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Abstract

This paper revisits the airline schedule-buffer choice problem analyzed by Brueckner, Czerny and Gaggero (2020) using a simpler model where the random shocks influencing flight times are discrete rather than continuous. The analysis yields closed-form solutions for the flight and ground buffers as well as full comparative-static results, neither of which were available in the earlier paper. The paper also explores several extensions to the model that were not present in the previous paper.

JEL-Codes: L900.

Keywords: flight delays, airlines, schedule buffers, delay propagation.

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Airline Schedule Buffers and Flight Delays: A Discrete Model

by

Jan K. Brueckner, Achim I. Czerny and Alberto A. Gaggero*

1. Introduction

Flight delays, fueled by the historic growth in air travel, represent a substantial problem for passengers and airlines worldwide. Flight times are influenced by many random daily factors, including weather, mechanical issues, and unanticipated congestion. Airline scheduling practices address these random influences through the use of "schedule buffers," which include flight buffers (denoted "block-time buffers" in the industry) and ground buffers. A buffer is the amount added to minimum feasible flight or ground time to get the scheduled flight or ground time. Flight buffers reduce the chance that an individual flight is late, and flight and ground buffers jointly address the problem of delay propagation, where a late inbound flight leads to late departure of the subsequent flight and then its late arrival. According to USDOT data, a late inbound aircraft is the primary cause of a subsequent arrival delay.[1] Ground buffers, which add extra time between flights, are especially well suited to addressing this problem.

Brueckner, Czerny and Gaggero (2020) (hereafter BCG) presented a stylized analysis of the choice of schedule buffers, using a model where the random shocks affecting flight durations are continuous random variables. They also offered empirical tests of some of the model's predictions. Because of its continuous formulation, their theoretical analysis was complex, although it yielded a number of intuitive conclusions. The purpose of the present short paper is to revisit the buffer-choice problem in a simpler model where the random shocks influencing flight times are discrete. In addition to providing greater transparency, the analysis yields closed-form solutions for the buffers as well as full comparative-static results, neither of which were available in the earlier paper. The study thus provides a fuller insights into a conceptually intriguing optimization problem.

* We thank Kangoh Lee for comments, but the usual disclaimer applies.

[1] See Brueckner, Czerny and Gaggero (2020) for details of the Department of Transportation data.

Previous theoretical work on the buffer-choice problem can be found in papers by Deshpande and Arikan (2012), Arikan, Despande and Sohoni (2013), and Kafle and Zou (2016), which also have empirical components. Studies by Hao and Hansen (2014) and Kang and Hansen (2017), which analyze the choice of scheduled flight times, are closely related. See BCG for additional references.

The analysis in section 2 of the paper presents a model with just a single flight, which serves as a benchmark for the main two-flight model developed in section 3. Section 4 presents extensions of the two-flight model by assuming that the random shocks to flight durations are correlated rather than independent and considering stochastic ground times. Section 5 offers conclusions.

2. Single-Flight Model

Consider first a model with just a single flight, denoted 1. The duration of an undisrupted flight is $f_1 = m$, with a random amount $\epsilon_1 \geq 0$ added to m to generate the actual flight time. With probability $1 - p$, no flight disruption occurs, so that $\epsilon_1 = 0$. With probability p, the flight is disrupted, with ϵ_1 taking a value $e > 0$ that reflects the influences of weather, mechanical issues and other factors. Therefore, the flight duration equals m with probability $1 - p$ and $m + e$ with probability p.

Because of the possibility of a flight disruption, the airline sets the scheduled duration of the flight to be longer than m by use of a flight buffer $b_1 > 0$. With departure at time zero, the scheduled arrival time is then given by $t_{a1} = m + b_1$. If there is no flight disruption, with the duration then equal to m, the flight arrives b_1 minutes early. If a disruption occurs, then the flight arrives $e - b_1$ minutes late if $b_1 < e$, while it arrives $b_1 - e$ minutes early if $b_1 > e$. Passengers dislike being late or early, with the parameters x and y capturing lateness and earliness costs, which depend on the squares of the times late or early.[2] For example, with no flight disruption, the early cost is yb_1^2, whereas a flight disruption when $b_1 < e$ leads to a late cost of $x(e - b_1)^2$. Since the inconvenience of a late flight is greater than that of an early flight,

[2] Using squared values generates the required nonlinearity in the optimization problem.

$x > y$ is assumed. The expected early/late cost is given by

$$(1-p)yb_1^2 \; + \; \begin{cases} px(e-b_1)^2 & \text{if } \; b_1 < e \\ py(b_1-e)^2 & \text{if } \; b_1 \geq e. \end{cases} \tag{1}$$

In addition to the expected value in (1), the airline considers other costs in choosing the magnitude of the flight buffer. These elements are the cost of operating the flight, which include expenditures on fuel and crew salaries, and the cost of ground time, which consists mainly of gate rental costs. To facilitate comparison with the two-flight model, where ground time is present, the single-flight model also includes ground time, as follows. Suppose that the airline has leased the aircraft for a fixed period T that more than covers the flight time. Scheduled flight time is $m+b_1$ and scheduled ground time following the flight equals $T-(m+b_1) > 0$, so that the airline's lease leaves "excess capacity." The leasing cost is fixed, but with c_f denoting the cost per minute of scheduled flight time and c_g denoting the cost of scheduled ground time, total operating costs are $c_f(m+b_1)+c_g(T-(m+b_1))$, which equals a constant plus $(c_f-c_g)b_1$. Realistically, the analysis assumes that flight time is more expensive than ground time, so that $c_f > c_g$, an assumption that also eliminates some complexity.

The profit-maximizing airline chooses b_1 to minimize the sum of (1) and $(c_f-c_g)b_1$.[3] Since this expression is increasing in b_1 when $b_1 \geq e$ (in which case the second line of (1) applies), the airline will not set b_1 at or above e, instead choosing $b_1 < e$. The first-order condition for b_1, which makes use of the first line of (1), is then

$$2(1-p)yb_1 \; - \; 2px(e-b_1) \; + \; c_f \; - \; c_g \;\; = \;\; 0, \tag{2}$$

which yields the solution[4]

$$b_1^* \;\; = \;\; \frac{px}{px+(1-p)y}e \; - \; \frac{c_f-c_g}{2[px+(1-p)y]}. \tag{3}$$

[3] Letting v denote the fixed benefit from air travel, a passenger's willingness-to-pay for a ticket equals v minus (1), or travel benefit minus expected late/early cost, which equals the airfare F. Normalizing the flight's passenger capacity to unity, revenue is then F, and profit equals v minus (1) minus $(c_f-c_g)b_1$ minus the constant $(c_f-c_g)m$. Choosing b_1 to minimize (1) plus $(c_f-c_g)b_1$ thus maximizes profit. Note that since this objective function represents social cost, a planner would make the same choice as the airline.

[4] Since (2) is increasing in b_1, the solution represents a minimum.

Parameter values are assumed to take values that make this b_1^* solution positive, an assumption that pertains to all subsequent buffer solutions. Note that the solution in (3) sets b_1 equal to a fraction of e minus a positive term involving c_f and c_g. Since $b_1^* < e$, *the buffer is then chosen so that the flight arrives late with probability* p. From (3), b_1^* is naturally increasing in e and decreasing in $c_f - c_g$. Since it is easily seen that the factor multiplying e is increasing in x and p, and since the ratio involving $c_f - c_g$ is decreasing in x and decreasing in p (given $c_f > c_g$), it follows that b_1^* increases with x and p as well (the buffer also decreases with y).[5] Thus, the buffer naturally rises with lateness cost and the probability of a flight disruption, and falls with earliness cost. Summarizing yields[6]

$$\frac{\partial b_1^*}{\partial e} > 0, \quad \frac{\partial b_1^*}{\partial c_f} < 0, \quad \frac{\partial b_1^*}{\partial c_g} > 0, \quad \frac{\partial b_1^*}{\partial x} > 0, \quad \frac{\partial b_1^*}{\partial y} < 0, \quad \frac{\partial b_1^*}{\partial p} > 0. \tag{4}$$

Noting that the variance of the flight disruption equals $p(1-p)e^2$, (4) implies that a higher variance, whether its source is a higher p or a higher e, raises b_1^* (this conclusion requires that p is realistically less than $1/2$).[7]

3. Two-Flight Model

3.1. The setup

The aircraft in the single-flight model is now assumed to make a second flight, carrying a different group of passengers from flight 1's destination city to a second destination. Note that, with the passenger groups on the two flights being separate, connecting passengers are absent. However, a group of passengers whose trips require a flight connection could be incorporated with only minor changes in the analysis.

The second flight has the same undisrupted duration as flight 1, with $f_2 = m$, so that actual flight time equals $m + \epsilon_2$, where ϵ_2 equals e with probability p and zero otherwise. The random terms ϵ_1 and ϵ_2 are assumed to be independent, so that the forces leading to

[5] The last conclusion follows from factoring out the common term in the denominators of (3), which is decreasing in y, and noting that the remaining expression is positive when $b_1 > 0$.

[6] The effects of x, y, c_f and c_g on b_1 in the model of BCG take the same signs as in (4).

[7] The variance is $\mathrm{E}\epsilon_1^2 - (\mathrm{E}\epsilon_1)^2 = pe^2 - (pe)^2$, yielding the expression in the text.

flight disruptions are not common across the flights (the correlated case is considered below).[8] Despite this independence, a late arrival of flight 1 can cause a late departure and possibly a late arrival for flight 2, leading to delay propagation. The resulting linkage between the performance of the two flights is central to the analysis.

The aircraft's scheduled ground time between the flights is denoted t_g, and it must be at least as large as the minimum feasible turnaround time for the aircraft, denoted $\overline{t}_g$. This minimum time equals the interval required for the deplaning and boarding of passengers as well as the cleaning and refueling of the plane. The ground buffer is the difference between t_g and $\overline{t}_g$, which equals the extra scheduled ground time beyond the minimum required, and it is denoted by $b_g = t_g - \overline{t}_g$. The ground buffer is an instrument for reducing delay propagation, as seen in the following analysis.

The scheduled departure time of flight 2, denoted t_{d2}, equals the scheduled arrival time of flight 1 plus the scheduled ground time, or $t_{d2} = m + b_1 + t_g$. If $\epsilon_1 = 0$, so that flight 1 is not delayed, instead arriving early, then flight 2 departs on time. However, if flight 1 arrives late, then flight 2's departure may be delayed. Late departure occurs if the earliest possible departure of flight 2, which equals flight 1's arrival time plus the minimum turnaround time, exceeds t_{d2}, flight 2's scheduled departure time. In other words, late departure occurs if

$$m \ + \ e \ + \ \overline{t}_g \quad > \quad m \ + \ b_1 \ + \ t_g \quad = \quad t_{d2}. \tag{5}$$

Recalling $b_g = t_g - \overline{t}_g$ and rearranging, (5) reduces to

$$b_1 \ + \ b_g \quad < \quad e. \tag{6}$$

Thus, if flight 1 is delayed, flight 2 departs late when b_1 and b_g satisfy (6), departing on time otherwise.

[8] Since flight 1's destination airport is flight 2's origin, weather at this airport could affect both flights, in which case ϵ_1 and ϵ_2 would be positively correlated. While this possibility is incorporated in the analysis in section 4, other sources of disruption are likely to be independent across the flights.

The focus of flight 2's passengers, however, is on their arrival time, not their departure time. If flight 2 departs on time, then the analysis of its arrival time follows the single-flight case. The expected late/early cost for its passengers is given by the expression that pertains to the single-flight case, equal to (1) with b_2 in place of b_1.

If flight 2 departs late, then derivation of its arrival time is more involved. Flight 2's scheduled arrival time is $t_{a2} = m + b_1 + t_g + m + b_2$, with the last two terms capturing the scheduled duration of flight 2. When flight 2 departs late, its actual arrival time is equal to the departure time $m + e + \overline{t}_g$ plus $m + \epsilon_2$. When $\epsilon_2 = 0$, late arrival occurs when

$$m \ + \ e \ + \ \overline{t}_g \ + \ m \ + \ 0 \ \ > \ \ m \ + \ b_1 \ + \ t_g \ + \ m \ + \ b_2 \ \ = \ \ t_{a2}, \tag{7}$$

or when

$$b_1 \ + \ b_2 \ + \ b_g \ \ < \ \ e, \tag{8}$$

with early arrival occurring when the inequality (8) is reversed. When ϵ_2 equals e instead of 0, the zero on the LHS of (7) is replaced by e, and the condition (8) for late arrival of flight 2 is replaced by

$$b_1 \ + \ b_2 \ + \ b_g \ \ < \ \ 2e. \tag{9}$$

Early arrival of flight 2 when $\epsilon_2 = e$ occurs when the inequality in (9) is reversed.

Table 1 shows how all this information can be used to build the airline's objective function for choosing b_1, b_2, and b_g. The first column of the table shows the different combinations of the random terms ϵ_1 and ϵ_2, with the second column showing the probabilities of the combinations. The third column shows the late/early cost for flight 1 passengers. The single-flight expression in (1) can be generated from the table by just focusing on flight 1. The different expressions in the "Flt. 1 Early/Late Cost" column would be multiplied by their associated probabilities and summed, an exercise that leads to (1). To incorporate flight 2, the expressions in the "Flt. 2 Early/Late Cost" column would also be weighted by their associated probabilities and summed, with the resulting expression added to (1) to get expected late/early cost for both flights.[9] While (1) itself is fairly simple, the resulting composite expression is much more

[9] Note that "Early/Late Cost" expressions for flights 1 and 2 that appear in the same row of Table 1 have no relation to one another aside from their appearance in the same "Random Outcome" block of the table.

complicated, involving many more conditional statements of the type $b_1 + b_g > e$, etc. But the same approach used in excluding the second line of (1) can be applied more broadly to generate a set of solutions for all three buffers.

To better understand the entries in the "Flt. 2 Late/Early Cost" column, observe that in rows 1–3, $\epsilon_1 = 0$ means that flight 2 departs on time (noted in the next column), which in turn implies that the early/late cost expressions for flight 2 are the same as those in the single-flight case. In row 4, $\epsilon_1 = e$, but with $b_1 + b_g \geq e$ assumed, flight 2 departs on time, and since $\epsilon_2 = 0$ in these rows, flight 2 is early, with early cost of yb_2^2. For the next two entries (in rows 5 and 6), $b_1 + b_g < e$ is assumed, so that Flight 2 departs late. Then, late/early arrival is governed by (8) and the reverse inequality, with late time equal to $e - (b_1 + b_2 + b_g)$ and early time the negative of this expression. In rows 7 and 8, $\epsilon_2 = e$, but flight 2 departs on time, so that the single-flight expressions apply for flight 2. In rows 9 and 10, flight 2 departs late, and late/early arrival is governed by (9), with the late/early times adjusted accordingly.

3.2. Derivation of b_1^* and b_2^* solutions

Excess capacity is absent in the two-flight model,[10] so that the cost of scheduled flight and ground time is $c_f(b_1 + b_2) + c_g b_g$ plus the constant $c_f(2m)$. The airline's goal is to minimize expect late/early cost plus the first of these expressions. It is useful to consider the first-order condition for b_g first. Differentiating the relevant expressions in Table 1, this condition is[11]

$$
\begin{aligned}
&- 2p(1-p)x(e - (b_1 + b_2 + b_g)) && \text{if } b_1 + b_g < e \text{ and } b_1 + b_2 + b_g < e \\
&+ 2p(1-p)y(b_1 + b_2 + b_g - e) && \text{if } b_1 + b_g < e \text{ and } b_1 + b_2 + b_g > e \\
&- 2p^2 x(2e - (b_1 + b_2 + b_g)) && \text{if } b_1 + b_g < e \text{ and } b_1 + b_2 + b_g < 2e \\
&+ 2p^2 y(b_1 + b_2 + b_g - 2e) && \text{if } b_1 + b_g < e \text{ and } b_1 + b_2 + b_g > 2e \\
&+ c_g \quad = \quad 0. && \qquad (10)
\end{aligned}
$$

[10] BCG explore the effects of changing the current excess-capacity assumptions by adding it to the two-flight model, so that extra ground time exists after flight 2. In the current setting the buffers would then be set to eliminate the chance of late arrival for flight 2, as in BCG. The effect of removing excess capacity from the single-flight model can also be investigated.

[11] Since the b_g-derivative of (10) is positive, the second-order condition is satisfied.

While (10) is not immediately useful in solving for b_g, it can be used to solve for b_1 and b_2, eventually leading to a b_g solution. In differentiating the flight-2 components of the objective function with respect to b_1, the derivative contains the first four lines of (10), as can be seen from differentiating the expressions in rows 5–6 and 9–10 of Table 1 with respect to b_1. But from (10) itself, the sum of the first four lines of (10) must equal $-c_g$ at the optimum. Therefore, the derivative of the flight-2 components of the objective function with respect to b_1 equals $-c_g$. With the derivative of the above cost function equal to c_f, it remains to add the derivative of the flight-1 components of the objective function. As noted above, these components equal the single-flight expression (1), which means that the derivative is equal to the first two terms in (2). Adding $c_f - c_g$, the first-order condition is then identical to the condition (2) from the single-flight model.[12] Therefore, *the optimal value of* b_1 *in the two-flight model is the same as the solution* b_1^* *in the single-flight model.*

This result also emerges in the analysis of BCG. The implication is that flight 1's buffer, being the same as in the single-flight model where delay propagation is absent, plays no role in addressing delay propagation in the two-flight model. As a result, delay propagation is dealt with entirely by flight 2's buffer and the ground buffer. The intuitive explanation is that, while a lengthening of flight 1's buffer to address delay propagation would distort its role in balancing early and late costs for flight 1, the ground buffer offers a superior, nondistorting instrument for addressing propagation. Therefore, reliance on flight 1's buffer is inefficient.

From the solution in (3), b_1^* is less than e. It is easy to see that $b_1^* + b_g^*$ must also be less than e, using (10). To proceed, suppose to the contrary that $b_1^* + b_g^* \geq e$. Then none of the conditions in the first four lines is satisfied, so that the derivative of the objective function with respect to b_g equals c_g. With the function thus increasing in b_g when $b_1 + b_g \geq e$, values that satisfy this inequality cannot be optimal, so that the optimum must satisfy $b_1^* + b_g^* < e$. As a result, *under the optimal ground buffer, flight 2 departs late when flight 1 arrives late, otherwise departing on time.* Note that the costliness of ground time means that it is not optimal to eliminate the chance of late departure for flight 2. The ground buffer, however, reduces the extent of the departure's lateness. While flight 1 arrives $e - b_1^*$ minutes late, flight

[12] The second-order condition is again satisfied here and in the choice of b_2 below.

2 departs $e - b_1^* - b_g^*$ minutes late, a smaller value.

In differentiating the objective function with respect to b_2, satisfaction of $b_1^* + b_g^* < e$ rules out the flight-2 cases in rows 4, 7, and 8 of Table 1, leaving only the cases where flight 2 departs late. As with b_1, the b_2 derivative of the flight-2 components in lines 5–6 and 9–10 is the same as the first four lines of (10). With b_g chosen optimally, this derivative is then again equal to $-c_g$. Initially assuming $b_2 \geq e$ and differentiating the remaining flight-two components in rows 1 and 3, the result is then added to c_f (the b_2-derivative of the cost term) minus c_g, yielding $2(1-p)^2 y b_2 + 2(1-p)py(b_2 - e) + c_f - c_g$. Positivity of this expression means that $b_2 < e$ must instead be optimal, and using rows 1 and 2 then yields the first-order condition

$$2(1-p)^2 y b_2 \;-\; 2(1-p)px(e - b_2) \;+\; c_f \;-\; c_g \;=\; 0. \tag{11}$$

and the solution

$$b_2^* \;=\; \frac{px}{px + (1-p)y} e \;-\; \frac{1}{1-p}\;\frac{c_f - c_g}{2[px + (1-p)y]}. \tag{12}$$

This solution is the same as the single-flight solution on the RHS of (3) except for the $1/(1-p)$ term in the second expression. This term, being larger than 1, makes the solution smaller than the RHS of (3). Therefore, *the optimal value of b_2 in the two-flight model is smaller than the single-flight value.* To understand this conclusion, recall that flight 2's buffer and the ground buffer are together responsible for addressing early/late arrival of flight 2. When flight 2 departs late, b_2 and b_g are in fact perfect substitutes in this task, given that they appear in summation form in lines 5–6 and 9–10 in Table 1. But since $c_f > c_g$, b_2 is a more expensive instrument than b_g, making usage of the ground buffer preferable and pushing b_2 toward zero. However, b_2 still plays a role in addressing flight 2's late arrival when the flight departs on time, as seen in lines 1-3 of Table 10. Therefore, b_2 is not set at zero, but downward pressure from the late-departure case makes it optimal to set b_2 below the single-flight value.

The effects of the parameters on b_2^* are the same as the effects on b_1^* with the exception of the effect of p, which is ambiguous. The reason for this ambiguity is that the increase in $1/(1-p)$ when p rises offsets the decrease in the last ratio in (12), leaving the net effect

unclear.[13] Summarizing yields

$$\frac{\partial b_2^*}{\partial e} > 0, \quad \frac{\partial b_2^*}{\partial c_f} < 0, \quad \frac{\partial b_2^*}{\partial c_g} > 0, \quad \frac{\partial b_2^*}{\partial x} > 0, \quad \frac{\partial b_2^*}{\partial y} < 0, \quad \frac{\partial b_2^*}{\partial p} > (<)\ 0. \tag{13}$$

In contrast to (13), the model of BCG, because of its greater complexity, was unable to generate any comparative-static results at all for b_2^*.

3.3. Solving for b_g^*

To solve for b_g^*, the first step is to note that the inequalities $b_2^* < e$ and $b_1^* + b_g^* < e$ imply $b_1^* + b_2^* + b_g^* < 2e$. This inequality means that the buffers take values smaller than the ones that would completely eliminate the chance of late arrival for flight 2 when it departs late. Note that satisfaction of $b_1^* + b_2^* + b_g^* \geq 2e$ would also imply $b_1^* + b_2^* + b_g^* \geq e$, so that neither of the conditions (8) and (9) for lateness of flight 2 when it departs late would hold.

Next, observe that $b_1^* + b_2^* + b_g^* < 2e$ allows the case in the fourth line of (10) to be ruled out. The b_g solution still depends, however, on whether $b_1^* + b_2^* + b_g^*$ is smaller or larger than e. In the first case, the second line of (10) is excluded, leaving the first, third and fifth lines. The first-order condition for b_g is then

$$-2p(1-p)x(e - (b_1 + b_2 + b_g)) \ - \ 2p^2x(2e - (b_1 + b_2 + b_g)) \ + \ c_g \ = \ 0. \tag{14}$$

Solving for $b_1 + b_2 + b_g$ then yields

$$b_1^* \ + \ b_2^* \ + \ b_g^* \ = \ (1+p)e \ - \ \frac{c_g}{2px} \ < \ e. \tag{15}$$

Alternatively, when $b_1^* + b_2^* + b_g^* \geq e$ holds, solving using the second, third and fifth lines of (10) yields

$$b_1^* \ + \ b_2^* \ + \ b_g^* \ = \ \left[1 + \frac{px}{px + (1-p)y}\right] e \ - \ \frac{c_g}{2p[px + (1-p)y]} \ \geq \ e. \tag{16}$$

[13] If $c_f - c_g$ is small, this ambiguous effect will be dominated by the positive p effect from the first term in (12), making $\partial b_2^*/\partial p > 0$.

By inspection, the inequality $b_1^* + b_2^* + b_g^* < 2e$ is validated by the solutions in (15) and (16). But in addition, the solutions in (15) (in (16)) must actually be less than (greater than or equal to) e. From inspection, the RHS of (15) is less than e when $e < c_g/2p^2x$, and rearrangment shows that the RHS of (16) is greater than or equal to e when the reverse of the previous inequality holds. Using this condition along with (15) and (16), the solution for b_g can then be written

$$b_g^* \;=\; \begin{cases} (1+p)e - c_g/2px - b_1^* - b_2^* & \text{if } e < c_g/2p^2x \\ \left[1 + \frac{px}{px+(1-p)y}\right] e \;-\; \frac{c_g}{2p[px+(1-p)y]} - b_1^* - b_2^* & \text{if } e \geq c_g/2p^2x, \end{cases} \tag{17}$$

with b_1^* and b_2^* given by (3) and (12).[14]

Turning to comparative-static effects, since an increase in x raises the first two terms in the solutions in (17) while also raising b_1^* and b_2^*, the net effect on b_g^* is unclear. Since b_1^* and b_2^* are decreasing in y, the first b_g^* solution is increasing in y, although y's effect on the second solution in (17) is unclear given the ambiguous reponse of the first part of the solution. Because b_1^* and b_2^* are decreasing in c_f, b_g^* increases with c_f. Moreover, because b_1^* and b_2^* increase with c_g and the second terms in (17) decrease with c_g, b_g^* is decreasing in c_g. The effects of c_f and c_g thus conform to intuition. Finally, because the effect of p on b_2^* is ambiguous, b_g^* also responds ambiguously to an increase in p. Summarizing yields

$$\frac{\partial b_g^*}{\partial c_f} > 0, \quad \frac{\partial b_g^*}{\partial c_g} < 0, \quad \frac{\partial b_g^*}{\partial x} < (>)\; 0, \quad \frac{\partial b_g^*}{\partial y} > (<)\; 0, \quad \frac{\partial b_g^*}{\partial p} > (<)\; 0. \tag{18}$$

The effect of the parameter e remains to be considered. Focusing just on the e terms in the solutions from (17) and using (3) and (12), the e term from the first line equals

$$(1+p)e \;-\; 2\frac{px}{px+(1-p)y}e \;=\; \frac{(1-p)((1+p)y - px)}{px+(1-p)y}e. \tag{19}$$

[14] While the inequality $b_1^* + b_2^* + b_g^* < 2e$ is validated by the solutions in (15)–(16) and conditions for $b_1^* + b_2^* + b_g^* < (\geq)\; e$ have been given, whether the inequality $b_1^* + b_g^* < e$ is validated by the actual solutions remains to be checked. Using the first line of (17) to solve for $b_1^* + b_g^*$ when $e < c_g/2p^2x$, the condition $b_1^* + b_g^* < e$ reduces to a complicated inequality involving all of the model's parameters, which is assumed to hold. When $e \geq c_g/2p^2x$, the inequality $b_1^* + b_g^* < e$ reduces to the condition $pc_f < c_g$, which must then hold along with the maintained assumption $c_f > c_g$.

The factor multiplying e is negative (positive) as $x > (<)\ ((1+p)/p)y$, so that the effect of e on b_g^* can take either sign (recall that $x > y$ is assumed). For the second solution in (17), the e effect is positive.[15] Therefore, the derivative $\partial b_g^*/\partial e$ can take either sign, indicating that *the ground buffer can be either increasing or decreasing in the size of the flight disruption*, as measured by e. Since this somewhat counterintuitive result appeared in a comparative-static simulation in the more complex model of BCG, its appearance here as well is noteworthy. Evidently, the positive responses of b_1^* and b_2^* to a higher e obviate the need for an unambiguous similar response in e. Despite this common conclusion, BCG's analysis produced no general comparative-static results for b_g^*, in contrast to the c_f and c_g effects in (18).

Even though the comparative statics for b_g^* are mostly ambiguous, parameter effects on the *sum of the buffers* are more often determinate. The $b_1^* + b_2^* + b_g^*$ solutions in (15) and (16) are increasing in p, e, x, decreasing in c_g, independent of c_f and either unaffected or ambiguously affected by y. Thus, letting S^* denote $b_1^* + b_2^* + b_g^*$,

$$\frac{\partial S^*}{\partial e} > 0, \quad \frac{\partial S^*}{\partial c_f} = 0, \quad \frac{\partial S^*}{\partial c_g} < 0, \quad \frac{\partial S^*}{\partial x} > 0, \quad \frac{\partial S^*}{\partial y} > (<)\ 0, \quad \frac{\partial S^*}{\partial p} > 0. \tag{20}$$

With S^* effectively capturing the airline's overall effort to address early/late arrivals and delay propagation via schedule buffers, it is natural that S^* increases with the size e and probability p of a flight disruption and with the cost x of lateness. Since the three buffers combined involve both flight and ground time, the effects of c_f and c_g on S^* are unclear a priori, although determinate effects are seen in (20).

As a final exercise, it is useful to compute the probability of late arrival for flight 2, making use of the preceding results. Consider first the case where $e < c_g/2p^2x$. When flight 2 departs on time, with $\epsilon_1 = 0$, it arrives late when $\epsilon_2 = e$, events that have probability $(1-p)p$. The flight also arrives late when $\epsilon_1 = e$ (implying late departure) and $\epsilon_2 = 0$, given that $b_1^* + b_2^* + b_g^* < e$ holds when e is small (these events have probability $p(1-p)$). In addition, flight 2 arrives late when $\epsilon_1 = e$ and $\epsilon_2 = e$ since $b_1^* + b_2^* + b_g^* < 2e$ holds, events that have

[15] The e expression equals the bracketed term in the second line with the second term in (19) again subtracted off. This difference equals e times $(1-p)y/(px + (1-p)y)$.

probability p^2. Therefore, the probability of late arrival for flight 2 when e is small equals $(1-p)p + p(1-p) + p^2 = p(2-p) > p$. When e is large, satisfying $e \geq c_g/2p^2x$, flight 2 arrives on time when $\epsilon_1 = e$ and $\epsilon_2 = 0$ since $b_1^* + b_2^* + b_g^* > e$ then holds. The middle term in the previous probability sum is then replaced with zero, while the other two terms remain the same, so that the sum becomes $(1-p)p + p^2 = p$. Thus, *the probability of late arrival for flight 2 exceeds (equals) p when e is small (large), so that the probability across the two e cases is at least as large flight 1's probability p of late arrival, a natural conclusion given that flight 2 is subject to delay propagation*. Note, however, the flight 2's probability of late arrival is larger when e is small than when e is large, a counterintuitive conclusion that is presumably related to the unexpected effects of e on the ground buffer.

4. Extensions

This section considers two extensions to the model that were not present in BCG's analysis. The first is correlation in the random shocks affecting flight durations and the second is stochastic ground times.

4.1. Correlation between ϵ_1 and ϵ_2

While the random factors affecting flight 1's and 2's durations have so far been assumed to be independent, it is useful to investigate the case where ϵ_1 and ϵ_2 are correlated. To this end, let R denote the covariance between ϵ_1 and ϵ_2, which will be positive when common factors affect the durations of flights 1 and 2. For example, because flight 1's destination airport is flight 2's origin, bad weather at that airport will add to the durations of both flights.

When the ϵ's are correlated, it can be shown that $\text{Prob}(\epsilon_1 = e, \epsilon_2 = e) = p^2 + R$, where p is now defined by $\text{E}(\epsilon_i) = pe$.[16] In addition, $\text{Prob}(\epsilon_1 = 0, \epsilon_2 = 0) = (1-p)^2 + R$ and $\text{Prob}(\epsilon_1 = 0, \epsilon_2 = e) = \text{Prob}(\epsilon_1 = e, \epsilon_2 = 0) = p(1-p) - R$.[17]

Substituting these probabilities in the probability column of Table 1, new buffer solutions can be computed. It is easy to see that the b_1 solution remains the same as before, given by

[16] In the uncorrelated case, the expected value of ϵ_i also equalled pe, but p was defined as $\text{Prob}(\epsilon_i = 0)$, a probability that is not relevant in the correlated case.

[17] See `https://math.stackexchange.com/questions/2329573/joint-probability-distribution-of-two-bernoulli-r-v-with-a-correlation-r`.

(3). The b_2 solution is now given by[18]

$$b_2^* \;=\; e\left(1 \;+\; \frac{(1-p)^2 \;+R}{(1-p)p - R}\,\frac{y}{x}\right)^{-1} \;-\; \frac{1}{1-p}\;\frac{c_f - c_g}{2[px + (1-p)y - (x-y)R/(1-p)]}. \quad (21).$$

Inspection of (21) shows that b_2^* is decreasing in R, so that moving from the independent case ($R = 0$) to the positive-covariance case, where $R > 0$, reduces flight 2's buffer, with $\partial b_2^*/\partial R < 0$. While it is natural that b_1^* is unaffected by R, a higher R raises the likelihood that both ϵ's are positive, making late departure and arrival for flight 2 more likely relative to the case where the flight departs on time (which occurs when only ϵ_2 is positive). As a result, the downward pressure on b_2 that arises in the late departure case (as discussed above) is strengthened, causing the buffer to fall as R increases.[19]

These forces are further revealed in the solution for the sum of the buffers, which is given by

$$b_1^* + b_2^* + b_g^* \;=\; \begin{cases} (p + 1 + R/p)e \;-\; c_g/2px & \text{if } e < \frac{c_g}{2x(p^2+R)} \\ e\left(1 + \left[1 + \frac{(1-p)p-R}{p^2+R}\,\frac{y}{x}\right]^{-1}\right) - \;\frac{1}{p}\;\frac{c_g}{2[px+(1-p)y+(x-y)R/p]} & \text{if } e \geq \frac{c_g}{2x(p^2+R)}. \end{cases} \quad (22)$$

Inspection of (22) shows that the buffer sum is increasing in R, so that *a greater covariance raises the airline's overall buffer-driven effort to address early/late arrivals and delay propagation.* This conclusion, combined with b_1^*'s independence of R and b_2^*'s inverse relationship, then implies that b_g^* must increase with R, so that $\partial b_g^*/\partial R > 0$. Note that these kinds of conclusions were well beyond the reach of BCG's analysis, given the greater complexity of their model.

Returning to the assumption of independent ϵ's, it also possible to investigate the effect of flight-specific p values, p_1 and p_2. Flight 1's buffer is naturally independent of p_2, while if x

[18] It is easily seen by rearrangement that the first expression in (21) reduces to the analogous expression in (12) when $R = 0$.

[19] Note that a positive R also increases the probability that both ϵ's are zero, an outcome under which a small b_2 is favored, so as to reduce earliness cost. Observe also that, if y were greater than x, the effect of R on b_2 would become ambiguous. Evidently, $x > y$ is needed for a determinate effect because more weight is then placed on the late-arrival as opposed to early-arrival outcomes.

is sufficiently close to y, an increase in p_2 raises b_2^* without affecting b_g^*, an intuitively sensible conclusion.

4.2. Random ground time

Suppose that instead of flight durations being random, aircraft ground time is stochastic, a result of unforeseen factors that slow the turnaround time between flights. Flight durations are now equal to m, but the minimum turnaround time equals $\overline{t}_g + e_g$ with probability q and $\overline{t}_g$ with probability $1-q$, where $e_g > 0$. With a flight disruption absent for flight 1, a buffer is unneeded, which means that its scheduled and actual arrival time is m. Flight 2's scheduled departure time is $m + t_g$, and it departs late if m plus the disrupted minimum turnaround time exceeds this value, or if $m + \overline{t}_g + e_g > m + t_g$, which reduces to $b_g < e_g$.

With a flight buffer potentially optimal, flight 2's scheduled arrival time is $m + t_g + m + b_2$. When no turnaround disruption occurs, flight 2's arrival time is $m + t_g + m$, making it b_2 minutes early and yielding a cost of yb_2^2, which occurs with probability $1-q$.

When a turnaround disruption occurs and $b_g \geq e_g$ holds, flight 2 departs on time and arrives early, leading again to an early cost of yb_2^2, which occurs with probability q. Adding $(1-q)yb_2^2$, qyb_2^2 and the buffer costs $c_f b_2 + c_g b_g$, overall expected cost is increasing in both b_2 and b_g, ruling out optimality of the case where $b_g \geq e_g$.

Thus, $e_g > b_g$ must hold, so that flight 2 departs late when a turnaround disruption occurs. Flight 2's actual arrival time is then $m + \overline{t}_g + e_g + m$, and this time is greater than (less than or equal to) the scheduled arrival time as $m + \overline{t}_g + e_g + m > (\leq)\ m + t_g + m + b_2$, or as $b_g + b_2 < (\geq)\ e_g$, yielding late (early) minutes equal to $e_g - b_g + b_2$ $(b_g + b_2 - e_g)$. Adding buffer costs and earliness cost when no flight disruption occurs, overall expected cost is then

$$c_f b_2 \ + \ c_g b_g \ + \ (1-q)yb_2^2 \ + \ \begin{cases} qx(e_g - (b_g + b_2))^2 & \text{if } \ b_g + b_2 < e_g \\ qy(b_g + b_2 - e_g)^2 & \text{if } \ b_g + b_2 \geq e_g. \end{cases} \tag{23}$$

Since (23) is increasing in b_g and b_2 when $b_g + b_2 \geq e_g$, the first line of the expression is relevant. Differentiating and solving for the buffers yields a negative solution for b_2^*, which is inadmissible. Therefore, b_2^* is optimally set at zero, in which case

$$b_g^* \ = \ e_g \ - \ \frac{c_g}{2qx}. \tag{24}$$

The ground buffer is increasing in the size e_g and probability q of the turnaround disruption and decreasing in c_g.

It makes intuitive sense that the potential turnaround disruptions are addressed entirely by the ground buffer. The explanation of this outcome is similar to that underlying the magnitude of b_2^* in the basic model, and it can be seen by considering the first line of (23) while ignoring the $(1-q)yb_2^2$ term. While the two buffers are perfect substitutes in reducing lateness cost (appearing as a sum), the flight buffer is more expensive. As a result, the ground buffer is favored, and this force is further amplified when the earliness-cost term $(1-q)yb_2^2$ is also considered, making $b_2 = 0$ optimal.

5. Conclusion

This paper has provided a simplified version of the schedule-buffer analysis of Brueckner, Czerny and Gaggero (2020), using a more transparent model that generates closed-form solutions and comparative-static results. This approach helps to generate fuller insights into a conceptually intriguing optimization problem while allowing exploration of several extensions not considered by BCG. Future work could perhaps build on this simpler approach by including more than two flights or two competing airlines.[20]

Beyond its theoretical interest, the analysis has real-world relevance. Some of its predictions are confirmed by the empirical results of BCG, which rely on voluminous USDOT data on the daily flight operations of individual aircraft to compute flight and ground buffers. For example, the results show that a higher flight-time variance (measured for the same flight in the previous year) raises flight buffers, consistent with the impacts of e and p in (3) and (12). In addition, mixed evidence shows that the variance's effect on ground buffers is sometimes positive and sometimes negative, consistent with (19). Flight buffers also rise with airport congestion, another factor that may increase flight-time variability. Therefore, the paper's theoretical analysis (like that of BCG) is closely linked to actual outcomes.

[20] To incorporate two carriers, the approach of Brueckner and Flores-Fillol (2007) could be used.

Table 1: Objective Function Components

Random Outcome	Probability	Flt. 1 Early/Late Cost	Flt. 1 Arrival	Flt. 2 Early/Late Cost	Flt. 2 dep. on time	Flt. 2 Arrival	row #
$\epsilon_1 = 0, \epsilon_2 = 0$	$(1-p)^2$	yb_1^2	early	yb_2^2	yes	early	1
$\epsilon_1 = 0, \epsilon_2 = e$	$(1-p)p$	yb_1^2	early	$x(e-b_2)^2$ if $b_2 < e$	yes	late	2
				$y(b_2-e)^2$ if $b_2 \geq e$	yes	early	3
$\epsilon_1 = e, \epsilon_2 = 0$	$p(1-p)$	$x(e-b_1)^2$ if $b_1 < e$	late	yb_2^2 if $b_1 + b_g \geq e$	yes	early	4
		$y(b_1-e)^2$ if $b_1 \geq e$	early	$x(e-(b_1+b_2+b_g))^2$ if $b_1+b_g < e$ and $b_1+b_2+b_g < e$	no	late	5
				$y(b_1+b_2+b_g-e)^2$ if $b_1+b_g < e$ and $b_1+b_2+b_g \geq e$	no	early	6
$\epsilon_1 = e, \epsilon_2 = e$	p^2	$x(e-b_1)^2$ if $b_1 < e$	late	$x(e-b_2)^2$ if $b_1+b_g \geq e$ and $b_2 < e$	yes	late	7
		$y(b_1-e)^2$ if $b_1 \geq e$	early	$y(b_2-e)^2$ if $b_1+b_g \geq e$ and $b_2 \geq e$	yes	early	8
				$x(2e-(b_1+b_2+b_g))^2$ if $b_1+b_g < e$ and $b_1+b_2+b_g < 2e$	no	late	9
				$y(b_1+b_2+b_g-2e)^2$ if $b_1+b_g < e$ and $b_1+b_2+b_g \geq 2e$	no	early	10

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