment defined as face value minus recovery value in case of default also deterministic, is can be found in real world CDS markets, in particular when risk of individual loans is to be transferred. We denote this payment by the deterministic variable  $\lambda$  which we normalize to 1. This creates no loss of generality because  $H$  can always be adjusted to meet the bank's preferences. After replacing  $\tilde{\theta}$  in (5) by  $\lambda\tilde{\delta} = \tilde{\delta}$  the profit function can be re-written as

$$\tilde{\Pi} = \left( (1 - \tilde{\theta})r_L - r \right) L - \tilde{\theta}L + rK + (r - r_D) D + (\tilde{\delta} - \bar{\theta})H - C(D, L), \quad (20)$$

where  $\bar{\theta}$  again denotes the given price of the (modified) CDS. The optimality condition (9) is replaced by

$$\mathbb{E} \left[ U'(\cdot)(\tilde{\delta} - \bar{\theta}) \right] = 0 \quad \Leftrightarrow \quad (21)$$

$$\frac{\text{Cov} \left[ U'(\cdot), \tilde{\delta} \right]}{\mathbb{E} \left[ U'(\cdot) \right]} + \mathbb{E}(\tilde{\delta}) = \bar{\theta}, \quad (22)$$

where  $\mathbb{E}(\tilde{\delta})$  is equal to the probability of the default event  $\Pr(\tilde{\delta} = 1)$  because  $\tilde{\delta} \in \{0, 1\}$ .

Condition (22) explicitly accounts for the role of the default probability when determining the optimal hedge against credit risk. We are now in a position to prove

**Proposition 3** *An optimal decision  $(D^*, L^*, H^*)$  with a CDS paying a pre-determined fixed amount in the credit event has the following properties:*

(a) *A perfect hedge is not possible, i.e., even with the optimal hedge position in the derivatives market the bank retains credit risk.*

(b) *Even for an unbiased CDS market, there is no simple hedge rule. Neither strong nor weak separation between deposit and loan volumes on the one hand and hedging volume on the other holds. The optimal values of all three decision variables depend on probabilities and risk preferences.*

(c) *If the market for the credit derivative is unbiased, the optimal hedge volume  $H^*$  is (i) in the interval  $(E(\tilde{\lambda})(1 + r_L)L^*, (1 + r_L)L^*]$ , if the utility function has the property  $U''' > 0$  (prudence), (ii) in the interval  $[0, E(\tilde{\lambda})(1 + r_L)L^*)$  for  $U''' < 0$ , and (iii) equal to  $E(\tilde{\lambda})(1 + r_L)L^*$ , if  $U''' = 0$  (quadratic utility function).*

**Proof** (a) It is almost trivial to state that with a fixed amount to be paid to the bank in case of default and a stochastic loss given default there can be no perfect hedge eliminating all risk.

(b) Using unbiasedness ( $E(\tilde{\delta}) = \bar{\theta}$ ) (22) can be simplified to  $\text{Cov}[U'(\tilde{\Pi}^*), \tilde{\delta}] = 0$  which in turn can be written as

$$\text{Cov} \left[ U' \left( \tilde{\delta}H^* - (1 + r_L)\tilde{\lambda}\tilde{\delta}L^* - \bar{\theta}H + \text{const} \right), \tilde{\delta} \right] = 0. \quad (23)$$

Contrary to (18) in the case with basis risk examined before, (23) does not lend itself to the derivation of a simple and general hedge rule. The first-order condition (21) for  $H^*$  can no longer be used to eliminate  $E[U'(\cdot)\tilde{\theta}]$  from (8).  $L^*$  depends on probabilities and risk preferences, and on the optimal value  $H^*$ . This in turn implies that for  $C''_{DL} \neq 0$   $D^*$  also cannot be determined without knowledge of probabilities, risk preferences and  $H^*$ . More than market data is required to decide the optimal deposit, loan and hedging volumes in this fully interdependent system.

(c) For a given  $L^*$  and for arbitrarily fixed  $U'(\cdot)$  we show that one can choose a volume  $H$  such as to obtain  $\text{Cov}(U'(\tilde{\Pi}), \tilde{\delta}) = 0$ , even though  $U'(\tilde{\Pi})$  obviously depends on  $\tilde{\delta}$ .<sup>4</sup> Moreover, an optimal value  $H^*$  must lie in the superset  $[0, (1 + r_L)L^*]$  which can be narrowed down by applying Jensen's Inequality, when appropriate conditions on the third derivative of the utility function are imposed. We write the profit definition (20) in the form  $\tilde{\Pi} = \tilde{\delta}H - (1 + r_L)\tilde{\lambda}\tilde{\delta}L^* - \bar{\theta}H + \text{const}$ . For given values  $\tilde{\delta} = 0$  and  $\tilde{\delta} = 1$  we conclude

$$\begin{aligned} \tilde{\delta} = 0 &\Rightarrow \tilde{\Pi}_{|\tilde{\delta}=0} = \text{const} - \bar{\theta}H, \\ \tilde{\delta} = 1 &\Rightarrow \tilde{\Pi}_{|\tilde{\delta}=1} = H - \tilde{\lambda}(1 + r_L)L^* + \text{const} - \bar{\theta}H. \end{aligned} \quad (24)$$

Assume  $E[U'(\tilde{\Pi}_{|\tilde{\delta}=1})]$  to be a twice differentiable and strictly decreasing function  $f(H)$ . Further, consider  $g(H) = E[U'(\text{const} - \bar{\theta}H)] = U'(\text{const} - \bar{\theta}H)$  as a twice differentiable function that is strictly increasing, as  $\bar{\theta} > 0$ . Let  $H_1 = 0$  and  $H_2 = (1 + r_L)L^*$ . For these values we get

$$\begin{aligned} f(H_1) &= E[U'(\text{const} - \bar{\theta}H_1 - \tilde{\lambda}(1 + r_L)L^*)] \\ &\stackrel{\tilde{\lambda} \in (0,1), U'' < 0}{>} U'(\text{const} - \bar{\theta}H_1) &= g(H_1) \\ f(H_2) &= E[U'((1 - \tilde{\lambda})(1 + r_L)L^* + \text{const} - \bar{\theta}H_2)] \\ &\stackrel{\tilde{\lambda} \in (0,1), U'' < 0}{\leq} U'(\text{const} - \bar{\theta}H_2) &= g(H_2). \end{aligned} \quad (25)$$

The intermediate value theorem of Bolzano implies  $\exists H \in [0, (1 + r_L)L^*] : f(H) - g(H) = 0$ , i.e.  $E[U'(\tilde{\Pi}_{|\tilde{\delta}=0})] = E[U'(\tilde{\Pi}_{|\tilde{\delta}=1})]$ . In addition,  $\forall H \notin [0, (1 + r_L)L^*]$  the necessary condition  $\text{Cov}(U'(\tilde{\Pi}), \tilde{\delta}) = 0$  does not hold, because  $f - g$  is strictly decreasing. Therefore, as  $H^*$  is an optimal value, we obtain  $[0, (1 + r_L)L^*]$  as a superset. This interval can be further narrowed down by using Jensen's Relationship  $E[U'(X)] > U'(E(X))$ ,  $E[U'(X)] < U'(E(X))$ , and  $E[U'(X)] = U'(E(X))$  for  $U''' >$

<sup>4</sup>Note that independence implies  $\text{Cov}(\cdot, \cdot) = 0$ , but the converse is not true in general. The proof exploits the relationship  $E(U'(\tilde{\Pi}_{|\tilde{\delta}=0})) = E(U'(\tilde{\Pi}_{|\tilde{\delta}=1})) \Leftrightarrow \text{Cov}(U'(\tilde{\Pi}), \tilde{\delta}) = 0$ .

0,  $U''' < 0$ , and  $U''' = 0$ , respectively. We focus our interest on the case of prudence, i.e.,  $U''' > 0$ ; results for the other two cases can be obtained analogously. For arbitrary  $H$  and  $\tilde{\delta} = 0$

$$\mathbb{E}[U'(\tilde{\Pi}_{|\tilde{\delta}=0})] = \mathbb{E}[U'(\text{const} - \bar{\theta}H)] \quad (26)$$

holds. Setting again  $H = \mathbb{E}(\tilde{\lambda})(1 + r_L)L^* + \varepsilon$  with  $\varepsilon \in \mathbb{R}$ , and  $\tilde{\delta} = 1$  leads to

$$\begin{aligned} \mathbb{E}[U'(\tilde{\Pi}_{|\tilde{\delta}=1})] &= \mathbb{E}[U'(H - \tilde{\lambda}(1 + r_L)L^* + \text{const} - \bar{\theta}H)] \\ &\stackrel{U''' > 0, \forall \varepsilon}{>} U'(H - \mathbb{E}(\tilde{\lambda})(1 + r_L)L^* + \text{const} - \bar{\theta}H) \\ &= U'(\varepsilon + \text{const} - \bar{\theta}H) \begin{cases} <_{\varepsilon > 0} \\ \geq_{\varepsilon \leq 0} \end{cases} \mathbb{E}[U'(\text{const} - \bar{\theta}H)]. \end{aligned} \quad (27)$$

Thus, equality can be achieved with  $\varepsilon > 0$  only. Therefore,  $H > \mathbb{E}(\tilde{\lambda})(1 + r_L)L^*$  holds. Taking into account the superset determined before, we arrive at  $H \in (\mathbb{E}(\tilde{\lambda})(1 + r_L)L^*, (1 + r_L)L^*]$ . q.e.d.

Despite this rather pessimistic result concerning simple rules for production and hedging decisions of a commercial bank, we note in passing that our first-order conditions suggest a numerical or simulation approach which could lead to deposit, loan and hedging volumes. (22) implicitly defines an optimal hedge  $H^*$  as a function (influenced by risk preferences and probabilities) of deposit and loan volumes. Denote this by a function  $H(D^*, L^*)$  and replace  $H^*$  in (8) by this function. (7)–(8) is then a system in  $D^*$  and  $L^*$  which depends, however, on the utility function (risk preference) and the probability distributions of  $\tilde{\delta}$  and  $\tilde{\lambda}$ . In a very weak sense this could be interpreted as return of our weak separation property, now in the framework of a simulation approach. But we need to emphasize that to make practical use of this insight, one would have to specify the bank management's von Neumann-Morgenstern utility function and to simulate the distributions of credit event and loss given default.

While we are not able to theoretically identify a simple and intuitive hedge rule (like the beta-hedge before) for a CDS with a pre-determined fixed payment in case of default, we can at least present intervals for the optimal hedge volume and thereby narrow down the bank's choice, if there is prior knowledge about the third derivative of the utility function. Notice that this derivative has meaningful and intuitive interpretation: The term “prudence” was introduced by Kimball (1990) for  $U''' > 0$  to capture the “propensity to prepare and forearm oneself in the face of uncertainty”. Menezes and Wang (2002) emphasize that prudence is equivalent to downside-risk aversion. A further useful interpretation has been offered by Huang (2002) who relates “risk aversion” to investment, “prudence” to the incentive for saving, and “cautiousness” to the hedging of risk. He derives that “if an investor is always more prudent than the others, given that her marginal utility of infinite

consumption is zero, then he/she will be always more risk-averse than the others". One can regard an investor with  $U''' > 0$  as downside-risk averse,  $U''' = 0$  as downside-risk neutral, and  $U''' < 0$  as downside-risk loving. Note that  $U''' = 0$  implies a quadratic utility function, and  $U''' > 0$  should be considered the normal case.

We conclude this section by observing that optimal decisions become much more difficult, if credit default swaps offer a pre-determined fixed payment as opposed to a payment (perfectly or imperfectly) correlated with the risky position. Readers may wonder why such a design of credit derivatives exists in real world markets, if it makes optimal behavior so cumbersome. For an intuitive explanation, return to the issues of information asymmetries tied to loan contracts. If there is a joint problem of sharing uncertainty (between the bank and the third party willing to take on credit risk) and providing incentives (for proper monitoring of the loan), conventional wisdom from incentive theory suggests that the bank in an optimal arrangement ought to retain some of the risk. This is exactly what happens for the from of CDS considered in this section. Our next section turns to macro derivatives as devices to hedge the systematic part of credit risk and investigates if and how their implications differ from those of credit derivatives as analyzed before.

## 4 Macro Derivatives

It is commonly acknowledged that systematic credit risk is primarily driven by macroeconomic conditions (cf. Wilson 1998), whereas unsystematic risk arises from specific properties of the debtor which are independent from the market. As the bank normally has a rather close relationship to a debtor, there is information asymmetry vis-a-vis the capital market and thus the bank has a comparative advantage in managing specific credit risk. Because of this, specific credit risk should not be sold and probably cannot be sold. To capture this idea, we now split off our random variable  $\tilde{\theta}$  capturing credit risk into a systematic and a specific part

$$\tilde{\theta} = \beta \tilde{\theta}_{syst} + \tilde{\theta}_{spec}, \quad (28)$$

where in analogy to capital market theory we include a parameter  $\beta$  to describe how closely the credit risk of a given loan or loan portfolio follows the systematic risk. Unfortunately, these components in general, and  $\tilde{\theta}_{syst}$  in particular, are not directly observable, but have to be inferred from macroeconomic indicators. Suppose, there is a vector of observable macroeconomic variables  $(\tilde{x}_1, \dots, \tilde{x}_k)$  which can be aggregated into a macroeconomic index  $\tilde{g}$  by an aggregation function  $A(\cdot)$  such that

$$\tilde{g} = A(\tilde{x}_1, \dots, \tilde{x}_k) \in [0, 1]. \quad (29)$$

At the moment we take this aggregation as given and carry on with the variable  $\tilde{g}$ . We interpret  $\tilde{g}$  as systematic risk, i.e.,  $\tilde{\theta}_{syst} = \tilde{g}$  and introduce a random variable



$\tilde{s}$  stochastically independent from  $\tilde{g}$  to capture specific risk, i.e.,  $\tilde{\theta}_{spec} = \tilde{s}$ . Our specification of credit risk then is

$$\tilde{\theta} = \beta\tilde{g} + \tilde{s}. \quad (30)$$

While this is an idealized assumption, in reality there should be at least a sufficient approximation  $\tilde{\theta}_{syst} \approx \tilde{g}$ . Since in our model of credit risk we require  $\tilde{\theta} \in [0, 1]$ , the logistic distribution looks like a natural choice, leading to the logistic regression.

Consider now a macro derivative which we call macro default swap (MDS) with underlying  $\tilde{g}$ . The bank's profit function is<sup>5</sup>

$$\tilde{\Pi} = \left( (1 - \tilde{\theta})r_L - r \right) L - \tilde{\theta}L + (r - r_D)D + rK + (\tilde{g} - \bar{g})H - C(D, L), \quad (31)$$

where  $\bar{g}$  is the given market price of the hedge instrument. Notice that (31) is identical to (16), i.e., the bank's decision problem with the macro derivative MDS is formally equivalent to the one with the credit default swap with basis risk. Assuming unbiasedness in the derivatives market which means  $E(\tilde{g}) = \bar{g}$ , the first-order conditions are therefore given by (7), (8), and (17), where we need to replace  $\tilde{\theta}$  in (8) by  $\beta\tilde{g} + \tilde{s}$ . This immediately leads to our

**Proposition 4** *Given an unbiased market and a macro derivative with an underlying matching the systematic part of credit risk (a) a perfect hedge is not possible because the specific part of credit risk is not sold, (b) the bank hedges a portion  $\beta$  of its risky position  $r_L L^* + L^*$  (beta-hedge rule), if  $\beta$  describes the influence of systematic risk on total credit risk according to (30). (c) Weak separation holds, i.e., the bank is able to decide deposit and loan volumes first, and then use optimal risk management. (d) In the absence of economies or diseconomies of scope, the optimal volume of deposits  $D^*$  can be determined as in the case of certainty.*

**Proof** Given the perfect analogy to proposition 2 the proof is almost identical to the one for proposition 2.

(a) Since only a derivative on the systematic part of credit risk is traded, the bank retains risk even if it uses its optimal hedge.

(b) Unbiasedness of the derivatives market, i.e.,  $E(\tilde{g}) = \bar{g}$  implies  $\text{Cov}[U'(\tilde{\Pi}^*), \tilde{g}] = 0$  which after inserting (31) and using (30) implies (18) as in the proof of proposition 2. As long as  $\tilde{g}$  and  $\tilde{s}$  are independent, this can only be true, if  $H^*$  is chosen such that (19) holds.

(c) See proof of part (b) of proposition 2.

(d) See proof of part (c) of proposition 2.

q.e.d.

We conclude that macro derivatives capturing the systematic component of credit risk from a bank's point of view create a setup analogous to the one with a credit

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<sup>5</sup>Thereby, we assume that an MDS is treated as an off-balance-sheet product like the CDS considered earlier.

derivative carrying basis risk. The full optimum  $(D^*, L^*, H^*)$  depends on probabilities and risk preferences. Separation in the strong sense does not hold. However, there is again the weaker notion of separation, enabling the bank to fix deposit and loan volumes, and use the beta-hedge rule afterwards no matter how deposits and loans were determined. In our model the bank can hedge systematic credit risk directly and retain the specific part of risk. As the macro index used as underlying of the MDS is publicly observable, no information asymmetry arises. Macro derivatives seem to be an innovation in financial markets which extend existing credit risk products in an interesting way.

Let us finally turn to the issues of liquidity in the markets for macro derivatives and of generating a macro index from macroeconomic indicators. The aggregation function  $A(\cdot)$  was introduced above to map observable macroeconomic variables  $(\tilde{x}_1, \dots, \tilde{x}_k)$  into the macroeconomic index  $\tilde{g}$  used as underlying of the macro derivative. Whereas liquidity in these new markets for macro derivatives is a problem that will be solved over time as a consequence of increased market demand, it will always be a challenging task to evaluate and maintain the optimal structure of a macro index, i.e., specify the aggregation function  $A(\cdot)$  and its arguments. Notice first that different banks may need different macro indices (and  $\beta$ s) because they may have different notions of systematic risk. Take banks focused on specific sectors of the economy as an example: Systematic risk for a bank financing car sales will not be identical to systematic risk from the point of view of a bank giving housing loans. A standardized derivative on  $\tilde{g}$  for all banks is hard to imagine. Bank-specific underlyings  $\tilde{g}$ , however, would lead to very illiquid markets for macro derivatives.

One way to get around this problem would be to establish standardized financial products on single macroeconomic indicators and let banks individually aggregate these indicators into their own macro indices. There could be reasonable liquidity in the markets for the single indicators and, if number and variety of macroeconomic indicators traded are sufficiently high and trading is sufficiently frequent, a bank would be able to dynamically replicate its individually optimal macro index by trading on the components of this index. Technically this could be done by using the following approximation of the aggregation function  $A(\cdot)$ :

$$\Delta A(x_1, \dots, x_k) \approx \sum_{i=1}^k \frac{\partial A}{\partial x_i} \cdot \Delta x_i = \nabla A^T \cdot \Delta x \quad (32)$$

$$\stackrel{\text{linear}}{\Leftrightarrow} A \Delta A(x_1, \dots, x_k) = m^T \cdot \Delta x \Leftrightarrow A(x_1, \dots, x_k) = m^T \cdot x + c,$$

where  $m$  denotes the constant vector of the gradient  $\nabla$  and  $\Delta$  is a delta-difference. This separation enables risk management to trade on the single indicators. Thereby each economic indicator  $x_i$  contributes the volume  $m_i \cdot H$  to the bank's macro index.

A bank would now have to determine empirically the weights  $m^T$  most appropriate for its macro index. When determining the macro index one can distinguish causally and statistically motivated relationships  $\tilde{\theta} = \beta A(\tilde{x}_1, \dots, \tilde{x}_k) + \tilde{s}$ . A causal

relationship allows to evaluate the relevant economic variables and the functional form of  $A(\cdot)$  based on a theoretical model like the CAPM. In this case, only the parameter estimation requires statistical methods. A statistical relationship does not allow to figure out the functional form and variables exactly, based on qualitative reasoning. This may be due to a lack of theoretical insight as no adequate theory has been developed yet. In this second case, the selection of proper explanatory variables, and sometimes even the decision for a functional relationship, has to be based on statistical methods.

The explanation of credit risk is an example for the second case. More precisely, even though we suggested already to use the logistic regression and may have pre-selected a set of potential variables  $(\tilde{x}_1, \dots, \tilde{x}_n)$ , with  $n \geq k$ , by qualitative reasoning, there is a need to further narrow down this set in order to receive an optimal vector of economic indicators. The logistic distribution allows for a linear transformation yielding the logistic regression model (cf. Gujarati 1995, p. 554):

$$\ln\left(\frac{\tilde{\theta}}{1-\tilde{\theta}}\right) = c + \sum_{i=1}^n m_i \cdot \tilde{x}_i + \tilde{s}. \quad (33)$$

A bank would use its internal data on credit depreciation as observations of  $\tilde{\theta}$  and use (33) to estimate optimal weights for macroeconomic indicators. To keep the exposition simple, assume  $\tilde{s}$  to be normally distributed in a homoskedastic manner, and additionally  $\tilde{\theta} \in (0, 1)$ . Under these assumptions one is able to apply ordinary least squares.<sup>6</sup> The restriction  $\tilde{\theta} \notin \{0, 1\}$  means that even though there is always some default, the bank will be able to recover a non-zero portion of its exposure. An efficient procedure to use (33) as a tool for the selection of proper indicators is a stepwise backward elimination of insignificant variables. The decision whether to eliminate a variable from further regressions can be based on the  $t^2$  statistics of the regression parameters.

## 5 Conclusion

Using the industrial organization approach to the microeconomics of banking, we analyzed the implications of credit risk, credit derivatives and macro derivatives on optimal behavior of a single large bank under risk aversion. In modelling credit risk we distinguished between the probability of a credit default and the loss given default. We were able to derive separation results and – in most cases – hedge rules. For a credit default swap perfectly correlated with the credit risk, decisions on deposit and loan volumes on the one hand and hedge volume on the other are (strongly) separable and can be made without knowledge of risk preferences and probabilities.

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<sup>6</sup>The critical assumption of a homoskedastic disturbance can be relaxed by the use of WLS, cf. Gujarati (1995), p. 557.

If in this situation the derivatives market is unbiased the bank optimally chooses to fully hedge its credit risk exposure. For the case of a credit derivative with basis risk we found a beta-hedge rule to be optimal, irrespective of the volumes of deposits and loans chosen. Whereas a full optimum now depends on risk preferences and probabilities, there still exists a (weak) form of separation between deposits and loans on the one hand and hedging on the other. If the derivative in case of default pays a pre-determined fixed amount, no simple hedge rule exists, neither weak nor strong separation holds, but we can relate optimal hedging to the notion of “prudence” and give intervals for the optimal hedge. We then examined macro derivatives as one very recent innovation in financial markets. We argue that macro derivatives are a valuable tool because as opposed to normal credit derivatives they enable lenders to sell the systematic part of credit risk which is what capital market theory suggests. The implications of a macro derivative for optimal bank behavior are formally equivalent to the ones of a credit derivative with basis risk. Finally we discussed how macro indices as underlyings of macro derivatives could be optimally designed from financial products on macroeconomics indicators, such as the products traded since fall 2002 mentioned in the introduction. Whereas the industrial economics approach to the bank does not consider issues of asymmetric information explicitly, we were able at several stages of our analysis to point out that aspects of asymmetric information were included in an implicit way.

Further work should be directed to generalizing the framework by modelling the derivatives market and moving closer to a general equilibrium framework. This should lead to an answer to the question of whether or not the existence of credit derivatives and macro derivatives leads to more or less risk in the financial system (see e.g. Instelfjord 2000 and Prato 2002 for some insights on this question).

## References

- 1 Altman, E. and A. Saunders (1997), *Credit Risk Measurement: Development Over the Last 20 Years*, *Journal of Banking and Finance* 21, 1721–1742.
- 2 Bank for International Settlements (2001), *Triennial Central Bank Survey. Foreign Exchange and Derivatives Market Activity in 2001*, Bank for International Settlements, Basle.
- 3 Benninga, S., R. Eldor, and I. Zilcha (1984), *The Optimal Hedge Ratio in Unbiased Futures Markets*, *Journal of Futures Markets* 4, 155–161.
- 4 British Bankers’ Association (2002), *Credit Derivatives Report 2002*, [www.bba.org.uk/public/services/publications/58309?field=execsummary](http://www.bba.org.uk/public/services/publications/58309?field=execsummary).
- 5 Chiappori, P.-A., D. Perez-Castrillo, and T. Verdier (1995), *Spatial Competition in*

- the Banking System: Localization, Cross Subsidies and the Regulation of Deposit Rates*, European Economic Review 39, 889–918.
- 6 Freixas, X. and J.-C. Rochet (1997), *Microeconomics of Banking*, Cambridge, MA: MIT Press.
  - 7 Froot, K., D. Scharfstein, and J. Stein (1993), *Risk Management: Coordinating Corporate Investment and Financing Policies*, Journal of Finance 48, 1629–1658.
  - 8 Froot, K. and J. Stein (1998), *Risk Management, Capital Budgeting, and Capital Structure for Financial Institutions: An Integrated Approach*, Journal of Financial Economics 47, 55–82.
  - 9 Gujarati, D. (1995), *Basic Econometrics*, New York: McGraw-Hill, 2. edn.
  - 10 Huang, J. (2002), *Relationships Between Risk Aversion, Prudence, and Cautiousness*, Working Paper, Lancaster University.
  - 11 Instelfjord, N. (2000), *Risk and Hedging: Do Credit Derivatives Increase Bank Risk?*, discussion paper, Fondation Banque de France.
  - 12 Jeanneau, S. (2002), *Derivatives Markets*, BIS Quarterly Review .
  - 13 Kimball, M. (1990), *Precautionary Saving in the Small and in the Large*, Econometrica 58, 53–73.
  - 14 Loubergé, H. and H. Schlesinger (2002), *Coping with Credit Risk*, Working Paper No. 36, Department of Economics, University of Manitoba.
  - 15 Marshall, J., V. Bansal, A. Herbst, and A. Tucker (1992), *Hedging Business Cycle Risk with Macro Swaps and Options*, Journal of Applied Corporate Finance 4, 103–108.
  - 16 Menezes, C. and H. Wang (2002), *The Precautionary Premium and the Risk-downside Risk Tradeoff*, Working Paper, Department of Economics, University of Missouri-Columbia.
  - 17 Neal, R. (1996), *Credit Derivatives: New Financial Instruments for Controlling Credit Risk*, Federal Reserve Bank of Kansas City Economic Review , 15–27.
  - 18 Neven, D. and L.-H. Röller (1999), *An Aggregate Structural Model of Competition in the European Banking Industry*, International Journal of Industrial Organization 17, 1059–1074.
  - 19 Pausch, T. and P. Welzel (2002), *Credit Risk and the Role of Capital Adequacy Regulation*, Volkswirtschaftliche Diskussionsreihe, Beitrag Nr. 224, Institut für Volkswirtschaftslehre, Universität Augsburg.

- 20 Prato, O. (2002), *Credit Derivatives: A New Source of Financial Instability?*, Financial Stability Review , 69–83.
  - 21 Schweimayer, G. (2003), *Risikomanagement mit Makroderivaten auf Basis zeitdiskreter stochastischer Prozesse*, Universität Augsburg: Dissertation.
  - 22 The Wall Street Journal (2002), *New Futures Could Help Folks Insure Against Economic Risks*, September 05, 2002.
  - 23 Topping, M. (2001), *Banks to Launch Derivatives on Economics Statistics*, Futures and Options Week, October 10, 2001, 12 .
  - 24 Wilson, T. (1998), *Portfolio Credit Risk*, Federal Reserve Bank of New York Economic Policy Review 4, 71–82.
  - 25 Wong, K. (1997), *On the Determinants of Bank Interest Margins Under Credit and Interest Rate Risk*, Journal of Banking and Finance 21, 251–271.
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