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Monitoring of the BTA Deep Hole Drilling Process Using Residual Control Charts

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Monitoring of the BTA Deep Hole Drilling Process Using Residual Control Charts

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Abstract

Deep hole drilling methods are used for producing holes with a high length-to-diameter ratio, good surface finish and straightness. The process is subject to dynamic disturbances usually classified as either chatter vibration or spiralling. In this work, we propose to monitor the BTA drilling process using control charts to detect chatter as early as possible and to secure production with high quality. These control charts use the residuals obtained from a model which describes the variation in the amplitude of the relevant frequencies of the process. The results showed that chatter is detected and some alarm signals are related to changing physical conditions of the process.

1 Introduction

Deep hole drilling methods are used for producing holes with a high length-to-diameter ratio, good surface finish and straightness. For drilling holes with a diameter of 20 mm and above, the BTA (Boring and Trepanning Association) deep hole machining principle is usually employed. Deep hole drilling means that $l/D \geq 3$, where l is the length and D is the diameter of the hole to be machined.

The machining of bore holes with high length-to-diameter ratio implies the use of slender tooling assemblies featuring low static and dynamic stiffness properties. This in turn leads to the process being susceptible to dynamic disturbances usually classified as either chatter vibration

or spiralling. Chatter is a form of self excited, mainly torsional vibration of the tool-boring bar assembly. The effect of chatter on the workpiece is usually restricted to radial chatter marks at the bottom of the bore hole, see Figure (1)a. In extreme cases it damages the boring wall by causing marks, called chatter marks, on the cylindrical surface of the bore hole, see Figure (1)b. The effect of chatter on the tool are more severe. It leads to excessive wear of the cutting edges

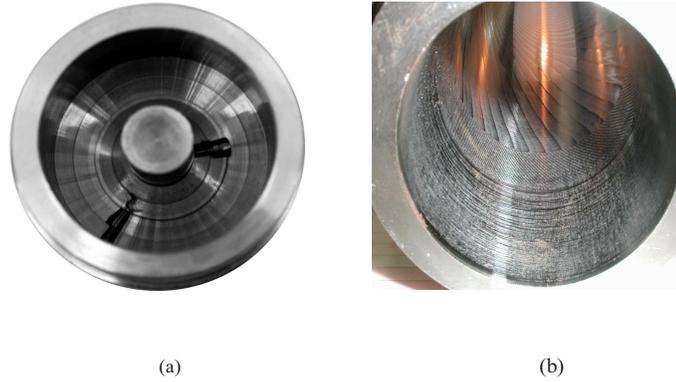


Figure 1: Radial chatter marks

and guiding pads of the tool which has an undesirable effect on the tool life. Spiralling damages the workpiece severely. It leads to a multi lobe-shaped deviation of the cross section of the hole from absolute roundness, see Figure (2).

The effect of chatter and spiralling are highly undesirable because the defect of form and surface quality constitute a significant impairment of the workpiece. As the deep hole drilling process is often used during the last production phases of expensive workpieces, it is necessary that a process monitoring system be devised to detect these disturbances during the process operation. The purpose of this work is to develop such a real time monitoring strategy by using statistical process control techniques. This strategy is used to detect the transition from stable operation to chatter.



Figure 2: Effect of spiralling on the bore hole wall

In section 2, models that describes the process are reviewed. In section 3, residual control charts are briefly introduced. The proposed monitoring strategy is discussed in section 4 and applied to real data in section 5.

2 Process models

Several drilling experiments are conducted in order to study the dynamics of the process. During these experiments several on-line measurements were sampled, see Weinert et al. (2001). Chatter is easily recognized in the on-line measurements by a fast increase of the dynamic part of the torque, force and acceleration signals. However, the drilling torque measurements yield the earliest and most reliable information about the transition from stable operation to chatter. For a complete discussion, see Weinert et al. (2002).

The spectrograms of the drilling torque, in different experiments, showed clearly that single frequencies play a key role in the discrimination between these states. Theis (2004) determined all the relevant frequencies of the process. He described the development of the amplitudes of these frequencies with respect to the cited states. In his work, the main features of the variation of the amplitudes of the amplitudes of the relevant frequencies are described, using a logistic function. He showed that his approximation is directly connected to the van der Pol equation proposed by Weinert et al. (2002). This equation is capable of describing the transition from stable operation to chatter in one frequency

$$\frac{d^2M(t)}{dt^2} + h(t)(b^2 - M(t)^2) \frac{dM(t)}{dt} + w^2M(t) = W(t), \quad (1)$$

where $t \in [0, \infty)$, $M(t)$ is the drilling torque, $b \in \mathbb{R}$, the frequency $w \in [200, 2500]$, $h(t) : \mathbb{R} \rightarrow \mathbb{R}$ is an integrable function and $W(t)$ is a white noise process. Theis (2004) considered $M(t)$ as a harmonic process

$$M(t) = R(t)\cos(w + \phi),$$

where ϕ is the corresponding phase. He showed that

$$2 \frac{dR(t)}{dt} + h(t)R(t) \left(b^2 - \frac{R(t)^2}{2} \right) = \frac{W(t)}{w} \quad (2)$$

is the amplitude-equation for the differential equation in (1) if there is only one frequency present in the process. He proved that if his proposed logistic function is the right form for $R(t)$, there is a function $h(t)$ so that equation (2) has a solution. From equation (2), the observed variation in amplitude of the relevant frequencies may be described by

$$R_t = \beta + (1 + a_t)R_{t-1} - a_t b_t R_{t-1}^3 + \varepsilon_t, \quad (3)$$

where a_t and b_t are time varying parameters and ε_t is normally distributed with mean 0 and variance σ_ε^2 .

3 The monitoring procedures: Residual control charts

Residual control charts are SPC procedures dealing with autocorrelated data in the SPC environment and have been suggested by several authors. For example, see Alwan and Roberts (1988), Montgomery and Mastrangelo (1991), among others. This procedure requires a model of the autocorrelative structure of the data which can be achieved by fitting an appropriate time series model to the observations. The typical time series model employed is the autoregressive integrated moving average (ARIMA) models

$$\Phi_p(B)\nabla^d R_t = \Theta_q(B)\varepsilon_t, \quad (4)$$

where $\Phi(B) = (1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p)$ is an autoregressive polynomial of order p , $\Theta(B) = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q)$ is a moving average polynomial of order q , ∇ is the backward difference operator, B is the backshift operator, and ε_t is a sequence of normally and independently distributed random “shocks” with mean zero and constant variance σ_ε^2 . If the time series model fits the data well, the residuals will be approximately uncorrelated. Then, traditional SPC charts to individual observations, such as Shewhart individual, CUSUM and EWMA can be applied to the residuals. For example, for an AR(1) process the residual AR(1) control chart is based on charting residuals

$$e_t \equiv R_t - \hat{R}_{t|t-1,t-2,\dots}$$

where $\hat{R}_{t|t-1,t-2,\dots} = E(R_t | R_{t-1}, R_{t-2}, \dots) = \mu + \phi(R_{t-1} - \mu)$. In practice, μ and ϕ have to be estimated from the data set that was obtained in a period where only common causes of variation were affecting the process. As long as the process is in control, observations are assumed to be generated by model (4). The quantity e_t that will be plotted in the residuals control chart satisfy

$$e_t = R_t - \hat{R}_{t|t-1,t-2,\dots} \approx \varepsilon_t \quad \text{for all } t.$$

In this paper, three control charts are used. Residual Shewhart and residual exponentially weighted moving average (EWMA) control charts are used to monitor the mean of the residuals. EWMA dispersion is used to monitor the variance of the residuals.

3.1 Residual Shewhart control charts

The residual Shewhart control chart operates by plotting residuals e_t given by equation (2). It signals that the process is out of control when e_t is outside UCL and LCL , that are defined to be

equal $LCL = \mu_e - k\sigma_\varepsilon$ and $UCL = \mu_e + k\sigma_\varepsilon$, where μ_e and σ_ε are, respectively, the mean and standard deviation of e_t when the process is in control and k is a constant.

3.2 Residual EWMA control charts (REWMA)

It is widely known that Shewhart charts are not sensitive to small and moderate changes in process parameters. The EWMA chart is considered in this work for improved detection of small and moderate parameter changes. The EWMA utilizes all previous observations, but the weight attached to observations is exponentially declining in the past. The EWMA of residuals at time t is denoted by $W_{e,t}$ and is computed as follows

$$W_{e,t} = \lambda_r e_t + (1 - \lambda_r)W_{e,t-1},$$

for $t \geq 2$, $W_{e,1} = \mu_e = 0$ and $0 < \lambda_r \leq 1$. Control limits for the EWMA residuals chart are of the form

$$LCL_t = \mu_e - c\sigma_{W_{e,t}}$$

$$UCL_t = \mu_e + c\sigma_{W_{e,t}}$$

where $\sigma_{W_{e,t}} = \sqrt{\lambda_r/(2 - \lambda_r)}\sigma_\varepsilon$ is the asymptotic standard deviation of $W_{e,t}$ under the assumption that the observations are independent and c is constant.

3.3 EWMA dispersion control charts (DEWMA)

EWMA dispersion charts for subgroups of size one plot the control statistic, for $t \geq 2$,

$$S_t^2 = (1 - \lambda_d)S_{t-1}^2 + \lambda_d(e_t - \mu)^2,$$

where $0 < \lambda_d < 1$ and $S_1^2 = \sigma_0^2$, where σ_0 represents the established value for the process standard deviation. The asymptotic control limits of this chart are

$$UCL = \sigma_\varepsilon^2 \left[1 + k_u \sqrt{\frac{2\lambda_d}{2 - \lambda_d}} \right]$$

$$LCL = \max \left\{ 0, \sigma_\varepsilon^2 \left[1 - k_l \sqrt{\frac{2\lambda_d}{2 - \lambda_d}} \right] \right\},$$

for constants k_u and k_l . For more details, see Acosta-Mejía and Pignatiello (2000).

4 Monitoring the variation in amplitude of relevant frequencies

As mentioned, single frequencies, mostly related to the eigenfrequencies of the boring bar, dominate the process when chatter vibrations are present. Therefore, we propose to monitor the variation in amplitude of the relevant frequencies of the process to detect chatter as early as possible. For the monitoring procedure, the model given by equation (3) is approximated by its linear autoregressive (AR) part

$$R_t = (1 + a_t)R_{t-1} + \varepsilon_t,$$

and this AR(1) model is used to calculate the residuals. In fact, it is known that the nonlinear term $-a_t b_t R_{t-1}^3$ only becomes important when there is chatter. The empirical evidence of this approximation is studied in the next section using real data. As noted, parameters of equation (3) are not constant. However, residual control charts are generally designed for processes where stationarity in the steady state is assumed, which means that a unique model parameters for the whole process is used. For this reason, a moving window of length T , defined in the time domain, is used to estimate the AR(1) parameters. Moving window techniques are useful to estimate model parameters which are time varying assuming stationarity only locally. The window moves in each period covering T observations $R_{t-T+1}, R_{t-T+2}, \dots, R_t$. In each window, parameters a , β and σ_ε of the linear regression model

$$R_t = \beta + (1 + a)R_{t-1} + \varepsilon_t. \quad (5)$$

are estimated and used to calculate the residuals, given by

$$e_t = R_t - (1 + \hat{a}_{t-1})R_{t-1} - \hat{\beta}_{t-1}, \quad (6)$$

where \hat{a}_{t-1} and $\hat{\beta}_{t-1}$ are estimates of the regression parameters a and β at time $t-1$. Note that β is included because there is a general shift in the amplitudes after depth 35 mm due to a change in the physical conditions of the process, see section 5.1. The estimated standard deviation of the process $\hat{\sigma}_{\varepsilon,t-1}$ at time $t-1$ is used to set the control limits of the three control charts at time t . These choices are motivated by the fact that using the estimated parameters at time t to calculate the residuals and to set the control limits may rather serve to mask changes than to detect them, see section 5.4.

5 Application

The three control charts are applied to real data of the change in amplitude for the frequency 703 Hz, which is among the eigenfrequencies of the boring bar, in an experiment with feed $f = 0.185$ mm, cutting speed $v_c = 90$ m/min and amount of oil $\dot{V}_{oil} = 300\text{L}/\text{min}$. For more details see Weinert et al. (2002). This frequency dominates chatter vibration in this experiment.

5.1 Transition from stable state to chatter

In order to investigate the ability of the different control charts to detect chatter, it is important to identify the transition from stable operation to chatter, which is expected to occur before depth 300 mm. Indeed, by eye inspection, the effect of chatter in this experiment is apparent on the bore hole wall after depth 300 mm.

The mean and variances of the amplitude of frequency 703 Hz are studied. Figure (3) shows the mean and variance, using the most recent 100 observations of the amplitudes of the frequency. It is clear that for depth ≥ 32 mm there is an increase in the process mean and process variance. In fact, it is known that approximately at this depth the guiding pads of the BTA tool leave the starting bush. From previous experiments, the process has been observed to either stay stable or start with chatter vibration; see Weinert et al. (2002). Also there is an increase in the mean and variance of the two frequencies at depth 110 mm and it is known that depth 110 mm is approximately the position where the tool enters the bore hole completely. Figure (3)a and (3)b show clearly that there are changes in the mean and variance of the amplitude of frequency 703 Hz at depth 252.91 mm. This conclusion is very important because we know that in this experiment chatter is observed with frequency 703 Hz, which makes depth 252.91 mm a candidate for the transition from stable operation to chatter.

Further investigation by studying the autocorrelation function, see next section, and the sign of the parameter a_t showed that the process changes at depth 252.91 mm. This change may indicate the presence of chatter or that chatter will start in few seconds.

5.2 Independence and normality assumptions of the residuals

In the previous section it is indicated that the variation in amplitude of the relevant frequencies, given by equation (1), is approximated by an AR(1) model within each time window. If this AR(1) model fits the data well, the residuals will be “approximately” uncorrelated and normally

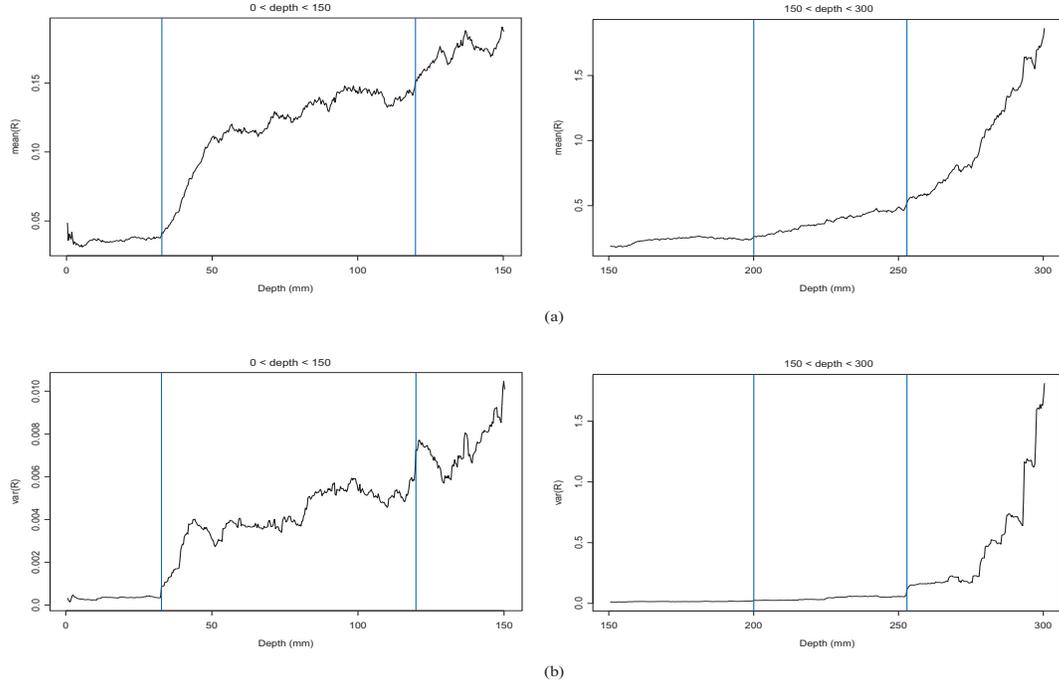


Figure 3: (a) Mean and (b) variance of the amplitude of frequency 703 Hz on 0-300 mm hole depth

distributed. Then standard control charts can be applied to the residuals. In order to check these two assumptions (important SPC assumptions) the quantiles of standard normal and sample autocorrelation function of the residuals over different time windows with length 150 are plotted in Figures (4) and (5), respectively. Figure (4) shows that at least for $270.64 \leq \text{depth} \leq 315.39$ mm the normality assumption of the residuals cannot be assumed. Also, for $0.30 \leq \text{depth} \leq 45.06$ mm the last points show a departure from the reference line. This departure may be explained by the sudden change in the model after depth 35 mm, see section 5.1. Figure (5) shows that the residuals are uncorrelated before the transition to chatter.

In conclusion, we assume that the variation in amplitude of the relevant frequencies can be approximated by the AR(1) model when the process is stable and that the nonlinear term $-a_t b_t R_{t-1}^3$ is not important before chatter, which is expected.

5.3 Choice of the control charts parameters

The performance of a control chart is usually evaluated on its run length or on the expectation of its run length, the average run length (ARL). The run length is defined as the number of observations that are needed to exceed the control limit for the first time. The ARL should be large

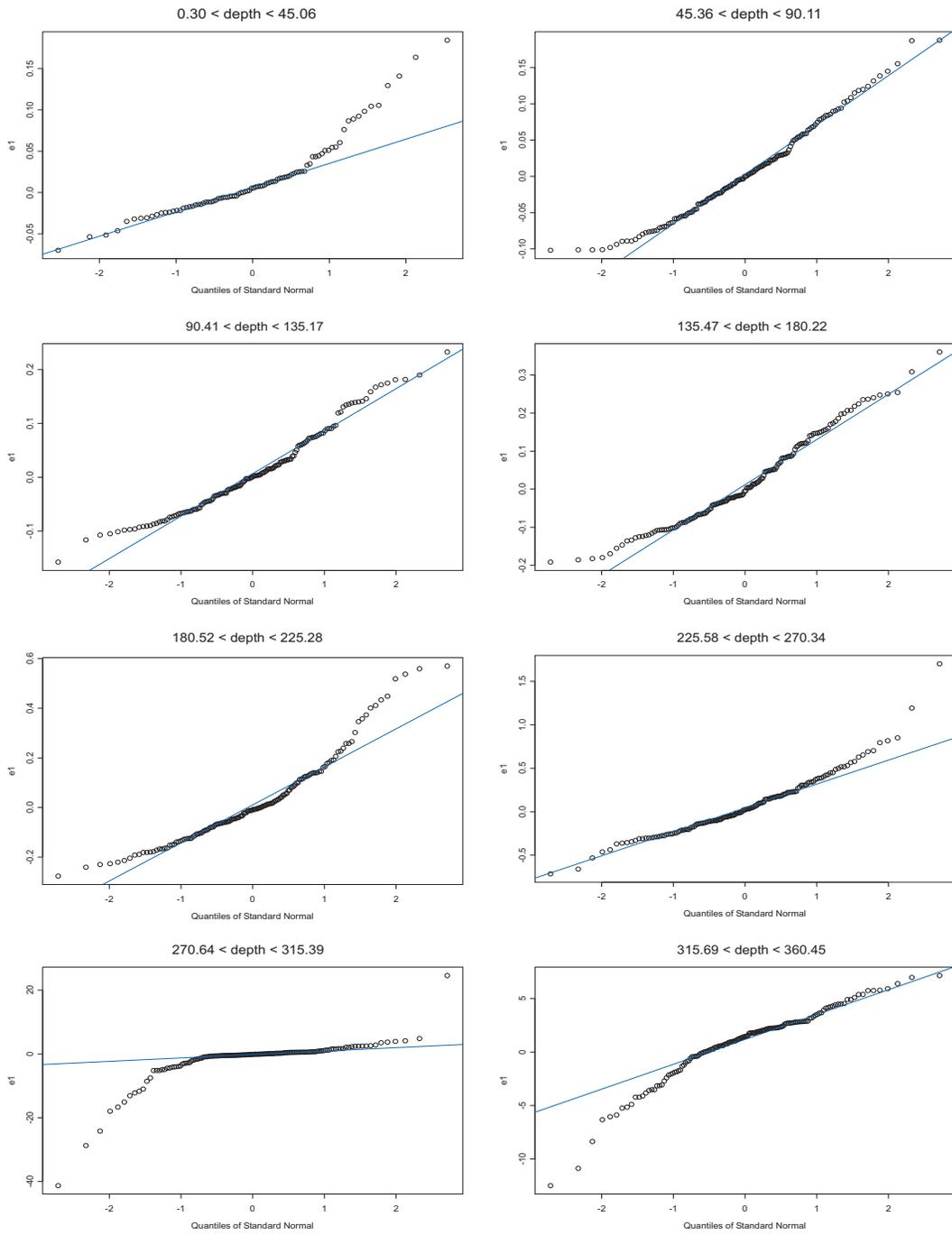


Figure 4: Quantiles of standard normal of the residuals of amplitude of frequency 703 Hz

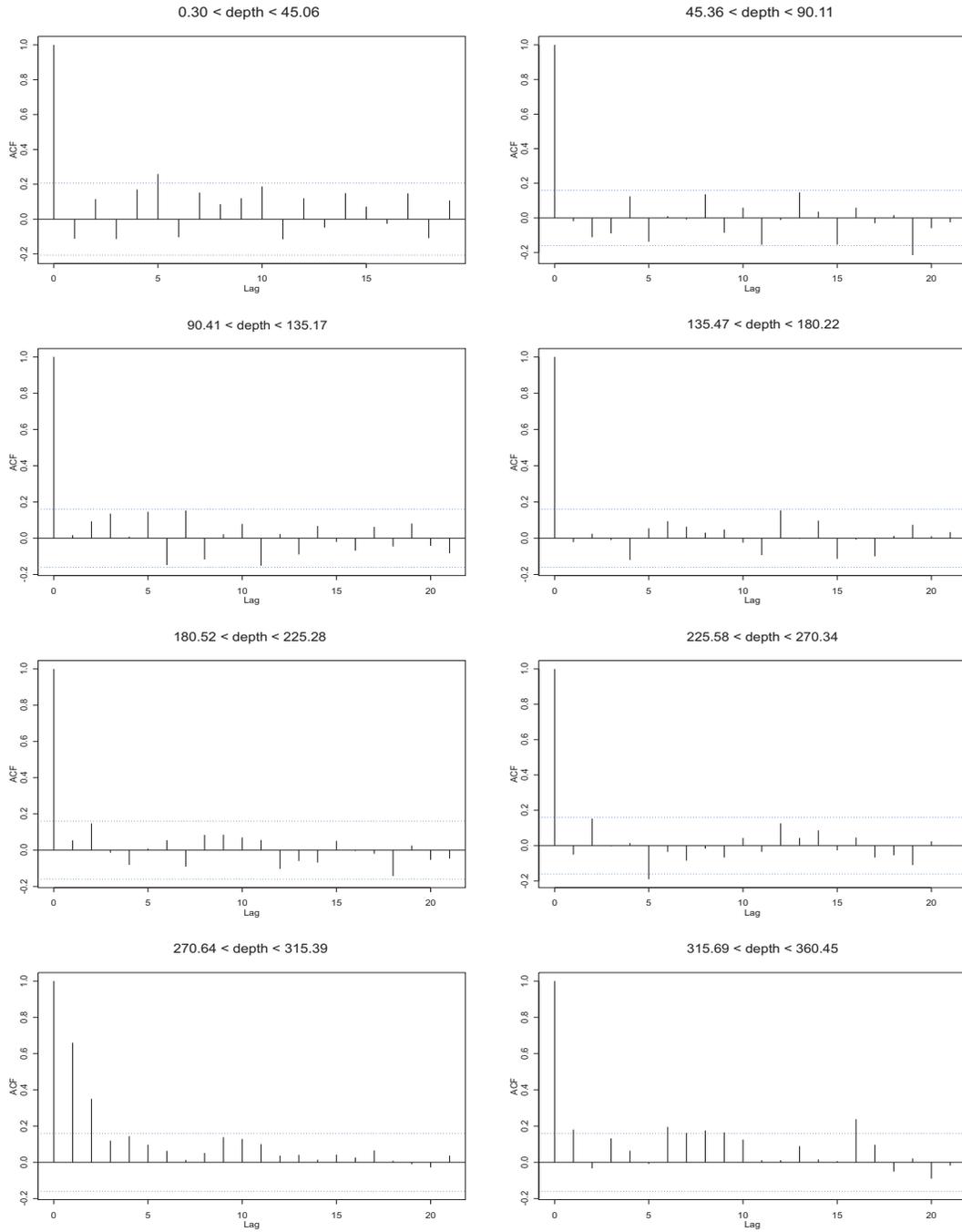


Figure 5: Autocorrelation function of the residuals of amplitude of frequency 703 Hz

when the process is statistically in-control (in-control ARL) and small when a shift has occurred (out-of-control ARL).

The parameters of the different control charts are chosen so that they have the same in-control average run length (ARL) equal to 500. This choice should not give a lot of signals because the control charts are applied to 1600 observations. For the residual Shewhart a value of 3.09 is chosen for k and for the REWMA values of 0.2, 0.4, and 0.75 are chosen for λ_r with the corresponding values of 2.962, 3.054, and 3.087, respectively, chosen for c . For the DEWMA, we used $\lambda_d = 0.1$, $k_u = 4.01$ and $k_l = 1.885$. The pair of values (k_u, k_l) produces an ARL-unbiased DEWMA chart. This concept is defined by Pignatiello et al. (1995). A control chart is said to be ARL-unbiased if its ARL curve achieves its maximum when the process parameter is equal to its in-control value. If the maximum occurs when the process parameter is equal to some other value, the control chart is said to be ARL-biased. Acosta-Mejía and Pignatiello (2000) described a search procedure to find the pair of values (k_u, k_l) that produces the ARL-unbiased DEWMA chart. A markov chain is used to approximate the in-control ARL of the DEWMA chart; see Appendix A.

It is important to note that when the control charts produce an out of control signal, we can conclude that there is a deviation from the stable process, which might implies that chatter is present. In fact, the residuals are calculated using an AR(1) model, which is an approximation of the variation in amplitude of the relevant frequencies when the process is stable.

5.4 Results

Table 1 shows the out of control signals for depth ≤ 270 mm . Table 1 shows that all control charts signal at depth 32.74 mm. As mentioned the guiding pads leave the starting bush approximately at depth 32 mm, which induce an increase in the process mean and variance for the amplitude of the two frequencies. This increase explains that all control charts have picked up these changes very quickly. All control charts signal at depth 119.84 and it is known that depth 110 mm is approximately the position where the tool enters the bore hole completely. Theis (2004) noted that this might lead to changes in the dynamic process because the boring bar is slightly thinner than the tool and therefore the pressures in the hole may change. The important out of control signal is found at depth 252.91 mm. As discussed, it is showed that the transition from stable operation to chatter may have occurred at this depth. This out of control signal means that a change occurred in the process. Thus, in this experiment chatter may be avoided if corrective actions are taken after this signal.

Table 1: Out of control signals of the different control charts applied to the amplitude of frequency 703 Hz using window length $T = 60$ (depth ≤ 270 mm)

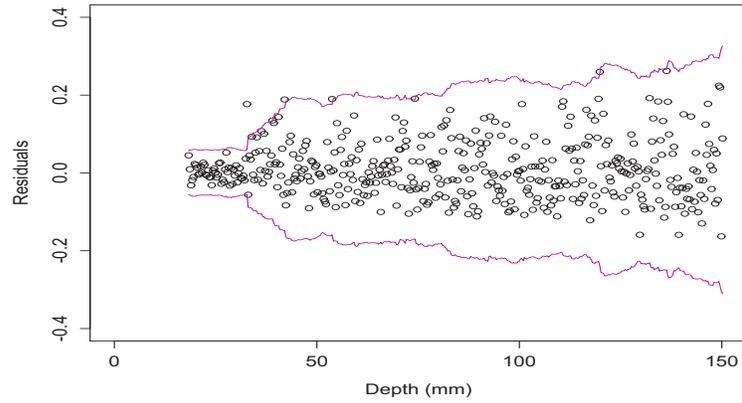
Residual Shewhart	REWMA			DEWMA	observations number
	$\lambda_r = 0.2$	$\lambda_r = 0.4$	$\lambda_r = 0.75$	$\lambda_d = 0.1$	
32.74	32.74	32.74	32.74	32.74	109
				33.04	110
				33.34	111
				33.64	112
				33.94	113
				34.24	114
				34.54	115
				34.84	116
				35.14	117
				35.44	118
				35.74	119
				36.34	121
				36.64	122
				36.94	123
				37.25	124
39.04	39.04	39.04	39.04	37.55	125
				39.04	130
				39.34	131
42.05	39.64	39.34	39.34	39.34	132
				40.55	135
				42.05	140
				42.35	141
				53.76	179
				74.19	247
				119.84	399
				153.19	510
				156.49	521
				191.33	637
200.05	200.05	200.05	200.05	200.05	666
				215.67	718
				224.97	750
				224.97	750
				225.28	750
				225.28	750
				252.91	842
				253.21	843
				253.51	844
				253.81	845
254.11	254.11	253.51	252.91	254.11	846
				254.41	847
				254.72	848
				255.02	849
				253.81	845
				253.51	844

Figures (6)a and b show the residual Shewhart control chart. It is clear from the two Figures that there are large out of control signals at depth 32.74 and 252.91 mm. The monitoring procedure started at observation $t = 61$ (depth=18.32 mm), that is after 60 observations are collected. The adaptive structure of the control limits allows for a detection of a change in the variance of the residuals. For example, Figure (6)b shows a sudden increase in the adaptive control limits (standard deviation of the residuals) at depth 252.91 mm caused by a sudden increase in the variance of the amplitude of frequency 703 Hz, see Figure (3)b. Knowing that there is a change in the process after depth 252.91 mm, the residual Shewhart control chart should produce many out-of-control signals after that depth. However, no out-of-control signals are produced for $252.91 < \text{depth} \leq 270$ mm. The reason is that the change is transferred to the adaptive estimated parameters. In fact, one limitation of the use of adaptive estimates to calculate the residuals is the “masking” or parameter adaptation problem. If an early process change is not quickly detected, then the parameter estimates may be adversely affected by the change, thus masking the shift from future detection. Finally some out of control signals are observed at $270 \leq \text{depth} \leq 300$ mm but are not considered in Table 1 because the normality and independence assumptions are not valid after the transition to chatter, see section 5.2.

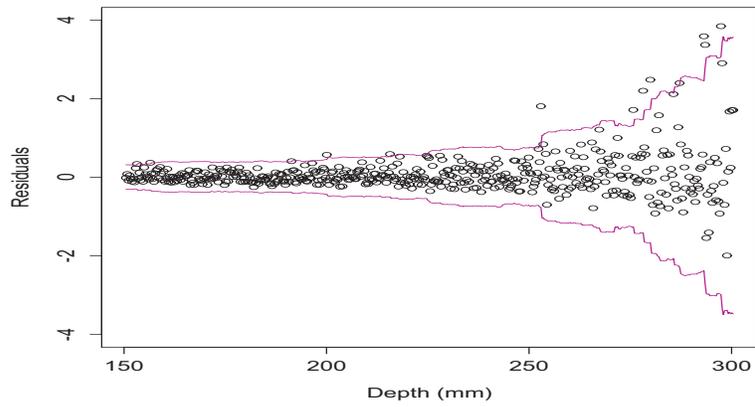
6 Discussion and future work

In this work the results showed that chatter can be detected only by monitoring the amplitude of frequency 703 Hz. This conclusion is expected because this frequency is the relevant frequency in this experiment. However, in practice there are more relevant frequencies and chatter may be observed at the beginning of the drilling process immediately after the guiding pads have left the starting bush, with high and low frequencies, see Weinert et al. (2001). Thus, an SPC procedure that monitors all the relevant frequencies is necessary. One solution is to classify all the relevant frequencies in different groups and to calculate a weighted mean of their amplitudes. The proposed univariate procedure can be used to monitor the variation of the weighted means of the different groups, separately. The resulting monitoring strategy signals an out-of-control condition when any univariate control chart produces an out-of-control signal. This strategy may lead to many out of control signals and it is difficult to interpret multiple control charts. Another solution is to use a multivariate approach, which is investigated by Messaoud et al. (2004).

In this work, it is supposed that the autocorrelation structure of the change in amplitude is an



(a)



(b)

Figure 6: Residual Shewhart control chart for the amplitude of frequency 703.13 Hz on (a) 0-150 mm and (b) 150-300 mm hole depth

inherent feature of the BTA process which cannot feasibly be removed. Many authors argued that the autocorrelation may itself indicate the presence of some variability which should be removed, rather than modelled. For the BTA drilling process the intrinsic autocorrelation can provide the basis for active process control as a tool for minimizing short term variability.

7 Conclusion

This work has focused on the application of statistical control procedures to monitor the BTA drilling process to detect chatter as early as possible. The different SPC procedures are based on residuals. This work showed that an approximated autoregressive model of the amplitude of relevant eigenfrequencies of the boring bar can be used to calculate the residuals. The results showed that the different SPC procedures can detect chatter and some alarm signals are related to changing physical conditions of the process (i. e. guiding pads leave the starting bush, the tool is completely in the hole). Based on the practical results, the different control charts have similar performances. However, we recommend the residual Shewhart because it is extremely simple to use compared to the others.

A Appendix: Markov Chain Approximation

For the in-control case, the ARLs of the DEWMA charts are approximated by a discrete Markov chain. To approximate the DEWMA statistic, the interval between the upper and lower control limits is partitioned into $2m + 1$ transient states, each of width $g = (UCL - LCL)/(2m + 1)$. The control statistic, S_t , is said to be in state j at time t if

$$LCL + (j - 0.5)g < S_t \leq LCL + (j + 0.5)g,$$

for $j = 1, 2, \dots, 2m + 1$. The control statistic S_t is in the absorbing state a if it falls outside the control limits ($S_t < LCL$ or $S_t > UCL$). The process is assumed to be in-control whenever S_t is in a transient state and is assumed to be out of control whenever the S_t is in the absorbing state.

The run-length distribution of the DEWMA is completely determined by its initial probability vector and the transition probability matrix. Let $p_{i,j}$ represent the probability that the control statistic S_t goes from state i to state j in one step, $i, j = 1, 2, \dots, 2m + 1$. To approximate this probability, it is assumed that the control statistic is equal to c_j whenever it is in state j , where

c_j represents the midpoint of the j th interval. This yields

$$\begin{aligned} p_{i,j} &= P[LCL + (j - 0.5)g < S_t \leq LCL + (j + 0.5)g \mid S_{t-1} = c_i] \\ &= P\left[\frac{LCL + (j - 0.5)g - (1 - \lambda_d)c_i}{\lambda_d \sigma_0^2} < \left(\frac{e_t - \mu_e}{\sigma_0^2}\right)^2 \leq \frac{LCL + (j + 0.5)g - (1 - \lambda_d)c_i}{\lambda_d \sigma_0^2}\right], \end{aligned}$$

where $\left(\frac{e_t - \mu_e}{\sigma_0^2}\right)^2 \sim \chi^2(1)$.

Let \mathbf{R} equal to the $(2m + 1) \times (2m + 1)$ transition probability matrix. The ARL is computed by

$$ARL = \mathbf{p}^T (\mathbf{I} - \mathbf{R})^{-1} \mathbf{1},$$

where $\mathbf{1}$ is a vector of ones and \mathbf{p} is $2m + 1$ vector with a one in the component that corresponds to the starting state and zero elsewhere. In this work it is assumed that the DEWMA is equal σ_0^2 at the onset of monitoring.

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