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Working Paper

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GLO Discussion Paper, No. 700

Provided in Cooperation with:

Global Labor Organization (GLO)

Suggested Citation: Mayr-Dorn, Karin (2020) : Adverse selection, learning, and competitive search, GLO Discussion Paper, No. 700, Global Labor Organization (GLO), Essen

This Version is available at:

<https://hdl.handle.net/10419/225515>

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Adverse selection, learning, and competitive search

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October 13, 2020

Abstract

I develop a dynamic version of the competitive search model with adverse selection in [Guerrieri, Shimer and Wright \(2010\)](#). My model allows for an analysis of the effects of firm learning on labor market efficiency in the presence of search frictions. I find that firm learning increases relative expected earnings in high-ability jobs and, thereby, enhances imitation incentives of low-ability workers. The net effect on the aggregate expected match surplus and unemployment is indeterminate a priori. Numerical results show that firm learning does not increase labor market efficiency.

Keywords: job search; on-the-job effort; asymmetric information; learning.

JEL classification: D82; D83; J64.

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1 Introduction

The inability of the labor market to allocate resources efficiently has been attributed to two important sources of frictions. First, search frictions may impose costs on the formation of suitable worker-firm matches. Second, asymmetric information may result in adverse selection of workers at the stage of hiring. The existing literature shows that these frictions together may seriously hamper efficient resource allocation in the labor market and increase the rate of unemployment.¹

However, firms can learn about their workers' unobserved abilities, as they observe them on the job (see [Farber and Gibbons \(1996\)](#) and [Altonji and Pierret \(2001\)](#)). Indeed, there is evidence that firms learn fast about workers' types (see, e.g., [Lange \(2007\)](#)).² New technologies, which allow to monitor workers ever more closely, can increase the rate of firm learning.³ This may reduce distortions due to imperfect information of firms. It suggests that firms have an incentive to monitor workers for efficiency reasons, and must be regulated in order to protect workers' privacy. Yet, little is known about the efficiency effects of firm learning in labor market equilibrium.⁴ In this paper, I develop a model that combines firm learning with state-of-the-art equilibrium theories of adverse selection and unemployment. I find that the marginal effect of firm learning is not unambiguously positive, and likely to be small. This can explain why firms may not always use improved ways of worker monitoring for the purpose of distinguishing between high- and low-ability types - not only for ethical reasons, but even for reasons of efficiency.

I propose a model that extends the framework of [Guerrieri et al. \(2010\)](#) to dynamic situations, integrating a dynamic version of adverse selection a la [Rothschild and Stiglitz](#)

¹I review some of this literature below.

²[Lange \(2007\)](#) finds that firms' initial expectation errors about the ability of their workers decline by 50% during the first three years of employment.

³These include, for example, monitoring programs, microchip implants, wristband trackers, and even sensors that can detect fatigue and depression.

⁴The existing literature on firm learning typically focuses on implications for worker turnover, as described below.

(1976) in a model of competitive search in labor markets a la Moen (1997). At the time of hiring, information about worker ability is asymmetric, and endogenous worker effort is unobservable. Due to asymmetric information, adverse selection is possible: workers may direct their search towards jobs designed for a different worker type, misreport their type at hiring, and receive a wage that exceeds the wage based on their true type. This is possible because firms cannot observe worker effort initially, and so cannot infer a worker's type from observed output. In consequence, effort on the job may be suboptimal. Within this framework, I address a number of questions, e.g.: How does firm learning affect the search behavior of workers and firms? How does it affect a worker's choice of effort on the job, the surplus of a worker-firm match⁵, (relative) wages and unemployment rates, and aggregate labor market efficiency?

If asymmetric information causes distortions in the labor market, then an increase in the rate at which firms learn about the true type of a worker can be expected to improve labor market outcomes. Interestingly, however, I find that in the presence of search frictions, the effect of firm learning on labor market efficiency is not necessarily positive and may, in fact, be negative. This is because faster firm learning increases the expected surplus in high-ability jobs and, therefore the job finding rate and average expected wage in these jobs, for given work effort. As a result, low-ability workers have a greater incentive to imitate high-ability workers, and the work effort of high-ability workers during probation may increase. If this effect is sufficiently large, the expected match surplus in high-ability jobs decreases, and unemployment increases, as firms learn faster about a worker's type.

More specifically, as in Moen (1997), firms post contracts, and workers decide where to search given those contracts. As in Guerrieri et al. (2010), information is asymmetric: workers are perfectly informed about their ability, but firms are not. However, in contrast

⁵Match surplus is the difference between the expected present value of the future incomes that the two parties to a match earn and the expected present value of income that they forgo by participating in the employment relationship (Mortensen and Nagypal, 2007, p. 330).

to these benchmark models, firms in my model gradually learn about a worker's ability upon observing his effort over time. The effort of workers on the job is endogenous; it is unobservable by the firm initially but affects worker output and, thereby, serves as a signal for worker ability. Job offers and expected earnings are based on the worker's current output (which depends on (true) worker ability and effort on the job) as well as the worker's future expected output (which depends on reported worker ability).

Workers decide to search for jobs based on their option value of search, which varies with reported ability. By misreporting his type, a low-ability worker benefits from a greater job finding rate and may earn a higher starting wage. In turn, he also faces a greater cost of effort. In case of adverse selection, relative job finding rates and expected earnings in high-ability jobs increase in the rate of firm learning, increasing imitation incentives for low-ability workers. Then, firm learning has two important counter-vailing effects on the expected match surplus of high-ability workers. On the one hand, given efforts, faster firm learning increases their surplus, since effort is suboptimal before firm learning but optimal thereafter (direct effect). On the other hand, faster firm learning decreases their surplus, because the (suboptimally high) effort before firm learning increases even further in response to greater imitation incentives of low-ability workers (indirect effect). The net effect is indeterminate a priori. In numerical simulations, I find that firm learning has no effect on labor market efficiency.

Related literature

There is a growing literature on the problem of worker-firm matching in the presence of costly search and asymmetric information. For example, [Lockwood \(1991\)](#) suggests that adverse selection increases inefficiency in a frictional labor market where firms test workers prior to hiring and unemployment is used as a signal of productivity. More recently, [Inderst \(2005\)](#) analyzes labor market equilibria in a model with random search and adverse selection where new participants enter the market. He derives conditions

for the existence of a unique separating equilibrium. [Guerrieri et al. \(2010\)](#) analyze equilibrium existence and efficiency in labor markets with directed search and adverse selection. They show that there always exists a separating equilibrium, which is not generally efficient. All of the above consider stationary environments without firm learning or wage dynamics. In comparison, I consider equilibria in labor markets with directed search, where firms are allowed to learn about workers' types. I show that a separating equilibrium exists, and that firm learning does not generally increase equilibrium efficiency.

The existing literature on firm learning typically focuses on implications for worker turnover. For example, [Carrillo-Tudela and Kaas \(2015\)](#) determine worker turnover and optimal wage contracts in a model with random on-the-job search. In their model, firms also gradually learn about their workers' types. Faster firm learning decreases incentives for workers to misreport, because they are less likely to move to another job before the firm learns their type and demotes them. In my model with directed search, in contrast, job finding rates and work efforts are endogenous and submarket-specific. Faster firm learning increases incentives for workers to misreport, because it increases the relative job finding rate and average expected wage in high-ability jobs. In response, the work effort of high-ability workers may increase, which decreases efficiency. Other examples for theoretical contributions include [Jovanovic \(1979\)](#), [Moscarini \(2005\)](#) and [Papageorgiou \(2018\)](#), where workers and firms jointly learn about match quality over time.⁶ Empirical contributions such as [Altonji and Pierret \(2001\)](#), [Lange \(2007\)](#) and [Kahn \(2013\)](#) provide evidence for the degree of asymmetric information and the speed of firm learning.

Another related strand of the literature focuses on wage dynamics in search models with asymmetric information. For example, [Moen and Rosen \(2006\)](#) analyze optimal wage contracts in a random search model where firms do not observe workers' effort nor their type. Similarly, [Moen and Rosen \(2011\)](#) and [Tsuyuhara \(2016\)](#) analyze optimal wage

⁶In [Gonzalez and Shi \(2010\)](#), there is worker learning about their own job-finding process in a directed search framework.

contracts with unobservable worker effort (and type) and directed search. These papers focus on the retention and incentive effects of wages in the presence of adverse selection or moral hazard. They do not, however, address the implications of firm learning for efficiency. My paper implements firm learning in a tractable search model with asymmetric information about both worker ability and worker effort. In this setting, I analyze the effect of firm learning on job search, effort on the job, match surplus, (relative) wages and unemployment rates, and aggregate labor market efficiency.

The rest of the paper is organized as follows. Section 2 describes the framework of the model. Sections 3-4 characterize equilibria in the case of perfect and imperfect information, respectively. Section 4.2 discusses the role of firm learning for equilibrium efficiency. Section 5 simulates the model numerically. Section 6 concludes.

2 Model framework

2.1 Workers and firms

Consider a continuous time economy with a continuum of workers and firms. Workers are either employed, or unemployed and searching for a job. They are one of two types, high-ability or low-ability, with ability p_i , $i \in \{H, L\}$, $p_H > p_L > 0$, and cost of effort e , $c(e)$, with $c(0) = 0$, $\partial c(e)/\partial e > 0$, and $\partial^2 c(e)/\partial e^2 > 0$.⁷ The measure of workers is normalized to one, and the population share of high- and low-ability workers is assumed constant and equal to $0 < a_H < 1$ and $a_L = 1 - a_H$, respectively. Employed workers are displaced into unemployment according to a Poisson process with parameter $\delta > 0$ due to job destruction shocks. When unemployed, workers receive a constant payoff b per period. Workers search for jobs only when unemployed (that is, there is no on-the-job search).

⁷The single-crossing condition, which ensures that the indifference curves of high- and low-ability workers in wage-effort space intersect only once, is fulfilled even though cost functions are homogeneous (see below).

Firms are homogeneous ex ante and each consist of one job, which is either filled or vacant. They must pay a cost $k \geq 0$ for keeping an open vacancy.⁸ The output of a job that is filled with a worker of type i who exerts effort e is equal to $y_i = p_i e$. That is, for any given level of effort $e > 0$, output of a high-ability worker is greater than output of a low-ability worker, $p_H e > p_L e$. Output is observable by both workers and firms, and contractible.

Each firm wants to hire exactly one worker, and each worker is searching for one job. Both firms and workers are risk-neutral. The objective of workers and firms is to maximize their present discounted value of expected income. Future values are discounted at rate r .

Workers are perfectly informed about their type, but a firm does not know its worker's ability at the hiring stage and only learns about it over time at exogenous Poisson rate $\psi > 0$.^{9,10}

2.2 The matching framework

There is an aggregate matching function $m(v, u)$ that gives the number of matches between searching firms and workers (divided by the fixed total labor force equal to 1) each period. The matching function is assumed to be non-negative, increasing and concave in both arguments, v , the number of vacancies, and u , the number of unemployed workers, and homogeneous of degree one. There are no matches, if there are no vacancies or unemployed workers, $m(0, u) = m(v, 0) = 0$. Vacancies are filled at rate $q(\frac{v}{u}) = m(1, \frac{u}{v}) = m(v, u)/v$, with $\partial[q(\frac{v}{u})]/\partial[\frac{v}{u}] < 0$. Unemployed workers find jobs at rate $f(\frac{v}{u}) = \frac{v}{u} q(\frac{v}{u}) = m(v, u)/u$, with $\partial[\frac{v}{u} q(\frac{v}{u})]/\partial[\frac{v}{u}] > 0$. The ratio $\frac{v}{u}$ is a measure of labor market tightness and is denoted in the following by $\theta \in [0, \infty]$.

⁸This cost, through free entry, determines the endogenous measure of jobs.

⁹The implicit assumption here is that it is too costly for firms to observe the effort (and implied ability) of a worker immediately.

¹⁰There is evidence for asymmetric (Kahn, 2013) as well as symmetric (Schönberg, 2007) learning in the labor market. In this paper, I focus on the former.

A submarket consists of a set of firms posting vacancies and a set of workers searching for those vacancies. I assume that workers and firms meet pairwise (i.e., there is a bilateral matching technology). Each firm in a given submarket posts (and commits to) a single contract s_i , $i \in \{H, L\}$, that is designed for a worker of type i and specifies wages and outputs before and after firm learning, $s_i = (w_i, y_i, \bar{w}'_i, \bar{y}'_i, \underline{w}'_i, \underline{y}'_i) \in S$, where $S \subset \mathbb{R}_+^6$ is the set of contracts.¹¹ Workers decide where to search after observing all posted contracts, conditional on the search behavior of other workers. They are free to search for contracts designed for workers of a different type. Before firm learning (i.e., during probation), workers with contract s_i receive wage w_i conditional on output y_i . After the type of a worker is revealed, he receives wage \bar{w}'_i for \bar{y}'_i , if he reported his type truthfully, and he receives \underline{w}'_i for \underline{y}'_i , if he misreported his type.¹² Wages are subject to a minimum wage regulation, as described in Section 4.1 below.¹³ Any contract s_i is associated with market tightness $\theta(s_i) = \frac{v(s_i)}{u(s_i)}$ and a share of workers applying to s_i that are of type i , $\alpha_i(s_i) \geq 0$, where $\sum_i \alpha_i(s_i) = 1$. Therefore, in submarket s_i , the probability that a worker matches with a firm is independent of type and given by $\theta(s_i)q(\theta(s_i))$, and the probability that a firm matches with a worker of type H (L) is given by $\alpha_H(s_i)q(\theta(s_i))$ ($\alpha_L(s_i)q(\theta(s_i))$). The probabilities of matching are determined in equilibrium.

2.3 Bellman equations

Consider a firm that posts a contract $s_i = (w_i, y_i, \bar{w}'_i, \bar{y}'_i, \underline{w}'_i, \underline{y}'_i)$ designed for a worker of type i , paying wage w_i for producing output y_i before firm learning, and wage \bar{w}'_i (\underline{w}'_i) for output \bar{y}'_i (\underline{y}'_i) after firm learning in the case where the worker truthfully reported

¹¹The same outcome would obtain, if firms posted menus $[s_H, s_L]$.

¹²Since my model allows for both adverse selection and learning, I need to distinguish between the after-learning wage and output of workers whose type matches the contract type ('truth-telling') and those of workers whose type does not match the contract type ('misreporting'). This is similar to [Carrillo-Tudela and Kaas \(2015\)](#), whose notation I borrow. However, unlike the latter, contracts in my model specify not only wages but also outputs. The two models are also very different in other important respects, as described in the introduction.

¹³Minimum wages ensure that low-ability workers have an incentive to imitate high-ability workers, i.e., that there is adverse selection.

(misreported) his type.¹⁴ The firm's expected value of a filled job of type i is $J(s_i)$. After the firm learns the worker's type, the wage and output may change, and the firm receives an expected continuation value of $J'(s_i)$. The expected value of a vacant job is V .¹⁵ These values are given implicitly by the following Bellman equations:

$$rJ(s_i) = y_i - w_i + \delta[V - J(s_i)] + \psi[J'(s_i) - J(s_i)], \quad (1)$$

$$rJ'(s_i) = \alpha_i(\bar{y}'_i - \bar{w}'_i) + (1 - \alpha_i)(\underline{y}'_i - \underline{w}'_i) + \delta[V - J'(s_i)], \quad (2)$$

$$rV = -k + q(\theta(s_i))[J(s_i) - V]. \quad (3)$$

Equation (1) shows that the expected value for a firm that offers a job of type i includes the firm's profit, $y_i - w_i$, where $y_i = p_i e_i$, plus the expected loss, if the match is destructed and the job becomes vacant, which happens at rate δ , plus the expected change in the job's value after the firm has learned the worker's type and the output and wage may change, which happens at rate ψ .¹⁶ Equation (2) shows that the expected continuation value of a job of type i , after the firm has learned the worker's type, equals the new expected profit, $\alpha_i(\bar{y}'_i - \bar{w}'_i) + (1 - \alpha_i)(\underline{y}'_i - \underline{w}'_i)$, $\bar{y}'_i = p_i e'_i$, $\underline{y}'_i = p_j e'_j$, plus the loss in case of job destruction. Equation (3) shows that the expected value of a vacant job of type i includes the cost of posting a vacancy (e.g., recruiting costs), k , plus the expected gain of filling the job (with a worker of type i or j), which happens with probability $q(\theta(s_i))$. A worker of type i obtains an expected value of $W_i(s_i)$ in a job with contract i , and an expected value of $W_i(s_j)$ in a job with contract $j \neq i$. In the former case, he chooses effort e_i , and in the latter case, he deviates to effort $\frac{p_j}{p_i} e_j$, so that the firm takes him to be a type- j worker, observing output $y_j = p_j e_j = p_i(\frac{p_j}{p_i} e_j)$, and pays him the wage w_j .¹⁷

¹⁴In the following, I call the job of a firm that posts contract s_i a 'job of type i '.

¹⁵Free entry ensures that the value of a job is equal to zero independent of type.

¹⁶I assume here that the match surplus after firm learning is strictly positive, such that workers and firms do not separate from a match voluntarily (also see footnote 28). In Section 5.4, I consider the case of voluntary separations.

¹⁷The difference in efforts that workers of different abilities are required to undertake in order to earn a given wage ensures that the marginal rate of substitution between effort and the wage (at any given

After the firm learns the worker's type, a worker of type i exerts effort e'_i and receives a continuation value of $W'_i(s_i)$ ($W'_i(s_j)$) in a job of type i ($j \neq i$). The worker receives an expected value of U_i , if unemployed. The corresponding Bellman equations are as follows:

$$rW_i(s_i) = w_i - c_i(s_i) + \delta[U_i - W_i(s_i)] + \psi[W'_i(s_i) - W_i(s_i)], \quad (4)$$

$$rW_i(s_j) = w_j - c_i(s_j) + \delta[U_i - W_i(s_j)] + \psi[W'_i(s_j) - W_i(s_j)], \quad (5)$$

$$rW'_i(s_i) = \bar{w}'_i - c'_i(s_i) + \delta[U_i - W'_i(s_i)], \quad (6)$$

$$rW'_i(s_j) = \underline{w}'_j - c'_i(s_j) + \delta[U_i - W'_i(s_j)], \quad (7)$$

$$rU_i = b + \max[\theta(s_i)q(\theta(s_i))(W_i(s_i) - U_i), \theta(s_j)q(\theta(s_j))(W_i(s_j) - U_i)]. \quad (8)$$

Equation (4) shows that the expected value of employment of a worker of type i in a job with contract i includes the wage w_i minus the cost of effort $c_i(s_i) = c(e_i)$ plus the expected loss of a separation to unemployment, which happens at rate δ , plus the expected gain after the firm has learned the worker's type and his wage is \bar{w}'_i , which happens at rate ψ . The expected value of a worker of type i being employed in a job with contract j includes the wage w_j minus the cost of effort $c_i(s_j) = c(\frac{p_j}{p_i}e_j)$ instead, while his expected value after firm learning includes the post-learning wage \underline{w}'_j minus the cost $c'_i(s_j) = c'_i(s_i) = c(e'_i)$, according to equations (5) and (7), respectively.¹⁸ Equation (8) shows that the expected value of unemployment for a worker of type i includes unemployment income b plus the option value of searching. The latter consists of the possibility of meeting a firm with a job i at rate $\theta(s_i)q(\theta(s_i))$, or the possibility of meeting a firm offering a job j at rate $\theta(s_j)q(\theta(s_j))$, times the expected increase in value associated with the offers, respectively.¹⁹ The arrival rates for workers of type i depend

effort and wage) is greater for L - than for H -workers. Therefore, the single-crossing condition is fulfilled, even though effort cost functions are the same.

¹⁸Post-learning effort, e'_i , is chosen to maximize output of a worker of type i net of his cost of effort and, therefore, is the same regardless of the type of contract (also see equations (37)-(38) below).

¹⁹In principle, offers may also be rejected, but we are interested in situations where $W > U$, and

not only on their own search behavior and that of firms, but also on the search behavior of workers of type $j \neq i$. In particular, if the incentive constraint of workers of type i is fulfilled, they will search for jobs of type $i \neq j$ (separating equilibrium). The incentive constraint for a worker of type i requires that the option value of searching for a job of type i is at least as large as that of searching for a job of type j , that is:

$$\theta(s_i)q(\theta(s_i))(W_i(s_i) - U_i) \geq \theta(s_j)q(\theta(s_j))(W_i(s_j) - U_i). \quad (9)$$

It describes the main trade-off faced by a worker of type i when searching for a job. Worker i may find a job with contract j with greater probability $\theta(s_j)q(\theta(s_j))$ than a job with contract i . Conditional on finding the job, he potentially earns a higher wage w_j during the probation period, but also has to exert a potentially greater level of effort $\frac{p_j}{p_i}e_j$ compared to a job of type i . Furthermore, his wage after probation, \underline{w}'_j , may be lower. Worker i will search for a job of type i and self-select into the right contract when the expected gain from searching for a job j (instead of job i) does not exceed the expected cost. He may mimick the other type j , otherwise.²⁰

3 Perfect information equilibrium

In the following, I determine equilibrium contracts in the case of perfect information, where both the worker and the firm know the worker's type from the time of hiring. This part is closely related to [Moen \(1997\)](#), which I refer to for details.²¹ It serves as a benchmark for the case of imperfect information, where firms do not know a worker's true type initially and only learn about it gradually over time (to be discussed in the next section).

With perfect information, both the worker and the firm know the worker's type. Firms

²⁰ $J > V$, so there is a strictly positive match surplus.

²¹However, pooling is not an equilibrium here, as discussed in Section 4.

²²In [Moen \(1997\)](#), both workers and firms are homogeneous, whereas workers are heterogeneous here.

can condition wages and outputs on a worker's type from the time of hiring (i.e., there is no probation period) and post contracts $s_i^* = (w_i^*, y_i^*)$, where $y_i^* = p_i e_i^*$. In this case, we have $\psi = 0$, $J(s_i^*) = J'(s_i^*) \equiv J_i^*$, $V \equiv V^*$, $W_i(s_i^*) = W_i'(s_i^*) \equiv W_i^*$, $U_i \equiv U_i^*$, and $\theta(s_i^*) \equiv \theta_i^*$.²² The Bellman equations for firms and workers are given by:

$$rJ_i^* = y_i^* - w_i^* + \delta[V^* - J_i^*] \quad (10)$$

$$rV^* = -k + q(\theta_i^*)[J_i^* - V^*], \quad (11)$$

$$rW_i^* = w_i^* - c(e_i^*) + \delta[U_i^* - W_i^*] \quad (12)$$

$$rU_i^* = b + \theta_i^* q(\theta_i^*)(W_i^* - U_i^*), \quad (13)$$

where $\theta_i^* = \frac{v_i^*}{w_i^*}$.

Firms in submarket i post contracts that maximize J_i^* subject to worker value U_i^* and the free-entry condition $V^* = 0$. This results in unique wages

$$w_i^* = y_i^*(e_i^*) - \frac{r + \delta}{q(\theta_i^*(e_i^*))} k \quad (14)$$

and labor market tightness $\theta_i^*(e_i^*)$ ²³, which is implicitly given by

$$k = q(\theta_i^*)\varepsilon(\theta_i^*)S_i^*(e_i^*), \quad (15)$$

where

$$S_i^*(e_i^*) = \frac{y_i^*(e_i^*) - c(e_i^*) - b}{r + \delta + \theta_i^* q(\theta_i^*)(1 - \varepsilon(\theta_i^*))} \quad (16)$$

is the match surplus and $\varepsilon(\theta_i^*) = \frac{\partial(\theta_i^* q(\theta_i^*))}{\partial \theta_i^*} / \frac{\theta_i^* q(\theta_i^*)}{\theta_i^*} \in [0, 1]$ is the elasticity of the job

²² $W_i(s_j^*)$ is not relevant in the case of perfect information, since workers of type i cannot pretend to be a different type.

²³Below, I use θ_i^* as short-hand notation for $\theta_i^*(e_i^*)$.

finding rate of workers, $\theta_i^* q(\theta_i^*)$, with respect to labor market tightness, θ_i^* .²⁴

Figure 1 illustrates the equilibrium characterized by equations (15) and (16), depicted in curves ZZ and SS, respectively. An increase in output net of the cost of effort, $y_i^*(e_i^*) - c(e_i^*)$, increases the value of the match according to (16), shifting up curve SS in the graph. In consequence, assuming $\frac{\partial \varepsilon(\theta_i)}{\partial \theta_i} \leq 0$, θ_i^* increases, and the arrival rate for workers (firms), $\theta_i^* q(\theta_i^*)$ ($q(\theta_i^*)$), increases (decreases). The net wage, $w_i^* - c(e_i^*)$, increases²⁵, and the unemployment rate, u_i^* , decreases (see equations (18) below). And vice versa.

Steady-state turnover implies that the flow into employment equals the flow out of employment:

$$\theta_H^* q(\theta_H^*) u_H^* = \delta (a_H - u_H^*) \quad \text{and} \quad \theta_L^* q(\theta_L^*) u_L^* = \delta (a_L - u_L^*). \quad (17)$$

The steady-state rates of unemployment for both types of workers in case of perfect information are, therefore, given by

$$\frac{u_H^*}{a_H} = \frac{\delta}{\delta + \theta_H^* q(\theta_H^*)} \quad \text{and} \quad \frac{u_L^*}{a_L} = \frac{\delta}{\delta + \theta_L^* q(\theta_L^*)}, \quad (18)$$

which decrease in θ_H^* and θ_L^* , respectively.

Let us next determine optimal efforts. Since worker types are observable, wage contracts can be made contingent on a worker's effort, which can be directly inferred from

²⁴This can be derived by using (10)-(11) together with $V^* = 0$ to solve for w_i^* and substituting this solution in an expression for U_i^* derived from (12)-(13), which is maximised with respect to θ_i^* . Note that it does not matter whether contracts are posted by firms or workers.

²⁵To see this, use (15) and (16) to substitute for $\frac{k}{q(\theta_i^*(e_i^*))}$ in (14) and subtract $c(e_i^*) + b$ from both sides. The result follows from the fact that $\frac{(r+\delta)\varepsilon(\theta_i^*)}{r+\delta+\theta_i^* q(\theta_i^*)(1-\varepsilon(\theta_i^*))}$ decreases in ε , if $\varepsilon' \leq 0$.

observable output. The optimal effort is the one that maximizes the match surplus (16):

$$e_i^* = \arg \max_e p_i e - c(e). \quad (19)$$

Therefore, under perfect information, wage contracts $(w_i^*, y_i(e_i^*))$ specify outputs $y_i^* = p_i e_i^*$, where efforts e_i^* are implicitly given by

$$p_i = \frac{\partial c(e)}{\partial e}. \quad (20)$$

From $p_H > p_L$ and $\partial^2 c(e)/\partial e^2 > 0$, it follows that high-ability workers exert greater effort than low-ability workers.

Proposition 1 *Perfect information.* *In the case of perfect information, equilibrium consists of the value functions J_i^*, V^*, W_i^*, U_i^* , $i \in \{H, L\}$, that satisfy the Bellman equations (10)-(13) and the free-entry condition $V^* = 0$, where efforts e_i^* satisfy (20), labor market tightness θ_i^* satisfies (15)-(16), and wages w_i^* satisfy (14).*

Proof. Analogously to Moen (1997)²⁶, competitive equilibrium is defined as an allocation in which firms in a given submarket cannot be made strictly better off without making workers worse off, and vice versa. Since expected firm and worker values can be written as:²⁷

$$J_i^* = \varepsilon(\theta_i^*) S_i^*(e_i^*), \quad (21)$$

$$W_i^* - U_i^* = (1 - \varepsilon(\theta_i^*)) S_i^*(e_i^*), \quad (22)$$

equilibrium efforts satisfy (20), thus maximizing the match surplus, S_i^* , which, together with labor market tightness, θ_i^* , is given by (15)-(16).²⁸

²⁶See Section III.C.

²⁷Equation (21) follows from $k = q(\theta_i^*) J_i^*$ (due to $V^* = 0$) together with (15). Equation (22) follows from (12)-(13), substituting for wages w_i^* using (14) and (15).

²⁸I assume in the following parameter values such that $p_L e_L^* - c(e_L^*) - b > 0$, which implies that both high- and low-ability workers are employable, consistent with footnote 19 (compare (16) together with (21) and (22)). However, I also consider the case where $p_L e_L^* - c(e_L^*) - b \leq 0$ in Section 5.4 below.

Corollary 1. *With perfect information, the match surplus and, in turn, the net wage and the job finding rate of high-ability workers is greater than that of low-ability workers.*

See (14)-(16) together with (20) and the fact that $p_H > p_L$.

4 Imperfect information equilibrium

In the case of imperfect information, firms do not know a worker's true type at the time of hiring. They can, however, screen workers via the choice between different contracts. In equilibrium, firms post profit-maximizing contracts subject to free entry, and workers direct their search to their preferred contract, conditional on the contracts posted and (beliefs about) the search behavior of other workers. I define the (steady-state) equilibrium as follows.²⁹

Definition 1. Equilibrium: A competitive search equilibrium is a set of expected firm and worker values, $J(s), J'(s), V, W_i(s), W'_i(s), U_i(s), i \in \{H, L\}$, tightness $\theta(s)$, efforts $e_i(s), e'_i(s)$, and market composition, $\alpha_i(s)$, defined over S , a cumulative distribution function $\lambda(s)$, and a set of posted contracts $S^P \subset S$ that satisfy the following conditions:

(i) firms' profit maximization and free entry,

$$q(\theta(s)) \sum_i \alpha_i(s) J(s) - k \leq 0 \quad \forall s \in S,$$

with equality, if $s \in S^P$;

(ii) workers' maximization,

$$U_i \geq U_i(s) \quad \forall s \in S \text{ and } i \in \{L, H\},$$

with equality, if $\theta(s) < \infty$ and $\alpha_i(s) > 0$, where

$$rU_i = \max_{s \in S^P} rU_i(s) = b + \max_{s \in S^P} \theta(s) q(\theta(s)) \frac{[(r+\delta)(w_i(s)-c_i(s))+\psi(\bar{w}'_i(s)-c'_i(s))-b(r+\delta+\psi)]}{(r+\delta+\psi)(r+\delta+\theta(s)q(\theta(s)))},$$

(iii) market clearing,

$$\int_{S^P} \frac{\alpha_i(s)}{\theta(s)} d\lambda(s) \leq a_i \quad \forall i,$$

²⁹This is a dynamic version of the equilibrium defined in [Guerrieri et al. \(2010\)](#).

with equality, if $rU_i - b > 0$;

(iv) and unemployment,

$$\theta(s)q(\theta(s))u(s) = \delta(\sum_i \alpha_i(s)a_i - u(s)) \quad \forall i.$$

The first set of conditions (i) determines wages in each submarket. Given wages, (ii) determines the corresponding values of labor market tightness θ and efforts e and e' . According to (iii), all type- i workers apply to some contract, if the expected gain from employment (see equation (8)) is strictly positive. Finally, (iv) determines the unemployment rate in each submarket.³⁰

4.1 Separating equilibrium

In separating equilibrium, workers of type i apply for the contract designed for type i , $s_i^S = (w_i^S, y_i^S, \bar{w}_i^{tS}, \bar{y}_i^{tS}, \underline{w}_i^{tS}, \underline{y}_i^{tS})$, where $y_i^S = p_i e_i^S$, $\bar{y}_i^{tS} = p_i e_i^{tS}$, $\underline{y}_i^{tS} = p_j e_j^{tS}$, such that $\alpha_i(s_i) = 1$ and $\alpha_j(s_i) = 0 \quad \forall i \neq j$. Their probability of matching with a firm differs, $\theta(s_i^S) \equiv \theta_i^S$, and, conditional on matching, they produce different levels of output and earn different average expected wages.³¹ In this case, the expected value functions of firms and workers are given by $J(s_i^S) \equiv J_i^S$, $J'(s_i^S) \equiv J_i^{tS}$, $V \equiv V^S$, $W_i(s_i^S) \equiv W_i^S$, $W_i'(s_i^S) \equiv W_i^{tS}$, $U_i \equiv U_i^S$:

$$rJ_i^S = y_i^S - w_i^S + \delta[V^S - J_i^S] + \psi[J_i^{tS} - J_i^S], \quad (23)$$

$$rJ_i^{tS} = \bar{y}_i^{tS} - \bar{w}_i^{tS} + \delta[V^S - J_i^{tS}], \quad (24)$$

$$rV^S = -k + q(\theta_i^S)[J_i^S - V^S], \quad (25)$$

³⁰It states that the inflow of workers into contract s is equal to the corresponding outflow of workers. Below, I show that different types of workers apply for different contracts in equilibrium, which implies that $\sum_i \alpha_i(s) = \alpha_i(s) = 1$. That is, submarket-specific unemployment rates (and wages) are also type-specific.

³¹The wage during probation may be different from the wage after the firm has learned a worker's type, see below.

$$rW_i^S = w_i^S - c(e_i^S) + \delta[U_i^S - W_i^S] + \psi[W_i'^S - W_i^S], \quad (26)$$

$$rW_i'^S = \bar{w}_i'^S - c(e_i'^S) + \delta[U_i^S - W_i'^S], \quad (27)$$

$$rU_i^S = b + \theta_i^S q(\theta_i^S)(W_i^S - U_i^S), \quad (28)$$

where $\theta_i^S = \frac{v_i^S}{u_i^S}$.

Note that, even though firms can distinguish between worker types from the start in a separating equilibrium, the values of filled jobs before and after firm learning, J_i^S and $J_i'^S$, are not necessarily the same. This is because the output that wage contracts must specify to separate workers of a given type from workers of the other type before firm learning may be different from output after firm learning.³² For the same reason, the values of employment before and after firm learning, W_i^S and $W_i'^S$, are not necessarily the same.

As shown below, the separating equilibrium can be derived as the solution to the following optimization problem: for any type $i \in \{H, L\}$,

$$\begin{aligned} \max_{\theta_i^S, e_i^S, e_i'^S} \quad & \theta_i^S q(\theta_i^S)(W_i^S(s_i^S) - U_i^S) \\ \text{s.t.} \quad & q(\theta_i^S)J_i^S = k \quad \text{and} \\ & \theta_L^S q(\theta_L^S)(W_L^S(s_L^S) - U_L^S) \geq \theta_H^S q(\theta_H^S)(W_L^S(s_H^S) - U_L^S). \end{aligned} \quad (29)$$

To solve this problem, I first derive wages from (23)-(25), using $V^S = 0$:³³

$$(r + \delta)w_i^S + \psi\bar{w}_i'^S = (r + \delta)p_i e_i^S + \psi p_i e_i'^S - \frac{k(r + \delta)(r + \delta + \psi)}{q(\theta_i^S)}. \quad (30)$$

Equation (30) jointly determines wages during and after probation, w_i^S and $\bar{w}_i'^S$. Firms can shift wages across periods accordingly without affecting the values of firms or truth-telling workers, which only depend on the match surplus, S_i^S , and on labor market

³²In consequence, workers' wages and firms' profits may not be the same before and after firm learning.

³³Below, I use θ_i^S , c_i^S , $c_i'^S$ as short-hand notation for $\theta_i^S(e_i^S, e_i'^S)$, $c(e_i^S)$, $c(e_i'^S)$, respectively.

tightness, θ_i^S (see equations (34) and (35) below).^{34,35}

Solving (26)-(28) for U_i^S and substituting for w_i^S using (30), the optimization problem with respect to θ_i^S , e_i^S , and $e_i'^S$ is

$$rU_i^S = b + \max_{\theta_i^S, e_i^S, e_i'^S} \frac{\theta_i^S q(\theta_i^S) [(r + \delta)(y_i^S - c_i^S) + \psi(y_i'^S - c_i'^S) - (r + \delta + \psi)b]}{(r + \delta + \psi)(r + \delta + \theta_i^S q(\theta_i^S))} - \frac{\theta_i^S (r + \delta)k}{(r + \delta + \theta_i^S q(\theta_i^S))}. \quad (31)$$

Taking the partial derivative of (31) with respect to θ_i^S and solving for the corresponding first-order condition, we derive an implicit expression for θ_i^S , given e_i^S and $e_i'^S$:

$$k = q(\theta_i^S) \varepsilon(\theta_i^S) S_i^S(e_i^S, e_i'^S), \quad (32)$$

where

$$S_i^S(e_i^S, e_i'^S) \equiv \frac{(r + \delta)(y_i(e_i^S) - c(e_i^S)) + \psi(y_i(e_i'^S) - c(e_i'^S)) - (r + \delta + \psi)b}{(r + \delta + \psi)(r + \delta + \theta_i^S q(\theta_i^S)(1 - \varepsilon(\theta_i^S)))}. \quad (33)$$

This equates firms' vacancy cost to their arrival rate times their share, $\varepsilon(\theta_i^S)$, of the present value of the expected surplus $S_i^S(e_i^S, e_i'^S)$.³⁶

From $k = q(\theta_i^S) J_i^S$ (due to $V^S = 0$) together with (32), it follows that the value of a match for firms, J_i^S , is:

$$J_i^S = \varepsilon(\theta_i^S) S_i^S(e_i^S, e_i'^S). \quad (34)$$

³⁴Wage shifting may, however, have a deterrence effect on imitating workers, as described below.

³⁵This implies that workers and firms can commit to remain in a match with a negative expected continuation value, respectively. Alternatively, we could impose limited-commitment constraints such that wages must be i) sufficiently high for the expected continuation value of workers to be at least as large as the expected value of unemployment, and ii) sufficiently low for the expected continuation value of firms to be non-negative. Then, post-learning wages would be subject to $rU_i^S + c(e_i'^S) \leq \bar{w}_i^S \leq p_i e_i'^S$ and $rU_j^S + c(e_j'^S) \leq w_i^S \leq p_j e_j'^S$. This would not change the results that follow.

³⁶Note that the case of $\psi = \infty$ corresponds to the case of perfect information, as in Moen (1997). In this case, the expressions for w_i^S , θ_i^S , and S_i^S in (30) and (32)-(33) collapse to the respective expressions under perfect information in (14) and (15)-(16). The case of $\psi = 0$ corresponds to the case of imperfect information, as in Guerrieri et al. (2010).

Similarly, we can express the match value for workers, $W_i^S - U_i^S$, as:

$$W_i^S - U_i^S = (1 - \varepsilon(\theta_i^S))S_i^S(e_i^S, e_i'^S). \quad (35)$$

Equations (34) and (35) show that match values for firms and workers can be expressed as shares of the expected match surplus, S_i^S , with the share of firms (workers) increasing (decreasing) in the elasticity of the job finding rate of workers with respect to labor market tightness, $\varepsilon(\theta_i^S)$.

Finally, the steady-state rate of unemployment for the two types of workers is given by

$$\frac{u_H^S}{a_H} = \frac{\delta}{\delta + \theta_H^S q(\theta_H^S)} \quad \text{and} \quad \frac{u_L^S}{a_L} = \frac{\delta}{\delta + \theta_L^S q(\theta_L^S)}. \quad (36)$$

Equilibrium is characterized by equations (30), (32)-(33) and (36) for any given levels of effort before and after firm learning, e_i^S and $e_i'^S$.

Result 1. *In separating equilibrium, with given effort levels e_i^S and $e_i'^S$, $i \in \{H, L\}$, an increase in the firm learning rate, ψ , increases expected values of workers and firms, if and only if output net of the cost of effort is greater after firm learning than before: $y_i(e_i'^S) - c(e_i'^S) > y_i(e_i^S) - c(e_i^S)$. Then, the surplus, S_i^S , and the job finding rate of workers, $\theta_i^S q(\theta_i^S)$, increase, while the unemployment rate, u_i^S , decreases. And vice versa. If $y_i(e_i'^S) - c(e_i'^S) = y_i(e_i^S) - c(e_i^S)$, then the firm learning rate has no effect on labor market outcomes.*

Proof. Expected values of workers and firms, W_i^S , U_i^S and J_i^S , increase in the surplus, S_i^S . For given effort levels e_i^S and $e_i'^S$, the surplus, S_i^S , in turn, increases in the firm learning rate, ψ , if and only if output net of the cost of effort is greater after firm learning than before.³⁷ And vice versa.

Let us next consider equilibrium efforts before and after firm learning, e_i^S and $e_i'^S$. In

³⁷This corresponds to an upward shift of the SS curve in Figure 1.

equilibrium, efforts maximize the surplus, S_i^S , subject to the incentive constraint (9). After firm learning, worker types are observable. Therefore, wage contracts can be conditioned on efforts $e_i'^S$ that maximize the surplus given in equation (33):

$$e_i'^S = \arg \max_e \frac{[(r + \delta)(y_i(e_i^S) - c(e_i^S)) + \psi(y_i(e_i'^S) - c(e_i'^S)) - (r + \delta - \psi)b]}{(r + \delta + \psi)(r + \delta + \theta_i^S q(\theta_i^S)(1 - \varepsilon(\theta_i^S)))}, \quad (37)$$

where θ_i^S is given by (32) and efforts before learning, e_i^S , are given.

As a result, optimal effort after firm learning, $e_i'^S$, is equal to the first-best and given implicitly by

$$p_i = \frac{\partial c(e)}{\partial e}. \quad (38)$$

Result 2. *In separating equilibrium, after firm learning, efforts are equal to first-best levels of effort: $e_H'^S = e_H^*$, $e_L'^S = e_L^*$.*

Proof. Analogously to the case of perfect information, first-best levels of effort after firm learning e_i^* maximize the surplus S_i^S and, therefore, expected worker and firm values, according to equations (32)-(33), for any given levels of effort before learning, e_i^S . It remains to show that a deviation of post-learning efforts $e_i'^S$ from the first best cannot increase the surplus via an effect on e_i^S . Such an effect is possible, if high-ability workers face imitation from low-ability workers, via a decrease in θ_H^S (see the following paragraphs for details).³⁸ However, overall, any change in effort levels always simultaneously increases or decreases S_i^S and θ_i^S according to equations (32)-(33). As a result, equilibrium levels of effort after firm learning are equal to the first best.

From Results 1-2, it follows that an increase in the firm learning rate is beneficial for both workers and firms, if and only if effort before learning deviates from the first-best:

$$e_i^S \neq e_i^*.$$

³⁸In particular, as $e_H'^S$ deviates from the first best, the arrival rate of high-ability jobs, $\theta_H^S q(\theta_H^S)$, decreases, according to equations (32)-(33) together with the fact that $\theta_H^S q(\theta_H^S)$ increases in θ_H^S . This diminishes imitation incentives, moving e_H^S closer to the first best and increasing S_H^S .

Before firm learning, firms cannot distinguish between worker types. Since the expected surplus and, therefore, the arrival rate of a job of type H is greater than that of a job of type L at any given effort e , low-ability workers have an incentive to imitate high-ability workers, unless their required cost of effort during probation is sufficiently high, or their average expected wage is sufficiently low.³⁹ Regarding the latter, note that, in contrast to truth-telling workers, the option value of search for imitating workers depends on the distribution of wages across periods. In particular, consider low-ability workers who search for high-ability jobs. Their option value of search, given in equation (41) below, increases in the wage of high types during probation, w_H^S , and in the wage of misreporting low types after probation, $w_H'^S$. Therefore, to minimize imitation incentives, firms will set w_H^S and $w_H'^S$ equal to the minimum wage, and compensate high-ability workers accordingly in the period after firm learning.⁴⁰ Assume in the following that, at given minimum wages, the incentive constraint (9) for low-ability workers is binding at $e_H^S = e_H^*$. This requires that minimum wages are greater than zero.⁴¹ For purposes of illustration, let us take a stand and assume that the minimum wage differs in high- and low-ability jobs during probation but is restricted to be the same in all periods for truth-telling workers.⁴² More precisely, assume that wages are $w_H^S = \bar{w}_H'^S = w_{H,min}$ and $w_L^S = \bar{w}_L'^S = w_L'^S = w_L^S = w_{L,min}$ where $w_{i,min} = w_i^S = \bar{w}_i'^S$ is derived from (30):

$$w_{i,min} = \frac{(r + \delta)p_i e_i^S + \psi p_i e_i'^S}{r + \delta + \psi} - \frac{k(r + \delta)}{q(\theta_i^S)}. \quad (39)$$

Then, wage contracts are $(w_i^S, y_i^S, \bar{w}_i'^S, \bar{y}_i'^S, w_i'^S, y_i'^S)$ with first-best efforts $e_L^S = e_L'^S = e_L^*$,

³⁹In turn, high-ability workers do not have an incentive to imitate low-ability workers, due to single-crossing (see footnote 17).

⁴⁰This way, firms maximally differentiate the wages of truth-telling and misreporting workers after probation, which maximizes the incentive for truthful reporting (compare Carrillo-Tudela and Kaas (2015)). This does not distort efficiency, unlike the specification of greater output (and, thus, effort) during probation in high-ability jobs.

⁴¹Otherwise, the incentive constraint for low-ability workers would be slack. See (40)-(42) below. Minimum wages must also be greater than those implied by the limited-commitment constraints described in footnote 35, as can be shown numerically.

⁴²I will consider an alternative minimum wage regulation in Section 5.4 below.

$e'_H^S = e_H^*$ as well as e_H^S that is defined to solve the incentive constraint (9) for low-ability workers with equality, given $e_L^S = e'_L^S = e_L^*$.⁴³⁴⁴ That is, the option value of search for low-ability workers when searching for high-ability jobs is equal to their option value of search when searching for low-ability jobs:

$$OVS_L^S(s_H^S) = OVS_L^S(s_L^S), \quad (40)$$

where

$$OVS_L^S(s_H^S) = \frac{\theta_H^S q(\theta_H^S)((r + \delta)(w_H^S - c(\frac{p_H}{p_L} e_H^S) - b) + \psi(w_H'^S - c(e_L'^S) - b))}{(r + \delta + \psi)(r + \delta + \theta_H^S q(\theta_H^S))}, \quad (41)$$

$$OVS_L^S(s_L^S) = \frac{\theta_L^S q(\theta_L^S)((r + \delta)(w_L^S - c(e_L^S) - b) + \psi(\bar{w}_L'^S - c(e_L'^S) - b))}{(r + \delta + \psi)(r + \delta + \theta_L^S q(\theta_L^S))}, \quad (42)$$

$e_L^S = e'_L^S = e_L^*$ is given by (20), $\theta_L^S = \theta_L^*$ is given by (32)-(33), $w_H^S = w_{H,min}$, and $w_L^S = \bar{w}_L'^S = w_H'^S = w_{L,min}$ as given by (39).

Hence, the incentive compatibility constraint (40) can be re-written as follows:

$$\frac{\theta_H^S q(\theta_H^S)((r + \delta)(w_{H,min} - c(\frac{p_H}{p_L} e_H^S) - b) + \psi(w_{L,min} - c(e_L^*) - b))}{(r + \delta + \psi)(r + \delta + \theta_H^S q(\theta_H^S))} = OVS_L^*(s_L^*), \quad (43)$$

where $OVS_L^*(s_L^*) \equiv \theta_L^* q(\theta_L^*)(W_L^*(w_L^*(e_L^*)) - U_L^*)$.

In sum, e_H^S is chosen so that a low-ability worker is just indifferent between finding a job at rate $\theta_H^S q(\theta_H^S)$ and, conditional on finding the job, earning the minimum wage $w_{H,min}$ ($w_{L,min}$) for effort $\frac{p_H}{p_L} e_H^S$ (e_L^*) during (after) probation, and finding a first-best low-ability job at rate $\theta_L^* q(\theta_L^*)$. There are at most two solutions e_{H1} and e_{H2} to (43),

⁴³Compare standard screening models, e.g., [Akerlof \(1970\)](#) and [Rothschild and Stiglitz \(1976\)](#).

⁴⁴Single-crossing ensures that, if one of the two incentive constraints is fulfilled with equality, then the other constraint is fulfilled strictly.

and only the maximum solution represents an equilibrium (see Appendix A).⁴⁵

Figure 2 illustrates an example of a separating equilibrium with an effort of high-ability workers greater than the first-best, $e_H^S = e_{H2} > e_H^*$. It depicts the option values of search for high- and low-ability workers, $OV S_H^S(s_H^S)$, $OV S_L^S(s_L^S)$ and $OV S_L^S(s_H^S)$, as functions of worker effort before firm learning, e . Note that, for a worker of type i , the function of the option value of searching for a contract s_i^S is concave and increasing (decreasing) in e , if $p_i > \frac{\partial c(e)}{\partial e}$ ($p_i < \frac{\partial c(e)}{\partial e}$). It attains its maximum at the first-best level of effort, e_i^* , where $OV S_i^S(s_i^S) = OV S_i^*$, $i \in \{H, L\}$. In separating equilibrium, $e_L^S = e_L^*$ is the effort of low-ability workers, and e_H^S is the effort of high-ability workers. At e_H^S , the option value of search for low-ability workers when searching for H-jobs is equal to their option value when searching for L-jobs.

Result 3. *In separating equilibrium, before firm learning, low-ability workers choose the first-best effort, $e_L^S = e_L^*$, and high-ability workers choose a (sub-)optimal level of effort greater than or equal to the first best, $e_H^S \geq e_H^*$.*

Proof. Equilibrium levels of effort before firm learning maximize the surplus S_i^S and, therefore, expected worker and firm values, according to equations (32)-(33), subject to the incentive constraint (9), for efforts after firm learning equal to the first best, $e_i'^S = e_i^*$ (see Result 2). It follows that low-ability workers choose $e_L^S = e_L^*$. High-ability workers choose $e_H^S = e_H^*$, if the incentive constraint for low-ability workers is slack at $e_H = e_H^*$. Otherwise, their effort is greater than the first best, $e_H^S > e_H^*$, as described above. The greater the benefit for low-ability workers when imitating high-ability workers, the greater must be the output (and implied worker effort) during probation specified in wage contracts for H-jobs to separate high-ability from low-ability workers.

Proposition 2 Imperfect information: Separating Equilibrium.

⁴⁵This does not require any specific assumptions regarding minimum wage regulations apart from the (sufficient) condition that they do not reward misreporting workers, i.e., $w_H'^S \leq \bar{w}_L^S$.

In separating equilibrium, low-ability and high-ability workers are unemployed with probabilities $u_L^S = u_L^*$ and u_H^S , given by (36). When employed, their wage contracts are $(w_i^S, y_i^S, \bar{w}_i^S, \bar{y}_i^S, w_i'^S, y_i'^S)$, $i \in \{H, L\}$, where

$w_H^S = w_{H,min}$, $w_H'^S = w_L^S = \bar{w}_L^S = w_L'^S = w_{L,min}$ as given by (39),

\bar{w}_H^S is (implicitly) given by (30),

θ_i^S is given by (32)-(33),

$$e_H^S = \max \left[e_H^*, e_H : OV S_L^*(s_L^*) = \frac{\theta_H^S q(\theta_H^S)((r+\delta)(w_{H,min}-c(\frac{p_H}{p_L}e_H^S)-b)+\psi(w_{L,min}-c(e_L^*)-b))}{(r+\delta+\psi)(r+\delta+\theta_H^S q(\theta_H^S))} \right],$$

$e_L^S = e_L'^S = e_L^*$ and $e_H'^S = e_H^*$ as given by (38).

The separating equilibrium described in Proposition 2 is the (unique) solution to the constrained optimization problem (29), given $S_i^S > 0 \forall i \in \{H, L\}$. This solution constitutes an equilibrium, since all of the conditions in Definition 1 hold for $\alpha_i(s_i^S) = 1$, $\alpha_j(s_i^S) = 0$, $j \neq i$, $d\lambda(s_i^S) = a_i\theta(s_i^S)$ (see (31)-(34) and (36)).

Note that, as in Guerrieri et al. (2010), a deviation to a pooling contract (see Appendix B) is never optimal here. This is because high-ability workers are more attracted to a deviating separating contract in the sense that they would be willing to accept a lower firm-worker ratio, because they have more to gain from the deviation.⁴⁶⁴⁷ Even though, collectively, high-ability workers might prefer to pool with low-ability workers to avoid costly screening, any individual high-ability worker prefers a separating contract. Therefore, pooling does not constitute an equilibrium.

4.2 Efficiency and the role of firm learning

In separating equilibrium, as characterized above, equilibrium wages, efforts, and market tightness for low-ability workers are equal to their respective first-best levels, $w_L^S = w_L'^S = w_L^*$, $e_L^S = e_L'^S = e_L^*$, $\theta_L^S = \theta_L^*$. The labor market outcome for high-ability workers may

⁴⁶In high-ability jobs, high-ability workers have a smaller cost of effort during probation, and a higher wage after probation compared to low-ability workers.

⁴⁷This depends crucially on the beliefs about the composition of workers searching for contracts that are not offered in equilibrium as imposed by the equilibrium definition.

also be first-best. This is the case, if the gain of imitation for low-ability workers is not greater than their cost⁴⁸, such that the incentive constraint for low-ability workers (9) is not binding at first-best efforts of high-ability workers:

$$OVS_L^S(s_H^*) \leq OVS_L^S(s_L^*). \quad (44)$$

We are interested in the cases where this constraint is violated. Then, the labor market outcome for high-ability workers is inefficient, and firm learning may have an effect on labor market efficiency.⁴⁹ The overall effect of firm learning is composed of a direct positive effect and an indirect negative effect, as summarized in Corollaries 2-3 to Proposition 2 below.

Corollary 2. *In separating equilibrium, faster firm learning increases the effort of high-ability workers during the probation period, if $e_H^S > e_H^*$; it does not affect worker effort, otherwise.*

From Results 2-3, it follows that an increase in the firm learning rate, ψ , does not affect the effort of low-ability workers, nor that of high-ability workers, if $e_H^S = e_H^*$. However, if $e_H^S > e_H^*$, faster firm learning increases the option value of searching for high-ability jobs for low-ability workers, $OVS_L^S(s_H^S)$ (see the left-hand side of equation (43)), while leaving their option value of searching for low-ability jobs unchanged, $OVS_L^S(s_L^S) = OVS_L^*$. In Figure 2, the curve denoted $OVS_L^S(s_H^S)$ shifts up, as ψ increases. There are three different channels. First, the relative arrival rate of high-ability jobs increases, since $\theta_H^S q(\theta_H^S)$ increases in ψ , while $\theta_L^S q(\theta_L^S) = \theta_L^* q(\theta_L^*)$ remains unchanged (see Results 1-2). Second, the wage during probation, w_H^S , increases due to an increase in the expected surplus S_H^S .

⁴⁸The net gain of imitation depends on the difference in productivity between high- and low-ability workers as well as the functional form of the effort cost function (see Section 5.4 below).

⁴⁹Note that an inefficient separating equilibrium is constrained efficient (second best), unless there are sufficiently few low-ability workers such that the match surplus for high types in a pooling contract that treats the two types identically, S^P , exceeds the one under separation, S_H^S (see Appendix B). In this case, the separating equilibrium would be Pareto dominated by the allocation under a pooling contract.

Third, the average expected net wage in H-jobs also increases due to a reduction in the expected duration of the probation period, where the net wage of low-ability workers is lower.⁵⁰ In response to an increase in the benefit of imitation for low-ability workers, the effort of high-ability workers, $e_H^S > e_H^*$, has to increase.

Corollary 3. *The overall effect of firm learning on aggregate efficiency in the labor market is ambiguous, if $e_H^S > e_H^*$; it is zero, otherwise.*

In equilibrium, an increase in the rate of firm learning, ψ , does not affect the expected match surplus of workers who choose their first-best effort both before and after firm learning. However, if $e_H^S > e_H^*$, firm learning increases the inefficiently high effort of high-ability workers even further, which decreases their expected match surplus. In this case, firm learning has two countervailing effects on the expected match surplus of high-ability workers, S_H^S . On the one hand, taking e_H^S as given, S_H^S increases in ψ (direct effect, see Result 1). On the other hand, e_H^S increases and, therefore, S_H^S decreases in ψ , as imitation incentives of low-ability workers are enhanced (indirect effect, see Corollary 2). Depending on which effect dominates, the expected match surplus, S_H^S , and, therefore, aggregate efficiency in the labor market, $a_H S_H^S + a_L S_L^*$, may increase or decrease, as firms learn faster:

$$\frac{dS_H^S}{d\psi} \Big|_{e_H^S \neq e_H^*} = \underbrace{\frac{\partial S_H^S}{\partial \psi}}_{>0} + \underbrace{\frac{\partial S_H^S}{\partial e_H^S}}_{<0} \underbrace{\frac{de_H^S}{d\psi}}_{>0} \geq 0. \quad (45)$$

The overall effect of an increase in firm learning depends on parameter values as well as the functional forms of the matching function and the cost-of-effort function. It also depends, importantly, on the rate of firm learning, ψ . In particular, note that, as

⁵⁰To see that the net wage of low-ability workers in high-ability jobs increases after probation, note that, if $\theta_H^S = \theta_L^S$, then $w_H^S = w_L^S = w_H^*$. Therefore, for the incentive constraint (43) to be fulfilled, e_H^S must be such that $\theta_H^S > \theta_L^S$ and $w_H^S - c(\frac{p_H}{p_L} e_H^S) < w_H^S - c(e_L^*)$.

ψ increases, marginal effects decrease in absolute size due to diminishing returns. In Section 5 below, I numerically simulate the model to quantify the effect of firm learning on the effort of high-ability workers during probation and, in turn, on their expected match surplus.

I find that, if the rate of firm learning is sufficiently low, the effort of high-ability workers during probation is optimal, $e_H^S = e_H^*$, and, therefore, $S_H^S = S_H^*$. In consequence, the match surplus of both high- and low-ability workers equals the surplus that would obtain, if firms had perfect information about workers' ability from the start, and faster firm learning does not affect labor market efficiency. If the rate of firm learning is sufficiently high, imitation incentives exist. However, marginal (direct and indirect) effects of firm learning have decreased to approximately zero.

5 Quantitative analysis

In this section, I calibrate the model above. I first calculate option values of search, as functions of effort during probation, for high- and low-ability workers under imperfect information. Then, I compute equilibrium efforts, and the corresponding expected match surplus of high- and low-ability workers. This allows me to compare the labor market equilibria with perfect and imperfect information. I also perform quantitative comparative statics exercises and calculate the efficiency impact of an increase in the rate at which firms learn about the productivity of their workers. Finally, I test the sensitivity of results to the choice of parameter values and functional forms.

5.1 Parameter values and functional forms

I use parameter values to match U.S. labor market facts as of 2019. The discount rate $r = 0.02$ is set at the annual real U.S. interest rate. The matching function is assumed to be Cobb-Douglas, $m(u, v) = m_0 u^\xi v^{1-\xi}$, where m is the number of jobs formed during one

period, m_0 is the matching constant, u is the number of unemployed workers looking for a job and v is the number of vacant jobs; $\xi = 0.5$ is the matching elasticity with respect to the number of unemployed workers.⁵¹ For the cost of effort, I use a quadratic functional form, $c(e) = e^2$, in the baseline scenario. Table 1 summarizes the parameter values and their respective sources, as well as the functional forms of the arrival rate for firms and workers (as implied by the matching function), $q(\theta)$ and $\theta q(\theta)$, and the cost of effort, $c(e)$. Given these, I target the average unemployment rate in the U.S. in the year 2019 of 3.7% (U.S. Labor Force Statistics (2020)). I also target the unemployment benefit to be 40% of the average wage of employed workers after firm learning, following [Shimer \(2005\)](#). Lastly, I target an average v-u ratio of 0.72 based on [Pissarides \(2009\)](#). I choose the parameter values for the productivities of high- and low-ability workers, p_H and p_L , the unemployment benefit b , the vacancy posting cost k , and the matching constant m_0 that most closely match the three target moments as well as the condition that the expected match surplus of high- and low-ability workers after probation is strictly positive.⁵² The Poisson rate of firm learning, ψ , is set at 100 in the baseline scenario, implying that, within a 10-year period, there are on average 10 events (e.g., firm inspections) per year that reveal a worker's type.⁵³ In the next subsection, I derive the labor market equilibria under perfect and imperfect information. Subsection 5.3 evaluates changes in the firm learning rate, and their effects on equilibrium efficiency. The sensitivity analysis in Subsection 5.4 assesses the importance of assumptions regarding the relative productivity of high- and low-ability workers, the effort cost function, and the minimum wage regime. It also considers the case of optimal separations after firm learning.

⁵¹Therefore, the arrival rate is $\theta q(\theta) = m_0 \theta^{1-\xi} = m_0 \theta^{0.5}$ for workers and $q(\theta) = m_0 \theta^{-\xi} = m_0 \theta^{-0.5}$ for firms.

⁵²This results in endogenous variable values close to their target values: an average unemployment rate of 4%, an average replacement rate of 0.3, and an average v-u ratio of 0.7.

⁵³In this case, the probability for a worker's type to be revealed within three months is $1 - e^{-10/4} = 92\%$. See Section 5.3 below for how results vary with the rate of firm learning.

5.2 Baseline results

Figure 3 plots the option values of search for high- and low-ability workers under imperfect information, $OV S_H^S(s_H^S)$, $OV S_L^S(s_L^S)$ and $OV S_L^S(s_H^S)$, as functions of worker effort before firm learning, e , replicating Figure 2.⁵⁴ $OV S_H^S(s_H^S)$ and $OV S_L^S(s_L^S)$ are concave functions with maximum values at efforts $e_H^* = 1.5$ and $e_L^* = 1$, respectively, where $OV S_H^S(s_H^S) = OV S_H^*$ and $OV S_L^S(s_L^S) = OV S_L^*$. The figure shows that low-ability workers benefit from imitating high-ability workers, $OV S_L^S(s_H^S) > OV S_L^S(s_L^S)$, only if effort and, therefore, the cost of imitation, $c(\frac{p_H}{p_L}e) - c(e)$, is sufficiently small. As e increases, the cost of imitation increases, such that $OV S_L^S(s_H^S)$ is smaller than $OV S_L^S(s_L^S)$ for sufficiently large values of e . At chosen numerical values, the maximum level of effort at which imitation pays off for low-ability workers – e_H^S as defined in the incentive compatibility constraint (40) – is equal to 1.5018 and, therefore, only marginally greater than $e_H^* = 1.5$. As a result, the labor market equilibrium under imperfect information is close to the first best. Table 2 summarizes equilibrium values under perfect and imperfect information.

5.3 Firm learning

Let us analyze the effect of changes in the rate of firm learning, ψ , in our numerical example. Figure 4 shows the effort of high-ability workers, $e_H^S(\psi)$, at which the incentive constraint for low-ability workers in separating equilibrium (IC_L , see equation (40)) is binding, as a function of ψ . The figure shows that e_H^S increases in ψ , as expected. It becomes greater than the first-best effort, $e_H^* = 1.5$, for values of ψ greater than around 100. For firm learning rates below this threshold, the IC_L is not binding, and faster learning has no effect on welfare, which is equal to the first best (also see the discussion of Corollary 2). At $\psi = 100$, welfare is close to the first best⁵⁵, and the overall effect of

⁵⁴The value of effort after learning is given and equal to its first-best, $e_i'^S = e_i^*$.

⁵⁵ $S_H^S - S_H^* = -8.7 \times 10^{-10}$.

firm learning on efficiency is close to zero⁵⁶. For rates above this threshold, the IC_L is binding, and faster learning increases the effort of high-ability workers during probation. This serves to decrease overall efficiency (at a decreasing rate), even as the duration of probation is reduced (not shown). In the limit, where $\psi = \infty$, welfare is again equal to the first best, and the effect of firm learning is zero.

5.4 Sensitivity analysis

In the following, I test for the sensitivity of the above results with respect to the productivity of high- and low-ability workers, the functional form of the effort cost function, $c(e)$, and the minimum wage regime. An increase in the relative productivity of workers, p_H/p_L , increases the gap between the first-best efforts of high- and low-ability workers, which increases the difference in the expected match surplus and job arrival rates of high- and low-ability jobs. This, in turn, increases the benefit of imitation for low-ability workers. However, the cost of imitation, $c(\frac{p_H}{p_L}e) - c(e_L^*)$, also increases. Assuming an increase in the relative productivity of H- and L-workers from 1.5 to 2, I find that the cost of imitation for low-ability workers increases by more than its benefit.⁵⁷ Now, low-ability workers prefer their unconstrained contract to the unconstrained contract of high-ability workers. Efforts of both types of workers are optimal, and the labor market is efficient.

Similarly, a decrease in the curvature of the effort cost function increases the gap between the efforts of high- and low-ability workers in the first best and, therefore, the benefit of imitation for low-ability workers. In turn, the cost of imitation may increase. Using a cost function of $c(e) = e^{1.2}$, I again find that the net benefit of imitation decreases.⁵⁸ Therefore, the separating equilibrium is equal to the first best, and firm learning does not affect labor market efficiency.

⁵⁶The exact value, in terms of expression (45), is $\frac{dS_H^S}{d\psi} = 5.7 \times 10^{-12} + (-7.8) \times 10^{-7} \frac{de_H^S}{d\psi} \approx 0$.

⁵⁷The numerical result is available upon request.

⁵⁸The numerical result is available upon request.

Next, I check for the sensitivity of results with respect to the minimum wage regime. For example, we could assume that minimum wages are the same for all workers independent of type, and constant over time. Then, the incentive constraint (43) can be re-written as follows:

$$\frac{\theta_H^S q(\theta_H^S)((r + \delta)(w_{min} - c(\frac{p_H}{p_L} e_H^S) - b) + \psi(w_{min} - c(e_L^*) - b))}{(r + \delta + \psi)(r + \delta + \theta_H^S q(\theta_H^S))} = OVS_L^*(s_L^*), \quad (46)$$

where the minimum wage

$$w_{min} = p_L e_L^* - \frac{k(r + \delta)}{q(\theta_L)} \quad (47)$$

is derived by substituting and solving for $w_{min} = w_L^S = \bar{w}_L^S$ in (30).

Note that imitation incentives may exist even though, in this case, wages for low-ability workers are the same in high- and low-ability jobs. This is because job finding rates are based on the expected match surplus, which is greater for high-ability than for low-ability jobs. The potential distortionary effect of firm learning is now smaller, as it increases imitation incentives (the left-hand side of equation (46)) only via two (rather than three) channels. As before, for high-ability jobs, the job finding rate increases, and the average expected wage increases due to a decrease in the expected duration of probation. However, the wage during probation, w_{min} , is now constant and independent of firm learning. I find that the incentive constraint for low-ability workers now becomes binding only for firm learning rates above around 250.⁵⁹ At the threshold, the overall effect of firm learning on efficiency is again zero.⁶⁰

Finally, it may be interesting to consider the case where firms optimally choose to separate from workers (and vice versa), once they are revealed to be low-ability. This would happen, if, unlike assumed so far, parameter values are such that $p_L e_L^* - c(e_L^*) - b \leq 0$ and, therefore, $S_L^* \leq 0$, and, in turn, $J_L^* \leq 0$ and $W_L^* - U_L^* \leq 0$. In this case, the Bellman

⁵⁹With 25 learning events per year on average over a ten-year period, the firm learns about a worker's type within three months almost with certainty.

⁶⁰The numerical results are available upon request.

equations of imitating workers (compare (5) and (8)) are as follows:

$$rW_i(s_j) = w_j - c_i(s_j) + (\delta + \psi)[U_i - W_i(s_j)], \quad (48)$$

$$rU_i = b + \theta(s_j)q(\theta(s_j))(W_i(s_j) - U_i). \quad (49)$$

Assuming that minimum wages are given again by equation (39), the incentive constraint (9) for low-ability workers is:

$$\frac{\theta_H^S q(\theta_H^S)(w_H^S - c(\frac{p_H}{p_L}e_H^S) - b)}{(r + \delta + \psi + \theta_H^S q(\theta_H^S))} \geq OV S_L^*(s_L^*), \quad (50)$$

where

$$w_H^S = w_{H,min} = \frac{(r + \delta)p_H e_H^S + \psi p_H e_H'^S}{r + \delta + \psi} - \frac{k(r + \delta)}{q(\theta_H^S)}.$$

As in the baseline scenario, faster firm learning increases any given imitation incentives (the left-hand side of equation (50)) via an increase in the job finding rate, $\theta_H^S q(\theta_H^S)$, as well as an increase in the wage during probation, w_H^S , in high-ability jobs due to an increase in the expected match surplus, S_H^S . However, voluntary separations after firm learning make a crucial difference for the third channel of firm learning effects: the average expected net wage for low-ability workers in high-ability jobs now decreases (rather than increases) due to a decrease in the expected duration of probation. I find that, as a result, imitation incentives for low-ability workers increase for sufficiently low rates of firm learning (below around 7), and decrease thereafter (not shown). However, the incentive constraint never becomes binding. In consequence, the separating equilibrium is equal to the first best, and firm learning does not affect labor market efficiency, independent of the rate of firm learning.⁶¹

⁶¹The numerical results are available upon request.

6 Conclusion

This paper studies labor market efficiency in a dynamic model of adverse selection with search frictions. Firms do not know worker ability at the time of hiring but gradually learn about it over time. I find that faster firm learning is unlikely to increase labor market efficiency, for two reasons.

First, if firm learning is slow, low-ability workers do not have an incentive to imitate high-ability workers, despite the fact that information about worker ability is asymmetric. In this case, worker effort is the same, as if firms had perfect information about a worker's type from the time of hiring, and the labor market is efficient independently of the rate of firm learning.

Second, if firm learning is sufficiently fast, imitation incentives exist, and high-ability workers exert an inefficiently high level of effort on the job during probation to separate themselves from low-ability workers. After firm learning, they choose the optimal level of effort. This (direct) effect of firm learning increases the expected match surplus of high-ability workers. However, firm learning also enhances the imitation incentives of low-ability workers, in turn increasing the effort of high-ability workers during probation. This is because, with faster firm learning, the job arrival rate and expected earnings for low-ability workers in high-ability jobs increases. This (indirect) effect of firm learning decreases the expected surplus of high-ability workers. Depending on the relative size of effects, faster firm learning may potentially harm labor market efficiency, decreasing the average expected match surplus and increasing unemployment in equilibrium.

Numerical results show that any non-zero marginal (direct and indirect) effects of firm learning are negligible. In sum, the effect of firm learning is approximately zero. It should be interesting to consider extensions that make imitation feasible already at smaller rates of firm learning.⁶² Then, quantitative effects can be expected to be greater. However,

⁶²This may be the case, if individual output cannot be perfectly observed, or if individual output is not fully determined by worker ability and effort (but also, for example, by a random element such as

any direct positive efficiency effect of firm learning will be attenuated by an upward pressure on effort during probation in high-ability jobs, which diminishes efficiency.

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Figures

Figure 1: Perfect information equilibrium.

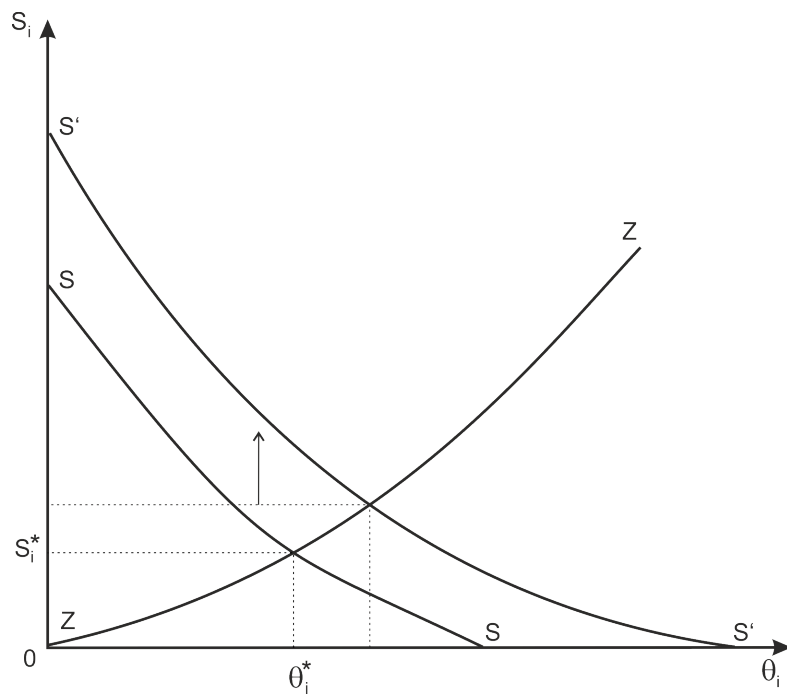


Figure 2: Imperfect information equilibrium.

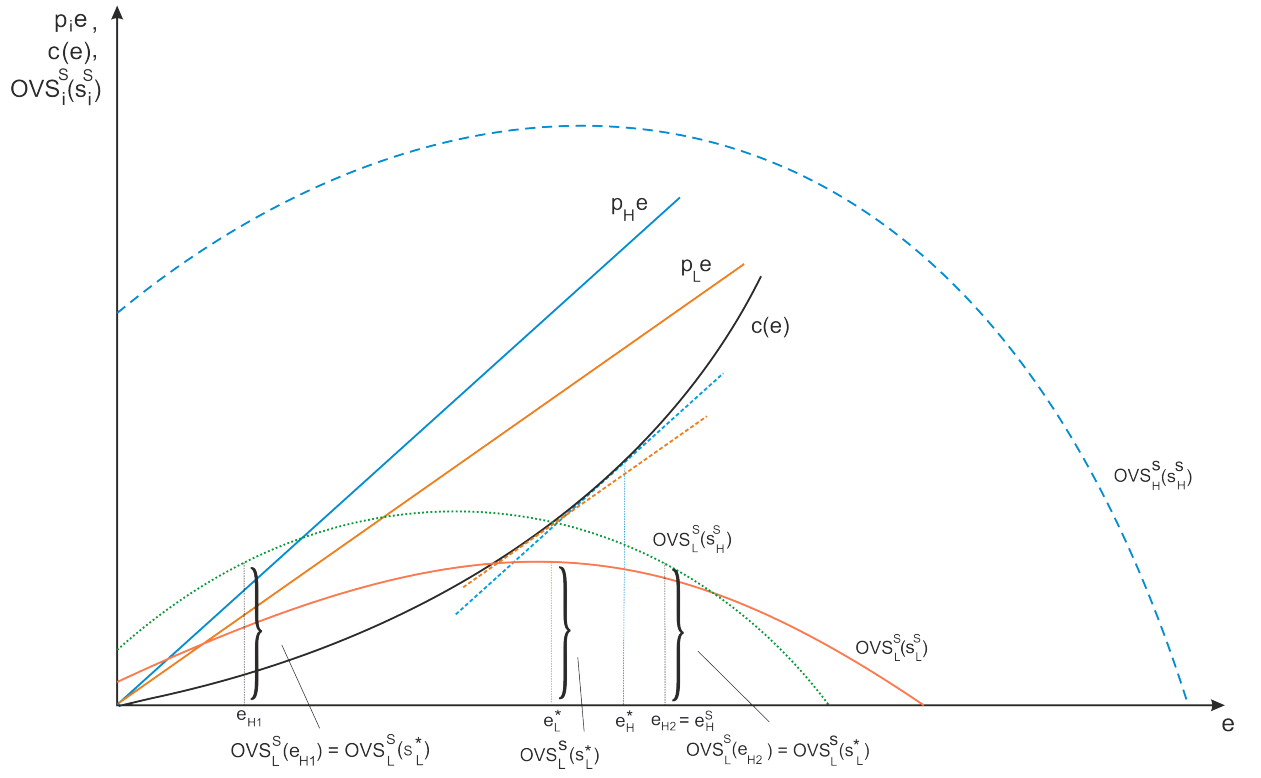


Figure 3: Option values of search

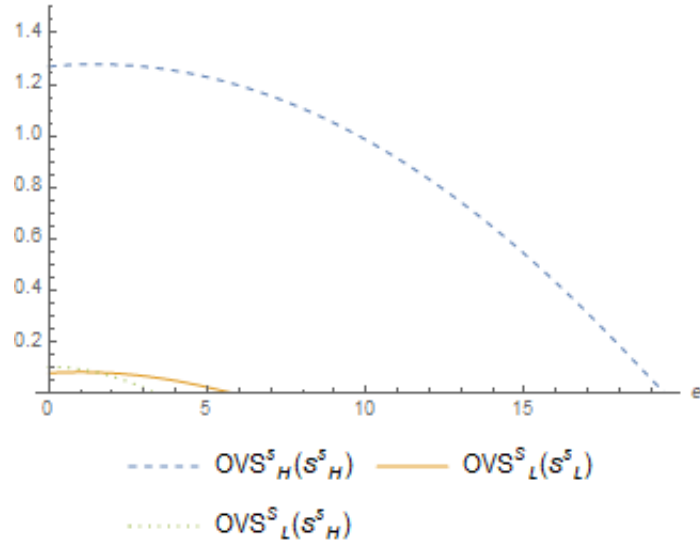
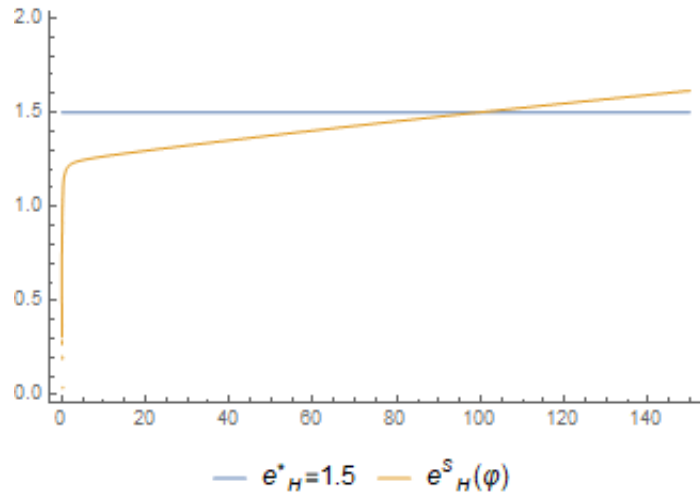


Figure 4: Firm learning and high-ability worker effort during probation



Tables

Table 1: Parameter values and functional forms.

Symbol	Description	Value	Source/Target
$q(\theta)$	Arrival rate for firms	$m_0\theta^{-\xi}$	
$\theta q(\theta)$	Arrival rate for workers	$m_0\theta^{1-\xi}$	
$c(e)$	Worker's effort cost function	e^2	
ψ	Poisson rate of firm learning	30	
r	Discount rate	0.02	U.S. Federal Reserve (2018)
δ	Job separation rate	0.4	Shimer (2005)
α	Share of high-ability workers	0.4	U.S. Labor Force Statistics (2016)
ξ	Matching function elasticity	0.5	Mortensen Nagypal (2007)
p_H	Productivity of a high-ability worker	3	Match targets: Unemployment rate: 0.037 Replacement rate: 0.4 v-u ratio: 0.72 $S_H^* > 0, S_L^* > 0$
p_L	Productivity of a low-ability worker	2	
b	Unemployment benefit	0.9	
k	Vacancy posting cost	0.8	
m_0	Matching constant	12	

NOTE: A matching function elasticity of 0.5 is well within the empirically-supported range reported by [Petrongolo and Pissarides \(2001\)](#).

Table 2: Labor market equilibria under perfect and imperfect information: $\psi = 100$.

Variable	High-ability workers:		Low-ability workers:	
	first-best	separating equilibrium	first-best	separating equilibrium
Effort on the job: e_i^*, e_i^S	1.5	1.5	1	1
Labor market tightness: θ_i^*, θ_i^S	1.59	1.59	0.10	0.10
Match surplus: S_i^*, S_i^S	0.16	0.16	0.04	0.04
Wage: w_i^*, w_i^S	4.46	4.46	1.99	1.99
Unemployment rate: u_i^*, u_i^S	0.01	0.01	0.06	0.06

NOTE: Values of efforts, match surplus, and wages in separating equilibrium denote values during probation, respectively. They only differ from first-best values for high-ability workers from the eighth decimal place onwards (not shown).

Appendix

A. Separating equilibrium: effort of high-ability workers during probation.

Equation (43) has at most two solutions⁶³, and only the maximum solution represents an equilibrium. This is because, among any two solutions e_{H1} and e_{H2} , $e_{H2} > e_{H1}$ (with corresponding contracts s_{H1} , s_{H2} , and labor market tightness θ_{H1} , θ_{H2}), the option value of search for high-ability workers is greater at effort e_{H2} than at effort e_{H1} :

$$\theta_{H2}q(\theta_{H2})(W_H^S(s_{H2}) - U_H^S(s_{H2})) > \theta_{H1}q(\theta_{H1})(W_H^S(s_{H1}) - U_H^S(s_{H1})). \quad (51)$$

To see this, note that equation (43) implies that

$$\begin{aligned} \theta_L^S q(\theta_L^S)(W_L^S(s_L^S) - U_L^S) = \\ \theta_{H1}q(\theta_{H1})(W_L^S(s_{H1}) - U_L^S(s_{H1})) = \theta_{H2}q(\theta_{H2})(W_L^S(s_{H2}) - U_L^S(s_{H2})). \end{aligned} \quad (52)$$

Inequality (51) follows from (52) together with the fact that i) $c(\frac{p_H}{p_L}e_{H2}) - c(e_{H2}) > c(\frac{p_H}{p_L}e_{H1}) - c(e_{H1})$, ii) $\theta_{H2} > \theta_{H1}$, and iii) $\bar{w}_H^S(e_{H2}) - c(e_{H2}') > \bar{w}_H^S(e_{H1}) - c(e_{H1}') > w_H^S - c(e_L^S)$.

Condition i) follows from the convexity of the effort cost function.

Condition ii) follows from the fact that, if $\theta_{H1} = \theta_{H2}$, then $p_H e_{H1} - c(e_{H1}) = p_L e_{H2} - c(e_{H2})$ and, therefore, $w(e_{H1}) - c(e_{H1}) = w(e_{H2}) - c(e_{H2})$.⁶⁴ Since $c(\frac{p_H}{p_L}e_{H2}) > c(\frac{p_H}{p_L}e_{H1})$, equation (52) implies that $\theta_{H2} > \theta_{H1}$.

Condition iii) follows from $\theta_{H2} > \theta_{H1} > \theta_L^S$ together with $w_H^S = w_L^S = \bar{w}_L^S$.⁶⁵

⁶³There may be one or two solutions, or none. See Figure 4 for numerical results.

⁶⁴For the latter, use (32) and (33) to substitute for $\frac{k}{q(\theta_i^S)}$ in (30) and subtract $c(e_i^S)$ from both sides.

⁶⁵For $\theta_{H1} > \theta_L^S$, note that, if $\theta_{H1} = \theta_L^S$, then $(r + \delta)(p_H e_{H1} - c(e_{H1})) + \psi(p_H e_H^S - c(e_H^S)) = (r + \delta)(p_L e_L^S - c(e_L^S)) + \psi(p_L e_L^S - c(e_L^S))$ and, therefore, $(r + \delta)(w_H^S(e_{H1}) - c(e_{H1})) + \psi(\bar{w}_H^S - c(e_H^S)) = (r + \delta)(w_L^S - c(e_L^S)) + \psi(\bar{w}_L^S - c(e_L^S))$. So, for condition (43) to be fulfilled, e_{H1} must adjust until $(r + \delta)(p_H e_{H1} - c(e_{H1})) + \psi(p_H e_H^S - c(e_H^S)) > (r + \delta)(p_L e_L^S - c(e_L^S)) + \psi(p_L e_L^S - c(e_L^S))$, which implies that $\theta_{H1} > \theta_L^S$.

B. Pooling.

In this section, I consider a pooling contract $s^P = (w^P, y^P, w'^P, y'^P, \theta^P)$, which maximizes the expected values of firms and workers when the probability that the worker is a high (low) type is equal to α ($1 - \alpha$). There is only one type of job, and wages, outputs, and profits are the same for both types of workers.⁶⁶

In pooling equilibrium, the expected value functions of firms and workers are given as follows:

$$rJ^P = y^P(e^P) - w^P + \delta[V^P - J^P] + \psi[J'^P - J^P], \quad (53)$$

$$rJ'^P = y'^P(e'^P) - w'^P + \delta[V^P - J'^P], \quad (54)$$

$$rV^P = -k + q(\theta^P)[J^P - V^P], \quad (55)$$

$$rW^P = w^P - c(e^P) + \delta[U^P - W^P] + \psi[W'^P - W^P], \quad (56)$$

$$rW'^P = w'^P - c(e'^P) + \delta[U^P - W'^P], \quad (57)$$

$$rU^P = b + \theta^P q(\theta^P)(W^P - U^P), \quad (58)$$

where $y^P = (\alpha p_H + (1 - \alpha)p_L)e^P$ and $y'^P = (\alpha p_H + (1 - \alpha)p_L)e'^P$.

From equations (53)-(55), together with $V^P = 0$, it follows that⁶⁷

$$w^P = y^P(e^P) + \frac{\psi}{r + \delta}(y'^P(e'^P) - w'^P) - \frac{k(r + \delta + \psi)}{q(\theta^P)}. \quad (59)$$

Solving (56)-(58) for U^P and substituting for w^P using (59), the optimization problem

⁶⁶Note that this is true for the period before as well as after firm learning, since firms post and (commit to) both.

⁶⁷Below, I use θ^P , c^P , c'^P as short-hand notation for $\theta^P(e^P, e'^P)$, $c(e^P)$, $c(e'^P)$, respectively.

is

$$rU^P = b + \max_{\theta^P} \frac{\theta^P q(\theta^P) [(r + \delta)(y^P - c^P) + \psi(y'^P - c'^P) - (r + \delta + \psi)b]}{(r + \delta + \psi)(r + \delta + \theta^P q(\theta^P))} - \frac{\theta^P (r + \delta)k}{r + \delta + \theta^P q(\theta^P)}. \quad (60)$$

The first-order condition gives an implicit expression for θ^P :

$$k = q(\theta^P) \varepsilon(\theta^P) S^P, \quad (61)$$

where

$$S^P \equiv \frac{(r + \delta)(y^P(e^P) - c(e^P)) + \psi(y'^P(e'^P) - c(e'^P)) - (r + \delta + \psi)b}{r + \delta + \theta^P q(\theta^P)(1 - \varepsilon(\theta^P))}. \quad (62)$$

The steady-state rate of unemployment is the same for both types of workers and given by

$$u^P = \frac{\delta}{\delta + \theta^P q(\theta^P)}. \quad (63)$$

Equilibrium is characterized by (59)-(63) for given levels of effort before and after firm learning, e^P and e'^P . Efforts are determined as follows.

The effort after firm learning, e'^P , is the effort that maximizes the expected surplus in pooling equilibrium, taking e^P as given:

$$e'^P = \arg \max_e S^P, \quad (64)$$

and, analogously, for effort before firm learning, e^P . It follows that $e^P = e'^P$ is implicitly defined by

$$\alpha p_H + (1 - \alpha) p_L = \frac{\partial c(e)}{\partial e}. \quad (65)$$

Note that, as the share of low-ability workers, α_L , converges to 0, S^P and θ^P converge

to the first-best levels S_H^* and θ_H^* . In turn, U^P converges to the first-best $U_H^* > U_L^*$. If the incentive constraint for low-ability workers (9) is binding, then $U_H^* > U_H^S$, and the separating equilibrium is Pareto dominated by the pooling allocation.