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Risk preferences, global market conditions and foreign debt: Is there any role for the currency composition of FX reserves?

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Abstract

I use a transition probability matrix associated with different global market conditions and I assume that it captures switches in central bank preferences between approximated constant relative risk aversion (CRRA) expected utility and approximated increasing relative risk aversion (IRRA) expected utility. I approximate CRRA and IRRA expected utility, to construct and propose constrained portfolio selection frameworks with skewness, for the currency composition of FX reserves over different global market conditions that influence central bank preferences. These portfolio selection frameworks account for portfolio rebalancing, they satisfy Pratt-Arrow measures of risk aversion and are constrained by the country's currency composition of foreign debt. Thus, for these portfolios, the currency composition of FX reserves is motivated by its country's currency composition of foreign debt. I propose these frameworks for 6 emerging market economies (EMEs) and this is only for a small portion of the total portfolio of FX reserves. These EMEs are Brazil, India, Indonesia, Mexico, South Africa and Turkey and five of these EMEs have been denoted as the "Fragile Five". Using different methods of computing expected FX reserves returns and different maturity structures on FX reserves, I validate my proposal using data over the 2010-2018 period on these EMEs by simulating optimal FX reserve weights for each EME; where each country's actual currency composition of foreign debt is a constraint.

JEL Codes: E58, F31, G11, G15.

Keywords: IRRA, CRRA Expected Utility, Global Market Conditions, Currency Composition of FX Reserves, Foreign Debt, Portfolio Selection, Skewness, Emerging Market Economies

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1 Introduction

I propose that one large portion of the total portfolio of FX reserves for a select set of emerging market economies (EMEs) is for policy and operational purposes such as FX intervention purposes, payment for goods and services of the country, provision of emergency liquidity assistance to distressed sectors of the economy and for any other precautionary purposes against negative current and capital account shocks, see also Caballero and Panageas (2006), Borio et al. (2008), Dominguez et al. (2012) and Bianchi et al. (2018). For the remaining small portion, I propose a formal and quantitative approach that aligns the currency composition of FX reserves with the country’s currency composition of foreign (external) debt.¹

For this small portfolio and generalizing for the EMEs, I characterize an EME central bank as a risk-averse investor. Thus, I assume that the central bank exhibits increasing risk aversion during crisis periods because of higher global economic and financial instability that originates in advanced economies (AEs) and filters to EMEs or instability that originates from other EMEs and it results in contagion effects. Such global market conditions are associated with large, and in some instances, unexpected EMEs real exchange rate depreciations; and these depreciations can be persistent. I also assume that the central bank exhibits increasing risk aversion during global pandemic and virus-related crisis periods, such as the recent COVID-19 pandemic. Thus, under such conditions, I approximate increasing relative risk aversion (IRRA) expected utility for the central bank using a third-order Taylor series expansion. This expansion captures the first three moments of expected FX reserves returns, it’s consistent with Pratt-Arrow measures of risk aversion (Pratt, 1964; Arrow, 1971) and implies the central bank prefers odd moments and dislikes even moments (Scott and Horvath, 1980). IRRA expected utility exhibits changes in the portfolio allocation for riskier assets, as portfolio return changes. Thus, it is suitable for periods of higher global economic and financial instability.

As a result, I construct and propose a constrained portfolio selection framework with skewness, such that the currency composition of FX reserves is consistent with its constraint in the form of the country’s currency composition of foreign debt. My proposal incorporates portfolio rebalancing transaction costs that explicitly reduce expected portfolio return. These are costs associated with the purchase and sale of FX reserves towards an allocation that matches the country’s currency composition of foreign debt and an allocation that maximizes the central bank’s approximated expected utility. However, during non-crisis periods, I assume the central bank switches to a constrained portfolio selection framework with skewness, based on approximated constant relative risk aversion (CRRA) expected utility. Under CRRA expected utility, the portfolio allocation towards riskier assets, re-

¹The allocation between the large and small portion of the total portfolio, will differ across the 6 EMEs and may be influenced by a trade-off between policy and operational purposes along with leeway and preferences towards active FX reserves management. Thus, throughout this paper, I generalize by using the term “small portion”, rather than pinning down the exact value for the small portion for each EME.

mains constant as portfolio return changes. Both portfolio selection frameworks account for portfolio rebalancing and they satisfy Pratt-Arrow measures of risk aversion.

Concerning tail risks, I model the central bank to prefer positive skewness as compared to negative skewness because this implies a low probability of obtaining a large negative return (Briec et al., 2007). Harvey et al. (2010) explain that risk averse investors prefer a high probability of an extreme positive outcome over a high probability of an extreme negative outcome. Thus, in my proposal, the central bank seeks to maximize portfolio skewness because this anchors its portfolio towards large gains relative to large losses. Athayde and Flóres (2004) interpret a preference for high values of odd moments by investors, as a mechanism to reduce extreme portfolio losses and anchoring towards gains. Whereas a dislike for high values of even moments, exhibits a dislike for dispersion and thus volatility because volatility increases the uncertainty of returns. Similarly, Arditti (1967) explains that risk averse investors, exhibit reluctance towards a portfolio with a small likelihood of a large loss - no matter how small - and only a limited gain.

Using different methods of computing expected FX reserves returns and different maturity structures on FX reserves, I validate my proposal using data for 6 EMEs over the 2010-2018 period by simulating optimal FX reserve weights for each EME; where each country's actual currency composition of foreign debt is a constraint. These EMEs are Brazil, India, Indonesia, Mexico, South Africa and Turkey and five of these EMEs have been denoted as the "Fragile Five" in the literature, see Aizenman et al. (2017). Using different maturity structures on FX reserves, allows me to potentially align FX reserves to the wide spectrum of external debt with respect to different maturity structures. I use these EMEs because they exhibit rising total and short-term external debt to GDP ratios and rising and significant FX reserves to GDP ratios over the 2010-2018 period. Furthermore, these EMEs exhibit concurrently rising public sector and private sector external debt to GDP ratios and all these ratios are comparable across the 6 EMEs that span 4 continents, see Figures 1, 2 and 3.²

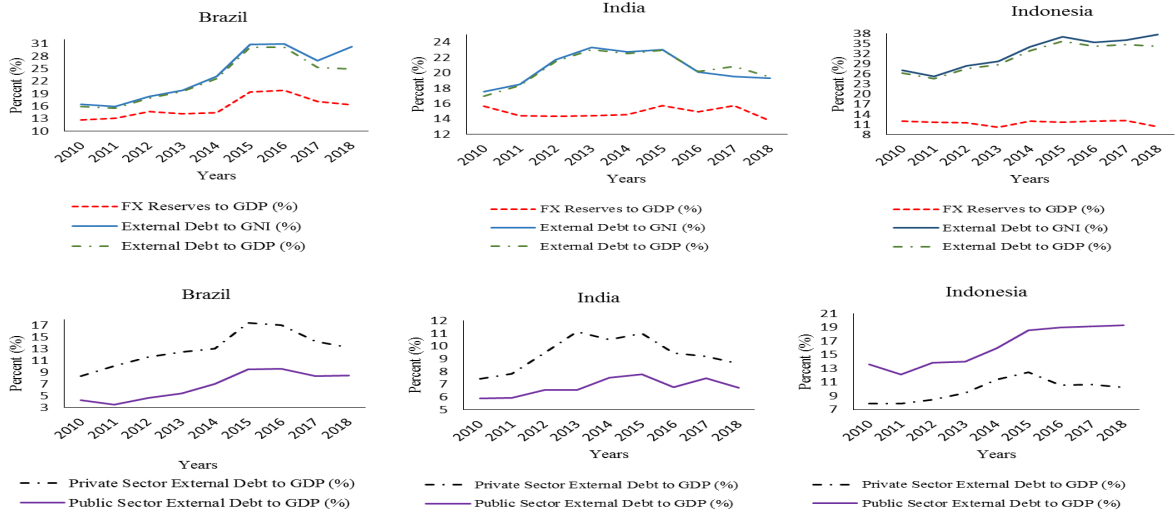
This paper makes three main contributions to the literature. The first contribution is to propose constrained portfolio selection frameworks with skewness and rebalancing, such that the currency composition of FX reserves is consistent with its country's currency composition of foreign debt; over different global market conditions along with maximizing (approximated) expected utility. Thus, to the best of my knowledge, this paper is the first to propose a quantitative framework for the currency composition of FX reserves, that incorporates skewness in a manner consistent with Pratt-Arrow measures of risk aversion.

The second contribution of this paper is as a FX reserves related prudential strategy. Thus, my proposal may allow a central bank to anchor its economy in the event of foreign debt obligations of

²This is external debt owed to non-residents and repayable in currency (World Bank: International Debt Statistics). Emphasis in my proposal is on foreign currency denomination of external debt and I refer to this as foreign debt. Significant FX reserves to GDP ratios are ratios that are above 10 %, see also Truman and Wong (2006), who argue that active reserve management and diversification is relevant when a country's FX reserves to GDP ratio is significant.

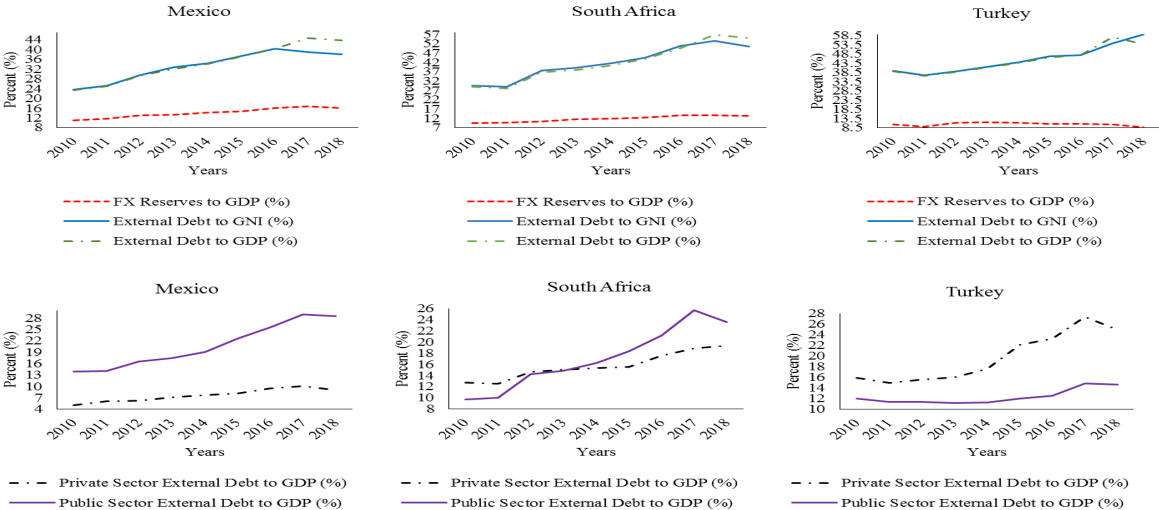
its country - whether short term or long - that are due soon and cannot be settled over different global market conditions; with a suitably managed FX reserve resource. Thus, this paper is the first to propose a progressive formal FX reserves risk-management practice over different global market conditions. The third contribution is to the currency composition of FX reserves literature because most studies examine whether the currency composition of FX reserves is optimal and consistent with variants of mean-variance models. Furthermore, beyond portfolio selection models for the currency composition of FX reserves, the literature presents and examines the transactions approach and generally using regressions.

Figure 1: Country relevant ratios (%)



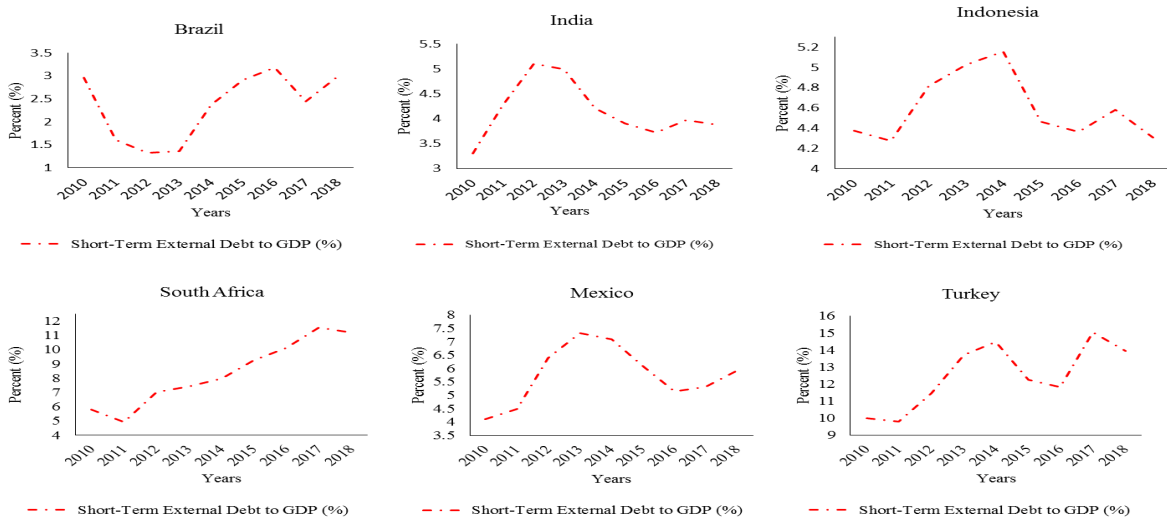
Source: IMF’s International Financial Statistics and World Economic Outlook, World Bank’s Global Development International Debt Statistics and authors’ calculations.

Figure 2: Country relevant ratios (%)



Source: IMF’s International Financial Statistics and World Economic Outlook, World Bank’s Global Development International Debt Statistics and authors’ calculations.

Figure 3: Country relevant ratios (%)



Source: IMF’s World Economic Outlook, World Bank’s Global Development International Debt Statistics and authors’ calculations.

Rising and significant FX reserves to GDP ratios provide policy authorities leeway to pursue active FX reserves management; along with the main policy and operational purposes of FX reserves. However, FX reserves are a limited national resource, and this reinforces the need for FX reserves to be managed in the best possible manner to benefit a country when the need arises; along with potentially building a country’s credibility internationally. My criteria of EMEs with rising external debt to GDP ratios, is aligned and informed by Mendoza and Yue (2012) who use data from 23 sovereign default events of a group of EMEs. They document that countries that default, have high external debt to GDP ratios and these ratios are amplified when countries default.³

³Emphasis in my proposal is on a small portion of the total portfolio of FX reserves because (i) proposing for the total portfolio would exclude other primary policy and operational purposes of FX reserves and this is inconsistent with central bank conduct. (ii) From a level effect and for many economies, total foreign debt is usually larger than FX reserves. Thus, it would be impractical to propose an outcome of matching currency compositions on the basis that the levels are relatively equal. Furthermore, this would generate the inconsistency explained in point (i). Thus, (iii) my proposal on a small portion of the total portfolio of FX reserves, may allow a central bank to anchor its economy because not all foreign debt obligations across all sectors of a country may be subject to default. Moreover, my proposal cannot be applicable to the possibility of foreign debt defaults generated by speculative borrowing or mismanagement on the part of borrowers. As a result, it is not the total scale of foreign debt of a country that is subject to default and that may potentially need central bank intervention. (iv) The realizations in (iii) anchor the fact that a small portion is suitable and reinforce the practical and feasible outcomes related to central bank intervention because of the negative consequences of foreign debt default. This is consistent with policy authorities fulfilling their mandate as crisis managers, preventers of potential crises and as authorities that seek to maintain the stability of their economy by following progressive risk management practices, rather than not responding when a suitably managed FX reserve resource is available. Default in one sector or institution, can negatively impact other sectors that are functional and that would have been able to continue operating and fulfil their debt obligations had there not been a default in another sector(s). Thus, my proposal may anchor a firm, institution or sector from a costly liquidation of an investment project due to unavoidable external factors; where the project could be profitable and sustainable in future periods.

These factors show that EMEs with rising and high external debt to GDP ratios are vulnerable to a foreign debt crisis, especially one triggered by large unexpected (real) exchange rate depreciations. The International Monetary Fund’s (IMF) Balance of Payments and International Investment Position (IIP), shows that the net IIP (net financial claims on the rest of the world) is negative for these 6 EMEs over the 2010-2018 period, except for South Africa over the 2015-2018 period. Furthermore, in section 3.2, I explain how a positive net foreign asset position of a country may not give a good reflection of the extent of vulnerability to foreign debt exposure, the extent of currency mismatches in individual sectors and liquidity risks between foreign currency assets and debt.⁴⁵

The rest of the article proceeds as follows. Section 2 describes related literature. Section 3 presents the model proposal and provides further motivation. Section 4 describes the data and the computation of expected FX reserves returns. Section 5 validates my proposal. Section 6 concludes.

2 Related literature

EMEs are vulnerable to economic and financial instability that originates from AEs and filters to EMEs, and they are also vulnerable to instability that originates from other EMEs and this results in contagion effects. Furthermore, EMEs are vulnerable to large exchange rate depreciations that can eventually moderate in a short period, however in some instances, the exchange rate depreciations

⁴For example, the net foreign currency assets of a country can be positive, but this may not be the case for all sectors. Chui et al. (2016) show this using country groups that combine Latin American, Asian, Other Asian and other EMEs. The net foreign currency assets are positive for all country groups over the 2010-2014 period. However, the net foreign currency assets of the non-government sectors are negative over the same period excluding Asian large EMEs.

⁵Avdjiev et al. (2020) note that a complete assessment of external and foreign currency debt vulnerability to exchange rate depreciations, needs to account for factors such as foreign currency revenues and assets, along with intragroup linkages. Chui et al. (2016) also provide a detailed discussion on currency mismatches based on stock and flow measures. Thus, such assessments need to account for liquidity mismatches between foreign currency assets and foreign currency debt; along with an assessment of the currency composition of foreign currency revenues and assets relative to the currency composition of foreign debt. Moreover, an assessment of external debt denominated in foreign currency and domestic debt denominated in foreign currency. Determining the foreign currency debt exposure of local affiliates of foreign owned companies across all sectors relative to foreign currency debt exposure of domestic owned companies. Determining hedged and unhedged foreign currency debt with natural hedges such as foreign currency revenues and derivative contracts, that do not eliminate FX risk because of maturity mismatches. Within this context and using detailed firm-level data on foreign currency liabilities, exports, and derivatives for three EMEs, Caballero (2020) finds that (i) many firms in these EMEs with foreign currency debt exposure are non-exporters and thus do not have natural hedges, (ii) the foreign currency debt exposure of firms is not perfectly matched with derivatives and (iii) for the firms using derivatives, these contracts do not significantly hedge foreign currency exposure to the extent to which they would insulate firms from the negative effects of exchange rate depreciations. Furthermore, Honig (2009) explains that not all FX hedging mechanisms such as forward contracts eliminate FX risk. Data on all these elements is not publicly available for EMEs. For some EMEs, aggregate data is available for some sectors however without a complete outline of the currency composition. Thus, I use the aggregate criteria I have outlined as a source of vulnerability for the EMEs in my sample which opens the possibility of foreign debt defaults. My approach is aligned with theoretical and empirical literature that emphasizes on foreign debt exposure as a source of vulnerability for EMEs.

can persist. These dynamics capture different market conditions associated with unavoidable external events that can destabilize EMEs and result in EMEs foreign debt defaults. Thus, in my proposal, the switching dynamics between IRRA and CRRA expected utility, are characterized with a transition probability matrix that corresponds to different global market conditions that influence and generate different central bank preferences. The transition probability matrix has four different phases that account for unavoidable external factors that can influence EMEs stability and their associated probabilities. These are expressed as follows:

$$\Pi = \begin{bmatrix} \Pi_{\text{AEs to EMEs}} & \Pi_{\text{EMEs to EMEs}} \\ \Pi_{\text{Pandemic}} & \Pi_{\text{Stability}} \end{bmatrix} \quad (1)$$

In the 1st phase, there is greater global economic and financial instability that originates in AEs and this spills over to EMEs. There are several transmission channels through which such instability spills over to EMEs, for example a reduction in global aggregate demand, lower export demand, lower commodity prices, sudden stops and capital flight from EMEs. An example of this, is the most recent Global Financial Crisis (GFC). Such events generate large and persistent EMEs currency depreciation.⁶ However, such events are not frequent. Thus, I assume a moderate $\Pi_{\text{AE to EMEs}}$ probability of such events over different global market conditions. Under such circumstances, I assume the central bank exhibits increasing risk aversion and uses IRRA expected utility for portfolio selection.⁷

The 2nd phase is one of greater economic and financial instability that originates in one or several EMEs and this spills over into other EMEs especially with contagion effects, that eventually destabilize EMEs and generate large and persistent EMEs currency depreciation. These events can coincide with lower export demand and lower commodity prices. Thus, generating a decrease in foreign currency receipts, reducing the ability of EMEs to service foreign currency debt obligations and reducing the ability to accumulate foreign currency assets. As a result, increasing the likelihood of foreign debt defaults. Examples of such instability originating in other EMEs and generating contagion effects, are the 1994-95 Tequila crisis, the 1997-98 Asian crisis and the 1998 Russian crisis, see for example Caballero and Panageas (2006), Barkbu et al. (2012), Catão and Milesi-Ferretti (2014) and Korinek and Mendoza (2014).⁸ As noted by Kose et al. (2020), the 1980s Latin American debt crisis is another example of instability originating in EMEs and generating contagion effects to other EMEs and developing economies in other regions such as Algeria, Nigeria and Niger.

⁶Upon impact these events are also associated with large and unexpected EMEs currency depreciation.

⁷I discuss the length of the portfolio holding period and associated portfolio rebalancing in section 3.

⁸This is not an exhaustive list of EMEs instability and contagion effects. Furthermore, over 1994-1998 some EMEs were pegged to the U.S. dollar and thus experienced devaluation pressure. There have been other episodes of less systemic crises, sovereign debt defaults and restructuring in EMEs such as Brazil, Argentina, Uruguay and Turkey in the late 1990s and early 2000s (Barkbu et al., 2012; Kose et al., 2020). Other channels of instability can be through geo-political factors in one or several EMEs that spills over into other EMEs. Furthermore, although not originating in EMEs, market outcomes such as the 2013 taper tantrum episode associated with the U.S. Federal Reserve, has generated currency depreciation for some EMEs and downturns in their equity markets (Aizenman et al., 2017; Caballero, 2020).

However, during this period, most Latin American EMEs currencies were pegged to the U.S. dollar, they were experiencing substantial downward currency pressure and substantial capital flight. These factors generated sharp currency devaluations in EMEs, especially Brazil, Mexico and Argentina (Kose et al., 2020). These events and contagion effects are common for EMEs, however, there has not been any *major* crises in EMEs in the past 20 years, see also Caballero (2020). Thus, I assume a high $\Pi_{\text{EMEs to EMEs}}$ probability of such events over different global market conditions. Under such circumstances, I assume the central bank exhibits increasing risk aversion and uses IRRA expected utility for portfolio selection because such events are usually associated with large EMEs currency depreciation, that in some instances can be persistent for several months. Else, the central bank can use CRRA approximated expected utility for portfolio selection, if it does not expect persistent currency depreciation for several months.

For the 3rd phase I assume that EMEs policy authorities exhibit increasing risk aversion during global pandemic and virus-related crisis periods, such as the recent COVID-19 outbreak that has generated large and persistent currency depreciation for EMEs. These events are rare, and I assume a low Π_{Pandemic} probability of such events over different global market conditions. With such events, I assume the central bank exhibits increasing risk aversion and uses IRRA expected utility for portfolio selection, if the central bank expects large and persistent currency depreciation for several months. Else, the central bank can use CRRA approximated expected utility for portfolio selection if currency depreciation does not persist for several months. Relatedly, Hofmann et al. (2020) discuss FX reserves related macroprudential policies and they argue that FX reserves can buffer EMEs from the COVID-19 shock that has generated EMEs currency depreciation and capital outflows. For the 4th phase, I assume that there is a high degree of global and EMEs stability. Thus, market conditions allow for regular EMEs currency fluctuations, rather than large and persistent EMEs currency depreciation. Thus, I assume a moderate $\Pi_{\text{Stability}}$ probability of such events over different global market conditions. Under such circumstances, I assume the central bank exhibits risk aversion, however, it uses CRRA approximated expected utility for portfolio selection with skewness.⁹

⁹I conduct two normality tests, the Jarque-Bera and Kolmogorov-Smirnov tests to establish whether the individual returns on FX reserves exhibit normality and whether it is appropriate to incorporate higher return moments. The tests consistently reject normality at all relevant levels of significance. I report these in section 5 along with the univariate statistics associated with returns on the individual FX reserves used in this paper. When using different methods of computing expected returns on FX reserves, the preliminary findings show that the individual returns and portfolio returns exhibit skewness for different allocations, including for when computing expected FX reserves returns that only account for yields on government bonds of major reserve currencies. However, this is not the case for kurtosis, especially when computing returns with the yield only. The kurtosis values are small and portfolios do not exhibit excess kurtosis. When computing expected returns accounting for yields and for expected exchange rate changes, skewness and kurtosis matters. However, the optimal portfolios selected are not significantly different to those generated where only tail risk in the form of skewness is accounted for; even when loosening constraints and varying risk aversion parameters to generate optimal portfolios and when comparing these portfolios relative to a naive equally weighted portfolio allocation and portfolio allocation generated with only a no short selling constraint. Thus, from a portfolio weights and moments perspective, including kurtosis does not generate any additional portfolio gains. However, the central bank's preference

My proposal is a procedure that may allow a central bank to anchor its country's ability to withstand foreign debt defaults over different global market conditions. More so, when unavoidable external factors result in large and in some instances, unexpected exchange rate depreciations that can persist and thus exacerbate negative balance sheet effects. This is relevant because Forbes and Warnock (2012) emphasize that factors related to crises - such as global risk and contagion - are beyond the control of policymakers in most countries. Within this context, Claessens and Kreuser (2007) emphasize that risks faced by sovereigns are broad and thus sovereigns account for risks in all sectors of the economy, rather than only accounting for government sector risks. Thus, in line with my proposal, this makes it more necessary for policy authorities to continuously assess a country's external debt; using a formal approach that anchors a country's ability to withstand foreign debt defaults. Such an approach is consistent with central banks having unique investment objectives and mandates (Cardon and Coche, 2004; Fisher and Lie, 2004). My proposal is aligned with Gopinath and Stein (2018) who construct a model consisting of households, banks and a central bank. They show that when a country's banking sector has greater dollar-denominated liabilities, this influences the central bank to accumulate more dollar reserves to facilitate its role as a lender of last resort.

As a FX reserves related prudential strategy, this paper is also aligned with Bocola and Lorenzoni (2020) who construct a model where the holding of FX reserves boosts the government's fiscal capacity to intervene ex-post a crisis and currency depreciation that generates banking sector distress. In their model, FX reserves can eliminate a bad equilibrium that is associated with financial instability and foreign currency debt exposure. Relative to Bocola and Lorenzoni (2020), my proposal focuses on the ex-ante preparedness hedging benefits of the currency composition of FX reserves for all sectors of the economy, against potential foreign debt defaults that can be triggered by large unexpected currency depreciations. Relatedly, Shousha (2017) examines whether the role of international reserves as implicit collateral for foreign borrowing, is a motive for international reserve accumulation for EMEs and whether this explains the high levels of EMEs international reserves. His assertion for international reserves as implicit collateral, is based on the negative reputational effects for a country following private or public sector external debt default, when a suitable stock of international reserves is available to honor these obligations.

My proposal is consistent with the econometric empirical evidence by Dooley et al. (1989), Eichengreen and Mathieson (2000), Hatase and Ohnuki (2009) and Aizenman et al. (2020). These studies show that a country's currency denomination of foreign debt, influences its reserve currency choice. Thus, I view my proposal as a mechanism that may increase a country's credibility and anchor foreign investor confidence and expectations, because a country's policy authorities are continuously assessing function (approximated expected utility) consistently delivers negative values when kurtosis is included for both CRRA and IRRA utility. This is inconsistent with maximizing expected utility. As a result, although I had initially constructed portfolio selection with kurtosis, the data, the constraints, optimal portfolios and the positive values of expected utility, show that portfolio selection with skewness is suitable.

the country's foreign debt and currency composition of foreign debt through a formal and quantitative approach. Furthermore, it might be the case that in some instances the central bank may not even use its FX reserves. However, having suitably managed FX reserves over different global market conditions, may anchor and reinforce a country's credibility to potentially meet its debt obligations, see also Bocola and Lorenzoni (2020) on unused reserves.

Eichengreen et al. (2003) emphasize and show that the currency composition of foreign debt, should not be neglected because this influences the output stability, volatility of capital flows and the country credit ratings of a wide array of countries, including developing economies and EMEs. In line with Gete and Melkadze (2020) who construct a quantitative model of international lending of last resort with partial financial assistance, I assume that the central bank may assist the distressed firm/institution or sector with FX reserves at an interest rate lower than what would be charged when foreign debt is rolled over by private lenders. Or, at an interest rate lower than what would be charged when the distressed sector does not have its debt rolled over and it seeks to raise proceeds in the market so that it can honor its debt that is due soon. Although my proposal is not concerned about the possibility of foreign debt defaults generated by speculative borrowing or mismanagement on the part of borrowers, I provide a discussion on moral hazard in section 3.2.

My proposal is feasible computationally and operationally because I validate it with unique modelling requirements for each EME based on their foreign currency debt characteristics and expected FX reserves returns. My proposal is also consistent with the limits on currency mismatches and this is an instrument in the macroprudential frameworks of EMEs central banks of Brazil, Colombia, India, Indonesia, Malaysia, Mexico, Russia, South Africa and Thailand, see Villar (2017). Relatedly, however using an econometric model to examine advanced economies and EMEs, Catão and Milesi-Ferretti (2014) find that higher FX reserves reduce a country's external crisis risk as compared to other foreign asset holdings; where they define external crises to include external defaults, payment re-scheduling and a country's ability to access sizeable support from the IMF.

Existing currency composition of FX reserves literature, examine whether the currency composition of FX reserves is optimal and consistent with variants of mean-variance models, see for example Ben-Bassat (1980), Dellas and Yoo (1991), Papaioannou et al. (2006), Mateane and Semmler (2016), Lu and Wang (2019) and Farhi and Maggiori (2018) who characterize world demand for reserve assets using investors that exhibit mean-variance preferences. Furthermore, beyond portfolio selection models for the currency composition of FX reserves, the literature presents and examines the transactions approach. In this approach, international transaction activities related to a country or country groups, are argued to influence the associated currency composition of FX reserves. Thus, country's composition of trade flows, country's currency composition of trade invoicing, exchange rate arrangements and movements, country's currency composition of foreign debt and a wide array of other economic variables are used for examining whether they influence the currency composition of FX reserves; and

generally with regressions, see Heller and Knight (1978), Dooley et al. (1989), Eichengreen and Mathieson (2000), Chinn and Frankel (2008), Hatase and Ohnuki (2009), Ouyang and Li (2013), McCauley and Chan (2014), Soesmanto et al. (2015), Ito et al. (2015), Eichengreen et al. (2015, 2019), Lu and Wang (2019), Ito and McCauley (2020) and Aizenman et al. (2020).¹⁰

My proposal is also motivated by several factors documented in recent literature. For example, following the recent GFC, there has been a rapid increase in foreign currency borrowing by EMEs. Avdjiev et al. (2020) and Asis et al. (2020) document that it is the non-financial corporate sector of EMEs that has exhibited a rapid increase in foreign currency borrowing following the GFC. Furthermore, Bank for International Settlements (BIS) data shows substantial international debt securities exposure for some EMEs financial corporations, non-financial corporations and the general government sector at the end of December 2019, especially for Brazil, Mexico, Turkey and Indonesia.¹¹

Although in recent times, EMEs sovereign bonds issuance has increasingly been denominated in domestic currency, there remains a relevant share of foreign currency exposure in loans and debt securities that contribute to public sector external debt. Furthermore, rising foreign currency debt of EMEs corporates, has generated economy wide foreign currency debt exposure and thus indicating weakening net external debt positions of some EMEs. Concerning the residence and nationality basis, Chui et al. (2016) show a high degree and a rapid increase in foreign currency exposure of the international debt securities of non-financial companies of Brazil, China, India, Russia, other major Asian EMEs and for South Africa over the 2010-2015 period, more so for the nationality basis. Coppola et al. (2020) show that using the nationality basis rather than the residence basis, substantially decreases the domestic currency share of external debt of EMEs and this exhibits a high degree of foreign currency exposure in EMEs external debt. Similarly, the BIS (2020) shows the international debt securities exposure of EMEs financial and non-financial corporations, and the nationality basis shows a higher degree of international debt exposure relative to the residence basis.

Cantú et al. (2020) show that some EMEs state-owned enterprises have significant foreign currency debt to GDP ratios, especially Malaysia, Mexico and South Africa. Thus, these factors exhibit EMEs vulnerability to a foreign debt crisis that could be triggered by large unexpected exchange rate depreciations. Chui et al. (2016) explain that the surge in global liquidity after the GFC was influenced by central banks globally lowering their policy rates and expanding their balance sheets through asset purchases. Thus, they document that such policies have increased foreign currency borrowing by EMEs along with generating a sharp increase in aggregate currency mismatches of EMEs non-financial and non-government sectors balance sheets since 2010.¹² All these factors have resulted in what Chui

¹⁰Country groups regressions are mostly based on the IMF's currency composition of official foreign exchange reserves (COFER) data which is available for the whole world, advanced economies and for emerging and developing economies country groups; however discontinued from 2015Q2 for emerging and developing economies.

¹¹<http://stats.bis.org/statx/srs/table/c1>

¹²Chui et al. (2016) specify that it is non-financial companies producing tradable goods and services and those producing non-tradable goods, that have exhibited a rapid increase in foreign currency debt after the GFC.

et al. (2016) refer to as a new and powerful dimension to currency mismatches in EMEs in recent times. Moreover, they note that this dimension has resulted in some reversal in the policy achievement in eliminating currency mismatches of EMEs government balance sheets with an associated rise in currency mismatches of EMEs non-financial companies.

Bocola and Lorenzoni (2020) explain that foreign currency debt is perceived as a source of financial instability in EMEs because the negative effects of foreign currency debt exposure are amplified during periods of substantial currency depreciation and these depreciations increase the real burden of foreign currency debt. Based on a sample of 1213 firms from 18 EMEs, Bruno and Shin (2020) find and emphasize that EMEs nonfinancial firms' vulnerability to currency depreciation, is not only generated by foreign currency debt exposure, instead it is generated by a combination of foreign currency debt and the usage of foreign currency debt proceeds to accumulate financial assets denominated in domestic currency. The adverse effects of currency depreciations are intensified when a country's foreign currency assets and foreign currency receipts do not match its foreign currency debt obligations. Furthermore, large unexpected exchange rate depreciations - typically triggered by unavoidable external factors - can generate negative effects by increasing EMEs foreign currency debt, increasing foreign debt to GDP ratios and increasing the possibility of many sectors experiencing foreign debt defaults. Such settings can generate devastating economic outcomes, including several years of output losses.

Concerning sovereign defaults, Levy Yeyati and Panizza (2011) argue that the negative effects of a default are likely to be influenced by the expectation of the event irrespective of whether or not a country defaults. They also point out that EMEs exhibit comparable vulnerability to external shocks that can result in capital account reversals, sudden stops and higher propensities to default on international debt. Asonuma et al. (2019) document that the economic costs of sovereign defaults are contingent on whether debt restructuring occurs before the default where payments have not been missed or whether restructuring takes place post default.

Using a small open economy model that incorporates a collateral constraint, Korinek (2018) characterizes the contractionary effects of real depreciations and the associated financial amplification effects. His model captures the typical dynamics of the exchange rate, the current account and economic activity during EMEs crises that may be triggered by an increase in international investors risk aversion or a negative output shock; along with the resulting feedback loop effects between these negative shocks, real depreciations and tightening financial constraints. Similarly, and using a small open economy model, Céspedes et al. (2017) show that the exchange rate can influence international credit constraints through a net worth and leverage effect because a real depreciation can result in negative balance sheet effects and thus tighten credit limits of banks. The negative economic effects of exchange rate depreciations and the associated transmission channels, are also documented in numerous studies, see for example, Krugman (1999), Aghion et al. (2001, 2004), Levy Yeyati (2006), Bordo et al. (2010), Hardy (2018), Shousha (2019) and Caballero (2020).

3 Model and further motivation

3.1 Model

For a small portion of the total portfolio of FX reserves, I characterize an EME central bank as a risk-averse investor. I incorporate portfolio rebalancing transaction costs so that they explicitly reduce expected portfolio return. Thus, I assume that the purchase and sale of FX reserves - typically foreign government securities - reduce expected portfolio return and I incorporate these transaction costs as a proportion of expected portfolio return.¹³ I make this assumption because it seems plausible to assume that the central bank may in general be concerned about total portfolio transaction costs rather than the transaction costs of each FX reserve.

I define IRRA and CRRA expected utility as follows, respectively:

$$E[U(\xi R_p)] = E[-e^{(-\lambda \xi R_p)}], \quad E[U(\xi R_p)] = E\left[\frac{(\xi R_p)^{1-\theta} - 1}{1-\theta}\right]. \quad (2)$$

Using a third order Taylor series expansion, I approximate IRRA and CRRA expected utility for the central bank, where $A_U(\xi R_p) = \frac{-U''(\xi R_p)}{U'(\xi R_p)} = \lambda > 0$ and $R_U(\xi R_p) = (\xi R_p) A(\xi R_p) = \xi R_p \lambda > 0$ are the respective parameters of absolute and relative risk aversion for IRRA utility $\forall \xi R_p > 0$, where ξR_p is portfolio return adjusted for portfolio rebalancing transaction costs.¹⁴ For CRRA utility, $A_U(\xi R_p) = \frac{-U''(\xi R_p)}{U'(\xi R_p)} = \frac{\theta}{\xi R_p} > 0$ and $R_U(\xi R_p) = \xi R_p A(\xi R_p) = \theta > 1$ are the respective parameters of absolute and relative risk aversion $\forall \xi R_p > 0$. Furthermore, IRRA utility exhibits $\frac{dA_U(\xi R_p)}{d(\xi R_p)} = 0$, $\frac{dR_U(\xi R_p)}{d(\xi R_p)} = \lambda > 0$ and CRRA utility exhibits $\frac{dA_U(\xi R_p)}{d(\xi R_p)} = \frac{-\theta}{(\xi R_p)^2} < 0$ and $\frac{dR_U(\xi R_p)}{d(\xi R_p)} = 0$, $\forall \xi R_p > 0$, see also Kane (1982), Varian (1988) and Meyer and Meyer (2005). For my proposal, IRRA and CRRA utility exhibit non-increasing absolute risk aversion and this property is a sufficient condition for a positive third derivative of a utility function that in turns implies a preference for positive skewness (Tsiang, 1972; Kraus and Litzenberger, 1976).¹⁵

For my purposes, I use IRRA and CRRA expected utility of portfolio return adjusted for portfolio rebalancing transactions costs. Thus, I construct an approximation around portfolio mean return

¹³Evident if policy authorities purchase foreign government securities at a premium and if they sell these securities at a discount and this can depend on prevailing market conditions, however both these actions can reduce expected portfolio return. Also, noting that FX reserves assets can include corporate and other debt securities that exhibit low credit and liquidity risk, see Wooldridge (2006) and the IMF's International Reserves and Foreign Currency Liquidity data template on the instrument composition of FX reserves and non-currency reserves.

¹⁴In this context, IRRA expected utility is the same as constant absolute risk aversion (CARA) utility.

¹⁵This follows from $\frac{d\left[\frac{-U''(\xi R_p)}{U'(\xi R_p)}\right]}{d(\xi R_p)} \leq 0$. Within this context, Pratt-Arrow measures of risk aversion are the desirable properties for an investors utility function and these are (i) positive marginal utility for return (i.e. non-satiety for return), (ii) decreasing marginal utility for return (i.e. risk aversion) and (iii) non-increasing absolute risk aversion (i.e. risky assets are not inferior goods), see also Kraus and Litzenberger (1976).

adjusted for portfolio rebalancing transactions costs.¹⁶ Using tensor notation for a portfolio of n FX reserves, in each period t , the central bank selects \mathbf{w} to maximize expected utility between period t and $t + 1$ and thus the central bank problem is as follows:

$$\max_{\mathbf{w}} E_{|\Pi} \left[-e^{(-\lambda\xi R_p)} \right] \approx \max_{\mathbf{w}} \left[\begin{array}{l} -e^{(-\lambda\xi\mathbf{w}'\boldsymbol{\mu})} - \frac{(\lambda\xi)^2 e^{(-\lambda\xi\mathbf{w}'\boldsymbol{\mu})}}{2} \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \\ + \frac{(\lambda\xi)^3 e^{(-\lambda\xi\mathbf{w}'\boldsymbol{\mu})}}{6} \mathbf{w}'\mathbf{M}_3(\mathbf{w} \otimes \mathbf{w}) \end{array} \right] \quad (5)$$

or

$$\max_{\mathbf{w}} E_{|\Pi} \left[\frac{(\xi R_p)^{1-\theta} - 1}{1-\theta} \right] \approx \max_{\mathbf{w}} \left[\begin{array}{l} \frac{(\xi\mathbf{w}'\boldsymbol{\mu})^{1-\theta} - 1}{1-\theta} - \frac{\theta\xi^2(\xi\mathbf{w}'\boldsymbol{\mu})^{-\theta-1}}{2} \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \\ + \frac{\theta(\theta+1)\xi^3(\xi\mathbf{w}'\boldsymbol{\mu})^{-\theta-2}}{6} \mathbf{w}'\mathbf{M}_3(\mathbf{w} \otimes \mathbf{w}) \end{array} \right] \quad (6)$$

$$s.t. \text{ (a) } \mathbf{i}'\mathbf{w} = \mathbf{1}, \mathbf{i}' = [1, 1, \dots, 1], \quad (7)$$

$$\text{(b) } 0.13 \leq w_h \leq 0.26, \quad 0.24 \leq w_i \leq 0.48, \quad 0.18 \leq w_j \leq 0.36,$$

$$\text{where } h, i, j = 1, 2, \dots, n, \text{ and } \xi = [1 - \tau], \quad 0 < \xi < 1, \quad 0 < \tau < 1,$$

where τ is the total portfolio rebalancing transaction cost rate between period t and $t + 1$ and this rate is a proportion of portfolio return, where R_p is portfolio return between period t and $t + 1$, \mathbf{w} is a $nx1$ FX reserves weight vector, $R_{i,t+1}$ is the return of each FX reserve $i = 1, 2, \dots, n$ between period t and $t + 1$ and \mathbf{R} is a $nx1$ return vector consisting of the n FX reserves and $\mathbf{w}'\mathbf{R} = R_p$. Thus, $E[\mathbf{R}] = \boldsymbol{\mu}$ is a $nx1$ mean return vector of the n FX reserves and $\mathbf{w}'\boldsymbol{\mu} = \mu_p$ is the portfolio mean return. $E_{|\Pi}[U(\xi R_p)]$ is the appropriate expected utility function used by the central bank for portfolio selection, depending on global market conditions that influence and generate different central bank preferences.

For each FX reserve weight w_i , where $h, i, j = 1, 2, \dots, n$, the allocation is bounded above by the associated currency share in a country's currency composition of foreign debt. Concerning lower bounds, each FX reserve cannot be allocated less than half the associated currency share in a country's currency composition of foreign debt. For example, *Euro* denominated FX reserves cannot be allocated less than 24 % and cannot have an allocation above 48 %, where 48 % is the actual weight of *Euro* denominated debt in a country's currency composition of foreign debt, i.e. $0.24 \leq w_i \leq 0.48$. Using generic notation, I define the covariance between returns on FX reserves i and j as $\sigma_{ij} = E[(R_i - E(R_i))(R_j - E(R_j))]$, for $i, j = 1, 2, \dots, n$ and the co-skewness between the returns on FX reserves i, j and k as $S_{ijk} = E[(R_i - E(R_i))(R_j - E(R_j))(R_k - E(R_k))]$, for $i, j, k = 1, 2, \dots, n$.

Thus, I express the portfolio moments as follows: $\boldsymbol{\Sigma} = \mathbf{E}[(\mathbf{R} - \boldsymbol{\mu})(\mathbf{R} - \boldsymbol{\mu})']$, $\sigma_p^2 = \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$, $\mathbf{M}_3 = \mathbf{E}[(\mathbf{R} - \boldsymbol{\mu})(\mathbf{R} - \boldsymbol{\mu})' \otimes (\mathbf{R} - \boldsymbol{\mu})']$, $S_p^3 = \mathbf{w}'\mathbf{M}_3(\mathbf{w} \otimes \mathbf{w})$, where $\boldsymbol{\Sigma}$ is a nxn covariance matrix, σ_p^2 is

¹⁶See also Makin (1971), Danthine and Donaldson (2005) and Bricc et al. (2007) for utility of portfolio return.

portfolio variance, \mathbf{M}_3 is a nxn^2 co-skewness matrix, S_p^3 is portfolio skewness and \otimes is the Kronecker product, see also Athayde and Flôres (2004), Jondeau and Rockinger (2006), Briec et al. (2007) and Harvey et al. (2010) on the usage of tensor notation in portfolio models.¹⁷ The optimal weight vector \mathbf{w} that maximizes each of the central bank’s preference function, eqns. (5) and (6), depending on global market conditions, eqn. (1), is determined on the basis of explicit IRRA or CRRA expected utility, taking into account the relevant parameters of risk aversion (θ, λ) and the portfolio rebalancing transaction cost rate τ and this optimal portfolio allocation is computed numerically, see also Jondeau and Rockinger (2006).¹⁸

Eqns. (5) and (6) show that the central bank’s preference functions, exhibit a trade-off between a low (positive) portfolio mean return, for a low portfolio variance. Furthermore, for a low portfolio mean return or for any given (positive) portfolio mean return, each model pursues high positive portfolio skewness and thus a trade-off between low portfolio mean return for high positive portfolio skewness. Low portfolio mean return and low portfolio variance, along with low portfolio mean return and high positive portfolio skewness, jointly interact to maximize expected utility. This trade-off is more emphasized with a smaller portfolio mean return and reinforced by a high-risk aversion profile.¹⁹

The extent to which a portfolio is diversified, influences its skewness. If investors prefer positive skewness, Simkowitz and Beedles (1978) and Conine and Tamarkin (1981) show that investors may prefer under-diversified portfolios that generate positive portfolio skewness. For my proposal, I limit the possibility of under-diversified portfolios by incorporating lower and upper bound currency composition of foreign debt constraints, however within a set-up of pursuing and maximizing positive portfolio skewness.²⁰ These constraints decrease the possibility of over-allocating towards FX reserves with moments and co-moments such as high expected returns, small variances, negative covariances and high positive co-skewness. Such moment characteristics can maximize the central bank’s expected utility; however they may not allow the currency composition of FX reserves to match the currency composition of foreign debt as best as possible. Thus, the constraints preclude over allocating FX reserve weights beyond the associated actual foreign debt weights and preclude under allocating FX

¹⁷I define skewness using central moments, see also Jondeau and Rockinger (2006) and Briec et al. (2007).

¹⁸Jondeau and Rockinger (2006) maximize approximated expected IRRA utility with respect to the weight vector and show that optimization can be based on moments or direct numerical optimization of expected utility. They also explore the optimization of CRRA utility in a similar manner, along with its associated optimality expression that may be solved numerically.

¹⁹Arditti (1967) provides a formal outline using a risk premium measure that decreases with wealth and establishes that the coefficient of the third moment must be positive. As a result, an investor is willing to accept lower expected return along with higher positive skewness and the same variance relative to other opportunities. Thus, he interprets that a risk averse investor exhibits reluctance towards a portfolio with a small likelihood of a large loss - no matter how small - and only a limited gain.

²⁰Based on the signs attached to each moment in my approximation, I assume a preference towards odd moments and aversion to even moments. However, I acknowledge Brockett and Garven’s (1998) study that shows that expected utility preferences never universally translate into moment preferences. Thus, moment ordering is a necessary but not a sufficient condition for utility ordering.

reserve weights lower than half the associated actual foreign debt weights.

Proposition 1 (*Operational elements for the central bank*).

Operationally, my model proposal is a maximization of the central bank's preference function in each period t , resulting in a portfolio held over period t and $t + 1$, where portfolio selection is based on information available up to the end of period $t - 1$.

Operationally, the central bank problem is as follows:

(i) in the period of initialization period t , the central bank has information on the country's (annual) currency composition of foreign debt up to the end of period $t - 1$. The central bank computes a sequence of ex-post one period ahead expected returns for each of the n FX reserves using different methods for computing expected returns and using monthly data.²¹ This is a sequence from previous periods up to the end of period $t - 1$ and is a sequence that is consistent with the period of initialization and the currency composition of foreign debt up to the end of period $t - 1$. Thus, the central bank has a vector of ex-post one period ahead expected returns for each of the n FX reserves.

(ii) With this information, the central bank computes moments and co-moments of the n FX reserves based on a large sample. Then, with the computed moments and co-moments, depending on prevailing global market conditions and thus the associated risk preferences aligned with either IRRA or CRRA expected utility and assigned values of the relevant parameters of risk aversion (θ, λ) , the central bank simulates its currency composition of FX reserves using the country's currency composition of foreign debt up to the end of period $t - 1$ as portfolio constraints.

(iii) The simulated weights generate a FX reserves portfolio that is held between period t and $t + 1$ and this portfolio generates returns over the holding period. This portfolio selection framework allows the central bank to align its currency composition of FX reserves to the most recently available and observed currency composition of foreign debt. Furthermore, this potentially covers for any outstanding foreign debt that is due soon, for example between period t and $t + 1$ and new foreign debt accumulated up to the end of period $t - 1$, due in later periods. Using ex-post one period ahead expected returns captures the actual dynamics of FX reserves returns. Thus, the central bank portfolio selection is informed by a meaningful and valid information set.

Remark 1 (*Portfolio rebalancing transaction costs*).

(i) In the period of initialization and optimization, the resulting purchase of the associated FX reserves that maximize the central bank's preference function, can include transaction costs that reduce expected portfolio return. Thus, in simulating FX reserve weights in period t , the central bank

²¹I provide more details in section 4 on the different methods I use for computing expected returns.

can assume and assign a value for the total portfolio rebalancing transaction cost rate τ , that may reflect the purchase of FX reserves *ex ante* and reduce expected portfolio return in period t .²²

(ii) After the initial portfolio holding period between t and $t + 1$, and with new data available about the country's currency composition of foreign debt at the end of period t , at the beginning of period $t + 1$ the central bank computes (ex-post) one period ahead expected returns for each of the n FX reserves.²³ Then, the central bank has an updated sequence of expected returns and it computes updated moments and co-moments. Thus, at the beginning of period $t + 1$, with new or unaltered risk preferences based on assigned values of the relevant parameters of risk aversion (θ, λ) , the central bank simulates its currency composition of FX reserves that maximizes its preference function; using the updated and most recently available currency composition of foreign debt as portfolio constraints.

(iii) However, in period $t + 1$, portfolio rebalancing is in the form of purchasing and selling FX reserves towards a FX reserve allocation that maximizes the central bank's preference function. Thus, in simulating FX reserve weights in this new period, the central bank can assume a value for the total portfolio rebalancing transaction cost rate τ , that may reflect the purchase and sale of FX reserves *ex ante*. The assumed portfolio rebalancing transaction cost rate reduces expected portfolio return in period $t + 1$.²⁴

(iv) As a result and going forward in each new period of portfolio selection, the central bank updates its relevant information set concerning expected FX reserves returns and the associated moments and co-moments, the country's currency composition of foreign debt, its risk preferences through the relevant parameter of risk aversion and a value for the total portfolio rebalancing transaction cost rate. Then, it simulates FX reserves weights that maximize its preference function; along with matching the country's currency composition of foreign debt.

To avoid excessive rebalancing, I propose a 12-month portfolio holding period, for given central bank preferences that are influenced by prevailing global market conditions and thus the usage of an appropriate expected utility function. In this set-up, the central bank uses the most recently available average (annual) currency composition of foreign debt as constraints, over a period such as 2010-2018. I validate my proposal using the average foreign debt weights over the 2010-2018 period. I view this as a better matching strategy because the average will most likely exhibit smaller changes as compared to period-by-period changes in the currency composition of foreign debt. Thus, this can prevent the central bank from having to substantially adjust its currency composition of FX reserves to match its country's currency composition of foreign debt. Moreover, using the average currency composition of

²²Where the assumed ex-ante total portfolio rebalancing transaction cost rate may differ from the realized ex-post total portfolio rebalancing transaction cost rate over period t and $t + 1$.

²³Noting that over each portfolio holding period, the central bank is also updating its information set about its country's currency composition of foreign debt and updating its information set about variables that are relevant for computing expected returns using different methods for each of the n FX reserves over the holding period; along with observing the actual generated returns over the holding period and the actual portfolio rebalancing transaction costs.

²⁴The ex-post transaction rebalancing cost rate, reduces expected portfolio return between period $t + 1$ and $t + 2$.

foreign debt potentially covers for any outstanding foreign debt that is due soon and new foreign debt due in later periods; irrespective of the debt maturity structure.

Nonetheless, there are other variants of portfolio models used in the literature on the currency composition of FX reserves and these differ to my proposal. For example, Makin (1971) uses cubic utility of returns in a portfolio model for reserve assets based on a choice between U.S. dollars and gold. Ben-Bassat (1980) uses a mean-variance model with a no short selling constraint to select an optimal FX reserves portfolio for semi-industrial and developing countries and thus compares the actual portfolios relative to the efficient portfolios. He also estimates an efficiency frontier for the Bank of Israel in terms of imports. Dellas and Yoo (1991) compare the actual currency composition of FX reserves of Korea relative to portfolios generated by a mean-variance model with a no short selling constraint. Ramaswamy (1999) proposes a framework in which a central bank's objective is minimizing the worst possible return outcome in different currency numeraires. Following this set-up, the currency allocation problem is solved using a multi-objective optimization problem within the context of fuzzy decision theory.

Gintschel and Scherer (2004) develop a currency allocation decision framework for central banks as a multi-objective optimization problem with different weights attached to a nominal wealth preservation benchmark and a liquidity benchmark; along with alternative risk regimes. In the context of four large EMEs, Papaioannou et al. (2006) use a mean-variance framework with portfolio rebalancing costs to simulate optimal portfolio weights comprising of major reserve currencies, with an objective function of maximizing expected portfolio return.²⁵ Mateane and Semmler (2016) minimize portfolio variance for the South African Reserve Bank and simulate optimal FX reserve weights by using lower and upper bound constraints that are South Africa's currency composition of foreign debt.

Farhi and Maggiori (2018) propose a model of the international monetary system; where they characterize world demand for reserve assets with international investors that exhibit mean-variance preferences. Their analysis examines a variety of scenarios such as a world with many relative to scarce reserve assets, a gold exchange standard relative to a floating exchange rate system and a case with only a single issuer of reserve assets or a few reserve currency issuers. Lu and Wang (2019) present a central bank's reserve portfolio choice, where they adopt a mean-variance model for the investment tranche portfolio and adopt an asset-liability framework for the liquidity tranche portfolio.

²⁵They simulate optimal weights for Brazil, China, India and Russia and incorporate constraints such that the composition of FX reserves is consistent with currencies of a country's peg, foreign debt and trade. In their model, they note that Russia had about 65 % of its external debt in U.S. dollar and 29 % in Deutsche marks. Thus, they impose a constraint that the Russian central bank would desire to hold at least 32.5 % of its reserves in U.S. dollars and 14.5 % of its reserves in Deutsche mark. They note that the 50 % thresholds are arbitrary. By only imposing lower bound constraints, their approach over allocates FX reserve weights beyond the associated actual foreign debt weights.

3.2 Further motivation

Beyond portfolio selection models for the currency composition of FX reserves, the literature presents and examines the transactions approach. In this approach, international transaction activities related to a country or country groups, are argued to influence the associated currency composition of FX reserves. Thus, country's composition of trade flows, country's currency composition of trade invoicing, exchange rate arrangements and movements, country's currency composition of foreign debt and a wide array of other economic variables are used for examining whether they influence the currency composition of FX reserves; and generally with regressions.²⁶

Stekler and Piekarz (1970) and Galati and Wooldridge (2009) outline that a reserve currency can serve as a medium of international exchange, a store of value and can be used for intervention in FX markets. In the context of exchange rate arrangements, trade flows and from a global perspective, Heller and Knight (1978) find that countries increase their share of FX reserves in the currency in which they peg relative to; and also countries increase their FX reserve shares associated with reserve-countries that are important trading partners. Across industrial and developing countries, Dooley et al. (1989) find that the composition of trade flows, the currency composition of foreign debt and currency pegs are some of the important determinants of the currency composition of FX reserves. Updating on this, Eichengreen and Mathieson (2000) find and conclude that this relationship is stable over time, however they mainly focus on emerging and transition economies.

Chinn and Frankel (2008) find that the size of a reserve currency country, the rate of return on a reserve currency and foreign exchange market turnover in the reserve currency country, influence and increase the associated reserve currency share in the global FX reserves portfolio. Using interwar data between 1924-1939, Hatase and Ohnuki (2009) find that Japan's reserve currency preferences are influenced by its trade structures and more so by its currency denomination of foreign debt. Ouyang and Li (2013) find that it is mainly the size of a reserve currency country, its share of world military expenditure and its share of international bonds, that positively influence the weight of the associated reserve currency in the global FX reserves portfolio. Using a wide array of 24 industrialized economies and EMEs, McCauley and Chan (2014) find that higher co-movement between the currency of a country with the U.S. dollar, anchors the U.S. dollar share in the country's currency composition of FX reserves. Ito et al. (2015) find that trade invoicing and currency movements influence the currency composition of FX reserves for five central and eastern European countries. Soesmanto et al. (2015) also find that transaction motives influence the currency composition of FX reserves of Australia.

From a global perspective, Eichengreen et al. (2015), find that inertia and policy credibility effects strongly influence the currency composition of FX reserves in the long run and network effects have

²⁶Country groups regressions are mostly based on the IMF's currency composition of official foreign exchange reserves (COFER) data which is available for the whole world, advanced economies and for emerging and developing economies country groups; however discontinued from 2015Q2 for emerging and developing economies.

weakened after the Bretton Woods era. Furthermore, they find that public debt to GDP ratios and fiscal balance to GDP ratios influence reserve currency choice. Using a data set of 19 countries prior to World War I, Eichengreen et al. (2019) examine the role of economic and military/security considerations in FX reserve currency choice. They find and conclude that the size of a reserve currency country, the credibility of a reserve currency country and geopolitical factors such as military alliances between a country and a reserve currency country, positively influence the weight of the associated reserve currency in the partner country's currency composition of FX reserves.

Using a sample of 22 emerging and developing countries, Lu and Wang (2019) find that the currency composition of imports invoicing and the currency composition of short-term foreign debt, significantly influence the currency composition of FX reserves. Using a sample of 58 countries consisting of advanced, emerging and developing economies, Aizenman et al. (2020) find that the composition of trade flows and the currency composition of foreign debt, influence the currency composition of FX reserves and more so for the emerging and developing economies in their sample. Lastly, using a constructed dataset for 58 countries, Ito and McCauley (2020) find that the currency composition of FX reserves of these economies, are strongly related to the bilateral exchange rate movement between domestic currency and reserve currencies, and the currency composition of trade invoicing.

Nugée (2000) discusses the role of FX reserves in servicing foreign currency obligations and the benefits of having FX reserves as compared to outright purchases of foreign currency with domestic currency in the market, when a country's foreign currency obligations fall due. This is important because countries can face challenges when trying to purchase foreign currency with domestic currency especially during difficult market conditions. These effects can be exacerbated by negative investor sentiment that also depends on prevailing global market conditions (Nugée, 2000; Claessens and Kreuser, 2007). Among others, Bianchi et al. (2018) note that holding reserves is costly and this is another reason that justifies my proposal because it may yield a trade-off for policy authorities because of the insurance benefits of a currency composition of FX reserves related prudential strategy over different global market conditions. Feldstein (1999) and Rodrik (2006) state the costs of reserves for EMEs. For example, the interest earned on the purchase of reserves is generally lower than the interest paid on bonds used for financing the purchase of reserves by EMEs, see also Levy Yeyati (2008) and Dominguez et al. (2012). There are also opportunity costs associated with holding reserves, rather than investing in the real economy (Ben-Bassat and Gottlieb, 1992a, b).²⁷

The relevance of my proposal is that foreign debt default by a sector or an institution, can negatively impact other public and private institutions that have foreign debt exposure because of worsening currency depreciation, worsening negative balance sheets effects and the destabilization of the economy through many channels (Asonuma et al., 2019). As a result, it is in the best interest of policy authorities to take responsibility for dealing with a potential crisis, where systemic risk assessments are also

²⁷These costs correspond to FX reserves accumulated through outright purchases of foreign currency in exchange for domestic currency and also through the issuance of an international debt security.

consistent with the historical role of policy authorities as crisis lenders and crisis managers (Fischer, 1999; Upper, 2017; Villar, 2017). Even if an EME's net foreign currency asset position is balanced or positive, liquidity mismatches pose as a risk and open the possibility of default. Furthermore, aggregate balanced or positive net foreign currency asset positions may conceal mismatches in the context of different sectors in the economy. For example, some sectors may have large positive net foreign currency asset positions, whereas other sectors may have large or small negative net foreign currency asset positions.

As a result, the country's net foreign currency asset positions may conceal both liquidity and sectoral mismatches. The net foreign currency assets positions also do not provide information about currency mismatches across individual currencies and this is relevant when foreign debt is not concentrated in one foreign currency. Thus, my proposal potentially deals with these issues by accounting for a country's total foreign debt and its currency composition of foreign debt. By using different maturity structures on the currency composition of FX reserves in my validation process, I potentially cover the wide spectrum of short and long term foreign debt. Such a prudential strategy reinforces the role of a central bank as a progressive risk manager for the benefit of its economy. In particular, a procedure that may allow a central bank to anchor its country's ability to withstand foreign debt defaults. More so, when unavoidable external factors result in large unexpected real exchange rate depreciations, that can generate foreign debt defaults.

The costs of foreign debt default are examined and discussed in numerous studies. For example, Ben-Bassat and Gottlieb (1992a) explain the costs of foreign debt defaults, and how a country's insolvency damages its relations with the rest of the world along with output reductions for several years. Motivated by the international debt default experience of Argentina with its associated negative economic outcomes, Arellano (2008) models the costs of defaults in the form of exclusion from international financial markets and direct output costs. Using a group of EMEs, Levy Yeyati and Panizza (2011) find that sovereign defaults are associated with output contractions. However, they focus on the timing of the default along with the timing of the associated economic effects. Thus, they argue that the negative effects of a default are likely to be influenced by the anticipation of the event irrespective of whether or not a country defaults. Based on Levy Yeyati and Panizza's (2011) study and to characterize several empirical regularities, Mendoza and Yue (2012) use data from 23 sovereign default events associated with a group of EMEs over the 1977-2009 period.

They document three key facts: (i) defaults are associated with large output contractions (ii) there is a negative correlation between interest rates on sovereign debt and output and (iii) countries that default have high external debt to GDP ratios and these ratios are amplified when countries default. Thus, Mendoza and Yue (2012) propose a general equilibrium model of sovereign default and business cycles and their model has quantitative predictions that are consistent with observed empirical regularities of actual sovereign defaults; along with resolving the disconnection between

sovereign debt models and EMEs business cycles. Using a group of developing economies and EMEs, Trebesch and Zabel (2017) examine the output costs of sovereign defaults and differentiate between hard and soft defaults. They are able to determine that the more severe are defaults, the greater are the output losses and thus establishing differences in debt crises and the associated output costs.

There is always a possibility of moral hazard related with foreign debt accumulation by the private and public sector because of the knowledge that the central bank may be willing to intervene to safeguard the domestic economy. Moreover, the private and public sector may take upon more foreign debt and possibly reduce their foreign currency exposure hedging outcomes. However, EMEs policy authorities typically implement measures that may allow them to assess the foreign currency positions of its country across all sectors, along with continuous collaborative efforts and updating on information about the country's currency composition of foreign debt. For example, all foreign borrowing by domestic institutions can be subject to policy authority approval and declaration on foreign currency positions along with limits on foreign currency positions, can be imposed on domestic institutions by policy authorities (Ostry et al., 2012; Upper, 2017; Villar, 2017).

Furthermore, policy authorities can continuously assess the expected profitability of domestic institutions - relative to other factors such as the cost of capital - of new investment projects financed by foreign debt so as to potentially offset moral hazard, see Corsetti et al. (1999) for an analysis within the context of the 1997-98 Asian crisis. This works well with an outcome that suitably managed FX reserves, may not be used for anchoring sectors or institutions that are likely to default on foreign debt because of speculative borrowing or mismanagement on the part of borrowers. Moreover, an approach of continuous assessment of foreign currency debt associated with a high degree of monitoring effort and supervisory effort on the part of policymakers, however without disruption of efficient and potentially profitable operational mechanisms of institutions or sectors, see also Jeanne (2000).

Within this context, Jeanne (2000) provides alternative policy recommendations about dealing with risks posed by EMEs foreign currency debt. Thus, one of his arguments based on his model, is that restricted forms of assistance on foreign currency debt obligations by policy authorities can be optimal given they contain moral hazard within appropriate bands. Chang and Velasco (2001) explain that incorporating an international lender of last resort in their model of financial crises in EMEs, can prevent avoidable credit crises and costly liquidation of investment. However, they emphasize that moral hazard must be accounted for and thus international assistance can be provided given that stringent conditions are put into place to potentially prevent risky incentives.²⁸

²⁸Relatedly, although my proposal is not about the total FX reserves portfolio being used as outright collateral for a country's foreign currency debt, Alfaro and Kanczuk (2009) discuss whether reserves can be used as collateral and point out that such an arrangement can work given there is government willingness along with credibility and provide an example with respect to Germany.

4 Data and computing expected FX reserves returns

4.1 Data

I use data over the 2010-2018 period for Brazil, India, Indonesia, Mexico, South Africa and Turkey. I use these 6 EMEs because they exhibit rising total and short-term external debt to GDP ratios and rising and significant FX Reserves to GDP ratios over the 2010-2018 period. Furthermore, they exhibit concurrently rising public sector and private sector external debt to GDP ratios. These features show the necessity of assessing the total external debt of these EMEs and accounting for all sectors, rather than only assessing one sector. Rising and significant FX Reserves to GDP ratios refers to ratios that are above 10 % and that exhibit an upward trajectory, see also Truman and Wong (2006). Rising FX reserves to GDP ratios, provide policy authorities leeway to pursue active FX reserves management; along with the main policy and operational purposes of FX reserves.

For each EME, I construct the annual FX reserves to GDP ratios with data sourced from the IMF's International Financial Statistics (IFS) (for the FX reserves data) and the IMF's World Economic Outlook (WOE) (for GDP data). Concerning external debt to GDP ratios, I use the annual external debt to GNI ratio (%) sourced from the World Bank's International Debt Statistics and for robustness I construct an external debt to GDP ratio (%) using World Bank and IMF WOE data. I also construct public sector external, private sector external and short-term external debt to GDP ratios (%) using World Bank and IMF WOE data and all these ratios exhibit an upward trajectory and are comparable across the 6 EMEs in this paper, see Figures 1, 2 and 3.²⁹ Concerning the 10 % threshold, Turkey and South Africa's FX reserves to GDP ratios are marginal cases and more so for Turkey because in 2011 and 2018, its FX reserves to GDP ratio dips marginally around 9 %, however for the other years over the 2010-2018 period, the ratio is above 10 %. For South Africa and for the years 2010 and 2011, its FX reserves to GDP ratio is about 9.5 %, however thereafter the ratio is increasing and above 10 % and almost reaching 14 %. For Brazil, India and Turkey, their respective private sector external debt to GDP ratios are greater than the public sector external debt to GDP ratios and are increasing at a more rapid rate over the 2010-2018 period.

In contrast, for Indonesia, Mexico and South Africa, their respective public sector external debt to GDP ratios are greater than the private sector external debt to GDP ratios and are increasing at

²⁹The World Bank's International Debt Statistics are defined as follows: (i) External debt is total external debt owed to non-residents repayable in currency, goods, or services. It is the sum of public, publicly guaranteed, and private nonguaranteed long-term debt, short-term debt, and use of IMF credit. (ii) Private sector external debt is private nonguaranteed long-term external debt obligations that are not guaranteed for repayment by a public entity. (iii) Public sector debt comprises long-term external debt obligations of public debtors, including the national government of all levels, political subdivisions (or an agency of either), autonomous public bodies such as Public Corporations, State Owned Enterprises, Development Banks and Other Mixed Enterprises. (iv) Short-term external debt is defined as debt that has an original maturity of one year or less and this data does not distinguish between public and private nonguaranteed short-term debt.

a more rapid rate over the 2010-2018.³⁰ Lastly, the short-term external debt to GDP ratios for all the EMEs, exhibit an upward trend and more so for Brazil, Mexico, South Africa and Turkey. I also use these EMEs because they exhibit a high degree of capital account openness, they are vulnerable to currency volatility, international capital flow volatility and foreign debt crises. These features are noted in the World Economic Forum Financial Development Report 2012 for the construction of the World Economic Forum’s Financial Development Index. In particular, the outline and ranking of factors such as the institutional environment and financial stability of the EMEs in this paper because these encompass financial sector liberalization, currency stability, banking system stability and risk of sovereign debt crisis.

Table 1 reports the annual average currency composition of foreign debt of the EMEs in this paper. For the currency composition of external debt for the EMEs in this paper, ideally, I should validate my proposal with two debt maturity structures in the form of (i) short term foreign currency debt and (ii) long term foreign currency debt of a country. However, this data is publicly unavailable for all 6 EMEs. Thus, I validate my proposal using the annual currency composition of external long-term public and publicly guaranteed debt contracts sourced from the World Bank’s International Debt Statistics as a proxy for the currency composition of short term and long term foreign debt of each respective country over the 2010-2018 period.³¹ This procedure allows me to potentially cover the full spectrum of total foreign currency debt and align it to the different maturity structures of the FX reserves.³² Dooley et al. (1989), Hatase and Ohnuki (2009), Aizenman et al. (2017) and Lu and Wang (2019) use similar proxy approaches on foreign currency debt.³³

³⁰Cantú et al. (2020) show that some EMEs state-owned enterprises have significant foreign currency debt to GDP ratios, especially Malaysia, Mexico and South Africa. Furthermore, BIS statistics show similar outcomes for the amounts outstanding of international debt securities at end of December 2019 for the financial and non-financial corporations and the general government sector of these 6 EMEs. <http://stats.bis.org/statx/srs/table/c1>

³¹I had initially included EMEs such as Argentina, Bulgaria, Chile, Croatia, Hungary, Malaysia, the Philippines and Venezuela in my proposal. However, for Chile, Croatia, Hungary and Malaysia, I cannot obtain data on their respective external debt statistics and the World Bank’s International Debt Statistics does not have records for these countries. Argentina and Venezuela do not have rising and significant FX reserves to GDP ratios over the 2010-2018 period and the FX reserves to GDP ratios are falling reaching levels below 4 %. Argentina and Venezuela also exhibit rapid FX reserves depletion and thus I exclude them. I exclude Bulgaria and the Philippines because they have falling total and short-term foreign currency debt to GDP ratios.

³²For the currency composition of external debt of South Africa, I use annual data reported by the South African Reserve Bank (SARB). The SARB reports the composition of foreign liabilities of South Africa by country of origin and not by currency denomination. I use the country composition of foreign liabilities as a proxy for the currency composition counterpart. This is a reasonable proxy because over the period 1996-2018 the country composition of foreign liabilities of South Africa, correspond to major reserve currency countries.

³³I use World Bank data on the currency composition of external debt because this data covers a wider array of the currency composition of foreign debt and accounts for all the main reserve currencies such as the US dollar, Euro, Pound sterling, Japanese yen and Swiss franc. This differs to BIS data on international debt securities by currency of denomination. BIS data on outstanding international debt securities by currency of denomination is available for a wide array of foreign currencies including the main reserve currencies. However, this is data where issuer residence and nationality are for all countries, rather than for individual countries based on residence and nationality of the issuer. For

Table 1: Actual currency composition of foreign debt of EMEs (weights in %)

<i>Currency</i> U.S. dollar	Brazil	India	Indonesia	Mexico	South Africa	Turkey
2010-2018*	91.51	79.15	75.75	55.32	25.24	68.18
2010-2018**	91.93	79.84	76.13	62.07	27.66	68.31
<i>Currency</i> Euro	Brazil	India	Indonesia	Mexico	South Africa	Turkey
2010-2018*	4.14	3.23	5.20	6.79	26.61	25.36
2010-2018**	4.55	3.91	5.58	13.55	29.03	25.48
<i>Currency</i> British pound	Brazil	India	Indonesia	Mexico	South Africa	Turkey
2010-2018*	0.34	0.70	0.31	0.69	32.33	0.003
2010-2018**	0.76	1.38	0.69	7.45	34.76	0.13
<i>Currency</i> Japanese yen	Brazil	India	Indonesia	Mexico	South Africa	Turkey
2010-2018*	1.83	13.26	16.64	2.99	1.74	5.69
2010-2018**	2.25	13.94	17.03	9.75	4.16	5.81
<i>Currency</i> Swiss franc	Brazil	India	Indonesia	Mexico	South Africa	Turkey
2010-2018*	0.09	0.25	0.18	0.42	1.97	0.15
2010-2018**	0.51	0.93	0.57	7.18	4.39	0.27

Source: World Bank's International Debt Statistics and authors' calculations. Notes: * - These are the average actual currency shares in foreign debt over the relevant period and when adding all of them up, the weights may not add to 100 % because of unidentified currencies. ** - These are the average actual currency shares in foreign debt over the relevant period however adjusted for and including unidentified currencies by equally distributing across the major currencies.

4.2 Computing Expected FX Reserves Returns

For robustness, I compute monthly expected returns using three different methods over the 2010:01-2018:12 period for each of the 6 EMEs. I compute expected FX reserves returns as follows:

$$R_{i,t+1}^e = r_{i,t} + \Delta s_{i,t+1}^e,$$

where $R_{i,t+1}^e$ is the expected return on FX reserve i between period t and $t + 1$ and $r_{i,t}$ is the individual countries - on the basis of residence and nationality of the issuer - the currency composition of international debt securities is only available for US dollar denomination and for the sum of ECU, Euro and legacy currencies now included in the Euro, see also BIS Quarterly Review September 2015 and Aizenman et al. (2017) and Aizenman et al. (2020).

nominal interest rate on FX reserve i . The log of the nominal exchange rate is denominated as the domestic currency units per foreign currency unit and is expressed as $s_{i,t}$ and $\Delta s_{i,t+1}^e$ is the expected percentage change in the exchange rate. For the first method of computing expected returns, I assume the exchange rate follows a random walk (RW model).³⁴ For the second method of computing expected returns, I assume the central bank has perfect foresight of exchange rate movements (PF model), see also Papaioannou et al. (2006). This method captures the impact of realized actual exchange rate changes on returns and the associated impact on the optimal currency composition of FX reserves. For the third method of computing expected returns, I assume expected exchange rate changes based on relative purchasing power parity (PPP model) because of the importance of PPP theory in international finance and because of its importance as a measure of exchange rate misalignment, see among others, Cheung et al. (2005), Rossi (2013) and Cheung et al. (2019). Thus, for the PPP model, I assume that one period ahead expected exchange rate changes are equal to domestic-to-foreign contemporaneous consumer price inflation differentials.

Proposition 2 (*Operational computation of expected FX reserves returns*).

Operational computation of expected FX reserves returns, the timing convention and the extent to which variables are observable.

(i) In the period of initialization, period t , for example January 2019, the central bank computes a sequence of ex-post one period ahead (monthly) expected returns for each of the n FX reserves. This is a sequence from previous periods up to the beginning of the period of initialization and thus up to the end of period $t - 1$, for example expected returns up to December 2018.

(ii) The average currency composition of foreign debt up to the end of period $t - 1$ is used as a constraint, that is in January 2019, the central bank has data on the country's currency composition of foreign debt up to December 2018 and thus can compute the average currency composition over a particular annual period up to December 2018. Noting that after the end of each year, when new data is available, the average currency composition of foreign debt is updated and used as a constraint over the new portfolio optimizing and holding annual period. Thus, in period t (in particular mid-to-end of January 2019), operationally, beginning of the month data on interest rates and monthly average exchange rates up to the end of period $t - 1$ (December 2018), are observable.

Remark 2 (*Feasibility of computing expected FX reserves returns*).

(i) The random walk and the perfect foresight model are feasible because of the availability of a series of ex-post one period ahead exchange rate changes computed on a monthly basis, along with the yield (interest rate) on a FX security that is known at the beginning of each period (month).³⁵

³⁴See also, among others, Dellas and Yoo (1991) and Baz et al. (2001).

³⁵Thus, for the RW model, at the beginning of the month of December 2018, the one month ahead expected return is equal to the interest rate on the FX asset at the beginning of the month. For the PF model, at the beginning of

Furthermore, for the PPP model, domestic and foreign consumer price inflation data up to the end of period $t - 1$, is available in period t . Inflation data up to December 2018 is available in mid-to-end of January 2019 and thus the PPP model is also feasible because of the availability of an ex-post inflation series.³⁶

I have assumed that the exchange rate follows a random walk in one instance and assumed perfect foresight in another. This is not without problems. Assuming random walk of the exchange rate, opens up the possibility of an optimal composition geared towards reserve currencies with the highest yield. Papaioannou et al. (2006) emphasize that this is an unlikely strategy for a central bank because it would imply continuous rebalancing. They also point out that assuming perfect foresight exhibits outcomes under which it is illogical to allocate substantial weight to reserves with negative returns unless a central bank needs to be invested in a reserve asset with its associated risk. My proposal addresses these problems because of (i) the lower and upper bound constraints, (ii) the central bank is not allocating reserves purely on the basis of the highest yield, instead it takes into account higher moments, co-moments and its risk preferences and (iii) even if some reserves may exhibit negative returns, the central bank is invested in such a reserve asset on the basis of constraints unique to a central bank's requirement; its country's currency composition of foreign debt. This in turn shows that my proposal is robust to the shortcomings associated with assuming an exchange rate that follows a random walk and a perfect foresight model.

Another problem is that although I do compute ex-post expected FX reserves returns, I make assumptions about the level and change in one period ahead expected exchange rates. However, forecasting exchange rates has proven to be difficult. This difficulty is well known and has been established through the seminal work of Meese and Rogoff (1983) who find that their candidate structural models do not perform any better than the random walk model at short to medium horizons. The candidate structural models in Meese and Rogoff (1983) consist of the flexible-price (Frenkel-Bilson) model, the sticky-price (Dornbusch-Frankel) monetary models and a sticky-price model that incorporates the current account (Hooper-Morton). Cheung et al. (2005) update the robustness of Meese and Rogoff's (1983) findings by examining exchange rate predictability using two parity conditions - purchasing power parity and the uncovered interest rate parity condition - and three models, namely the sticky-price monetary model, a productivity based differential exchange rate model and a behavioral equilibrium exchange rate model. They find that exchange rate forecasts based on the month of December 2018, the one month ahead expected return is equal to the interest rate plus the change in the monthly average exchange rate between the December 2018 average exchange rate - not observable at beginning of month - minus the November 2018 average exchange rate that is observable at the beginning of the month of December.

³⁶At the beginning of the month of December 2018, the one month ahead expected return is equal to the interest rate on the FX asset at the beginning of the month (observable) plus the inflation rate of the EME for November 2018 minus the inflation rate of the reserve currency country for November 2018 because of assuming that one period ahead expected exchange rate changes are equal to the contemporaneous inflation differentials. At the beginning of December 2018, inflation of November 2018 is observable (in the strictest sense usually by the middle of the following month).

their structural models, do not consistently perform any better than the random walk model. However, Gourinchas and Rey (2005, 2007) find modest predictive power for some bilateral exchange rates in relation to the U.S. dollar and predictive power for a FDI and Federal Reserve trade-weighted U.S. dollar, respectively.³⁷

Within this context, Rossi (2013) conducts a comprehensive analysis of the literature and data, to examine exchange rate predictability. She concludes that there is no consensus outcome concerning exchange rate predictability. Instead, exchange rate predictability depends on a wide array of factors such as the model, the choice of predictors, forecast horizon, sample period and forecast evaluation method. Lastly, Cheung et al. (2019) expand the exchange rate prediction models in Cheung et al. (2005) by including Taylor rule fundamentals, yield curve factors, shadow rates and risk and liquidity factors. They find that these expanded and more recent models, do not consistently outperform the older models and do not consistently outperform the random walk model. Thus, they conclude that exchange rate predictability remains unresolved and there is no consensus outcome. For my purposes, without further discussing the merits of different exchange rate models and to prevent deviating from the theme of my proposal, I use the RW model, PF model and the PPP model based on the guidance provided by the literature.

Dominguez et al. (2012) and Dominguez (2012) use monthly data on 10-year government bond yields and monthly data on 3-month inter-bank yields as proxies for returns on securities and deposits respectively. They use ex-post data to simulate the level of foreign currency reserves in period $t + 1$. In their approach, simulated foreign currency reserves in period $t + 1$ are computed as foreign currency reserves in period t , plus estimated interest income - 3-month inter-bank yields on deposits in period t and 10-year government bond yields on securities in period t - and valuation changes on existing assets that occur between period t and $t + 1$. In my approach, I use monthly data on 3-month treasury bill rates and monthly data on 10-year government bond yields as proxies for rates on short and long-term FX reserves assets at the beginning of the month, respectively. I use monthly data over the 2010:01-2018:12 period because this allows for more variation in the return series and captures the actual returns dynamics over a longer sample period. I compute expected FX reserves returns using five

³⁷Gourinchas and Rey (2007) find that a theoretically defined measure of cyclical external imbalances contains strong predictive power for a Foreign Direct Investment (FDI) weighted effective exchange rate of the U.S. and for the Federal Reserve trade-weighted multilateral exchange rate for major currencies. Furthermore, their results are robust to the inclusion of a three-month interest rate differential and a stationary component of the trade balance. Thus, they conclude that, as compared to traditional exchange rate determination models at quarterly-yearly frequencies which perform badly, their approach exhibits predictability and significant predictive power at short, medium and long horizons, particularly for the FDI weighted effective exchange rate. They also perform out-of-sample tests for their model and find that it outperforms the random walk model. However, these results are for the FDI and Federal Reserve trade-weighted U.S. dollar and at a quarterly frequency. In their earlier working paper, Gourinchas and Rey (2005) also find modest predictive power of the cyclical external imbalances for the U.S. dollar-yen, U.S. dollar-Euro, U.S. dollar-Swiss franc bilateral exchange rates. However, no predictive power for the U.S. dollar-Canadian dollar and weak predictive power for the U.S. dollar-Sterling pound exchange rates.

main reserve currencies and these currencies constitute as the majority in the currency composition of foreign debt of the EMEs in this paper.

I compute expected FX reserves returns using the U.S. dollar, British pound, Euro, Japanese yen and Swiss franc. I source this data from the IMF’s International Financial Statistics (IFS), Organization for Economic Co-operation and Development (OECD) and the Federal Reserve Bank of St. Louis (FRED). Thus, the short-term rates for British pound, U.S. dollar and Japanese yen FX reserves, are the respective 3-month treasury bill rates.³⁸ For the Euro and Swiss franc, I use monthly data on 3-month interbank yields for the Euro area and Switzerland respectively, as proxies for the short-term rates. For long-term rates associated with the FX reserves, I use monthly data on the 10-year government bond yields of the UK, Euro area, U.S., Japan and Switzerland and I source this from the OECD and FRED.

For each EME in this paper, I extract consumer price index and bilateral exchange rate data from the IMF’s IFS to allow for a consistent series. The IFS has a consistent domestic currency units per U.S. dollar and domestic currency units per Euro series for all the EMEs in this paper. For most of the EMEs, bilateral exchange rate data with the Japanese yen, British Pound and Swiss franc is not available, except for the South African rand per British Pound, South African rand per Swiss franc, Brazilian real per Euro and Brazilian real per Swiss franc. Thus, to construct a consistent series for all the EMEs, I use cross rates based on the U.S. dollar per Swiss franc, U.S. dollar per Japanese yen and U.S. dollar per British pound and the respective domestic currency units per U.S. dollar for the EMEs. This allows me to construct an exchange rate series of domestic currency units per Japanese yen, British Pound and Swiss franc. This series is very similar and exhibits the same trajectory to the few data series that is available for the South African rand per British Pound, South African rand per Swiss franc, Brazilian real per Euro and Brazilian real per Swiss franc.

5 Validation of proposed framework

This section reports simulated optimal FX reserve weights based on my model proposal in section 3. Table 2 and 3 report the univariate statistics of the FX reserves with a random walk model and the associated covariance matrices using short and long-term (ex-post expected) FX reserves returns. I conduct two normality tests, namely the Jarque-Bera test and the Kolmogorov-Smirnov test to establish whether the individual returns on FX reserves exhibit normality. This allows me to determine whether it is appropriate to incorporate higher return moments. Table 2 and 3 show that

³⁸For British pound 3 month treasury bill rates, IFS data is only available up until August 2016 and Bank of England data is only available up to June 2017. Thereafter the data is discontinued. Thus, I augment the treasury bill rates data with 3 month interbank yields for the UK sourced from the OECD up to the end of 2018. I follow the same procedure for the Japanese yen because IFS data on treasury bill rates is only available up until June 2017 and thus I augment it with 3 month interbank yields for Japan sourced from the OECD up to the end of 2018.

the tests consistently reject normality at all relevant levels of significance. Furthermore, Table A1-A5 in the appendix report the co-skewness matrices using short and long-term returns on the FX reserves. All the EMEs have a common covariance and co-skewness matrix for the random walk model when using short and long-term returns on FX reserves. For brevity I do not report the covariance and co-skewness matrices for other methods of computing expected returns because that requires reporting 4 tables for each method of computing expected returns and for each of the 6 EMEs in this paper.

To optimize on space and report validation results for all 6 EMEs, I report and interpret simulated optimal FX reserve weights based on CRRA and IRRA expected utility for Brazil using the random walk (RW) model for short-term returns (Table 4), for Indonesia using the RW model for long-term returns (Table 5), for South Africa using the perfect foresight (PF) model for short-term returns (Table 6), for Mexico using the PF model for long-term returns (Table 7), for India using the Purchasing Power Parity (PPP) model for short-term returns (Table 8) and for Turkey using the PPP model for long-term returns (Table 9). I also report an online appendix for the covariance and co-skewness matrices associated with the results in Table 5 – 9; along with the univariate statistics and normality tests for each expected FX reserves returns.

Table 2: Variance-covariance matrix of short-term FX reserves returns using random walk

2010:01-2018:12	U.S. dollar	Euro	British pound	Japanese yen	Swiss franc
Mean return (%)	0.41	0.25	0.43	0.02	-0.29
Variance	0.40	0.33	0.02	0.015	0.18
Skewness	0.47	0.16	0.003	-0.003	-0.01
Jarque-Bera stat	85.51 (0.001)	13.61 (0.008)	40.60 (0.001)	77.95 (0.001)	16.28 (0.006)
Kolmogorov-Smirnov stat	0.5040 (0.000)	0.3705 (0.000)	0.59 (0.000)	0.45 (0.000)	0.40 (0.000)
Variance-covariance					
U.S. dollar	0.40				
Euro	-0.19	0.33			
British pound	0.06	-0.004	0.02		
Japanese yen	-0.001	0.04	0.003	0.015	
Swiss franc	-0.15	0.19	-0.01	0.03	0.18

Notes: Values in parantheses are p-values of the associated test statistic

Table 3: Variance-covariance matrix of long-term FX reserves returns using random walk

2010:01-2018:12	U.S. dollar	Euro	British pound	Japanese yen	Swiss franc
Mean return (%)	2.43	2.34	2.17	0.52	0.55
Variance	0.31	1.52	0.72	0.19	0.50
Skewness	0.09	0.59	0.40	0.013	0.17
Jarque-Bera stat	5.21 (0.054)	10.47 (0.01)	8.89 (0.02)	7.33 (0.029)	7.06 (0.032)
Kolmogorov-Smirnov stat	0.93 (0.000)	0.77 (0.000)	0.83 (0.000)	0.44 (0.000)	0.32 (0.000)
Variance-covariance					
U.S. dollar	0.31				
Euro	0.29	1.52			
British pound	0.32	0.88	0.72		
Japanese yen	0.1	0.51	0.33	0.19	
Swiss franc	0.24	0.81	0.56	0.29	0.50

Notes: Values in parantheses are p-values of the associated test statistic

Concerning upper and lower bound constraints each FX reserve weight is bounded above by the associated currency share in a country's currency composition of foreign debt, see Table 1. For the lower bounds, each FX reserve cannot be allocated less than half the associated currency share in a country's currency composition of foreign debt. Table 4 reports the simulated optimal FX reserve weights for Brazil using the RW model for short-term returns on FX reserves. For CRRA and IRRA expected utility, risk aversion parameters $(\theta, \lambda) = 10$, $(\theta, \lambda) = 15$ and portfolio transaction rebalancing cost rate $\tau = 0.05$, the optimal portfolio allocation constructed with currency composition of foreign debt constraints, is superior to a naive equally weighted portfolio allocation.³⁹

This is because the optimal portfolio generates higher portfolio mean return and positive portfolio skewness as compared to negative portfolio skewness generated by the naive equally weighted portfolio. However, the optimal portfolio has a higher variance. Furthermore, the optimal portfolio generates higher positive expected utility and more so for CRRA expected utility, see columns 2 and 3 of Table 4 relative to column 5. This is a consistent result for both CRRA and IRRA expected utility for alternative risk aversion parameters from $(\theta, \lambda) = 10$, $(\theta, \lambda) = 15$ to $(\theta, \lambda) = 20$. Thus, even with higher risk aversion, both optimal portfolios generated with CRRA and IRRA expected utility are superior relative to a naive equally weighted portfolio and a portfolio generated only with a no short selling constraint. This result also holds when varying the risk aversion parameters to $(\theta, \lambda) = 30$, 40 and 50, however for brevity I do not report these results and they are available upon request.

Table 4 also shows that when varying the risk aversion parameters, from $(\theta, \lambda) = 10$ to $(\theta, \lambda) = 15$,

³⁹In all cases, parameters and their associated assigned values, for example $(\theta, \lambda) = 10$ refers to $\theta = 10$ for CRRA expected utility and $\lambda = 10$ for IRRA expected utility when simulating optimal FX reserve weights for each EME, where $\tau = 0.05$ is common for both CRRA and IRRA expected utility.

both CRRA and IRRA expected utility, generate optimal portfolios that match Brazil’s currency composition of foreign debt. As a result, both CRRA and IRRA expected utility fulfil the mandate of generating a portfolio of FX reserves that is consistent with the country’s currency composition of foreign debt. To allow a bit more variation in the upper and lower bound constraints and potentially more variation in the optimal portfolio allocation, I loosen the upper bound constraint slightly above the associated actual currency composition of foreign debt. This allows me to establish whether the model can generate an allocation that matches the currency composition of foreign debt.

In loosening constraints, I allow for marginal over allocations towards some FX reserves however, no under allocation below half the upper bound constraint. Thus, for Brazil (Table 4), the looser upper bound constraint (%) = [91.93, 9.00, 2.00, 5.00, 1.00] and this corresponds to the (U.S.) dollar, Euro, (British) pound, (Japanese) yen and (Swiss) franc shares and the looser lower bound (%) = [45.965, 4.50, 1.00, 2.50, 0.5]. Furthermore, in this set up, I vary and increase the risk aversion parameters and thus $(\theta, \lambda) = 20$, across the looser constraints for the naive equally weighted portfolio allocation and a portfolio allocation constructed with only a no short selling constraint.

With looser constraints and higher risk aversion from $(\theta, \lambda) = 10$ or $(\theta, \lambda) = 15$ to $(\theta, \lambda) = 20$ and under a RW model of short-term returns for Brazil, this delivers a higher allocation towards the Euro and yen and a reduction of the dollar in the optimal portfolio for CRRA expected utility, see column 3 of Table 4. The shift in allocation is due to several factors that are exhibited in the univariate and covariate statistics of the FX reserves (Table 2 and Table A1). For example, the yen exhibits modest positive returns and thus a modest positive mean return; however, it has the lowest variance relative to other FX reserves and this anchors its weight. The Euro exhibits higher returns relative to the yen, along with the second highest positive skewness and these elements anchor its weight.

Concerning the dollar, the large negative co-skewness of the dollar with other FX reserves (especially the dollar, Euro, dollar co-skewness in Table A1) results in a slight reduction in its allocation especially for CRRA expected utility. However, the largest negative covariance is between the dollar and Euro and the negative covariance between the dollar and yen, anchor Euro and yen allocations towards their upper bounds with a slight reduction in the dollar to anchor for higher positive skewness in the Euro. Higher Euro and yen combinations along with a marginal reduction in the dollar, jointly interact to increase portfolio mean return, reduce portfolio variance and generate positive skewness; along with maximizing expected utility for a given risk profile.

This feature is evident for CRRA expected utility. However, there is an adjustment element between the dollar and Euro for IRRA expected utility because of the higher risk aversion associated with IRRA utility (column 4 of Table 4). Thus, although for both CRRA and IRRA expected utility, for $(\theta, \lambda) = 20$ and under looser upper bound constraints, there is a marginal reduction in the dollar allocation, IRRA utility, exhibits a higher allocation to the dollar. This higher allocation is generated by the higher risk aversion of IRRA expected utility relative to CRRA expected utility, where the dollar

has positive skewness that is 3 times larger than that of the Euro and their individual variances are not significantly different from each other, see Table 2. Thus, although there is a marginal reduction in the dollar, IRRA expected utility maintains a higher dollar allocation because of its individual risk averse appealing properties relative to the Euro; over and above the associated co-moment structures between the FX reserves returns.

Table 4: Brazil FX reserve weights for CRRA and IRRA utility using short-term returns - RW model

$\tau = 0.05$	$\theta = 10$	$\theta = 15$	$\theta = 20^*$	$\theta = 20^{**}$	$\theta = 20^\dagger$
Port. Mean (%)	0.37	0.37	0.35	0.16	0.09
Port. Variance	0.33	0.33	0.26	0.03	0.08
Port. Skewness	0.35	0.35	0.25	$-5.4*10^{-5}$	0.02
$E \left[\frac{(\xi R_p)^{1-\theta} - 1}{1-\theta} \right]_{\mathbf{w}}$	$7.9*10^5$	$2.67*10^8$	$1.69*10^{11}$	$-2.8*10^{16}$	$5.1*10^{23}$
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	91.93	91.93	84.00	20.00	53.87
Euro	4.55	4.55	9.00	20.00	0.39
British pound	0.76	0.76	1.00	20.00	0.23
Japanese yen	2.25	2.25	5.00	20.00	0.00
Swiss franc	0.51	0.51	1.00	20.00	45.51
$\tau = 0.05$	$\lambda = 10$	$\lambda = 15$	$\lambda = 20$	$\lambda = 20$	$\lambda = 20$
Port. Mean (%)	0.37	0.37	0.36	0.16	0.12
Port. Variance	0.33	0.33	0.30	0.03	0.1
Port. Skewness	0.35	0.35	0.31	$-5.4*10^{-5}$	0.05
$E \left[-e^{(-\lambda \xi R_p)} \right]_{\mathbf{w}}$	0.9	0.54	0.25	-0.32	3.158
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	91.93	91.93	88.50	20.00	59.39
Euro	4.55	4.55	4.50	20.00	0.00
British pound	0.76	0.76	1.00	20.00	0.00
Japanese yen	2.25	2.25	5.00	20.00	0.00
Swiss franc	0.51	0.51	1.00	20.00	40.61

Notes: * refers to a portfolio constructed with marginally looser constraints, where upper bounds (%) = [91.93, 9.00, 2.00, 5.00, 1.00] lower bounds (%) = [45.965, 4.50, 1.00, 2.50, 0.50], at the same risk aversion parameter, ** is the naive equally weighted portfolio. and † is a portfolio constructed with only a no short selling constraint.

FX reserves that jointly interact to generate positive skewness and maximize skewness, anchor the portfolio towards large gains relative to large losses. More importantly, the optimal portfolio generated with the looser constraints, also matches the country's actual currency composition of foreign debt, with marginal over allocations relative to the actual foreign currency debt weights. However, no under allocation for any FX reserves. When comparing to a portfolio constructed with only a no short

selling constraint, column 6 of Table 4, we observe similar patterns under a RW model with short term returns for Brazil. For CRRA and IRRA expected utility, the no short selling constraint portfolio, allocates 53.87 % and 59.39 % to the dollar respectively, 45.51 % and 40.61 % to the franc respectively and close to a 0.62 % allocation to the Euro, pound and yen combined for CRRA expected utility. IRRA expected utility maintains a higher dollar share because of the dollar’s individual risk averse appealing properties relative to the other FX reserves.

The large dollar allocation in the no short selling constraint portfolio is anchored by its high returns and the dollar exhibiting the highest positive skewness. Furthermore, the dollar, franc, franc returns combinations, generate positive co-skewness. Thus, showing that the joint interaction between these FX reserves generates positive portfolio skewness and thus maximizes portfolio skewness. In addition, the covariance between the dollar and franc is large and negative and this reinforces how jointly these variables interact to reduce portfolio variance and jointly interact to maximize expected utility. As a result, generating the substantial allocation towards the dollar and franc FX reserves.

Next, concerning long-term returns under the RW model, I interpret results for Indonesia and these are reported in Table 5. The simulated optimal FX reserve weights for Indonesia, exhibit similar patterns to those in Table 4 for Brazil. For example, in Table 5, for risk aversion parameters $(\theta, \lambda) = 10$, $(\theta, \lambda) = 15$, the optimal portfolios constructed with the currency composition of foreign debt constraints, are superior because they generate higher portfolio mean return and lower portfolio variance as compared to a naive equally weighted portfolio allocation, see columns 2, 3 and 5 of Table 5. However, both models generate positive portfolio skewness and expected utilities that are not significantly different from each other, especially for CRRA expected utility. Furthermore, IRRA expected utility, generates lower expected utility values. More importantly, the optimal portfolios generate an allocation that is the same as Indonesia’s currency composition of foreign debt, even when varying the risk aversion parameter to higher values, from $(\theta, \lambda) = 10$ to $(\theta, \lambda) = 15$.

With marginally looser constraints and higher risk aversion from $(\theta, \lambda) = 10$ or $(\theta, \lambda) = 15$ to $(\theta, \lambda) = 20$, there is a reduction in the dollar allocation and a higher allocation towards the other FX reserves.⁴⁰ The marginally looser constraints in Table 5 correspond to an upper bound (%) = [76.13, 9.00, 5.00, 20.00, 5.00] and this corresponds to the dollar, Euro, pound, yen and franc shares and the lower bound (%) = [38.065, 2.79, 0.345, 8.515, 0.285]; where this is half the allocation of the original upper bound constraint. Thus, in this set-up of higher risk aversion and looser constraints, IRRA expected utility, exhibits a higher allocation to the dollar, see column 4 Table 5. The higher dollar allocation is generated by the higher risk aversion of IRRA expected utility relative to CRRA expected utility, where the dollar exhibits the highest mean return and greater individual risk averse appealing properties relative to the other FX reserves and especially relative to the yen; over and above the associated co-moment structures between the FX reserves returns, see Table 3 and the co-skewness

⁴⁰This result also holds when varying the risk aversion parameters to $(\theta, \lambda) = 30, 40$ and 50 , however for brevity I do not report these results.

matrices in the appendix. The yen share is associated with a higher dollar share trade-off.

Comparing to a portfolio constructed with only a no short selling constraint, column 6 of Table 5, shows that for CRRA expected utility, the no short selling constraint portfolio, allocates 100 % to the franc. Whereas, IRRA expected utility generates a portfolio that is the same as the naive equally weighted portfolio and thus a substantial shift from the 100% franc allocation. The equally weighted allocation is generated because it maximizes expected utility. Concerning the 100% franc allocation, although the franc exhibits low mean return, the Swiss franc has lower variance relative to the Euro and pound. The franc also has the third highest positive skewness after the Euro and pound respectively, where the Euro and pound exhibit the highest variance and this reduces the Euro and pound allocation because higher variance reduces the central bank's preference function, see Table 3.

Co-skewness combinations involving the franc and Euro returns along with other FX reserves, are positive and larger than the co-skewness exhibited by other combinations (Table A2 and A5). Thus, these factors anchor the franc allocation in the portfolio, and the lower franc variance relative to the Euro and pound, result in the 100 % allocation towards the franc because its return, variance and its skewness, jointly contribute towards maximizing expected utility as compared to any other portfolio composition. On the other hand, for IRRA expected utility and a portfolio constructed with a no short selling constraint, this delivers an allocation that is equal to a naive equally weighted portfolio.

Table 5 shows that for both CRRA and IRRA expected utility, the optimal portfolios generate higher portfolio mean and lower portfolio variance as compared to a naive equally weighted portfolio allocation and relative to a portfolio constructed with only a no short selling constraint. However, all these portfolios generate positive skewness that are not significantly different from each other. More importantly, the portfolio allocation constructed with a no short selling constraint and the naive equally weighted portfolio allocation, are not diversified to fulfil the objective of matching the currency composition of foreign debt as best as possible. Thus, these portfolios are inferior relative to the optimal portfolios constructed with Indonesia's currency composition of foreign debt as lower and upper bound constraints and for both CRRA and IRRA expected utility.

These findings are consistent for optimal portfolios under a RW model for short and long-term returns on FX reserves of Brazil and Indonesia respectively, along with varying risk aversion parameters and with looser constraints. Furthermore, because all the EMEs have a common covariance and co-skewness matrices for the random walk model when using short and long-term returns on FX reserves, however with different currency compositions of foreign debt constraints, these results can be generalized for the remaining EMEs. This is reinforced by the fact that the other EMEs have less tighter constraints as compared to Brazil. These results show the feasibility of the model proposal for different global market conditions and the findings do show that with higher risk aversion and when using IRRA expected utility, this portfolio selection model with skewness allocates to less riskier assets based on their moment and co-moments characteristics, as compared to CRRA expected utility.

Table 5: Indonesia FX reserve weights for CRRA and IRRA utility using long-term returns - RW model

$\tau = 0.05$	$\theta = 10$	$\theta = 15$	$\theta = 20^*$	$\theta = 20^{**}$	$\theta = 20^\dagger$
Port. Mean (%)	1.99	1.99	1.85	1.52	0.52
Port. Variance	0.26	0.26	0.27	0.48	0.50
Port. Skewness	0.09	0.09	0.11	0.18	0.17
$E \left[\frac{(\xi R_p)^{1-\theta} - 1}{1-\theta} \right]_{\mathbf{w}}$	0.11	0.071	0.0526	0.053	$1.36 \cdot 10^7$
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	76.13	76.13	62.70	20.00	0.00
Euro	5.58	5.58	8.10	20.00	0.00
British pound	0.69	0.69	4.57	20.00	0.00
Japanese yen	17.03	17.03	19.80	20.00	0.00
Swiss franc	0.57	0.57	4.83	20.00	100.00
$\tau = 0.05$	$\lambda = 10$	$\lambda = 15$	$\lambda = 20$	$\lambda = 20$	$\lambda = 20$
Port. Mean (%)	1.99	1.99	1.96	1.52	1.52
Port. Variance	0.26	0.26	0.27	0.48	0.48
Port. Skewness	0.09	0.09	0.1025	0.18	0.18
$E \left[-e^{(-\lambda \xi R_p)} \right]_{\mathbf{w}}$	$1.36 \cdot 10^{-9}$	$1.99 \cdot 10^{-12}$	$6.49 \cdot 10^{-16}$	$6.92 \cdot 10^{-12}$	$6.92 \cdot 10^{-12}$
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	76.13	76.13	71.90	20.00	20.00
Euro	5.58	5.58	6.40	20.00	20.00
British pound	0.69	0.69	2.90	20.00	20.00
Japanese yen	17.03	17.03	15.90	20.00	20.00
Swiss franc	0.57	0.57	2.90	20.00	20.00

Notes: * refers to a portfolio constructed with marginally looser constraints, where upper bounds (%) = [76.13, 9.00, 5.00, 20.00, 5.00] lower bounds (%) = [38.065, 2.79, 0.345, 8.515, 0.285], at the same risk aversion parameter, ** is the naive equally weighted portfolio. and † is a portfolio constructed with only a no short selling constraint.

Next, Table 6 reports simulated optimal FX reserve weights for South Africa using the perfect foresight (PF) model for short-term returns. Consistent with the random walk model for Brazil and Indonesia, Table 6 shows that incorporating exchange rate changes, delivers similar portfolio allocation outcomes based on the univariate statistics and co-moments of South African expected FX reserves returns. However, incorporating exchange rate changes, delivers higher portfolio moments. Increasing the parameter of risk aversion from $(\theta, \lambda) = 10$ or $(\theta, \lambda) = 15$ to $(\theta, \lambda) = 20$, results in a reduction in the pound allocation because it exhibits the largest negative skewness and second largest variance, however, the pound has the second largest mean return, see columns 2, 3 and 4 of Table 6 and column 4 of Table B1 in the online appendix for the pound's statistics.

For CRRA expected utility, there is a resulting increase in the Euro, yen and franc allocations,

because their univariate statistics exhibit high positive skewness in excess of 10, especially the yen and franc. Furthermore, the large positive co-skewness between the yen and dollar, yen and Euro and yen and franc combinations with other FX reserves, anchor the risk aversion appeal of these FX reserves relative to combinations including the pound, see online appendix B. This also holds for IRRA expected utility when increasing the parameter of risk aversion, however, there is a further downward adjustment from the pound with an additional and higher allocation towards the dollar and with the Euro maintaining the same allocation (column 4 of Table 6). The adjustment under IRRA is due to the dollar and Euro exhibiting lower variance relative to the pound and yen, the positive skewness of the dollar and Euro, both in excess of 5 and the modest positive mean return of the dollar and Euro. In contrast, the pound exhibits the highest negative skewness, second largest variance after the yen, however a modest positive mean return that comes second to the dollar. Furthermore, the positive co-skewness between dollar, yen combinations with other FX reserves and the positive co-skewness between Euro, yen combinations with other FX, where individually the yen has the largest positive skewness, anchor the dollar and Euro allocation because of their risk aversion appealing profile. This outcome is compatible with IRRA expected utility.

Table 6 shows that for both CRRA and IRRA expected utility, the optimal portfolios generate higher portfolio mean as compared to a naive equally weighted portfolio allocation and relative to a portfolio constructed with only a no short selling constraint. However, all these portfolios generate portfolio variance that are not significantly different from each other.⁴¹ In contrast, the naive equally weighted portfolio and portfolio constructed with only a no short selling constraint, generate larger positive skewness as compared to the optimal portfolios and this is due to their lower exposure to the pound; where the pound exhibits the largest negative skewness. More importantly, the portfolio allocation constructed with a no short selling constraint and the naive equally weighted portfolio allocation, do not fulfil the objective of matching the currency composition of foreign debt as best as possible. Thus, these portfolios are inferior relative to the optimal portfolios constructed with South Africa’s currency composition of foreign debt as lower and upper bound constraints and for both CRRA and IRRA expected utility.

Tables 4, 5 and 6 show that the central bank’s preference functions, exhibit a trade-off between a low (positive) portfolio mean return, for a low portfolio variance. Furthermore, for a low portfolio mean return or for any given (positive) portfolio mean return, IRRA and CRRA expected utility pursue high positive portfolio skewness and thus a trade-off between low portfolio mean return for high positive portfolio skewness, and more so in Tables 5 and 6. Low portfolio mean return and low portfolio variance, along with low portfolio mean return and high positive portfolio skewness, jointly interact to maximize expected utility. This trade-off is more emphasized with a smaller portfolio mean return and reinforced by a high-risk aversion profile (higher risk aversion parameter). My proposal

⁴¹These features also holds for higher risk aversion parameters, i.e. $(\theta, \lambda) = 30, 40$ and 50 , for simulated optimal FX reserve weights for South Africa using the PF model for short-term returns.

also shows that imposing lower and upper bound constraints, allows for diversification benefits that lower portfolio downside risk that can be influenced by over exposure to FX reserves with negative skewness. Thus, the generated optimal composition, increases portfolio mean return, it generates modest portfolio variance and generates positive portfolio skewness; along with maximizing expected utility for a given risk profile and across the two portfolio selection frameworks with skewness.

Table 6: South Africa FX reserve weights for CRRA and IRRA utility using short-term returns - PF model

$\tau = 0.05$	$\theta = 10$	$\theta = 15$	$\theta = 20^*$	$\theta = 20^{**}$	$\theta = 20^\dagger$
Port. Mean (%)	0.73	0.73	0.70	0.6	0.32
Port. Variance	8.84	8.84	8.81	8.51	9.35
Port. Skewness	2.31	2.31	3.22	7.42	10.88
$E \left[\frac{(\xi R_p)^{1-\theta} - 1}{1-\theta} \right]_{\mathbf{w}}$	353.26	8.58×10^3	3.48×10^5	3.3×10^7	4.95×10^{13}
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	27.66	27.66	23.47	20.00	0.00
Euro	29.03	29.03	35.00	20.00	0.00
British pound	34.76	34.76	31.53	20.00	0.00
Japanese yen	4.16	4.16	5.00	20.00	0.00
Swiss franc	4.39	4.39	5.00	20.00	100.00
$\tau = 0.05$	$\lambda = 10$	$\lambda = 15$	$\lambda = 20$	$\lambda = 20$	$\lambda = 20$
Port. Mean (%)	0.73	0.73	0.73	0.6	0.33
Port. Variance	8.84	8.84	8.58	8.51	9.10
Port. Skewness	2.31	2.31	4.35	7.42	12.05
$E \left[-e^{(-\lambda \xi R_p)} \right]_{\mathbf{w}}$	-0.05	0.0041	0.002	0.04	18.35
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	27.66	27.66	34.95	20.00	0.00
Euro	29.03	29.03	35.00	20.00	0.00
British pound	34.76	34.76	21.50	20.00	0.00
Japanese yen	4.16	4.16	4.16	20.00	8.36
Swiss franc	4.39	4.39	4.39	20.00	91.64

Notes: * refers to a portfolio constructed with marginally looser constraints, where upper bounds (%) = [35.00, 35.00, 35.00, 5.00, 5.00] lower bounds (%) = [13.83, 14.515, 17.38, 2.08, 2.195], at the same risk aversion parameter, ** is the naive equally weighted portfolio. and † is a portfolio constructed with only a no short selling constraint.

Table 7 reports simulated optimal FX reserve weights for Mexico using the PF model for long-term returns, Table 8 reports for India using the PPP model for short-term returns and Table 9 reports for Turkey using the PPP model for long-term returns. The optimal portfolio composition and interpretations exhibit similar outcomes for Mexico, India and Turkey. For example, in the context of Mexico, for a higher risk aversion profile and comparing CRRA relative to IRRA expected utility,

there is a shift out of the Euro and pound because of their negative skewness and a higher allocation towards the dollar and franc because of their positive skewness; where this exhibits individual risk averse appealing properties relative to the Euro and pound. This outcome, is over and above the associated co-moment structures between the FX reserves returns, see column 4 of Table 7 and online appendix C. Furthermore, portfolios constructed with long-term expected returns on FX reserves under the PF model, fulfil the objective of matching as best as possible Mexico's currency composition of foreign debt; along with reasonable portfolio performance that is consistent with risk averse appealing outcomes.

Concerning India's FX reserves and using short term returns with a PPP model, Table 8 shows that the dollar maintains its upper bound allocation. This allocation is anchored by the dollar exhibiting the second largest positive skewness and a variance that is not substantially different to the Euro and franc. Furthermore, the high dollar allocation is reinforced by the dollar being the only FX reserve that exhibits negative covariance with the other FX reserves. This increases its risk aversion appeal along with anchoring expected utility and these univariate and covariate statistics generate a unique feature of maintaining the same optimal allocation under CRRA and IRRRA expected utility with marginally looser constraints, see column 4 of Table 8 and online appendix D. Using marginally looser constraints, generates a substantial reduction in the yen allocation relative to its upper bound because the yen exhibits a modest positive mean return, however, the lowest positive skewness. The pound and franc achieve higher allocations, especially the franc and this allocation is anchored by the franc exhibiting the highest positive skewness, however a modest positive mean return and variance that is not substantially different relative to the other FX reserves. Excluding the pound, franc and dollar co-skewness combination and relative to other co-skewness combinations, pound and franc, yen and franc and in general, franc co-skewness combinations are positive and large, see online appendix D.

These factors reinforce the risk aversion appealing profile of the franc and thus deliver a substantially large franc allocation at the higher risk aversion parameters for India's optimal allocation of FX reserves. Furthermore, none of the FX reserves are allocated less than half their counterparty in the actual currency weight of India's foreign debt. Moreover, the optimal FX reserves weights based on CRRA and IRRRA expected utility, are superior relative to portfolios constructed with a no short selling constraint and a naive equally weighted portfolio because they fulfil the mandate of matching India's currency composition of foreign debt as best as possible, they generate higher expected utility and higher positive skewness; along with portfolio mean returns and variance that are not substantially different to that of the no short selling constraint portfolio and naive equally weighted portfolio, see columns 4, 5 and 6 of Table 8.

Table 7: Mexico FX reserve weights for CRRA and IRRA utility using long-term returns - PF model

$\tau = 0.05$	$\theta = 10$	$\theta = 15$	$\theta = 20^*$	$\theta = 20^{**}$	$\theta = 20^\dagger$
Port. Mean (%)	2.31	2.31	2.30	1.80	2.45
Port. Variance	6.23	6.23	6.21	6.69	6.08
Port. Skewness	-0.63	-0.63	-0.69	-0.69	-1.28
$E \left[\frac{(\xi R_p)^{1-\theta} - 1}{1-\theta} \right]_{\mathbf{w}}$	0.11	0.07	0.053	0.0523	0.0526
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	62.07	62.07	62.30	20.00	54.33
Euro	13.55	13.55	13.29	20.00	31.54
British pound	7.45	7.45	6.77	20.00	7.17
Japanese yen	9.75	9.75	8.88	20.00	3.60
Swiss franc	7.18	7.18	8.76	20.00	3.36
$\tau = 0.05$	$\lambda = 10$	$\lambda = 15$	$\lambda = 20$	$\lambda = 20$	$\lambda = 20$
Port. Mean (%)	2.31	2.31	2.30	1.80	1.80
Port. Variance	6.23	6.23	6.21	6.69	6.69
Port. Skewness	-0.63	-0.63	-0.65	-0.48	-0.48
$E \left[-e^{(-\lambda \xi R_p)} \right]_{\mathbf{w}}$	$-3.5 \cdot 10^{-8}$	$-8.4 \cdot 10^{-13}$	$-1.8 \cdot 10^{-17}$	$-4.2 \cdot 10^{-13}$	$-4.2 \cdot 10^{-13}$
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	62.07	62.07	62.94	20.00	20.00
Euro	13.55	13.55	12.81	20.00	20.00
British pound	7.45	7.45	6.52	20.00	20.00
Japanese yen	9.75	9.75	8.90	20.00	20.00
Swiss franc	7.18	7.18	8.83	20.00	20.00

Notes: * refers to a portfolio constructed with marginally looser constraints, where upper bounds (%) = [63.00, 14.00, 7.50, 10.00, 10.00] lower bounds (%) = [31.035, 6.775, 3.725, 4.875, 3.59], at the same risk aversion parameter, ** is the naive equally weighted portfolio. and † is a portfolio constructed with only a no short selling constraint.

Table 8: India FX reserve weights for CRRA and IRRA utility using short-term returns - PPP model

$\tau = 0.05$	$\theta = 10$	$\theta = 15$	$\theta = 20^*$	$\theta = 20^{**}$	$\theta = 20^\dagger$
Port. Mean (%)	0.11	0.11	0.10	0.43	0.03
Port. Variance	0.57	0.57	0.56	0.32	0.96
Port. Skewness	0.27	0.27	0.27	0.08	0.44
$E \left[\frac{(\xi R_p)^{1-\theta} - 1}{1-\theta} \right]_{\mathbf{w}}$	$1.28 \cdot 10^{12}$	$1.77 \cdot 10^{17}$	$1.26 \cdot 10^{23}$	$4.3 \cdot 10^8$	$7.26 \cdot 10^{35}$
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	79.84	79.84	79.84	20.00	93.38
Euro	3.91	3.91	2.50	20.00	1.46
British pound	1.38	1.38	2.50	20.00	0.10
Japanese yen	13.94	13.94	10.16	20.00	0.06
Swiss franc	0.93	0.93	5.00	20.00	5.00
$\tau = 0.05$	$\lambda = 10$	$\lambda = 15$	$\lambda = 20$	$\lambda = 20$	$\lambda = 20$
Port. Mean (%)	0.11	0.11	0.10	0.43	0.25
Port. Variance	0.57	0.57	0.56	0.32	1.13
Port. Skewness	0.27	0.27	0.27	0.08	0.58
$E \left[-e^{(-\lambda \xi R_p)} \right]_{\mathbf{w}}$	4.1	13.96	27.30	0.007	3.21
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	79.84	79.84	79.84	20.00	0.00
Euro	3.91	3.91	2.50	20.00	0.00
British pound	1.38	1.38	2.50	20.00	0.00
Japanese yen	13.94	13.94	10.16	20.00	0.00
Swiss franc	0.93	0.93	5.00	20.00	100.00

Notes: * refers to a portfolio constructed with marginally looser constraints, where upper bounds (%) = [79.84, 5.00, 5.00, 15.00, 5.00] lower bounds (%) = [39.92, 2.50, 2.50, 7.50, 2.50], at the same risk aversion parameter, ** is the naive equally weighted portfolio. and † is a portfolio constructed with only a no short selling constraint.

Table 9 shows that Turkey’s optimal portfolio allocation of FX reserves using long term returns with a PPP model, exhibit an optimal portfolio that matches the country’s currency composition of foreign debt for parameters of risk aversion $(\theta, \lambda) = 10$ and $(\theta, \lambda) = 15$ and the optimal allocation maximizes the central bank’s preference functions, see columns 2 and 3 of Table 9. Increasing the parameters of risk aversion to $(\theta, \lambda) = 20$, with IRRA expected utility, there is a slight reduction in the Euro, pound, yen and franc allocation towards the dollar because of the risk aversion appealing univariate and covariate statistics of the dollar. For example, the dollar has the highest mean return, modest variance that is smaller than that of the Euro and pound, however, not significantly different to the franc’s variance. Furthermore, the dollar has the largest positive skewness, lowest positive covariance with other FX reserves as compared to Euro and pound covariance with the other FX reserves.

Table 9: Turkey FX reserve weights for CRRA and IRRA utility using short-term returns - PPP model

$\tau = 0.05$	$\theta = 10$	$\theta = 15$	$\theta = 20^*$	$\theta = 20^{**}$	$\theta = 20^\dagger$
Port. Mean (%)	2.80	2.80	2.72	2.18	1.19
Port. Variance	1.28	1.28	1.27	1.25	1.04
Port. Skewness	1.94	1.94	1.87	1.49	1.32
$E \left[\frac{(\xi R_p)^{1-\theta} - 1}{1-\theta} \right]_{\mathbf{w}}$	0.1112	0.0714	0.053	0.053	1.45
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	68.31	68.31	61.64	20.00	0.00
Euro	25.48	25.48	25.28	20.00	0.00
British pound	0.13	0.13	2.92	20.00	0.00
Japanese yen	5.81	5.81	7.21	20.00	100.00
Swiss franc	0.27	0.27	2.95	20.00	0.00
$\tau = 0.05$	$\lambda = 10$	$\lambda = 15$	$\lambda = 20$	$\lambda = 20$	$\lambda = 20$
Port. Mean (%)	2.80	2.80	2.73	2.18	2.18
Port. Variance	1.28	1.28	1.27	1.25	1.25
Port. Skewness	1.94	1.94	1.88	1.49	1.49
$E \left[-e^{(-\lambda \xi R_p)} \right]_{\mathbf{w}}$	$1.52 \cdot 10^{-10}$	$4.7 \cdot 10^{-16}$	$3.9 \cdot 10^{-21}$	$1.9 \cdot 10^{-16}$	$1.9 \cdot 10^{-16}$
	Weights (%)	Weights (%)	Weights (%)	Weights (%)	Weights (%)
U.S. dollar	68.31	68.31	62.64	20.00	20.00
Euro	25.48	25.48	24.62	20.00	20.00
British pound	0.13	0.13	2.83	20.00	20.00
Japanese yen	5.81	5.81	7.04	20.00	20.00
Swiss franc	0.27	0.27	2.87	20.00	20.00

Notes: * refers to a portfolio constructed with marginally looser constraints, where upper bounds (%) = [68.31, 30.00, 5.00, 10.00, 5.00] lower bounds (%) = [34.155, 12.74, 0.1, 2.905, 0.20], at the same risk aversion parameter, ** is the naive equally weighted portfolio. and † is a portfolio constructed with only a no short selling constraint.

Lastly, dollar, dollar and other FX reserves co-skewness combinations are positive and are always the largest. These risk aversion appealing univariate and covariate statistics maintain a high dollar allocation and generate a moderately lower Euro, pound, yen and franc weights in the optimal allocation of FX reserves for IRRA expected utility relative to CRRA, see column 4 in Table 9. More importantly, the CRRA and IRRA portfolios are superior to a naive equally weighted portfolio and portfolio constructed with a no short selling constraint because they maximize expected utility, they generate higher portfolio mean, generate portfolio variance that is not significantly different and generate higher positive skewness; along with fulfilling the mandate of matching Turkey's currency composition of foreign debt as best as possible, see columns 4, 5 and 6 in Table 9.

6 Conclusion

Using approximated CRRA and IRRA expected utility, using different methods of computing expected FX reserves returns and different maturity structures on FX reserves, I validate my proposal on 6 EMEs. Thus, with alternative risk aversion parameters and associated portfolio rebalancing transaction costs, I simulate optimal FX reserves weights using the actual currency composition of foreign debt as constraints and maximize the central bank's expected utility. My proposal incorporates higher FX reserves returns moments in a manner consistent with Pratt-Arrow measures of risk aversion and is a progressive risk-management practice over different global market conditions. Thus, based on a transition probability matrix associated with different global market conditions that influence and generate different central bank preferences, the central bank can switch between CRRA and IRRA expected utility for portfolio selection with skewness.

My proposal is relevant for EMEs with characteristics of rising total and short-term foreign currency debt to GDP ratios and rising and significant FX reserves to GDP ratios. Furthermore, EMEs with concurrently rising public sector and private sector external debt to GDP ratios and EMEs that are vulnerable to currency volatility, foreign debt crises and capital flow volatility. My proposal is a procedure that may allow a central bank to anchor its country's ability to withstand foreign debt defaults over different global market conditions. More so, when unavoidable external factors result in large and in some instances, unexpected, currency depreciations that can be persistent and thus exacerbate negative balance sheet effects. My proposal allows for systemic risk mitigation and is consistent with the limits on currency mismatches and this is an instrument in the macroprudential frameworks of several EMEs.

My proposal exhibits a trade-off between a low portfolio mean return, for a low portfolio variance. Furthermore, for a low portfolio mean return, the model pursues high positive portfolio skewness and thus a trade-off between lower portfolio mean return for higher positive portfolio skewness. Low portfolio mean return and low portfolio variance along with low portfolio mean return and high positive portfolio skewness, jointly interact to maximize expected utility. This trade-off is more emphasized with a smaller portfolio mean return and reinforced by a high-risk aversion profile.

References

- Aghion, P., Bacchetta, P., Banerjee, A., 2001. Currency crises and monetary policy in an economy with credit constraints. *European Economic Review* 45, 1121-1150.
- Aghion, P., Bacchetta, P., Banerjee, A., 2004. A corporate balance-sheet approach to currency crises. *Journal of Economic Theory* 119, 6-30.
- Aizenman, J., Chinn, M., Ito, H., 2017. Balance sheet effects on monetary and financial spillovers: The East Asian crisis plus 20. *Journal of International Money and Finance* 74, 258-282.

Aizenman, J., Cheung, Y-W., Qian, X., 2020. The currency composition of international reserves, demand for international reserves and global safe assets. *Journal of International Money and Finance* 102, 102120.

Alfaro, L., Kanczuk, F., 2009. Optimal reserve management and sovereign debt. *Journal of International Economics* 77, 23-36.

Arditti, F. D., 1967. Risk and the Required Return on Equity. *Journal of Finance* 22, 19-36.

Arellano, C., 2008. Default Risk and Income Fluctuations in Emerging Economies. *American Economic Review* 98, 690-712.

Arrow, K., 1971. *Essays in the Theory of Risk-Bearing*. Chicago: Markham Publishing Co.

Asis, G., Chari, A., Haas, A., 2020. Emerging Markets at Risk. *AEA Papers and Proceedings* 110, 493-498.

Asonuma, T., Chamon, M., Erce, A., Sasahara, A., 2019. Costs of Sovereign Defaults: Restructuring Strategies, Bank Distress and the Capital Inflow-Credit Channel. *IMF WP/19/69*.

Athayde, G., Flôres, R., 2004. Finding a maximum skewness portfolio—a general solution to three-moments portfolio choice. *Journal of Economic Dynamics and Control* 28, 1335–1352.

Avdjiev, S., McGuire, P., von Peter, G., 2020. International dimensions of EME corporate debt. *BIS Quarterly Review*, June, 1-13.

Bank for International Settlements., 2015. *BIS Quarterly Review - Sept 2015*. International banking and financial market developments.

Bank for International Settlements., 2020. *BIS Quarterly Review - June 2020*. International banking and financial market developments.

Barkbu, B., Eichengreen, B., Mody, A., 2012. Financial crises and the multilateral response: What the historical record shows. *Journal of International Economics* 88, 422-435.

Baz, J., Breedon, F., Naik, V., Peress, J., 2001. Optimal Portfolios of Foreign Currencies: Trading on the forward bias. *Journal of Portfolio Management* 28, 102-111.

Ben-Bassat, A., 1980. The Optimal Composition of FX Reserves. *Journal of International Economics* 10, 285-295.

Ben-Bassat, A., Gottlieb, D., 1992a. Optimal international reserves and sovereign risk. *Journal of International Economics* 33, 345-362.

Ben-Bassat, A., Gottlieb, D., 1992b. On the Effect of Opportunity Cost on International Reserve Holdings. *Review of Economics and Statistics* 74, 329-332.

Bianchi, J., Hatchondo, J., Martinez, L., 2018. International Reserves and Rollover Risk. *American Economic Review* 108, 2629-2670.

Bocola, L., Lorenzoni, G., 2020. Financial Crises, Dollarization and Lending of Last Resort in Open Economies. *American Economic Review* 110, 2524–2557.

Bordo, M., Meissner, C., Stuckler, D., 2010. Foreign currency debt, financial crises and economic growth: A long-run view. *Journal of International Money and Finance* 29, 642-665.

Borio, C., Ebbesen, J., Galati, G., Heath, A., 2008. FX reserve management: elements of a framework. *BIS Papers* No 38.

Briec, W., Kerstens, K., Jokung, O., 2007. Mean-Variance-Skewness Portfolio Performance Gauging: A General Shortage Function and Dual Approach. *Management Science* 53, 135-149.

Brockett, P., Garven, J., 1998. A Reexamination of the Relationship Between Preferences and Moment Orderings by Rational Risk-Averse Investors. *Geneva Papers on Risk and Insurance Theory* 23, 127-137.

Bruno, V., Shin, H-S., 2020. Currency Depreciation and Emerging Market Corporate Distress. *Management Science* 66, 1935-1961.

Caballero, J., 2020. Corporate Dollar Debt and Depreciations: All's Well that Ends Well? *BIS WP* No 879.

Caballero, R., Panageas, S., 2006. Contingent Reserves Management: An Applied Framework. *External Vulnerability and Preventive Policies*, edited by R. Caballero, R., Calderón, C., Céspedes, L., Santiago, Chile. © 2006 Central Bank of Chile.

Calvo, G., Izquierdo, A., Talvi, E., 2006. Sudden Stops and Phoenix Miracles in Emerging Markets. *AEA Papers and Proceedings* 96, 405-410.

Cantú, C., Goel, T., Schanz, J., 2020. EME government debt: cause for concern? *BIS Quarterly Review*, June, 15-16.

Cardon, P., Coche, J., 2004. Strategic asset allocation for FX reserves. In: Bernadell, C., Cardon, P., Coche, J., Diebold, F.X., Manganelli, S., *Risk Management for Central Bank FX*. European Central Bank, Frankfurt, Germany.

Catão, L., Milesi-Ferretti, G., 2014. External liabilities and crises. *Journal of International Economics* 94, 18-32.

Céspedes, L., Chang, R., Velasco A., 2017. Financial intermediation, real exchange rates, and unconventional policies in an open economy. *Journal of International Economics* 108, S76-S86.

Chang, R., Velasco A., 2001. A Model of Financial Crises in Emerging Markets. *Quarterly Journal of Economics* 116, 489-517.

Cheung, Y-W., Chinn, M., Pascual, A., 2005. Empirical exchange rate models of the nineties: Are any fit to survive? *Journal of International Money and Finance* 24, 1150-1175.

Cheung, Y-W., Chinn, M., Pascual, A., Zhang, Y., 2019. Exchange rate prediction redux: New

models, new data, new currencies. *Journal of International Money and Finance* 95, 332-362.

Chinn, M., Frankel, J., 2008. The Euro May Over the Next 15 Years Surpass the Dollar as Leading International Currency. NBER WP 13909.

Chui, M., Kuruc, E., Turner, P., 2016. A new dimension to currency mismatches in the emerging markets: non-financial companies. BIS WP No 550.

Claessens, S., Kreuser, J., 2007. Strategic foreign reserves risk management: Analytical framework. *Ann Oper Res* 152, 79-113.

Conine, T., Tamarkin, M., 1981. On Diversification Given Asymmetry in Returns. *Journal of Finance* 36, 1143-1155.

Coppola, A., Maggiori, M., Nieman, B., Schreger, J., 2020. Redrawing the Map of Global Capital Flows: The Role of Cross-Border Financing and Tax Havens. CMNS Paper.

Corsetti, G., Pesenti, P., Roubini, N., 1999. What caused the Asian currency and financial crisis? *Japan and the World Economy* 11, 305-373.

Danthine, J-P., Donaldson, J., 2005. *Intermediate Financial Theory*. 2nd Edition, Elsevier Academic Press Advanced Finance Series.

Dellas, H., Yoo, C., 1991. Reserve currency preferences of central banks: the case of Korea. *Journal of International Money and Finance* 10, 406-419.

Dominguez, K., 2012. Foreign reserve management during the global financial crisis. *Journal of International Money and Finance* 31, 2017-2037.

Dominguez, K., Hashimoto, Y., Ito, T., 2012. International reserves and the global financial crisis. *Journal of International Economics* 88, 388-406.

Dooley, M., Lizondo J., Mathieson, D., 1989. The Currency Composition of FX Reserves. *Staff Papers - IMF* 36, 385-434.

Eichengreen, B., Mathieson, D., 2000. The Currency Composition of FX Reserves: Retrospect and Prospect. IMF WP/00/131.

Eichengreen, B., Hausmann, R., Panizza, U., 2003. The Pain of Original Sin. Working Paper, August 2003.

Eichengreen, B., Chițu, L., Mehl, A., 2015. Stability or Upheaval? The Currency Composition of International Reserves in the Long Run. *IMF Economic Review* 00, 1-27.

Eichengreen, B., Mehl, A., Chițu, L., 2019. Mars or Mercury? The geopolitics of international currency choice. *Economic Policy* 34, 315-363.

Farhi, E., Maggiori, M., 2018. A Model of the International Monetary System. *Quarterly Journal of Economics* 133, 295-355.

- Feldstein, M., 1999. A Self-Help Guide for Emerging Markets. *Foreign Affairs* 78, 93-109.
- Fischer, S., 1999. On the Need for an International Lender of Last Resort. *Journal of Economic Perspectives* 13, 85-104.
- Fisher, S., Lie, M., 2004. Asset allocation for central banks: optimally combining liquidity, duration, currency and non-government risk. In: Bernadell, C., Cardon, P., Coche, J., Diebold, F.X., Manganelli, S., Risk Management for Central Bank FX. European Central Bank, Frankfurt, Germany.
- Forbes, K., Warnock, F., 2012. Capital flow waves: Surges, stops, flight, and retrenchment. *Journal of International Economics* 88, 235-251.
- Galati, G., Wooldridge, P., 2009. The Euro As A Reserve Currency: A Challenge To The Pre-Eminence Of The US Dollar? *International Journal Of Finance & Economics* 14, 1-23.
- Gete, P., Melkadze, G., 2020. A quantitative model of international lending of last resort. *Journal of International Economics* 123, 103290.
- Gintschel, A., Scherer, B., 2004. Currency reserve management by dual benchmark optimisation. In: Bernadell, C., Cardon, P., Coche, J., Diebold, F.X., Manganelli, S., Risk Management for Central Bank FX. European Central Bank, Frankfurt, Germany.
- Gopinath, G., Stein, J., 2018. Trade Invoicing, Bank Funding, and Central Bank Reserve Holdings. *AEA Papers and Proceedings* 108, 542-546.
- Gourinchas, P-O., Rey, H., 2005. International Financial Adjustment. *CIDER WP No. C05-139*.
- Gourinchas, P-O., Rey, H., 2007. International Financial Adjustment. *Journal of Political Economy* 115, 665-703.
- Hardy, B., 2018. Foreign currency borrowing, balance sheet shocks and real outcomes. *BIS WP No 758*.
- Harvey, C., Liechty, J., Liechty, M., Müller, P., 2010. Portfolio selection with higher moments. *Quantitative Finance* 10, 469-485.
- Hatase, M., Ohnuki, M., 2009. Did the structure of trade and foreign debt affect reserve currency composition? Evidence from interwar Japan. *European Review of Economic History* 13, 319-347.
- Heller, R., Knight, M., 1978. *Reserve-Currency Preferences of Central Banks*. *Essays in International Finance* 131, 1-35, Princeton, New Jersey: Princeton University.
- Hofmann, B., Shim, I., Shin, H-S., 2020. Emerging market economy exchange rates and local currency bond markets amid the Covid-19 pandemic. *BIS Bulletin no. 5*, 7 April.
- Honig, A., 2009. Dollarization, exchange rate regimes and government quality. *Journal of International Money and Finance* 28, 198-214.
- Ito, H., McCauley, R, Chan, T., 2015. Currency composition of reserves, trade invoicing and

currency movements. *Emerging Markets Review* 25, 16–29.

Ito, H., McCauley, R., 2020. Currency composition of foreign exchange reserves. *Journal of International Money and Finance* 102, 102104.

Jeanne, O., 2000. Foreign currency debt and the global financial architecture. *European Economic Review* 44, 719-727.

Jondeau, E., Rockinger, M., 2003. How Higher Moments Affect the Allocation of Assets. © Global EcoFinance™, 1-4.

Jondeau, E., Rockinger, M., 2006. Optimal Portfolio Allocation under Higher Moments. *European Financial Management* 12, 29-55.

Kane, A., 1982. Skewness Preference and Portfolio Choice. *Journal of Financial and Quantitative Analysis* 17, 15-25.

Korinek, A., Mendoza, E., 2014. From Sudden Stops to Fisherian Deflation: Quantitative Theory and Policy. *Annu. Rev. Econ.* 6, 299-332.

Korinek, A., 2018. Regulating capital flows to emerging markets: An externality view. *Journal of International Economics* 111, 61-80.

Kose, M., Nagle, P., Ohnsorge, F., Sugawara, N., 2020. Global Waves of Debt: What Goes up Must Come Down? Chp 3. In: *Global Waves of Debt: Causes and Consequences*. Advance Edition, Kose, M., Nagle, P., Ohnsorge, F., Sugawara, N., 2020. Washington, DC: World Bank. License: Creative Commons Attribution CC BY 3.0 IGO.

Kraus, A., Litzenberger, R., 1976. Skewness preference and the valuation of risk assets. *Journal of Finance* 31, 1085-1100.

Krugman, P., 1999. Balance Sheets, the Transfer Problem, and Financial Crises. *International Tax and Public Finance* 6, 459-472.

Levy Yeyati, E., 2006. Financial dollarization: evaluating the consequences. *Economic Policy* 21, 61-118.

Levy Yeyati, E., 2008. The cost of reserves. *Economic Letters* 100, 39-42.

Levy Yeyati, E., Panizza, U., 2011. The elusive costs of sovereign defaults. *Journal of Development Economics* 94, 95-105.

Lu, Y., Wang, Y., 2019. Determinants of currency composition of reserves: a portfolio theory approach with an application to RMB. *IMF WP/19/52*.

Makin, J., 1971. The Composition of International Reserve Holdings: A Problem of Choice Involving Risk. *American Economic Review* 61, 818-832.

Mateane, L., Semmler, W., 2016. A Unifying Framework for the Evaluation of the Composition

of FX reserves: The case for South Africa. In: Theoretical foundations of macroeconomic policy: Growth, productivity, and public finance, Giovanni Di Bartolomeo and Enrico Saltari (eds.), Routledge, London.

McCauley, R., Chan, T., 2014. Currency movements drive reserve composition. *BIS Quarterly Review*, December, 23-36.

Meese, R., Rogoff, K., 1983. Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Economics* 14, 3-24.

Mendoza, E., Yue, V., 2012. A General Equilibrium Model of Sovereign Default and Business Cycles. *Quarterly Journal of Economics* 127, 889-946.

Meyer, D., Meyer, J., 2005. Relative Risk Aversion: What Do We Know? *The Journal of Risk and Uncertainty* 31, 243-262.

Nugée, J., 2000. Foreign Exchange Reserves Management. *Handbooks in Central Banking* 19, Bank of England Centre for Central Banking Studies.

Ostry, J., Ghosh, A., Chamon, M., Qureshi, M., 2012. Tools for managing financial-stability risks from capital inflows. *Journal of International Economics* 88, 407-421.

Ouyang, A., Li, J., 2013. Too Big to Change: The Stabilizing Force of Reserve Currency Preferences in the International Monetary System. *Emerging Markets Finance and Trade* 49, 120-133.

Papaioannou, E., Portes, R., Siourounis, G., 2006. Optimal currency weights in international reserves: The impact of the euro and the prospects for the dollar. *Journal of the Japanese and International Economies* 20, 508-547.

Pratt, J., 1964. Risk Aversion in the Small and in the Large. *Econometrica* 32, 122-136.

Ramaswamy, S., 1999. Reserve Currency Allocation: An Alternative Methodology. *BIS WP* 72.

Rodrik, D., 2006. The Social Cost of FX Reserves. *NBER, WP-11952*.

Rossi, B., 2013. Exchange Rate Predictability. *Journal of Economic Literature* 51, 1063-1119.

Scott, R., Horvath, P., 1980. On the direction of preference for moments of higher order than the variance. *Journal of Finance* 35, 915-919.

Simkowitz, M., Beedles, W., 1978. Diversification in a Three-Moment World. *Journal of Financial and Quantitative Analysis* 13, 927-941.

Shousha, S., 2017. International Reserves, Credit Constraints, and Systemic Sudden Stops. *International Finance Discussion Papers* 1205, Board of Governors of the Federal Reserve System.

Shousha, S., 2019. The Dollar and Emerging Market Economies: Financial Vulnerabilities Meet the International Trade System. *International Finance Discussion Papers* 1258, Board of Governors of the Federal Reserve System.

Soesmanto, T., Selvanath, E., Selvanath, S., 2015. Analysis of the management of currency composition of foreign exchange reserves in Australia. *Econ Analysis Policy* 47, 82-89.

Stekler, L., Pierkarz, R., 1970. Reserve Asset Composition for Major Central Banks. *Oxford Economic Papers New Series* 22, 260-274.

Tsiang, S., 1972. The Rationale of the Mean-Standard Deviation Analysis, Skewness Preference, and the Demand for Money. *American Economic Review* 62, 354-371.

Trebesch, C., Zabel, M., 2017. The output costs of hard and soft sovereign default. *European Economic Review* 92, 416-432.

Truman, E., Wong, A., 2006. The Case for an International Reserve Diversification Standard. *Institute for International Economics, WP 06-2*.

Upper, C., 2017. Macroprudential frameworks, implementation and relationship with other policies. *BIS background paper, BIS Papers No 94*.

Varian, H., 1988. Estimating Risk Aversion from Arrow-Debreu Portfolio Choice. *Econometrica* 56, 973-979.

Villar, A., 2017. Macroprudential frameworks: objectives, decisions and policy interactions. *BIS background paper, BIS Papers No 94*.

Wooldridge, P., 2006. The changing composition of official reserves. *BIS Quarterly Review*, September, 25-38.

World Economic Forum 2012. *The Financial Development Report*. WEF USA Inc.

Appendix: Co-Skewness matrices for short and long-term returns using the random walk model

Table A1: Co-Skewness U.S. dollar matrix of short and long-term FX returns using random walk

	U.S. dollar	Euro	British pound	Japanese yen	Swiss franc
U.S. dollar	0.47 (0.09)	-0.165 (0.19)	0.09 (0.15)	0.02 (0.08)	-0.13 (0.13)
Euro	-0.165 (0.19)	-0.0006 (0.21)	-0.03 (0.34)	-0.01 (0.095)	0.023 (0.21)
British pound	0.09 (0.15)	-0.03 (0.34)	0.018 (0.32)	0.003 (0.14)	-0.0204 (0.24)
Japanese yen	0.02 (0.08)	-0.01 (0.095)	0.003 (0.14)	0.00006 (0.04)	-0.009 (0.085)
Swiss franc	-0.13 (0.13)	0.023 (0.21)	-0.0204 (0.24)	-0.009 (0.085)	0.009 (0.17)

Notes: The values in the parentheses are co-skewness values for long term returns

Table A2: Co-Skewness Euro matrix of short and long-term FX returns using random walk

	U.S. dollar	Euro	British pound	Japanese yen	Swiss franc
U.S. dollar	-0.16 (0.19)	-0.00056 (0.21)	-0.031 (0.34)	-0.011 (0.095)	0.023 (0.21)
Euro	-0.00056 (0.21)	0.16 (0.59)	0.014 (0.36)	0.008 (0.17)	0.034 (0.30)
British pound	-0.03 (0.34)	0.014 (0.36)	-0.006 (0.38)	-0.0015 (0.12)	0.009 (0.30)
Japanese yen	-0.011 (0.095)	0.008 (0.17)	-0.0015 (0.12)	-0.005 (0.05)	-0.0003 (0.096)
Swiss franc	0.023 (0.21)	0.034 (0.30)	0.009 (0.30)	-0.0003 (0.096)	-0.0055 (0.21)

Notes: The values in the parentheses are co-skewness values for long term returns

Table A3: Co-Skewness British Pound matrix of short and long-term FX returns using random walk

	U.S. dollar	Euro	British pound	Japanese yen	Swiss franc
U.S. dollar	0.088 (0.15)	-0.031 (0.34)	0.018 (0.32)	0.0032 (0.14)	-0.020 (0.24)
Euro	-0.031 (0.34)	0.014 (0.36)	-0.006 (0.38)	-0.002 (0.12)	0.009 (0.30)
British pound	0.018 (0.32)	-0.006 (0.38)	0.003 (0.40)	0.0007 (0.14)	-0.0037 (0.31)
Japanese yen	0.003 (0.14)	-0.002 (0.12)	0.0007 (0.14)	-0.0006 (0.04)	-0.002 (0.11)
Swiss franc	-0.02 (0.24)	0.009 (0.30)	-0.004 (0.31)	-0.002 (0.11)	0.0036 (0.24)

Notes: The values in the parentheses are co-skewness values for long term returns

Table A4: Co-Skewness Japanese yen matrix of short and long-term FX returns using random walk

	U.S. dollar	Euro	British pound	Japanese yen	Swiss franc
U.S. dollar	0.02 (0.08)	-0.012 (0.095)	0.003 (0.14)	0.00006 (0.04)	-0.009 (0.09)
Euro	-0.012 (0.095)	0.008 (0.17)	-0.002 (0.12)	-0.005 (0.05)	-0.0003 (0.10)
British pound	0.003 (0.14)	-0.002 (0.12)	0.0007 (0.14)	-0.0006 (0.04)	-0.002 (0.11)
Japanese yen	0.00006 (0.04)	-0.005 (0.05)	-0.0006 (0.04)	-0.003 (0.013)	-0.004 (0.032)
Swiss franc	-0.009 (0.09)	-0.0003 (0.10)	-0.002 (0.11)	-0.004 (0.032)	-0.002 (0.08)

Notes: The values in the parentheses are co-skewness values for long term returns

Table A5: Co-Skewness Swiss franc matrix of short and long-term FX returns using random walk

	U.S. dollar	Euro	British pound	Japanese yen	Swiss franc
U.S. dollar	-0.13 (0.13)	0.023 (0.21)	-0.02 (0.24)	-0.01 (0.09)	0.009 (0.17)
Euro	0.023 (0.21)	0.034 (0.30)	0.009 (0.30)	-0.0003 (0.10)	-0.006 (0.21)
British pound	-0.02 (0.24)	0.009 (0.30)	-0.004 (0.31)	-0.002 (0.11)	0.004 (0.24)
Japanese yen	-0.01 (0.09)	-0.0003 (0.10)	-0.002 (0.11)	-0.004 (0.032)	-0.002 (0.08)
Swiss franc	0.009 (0.17)	-0.006 (0.21)	0.004 (0.24)	-0.002 (0.08)	-0.012 (0.17)

Notes: The values in the parentheses are co-skewness values for long term returns