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Working Paper

Decomposing Redistributive and Reranking Effects to Reveal Contributions of Taxes and Benefits

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The wiiw Balkan Observatory

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The wiiw Balkan Observatory

www.balkan-observatory.net

About

Shortly after the end of the Kosovo war, the last of the Yugoslav dissolution wars, the Balkan Reconstruction Observatory was set up jointly by the Hellenic Observatory, the Centre for the Study of Global Governance, both institutes at the London School of Economics (LSE), and the Vienna Institute for International Economic Studies (wiiw). A brainstorming meeting on Reconstruction and Regional Co-operation in the Balkans was held in Vouliagmeni on 8-10 July 1999, covering the issues of security, democratisation, economic reconstruction and the role of civil society. It was attended by academics and policy makers from all the countries in the region, from a number of EU countries, from the European Commission, the USA and Russia. Based on ideas and discussions generated at this meeting, a policy paper on Balkan Reconstruction and European Integration was the product of a collaborative effort by the two LSE institutes and the wiiw. The paper was presented at a follow-up meeting on Reconstruction and Integration in Southeast Europe in Vienna on 12-13 November 1999, which focused on the economic aspects of the process of reconstruction in the Balkans. It is this policy paper that became the very first Working Paper of the wiiw Balkan Observatory Working Papers series. The Working Papers are published online at www.balkan-observatory.net, the internet portal of the wiiw Balkan Observatory. It is a portal for research and communication in relation to economic developments in Southeast Europe maintained by the wiiw since 1999. Since 2000 it also serves as a forum for the Global Development Network Southeast Europe (GDN-SEE) project, which is based on an initiative by The World Bank with financial support from the Austrian Ministry of Finance and the Oesterreichische Nationalbank. The purpose of the GDN-SEE project is the creation of research networks throughout Southeast Europe in order to enhance the economic research capacity in Southeast Europe, to build new research capacities by mobilising young researchers, to promote knowledge transfer into the region, to facilitate networking between researchers within the region, and to assist in securing knowledge transfer from researchers to policy makers. The wiiw Balkan Observatory Working Papers series is one way to achieve these objectives.



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This study has been developed in the framework of research networks initiated and monitored by wiiw under the premises of the GDN–SEE partnership.

The Global Development Network, initiated by The World Bank, is a global network of research and policy institutes working together to address the problems of national and regional development. It promotes the generation of local knowledge in developing and transition countries and aims at building research capacities in the different regions.

The Vienna Institute for International Economic Studies is a GDN Partner Institute and acts as a hub for Southeast Europe. The GDN–wiiw partnership aims to support the enhancement of economic research capacity in Southeast Europe, to promote knowledge transfer to SEE, to facilitate networking among researchers within SEE and to assist in securing knowledge transfer from researchers to policy makers.

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Decomposing redistributive and reranking effects to reveal contributions of taxes and benefits

Ivica Urban

Abstract

New decompositions of redistributive effects are developed, revealing the relative contributions of different tax and benefit instruments. The methodology uses a “micro-approach” in the study of income inequality, where different indicators are first calculated for pairs of income units, and then aggregated to the population level. This approach is much more suitable for scrutinizing various income transitions emerging from fiscal activities than the usual apparatus of Lorenz and concentration curves. Emerging within this framework are also the concepts of fiscal deprivation, distance narrowing deprivation from reranking; their connections with widely known indices of redistributive, vertical and reranking effects are established. The methodology is applied in an analysis of the Croatian system of direct taxes and cash social benefits.

Keywords: redistributive effect, reranking effect, taxes, benefits

JEL classification: D63, H22, H23

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Decomposing redistributive and reranking effects to reveal contributions of taxes and benefits

1. Introduction

Since the 1970s, when major methodological advancements in the measurement of inequality and progressivity emerged, there has been a huge empirical interest in evaluating how fiscal systems affect income distribution. A large number of studies concentrated on single tax or benefit instruments. However, many researchers decided to obtain a larger picture by analysing whole fiscal systems, although more usually they studied fiscal subsystems, such as those comprising personal taxes and cash benefits.

The question naturally posed by these researchers is how different fiscal instruments interact to produce the given redistributive effect. In other words, what are the contributions of individual tax and benefit instruments? The answers to these questions may, among other things, help in the forecasting of how overall redistribution would be affected if certain instruments were redesigned, for example, in tax reforms introducing a single-rate personal income tax.

One of the most often used measures of income redistribution is equal to a difference between Gini coefficients of pre- and post-fiscal income, and is simply called the redistributive effect (RE). One weakness of RE is its neglect of changes in ranks of income units in the transition from pre- to post-fiscal income distribution, which is known as reranking. Reranking was first recognized and measured by Atkinson (1980) and Plotnick (1981); hence the Atkinson-Plotnick index of reranking R^{AP} . Kakwani (1984) decomposed redistributive effect RE into the Kakwani (1977) vertical effect V^K and R^{AP} ; the formula $RE = V^K - R^{AP}$, aiming to portray in an integrated way both the ‘vertical’ or progressivity (by V^K) and ‘horizontal’ (by R^{AP}) characteristics of the fiscal system, became one of the most popular tools in research into income redistribution.

A methodological apparatus to decompose RE into the contributions of taxes and benefits was first proposed by Lambert (1985), but it assumed the absence of reranking, and in fact, it decomposed only the Kakwani vertical effect V^K . There were two later attempts to decompose reranking, by Jenkins (1988) and Duclos (1993). However, each has its own

limitations, calling for another attempt to decompose reranking, thus enabling the full decomposition of RE .

The above mentioned indices of redistributive, vertical and reranking effects and their decompositions are usually derived using Lorenz and concentration curves. For example, to decompose the vertical effect V^K , the area between the pre-fiscal income Lorenz curve and post-fiscal income concentration curve is divided into sub-areas delineated by corresponding concentration curves, these areas representing the contributions of individual taxes and benefits.

This paper uses a different way to calculate indices and their decompositions: analysing differences in incomes, taxes and benefits for pairs of income units; afterwards, these differences are aggregated across the population and averaged. This “micro-approach” has its roots in Yitzhaki (1979), who recognized the Gini coefficient as an index of relative deprivation, and Duclos (2000), who extended the principle to fiscal process phenomena. As we shall see, it enables detailed examination of various income transitions caused by taxes and benefits.

This beneficial side of the ‘micro-approach’ was revealed in attempts to evaluate contributions of taxes and benefits to redistributive effects. The decomposition of Lambert (1985) was reinvented during the process, and several new upgrades were introduced, all of which will be shown in the paper.

The ‘micro-approach’ enables us to create indicators grounded in peoples’ ‘feelings’ about the incomes of other people and the equity of the fiscal process. In this work, several new concepts are invented in the spirit of Duclos (2000) and their relationship with existing indices is established. They may provide researchers with appealing normative interpretations for the existing indices and decompositions.

The paper contains the following parts. Section *two* introduces new indicators of fiscal deprivation, deprivation from reranking and distance narrowing, all based on the ‘micro-approach’. In section *three* these indicators are decomposed to show contributions of different taxes and benefits. Since all the concepts derived in sections two and three are defined for a pair of income units, to obtain population indices, an aggregation procedure must be employed, which is described in section *four*. Using the new methodology, section *five* analyses the redistributive effects of Croatian direct taxes and cash social benefits. The final section concludes.

2. Fiscal deprivation, distance narrowing and deprivation from reranking

Let \mathbf{X} and \mathbf{N} be the pre-fiscal and post-fiscal income vectors with s elements; X_i^x and N_i^x are pre-fiscal and post-fiscal incomes of units with pre-fiscal income rank i . Here, superscript x signifies that units of income vectors are sorted in ascending order of *pre-fiscal* income, whereas subscript i denotes pre-fiscal income rank, which takes the value of 1 (s) for the unit with the lowest (highest) pre-fiscal income; hence $i > j$ if $X_i^x > X_j^x$.

We usually consider *pre-fiscal* income as a result of the market process. Since working efforts, capital investments, and factors that determine factor productivity vary across the population of individuals, we observe income inequality among income units. Imposing various fiscal instruments the government brings about a transition from pre-fiscal to *post-fiscal* income. Amounts of individuals' incomes change as may their positions on the income scale. How the fiscal process affects the incomes of two individuals can be illustrated by the hypothetical example shown in Table 1.

Table 1: Hypothetical example

	All cases	Case A	Case B	Case C	Case D	Case E
Unit i	$X_i^x = 100$	$N_i^x = 95$	$N_i^x = 85$	$N_i^x = 80$	$N_i^x = 70$	$N_i^x = 110$
Unit j	$X_j^x = 80$	$N_j^x = 85$	$N_j^x = 95$	$N_j^x = 100$	$N_j^x = 110$	$N_j^x = 70$
Income supremacy	20	10	-10	-20	-40	40
Fiscal deprivation		10	30	40	60	-20
Income distance	20	10	10	20	40	40
Distance narrowing		10	10	0	-20	-20
Deprivation from reranking		0	20	40	80	0

From the hypothetical distribution of incomes we take two units, with pre-fiscal incomes $X_i^x = 100$ and $X_j^x = 80$. We hold these incomes constant during the exercise, experimenting with different possibilities for post-fiscal incomes (cases A to E): for example, in case A we have $N_i^x = 95$ and $N_j^x = 85$, etc.

The first concept we define is *income supremacy*. This is the difference between the income of the pre-fiscally higher ranked unit i and the pre-fiscally lower-ranked unit j , and is calculated both for pre- and post-fiscal income. Pre-fiscal income supremacy in our example is $X_i^x - X_j^x = 20$, while post-fiscal income supremacy in case A is $N_i^x - N_j^x = 10$. Thus, in case A, income supremacy fell from 20 to 10, or by 10. The change of income supremacy in the transition from pre- to post-fiscal income is called *fiscal deprivation*, and is obtained as in (1).

$$(1) \quad v_{i,j}^x = (X_i^x - X_j^x) - (N_i^x - N_j^x)$$

In case A, unit i has lost a part of its income supremacy, but it still has a higher income than unit j . However, in case B, income supremacy goes from 20 to -10; unit i , which had a higher pre-fiscal income than unit j , now has a lower post-fiscal income. If we drew the post-fiscal income scale (reordering elements of \mathbf{N} according to post-fiscal income), we would observe that unit j would have a higher post-fiscal rank than unit i ; the two units have changed their ranks in the transition from pre- to post-fiscal income distribution. The phenomenon is therefore simply called reranking. In our terminology, reranking has occurred whenever post-fiscal income supremacy is negative (cases B, C and D).

In case C, two units ‘swapped’ their incomes; unit i had a pre-fiscal income of 100, and now has a post-fiscal income of 80, while for unit j just the opposite is true. It follows that the absolute difference between their incomes has not changed – it has remained 20. We will call the latter term income distance, which, like income supremacy, can be obtained for both pre- and post-fiscal income; indeed, it is equal to the absolute amount of income supremacy. The change of income distance in the transition from pre- to post-fiscal income is called *distance narrowing*, and is calculated as in (2).

$$(2) \quad \delta_{i,j}^x = |X_i^x - X_j^x| - |N_i^x - N_j^x|$$

When the fiscal system reduces distance between two units, distance narrowing is positive (case A), but it may occur that the system also increases some income distances (cases D and E), where we might actually talk about ‘distance widening’, which is signified by a negative value of distance narrowing.

Let us now turn back again to the cases B, C and D, where reranking has occurred. Notice that in these cases fiscal deprivation is higher than distance narrowing, whereas in the no-reranking cases A and E, fiscal deprivation is just equal to distance narrowing. The part of fiscal deprivation above the level of distance narrowing will be called *deprivation from reranking*, and it is equal to twice the negative value of post-fiscal income supremacy; it is obtained as in (3).

$$(3) \quad \kappa_{i,j}^x = |N_i^x - N_j^x| - (N_i^x - N_j^x)$$

In the last definition we stated that the three concepts – fiscal deprivation, deprivation from reranking and distance narrowing – are connected. This is confirmed by the following equation (4), which says that fiscal deprivation is the sum of distance narrowing and deprivation from reranking.

$$(4) \quad v_{i,j}^x = \delta_{i,j}^x + \kappa_{i,j}^x$$

We can use this equation to illustrate the meaning of the concepts described above. Assume that, in order to improve welfare, the social decision maker decides to reduce income distance between i and j by $\tilde{\delta}_{i,j}^x \leq |X_i - X_j|$. What may the consequences of this action be to the income supremacy of i , i.e. how large should its fiscal deprivation be? In the case of no reranking, fiscal deprivation will be $\tilde{\delta}$; in the presence of reranking, it increases to $\tilde{\delta}_{i,j}^x + \kappa_{i,j}^x$. Thus, for a given distance narrowing, reranking enlarges the units i 's income supremacy sacrifice.

Suppose that society agrees that a certain distance narrowing between i and j is desirable, i.e. that i must sacrifice part of its income supremacy. However, presume that society also considers that the fiscal process should not affect income rankings, i.e. unit i must remain ‘the richer’, and unit j “the poorer”. In this light, we may treat deprivation from reranking ($\kappa_{i,j}^x$) as a measure of the *excess fiscal deprivation* felt by unit i needed to achieve the given distance narrowing.

3. Contributions of tax and benefit instruments

We have mentioned that the transition from pre-fiscal to post-fiscal income is induced by different fiscal instruments, but two types of them overwhelmingly tend to be studied: taxes and

benefits (henceforth: T&Bs). Post-fiscal income is obtained simply by subtraction of taxes from pre-fiscal income and addition of benefits. If we have P different taxes and Q benefits, the post-fiscal income of unit i is obtained as in (5), where $T_{i,p}^x$ ($B_{i,q}^x$) is the amount of the p th tax paid (q th benefit received) by the unit with pre-fiscal income rank i .

$$(5) \quad N_i^x = X_i^x - \sum_{p=1}^P T_{i,p}^x + \sum_{q=1}^Q B_{i,q}^x$$

By simple rearranging of (5) we obtain the following equation (6).

$$(6) \quad X_i^x - N_i^x = \sum_{p=1}^P T_{i,p}^x - \sum_{q=1}^Q B_{i,q}^x$$

In the analysis of real fiscal systems the aggregate sum of taxes is either higher than the sum of all benefits (therefore $\bar{N} < \bar{X}$), or vice versa (therefore $\bar{N} > \bar{X}$), where $\bar{X} = s^{-1} \sum_i X_i^x$ and $\bar{N} = s^{-1} \sum_i N_i^x$, but rarely will the two be equal. Since the models in this paper require that aggregate post-fiscal income equals pre-fiscal income (total taxes equal total benefits), we typically have to introduce counterfactual tax and benefit variables. Since there are two different models, we need two types of these ‘fill-in’ variables.

A. The fill-in variables of *type 1* are defined in (7),

$$(7) \quad B_{i,Q+1}^x = \begin{cases} \bar{X} - \bar{N} & \text{if } \bar{N} < \bar{X} \\ 0 & \text{otherwise} \end{cases} \quad T_{i,P+1}^x = \begin{cases} \bar{N} - \bar{X} & \text{if } \bar{N} > \bar{X} \\ 0 & \text{otherwise} \end{cases}$$

Adding (subtracting) a fill-in tax (benefit) defined in (7) to (from) both sides of (6) and substituting \tilde{X}_i^x for $X_i^x + T_{i,P+1}^x - B_{i,Q+1}^x$ on the left hand, in this case we obtain (8).

$$(8) \quad \tilde{X}_i^x - N_i^x = \left(T_{i,P+1}^x + \sum_{p=1}^P T_{i,p}^x \right) - \left(B_{i,Q+1}^x + \sum_{q=1}^Q B_{i,q}^x \right)$$

B. The fill-in variables of *type 2* are defined in (9).

$$(9) \quad B_{i,Q+1}^x = \begin{cases} X_i^x (1 - \bar{N} / \bar{X}) & \text{if } \bar{N} < \bar{X} \\ 0 & \text{otherwise} \end{cases} \quad T_{i,P+1}^x = \begin{cases} X_i^x (1 - \bar{N} / \bar{X}) & \text{if } \bar{N} > \bar{X} \\ 0 & \text{otherwise} \end{cases}$$

Adding (subtracting) a fill-in benefit (tax) variable defined in (9) to (from) both sides of (6) and substituting \widehat{X}_i^x for $X_i^x + T_{i,P+1}^x - B_{i,Q+1}^x$ on the left side we obtain (10).

$$(10) \quad \widehat{X}_i^x - N_i^x = \left(T_{i,P+1}^x + \sum_{p=1}^P T_{i,p}^x \right) - \left(B_{i,Q+1}^x + \sum_{q=1}^Q B_{i,q}^x \right)$$

Counterfactual systems containing fill-in tax and benefit variables, described in (8) and (10), now satisfy the “balanced budget” condition required for our models. The fill-in tax (benefit) of type 1 has equal absolute amounts for all units, while the fill-in tax (benefit) of type 2 is proportional to pre-fiscal income. In the ‘deviations’ approach, the Gini coefficient of counterfactual pre-fiscal income (\widehat{X}_i^x) will be the same as the Gini of actual pre-fiscal income (X_i^x), but in the ‘amounts’ case, the Gini of counterfactual pre-fiscal income (\check{X}_i^x) will be different from the one obtained for X_i^x . That is, if $\bar{N} < \bar{X}$, a fill-in benefit, formed according to (7), is subtracted from actual pre-fiscal income, this must *increase* inequality. The opposite will happen if $\bar{N} > \bar{X}$, when a fill-in tax is introduced; the inequality of counterfactual pre-fiscal income is *lower* than actual inequality. We will deal with this issue later.

Before turning to the main problem, we need to define another set of variables that will be required in the analysis. The share of the p th tax in pre-fiscal income is equal to $t_p^x = \sum_i T_{i,p}^x / \sum_i X_i^x$, and $b_q^x = \sum_i B_{i,q}^x / \sum_i X_i^x$ is the share of the q th benefit in pre-fiscal income. Now, we have that:

$$(11) \quad \ddot{T}_{i,p}^x = T_{i,p}^x - t_p^x X_i^x \quad \ddot{T}_{i,P+1}^x = T_{i,P+1}^x - t_{P+1}^x X_i^x$$

$$(12) \quad \ddot{B}_{i,q}^x = B_{i,q}^x - b_q^x X_i^x \quad \ddot{B}_{i,Q+1}^x = B_{i,Q+1}^x - b_{Q+1}^x X_i^x$$

The term $t_p^x X_i^x$ in (11) is the amount of p th tax paid by unit i if this tax instrument is proportional with pre-fiscal income. Consequently, the term $\ddot{T}_{i,p}^x$ measures how the actual values of the p th tax *deviate from proportionality*. Analogously, $\ddot{B}_{i,q}^x$ represent deviations of the values of the q th benefit from proportionality. The following equation (13) introduces these new terms.

$$(13) \quad \widehat{X}_i^x - N_i^x = \left(\ddot{T}_{i,P+1}^x + \sum_{p=1}^P \ddot{T}_{i,p}^x \right) - \left(\ddot{B}_{i,Q+1}^x + \sum_{q=1}^Q \ddot{B}_{i,q}^x \right)$$

Using (8) and (13), we calculate fiscal deprivation from (1) for a pair of units (i, j) , as shown by (14) and (15).

$$(14) \quad v_{i,j}^x = (\check{X}_i^x - \check{X}_j^x) - (N_i^x - N_j^x) = \\ = \left((T_{i,P+1}^x - T_{j,P+1}^x) + \sum_{p=1}^P (T_{i,p}^x - T_{j,p}^x) \right) - \left((B_{j,Q+1}^x - B_{i,Q+1}^x) + \sum_{q=1}^Q (B_{j,q}^x - B_{i,q}^x) \right)$$

$$(15) \quad v_{i,j}^x = (\widehat{X}_i^x - \widehat{X}_j^x) - (N_i^x - N_j^x) = \\ = \left((\ddot{T}_{i,P+1}^x - \ddot{T}_{j,P+1}^x) + \sum_{p=1}^P (\ddot{T}_{i,p}^x - \ddot{T}_{j,p}^x) \right) - \left((\ddot{B}_{j,Q+1}^x - \ddot{B}_{i,Q+1}^x) + \sum_{q=1}^Q (\ddot{B}_{j,q}^x - \ddot{B}_{i,q}^x) \right)$$

It can be shown that the differences $T_{i,P+1}^x - T_{j,P+1}^x$, $B_{j,Q+1}^x - B_{i,Q+1}^x$, $\ddot{T}_{i,P+1}^x - \ddot{T}_{j,P+1}^x$ and $\ddot{B}_{j,Q+1}^x - \ddot{B}_{i,Q+1}^x$ are all equal to zero; therefore, we can rewrite (14) and (15) as (16) and (17).

$$(16) \quad v_{i,j}^x = (\check{X}_i^x - \check{X}_j^x) - (N_i^x - N_j^x) = \sum_{p=1}^P (T_{i,p}^x - T_{j,p}^x) - \sum_{q=1}^Q (B_{j,q}^x - B_{i,q}^x)$$

$$(17) \quad v_{i,j}^x = (\widehat{X}_i^x - \widehat{X}_j^x) - (N_i^x - N_j^x) = \sum_{p=1}^P (\ddot{T}_{i,p}^x - \ddot{T}_{j,p}^x) - \sum_{q=1}^Q (\ddot{B}_{j,q}^x - \ddot{B}_{i,q}^x)$$

After substituting $\check{v}_{i,j,p}^{x,T} = T_{i,p}^x - T_{j,p}^x$, $\check{v}_{i,j,q}^{x,B} = B_{j,q}^x - B_{i,q}^x$, $\widehat{v}_{i,j,p}^{x,T} = \ddot{T}_{i,p}^x - \ddot{T}_{j,p}^x$ and $\widehat{v}_{i,j,q}^{x,B} = \ddot{B}_{j,q}^x - \ddot{B}_{i,q}^x$, we can write:

$$(18) \quad v_{i,j}^x = \sum_{p=1}^P \check{v}_{i,j,p}^{x,T} + \sum_{q=1}^Q \check{v}_{i,j,q}^{x,B}$$

$$(19) \quad v_{i,j}^x = \sum_{p=1}^P \widehat{v}_{i,j,p}^{x,T} + \sum_{q=1}^Q \widehat{v}_{i,j,q}^{x,B}$$

The last two equations decompose fiscal deprivation into contributions of individual T&Bs. For example, the contribution of the q th benefit to fiscal deprivation in the ‘amounts’ approach is equal to $\check{v}_{i,j,q}^{x,B}$. It is clear that we have two competing approaches to decompose fiscal deprivation: the first, from (16), uses simple amounts of T&Bs, while the other, shown in (17),

employs deviations of T&Bs from proportionality. The former can thus be called the ‘amounts’, and the latter, the ‘deviations’, approach. The differences between the two approaches will be discussed later.

Thus, we have decomposed the first of the three concepts – fiscal deprivation – while deprivation from reranking and distance narrowing are yet to be dealt with. Since they are all connected by (4), we need to discover how to decompose *one* of the remaining two, whereas the third will then be obtained simply as a resultant of the other two. We turn now to decomposition of the deprivation from reranking.

Deprivation from reranking, as presented by (3), cannot be straightforwardly decomposed in the same manner as fiscal deprivation in (1). The intuitive way is as follows: for the pair of units (i, j) , multiply $\kappa_{i,j}^x$ by a weight that represents the share of tax or benefit in total T&Bs – at the level of the pair (i, j) . It is easier to understand this from formulas (20) and (21). The former calculates the contribution of the p th tax ($\tilde{\kappa}_{i,j,p}^{x,T}$) and the q th benefit ($\tilde{\kappa}_{i,j,q}^{x,B}$) to deprivation from reranking, according to the ‘amounts’ approach, while the latter, with $\hat{\kappa}_{i,j,p}^{T,x}$ and $\hat{\kappa}_{i,j,q}^{B,x}$, does the same for the ‘deviations’ approach.

$$(20) \quad \tilde{\kappa}_{i,j,p}^{x,T} = \kappa_{i,j}^x \frac{\tilde{v}_{i,j,p}^{x,T}}{\Phi_{i,j}^x} \quad \tilde{\kappa}_{i,j,q}^{x,B} = \kappa_{i,j}^x \frac{\tilde{v}_{i,j,q}^{x,B}}{\Phi_{i,j}^x}$$

$$(21) \quad \hat{\kappa}_{i,j,p}^{T,x} = \kappa_{i,j}^x \frac{\hat{v}_{i,j,p}^{x,T}}{\Theta_{i,j}^x} \quad \hat{\kappa}_{i,j,q}^{B,x} = \kappa_{i,j}^x \frac{\hat{v}_{i,j,q}^{x,B}}{\Theta_{i,j}^x}$$

The new terms in (20) and (21) are $\Phi_{i,j}^x = \sum_p^P \tilde{v}_{i,j,p}^{x,T} + \sum_q^Q \tilde{v}_{i,j,q}^{x,B}$, and

$\Theta_{i,j}^x = \sum_p^P \hat{v}_{i,j,p}^{x,T} + \sum_q^Q \hat{v}_{i,j,q}^{x,B}$. To illustrate the construction of the weights better, we “recompress” one of them, $\tilde{v}_{i,j,p}^{x,T} / \Phi_{i,j}^x$, and show it in (22).

$$(22) \quad \frac{\tilde{v}_{i,j,p}^{x,T}}{\Phi_{i,j}^x} = \frac{T_{i,p}^x - T_{j,p}^x}{\sum_{p=1}^P (T_{i,p}^x - T_{j,p}^x) + \sum_{q=1}^Q (B_{j,q}^x - B_{i,q}^x)}$$

Overall deprivation from reranking for the pair of income units (i, j) is decomposed as in (23) and (24), for the ‘amounts’ and ‘deviation’ approaches, respectively.

$$(23) \quad \kappa_{i,j}^x = \sum_{p=1}^P \kappa_{i,j}^x \frac{\tilde{v}_{i,j,p}^{x,T}}{\Phi_{i,j}^x} + \sum_{q=1}^Q \kappa_{i,j}^x \frac{\tilde{v}_{i,j,q}^{x,B}}{\Phi_{i,j}^x}$$

$$(24) \quad \kappa_{i,j}^x = \sum_{p=1}^P \kappa_{i,j}^x \frac{\hat{v}_{i,j,p}^{x,T}}{\Theta_{i,j}^x} + \sum_{q=1}^Q \kappa_{i,j}^x \frac{\hat{v}_{i,j,q}^{x,B}}{\Theta_{i,j}^x}$$

What we are still missing is the decomposition of distance narrowing. From (4) we know that $\delta_{i,j}^x = v_{i,j}^x - \kappa_{i,j}^x$. Therefore, to decompose distance narrowing, we can just subtract reranking components, shown in (23) and (24), from the fiscal deprivation ones, as found in (18) and (19).

$$(25) \quad \delta_{i,j}^x = v_{i,j}^x - \kappa_{i,j}^x = \left(\sum_{p=1}^P \tilde{v}_{i,j,p}^{x,T} + \sum_{q=1}^Q \tilde{v}_{i,j,q}^{x,B} \right) - \left(\sum_{p=1}^P \kappa_{i,j}^x \frac{\tilde{v}_{i,j,p}^{x,T}}{\Phi_{i,j}^x} + \sum_{q=1}^Q \kappa_{i,j}^x \frac{\tilde{v}_{i,j,q}^{x,B}}{\Phi_{i,j}^x} \right)$$

$$(26) \quad \delta_{i,j}^x = v_{i,j}^x - \kappa_{i,j}^x = \left(\sum_{p=1}^P \hat{v}_{i,j,p}^{x,T} + \sum_{q=1}^Q \hat{v}_{i,j,q}^{x,B} \right) - \left(\sum_{p=1}^P \kappa_{i,j}^x \frac{\hat{v}_{i,j,p}^{x,T}}{\Theta_{i,j}^x} + \sum_{q=1}^Q \kappa_{i,j}^x \frac{\hat{v}_{i,j,q}^{x,B}}{\Theta_{i,j}^x} \right)$$

After rearrangement of (25) and (26), we obtain more elegant formulas

$$(27) \quad \delta_{i,j}^x = v_{i,j}^x - \kappa_{i,j}^x = \left(1 - \frac{\kappa_{i,j}^x}{\Phi_{i,j}^x} \right) \sum_{p=1}^P \tilde{v}_{i,j,p}^{x,T} + \left(1 - \frac{\kappa_{i,j}^x}{\Phi_{i,j}^x} \right) \sum_{q=1}^Q \tilde{v}_{i,j,q}^{x,B}$$

$$(28) \quad \delta_{i,j}^x = v_{i,j}^x - \kappa_{i,j}^x = \left(1 - \frac{\kappa_{i,j}^x}{\Theta_{i,j}^x} \right) \sum_{p=1}^P \hat{v}_{i,j,p}^{x,T} + \left(1 - \frac{\kappa_{i,j}^x}{\Theta_{i,j}^x} \right) \sum_{q=1}^Q \hat{v}_{i,j,q}^{x,B}$$

Example

In the last section we have presented decompositions of fiscal deprivation, deprivation from reranking and distance narrowing obtained for a pair of income units (i,j) . Of course, the principles can be extended to the whole population of income units. Table 2 presents new hypothetical data, this time for a population of five people. Figure 1 introduces an analytical tool that we may call “triangular matrices”, containing income pair differences for various variables needed to calculate our indicators.

Thus, the first matrix named $v_{i,j}^x$ calculates the values of fiscal deprivation for our hypothetical example from Table 2. For example, the value 120 on the intersection of row 4 and column 3 is obtained for a pair of units $v_{i=4,j=3}^x$ and is calculated, according to (1), as $v_{4,3}^x = (70 -$

30)–(20–100); the value 66.7 in the matrix $\tilde{\kappa}_{i,j,p}^{x,T}$ is obtained for the same pair of units following (21), as $\tilde{\kappa}_{4,3,1}^{x,T} = 160 \cdot 50 / 120$ (where all the values: $\kappa_{4,3}^x = 160$, $\tilde{v}_{4,3,1}^{x,T} = 50$, $\Phi_{4,3}^x = \tilde{v}_{4,3,1}^{x,T} + \tilde{v}_{4,3,1}^{x,B} = 120$, can be found in the relevant matrices)

Table 2: Hypothetical data

i	X_i^x	$T_{i,1}^x$	$B_{i,1}^x$	N_i^x
1	8	0	32	40
2	12	0	48	60
3	30	10	80	100
4	70	60	10	20
5	180	100	0	80

Figure 1: Triangular matrices

$v_{i,j}^x$						$\delta_{i,j}^x$					
1	0					1	0				
2	-16	0				2	-16	0			
3	-38	-22	0			3	-38	-22	0		
4	82	98	120	0		4	42	18	-40	0	
5	132	148	170	50	0	5	132	148	130	50	0
i/j	1	2	3	4	5	i/j	1	2	3	4	5
$\kappa_{i,j}^x$						$\tilde{v}_{i,j,1}^{x,T}$					
1	0					1	0				
2	0	0				2	0	0			
3	0	0	0			3	10	10	0		
4	40	80	160	0		4	60	60	50	0	
5	0	0	40	0	0	5	100	100	90	40	0
i/j	1	2	3	4	5	i/j	1	2	3	4	5
$\tilde{\kappa}_{i,j,1}^{x,T}$						$\tilde{v}_{i,j,1}^{x,B}$					
1	0					1	0				
2	0	0				2	-16	0			
3	0	0	0			3	-48	-32	0		
4	29.3	49.0	66.7	0		4	22	38	70	0	
5	0	0	21.2	0	0	5	32	48	80	10	0
i/j	1	2	3	4	5	i/j	1	2	3	4	5
$\tilde{\kappa}_{i,j,1}^{x,B}$						$\tilde{\kappa}_{i,j,1}^{x,B}$					
1	0					1	0				
2	0	0				2	0	0			
3	0	0	0			3	0	0	0		
4	10.7	31.0	93.3	0		4	10.7	31.0	93.3	0	
5	0	0	18.8	0	0	5	0	0	18.8	0	0
i/j	1	2	3	4	5	i/j	1	2	3	4	5

$\widehat{v}_{i,j,1}^{x,T}$					
1	0.0				
2	-2.3	0.0			
3	-2.5	-0.2	0.0		
4	24.9	27.1	27.3	0.0	
5	2.5	4.8	5.0	-22.3	0.0
<i>i/j</i>	1	2	3	4	5

$\widehat{v}_{i,j,1}^{x,B}$					
1	0.0				
2	-13.7	0.0			
3	-35.5	-21.8	0.0		
4	57.1	70.9	92.7	0.0	
5	129.5	143.2	165.0	72.3	0.0
<i>i/j</i>	1	2	3	4	5

$\widehat{\kappa}_{i,j,1}^{x,T}$					
1	0				
2	0	0			
3	0	0	0		
4	12.1	22.2	36.4	0	
5	0	0	1.2	0	0
<i>i/j</i>	1	2	3	4	5

$\widehat{\kappa}_{i,j,1}^{x,B}$					
1	0				
2	0	0			
3	0	0	0		
4	27.9	57.9	123.6	0	
5	0	0	38.8	0	0
<i>i/j</i>	1	2	3	4	5

4. Aggregation to population indices

The next intuitive step in our investigation is to aggregate values inside triangular matrices across their two dimensions. Doing that, we will obtain single-index indicators of fiscal deprivation, deprivation from reranking and distance narrowing, and their components which show contributions of individual T&Bs. We know that the most well-known index of income inequality, the Gini coefficient (of pre-fiscal income here), among many other ways, can be calculated as:

$$(29) \quad G_X = \frac{1}{s^2 \bar{X}} \sum_{i=2}^s \sum_{j=1}^{i-1} (X_i^x - X_j^x)$$

The formula (29) is created in such a way that the summation loop $\sum_{i=2}^s \sum_{j=1}^{i-1} (\cdot)$ actually sums the values of “triangular” matrix presented above. This sum is then “normalized” by dividing by $s^2 \bar{X}$, the product of the squared number of units in the population (s) and the mean pre-fiscal income (\bar{X}). Now, assume that the aggregate sums of pre- and post-fiscal income are equal, i.e. $\bar{X} = \bar{N}$, which holds for our counterfactual system. We can create a whole range of indices aggregating analogously to (29), with the factor in front of the summation loops always equal to $c = s^{-2} \bar{X}^{-1}$.

The new indices are in fact equal to well-known and widely used indices of redistributive, vertical and reranking effects. The index of fiscal deprivation (30) is obtained

from (1) and is equal to the Kakwani (1977) index of vertical effect (V^K). The index of distance narrowing (31) derived from (2) corresponds to the index of redistributive effect (RE). The index of deprivation from reranking (32), based on (3), is identical to the Atkinson (1980) and Plotnick (1981) index of reranking (R^{AP}).

$$(30) \quad V^x = c \sum_{i=2}^s \sum_{j=1}^{i-1} ((X_i^x - X_j^x) - (N_i^x - N_j^x)) = V^K \quad (\text{if } \bar{X} = \bar{N})$$

$$(31) \quad \Delta^x = c \sum_{i=2}^s \sum_{j=1}^{i-1} (|X_i^x - X_j^x| - |N_i^x - N_j^x|) = RE \quad (\text{if } \bar{X} = \bar{N})$$

$$(32) \quad R^x = c \sum_{i=2}^s \sum_{j=1}^{i-1} (|N_i^x - N_j^x| - (N_i^x - N_j^x)) = R^{AP} \quad (\text{if } \bar{X} = \bar{N})$$

Are the above formulas still valid if $\bar{X} \neq \bar{N}$? First, equation (32) does not depend on pre-fiscal income, so the condition $\bar{X} \neq \bar{N}$ is irrelevant here. Next, if we use the ‘deviations’ approach, after the creation of the fill-in variable we obtain \hat{X}_i^x ; substituting \hat{X}_i^x for X_i^x in (30) and (31) would not change the result. However, if we use ‘amounts’ approach, and substitute \check{X}_i^x for X_i^x in the same two equations, the results are no longer the same. Hence, properly to estimate V^x and Δ^x we have to rely on the ‘deviations’ approach. Therefore we can write (30) and (31) as:

$$(33) \quad \hat{V}^x = c \sum_{i=2}^s \sum_{j=1}^{i-1} ((\hat{X}_i^x - \hat{X}_j^x) - (N_i^x - N_j^x)) = V^K$$

$$(34) \quad \hat{\Delta}^x = c \sum_{i=2}^s \sum_{j=1}^{i-1} (|\hat{X}_i^x - \hat{X}_j^x| - |N_i^x - N_j^x|) = RE$$

In the manner described above, we can aggregate all the “sub-indices” defined earlier – the components of the basic indicators, which show contributions of T&Bs. These components are shown for the ‘amounts’ and ‘deviations’ approaches in columns 1 and 3 of Table 3. We have seen that the ‘amounts’ approach incorrectly estimates the index of fiscal deprivation; the same is true for the sum of its components, shown in (35) and (38). However, the relative shares of tax and benefit instruments are correct; so, to obtain proper component values, we have to use these

relative shares as weights that multiply the correctly estimated index of fiscal deprivation (\widehat{V}^x). This is done in the second column of Table 3.

Contributions to distance narrowing can be simply obtained as differences between the corresponding fiscal deprivation and deprivation from reranking components, as presented in the bottom two rows of Table 3.

Table 3: Components of fiscal deprivation and deprivation from reranking

‘Amounts’ approach		‘Deviations’ approach
Basic	Adapted	
(35) $\check{V}_p^{x,T} = c \sum_{i=2}^s \sum_{j=1}^{i-1} v_{i,j,p}^{x,T}$	(36) $\check{V}_p^{x,T} = \frac{\widehat{V}^x \cdot \check{V}_p^{x,T}}{\sum_p^P \check{V}_p^{x,T} + \sum_q^Q \check{V}_q^{x,B}}$	(37) $\widehat{V}_p^{x,T} = c \sum_{i=2}^s \sum_{j=1}^{i-1} \widehat{v}_{i,j,p}^{x,T}$
(38) $\check{V}_q^{x,B} = c \sum_{i=2}^s \sum_{j=1}^{i-1} v_{i,j,q}^{x,B}$	(39) $\check{V}_q^{x,B} = \frac{\widehat{V}^x \cdot \check{V}_q^{x,B}}{\sum_p^P \check{V}_p^{x,T} + \sum_q^Q \check{V}_q^{x,B}}$	(40) $\widehat{V}_q^{x,B} = c \sum_{i=2}^s \sum_{j=1}^{i-1} \widehat{v}_{i,j,q}^{x,B}$
(41) $\check{R}_p^{x,T} = c \sum_{i=2}^s \sum_{j=1}^{i-1} \kappa_{i,j}^x \frac{v_{i,j,p}^{x,T}}{\Phi_{i,j}^x}$		(42) $\widehat{R}_p^{x,T} = c \sum_{i=2}^s \sum_{j=1}^{i-1} \kappa_{i,j}^x \frac{\widehat{v}_{i,j,p}^{x,T}}{\Theta_{i,j}^x}$
(43) $\check{R}_q^{x,B} = c \sum_{i=2}^s \sum_{j=1}^{i-1} \kappa_{i,j}^x \frac{v_{i,j,q}^{x,B}}{\Phi_{i,j}^x}$		(44) $\widehat{R}_q^{x,B} = c \sum_{i=2}^s \sum_{j=1}^{i-1} \kappa_{i,j}^x \frac{\widehat{v}_{i,j,q}^{x,B}}{\Theta_{i,j}^x}$
	(45) $\check{\Delta}_p^{x,T} = \check{V}_p^{x,T} - \check{R}_p^{x,T}$	(46) $\widehat{\Delta}_p^{x,T} = \widehat{V}_p^{x,T} - \widehat{R}_p^{x,T}$
	(47) $\check{\Delta}_q^{x,B} = \check{V}_q^{x,B} - \check{R}_q^{x,B}$	(48) $\widehat{\Delta}_q^{x,B} = \widehat{V}_q^{x,B} - \widehat{R}_q^{x,B}$

Finally, we can write the following two ‘total’ decompositions of distance narrowing, (49) for the ‘amounts’ and (50) for the ‘deviations’ approach.

$$(49) \quad \sum_{p=1}^P \check{\Delta}_p^{x,T} + \sum_{q=1}^Q \check{\Delta}_q^{x,B} = \left(\sum_{p=1}^P \check{V}_p^{x,T} + \sum_{q=1}^Q \check{V}_q^{x,B} \right) - \left(\sum_{p=1}^P \check{R}_p^{x,T} + \sum_{q=1}^Q \check{R}_q^{x,B} \right)$$

$$(50) \quad \sum_{p=1}^P \widehat{\Delta}_p^{x,T} + \sum_{q=1}^Q \widehat{\Delta}_q^{x,B} = \left(\sum_{p=1}^P \widehat{V}_p^{x,T} + \sum_{q=1}^Q \widehat{V}_q^{x,B} \right) - \left(\sum_{p=1}^P \widehat{R}_p^{x,T} + \sum_{q=1}^Q \widehat{R}_q^{x,B} \right)$$

Example

Turning to our hypothetical example from Table 2 and Figure 1, we can easily calculate all the indices and components using sums within triangular matrices and multiplying them with c , which is here equal to $c = s^{-2} \bar{X}^{-1} = 1/(25 \cdot 60)$. Thus, we obtain:

$$V^x = 0.4827, \Delta^x = 0.2693, R^x = 0.2133$$

$$\tilde{V}_1^{x,T} = 0.3467, \tilde{V}_1^{x,B} = 0.1360, \tilde{R}_1^{T,x} = 0.1107, \tilde{R}_1^{B,x} = 0.1026, \tilde{\Delta}_1^{x,T} = 0.2359, \tilde{\Delta}_1^{x,B} = 0.0334$$

$$\hat{V}_1^{x,T} = 0.0429, \hat{V}_1^{x,B} = 0.4397, \hat{R}_1^{T,x} = 0.0479, \hat{R}_1^{B,x} = 0.1654, \hat{\Delta}_1^{x,T} = -0.0050, \hat{\Delta}_1^{x,B} = 0.2743$$

Observe that the two approaches give quite opposite results concerning the roles of T&Bs: most of the distance narrowing is achieved by *taxes* in the ‘amounts’ approach (88% of Δ^x), while in the ‘deviations’ approach benefits create the whole distance narrowing (102% of Δ^x), whereas the role of taxes is actually even negative. The reasons for such a large discrepancy, which is not just coincidental in this hypothetical example, will be explained in the following section.

Again we will resort to a hypothetical example. The system depicted in Table 4 has a proportional tax of 50% on pre-fiscal incomes, and a benefit that is equal in absolute amounts for all units. This system narrows the distances between income units, and does not introduce reranking. Therefore, fiscal deprivation is equal to distance narrowing for all pairs of units, while the sum of elements in the triangular matrix $v_{i,j}^x = \delta_{i,j}^x$ is positive. The fiscal system evidently reduces inequality in the transition from pre- to post-fiscal income ($G_X = 0.536 \rightarrow G_N = 0.268$). But what produces this effect – the tax or the benefit?

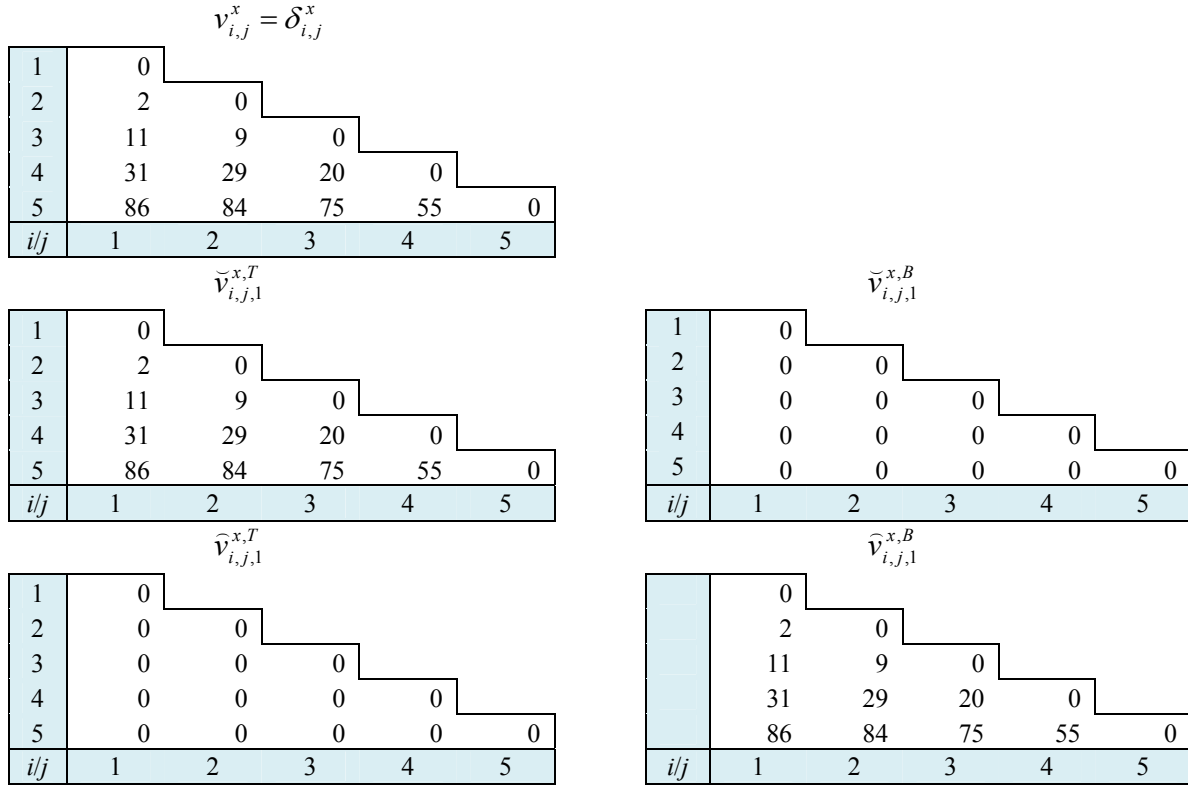
If we look at the result for the ‘amounts’ approach, we see that tax is to be credited with the whole of the effect: $\tilde{v}_{i,j,1}^{x,T} = v_{i,j}^x$ for all pairs (i, j) , while the elements of $\tilde{v}_{i,j,1}^{x,B}$ are all zeros, readily explicable if we recall that $\tilde{v}_{i,j,q}^{x,B} = B_{j,q}^x - B_{i,q}^x$. It is completely the opposite for the ‘deviations’ approach, where the benefit is responsible for the whole effect; thus, $\hat{v}_{i,j,1}^{x,B} = v_{i,j}^x$ for all pairs (i, j) , whereas all values of $\hat{v}_{i,j,1}^{x,T}$ are zero since $\hat{v}_{i,j,p}^{x,T} = \ddot{T}_{i,p}^x - \ddot{T}_{j,p}^x$ and $\ddot{T}_{i,p}^x = T_{i,p}^x - t_p^x X_i^x$

($\ddot{T}_{i,1}^x = t_1^x X_i^x$, in our case, because tax is proportional and the statutory rate is identical to the average rate t_p^x).

Table 4: Hypothetical data

i	X_i^x	$T_{i,1}^x$	$B_{i,1}^x$	N_i^x	τ_i^x	β_i^x
1	8	4	30	34	0.5	3.8
2	12	6	30	36	0.5	2.5
3	30	15	30	45	0.5	1.0
4	70	35	30	65	0.5	0.4
5	180	90	30	120	0.5	0.2

Figure 2: Triangular matrices



The issue is puzzling. On one side, we can say that the tax is *reducing income distances* – because people with higher income pay higher amounts of tax (in absolute amounts), while the benefit is not – since all people receive equal benefit (again, in absolute amounts). On the other hand, we can say that the benefit *deviates from proportionality*; this is confirmed in column β_i^x of Table 4, which contains shares of benefit in pre-fiscal income for each unit – this share decreases in income, meaning that lower income units receive larger benefits in proportion to

their pre-fiscal income; at the same time, tax is paid in the same proportion to income by all units, as shown in column τ_i^x .

In both circumstances we have observed a hypothetical fiscal system as a whole. Tax and benefit are here two blades of the scissors – they cannot work independently – what is given to people in benefits, must be somehow collected through taxes. If we measure the influence of one instrument on the distribution of income, we have to make an assumption of the distribution of the other instrument. Thus, if an assumption is made that the benefit is distributed in equal amounts to all people (as in the current example), then a proportional tax *decreases inequality*; however, if we assume that the benefit is proportional to income, then a proportional tax leaves income distribution unchanged. Similarly, if we assume that tax is proportional (as in our example), then the equally distributed benefit *decreases inequality*; if we assume a tax which is distributed equally (such as a poll tax), then the benefit does not affect inequality.

When the *role of a tax* is investigated, which assumption on the distribution of the benefit is appropriate: equal amounts of benefit for all or the amounts proportional to pre-fiscal income? What about the assumed distribution of tax, when the *contribution of the benefit* is explored? Which should be our reference points for discovering the influence of taxes, benefits, and taxes & benefits together on income distribution? Duclos and Araar (2006:137) explain:

“Note that [some] progressivity comparisons have as a reference point the initial Lorenz curve. In other words, a tax is progressive if the poorest individuals bear a share of the total tax burden that is less than their share in total gross income. As mentioned above, an alternative reference point would be the cumulative shares in the population. This is often argued in the context of state support – the reference point to assess the equity of public expenditures is population share. The analytical framework above can easily allow for this alternative view [...] This will make more stringent the conditions to declare a benefit to be progressive, but it will also make it easier for a tax to be declared progressive...”

What we consider ‘progressive’, ‘regressive’ or ‘neutral’ depends on the “reference point” chosen. In the ‘amounts’ approach, the ‘neutral’ distribution of tax (benefit) is when everybody pays (receives) the same absolute amount. In the ‘deviations’ approach, ‘neutrality’ requires proportionality with pre-fiscal income. Thus, the former has “cumulative shares in the population” as a reference point, and the latter has “cumulative shares in total gross income”.

Concluding this section on the interpretation of the two approaches, we could tentatively say that the ‘amounts’ approach decomposes distance narrowing, while the ‘deviations’ approach decomposes departure from proportionality, or progressivity.

5. Empirical application: direct taxes and social benefits in Croatia

In this section we present estimations of various indicators for Croatia in the period from 2001 to 2008. The results are obtained using methodological tools developed in sections 3 and 4. Basic indicators are calculated for two income-tax-benefit definitions (ITBDs; see below) and six income equivalence scales (IES). This gives us overall twelve “scenarios” for the basic indicators. For decompositions of redistributive effects that reveal contributions of T&Bs, two scenarios are presented, one for each.

Data

The empirical research is based on the national household budget survey, called *Anketa o potrošnji kućanstava* (APK), administered by the Croatian Bureau of Statistics. The samples for the period from 2001 to 2008 are taken into account. APK contains the relevant data on incomes at individual levels, on consumption at the household level, and many other indicators, for a sample of Croatian households. The average size of the yearly samples is about 8.500 persons.

The data on incomes are registered net of personal income tax (PIT) and social security contributions (SSC). Therefore, in order to make the analysis of these two instruments possible, a microsimulation model had to be developed, one that applies tax code to the data and transforms net incomes into gross incomes, identifying the amounts of PIT and SSC for each individual. This model uses all the data on individuals and their household members available in APK: working status, number of children and dependent spouses; place of living; net incomes by source (wages, pensions, self-employment income, capital income, rents, etc.); outlays on items such as mortgage interest rate, life insurance (needed for calculation of PIT deductions). The data on social transfers are already available in APK.

Income definitions

Analysts in the field of fiscal incidence use different definitions of pre-fiscal and post-fiscal income, taxes and benefits, which depend on various assumptions concerning the

economic role and incidence of taxes and benefits. In this research, three important questions were posed:

- (a) Can employers shift the burden of SSC onto employees? In other words, who really bears the burden of those SSC for which the employer is legally responsible?
- (b) Should public pensions be treated as social benefits or as a market income?
- (c) Should SSC to the 1st pillar of the pension system be treated as personal savings or as taxes?

Instead of attempting to provide definitive answers to these questions, we are going to employ different income-tax-benefit definitions (henceforth ITBD) reflecting various choices on the appropriate assumptions. Out of six definitions from the original research, we present here two of them, *ITBD-A* and *ITBD-B*. Table 5 presents these definitions as sums of (or differences between) different income variables. Pre-fiscal income, taxes, benefits and post-fiscal income are given by (53) to (56) for *ITBD-A*, and by (57) to (60) for *ITBD-B*. Detailed explanations of variables can be found in the Appendix.

In Croatia, employers are legally required to pay SSC for the health system (*ssch*) and SSC for the unemployment protection system (*sscu*), while pension contributions are paid by employees. The *ITBD-A* assumes that SSC paid by employers (*ssch* and *sscu*) are fully shifted onto employees, which conforms to a classical view of tax incidence theory. Therefore, *ssch* and *sscu* are included in pre-TB income; see (53). Consequently, they are also included into taxes; see (54). On the other hand, *ITBD-B* assumes that the SSCs paid by employers are actually economically borne by the employers; therefore, they are neither income nor taxes for employees, and hence they are not mentioned in income (57) and tax (58) definitions.

Then we have the issue of public pensions. We have two extreme choices: to treat public pensions as social benefits, or to treat them as market income. In *ITBD-A*, public pensions (*npyo* and *npol*) are regarded as benefits, as can be seen in (55). In *ITBD-B*, public pensions (*xpyo* and *xpol*) are a part of pre-TB income, as defined in (57).

Finally, the issue whether SSC to the 1st pillar of the pension system are taxes or savings is solved in the following way. *ITBD-B* does not treat these contributions (*sscp*) as taxes, but as a form of saving; see (58). Therefore, they add to disposable income; see (60). On the other hand *ITBD-A* treats SSC to the 1st pillar as taxes; see (54).

Table 5: Definitions of incomes, taxes and benefits

(51)	$f = xtmi + pfcp + nntx + ownu + trnk$
(52)	$g = unem + sick + chbn + bsba + matr + rehb$
<i>ITBD-A</i>	
(53)	$X_1 = f + (sscp + ssch + sscu)$
(54)	$T_1 = (sscp + ssch + sscu) + pito$
(55)	$B_1 = g + (npyo + npol)$
(56)	$N_1 = X_1 - T_1 + B_1 =$ $= (f + sscp + ssch + sscu) - (sscp + ssch + sscu + pito) + (g + npyo + npol) =$ $= f + g + (npyo + npol) - pito$
<i>ITBD-B</i>	
(57)	$X_2 = f + sscp + (xpyo + xpol)$
(58)	$T_2 = pitt$
(59)	$B_2 = g$
(60)	$N_2 = X_2 - T_2 + B_2 =$ $= (f + sscp + xpyo + xpol) - pitt + g =$ $= f + g + sscp + (xpyo + xpol) - pitt$

Equivalence scales

The analysis also takes into account three types of the most exploited equivalence scales in research into redistributive effects: the “OECD” scale (E_3), the Cutler and Katz scale (E_2) and the “power/root” scale (E_1). Table 6 shows how the number of equivalent adults is obtained in each equivalence scale configuration. Each type of scale appears with two different sets of parameters: (a) the so-called “modified OECD scale”, $E_3(.5,3)$, and the “original OECD scale”, $E_3(.7,5)$; (b) the “Cutler and Katz scale” features in the common configuration of $E_2(.5,5)$, and the alternative one, $E_2(.7,6)$; (c) the “power scale” appears with typical square-root set of $E_1(.5)$, and also $E_1(1)$, which is actually a setup that ignores both economies of scale and the differences in age of household members.

Table 6: Equivalence scales

Symbol	Formula	Name
$E_1(\sigma)$	$m = (n)^\sigma$	Power/root scale
$E_2(\theta, \alpha)$	$m = (n_a + \theta n_c)^\alpha$	“Cutler and Katz” scale
$E_3(\delta, \varepsilon)$	$m = 1 + \delta(n_a - 1) + \varepsilon n_c$	“OECD” scale

Notes: m = equivalent adults; n = adults and children; n_a = adults; n_c = children.

Distance narrowing

Table 7 summarizes the indices of income inequality and redistribution in Croatia 2008. Gini coefficients of pre-TB income G_X and post-TB income G_N , and the index of distance narrowing Δ^x are calculated for twelve different scenarios, as explained above. The scales $E_2(.5, .5)$ and $E_1(.5)$ show the largest Gini coefficients for both ITBDs, the opposite of $E_3(.7, .5)$ and $E_1(1)$, which result in the smallest inequalities, both for G_X and G_N . Looking at distance narrowing as a percentage of pre-TB income Gini coefficient, represented by the column ‘ Δ^x ($\%G_X$)’ in Table 7, we observe small differences between the results obtained for different equivalence scales. We proceed with a comparison of results across ITBDs.

Recall from Table 5 that *ITBD-A* and *ITBD-B* do not have identical post-TB income definitions. For the former, post-TB income is N_1 , and N_2 for the latter. For all equivalence scales N_1 shows smaller inequality than N_2 , which is expected, because N_2 contains *sscp*, unlike N_1 . The reason is that *sscp* are part of the income earned only by working people, who are, on average, richer than pensioners and non-working individuals.

On the other hand, differences between Gini coefficients of pre-TB income for two ITBDs are very large: pre-TB income for *ITBD-A* (X_1) contains employer’s contributions, but does not include public pensions; for pre-TB income in *ITBD-B* (X_2) it is exactly the opposite. These distinctions are expected to lead to higher inequality of the pre-TB income for the former definition, and this is demonstrated in Table 7, where G_X for *ITBD-A* exceeds G_X for *ITBD-B* by 15 percentage points.

Consequently, there is a large discrepancy in estimates of distance narrowing Δ^x . For *ITBD-A* system it amounts to more than 40% of G_X ; for *ITBD-B* it goes down to a modest 10-12% of G_X . Thus, the estimate of distance narrowing crucially depends on how the fiscal system is defined.

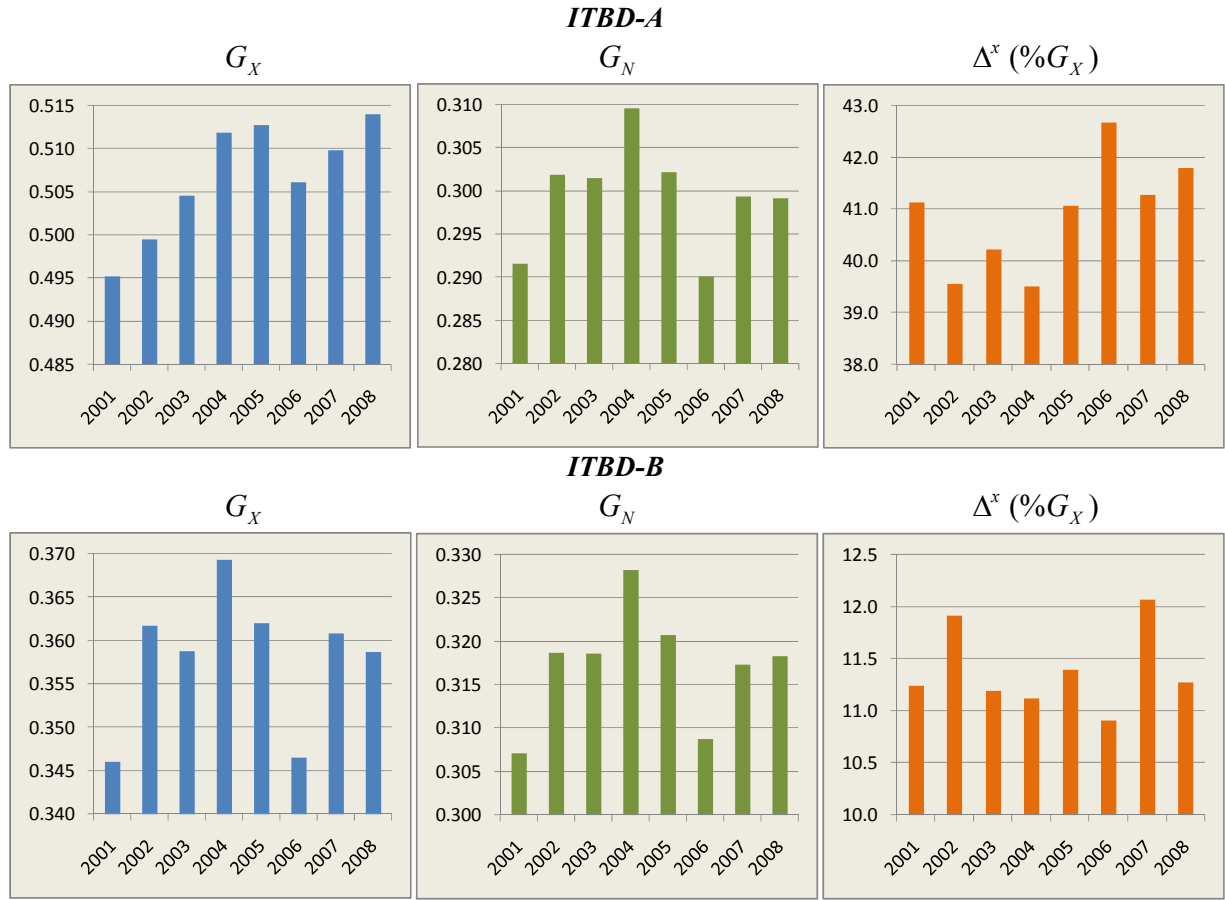
Table 7: Income inequality and distance narrowing in Croatia, 2008

	$E_3(.5,.3)$	$E_3(.7,.5)$	$E_2(.5,.5)$	$E_2(.7,.6)$	$E_I(.5)$	$E_I(1)$
<i>ITBD-A</i>						
G_X	0.5140	0.5048	0.5280	0.5182	0.5255	0.4991
G_N	0.2991	0.2945	0.3119	0.3022	0.3103	0.3015
Δ^x	0.2149	0.2104	0.2161	0.2159	0.2152	0.1975
$\Delta^x (\%G_X)$	41.8	41.7	40.9	41.7	40.9	39.6
<i>ITBD-B</i>						
G_X	0.3587	0.3536	0.3704	0.3618	0.3695	0.3579
G_N	0.3183	0.3118	0.3324	0.3220	0.3304	0.3155
Δ^x	0.0404	0.0418	0.0379	0.0398	0.0391	0.0424
$\Delta^x (\%G_X)$	11.3	11.8	10.2	11.0	10.6	11.9

Now we compare the results for 2008 with those for earlier years. Figure 3 indicates the inequality of pre-TB and post-TB income, and distance narrowing relative to pre-TB inequality, for two ITBDs in the period 2001-2008. A careful inspection may offer us the following interpretation: the inequality is fairly similar in most of the observed years, that is, in 2002, 2003, 2005, 2007 and 2008; the years 2001 and 2006 registered a lower inequality, by 1.5 Gini percentage points, while in 2004 inequality was higher by 1 Gini point. Without further research we cannot say whether these differences within the period reflect real swings in inequality, or they are perhaps a consequence of sampling.

For *ITBD-A* we can observe a certain rise of distance narrowing, as presented by graph ' $\Delta^x (\%G_X)$ ', after an initial decline in 2002. For *ITBD-B* we see no recognizable trend in magnitude of distance narrowing.

Figure 3: Income inequality and distance narrowing in Croatia, $E_3(.5,.3)$, 2001-2008



Deprivation from reranking

Table 8 shows values of the index of deprivation from reranking for twelve scenarios. Differences in the amount of reranking between *ITBD-A* and *ITBD-B* systems are huge: for the latter definition, reranking makes less than 1% of post-TB income inequality, as measured by a share ' $R^x (\%G_N)$ '; for the former definition, this share is more than 15 times higher.

Looking within *ITBD-B*, which describes the low-reranking system, all equivalence scales reveal similar magnitudes of reranking index, as expressed in terms of both G_N and Δ^x . On the other hand, reranking indices for *ITBD-A* show high divergence across the equivalent scales. Thus, R^{AP} ranges from 13% of G_N (18.7% of Δ^x) for $E_2(.5,.5)$, to 22% of G_N (33.5% of Δ^x) for $E_1(1)$.

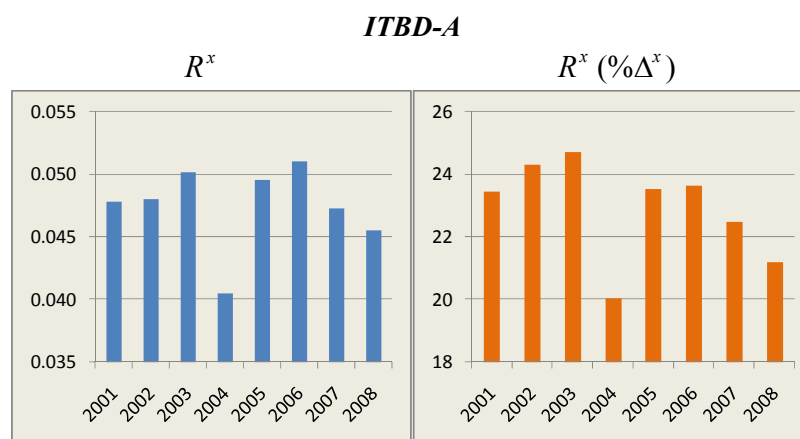
What might cause such large differences in the amount of reranking between these two ITBDs? A possible causal factor can be found by a simple comparison of these definitions: *ITBD-A* treats public pensions as benefits and SSCs as taxes, while *ITBD-B* does not include them into benefits and taxes. The role of public pensions and SSCs in creation of reranking will be analyzed soon.

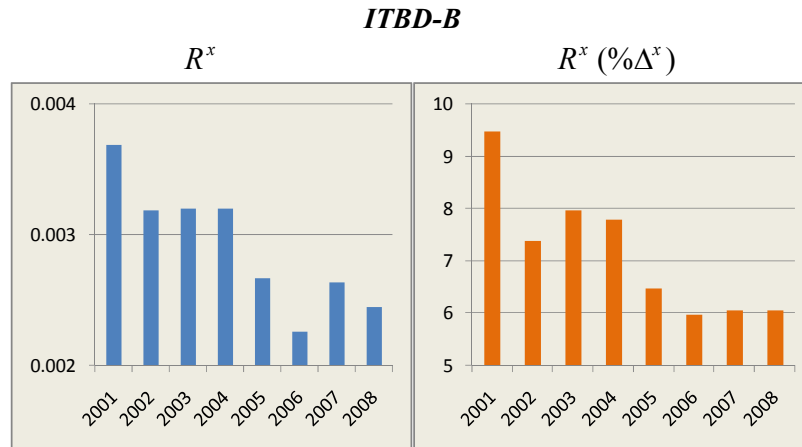
Table 8: Deprivation from reranking in Croatia, 2008

	$E_3(.5,.3)$	$E_3(.7,.5)$	$E_2(.5,.5)$	$E_2(.7,.6)$	$E_I(.5)$	$E_I(1)$
<i>ITBD-A</i>						
R^x	0.0455	0.0525	0.0404	0.0430	0.0418	0.0662
$R^x (\%G_N)$	15.2	17.8	13.0	14.2	13.5	22.0
$R^x (\%\Delta^x)$	21.2	24.9	18.7	19.9	19.4	33.5
<i>ITBD-B</i>						
R^x	0.0024	0.0023	0.0027	0.0025	0.0025	0.0021
$R^x (\%G_N)$	0.8	0.7	0.8	0.8	0.8	0.7
$R^x (\%\Delta^x)$	6.1	5.5	7.1	6.3	6.4	5.0

Figure 4 presents the index of deprivation from reranking R^x for two ITBDs, and its share in the corresponding index of distance narrowing, ' $R^x (\%\Delta^x)$ ' in the period 2001-2008. For *ITBD-A* reranking sustains its high level during the period, with 2004 acting as an outlier. For *ITBD-B* we notice a decreasing trend; thus, the share ' $R^x (\%\Delta^x)$ ' falls by more than three percentage points if we compare 2002 and 2008.

Figure 4: Deprivation from reranking in Croatia, $E_3(.5,.3)$, 2001-2008





Contributions of taxes and benefits

We have arrived at the central topic of this empirical part: decompositions of fiscal deprivation, deprivation from reranking and distance narrowing that reveal the roles of various T&Bs. The methodology section presented two approaches to dissect the basic measures: one based on absolute differences between amounts of T&Bs, and the other on deviations of T&Bs from proportionality. For the sake of brevity, they were called ‘amounts’ and ‘deviations’ approach.

Table 9 shows the results obtained for two ITBDs and $E_3(.5,.3)$. Values of indicators are omitted for convenience and, instead, percentage contributions are given. Subtotals for all taxes and all benefits are shown in rows ‘*taxes*’ and ‘*benefits*’.

Naturally, major instruments, like public pensions, will be the largest contributors to redistributive effects, simply because of their magnitudes. Therefore, we can compare each contribution of an instrument to V^x , R^x or Δ^x , with the instrument’s share in total sum of T&Bs, obtained in column ‘% *T&Bs*’ of Table 9. For easier analysis, we calculated ratios between ‘ $\Delta^x(\%)$ ’ and ‘% *T&Bs*’, and presented them in column ‘ $\Delta^x(\div)$ ’. Numbers in this column greater than 1 mean that the tax or benefit instrument creates an above-proportional contribution to distance narrowing.

We have seen that the system described by *ITBD-A* caused large amount of distance narrowing, which was indeed followed by a huge amount of reranking. Now we can understand what has caused this reranking. Public pensions (*pyo* and *pol* together) create about 88-98% of

total reranking, as shown in columns 6 and 11, titled ' R^x ($\% R^x$)', which is more than twice their share in total amount of T&Bs for *ITBD-A*. At the other pole, there is PIT whose contribution to *ITBD-A* is negative: PIT reduces reranking in this fiscal system, if the 'deviations' approach is considered; in the 'amounts' approach, PIT contributes 0.5% to overall R^x . For *ITBD-B*, the contribution of PIT to reranking is relatively larger, but still many times lower than the share of PIT in total amount of T&Bs.

SSC also contribute under-proportionally to reranking, but their overall size is high, and together they are the second contributor to reranking, after public pensions. For *ITBD-A*, they create about 12% of R^{AP} .

The role of cash benefits (*unem*, *sick*, *chbn*, *bspa*, *matr*, *rehb*) in reranking crucially depends on the choice of scenario. For *ITBD-A*, they even reduce reranking or contribute significantly below their proportion in overall T&Bs. For *ITBD-B*, it is quite the opposite: where cash benefits, while contributing 28.7% of T&Bs, they create more than 90% of reranking in this system. However, we should recall that overall reranking for *ITBD-B* was quite modest.

Recall (49) and (50) which represent decompositions of distance narrowing. Table 9 contains contributions of T&Bs to V^x , R^x and Δ^x obtained by these formulas and expressed as percentages of Δ^x . Now, we can fully decompose distance narrowing, both into fiscal deprivation and deprivation from reranking, *and* across fiscal instruments. In columns 2 to 4 of Table 9 we find components for the 'amounts' approach, while columns 7 to 9 present the results for the 'deviations' approach. Which instruments contribute most to fiscal deprivation, deprivation from reranking and distance narrowing?

The answers, as we expected, crucially depend on the approach. Let us first analyze the 'amounts' approach. Taxes – PIT and SSC combined – create 75% of distance narrowing for *ITBD-A*, where PIT, pension and health SSCs have similar contributions. For *ITBD-B*, PIT takes most of the reward, 87% of Δ^x , which is above its share in total T&Bs.

For the 'deviations' approach, the results are quite the reverse. Taxes bring only about 12% of *deviation from proportionality* for *ITBD-A*. This approach reveals benefits as overwhelming contributors.

Table 9: Decompositions of V^x , R^x and Δ^x , $E_3(.5,.3)$, 2008

		'amounts'					'deviations'				
	% <i>T&Bs</i>	V^x (% Δ^x)	R^x (% Δ^x)	Δ^x (% Δ^x)	$\Delta^x(\div)$	R^x (% R^x)	V^x (% Δ^x)	R^x (% Δ^x)	Δ^x (% Δ^x)	$\Delta^x(\div)$	R^x (% R^x)
	1	2	3	4	5	6	7	8	9	10	11
<i>ITBD-A</i>											
sscp	19.7	26.0	1.1	24.9	1.3	5.6	2.2	0.3	1.9	0.1	1.5
ssch	20.4	26.9	1.1	25.7	1.3	5.8	2.2	0.3	1.9	0.1	1.6
sscu	2.0	2.8	0.1	2.6	1.3	0.5	0.3	0.0	0.3	0.1	0.1
pit	11.8	21.3	0.1	21.2	1.8	0.5	7.6	-0.4	8.1	0.7	-2.1
unem	0.7	0.2	0.0	0.2	0.3	-0.2	1.1	0.0	1.1	1.6	0.0
sick	0.6	0.0	0.1	-0.1	-0.1	0.3	0.8	0.1	0.7	1.1	0.4
chbn	1.6	0.6	-0.2	0.8	0.5	-0.9	2.7	-0.1	2.9	1.8	-0.6
bspa	0.8	0.9	-0.2	1.1	1.3	-0.8	2.2	-0.1	2.3	2.7	-0.6
matr	0.9	0.0	0.1	-0.1	-0.1	0.4	1.2	0.1	1.1	1.2	0.6
rehb	0.4	0.3	0.1	0.2	0.5	0.5	0.9	0.1	0.8	1.8	0.6
pyo	14.9	10.1	6.0	4.0	0.3	31.6	31.3	7.5	23.8	1.6	35.3
pol	26.1	30.1	10.8	19.3	0.7	56.6	68.6	13.4	55.2	2.1	63.2
taxes	54.0	76.9	2.4	74.5	1.4	12.5	12.4	0.2	12.1	0.2	1.1
benefits	46.0	42.2	16.7	25.5	0.6	87.5	108.8	20.9	87.9	1.9	98.9
<i>ITBD-B</i>											
sscp											
ssch											
sscu											
pit	71.4	87.4	0.4	87.0	1.2	8.7	60.7	0.4	60.3	0.8	6.6
unem	4.1	1.8	0.5	1.3	0.3	10.4	5.6	0.6	5.0	1.2	10.6
sick	3.4	1.2	0.8	0.4	0.1	17.9	4.2	1.1	3.1	0.9	18.3
chbn	8.9	5.8	1.0	4.8	0.5	21.5	14.7	1.3	13.4	1.5	22.0
bspa	4.8	5.5	0.4	5.0	1.0	9.3	11.1	0.6	10.5	2.2	9.6
matr	5.0	1.7	1.1	0.6	0.1	24.3	6.2	1.5	4.7	0.9	24.7
rehb	2.5	1.2	0.4	0.8	0.3	8.0	3.5	0.5	3.0	1.2	8.2
pyo											
pol											
taxes	71.4	87.4	0.4	87.0	1.2	8.7	60.7	0.4	60.3	0.8	6.6
benefits	28.6	17.1	4.1	13.0	0.5	91.3	45.3	5.6	39.7	1.4	93.4

The same analysis is made for the whole period. We have observed steady contributions over the period for all instruments and both ITBDs. The tables are relegated to the Appendix.

6. Conclusion

This paper uses a “micro-approach” in the study of income inequality, where different indicators are first calculated for pairs of income units, and then aggregated to the population level. This approach is much more suitable for scrutinizing various income transitions emerging from fiscal activities than the usual apparatus of Lorenz and concentration curves. A well-known use of the “micro-approach” can be found in the measurement of relative deprivation first

proposed by Yitzhaki (1979), and then extended to an analysis of fiscal processes by Duclos (2000).

In this framework, new concepts of income supremacy, income distance, fiscal deprivation, deprivation from reranking and distance narrowing have emerged. An income unit with a higher pre-fiscal income has pre-fiscal *income supremacy* over a unit with lower pre-fiscal income, equal to the difference of their pre-fiscal incomes. The fiscal process usually bites a fraction of this income supremacy, transferring money from the higher- to lower-income unit, causing *fiscal deprivation* for the former unit; at the same time, *income distance*, an absolute difference between the two units' incomes, decreases – this is *distance narrowing*.

However, the redistributive transfer may be even larger than pre-fiscal income supremacy; in this case, the pre-fiscally richer unit becomes post-fiscally poorer – a situation known as reranking. When there is no reranking, distance narrowing is equal to fiscal deprivation. But, when reranking occurs, fiscal deprivation is higher than distance narrowing; this excess of fiscal deprivation imposed upon the pre-fiscally higher income unit is called *deprivation from reranking*. Consequently, fiscal deprivation can be decomposed into distance narrowing and deprivation from reranking.

The interesting feature is that when appropriately aggregated these indices collide with the well-known indices of redistributive effects mentioned in the introduction. The Kakwani vertical effect V^K corresponds to the index of fiscal deprivation V^x , the Atkinson-Plotnick reranking effect R^{AP} matches the index of deprivation from reranking R^x , and redistributive effect RE equals the index of distance narrowing Δ^x . Knowing these relationships may help us better to understand and interpret these widely used indices.

The largest part of the paper is devoted to decompositions of those three effects to reveal the contributions of the different tax and benefit instruments that make up the fiscal system. The use of a “micro-approach” has proved an indispensable tool in this task. New derivations started from the beginning, using *amounts* of T&Bs, on one side, and their *deviations* from proportionality on the other side, to describe the wedge between pre- and post-fiscal income. Use of two sets of tax/benefit variables led to two distinct approaches, named, for the sake of convenience, the ‘amounts’ and ‘deviations’ approaches. Only at the end was it realized that decomposition of fiscal deprivation using the ‘deviations’ approach is equivalent to the Lambert

(1985) decomposition of the Kakwani vertical effect. However, the present model also enables us, for the first time, to decompose the Atkinson-Plotnick index of reranking in a transparent and intuitive way.

The ‘amounts’ approach, on the other hand, produces completely new decompositions. Comparison of the ‘amounts’ and ‘deviations’ approaches indicates completely different results on the relative contributions of T&Bs. The former approach is inclined toward taxes, while the latter favours benefits. The example of a hypothetical system with proportional tax and equal amount of benefits for all income units, shows that the ‘amounts’ approach ascribes the full contribution for achieving fiscal deprivation to the *tax*, while the *benefit* is fully responsible in the ‘deviations’ approach. This puzzle is certainly not new in the field of fiscal incidence and has its root in the choice of the reference point or benchmark for the measurement. The conclusion is that the ‘deviations’ approach has ‘deviations from proportionality as a benchmark, while the ‘amounts’ approach has deviations from the mean as a reference point. Other benchmarks could also be devised: this is a topic for further research.

The research behind this paper had originally been intended solely to measure the redistributive effects of direct taxes and cash social benefits in Croatia. However, in view of the methodological issues, the empirical part was relegated to the background at the outset. Nonetheless, empirical topics did receive due attention later in the research, as demonstrated in this paper. Empirical investigation has posed many difficult questions that do not readily yield unambiguous answers. The most important question considers the role of public pensions: are they market income or a social benefit? We know they are both and neither at the same time. Ideally, we would divide each pension into ‘market’ and ‘fiscal’ components. In this research we decided to employ two different scenarios: in one, public pensions are benefits; in the other they are treated as market income. Other issues are related to the incidence of the SSC: who really pays the employers’ SSC – workers or employers, and are the mandatory SSCs paid to the pension fund taxes or a form of saving? These questions too have been dealt with using different scenarios.

Two fiscal system scenarios are explored: the ‘mini system’, consisting only of PIT and cash social benefits (other than public pensions), and the ‘full system’, containing the instruments from the mini system together with SSC and public pensions. The main result is that

the Croatian system of direct taxes and social benefits significantly reduces income inequality. The magnitude of distance narrowing crucially depends on the chosen scenario: the ‘full system’, reduces distances by 40%, the ‘mini system’ only by 12%. Deprivation from reranking is huge for the ‘full system’, amounting to over 20% of distance narrowing, but for the other system is quite modest.

We have learnt that all fiscal instruments included in the analysis reduce distances between income units; they are inequality-reducing. The most important are public pensions, SSC for pension and health funds, and PIT. Looking at the ‘full system’, the ‘amounts’ approach says that SSC and PIT account for 75% of distance narrowing, while the ‘deviations’ approach says that public pensions deliver 98% of deviations from proportionality. This is the empirical confirmation of the puzzle about how the two approaches ascribe relative importance to taxes and benefits. Concerning deprivation from reranking, the picture is much less ambiguous: public pensions are overwhelmingly responsible for it, independently of the chosen approach. On the other hand, instruments such as PIT, basic personal allowance and child allowance create quite acceptable quantities of reranking.

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Appendix

Application issues

The decompositions based on distance narrowing, as developed in this chapter, are ready-for-use only when observations have equal weights (for example, in case of a random sample drawn from a population of individual taxpayers). The reason is the following: the basic operation underlying these decompositions is the calculation of the difference in income (tax, benefit) of some units i and j ; now, if they have different weights, *what should be the weight of the difference?* Unfortunately, most empirical applications in this field employ household budget survey data and observation units are households, thus leading to two kinds of weights: (a) the sampling weights; (b) the weights arising from the use of equivalence scales. Therefore, some adaptations of original data sets are needed before the decompositions are applied.

Each sample household i represents w_i households drawn from the total population of $W = \sum_i w_i$. Household i has m_i equivalent adults; thus, there are $M = \sum_i m_i w_i$ equivalent adults in the population. Here, values w_i are sampling weights defined by sample designers and values m_i are obtained by equivalence scale formulas. The approach is as follows. Let y_i be the income of household i (the same procedure goes for T&Bs). Then, $y_i^e = y_i / m_i$ is the equivalent income of this household. Now, we form a new sample of incomes y_i^e replicating each income $m_i w_i / k$ times, where k serves as a parameter that will decide about the total number of

observations in the new sample. The smaller the value of k , the larger the sample: greater precision is achieved, but at the cost of computation time.

For example, household budget samples for Croatia consist of about 2,900 households on average which gives around 5,200 equivalent adults, if $E_3(.5,3)$ is used; for given sampling weights we obtain W of about 2.6 million. This sample size is prohibitively large, concerning the amount of needed calculations. Therefore, we set $k \approx 100$ and form the sample of about 25,000 observations. However, despite the fact that only 1% of “population” is covered, the 25k samples gave values of the main indicators and comparable decompositions quite close to those obtained from the original sample. Exercises with 50k samples have shown little improvement in precision, at a large cost of additional computation time.

Overview of income, tax and benefit variables

Table 10 overviews the variables used in empirical research. Three types of market income are recognized: market income taxable by PIT ($xtmi$), market income not taxable by PIT ($nntx$) and obligatory contributions to the private pension fund ($pfcp$). The latter item is actually “SSC for pension system – the 2nd pillar”; inclusion of this item into market incomes means that contributions to the 2nd pillar of the pension system will always be treated as private outlay of an individual, and not as a tax.

Notice that public pensions are presented by four different variables. The first division is inspired by Immervoll et al (2005), who introduced separate treatment of two groups of pensioners: those aged less than 65, and those aged 65 and more. The second division is between the pre-PIT and post-PIT pension income. The use of pre-PIT pensions ($xpyo$ and $xpol$), when public pensions are not part of pre-TB income, creates an anomaly that would prevent the proper estimation of redistributive effect and reranking of this fiscal instrument, as Urban (2008) explains. Therefore, the variables for post-PIT pensions ($npyo$ and $npol$) are also included.

Taxes are presented by six variables. SSC for the pension system ($sscp$) relate only to contributions to the 1st pillar. Because the distinction between pre-PIT and post-PIT pensions has to be made, as explained above, we also needed to create separate PIT variables ($pito$ and $pitp$), besides the main PIT variable that presents total PIT and surtax ($pitt$).

Finally, the six benefit items are defined according to the previous discussions. Note only that maternity allowance and layette supplement are joined into one variable (*matr*), while support for rehabilitation and employment of people with disabilities is merged with supplement for the injured to create another new variable (*rehb*).

Table 10: Variables of income, taxes and benefits

Notation	Description
<i>General</i>	
<i>X</i>	Pre-TB income
<i>N</i>	Post-TB income
<i>T</i>	Total individual taxes
<i>B</i>	Total cash benefits
<i>Market incomes</i>	
<i>xtmi</i>	Market income taxable by PIT: wages and salaries, self-employment income, income from part-time and contractual work, rental income and income from property rights
<i>nntx</i>	Non-taxable market income (interest on private saving and investment)
<i>pfcp</i>	Obligatory contributions to the private pension fund (i.e. SSC for pension system – the 2 nd pillar)
<i>Non-market-non-fiscal incomes</i>	
<i>ownu</i>	Value of production for own use
<i>trnk</i>	Periodic transfers from private persons: gifts, alimonies
<i>Public pensions</i>	
<i>xpyo</i>	Public pensions to persons aged less than 65, before PIT
<i>npyo</i>	Public pensions to persons aged less than 65, after PIT (hence also „Pensions (<65)“)
<i>xpol</i>	Public pensions to persons aged 65 and more, before PIT
<i>npol</i>	Public pensions to persons aged 65 and more, after PIT (hence also „Pensions (65&>)“)
<i>Taxes</i>	
<i>sscp</i>	SSC for the pension system - the 1 st pillar
<i>ssch</i>	SSC for the health system
<i>sscu</i>	SSC for the unemployment protection system
<i>pitt</i>	PIT and surtax, total
<i>pito</i>	PIT and surtax, on <i>xtmi</i>
<i>pitp</i>	PIT and surtax, on <i>xpyo</i> and <i>xpol</i>
<i>Benefits</i>	
<i>bspa</i>	Basic support allowances
<i>unem</i>	Unemployment benefit
<i>chbn</i>	Child allowance
<i>sick</i>	Sick-leave benefit
<i>matr</i>	Maternity allowance and layette supplement
<i>rehb</i>	Supplement for the injured and support for rehabilitation and employment of people with disabilities

Note: Observe the following relationships: $(xpyo + xpol) - pitp = npyo + npol$; $pitp = pitt - pito$; $xpyo + xpol - pitt = npyo + npol - pito$

Contributions of taxes and benefits: 2001, 2004 and 2008

For presentation of results in Table 11 and Table 12 we choose three years: 2001 and 2008, as a start and the end of the whole period, and also 2004 as a middle year, especially interesting because this year was a kind of outlier in previous analysis. Table 11 shows the results for the ‘amounts’ approach, and Table 12 for the ‘deviations’ approach.

Table 11: Decompositions of V^x , R^x and Δ^x , ‘amounts’, $E_3(.5,.3)$, 2001-2008

	% T&Bs			V^x (% Δ^x)			R^x (% Δ^x)			$\Delta^x(\div)$		
	2001	2004	2008	2001	2004	2008	2001	2004	2008	2001	2004	2008
ITBD 1												
sscp	19.3	20.7	19.7	26.1	26.4	26.0	1.4	1.1	1.1	1.3	1.2	1.3
ssch	20.0	21.3	20.4	26.9	27.3	26.9	1.5	1.1	1.1	1.3	1.2	1.3
sscu	1.8	2.1	2.0	2.6	2.7	2.8	0.1	0.1	0.1	1.4	1.3	1.3
pit	11.7	12.4	11.8	22.0	22.1	21.3	0.1	0.1	0.1	1.9	1.8	1.8
unem	1.0	1.0	0.7	-0.1	0.3	0.2	0.1	0.0	0.0	-0.2	0.3	0.3
sick	0.8	0.7	0.6	0.1	-0.1	0.0	0.3	0.3	0.1	-0.3	-0.6	-0.1
chbn	2.0	1.6	1.6	0.2	0.5	0.6	-0.2	-0.2	-0.2	0.2	0.4	0.5
bspa	0.9	0.9	0.8	1.1	1.1	0.9	-0.1	-0.1	-0.2	1.4	1.4	1.3
matr	1.3	1.0	0.9	-0.3	-0.3	0.0	0.1	0.1	0.1	-0.3	-0.5	-0.1
rehb	0.4	0.6	0.4	0.1	0.4	0.3	0.1	0.1	0.1	0.1	0.6	0.5
pyo	18.6	15.6	14.9	15.5	10.0	10.1	8.3	5.3	6.0	0.4	0.3	0.3
pol	22.1	22.3	26.1	27.6	26.5	30.1	10.1	9.1	10.8	0.8	0.8	0.7
taxes	52.9	56.5	54.0	77.7	78.6	76.9	3.1	2.4	2.4	1.4	1.3	1.4
benefits	47.1	43.5	46.0	44.2	38.3	42.2	18.7	14.5	16.7	0.5	0.5	0.6
ITBD 6												
sscp												
ssch												
sscu												
pit	65.9	69.7	71.4	91.6	89.1	87.4	0.6	0.5	0.4	1.4	1.3	87.0
unem	5.5	5.1	4.1	0.7	2.6	1.8	0.8	0.8	0.5	0.0	0.4	1.3
sick	4.2	3.6	3.4	1.6	0.7	1.2	1.8	1.8	0.8	0.0	-0.3	0.4
chbn	10.5	8.7	8.9	5.2	5.3	5.8	1.3	0.7	1.0	0.4	0.5	4.8
bspa	4.6	4.9	4.8	6.2	6.2	5.5	0.7	0.5	0.4	1.2	1.2	5.0
matr	6.9	5.1	5.0	1.3	-0.1	1.7	1.7	1.1	1.1	-0.1	-0.2	0.6
rehb	2.3	3.0	2.5	0.9	2.0	1.2	0.6	0.5	0.4	0.1	0.5	0.8
pyo												
pol												
taxes	65.9	69.7	71.4	91.6	89.1	87.4	0.6	0.5	0.4	1.4	1.3	87.0
benefits	34.1	30.3	28.6	15.9	16.8	17.1	6.9	5.3	4.1	0.3	0.4	13.0

Table 12: Decompositions of V^x , R^x and Δ^x , ‘deviations’, $E_3(.5,.3)$, 2001-2008

	% T&Bs			V^x (%RE)			R^x (%RE)			$\Delta^x(\div)$		
	2001	2004	2008	2001	2004	2008	2001	2004	2008	2001	2004	2008
ITBD 1												
sscp	19.3	20.7	19.7	1.8	2.3	2.2	0.5	0.4	0.3	0.1	0.1	0.1
ssch	20.0	21.3	20.4	1.9	2.3	2.2	0.5	0.4	0.3	0.1	0.1	0.1
sscu	1.8	2.1	2.0	0.3	0.3	0.3	0.0	0.0	0.0	0.2	0.1	0.1
pit	11.7	12.4	11.8	7.8	8.7	7.6	-0.5	-0.4	-0.4	0.7	0.7	0.7

	% T&Bs			V^x (%RE)			R^x (%RE)			$\Delta^x(\div)$		
	2001	2004	2008	2001	2004	2008	2001	2004	2008	2001	2004	2008
<i>unem</i>	1.0	1.0	0.7	1.3	1.6	1.1	0.1	0.0	0.0	1.1	1.7	1.6
<i>sick</i>	0.8	0.7	0.6	1.1	0.8	0.8	0.4	0.4	0.1	1.0	0.6	1.1
<i>chbn</i>	2.0	1.6	1.6	2.9	2.8	2.7	-0.1	-0.2	-0.1	1.5	1.9	1.8
<i>bspa</i>	0.9	0.9	0.8	2.4	2.6	2.2	0.0	-0.1	-0.1	2.8	3.0	2.7
<i>matr</i>	1.3	1.0	0.9	1.4	0.9	1.2	0.2	0.2	0.1	0.9	0.8	1.2
<i>rehb</i>	0.4	0.6	0.4	0.7	1.2	0.9	0.1	0.1	0.1	1.4	2.1	1.8
<i>pyo</i>	18.6	15.6	14.9	42.1	33.7	31.3	10.0	7.1	7.5	1.7	1.7	1.6
<i>pol</i>	22.1	22.3	26.1	59.8	62.6	68.6	12.2	12.0	13.4	2.2	2.3	2.1
<i>taxes</i>	52.9	56.5	54.0	11.8	13.7	12.4	0.6	0.5	0.2	0.2	0.2	0.2
<i>benefits</i>	47.1	43.5	46.0	111.8	106.3	108.8	22.9	19.5	20.9	1.9	2.0	1.9
ITBD 6												
<i>sscp</i>												
<i>ssch</i>												
<i>sscu</i>												
<i>pit</i>	65.9	69.7	71.4	60.5	60.3	60.7	0.5	0.6	0.4	0.9	0.9	0.8
<i>unem</i>	5.5	5.1	4.1	5.6	7.8	5.6	1.1	1.1	0.6	0.8	1.3	1.2
<i>sick</i>	4.2	3.6	3.4	5.6	3.9	4.2	2.3	2.4	1.1	0.8	0.4	0.9
<i>chbn</i>	10.5	8.7	8.9	15.4	14.2	14.7	1.7	1.0	1.3	1.3	1.5	1.5
<i>bspa</i>	4.6	4.9	4.8	11.8	12.2	11.1	0.9	0.7	0.6	2.3	2.4	2.2
<i>matr</i>	6.9	5.1	5.0	7.5	4.2	6.2	2.3	1.4	1.5	0.8	0.5	0.9
<i>rehb</i>	2.3	3.0	2.5	3.1	5.2	3.5	0.8	0.7	0.5	1.0	1.5	1.2
<i>pyo</i>												
<i>pol</i>												
<i>taxes</i>	65.9	69.7	71.4	60.5	60.3	60.7	0.5	0.6	0.4	0.9	0.9	0.8
<i>benefits</i>	34.1	30.3	28.6	49.0	47.4	45.3	8.9	7.2	5.6	1.2	1.3	1.4

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