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## Value at risk, Equity and Diversification

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# Value at risk, Equity and Diversification

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## Abstract:

The value at risk measure attempts to summarize in a single number market value risk of a portfolio of financial assets. The paper focuses on the interaction between the solvency probability of a bank, on one hand, and the diversification potential of its portfolio, on the other hand, when optimum endowment of equity capital is to be determined. Given the necessity to achieve some confidence level of solvency we demonstrate that diversification pays when optimizing the use of the equity resource.

JEL-Classification: G21, G28, G38

Keywords: equity capital; banking; value at risk; diversification; risk management; asset-liability management

## 1 Introduction

The value at risk (VaR) of a portfolio measures the loss in its market value over a risk horizon that is exceeded with a small probability. Bank management can apply VaR to set capital requirements based on an estimate of capital loss due to market and credit risk.<sup>1</sup> The aim of our note is to determine the *optimum* amount of equity capital of a banking firm.

Institutionally, bank regulation mandates that banks using VaR-models to set aside equity capital for market risk of their financial operations use a relatively short risk horizon and a significantly high confidence level. Risk management has to link this issue with financial views on risks and profitability. In our decision model of a banking firm the financial objective is to maximize the value of the firm in a competitive financial market environment.<sup>2</sup> The return on the bank's portfolio of assets is risky. The banking firm is exposed to market risk and may, therefore, not be able to meet its debt obligations. Instead of coping with the exposure of the firm to risk by using hedging instruments such as financial derivatives (Broll, Wahl, Zilcha (1999)), in the present model we incorporate the VaR concept as a risk management tool to address the solvency status of the banking firm. We demonstrate that by considering an asset portfolio market and also institutional factors determine the bank's optimal equity policy and asset-liability management. Given the institutional requirement to achieve some high confidence level of solvency the banking firm can either increase its equity capital base or improve on the diversification potential of its asset portfolio.

## 2 The model

Let a banking firm invest in two risky assets to the extent  $A_1$  and  $A_2$ , respectively. At the beginning of the period the returns on the assets,  $\tilde{r}_{A_1}$  and  $\tilde{r}_{A_2}$ , are random. The bank's portfolio is financed by deposits and equity capital,  $D$  and  $K$ , respectively. Intermediation costs of the bank occur at the beginning of the period. The compounded cost function  $C(D)$  is twice continuously differentiable and has properties  $C'(\cdot) > 0$  and  $C''(\cdot) > 0$  whenever

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<sup>1</sup>See, e.g., Duffie/Pan (1997), Jorion (2000), Simons (2000), Saunders/Allen (2002), and Frenkel/Hommel/Rudolf (2005).

<sup>2</sup>See, e.g., Greenbaum and Thakor (1995), Freixas and Rochet (1997), Wong (1997).

$D > 0$ .

Equity is held by shareholders. Optimum decision making of bank's management has to satisfy the balance sheet identity:  $A_1 + A_2 = D + K$ . Given that the bank's assets have risky outcomes there is some probability for insolvency of the banking firm.

If the bank's loss in market value of its asset portfolio does not exceed equity capital at some confidence level  $1 - \alpha$ , then  $\text{VaR}_\alpha$  measures the maximum size of that loss in the next period. Therefore,  $K = \text{VaR}_\alpha$  implies that the bank is not able to meet its debt obligations with probability  $\alpha$ . Hence,  $\alpha$  measures the probability of bankruptcy of the banking firm. In case of insolvency equity holders have to turn their property rights over to depositors.

The banking firm offers the contractually fixed market rate of deposits,  $r_D$ , of the competitive banking sector. Its risky end-of-period profit,  $\tilde{\Pi}$ , can be stated as follows:

$$\tilde{\Pi} = \tilde{r}_{A_1}A_1 + \tilde{r}_{A_2}A_2 - r_D D - C(D). \quad (1)$$

Bank management maximizes the value of the banking firm satisfying the bank's balance sheet identity. Applying the VaR risk management approach, i.e.  $K = \text{VaR}_\alpha$ , management chooses the amount of equity capital by

$$\max_K E(\tilde{\Pi}) \quad (2)$$

$$\text{s.t.} \quad A_1 + A_2 - D = \text{VaR}_\alpha, \quad (3)$$

where  $E$  denotes the expectation operator and the banking firm's risky profit is determined by equation (1). We assume that bank management assesses market risk by presuming a binormal distribution of the random returns with expected returns  $\mu_{A_1}$  and  $\mu_{A_2}$ , respectively, standard deviations of returns  $\sigma_{A_1}$  and  $\sigma_{A_2}$ , respectively, and correlation of risky returns  $\rho$ .

Let us derive the value at risk measure in our economic setting. The probability of bankruptcy is given by  $\text{Prob}(-(\tilde{r}_1 A_1 + \tilde{r}_2 A_2) > K) > 0$ , where  $\tilde{r}_j = (\tilde{r}_{A_j} - r_D)/(1 + r_D)$ ,  $j = 1, 2$  (Appendix (I)). Hence, there is a positive probability that, at the end of the period the loss in market value of the bank's asset portfolio may exceed the volume of equity capital of the banking firm. The degree of this probability has to be chosen by management and/or is given by bank regulation.

We obtain the following solvency condition:

$$\text{Prob}(-(\tilde{r}_1 A_1 + \tilde{r}_2 A_2) \leq K) = 1 - \alpha. \quad (4)$$

The random variables  $\tilde{r}_1$  and  $\tilde{r}_2$  are binormally distributed with expected values  $\mu_j = E(\tilde{r}_j) = (\mu_{A_j} - r_D)/(1 + r_D)$  and standard deviations  $\sigma_j = S(\tilde{r}_j) = \sigma_{A_j}/(1 + r_D)$ ,  $j = 1, 2$ , where  $S$  denotes the standard deviation operator.

*Assumption (A.1).* Suppose that both assets of the banking firm exhibit identical expected returns, i.e.  $\mu_{A_1} = \mu_{A_2}$ , and identical standard deviations of return, i.e.  $\sigma_{A_1} = \sigma_{A_2}$ .

This assumption implies that:  $\mu_1 = \mu_2 \equiv \mu$  and  $\sigma_1 = \sigma_2 \equiv \sigma$ . It follows that the solvency condition (4) can be stated as

$$r_{\alpha\rho}(A_1 + A_2) = K, \quad (5)$$

where  $r_{\alpha\rho} = -(\mu + u_\alpha \sigma \sqrt{\frac{1+\rho}{2}}) > 0$  and  $u_\alpha$  is the  $\alpha$ -fractile of the unit normal distribution (Appendix (II)).

The magnitude  $r_{\alpha\rho}$  represents value at risk of a risky investment of one dollar in the portfolio. This magnitude is decreasing in the probability of bankruptcy,  $\partial r_{\alpha\rho}/\partial \alpha < 0$ , and increasing in the correlation coefficient,  $\partial r_{\alpha\rho}/\partial \rho > 0$ . VaR is determined by multiplying the portfolio investment amount with the unit VaR:  $\text{VaR}_\alpha = r_{\alpha\rho}(A_1 + A_2)$ .

### 3 Optimum equity capital

In the following we would like to discuss especially the impact of diversification on the optimal amount of equity capital. In order to arrive at a reduced form of optimal equity capital volume let us specify the intermediation cost function.

*Assumption (A.2).* Suppose that the intermediation cost function be quadratic and of the form  $C(D) = \theta D^2/2$ ,  $\theta > 0$ .

Taking into account all definitions and constraints of the above section and the specified cost function the banking firm's expected risky profit (1) reads ( $\mu_{A_1} = \mu_{A_2} \equiv \mu_A$ , see (A.1)):

$$E(\tilde{\Pi}) = \frac{1}{r_{\alpha\rho}} [\mu_A - (1 - r_{\alpha\rho})r_D] K - \frac{\theta}{2} \left( \frac{1 - r_{\alpha\rho}}{r_{\alpha\rho}} \right)^2 K^2. \quad (6)$$

In our economic setting expected profit can be stated as a function of the bank's equity capital, only.

We claim the following propositions:

*Proposition 1* (Equity) Risk management by VaR under market value maximization implies that optimum amount of equity capital depends upon

- (i) market factors such as the deposit rate and the intermediation costs and
- (ii) institutional factors such as the confidence level and the bivariate probability distribution of returns on risky assets.

Note that the correlation between the assets' returns is of special interest in our study.

Proof. Maximizing equation (6) with respect to equity  $K$  leads to:

$$K^* = r_{\alpha\rho} \frac{\mu_A - (1 - r_{\alpha\rho})r_D}{\theta(1 - r_{\alpha\rho})^2}. \quad (7)$$

This proves the claim.

Our first result reveals that optimum equity  $K^*$  can be determined explicitly. Herein unit VaR  $r_{\alpha\rho}$  plays a crucial role.

*Proposition 2* (Solvency) If the required confidence level  $1 - \alpha$  is augmented, then (cet. par.) equity base  $K^*$  has to be increased in the optimum.

Proof. From equation (7) it follows that interior solutions require a positive expected margin  $\mu_A > (1 - r_{\alpha\rho})r_D$  and  $r_{\alpha\rho} < 1$ . Since  $r_{\alpha\rho}$  decreases in  $\alpha$ , it follows that  $K^*$  decreases in  $\alpha$ , i.e.  $\partial K^* / \partial \alpha < 0$ .

If bank regulation institutionally sets a higher (lower) confidence level the owners of the bank have to contribute additional (less) equity capital in order to enable bank management to realize an optimal asset-liability policy. Furthermore, in a optimum bank policy under VaR, assets and liabilities management has to be undertaken simultaneously. Note that  $D^* = K^*(1 - r_{\alpha\rho})/r_{\alpha\rho}$ .

*Proposition 3* (Diversification) If the correlation of assets' returns  $\rho$  decreases, then (cet. par.) the optimally required amount of equity capital  $K^*$  decreases, too.



Proof. Given interior solutions (see Proof of Proposition 2) from equation (7) we obtain  $\partial K^*/\partial \rho > 0$ .

The correlation coefficient measures the diversification potential of the assets portfolio. If the risky returns correlation of assets  $A_1$  and  $A_2$  becomes smaller, then portfolio risk diminishes. Therefore a given confidence level can be achieved with less equity capital. Hence there exists a tradeoff between the optimum amount of equity capital and the risk minimizing potential of the assets portfolio of the banking firm.

On the other, if a higher confidence level is to be satisfied institutionally, instead of increasing the equity base a banking firm could adjust its assets portfolio. In this case the bank would have to exchange its assets by other assets whose returns are less correlated in order to holding constant the required optimal amount of equity capital.

Note that our analysis is based on risk neutrality. Nonetheless diversification is important for the discussion of the optimal amount of equity capital under the VaR approach.

## 4 Concluding remarks

Our study uses the VaR concept to analyze how much equity capital a competitive banking firm should have. The optimum amount of equity depends upon market and institutional factors, where the confidence level set by the regulator and the correlation of risky returns of the assets within the bank's portfolio have a nontrivial relationship with this amount. Finally, note that the VaR concept excludes a separation of equity policy and asset-liability management.

## Appendix

(I) Bankruptcy risk is defined by

$$\text{Prob}\left(A_1(1 + \tilde{r}_{A_1}) + A_2(1 + \tilde{r}_{A_2}) - D(1 + r_D) < 0\right) > 0.$$

From the balance sheet constraint this condition is equivalent to

$$\text{Prob}\left(A_1(1 + \tilde{r}_{A_1}) + A_2(1 + \tilde{r}_{A_2}) - (A_1 + A_2 - K)(1 + r_D) < 0\right) > 0,$$

which in turn becomes

$$\text{Prob}\left(A_1(1 + \tilde{r}_1) + A_2(1 + \tilde{r}_2) - A_1 - A_2 + K < 0\right) > 0,$$

using  $1 + \tilde{r}_j = (1 + \tilde{r}_{A_j})/(1 + r_D)$ ,  $j = 1, 2$ . It follows  $\text{Prob}(-(\tilde{r}_1 A_1 + \tilde{r}_2 A_2) > K) > 0$ .

(II) If  $\tilde{x}$  is normally distributed with expected value  $\mu_x = E(\tilde{x})$  and variance  $\sigma_x^2 = V(\tilde{x})$ , then the  $N(\mu_x, \sigma_x^2)$ -fractile of order  $\alpha$  is defined by

$$\text{Prob}(\tilde{x} \geq x_\alpha) = 1 - \alpha,$$

where  $x_\alpha = \mu_x + u_\alpha \sigma_x$  and  $u_\alpha$  is the  $N(0, 1)$ -fractile of order  $\alpha$ .

Since  $\mu_x$  represents in our model the expected portfolio return and  $\sigma_x$  the standard deviation of the portfolio return, from our assumption (A.1) it follows: (i) the expected portfolio return in dollar reads  $\mu(A_1 + A_2)$  and (ii) the standard deviation of the portfolio return in dollar reads  $\sigma(A_1^2 + A_2^2 + 2\rho A_1 A_2)^{1/2} = (\frac{1+\rho}{2})^{1/2}(A_1 + A_2)$ , for in the optimum we have  $A_1 = A_2$ .

Hence the solvency condition (4) for a normally distributed  $\tilde{r}_1 A_1 + \tilde{r}_2 A_2$  becomes

$$\text{Prob}\left(\frac{\tilde{r}_1 A_1 + \tilde{r}_2 A_2}{A_1 + A_2} \geq -\left(\frac{K}{A_1 + A_2}\right)_\alpha\right) = 1 - \alpha.$$

It follows that  $-(K/(A_1 + A_2))_\alpha = \mu + u_\alpha \sigma \sqrt{\frac{1+\rho}{2}}$ , where  $\mu = E(\tilde{r}_1) = E(\tilde{r}_2)$  and  $\sigma = S(\tilde{r}_1) = S(\tilde{r}_2)$ . Defining  $r_{\alpha\rho} = -(\mu + u_\alpha \sigma \sqrt{\frac{1+\rho}{2}})$  yields equation (5).

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