

Bernales, Alejandro; Garrido, Nicolás; Sagade, Satchit; Valenzuela, Marcela;
Westheide, Christian

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Alejandro Bernales – Nicolás Garrido – Satchit Sagade
– Marcela Valenzuela – Christian Westheide

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A cooperation of the Center for Financial Studies and Goethe University Frankfurt

House of Finance | Goethe University
Theodor-W.-Adorno-Platz 3 | 60323 Frankfurt am Main

Tel. +49 69 798 30080 | Fax +49 69 798 33910
info@safe-frankfurt.de | www.safe-frankfurt.de

Trader Competition in Fragmented Markets: Liquidity Supply versus Picking-off Risk

Alejandro Bernales Nicolás Garrido Satchit Sagade
Marcela Valenzuela Christian Westheide*

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Abstract

By employing a dynamic model with two limit order books, we show that fragmentation is associated with reduced competition among liquidity suppliers and lower picking-off risk of limit orders. Due to these countervailing channels, the impact of fragmentation on liquidity and welfare differs with asset volatility: when volatility is high (low), liquidity and aggregate welfare in a fragmented market are higher (lower) than in a single market. However, fragmentation always shifts welfare away from agents with exogenous trading motives and towards intermediaries. We empirically corroborate our model's predictions about liquidity. Our model reconciles the mixed results in the empirical literature.

Keywords: Fragmentation, Competition, Liquidity, Price Efficiency

JEL Classification: G10, G12

*Alejandro Bernales is at Universidad de Chile (Facultad de Economía y Negocios, Departamento de Administración), email: abernales@fen.uchile.cl. Nicolas Garrido is at London School of Economics, email: N.garrido-sureda@lse.ac.uk. Satchit Sagade is at Goethe University and Research Center SAFE, email: sagade@safe.uni-frankfurt.de. Marcela Valenzuela is at the School of Management, Pontificia Universidad Católica de Chile, email: mavalenb@uc.cl. Christian Westheide is at University of Vienna, email: christian.westheide@univie.ac.at. For helpful comments and discussions we thank Markus Baldauf, Evangelos Benos, Jonathan Brogaard, Sabrina Buti, Jean-Edouard Colliard, Thierry Foucault, Peter Gomber, Thomas Johann, Jan-Pieter Krahnen, Raman Kumar, Jens Lausen, Katya Malinova, Albert Menkveld, Josua Mollner, Robert Neal, Andreas Park, Seongkyu Gilbert Park, Christine Parlour, Giovanni Petrella, Talis Putnins, Uday Rajan, Yves Rannou, Ioanid Rosu, Patrik Sandas, Andriy Shkilko, Erik Theissen, Vincent van Kervel, Clara Vega, Shixuan Wang, Marius Zoican, conference participants at the 2014 Market Microstructure: Confronting Many Viewpoints Conference, 2016 SAFE Market Microstructure Workshop, 2016 CMStatistics Conference, 2016 India Finance Conference, 2017 Securities Markets: Trends, Risks and Policies Conference, 2017 European Retail Investment Conference, 2017 Stern Microstructure Meeting, 2017 FIRS Conference, 2017 Spring International Conference of the French Finance Association, 2017 International Workshop in Financial Markets and Nonlinear Dynamics, 2017 FMA European Conference, BWL2017 Conference, 2017 AsianFA Conference (best paper award), 2017 Verein für Socialpolitik Jahrestagung, 2017 Northern Finance Association Conference, 2017 Annual Central Bank Conference on the Financial Microstructure of Financial Markets, 2017 Annual Meeting of the German Finance Association (DGF), 2018 SFS Cavalcade, 2018 FMA Conference, and seminar participants at University of Birmingham, University of Mannheim, University of Manchester, University of Frankfurt, University of Chile, Pontifical Catholic University of Chile. Sagade and Westheide gratefully acknowledge research support from the Research Center SAFE, funded by the State of Hessen initiative for research LOEWE. Sagade also gratefully acknowledges financial support from Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Projektnummer 329107530. Bernales and Valenzuela acknowledge the support of Fondecyt Project No. 1190477 and No. 1190162, and the support of Instituto Milenio ICM IS130002. The support of the Economic and Social Research Council (ESRC) in funding the SRC is gratefully acknowledged [grant number ES/R009724/1].

1. Introduction

In recent years, equity markets in the United States, the European Union, and elsewhere have evolved from national/regional stock exchanges being the dominant liquidity pools to a fragmented environment, where the same stock trades on multiple limit order books. In such an environment, market outcomes are shaped by trader competition. Specifically, traders compete with each other by dynamically choosing whether to supply or consume liquidity, and under what terms, across fragmented markets. These choices are determined by, among other things, investors' trading motives, market conditions, adverse selection, and security characteristics. Over the last 40 years, the literature examining the effects of fragmentation has provided mixed results. Hence, relevant questions remain unanswered. For example, how does fragmentation shape agents' trading behavior? Is fragmentation good or bad for liquidity? What are the effects of fragmentation on welfare? In this paper, we investigate these questions while allowing for the possibility that the answers may differ depending on market conditions.

We show that two intertwined channels drive traders' behavior, producing opposing effects on liquidity and welfare. First, as time priority applies only within but not across limit order books, traders can circumvent time priority in one order book by submitting an identically-priced limit order in the second order book. In a single market setting, traders can jump ahead of the queue only by improving upon the existing price. This form of queue jumping leads to increased price competition among liquidity providers in a single market setting as compared to a multi market setup. We call this channel the *competition for liquidity provision* channel. Yet, upon a change in the asset's fundamental value, limit orders cannot be instantaneously modified and hence can be picked off by incoming traders. As these traders can potentially choose between multiple stale orders (one on each limit order book), the execution probability of individual limit orders in fragmented markets is lower allowing them, in expectation, more time to update their orders. Hence, compared to a single market setting, a multi-market environment offers more protection against picking-off

risk. We call this channel the *adverse selection channel*.

Our model builds on the single market models of [Goettler et al. \(2005, 2009\)](#), which we extend to a multi-market setting. It is set up as a stochastic trading game in which a single asset, with a stochastically evolving fundamental value, can be traded in two limit order books by diverse (in their private values) and competing agents. Moreover, agents face a cost of delaying their trades, which represents the opportunity cost and the cost of monitoring the market until an order is executed, and incentivizes traders' competition. Traders also have the possibility of reentering the market to revise existing limit orders, though they cannot do so instantaneously when the fundamental value changes. Hence, our theoretical setting allows for picking-off risk.

[Goettler et al. \(2005, 2009\)](#) parameterize their models based on [Hollifield et al. \(2006\)](#), who in turn use order-level data from the early-1990s to compute trader arrival rates, fundamental value volatility, and private value distributions. We solve our model numerically by employing the same parameters used by [Goettler et al. \(2009\)](#) and compare a multi-market environment to a single market setup under two scenarios involving different levels of asset volatility with low (high) values corresponding to a low (high) level of picking-off risk. Furthermore, we also compare our parameters with estimates obtained from order-level data for stocks traded on the London Stock Exchange in January 2015, and confirm that the parameters from [Goettler et al. \(2009\)](#) are not too dissimilar from those in modern markets.

Agents endogenously decide whether to provide or consume liquidity on a limit order book of their choice and in the presence of discrete prices and picking-off risk. Those with an intrinsic motive to trade balance the delay costs associated with limit orders and the immediacy costs of market orders. Those without any intrinsic trading motives generate their gains from liquidity provision or from arbitrage.

We provide evidence consistent with both channels of trader behavior. Consistent with the competition for liquidity provision channel, we find that agents submit limit orders at

the best quotes more frequently in the single market setting. At the same time, consistent with the adverse selection channel, we also find that picking-off risk is lower in the multi-market setting. Furthermore, these two effects interact with each other: in the single market setting, the increased aggressiveness of limit orders resulting from more competition for liquidity provision increases their exposure to the picking-off risk due to a higher probability of such orders becoming mispriced upon a change in the fundamental value of the asset.

The equilibrium impact on liquidity depends on which of two channels dominates. The competition for liquidity provision channel is more important when fundamental value volatility, or picking-off risk, is low. In this setting, liquidity, as measured by quoted and effective spreads, is higher in the single market as compared to the multi-market scenario. Conversely, when fundamental value volatility, or picking-off risk, is high, the adverse selection channel dominates. In this setting, liquidity is higher in the multi-market scenario as compared to the single market. The interplay between these two competing channels likely helps explain the mixed findings in the empirical literature regarding the relationship between liquidity and fragmentation.

Consistent with the liquidity effects, aggregate welfare is marginally higher in the consolidated (fragmented) market as compared to the fragmented (consolidated) market in the low (high) volatility setting. However, independent of the market conditions, intermediaries always extract higher welfare gains in fragmented markets. Conversely, agents with intrinsic trading motives are always better off in consolidated markets. The higher revenues earned by intermediaries in fragmented markets without a commensurate increase in total welfare strongly suggests that costly investments in intermediation capacities, such as the high-speed connections to venues and subscriptions to exchanges' real-time data feeds,¹ are socially wasteful. This raises the question whether restricting fragmentation would lead to improvements in social welfare.²

¹Cespa and Foucault (2013) and Easley et al. (2016) further highlight the adverse effects associated with exchanges providing differential access to market data feeds.

²For instance, due to national and international mergers between exchanges, individual market operators

We empirically test the model implications related to the liquidity effects of fragmentation by using data from the second half of 2012 for German and French large-cap and mid-cap stocks. We employ panel regressions to determine how quoted and effective spreads depend on fragmentation – measured across the primary listing venue (Deutsche Börse or Euronext Paris) and the largest rival exchange, Chi-X – and within-stock variation in volatility. Consistent with our model, we find that, while there is an inverse relation between volatility and liquidity, an increase in fragmentation is associated with lower (higher) quoted and effective spreads on high (low) volatility days. As none of the venues implemented any major changes to their market structure during our sample period, these results lend empirical support to our model.

Our model allows for a potential explanation of the conflicting empirical results observed in the literature. For example, studies find that: (i) fragmentation increases liquidity (see, for example [Boehmer and Boehmer, 2003](#); [Chlistalla and Lutat, 2011](#); [Hengelbrock and Theissen, 2009](#); [Nguyen et al., 2007](#); [He et al., 2015](#); [Fink et al., 2006](#); [Menkveld, 2013](#)); (ii) fragmentation harms liquidity (see, for example [Bessembinder and Kaufman, 1997](#); [Arnold et al., 1999](#); [Hendershott and Jones, 2005](#); [Bennett and Wei, 2006](#); [Nielsson, 2009](#)); and (iii) fragmentation has mixed effects on liquidity (see, for example [Boneva et al., 2016](#); [Haslag and Ringgenberg, 2015](#)). In our model, when volatility is high (low), the picking-off risk (competition for liquidity provision) effect is dominant, and, as a result, fragmentation is associated with smaller (higher) bid-ask spreads. Consistent with this intuition, [O’Hara and Ye \(2011\)](#) find that fragmentation is associated with significantly lower effective spreads only for small-cap stocks in the US.³

Our paper is also related to the theoretical literature on market fragmentation in limit

routinely operate several limit order books. In the United States, the three largest exchange operators – Intercontinental Exchange, Nasdaq OMX, and Cboe – currently operate a total of eleven lit equity exchanges. Our results indicate that such within-operator fragmentation in the absence of, or under minimal, venue competition is harmful.

³Small-cap stocks are associated with higher volatility than large-cap stocks (see, for example, Table 1 in [Brogaard et al., 2014](#)).

order markets.⁴ [Foucault and Menkveld \(2008\)](#) model fee-based competition between two operators and predict that the entry of a second exchange increases consolidated depth and the increased use of smart order routers increases liquidity in the entrant market. However, their model does not feature picking-off risk. [Baldauf and Mollner \(2018\)](#) show that, while exchange competition induces a downward pressure on trading fees, the resulting fragmentation intensifies adverse-selection. However, their model exogenously determines who provides and consumes liquidity and does not feature competition for liquidity provision. In other studies, [Parlour and Seppi \(2003\)](#) examine competition between a specialist market and a pure limit order book, [Pagnotta and Philippon \(2018\)](#) investigate the joint role of trading fees and speed of market access across competing venues, and [Chao et al. \(2018\)](#) focus on the role of tick sizes on the dispersion of fee schedules in fragmented markets.

In contrast to these studies, we focus exclusively on trader competition. Our model simultaneously features competition for liquidity provision and picking-off risk in fragmented markets. Consequently, it allows for more flexible agent behavior and better captures the dynamics of these two channels. Specifically, it allows for endogenous liquidity provision and consumption in the presence of real world frictions (such as price discreteness) and in the absence of perfect competition between agents (as in [Glosten, 1998](#)). Using a dynamic equilibrium model for market fragmentation, we show that trader competition and picking off risk, independently and through interaction with each other, can lead to heterogeneous effects of fragmentation on investor welfare and market quality.

⁴Early theories of fragmentation such as [Mendelson \(1987\)](#), [Pagano \(1989\)](#), and [Chowdhry and Nanda \(1991\)](#), while not explicitly modeling limit order markets, highlight the positive network externalities associated with consolidating trading in a single venue. However, such a consolidated market is no longer the equilibrium outcome in the absence of post-trade transparency ([Madhavan, 1995](#)) and in the presence of real world frictions such as differences in markets' absorptive capacity and institutional mechanisms ([Pagano, 1989](#)), order splitting behavior ([Chowdhry and Nanda, 1991](#)), or trader heterogeneity ([Harris, 1993](#)).

2. Multi-Market Model

Our aim is to study the effect of fragmentation on liquidity and welfare in the presence of competition for liquidity provision and adverse selection (picking-off) risk. We set up a dynamic trading game in which agents make endogenous decisions to maximize their expected payoffs, taking into account their private reasons for trading the asset, market conditions, and the strategies employed by agents expected to arrive in the future.

2.1 Model Setting

We consider an economy in continuous time with a single financial asset that trades on two limit order books. The fundamental value of the asset, v_t , is stochastic, and its innovations follow a Poisson process with parameter λ_v . In the case of an innovation, the fundamental value increases or decreases by one tick, d , with equal probability. There is competition among agents. The economy is populated by agents who arrive sequentially following a Poisson process with intensity λ_a . All agents observe both limit order books (i.e., prices and depths at each price) and the fundamental value of v_t . Agents can submit limit or market orders to either book. Moreover, agents can reenter the market to modify unexecuted limit orders. There is adverse selection. In other words, agents cannot instantaneously modify their unexecuted limit orders after a change in market conditions, but instead reenter the market following a Poisson process with parameter λ_r . Thus, agents submitting limit orders face picking-off risk.

The limit order book at time t and in market m with $m \in \{1, 2\}$, $L_{m,t}$, is characterized by a set of discrete prices denoted by $\{p_m^i\}_{i=-N}^N$, where $p_m^i < p_m^{i+1}$ and N is a finite number. Let d be the distance between any two consecutive prices, which we refer to as the tick size (i.e., $d = p_m^{i+1} - p_m^i$). The tick size is the same in both limit order books. Let $l_{m,t}^i$ be the queue of unexecuted limit orders in order book m at time t and price p_m^i . A positive (negative) $l_{m,t}^i$ denotes the number of buy (sell) limit orders, and represents the depth of the book $L_{m,t}$ at

price p_m^i . In the book $L_{m,t}$ at time t , the best bid price is $B(L_{m,t}) = \sup\{p_m^i | l_{m,t}^i > 0\}$ and the best ask price is $A(L_{m,t}) = \inf\{p_m^i | l_{m,t}^i < 0\}$. $B(L_{m,t}) = -\infty$ or $A(L_{m,t}) = \infty$ if the order book $L_{m,t}$ is empty at time t on the buy side or on the sell side, respectively. Each limit order book independently respects price and time priority while executing the limit orders, i.e., buy (sell) limit orders at higher (lower) prices have priority in the queue and limit orders submitted earlier at the same price are executed first.

Agents are risk-neutral, but heterogeneous in terms of their intrinsic economic motives for trading the asset. These motives are reflected in their private values. Each agent has a private value α , which is known to her. α is drawn from the vector $\Psi = \{\alpha_1, \alpha_2, \dots, \alpha_g\}$ based on the cumulative distribution F_α , where g is a finite integer. Private values reflect the fact that agents want to trade for various reasons unrelated to the fundamental value of the asset (e.g. hedging needs, tax exposure, and/or wealth shocks). They are idiosyncratic and constant for each agent. Furthermore, agents face a cost of delaying, which is a cost of not executing immediately. This delay cost is not the time value of money, but instead, it represents the opportunity costs and the cost of monitoring the market until an order is executed. It is denoted by $\rho \in [0, 1]$, is constant across agents and order books, and applies to agents' total payoff.

Agent heterogeneity, delay costs, and the fundamental value of the asset determine agents' trading behavior. On the one hand, suppose agent x with a positive private value (i.e., $\alpha > 0$) arrives at time t_x . This agent is likely to be a buyer because she would like to have the asset to obtain the intrinsic benefit reflected by α . In this case, the agent's expected payoff is $(\alpha + v_{t'} - p)e^{-\rho(t' - t_x)}$, where p is the transaction price, t' is the expected time of the transaction, and $v_{t'}$ is the expected fundamental value of the asset at time t' . Moreover, if α is very high, her delay cost, denoted by $(e^{-\rho(t' - t_x)} - 1)\alpha$, is correspondingly high, and she may therefore prefer to buy the asset as soon as possible by using a market order. In this case, the agent will pay an immediacy cost denoted by $(v_{t'} - p)e^{-\rho(t' - t_x)}$. The agent will accept this immediacy cost because she is mainly generating her profits from the large

private value, α , rather than from the trading process *per se*. Accordingly, an agent with a high absolute private value will probably be a liquidity taker.⁵

On the other hand, suppose an agent y with a private value equal to zero (i.e., $\alpha = 0$) arrives at time t_y . This agent needs to find a profitable opportunity purely in the trading process, by obtaining a good price relative to the fundamental value, because she does not obtain any intrinsic economic benefit from trading. Consequently, she may be patient and prefer to act as a liquidity provider, in turn receiving the immediacy cost (bid-ask spread) paid by a liquidity taker. Alternatively, she may trade aggressively against a standing limit order that is mispriced relative to the fundamental value. Note that agents with $\alpha = 0$ are indifferent with respect to the side of the market they take because they can maximize their gains by either selling or buying the asset.

Agents are exposed to the risk of being picked off, because limit orders can generate a negative payoff if they are in an unfavorable position relative to the fundamental value. For example, suppose an agent x with $\alpha = 0$ first arrives at time $t = 0$ and submits a buy limit order to set the best bid price, B , in market $m = 1$. Suppose further that, at time t^* , the fundamental value of the asset decreases to level v_{t^*} , such that $v_{t^*} < B$, and simultaneously another agent, denoted by y , with private value $\alpha = 0$, arrives. Since agent x cannot immediately modify her unexecuted limit order, agent y can submit a market sell order and pick off agent x 's order, generating an instantaneous profit equal to $(B - v_{t^*})$. Agent x , on the other hand, has a negative realized payoff given by $(v_{t^*} - B)e^{-\rho t^*}$.

Each agent takes three main trading decisions upon arrival: (i) to submit an order to $L_{1,t}$ or $L_{2,t}$, (ii) to submit a buy or a sell order, and (iii) to choose the limit price, which implies the decision to submit either a market or a limit order.⁶ As mentioned above, an agent can reenter the market and modify her unexecuted limit order. Hence, she has to take

⁵Analogously, a similar example can be produced in the other direction in the case of an agent with a negative private value (i.e., $\alpha < 0$) having a preference to sell.

⁶A buy (sell) limit order priced at or above (below) the best ask (bid) is equivalent to a market buy (sell) order.

the following additional trading decisions after reentering: (i) to keep her unexecuted limit order unchanged or to cancel it, (ii) in case of a cancellation, to submit a new order to $L_{1,t}$ or $L_{2,t}$, (iii) to choose whether the new order will be a buy or a sell order, and (iv) to choose the price of the new order.

The decision to leave the order unchanged has the advantage of maintaining its time priority in the respective queue. The disadvantage is the increased exposure to picking-off risk or non-execution risk depending on the direction of the change in the fundamental value. Once a trader has submitted a limit order, she remains part of the trading game by revising her order until it is executed; however, the trader exits the market permanently after the execution of her order.

2.2 Agents' Dynamic Maximization

There is a set of states $s \in \{1, 2, \dots, S\}$ that describes the market conditions in the economy. These market conditions are observed by each agent before she makes any decision. The state s that an agent observes is described by the contemporaneous limit order books, L_1 and L_2 , the agent's private value α , and in case the agent has previously submitted a limit order to any of the books, the status of that order in L_1 or L_2 , i.e., its original submission price, its queue priority in the book, and its type (i.e., buy or sell). The fundamental value of the asset, v , is implicitly one of the variables that describe the state s , since agents interpret limit order prices relative to the fundamental value. For convenience, we set the arrival time of an agent to zero in the following discussion.

Let $a \in \Theta(s)$ be the agent's potential trading decision, where $\Theta(s)$ is the set of all possible decisions that an agent can take in state s . Suppose that the optimal decision given state s is $\tilde{a} \in \Theta(s)$. Let $\eta(h|\tilde{a}, s)$ be the probability that the optimally submitted order is executed at time h . Let $\gamma(v|h)$ be the density function of v at time h , which is exogenous and characterized by the Poisson process of the fundamental value of the asset with rate λ_v .

Thus, the expected value of the optimal order submission $\tilde{a} \in \Theta(s)$, if the order is executed prior to the agent's reentry time h_r , is

$$\pi(h_r, \tilde{a}, s) = \int_0^{h_r} \int_{-\infty}^{\infty} e^{-\rho h} ((\alpha + v_h - \tilde{p})\tilde{x}) \cdot \gamma(v_h|h) \cdot \eta(h|\tilde{a}, s) dv_h dh \quad (1)$$

where \tilde{p} is the submission price and \tilde{x} is the order direction indicator (i.e., $\tilde{x} = 1$ if the agent buys and $\tilde{x} = -1$ if the agent sells) and both are components of the optimal decision \tilde{a} . The expression $(\alpha + v_h - \tilde{p})\tilde{x}$ is the instantaneous payoff, which is discounted back to the trader's arrival time at rate ρ .

Let $\psi(s_{h_r}|h_r, \tilde{a}, s)$ be the probability that state s_{h_r} is observed by the agent at her reentry time h_r , given her decision \tilde{a} taken in the previous state s . The probability $\psi(\cdot)$ depends on the states and potential optimal decisions taken by other agents up to time h_r . In addition, let $R(h_r)$ be the cumulative probability distribution of the agent's reentry time, which is exogenous and described by the Poisson process λ_r . Thus, the Bellman equation that describes the agent's problem of maximizing her total expected value, $V(s)$, after arriving in state s , is given by

$$V(s) = \max_{\tilde{a} \in \Theta(s)} \int_0^{\infty} \left[\pi(h_r, \tilde{a}, s) + e^{-\rho h_r} \int_{s_{h_r} \in S} V(s_{h_r}) \cdot \psi(s_{h_r}|h_r, \tilde{a}, s) ds_{h_r} \right] dR(h_r) \quad (2)$$

where S is the set of possible states. The first term is defined in Equation (1), and the second term describes the subsequent payoffs in the case of reentries.

2.3 Equilibrium

In equilibrium, each agent behaves optimally by maximizing her expected utility, based on the observed state that describes the market conditions (as in Equation (2)). Thus, optimal decisions are state dependent. They are also Markovian, because the state observed by an agent is a consequence of the previous states and the historical optimal decisions taken in the trading game. As there is competition between agents, the equilibrium is competitive (although there is no competition between venues). A competitive equilibrium means that

competing agents will respond to an agent’s local deviation in a way that leads to a reduction in the deviating agent’s expected utility. We obtain a stationary and symmetric equilibrium, as in [Doraszelski and Pakes \(2007\)](#). In such an equilibrium, optimal decisions are time independent, i.e., they stay the same when an agent faces the same state in the present or in the future.

The trading game is also Bayesian in the sense that an agent knows her intrinsic private value to trade, α , but does not know the private values of other agents who are part of the game. Hence, our solution concept is a Markov perfect Bayesian equilibrium (see [Maskin and Tirole, 2001](#)). In the trading game, there is a state transition process in which the probability of arriving in state s_{h_r} from state s is given by $\psi(s_{h_r}|\tilde{a}, s, h_r)$. Thus, two conditions must hold in the equilibrium: agents solve equation (2) in each state s , and the equilibrium beliefs are consistent for each state over time.

As mentioned earlier, the state s is defined by the four-tuple $(L_{1,t}, L_{2,t}, \alpha, \text{status of previous limit order})$, where all variables that describe the state are discrete. Moreover, each agent’s potential decision a is taken from $\Theta(s)$, which is the set of all possible decisions that can be taken in state s . This set of possible decisions is discrete and finite, given the features of the model. Consequently, the state space is countable and the decision space is finite; thus, the trading game has a Markov perfect equilibrium (see [Rieder, 1979](#)).

2.4 Solution Approach

Given the large dimension of the state space, we use the [Pakes and McGuire \(2001\)](#) algorithm to compute a stationary and symmetric Markov perfect equilibrium. The intuition behind this algorithm is that the trading game by itself can be used initially, as a tool in which agents learn how to behave in each state. Thus, we set the initial beliefs about the expected payoffs of potential decisions in each state. Agents take the trading decision that provides the highest expected payoff conditional on the state they observe. Subsequently, agents

dynamically update their beliefs by playing the game and observing the realized payoffs of their trading decisions. In this sense, the algorithm is based on agents following a learning-by-doing mechanism. The [Pakes and McGuire \(2001\)](#) algorithm is able to deal with a large state space because it reaches the equilibrium only on the recurring states class.

The equilibrium is reached when there is nothing left to learn, i.e., when the beliefs about the expected payoffs have converged. We apply the same procedure as was used by [Goettler et al. \(2009\)](#) to determine whether the equilibrium has been reached. Once we reach the equilibrium, after making the agents play the game for at least 10 billion trading events, we fix the agents' beliefs and simulate a further 20 million events. All theoretical results presented in Section 3 are computed from the latter. Appendix A describes the implementation details of the [Pakes and McGuire \(2001\)](#) algorithm.

2.5 Model Parameterization

We use the same parameters as [Goettler et al. \(2009\)](#) who rely on the empirical findings of [Hollifield et al. \(2006\)](#) to identify parameters that reasonably describe real market features. We set the intensity of the Poisson process followed by agents' arrival, λ , to one. The intensity of the Poisson process followed by agents' reentry, λ_r , is set to 0.25; the intensity of the Poisson process followed by the innovations to the fundamental value, λ_v , is set to 0.125 and 0.625, to simulate scenarios of low and high volatility, respectively. We set the tick size, d , in both order books to one, and the number of discrete prices available on each side of both order books to $N = 31$. The delay cost, ρ , is set to 0.05. The private value, α , is drawn from the discrete vector $\Psi = \{-8, -4, 0, 4, 8\}$ using the cumulative probability distribution $F_\alpha = \{0.15, 0.35, 0.65, 0.85, 1.0\}$.

To further alleviate concerns surrounding the suitability of our parameter choices, we employ the [Hollifield et al. \(2006\)](#) approach to compute the trader arrival rate, fundamental value volatility, and the distribution of private values using message-level data for the month

of January 2015 from the LSE for two FTSE-100 stocks. Appendix B contains the implementation details of the Hollifield et al. (2006) approach, and the mean estimated parameters across the 21 days in our sample period. These estimates are not substantively different from our model parameter choices.

3. Theoretical Implications

Our aim is to examine the effect of fragmentation on liquidity and welfare, specifically via the competition for liquidity provision and adverse selection channel. To this end, we generate simulated data sets for the following two cases. First, we consider an environment with low volatility, or picking-off risk, by setting $\lambda_v = 0.125$. In this simulated environment the importance of competition for liquidity provision channel is relatively high. Second, we consider an environment with high asset volatility by setting $\lambda_v = 0.625$. In this simulated environment the adverse selection channel is relatively more important than the competition for liquidity provision channel. For both scenarios, we analyze two market setups: (i) a consolidated market with one limit order book and (ii) a fragmented market with two identical limit order books. We compute mean levels of the variables of interest for each of the four settings.⁷

3.1 Trading Behavior

Agents' order submission strategies determine the liquidity characteristics and the price discovery process of the asset, and, as a consequence, determine the welfare of different agent types and that of the economy as a whole. Hence, we start by investigating the agents' trading behavior in consolidated and fragmented markets under the two volatility scenarios. Throughout this subsection, we report trader statistics for sell orders. The results for buy orders are analogous and omitted for brevity.

⁷We do not report standard errors because a large number of trader arrivals ensures that standard errors are sufficiently low and a difference in means even to the order of 10^{-2} being statistically significant.

Insert Table 1 about here

Table 1 presents the results. In Panel A, we present the distribution of executed limit and market orders by each agent type for low and high levels of volatility. When volatility is low, agents with private value $\alpha = 0$ execute 77.1% of their trades using limit orders in the single market setting, whereas agents with private value $|\alpha| = 8$ execute 80.5% of their trades using market orders. Agents with private value $|\alpha| = 4$ use limit and market orders roughly equally. These frequencies remain largely unchanged when the market is fragmented. These results are consistent with the idea that agents with private value $\alpha = 0$ act as market makers and try to earn the bid-ask spread, whereas those with private reasons to trade tend to consume liquidity as their cost of waiting is relatively high.

When volatility is high, the behavior changes substantially. Speculators increase their frequency of market orders due to the increased exposure to picking-off risk, especially in the single market where they predominantly generate their trading gains by picking-off mispriced limit orders. Conversely, agents with private value $|\alpha| = 8$ increase their frequency of limit orders, also to a larger extent in the single market than in the fragmented market. This is because their large private value allows them to bear some picking-off risk, especially when the costs of taking liquidity are relatively high due to reduced liquidity supply by other traders. The liquidity supplied by agents with private value $|\alpha| = 8$ in turn is profitably picked off by the speculators. This is consistent with studies examining the behaviour of high-frequency trading (HFT) firms across stocks with different volatility. For example, [Brogaard et al. \(2014\)](#) find that, in the US, the ratio of liquidity consuming and supplying trades of HFT firms for large-cap (or low volatility) stocks versus small-cap (or high volatility) stocks is 1.0 and 2.3, respectively. The differences in order choice between the low and high volatility scenarios is less pronounced in the multi-market setting, as it provided higher protection against picking-off risk, thereby making speculators more willing to submit limit orders as compared to the single market setting. In conclusion, in a high volatility state, agents submit less aggressive orders in a consolidated market due to the higher adverse selection risk.

Panel A also reports the probability of submitting an aggressive limit order for each setting. This probability is computed as the proportion of sell limit orders that are submitted at the best ask price. For both volatility scenarios, fragmented markets decrease the level of traders' competition. In the low volatility scenario, 35.9% of limit orders are aggressive in the consolidated market, whereas only 28.5% of limit orders are aggressive in the fragmented market. The difference persists even in the high volatility scenario, where 24.7% (22.8%) of limit orders are aggressive in consolidated (fragmented) markets. These findings confirm that competition for liquidity provision is less intense in fragmented than consolidated markets.

In Panel B, we report the level of picking-off risk for each of the two market settings and volatility scenarios for all agent types. Picking-off risk is measured as the proportion of sell limit orders executed below the fundamental value. In our model, high (low) volatility corresponds to a higher (lower) frequency of fundamental price jumps which, in turn, translates to higher (lower) picking-off risk. Consider a limit sell order submitted by agent one at price p . A second agent entering the market after the first agent could place a limit sell order at the same price p in the second order book. If, subsequently, the fundamental value of the asset moves against these two unexecuted limit orders, an incoming trader can choose to pick off the second limit order, giving the first limit order trader additional time to react and modify her order. As such, the picking-off risk of the first agent is reduced as there is a higher probability that she will be able to reenter the market before her order is picked off.

Consistent with this intuition, we find that the aggregate level of picking-off risk under the low volatility setting is 21.8% in a single market and 20.8% in a fragmented market, whereas it increases to 59.3% in a single market and 42.3% in a fragmented market under the high volatility setting. The picking-off risk is lower in fragmented market for each agent type. For example, when asset volatility is low, agents with $\alpha = 0$ and $|\alpha| = 8$ have a picking-off risk of 4.1% (3.0%) and 73.9% (72.1%), respectively, in consolidated (fragmented) markets. The corresponding numbers in the high asset volatility setting are 13.8% (10.9%) and 85.4% (75.7%), respectively. This shows that fragmented markets provide more protection against

picking-off risk under both volatility settings.

Panels A and B of Table 1 also highlight the relative importance of the competition for liquidity provision and picking-off risk channel under different volatility scenarios. The difference in the probability of submitting aggressive limit orders between the single and multi market setting is *less* pronounced (7.4% versus 1.9%), and the difference in aggregate picking-off risk is *more* pronounced (1.0% versus 17%) when the fundamental volatility is high. These differences suggest that the competition for liquidity provision dominates the picking-off risk channel when asset volatility is low, whereas the effects associated with picking-off risk channel dominate the effects associated with competition for liquidity provision channel when asset volatility is high.

Finally, in Panel C, we report the time to execution of limit orders in aggregate and for each agent type. We define time to execution as the difference between the time an order executes and the market entry time of the agent. The time to execution of agents with private value $\alpha = 0$ decreases from 14.9 units in a consolidated market to 11.5 units in a fragmented market when volatility is low because the need to reprice existing limit orders is reduced when queues are shorter and trader competition is lower. The difference is even more significant in a high-volatility scenario, where it decreases from 23.8 units in a consolidated market to 12.1 units in a fragmented market. For agents with private value $\alpha \neq 0$, there is little difference in the time to execution between the consolidated and fragmented market settings under both volatility scenarios. Specifically, for the agents with $|\alpha| = 4$, time to execution in consolidated (fragmented) markets is 3.5 (3.7) units in the low volatility setting and 2.9 (3.7) units in the high volatility setting. For the agents with $|\alpha| = 8$, time to execution in consolidated (fragmented) markets is 2.1 (1.9) units in the low volatility setting and 1.2 (1.7) units in the high volatility setting.

When we compare the two volatility scenarios, the time to execution for agents with $\alpha = 0$ ($\alpha \neq 0$) is lower in the low (high) volatility setting. This relates directly to the results in Panel A discussed above. In the high volatility scenario, intermediaries drastically reduce

the frequency with which they provide liquidity. When they do submit limit orders, they quote very wide prices leading to an increase in the time to execution. The opposite holds for agents with $\alpha \neq 0$. They provide liquidity in the high volatility setting at attractive prices leading to a lower time to execution.

3.2 Impact on Quoted and Effective Spreads

We compute time-weighted quoted bid-ask spreads based on *local* and *inside* quotes, where the former comprise the bid and ask prices in one of the two markets, whereas the latter comprise the highest bid and the lowest ask across the two limit order books. The two versions are obviously identical in a single-market setting. Differences in quoted spreads do not necessarily translate into commensurate differences in transaction costs for traders submitting market orders. Thus, we next compare the mean effective spread in the single and fragmented markets, which captures the actual transaction costs incurred by traders submitting market orders. We calculate the effective spread as follows:

$$\text{effective spread} = 2x_t(p_t - m_t)/m_t, \quad (3)$$

where x_t is +1 for a buyer-initiated order and -1 for a seller-initiated order, p_t is the transaction price, and m_t is the mid-quote immediately before the transaction. Table 2 reports the results.

Insert Table 2 about here

As expected, there is a direct relationship between asset volatility and bid-ask spreads (as in [Stoll, 2000](#)). Quoted bid-ask spreads are higher by approximately a factor of three (two) in a consolidated (fragmented) market in the high-volatility setting. This relationship also holds for effective spreads, though the differences between consolidated and fragmented markets are smaller in magnitude.

We also observe that, in the low volatility setting, local and inside quoted bid-ask spreads are higher in fragmented markets by 1.06 and 0.36 ticks, respectively. However, when asset volatility is high, local and inside quoted bid-ask spreads are lower by 0.2 and 1.47 ticks, respectively, in fragmented markets. The same relationship also holds for effective spreads. In fragmented markets combined with low (high) picking-off risk, the local and inside effective spreads are 1.89 ticks and 1.57 ticks (3.53 ticks and 2.89 ticks), respectively, whereas the corresponding numbers in consolidated markets are lower (higher) at 1.34 ticks (3.58 ticks).

In our model, fragmentation reduces price competition because agents can gain priority by jumping the queue (with probability $1/2$, as liquidity takers are indifferent between taking liquidity on either of the two order books) and entering orders in the second order book, thereby inducing an increase in the bid-ask spread. At the same time, fragmentation reduces the picking-off risk, since the probability that an order will be picked off is lower in the presence of a second limit order book, resulting in a lower bid-ask spread. As described in subsection 3.1, the level of asset volatility directly influences the relative importance of the competition for liquidity provision and the adverse selection channels. The former is more important when asset volatility is low, whereas the latter is more important when asset volatility is high. As a result, higher (lower) asset volatility is associated with lower (higher) bid-ask spreads in fragmented markets. In this sense, our model can potentially explain the mixed findings in the empirical literature regarding the relationship between liquidity and fragmentation. For example, O'Hara and Ye (2011) find that fragmentation is associated with lower effective spreads for small-cap (or high-volatility) stocks in the US. In Section 4, we further test this model implication by examining the relationship between fragmentation and liquidity for different levels of volatility for European stocks.

3.3 Welfare Analysis

We next analyze the economic benefits per agent and for the whole market by examining the effect of fragmentation on welfare in the high- and low-volatility scenarios. Welfare is

measured as the average realized payoff per agent. In addition, we decompose the realized payoffs into gains and losses associated with agents' private values and the trading process.

Suppose that an agent with a private value α enters the market at time t . She submits an order (a limit order or a market order) to any of the books at price \tilde{p} , with order direction \tilde{x} (to buy or to sell). Suppose further that the agent does not modify the order and that it is finally executed at time t' when the fundamental value is $v_{t'}$. The agent's realized payoff is then given by

$$\Pi = e^{-\rho(t'-t)} (\alpha + v_{t'} - \tilde{p}) \tilde{x}. \quad (4)$$

We can decompose the agents' payoffs and rewrite Equation (4) as

$\Pi = \text{gains from private value} + \text{waiting cost} + \text{money transfer}$, where

$$\text{gains from private value} = \alpha \tilde{x}$$

$$\text{waiting cost} = (e^{-\rho(t'-t)} - 1) \alpha \tilde{x}$$

$$\text{money transfer} = e^{-\rho(t'-t)} (v_{t'} - \tilde{p}) \tilde{x} \quad (5)$$

The first term in (5), *gains from private value*, represents the gains obtained directly from the intrinsic reasons for trading the asset, $\alpha \tilde{x}$. The second term in Equation (5), *waiting cost*, reflects the cost associated with delaying the realization of the *gains from private value*. Agents submitting limit orders do not trade immediately after arriving, but have to wait until the orders are executed. This is costly, due to the delay cost ρ . The third term in Equation (5), *money transfer*, reflects the difference between the fundamental value $v_{t'}$ and the transaction price \tilde{p} , discounted back to the arrival time of the agent, which represents the welfare gain (or loss) associated with the trading process. In general, *money transfer* is related to the immediacy cost incurred when an agent wants to realize her private value immediately. For example, an agent who submits a market order realizes her intrinsic private value without a delay, but she may have to pay a price for demanding immediacy, which

would be reflected in a negative value of *money transfer*. Alternatively, an agent submitting a limit order earns the half bid-ask spread but may generate negative *money transfer* gains if her order is picked off.

Insert Table 3 about here

Table 3 presents the results. When volatility is high, aggregate welfare is higher in a fragmented market (3.662 ticks vs. 3.652 ticks), while when volatility is low, aggregate welfare is lower in a fragmented market (3.740 ticks vs. 3.745 ticks). This result complements our previous finding; when the adverse selection effect dominates the competition for liquidity provision effect, fragmented markets are beneficial in terms of liquidity and, consequently, aggregate welfare is higher. Conversely, when the competition for liquidity provision effect is more relevant, liquidity is reduced in a fragmented market and aggregate welfare is also lower.

In addition, we find that the welfare shifts among the three categories of agents are substantial. Agents with no intrinsic motives for trading (i.e., $\alpha = 0$) generate larger payoffs in fragmented markets in the low volatility setting (0.54 ticks vs. 0.63 ticks when volatility is low). Their payoffs are even larger in fragmented markets combined with the high-volatility setting (0.61 ticks vs. 0.82 ticks). The difference between the single and fragmented markets arises for three reasons. First, there is less limit order price competition in a fragmented market because time priority does not apply across order books; second, the risk of being picked off is reduced in a fragmented market; third, intermediaries' expected time to execution in a fragmented market is lower than in a consolidated market, which reduces the delay-cost component of their welfare.

Fragmentation has detrimental effects in terms of expected payoffs for agents with $|\alpha| = 4$ and $|\alpha| = 8$, for both levels of picking-off risk. In terms of the absolute waiting cost, they do not exhibit any significant difference between the single and fragmented settings. However, the negative welfare effects are generated due to higher money transfer losses. The underlying

reasons for these money transfer losses depend on the composition of limit versus market orders used by these agents in the two volatility scenarios. In the low volatility setting, the higher fraction of market orders used by agents with $|\alpha| = 8$ combined with lower price competition leads to higher immediacy costs in fragmented markets. The same mechanism also works in the high volatility setting for $|\alpha| = 8$ traders but the higher bid-ask spreads due to higher picking-off risk lead to substantially higher money transfer losses. In the single market setting combined with high asset volatility, these same agents predominantly use limit orders and earn the bid-ask spread, but are adversely selected by $|\alpha| = 0$ traders leading to money transfer losses. Nevertheless, they are still better off compared to the multi-market setting with high asset volatility.⁸

In conclusion, fragmentation induces a shift in trading gains away from agents with intrinsic motives to trade and towards intermediaries without a commensurate increase in total welfare. This indicates that costly investments in intermediation capacities such as colocation charges, subscriptions to data feeds, and investments in high-speed networks are socially wasteful. These investments are outside the scope of our model but in reality are endogenously determined by investors. In a computationally simpler alternative, we double the population of agents with $\alpha = 0$ in a multi-market setting combined with low asset volatility to mimic the increased participation by intermediaries resulting from their high profitability in fragmented markets. These results further confirm that intermediaries' entry reduces welfare of agents with private reasons to trade. The trading gains of intermediaries as a group are higher, even though individually they extract lower welfare.^{9,10}

⁸Note that, in Table 3, the total money transfers do not add up to zero, as the expected payoff in a single transaction of the limit order and corresponding market order is discounted back to different times. This is due to the asynchronous arrivals of the agents who submit these two orders. However, the instantaneous money transfer before it is discounted back, is equal to zero.

⁹The welfare decomposition results under this parameterization are reported in Appendix Table A2.

¹⁰In Appendix B, consistent with this alternative parameter choice, the population of agents with $\alpha = 0$ is higher for the two FTSE-100 stocks trading on the LSE.

4. Empirical Application

In this section, we empirically analyze the main implications relating the impact of fragmentation on liquidity generated by our model. The goal is to demonstrate equilibrium outcomes rather than causally identifying effects of any exogenous variation of one of the variables.

Our data comprise the constituents of the French and German large and mid-cap indices (CAC40, Next20, DAX, MDAX). After removing the smallest MDAX stocks to get a sample that is balanced across the two countries and eliminating stocks with incomplete data, we obtain a sample of 111 stocks. We employ high-frequency data from EUROFIDAI BEDOFIH for Euronext Paris, Deutsche Börse, and Chi-X. Euronext Paris and Deutsche Börse are the main listing exchanges for French and German firms, respectively. Chi-X established itself as the largest competitor to the incumbent exchanges subsequent to the implementation of the Markets in Financial Instruments Directive (MiFID) in 2007 which eliminated all rules prohibiting trading outside the national markets in the European Union.¹¹ Our data contains all lit order messages, allowing us to also accurately sign and match trades to the prevailing aggregate order book.¹² Furthermore, compared to some other commonly used high-frequency databases, our data have the advantage of containing accurate exchange time-stamps, which is important when consolidating limit order books to obtain the best prices across multiple venues. We use data from the continuous trading session from 9:00 to 17:30 CET for each day. In other words, our data begins immediately after the opening auction clears, ends immediately before the closing auction starts, and also excludes the period of the mid-day call auction that takes place for German stocks. Our sample period comprises the second half of 2012. To the best of our knowledge, none of the three exchanges implemented any changes to their market structures during this period.¹³ Hence, our analysis can be

¹¹Chi-X is operated by Cboe today.

¹²We do not use data from other smaller lit trading venues or off-exchange trading, for which equally precise data is unavailable. However, the trading activity contained in our data comprise the large majority of lit trading for the stocks contained in our sample.

¹³In October 2012, a commercial microwave link between Frankfurt and London became operational. Our results remain unchanged after controlling for this. [Sagade et al. \(2019\)](#) analyse the market quality implications of this event.

interpreted as what happens when exchange competition remains unchanged.

4.1 Empirical Approach

Our theoretical findings indicate that fragmentation has beneficial effects on the quoted and effective bid-ask spread when volatility is high, whereas the effect is the opposite when volatility is low. Therefore, we empirically examine the impact of fragmentation on these two spreads measures while conditioning on different levels of asset volatility.

To this end, we compute the quoted bid-ask spread for each stock i on day t ($\text{Quoted}_{i,t}$) as the time-weighted difference between the best ask and bid prices divided by their midpoint, and express it in basis points. We compute this measure both for the consolidated order book, i.e., using the best prices across both LOBs, and locally, i.e., considering only the best bid and ask prices on one order book. We also compute effective spreads ($\text{Effective}_{i,t}$) as two times the difference between the trade price and the prevailing midpoint time the trade indicator variable and aggregate the observations by value-weighting across all trades executed during the day t , also expressed in basis points. Effective spreads are also computed both with respect to the inside midpoint and the local midpoint.

To measure the level of fragmentation ($\text{Fragmentation}_{i,t}$), we follow previous literature (e.g., [Degryse et al., 2015](#); [Haslag and Ringgenberg, 2016](#)) and first compute the Herfindahl-Hirschman Index of traded volumes for every stock i and day t ($\text{HHI}_{i,t}$) across the two exchanges. As HHI measures the concentration of trading, we define $\text{Fragmentation}_{i,t}$ as one minus $\text{HHI}_{i,t}$. Specifically, we compute the measure as follows:

$$\text{Fragmenation}_{i,t} = 1 - \sum_k s_{i,k,t}^2, \quad (6)$$

where $s_{i,k,t}^2$ is the share of the total trades of stock i in venue k at time t .

We also estimate volatility as the standard deviation of one-second mid-quote returns, denoted by $\sigma_{i,t}$, for each stock i and day t . To eliminate the effects of any other stock

characteristics, we consider the variation of volatility over time within each stock rather than across stocks. We do so by creating five dummy variables that indicate the five volatility quintiles of a stock on each day:

$$\text{Volatility } j\text{-th quintile } i, t = \begin{cases} 1 & \text{if } \sigma_{i,t} \text{ is between the } 20 \times (j-1)\% \text{ and } 20 \times j\% \text{ percentiles} \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

Finally, we include the natural logarithm of the stock's trading euro volume across the two exchanges ($\text{Volume}_{i,t}$), as an additional explanatory variable, which proxies for variations in the arrival rate of traders.¹⁴

We estimate panel regressions where we regress the liquidity measures LIQ, the quoted bid-ask spread and the effective spread, on the level of fragmentation, volatility quintile dummies, the interaction between fragmentation and the volatility dummies, and the trading volume, as follows:

$$\begin{aligned} \text{LIQ}_{i,t} = & \alpha + \beta \text{ Fragmentation}_{i,t} + \sum_{j=1, j \neq 3}^{j=5} \gamma_j \text{ Volatility } j\text{-th quintile}_{i,t} \\ & + \sum_{j=1, j \neq 3}^{j=5} \delta_j \text{ Fragmentation}_{i,t} \times \text{ Volatility } j\text{-th quintile}_{i,t} \\ & + \nu_i [+ \eta_t] + \varepsilon_{i,t+1} \end{aligned} \quad (8)$$

where ν_i and η_t denote the stock and day fixed effects, respectively. To avoid multicollinearity, we exclude the dummy variable for the middle quintile. All estimations include stock-fixed effects to control for unobserved heterogeneity across stocks whereas we estimate models both with and without time-fixed effects, the latter because time-fixed effects remove much of the variation in volatility. We cluster standard errors by both stock and day.

We estimate all models both for local liquidity measures on the primary listing exchange (Euronext Paris or Deutsche Börse), which is empirically the larger of the two exchanges under consideration for each stock, and for inside liquidity measures, i.e., considering the

¹⁴The exclusion of trading volume from the regressions does not qualitatively affect the results.

highest bid and lowest ask prices across the primary listing exchange and Chi-X.

4.2 Empirical Results

Table 4 shows the results for the panel regression introduced in (8) for the local liquidity measures. We examine how fragmentation affects the quoted bid-ask spread and the effective spread for different levels of volatility.

Insert Table 4 about here

Our empirical results reveal that, unconditionally, fragmentation does not significantly affect either the bid-ask spread or the effective spread. We find that volatility increases quoted and effective spreads, consistent with the theoretical prediction that the increased exposure to adverse selection in high-volatility states leads to wider spreads. In particular, quoted bid-ask spreads monotonically increase with the volatility quintiles and the difference between the coefficients for the most and least volatile quintiles suggests that bid-ask spreads are higher by, dependent on the regression specification, between 3.5 and 4 basis points for a stock on its most volatile quintile of trading days compared to its least volatile quintile of trading days. The effects for effective bid-ask spreads are similar, with differences between the most and least volatile trading days of between 4.2 and 4.5 basis points. These effects are economically large compared to average quoted bid-ask spreads of 7.3 basis points and average effective bid-ask spreads of 6.2 basis points in our sample (untabulated).

Most importantly, we find that fragmentation has a varying impact on the spread, depending on the level of asset volatility. Specifically, the coefficient of the interaction term between fragmentation and the volatility dummies is positive (negative) for low (high) levels of volatility. The tests for equality of the coefficients for the first and fifth quintiles, shown at the bottom of the table, show that the differences are statistically significant. Note that the results do not suggest that more fragmented stocks become more liquid as volatility rises,

as the variation in fragmentation is not that large, but instead they show that fragmented markets compare more favorably to consolidated markets when volatility is high compared to when it is low.

Insert Table 5 about here

Table 5 reports the analogous results for the inside liquidity measures. Fragmentation here is unconditionally associated with smaller quoted and effective spreads. The fact that this result differs from our model predictions is not surprising: it can be explained by the fact that our empirical data features competing trading venues, whereas the venues in the model are identical. The theoretical prediction also differs, not surprisingly, from papers studying the causal effect of fragmentation resulting from exchange competition (O’Hara and Ye (2011) and Haslag and Ringgenberg (2016)). It has been argued in the literature that fragmentation can be positive for liquidity since it increases competition in exchanges, which reduces trading costs (Rust and Hall (2003)). In our theoretical model, we keep exchange competition constant, and focus on two other effects that influence the effect of fragmentation.¹⁵ The results for the effect of volatility on inside liquidity are similar to those observed for local liquidity. Quoted (Effective) spreads are between 2.7 (3.4) basis points and 3.2 (3.7) basis points larger when stocks are most versus least volatile, which is large compared to average inside quoted (effective) spreads of 6.4 (3.7) basis points in our sample. The results for the interaction effect between volatility and fragmentation are also similar to those for local liquidity measures. In each model, higher fragmentation is associated with lower spreads on volatile days compared to low-volatility days, and the difference is statistically significant.

These findings provide empirical support for our main theoretical implications concerning

¹⁵An earlier version of this paper considered an empirical analysis of Euronext merging order books for stocks that had listings on more than one of the Euronext constituent exchanges. That event did not feature a change in competition between exchanges and the results were consistent with the idea that fragmentation harms liquidity. We have removed that application from this paper for reasons of its small sample size and thus limited statistical power.

the impact of fragmentation on liquidity. The effect of fragmentation is heterogeneous and greatly depends on the level of picking-off risk resulting from the volatility of the stock's value. High picking-off risk can exceed the effect of price competition, and agents respond by submitting less aggressive orders. As fragmented markets provide greater protection against adverse selection, agents are able to submit more aggressive orders in a fragmented market than in a single market, leading to a reduction in the bid-ask spread. On the other hand, if we only consider the effect of traders' competition, fragmentation will lead to an increase in the bid-ask spread since more fragmented markets lead to reduced price competition. Hence, when the level of picking-off risk is low, the competition for liquidity provision effect becomes stronger, leading to an increase in the bid-ask spread in fragmented markets.

5. Conclusion

We model a fragmented market for an asset that trades in two limit order books populated by heterogeneous agents who endogenously choose to supply or consume liquidity, and compare the results with a single-market setting. Two channels – competition for liquidity provision and adverse selection – drive the results. As time priority is not enforced across markets, fragmentation leads to reduced price competition between intermediaries. At the same time, fragmentation provides better protection from adverse selection risk for limit orders, as incoming arbitrageurs can choose to trade against mispriced orders in either of the two order books.

The former effect dominates when asset volatility is low, whereas the latter effect dominates when asset volatility is high. Hence, market fragmentation can be beneficial or harmful for market liquidity depending on the level of asset volatility. These heterogeneous liquidity effects associated with market fragmentation also affect total welfare in our model: a consolidated (fragmented) market is associated with higher total welfare when asset volatility is low (high). However, in both volatility settings, the distribution of welfare across the heterogeneous agent types differs markedly across consolidated and fragmented markets. Agents

with intrinsic trading motives extract lower payoffs in fragmented markets whereas agents acting as intermediaries are better off in fragmented markets.

Our central theoretical prediction concerning the relationship between market fragmentation and liquidity can potentially reconcile the differences in the empirical literature. We also provide empirical support for this prediction by investigating the relationship between liquidity and fragmentation for French and German stocks in the second half of 2012.

Overall our results suggest that the positive effects of consolidating order flow in a single (or fewer) location(s) still exist even in modern electronic limit order markets where the activities of high-frequency traders serve to integrate fragmented order books. The adverse effects of fragmentation are borne by investors who trade for intrinsic trading motives. Our results also have important policy implications. Regulators may be able to improve the welfare of investors who trade for intrinsic motives by: (i) preventing individual market operators from keeping an artificially high(er) level of order flow fragmentation in the absence of commensurate benefits; and (ii) limiting excessive investment in intermediation capacities necessary to link multiple order books which come at a cost to end investors.

A. Pakes and McGuire Algorithm

In this appendix we describe the implementation details associated with the [Pakes and McGuire \(2001\)](#) algorithm and some additional steps taken to reduce the state-space of our model. In order to reduce the dimensionality of the state space, we center each limit order book at the contemporaneous fundamental value of the asset, i.e., by setting $p_m^0 = v_t$. Suppose, at time $t = 0$, the fundamental value is v_0 , but after a period τ the fundamental value has experienced some innovations and is now v_τ , with $v_\tau - v_0 = qd$, where q is a positive or negative integer. In this case, we shift both books by q ticks to center them at the new level of the fundamental value v_τ . Thus, we move the queues of existing limit orders in both books to take the relative difference with respect to the new fundamental value into account. This implies that the prices of all orders are always relative to the current fundamental value of the asset and agents always make decisions in terms of prices relative to the fundamental value.

A.1 The updating process to reach the equilibrium

For any state s of the economy, there is a set of possible actions, $\Theta(s)$, that a trader can take. Suppose that a given trader arrives for the first time or re-enters the market at time t and observes the state s . In our model setup, the trader has beliefs about the expected payoff of each possible action that could be taken given the observed state s . Suppose that $U_t(\tilde{a}|s)$ is the expected payoff at time t that is associated with the action $\tilde{a} \in \Theta(s)$. Suppose that the trader decides at time t to take the optimal action \tilde{a}^* that provides the maximum expected payoff out of all possible actions. As a first case, suppose that the optimal action \tilde{a}^* is not a market order (e.g. a limit order, or a cancellation and resubmission). Later on, at time t_r the same trader re-enters the market, but the market conditions have changed. The trader observes a new state s_{t_r} in which she follows the optimal strategy \tilde{a}^{**} that also gives a maximum payoff given the new market conditions. Consequently, the original decision

\tilde{a}^* induces a realized continuation of optimal actions and expected payoffs; and thus the updating process of beliefs can be written as:

$$U_{t_r}(\tilde{a}^*|s) = \frac{n_{\tilde{a}^*,s}}{n_{\tilde{a}^*,s} + 1} U_t(\tilde{a}^*|s) + \frac{1}{n_{\tilde{a}^*,s} + 1} e^{-\rho(t_r-t)} (U_{t_r} \tilde{a}^{**}|s_{t_r}),$$

where $n_{\tilde{a}^*,s}$ is a counter that increases by one when the action \tilde{a}^* is taken in the state s .¹⁶

Alternatively, as a second case, suppose that the optimal decision \tilde{a}^* is a market order (i.e. there is no future time t_r as in the previous case). Then, the updating process of the expected payoff of the optimal action \tilde{a}^* in this scenario can be expressed as:

$$U_{t_r}(\tilde{a}^*|s) = \frac{n_{\tilde{a}^*,s}}{n_{\tilde{a}^*,s} + 1} U_t(\tilde{a}^*|s) + \frac{1}{n_{\tilde{a}^*,s} + 1} (\alpha + v_t - \tilde{p}) \tilde{x}.$$

Here \tilde{p} is the submission price, α is the private value of the trader, v_t is the fundamental value of the asset, and \tilde{x} is equal to one (minus one) when the trader submits a buy (sell) order.

As a third case, suppose that the optimal decision \tilde{a}^* is a limit order; however, later on at time t_r this limit order is executed because another trader submits a market order. The updating process for the first trader with the optimal action \tilde{a}^* can be reflected in the following equation:

$$U_{t_r}(\tilde{a}^*|s) = \frac{n_{\tilde{a}^*,s}}{n_{\tilde{a}^*,s} + 1} U_t(\tilde{a}^*|s) + \frac{1}{n_{\tilde{a}^*,s} + 1} e^{-\rho(t_r-t)} (\alpha + v_{t_r} - \tilde{p}) \tilde{x},$$

where α is the private value for the first trader. Similarly, for the second trader who submits the market order that executes the limit order of the first trader, the updating process can be expressed as:

¹⁶The value of $n_{\tilde{a}^*,s}$ affects how quickly we reach the model equilibrium (a large value in $n_{\tilde{a}^*,s}$ is associated with a slow convergence). Therefore, we reset $n_{\tilde{a}^*,s}$ intermittently to improve the convergence speed.

$$U_{t_r}(\tilde{a}'|s_{t_r}) = \frac{n_{\tilde{a}', s_{t_r}}}{n_{\tilde{a}', s_{t_r}} + 1} U_t(\tilde{a}'|s_{t_r}) + \frac{1}{n_{\tilde{a}', s_{t_r}} + 1} (\alpha' + v_{t_r} - \tilde{p}) \tilde{x},$$

where α' and \tilde{a}' are the private value and the optimal decision of the second trader, respectively. In this case, \tilde{a}' is a market order which is chosen at time t_r by the second trader when the state s_{t_r} is found. In this last case, it is important to observe that any market order implies the execution of a previously submitted limit order. Thus, in the presence of market orders the updating process in beliefs always involves two traders: the trader who submits the market order, and the trader who submitted the limit order which is executed by the market order.¹⁷

A.2 Convergence Criteria

We check for convergence after running the trading game for at least 10 billion trading events. Subsequently, we check the evolution of agents' beliefs after every 500 million simulations for convergence. Let's assume that the first group of 500 million simulations after we start checking for convergence finishes at time t_1 and the second group of 500 million simulations finishes at time t_2 . Let $U_{t_1}(\tilde{a}|s)$ and $U_{t_2}(\tilde{a}|s)$ be the expected payoffs that are associated with the action \tilde{a} when the state s is present at times t_1 and t_2 , respectively. In addition, suppose that $k_{\tilde{a},s}^{t_1,t_2}$ is the number of times that the action \tilde{a} was taken between t_1 and t_2 when traders face s . We evaluate the change in the expected value of the expression $|U_{t_2}(\tilde{a}|s) - U_{t_1}(\tilde{a}|s)|$ for all pairs (\tilde{a}, s) weighted by $k_{\tilde{a},s}^{t_1,t_2}$ after every 500 million simulations. Once this weighted absolute difference is smaller than 0.01 (which suggests that the model has converged), we

¹⁷The initial beliefs about the expected payoffs $U_0(\tilde{a}|s)$ of the possible actions $\tilde{a} \in \Theta(s)$ that a trader can take given that she faces state s are set as follows. Suppose one of the possible actions for a trader with private value α in the state s is to submit a limit sell order at price p when the fundamental value is v . We set the initial expected payoff of this action as $p - v - \alpha$ discounted by ρ until the expected time that a new fast trader arrives at the market. This value is only a first approximation since we assume that v is constant, which is not true in the model, and there is a chance that the next trader may submit another limit order instead of a market order that executes the limit order of the previous trader. In the case of a market sell order the expected payoff is simply $p - v - \alpha$ without any discount. Similar values are obtained for buy orders.

apply two further convergence criteria in line with [Pakes and McGuire \(2001\)](#) and [Goettler et al. \(2009\)](#).

After reaching a small weighted absolute difference in the change in the expected values as described in the previous paragraph, we fix the agents' beliefs concerning the expected payoffs, $U^*(\cdot)$, and simulate the trading game for another 500 million events. Then, we calculate the realized payoffs of all order submissions after they have been executed. Let us denote these realized payoffs as $\tilde{J}(\cdot)$. $\tilde{J}(\cdot)$ is a direct measure of realized benefits to trade. First, we require that the correlation between beliefs $U^*(\cdot)$ and realized outcomes $\tilde{J}(\cdot)$ is higher than 0.99. Second, we require that the mean absolute error in beliefs, i.e. the difference between $U^*(\cdot)$ and $\tilde{J}(\cdot)$ weighted by the number of times that a specific action is selected in a given state within the last 500 million simulated events, is less than 0.01 (i.e. in a similar way to the previous paragraph when we evaluated the change in the expected value between $U_{t_2}(\tilde{a}|s)$ and $U_{t_1}(\tilde{a}|s)$ weighted by $k_{\tilde{a},s}^{t_1,t_1}$). If any convergence criterion is not reached, we continue simulating the trading game and updating the beliefs until all convergence criteria are satisfied.

B. Parameter Benchmarking

In order to ascertain the suitability of our model parameters choices, we employ the [Hollifield et al. \(2006\)](#) approach for empirically estimating trader arrival rate, asset volatility, and private value distributions in limit order markets for two FTSE-100 stocks listed on the London Stock Exchange (LSE): Vodafone (VOD) and Hargreaves Lansdown (HRGV). The two stocks belong to the largest and smallest market value quartile of the FTSE-100 index, respectively. We use message-level data from LSE's rebuild order book service for the month of January 2015. The data contains a record of the submission, cancellation, and execution of each visible order submitted to the LSE. Each message is time-stamped to the millisecond.

Hollifield et al. (2006) model traders' order submission decisions in limit order markets as a function of their private valuations, expected execution probabilities and picking-off risk, and market conditions. They employ a two-step process to compute the model parameters. In the first step, they estimate the execution probabilities and picking-off risk of different orders as a function of variables capturing the state of the limit order book, past order submission activity, order characteristics, and market conditions. In the second step, they use the execution probabilities and picking-off risk to estimate the trader arrival rates, private value distributions, and execution costs using maximum likelihood estimation.

Hollifield et al. (2006) estimate the gains from trade using this approach using data from the Vancouver Stock Exchange – primarily a venture capital exchange – for the period between May 1990 and November 1993. Due to differences in the specific markets, data, and time periods analyzed, we deviate from their approach in the following ways:

1. We identify all unexecuted immediate-or-cancel and fill-or-kill limit orders as those that have the same order submission and order cancellation timestamp and exclude them from our analysis.
2. We exclude multi-day orders from our analysis.
3. For computational reasons, we estimate the parameters independently for each day of our sample. Consequently, we exclude the exogenous variables used by Hollifield et al. (2006) as these are only updated on a daily basis.
4. While estimating the conditional distributions of time to execution and time to cancellation, we treat all limit orders that survive longer than two hours as censored observations. The corresponding number in Hollifield et al. (2006) is two days.
5. Similarly, we compute the probability of execution within two hours instead of two days for each limit order type.
6. We set execution costs to zero.

7. We exclude all opening, closing, intraday, and volatility auctions from our analysis and focus only on the continuous trading session.

Besides these deviations, we exactly follow the [Hollifield et al. \(2006\)](#) two-step estimation approach. We do not compute individual and market-wide trading gains, as our focus is in identifying the optimized model parameters. Specifically, we compute the trader arrival rate λ , fundamental volatility λ_v , and private value distribution F_α .

We start by calibrating the speed of time clock update. In our model, this value is linked with the trader arrival rate for the asset. The fundamental value volatility and traders' reentry rate are further determined relative to this trader arrival rate. [Goettler et al. \(2009\)](#), based on data from [Hollifield et al. \(2006\)](#), use a time clock update frequency of approximately one minute and link this rate to the daily number of trades. We rely on the same intuition, but instead use the daily number of trades to compute the *new* trader arrival rate in seconds. In our model – similar to [Goettler et al. \(2005, 2009\)](#) – all traders remains part of the trading game until their orders are executed. Hence the number of trades acts as a proxy for the number of *new* trader arrivals. We compute the average time clock speed, $ClockSpeed_t$ on day t as:

$$ClockSpeed_t = \frac{8.5 \times 3600}{2 \times No. Of Trades_t} \quad (9)$$

where the numerator corresponds to the number of seconds in a trading day. We multiply the number of trades in the denominator by 2 as each trade involves an aggressive order trading against a passive order sitting in the limit order book.

The intensity of the Poisson process governing the change in fundamental value λ_v corresponds to the number of time units, after which the fundamental value v changes by one tick. This translates to return volatility on day t denoted by:

$$Volatility_t = \sqrt{\frac{8.5 \times 3600}{ClockSpeed_t \times k}} \times Rel. Tick Size_t \quad (10)$$

where k denotes the frequency with which the fundamental value changes. $k = 8$ ($k = 1.6$) in the low (high) volatility setting i.e., the fundamental value changes after every eight (four) trader arrivals. We compare this with the standard deviation of realized one-minute returns on day t .

In our model, the distribution of private values F_α is discrete. However, [Hollifield et al. \(2006\)](#) parameterize the agents' private values as a mixture of two normal distributions with standard deviation denoted by σ_1 and σ_2 , and their corresponding weights denoted by ρ and $(1 - \rho)$, respectively. Specifically, the private value distribution F_α is parameterized as:

$$F_\alpha = \rho \left(\frac{\alpha}{y_t \sigma_1} \right) + (1 - \rho) \left(\frac{\alpha}{y_t \sigma_2} \right) \quad (11)$$

where y_t is the common value of the stock on day t proxied by the opening price. We assign the probability mass for all private values in the interval $[-\infty, -6]$, $[-6, -2]$, $[-2, 2]$, $[2, 6]$, and $[6, \infty]$, respectively to the five discrete values of $\alpha \in \{-8, -4, 0, 4, 8\}$ in our model.

Insert Table [A1](#) about here

Table [A1](#) contains the parameter values. Consistent with VOD and HRGV belonging to largest and smallest market value quartile of the FTSE-100 index, the former has a lower time between trader arrivals and lower volatility as compared to the latter. For VOD (HRGV), the average time between new trader arrivals λ_t is 1.3 (7.7) seconds. Based on these estimates, we obtain average low (high) intraday volatility estimates of 1.2% (2.7%) for VOD and 1.4% (3.2%) for HRGV.¹⁸ These estimates are comparable to realized intraday volatility of 1.4% for VOD and 1.8% for HRGV and suggest that our arrival rate and fundamental value volatility scale reasonably well under the assumption of a unit of time in our model being equivalent to few seconds in calendar time. This is consistent with an increase in trading speed due to widespread electronification, investments in technology by exchanges

¹⁸These estimates are based on the actual value of one tick, which is approximately 2bps for VOD and 6bps for HRGV.

and market participants, and use of algorithmic trading in global equity markets. Finally, the fraction of traders with absolute private value $|\alpha| = 8$, $|\alpha| = 4$, and $|\alpha| = 0$ is respectively equal to 24%, 42%, and 34% for VOD, and 14%, 38%, and 48% for HRGV. On the one hand, in our parameter set, traders with $|\alpha| = 8$ ($|\alpha| = 0$) have a higher (lower) weight as compared to the corresponding fractions for VOD and HRGV. On the other hand, our chosen value for traders with $|\alpha| = 4$ is close the corresponding values for VOD and HRGV. In conclusion, our chosen parameters seem largely comparable to modern electronic equity exchanges such as the LSE.

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Table 1. Trading Behavior

Panel A reports the distribution of sell limit and sell market orders executed by each agent type and the probability of submitting a limit order at the best ask price. Panel B reports the picking-off risk, defined as the proportion of sell limit orders executed below the fundamental value, for limit orders executed by each agent type. Panel C reports the average time to execution of limit orders, defined as the difference between the order execution time and the agent’s market entry time, executed by each agent type. We generate a simulated dataset under the following two cases: (i) an environment with low picking-off risk, i.e., low volatility, in which we employ $\lambda_v = 0.125$ and (ii) an environment with high picking-off risk, i.e., high volatility, in which we employ $\lambda_v = 0.625$. We report these frequencies for a single and fragmented market. We report all statistics for a single and fragmented market when volatility is low and when volatility is high.

Order Type		Low volatility: $\lambda_v = 0.125$ (Low levels of picking-off risk)						High volatility: $\lambda_v = 0.625$ (High levels of picking-off risk)					
		Aggr.			Private value $ \alpha $			Aggr.			Private value $ \alpha $		
		Prob.	0	4	8	Total		Prob.	0	4	8	Total	
Panel A: Order Submissions													
Single Market	Limit	35.9%	77.1%	52.5%	19.5%	–	–	24.7%	21.4%	65.4%	58.2%	–	–
	Market	–	22.9%	47.5%	80.5%	–	–	–	78.6%	34.7%	41.9%	–	–
Frag Market	Limit	28.5%	78.4%	50.7%	20.7%	–	–	22.8%	51.3%	57.5%	38.7%	–	–
	Market	–	21.6%	49.3%	79.3%	–	–	–	48.7%	42.5%	61.3%	–	–
Panel B: Picking-Off Risk													
Single Market	–	–	4.1%	26.8%	73.9%	21.8%	–	–	13.8%	53.0%	85.4%	59.3%	–
	–	–	3.0%	25.8%	72.1%	20.8%	–	–	10.9%	46.4%	75.7%	42.3%	–
Panel C: Time to Execution													
Single Market	–	–	14.9	3.5	2.1	8.6	–	–	23.8	2.9	1.2	4.9	–
	–	–	11.5	3.7	1.9	7.1	–	–	12.1	3.7	1.7	5.8	–

Table 2. Impact on Bid-Ask Spread

This table compares the impacts of fragmentation on the bid-ask spread. We generate a simulated dataset under the following two cases: (i) an environment with low picking-off risk, i.e., low volatility, in which we employ $\lambda_v = 0.125$, and (ii) an environment with high picking-off risk, i.e., high volatility, in which we employ $\lambda_v = 0.625$. We report the picking-off risk as the proportion of orders executed below the fundamental value for the sell side and above the fundamental value for the buy side. We also report the probability of submitting an aggressive limit order as the proportion of orders submitted at the best ask price over the total number of limit orders submitted on the sell side. We compute local and inside versions of spread measures. The former is based on the local quotes, i.e., the bid and ask prices of a local market, whereas the latter is based on inside quotes, i.e., the highest bid and the lowest ask across the two limit order books. In the single-market setting, the local and inside quotes are identical. Quoted spread is the difference between the best bid and best ask prices. Effective spread is defined in Equation (3). We report all statistics for a single and fragmented market when volatility is low and when volatility is high. We omit standard errors for the differences because a large number of trader arrivals leads to a difference in means to the order of 10^{-2} being statistically significant.

	Low volatility: $\lambda_v = 0.125$			High volatility: $\lambda_v = 0.625$		
	(Low levels of picking-off risk)			(High levels of picking-off risk)		
	Single Market (1)	Frag Market (2)	Difference (2)-(1)	Single Market (1)	Frag Market (2)	Difference (2)-(1)
Quoted Spread: Local	1.50	2.56	1.06	5.13	4.93	-0.20
Quoted Spread: Inside	1.50	1.86	0.36	5.13	3.66	-1.47
Effective Spread: Local	1.34	1.89	0.55	3.58	3.53	-0.05
Effective Spread: Inside	1.34	1.57	0.23	3.58	2.89	-0.70

Table 3. Decomposition of Welfare by Agent Type

This table reports the welfare, defined as the average realized payoff. In addition, this table presents the waiting cost and money transfer as defined in Equation 5. These measures are reported for each agent type. We report all statistics for a single and fragmented market under high ($\lambda_v = 0.125$) and low ($\lambda_v = 0.625$.) levels of volatility. The last column reports the average aggregate welfare per period. All measures are reported in ticks.

	Average welfare per trader			Waiting cost per trader			Money transfer per trader			Aggr. Welfare per period			
	Abs. Private Value		Total	Abs. Private Value		Total	Abs. Private Value		Total				
	0	4	8	0	4	8	0	4	8				
Panel A: Low levels of picking-off risk													
Single Market	0.543	3.510	7.265	3.745	0.000	-0.350	-0.162	-0.189	0.543	-0.140	-0.572	-0.065	3.745
Frag Market	0.626	3.479	7.202	3.740	0.000	-0.355	-0.172	-0.193	0.626	-0.166	-0.626	-0.066	3.740
Panel B: High levels of picking-off risk													
Single Market	0.606	3.398	7.039	3.652	0.000	-0.367	-0.270	-0.228	0.606	-0.235	-0.691	-0.119	3.652
Frag Market	0.817	3.389	6.871	3.662	0.000	-0.417	-0.284	-0.252	0.817	-0.192	-0.845	-0.085	3.662

Table 4. Panel Regressions of Local Liquidity on Fragmentation and Volatility

This table presents the results of panel regressions of relative quoted and effected local spreads on the listing exchange for the sample of French and German stocks. *Fragmentation* is 1 minus the Herfindahl-Hirschman Index of trading volumes on the listing exchange and Chi-X. *Volatility* quintiles are computed within stock. *Volume(log)* is the natural logarithm of the euro trading volume. All regressions include stock-fixed effects and the first four models also include day-fixed effects. We double cluster standard errors by stock and day. *, **, *** denote significance at 10%, 5%, and 1%, respectively.

	(1) Quoted	(2) Effective	(3) Quoted	(4) Effective
Fragmentation	1.310 (0.74)	0.067 (0.05)	1.722 (1.05)	0.489 (0.36)
Volatility 1st quintile	-1.854*** (-3.12)	-1.632*** (-2.79)	-1.516*** (-2.69)	-1.658*** (-3.01)
Volatility 2nd quintile	-0.759*** (-2.99)	-0.133 (-0.43)	-0.666** (-2.52)	-0.142 (-0.45)
Volatility 4th quintile	0.644 (0.99)	1.252** (2.28)	0.627 (1.05)	1.183** (2.40)
Volatility 5th quintile	2.124** (2.47)	2.870*** (3.59)	1.967** (2.33)	2.577*** (3.44)
Volatility 1st quintile x Fragmentation	2.207 (1.62)	2.327* (1.75)	2.423* (1.84)	2.757** (2.20)
Volatility 2nd quintile x Fragmentation	0.928* (1.69)	-0.284 (-0.41)	1.048* (1.80)	-0.153 (-0.22)
Volatility 4th quintile x Fragmentation	-0.925 (-0.64)	-2.635** (-2.11)	-1.108 (-0.84)	-2.565** (-2.32)
Volatility 5th quintile x Fragmentation	-3.259* (-1.68)	-5.511*** (-2.96)	-3.701* (-1.96)	-5.211*** (-3.05)
Volume (log)	-1.306*** (-7.77)	-0.321** (-2.10)	-1.453*** (-8.15)	-0.219 (-1.16)
Constant	29.105*** (10.80)	11.645*** (4.36)	31.441*** (11.09)	9.737*** (2.88)
Vola 5 x Frag – Vola 1 x Frag	-5.466	-7.838	-6.124	-7.968
p-value	0.016	<0.001	0.009	<0.001
Stock FE	Yes	Yes	Yes	Yes
Day FE	No	No	Yes	Yes
Observations	14097	14097	14097	14097

Table 5. Panel Regressions of Inside Liquidity on Fragmentation and Volatility

This table presents the results of panel regressions of relative quoted and effected inside spreads for the sample of French and German stocks. *Fragmentation* is 1 minus the Herfindahl-Hirschman Index of trading volumes on the listing exchange and Chi-X. *Volatility* quintiles are computed within stock. *Volume(log)* is the natural logarithm of the euro trading volume. All regressions include stock-fixed effects and the first four models also include day-fixed effects. We double cluster standard errors by stock and day. *, **, *** denote significance at 10%, 5%, and 1%, respectively.

	(1) Quoted	(2) Effective	(3) Quoted	(4) Effective
Fragmentation	-2.505** (-2.39)	-4.860*** (-4.64)	-1.983** (-2.13)	-4.363*** (-4.20)
Volatility 1st quintile	-1.816*** (-3.57)	-2.089*** (-3.51)	-1.560*** (-3.14)	-2.095*** (-3.57)
Volatility 2nd quintile	-0.994** (-2.59)	-1.472*** (-2.78)	-0.918** (-2.45)	-1.475*** (-2.65)
Volatility 4th quintile	0.125 (0.70)	-0.114 (-0.41)	0.145 (0.84)	-0.149 (-0.48)
Volatility 5th quintile	1.408*** (3.05)	1.598*** (3.02)	1.207** (2.56)	1.342** (2.54)
Volatility 1st quintile x Fragmentation	2.502** (2.20)	3.715*** (2.73)	2.755** (2.42)	4.021*** (3.02)
Volatility 2nd quintile x Fragmentation	1.657* (1.87)	2.966** (2.43)	1.769** (2.02)	3.068** (2.40)
Volatility 4th quintile x Fragmentation	0.163 (0.35)	0.508 (0.72)	-0.057 (-0.13)	0.514 (0.67)
Volatility 5th quintile x Fragmentation	-1.894* (-1.80)	-2.831** (-2.24)	-2.080* (-1.96)	-2.551** (-2.04)
Volume (log)	-1.132*** (-7.69)	-0.310** (-2.34)	-1.254*** (-8.32)	-0.231 (-1.39)
Constant	26.765*** (10.64)	12.600*** (5.21)	28.616*** (11.26)	11.051*** (3.71)
Vola 5 x Frag – Vola 1 x Frag	-4.396	-6.546	-4.836	-6.571
p-value	0.022	0.001	0.010	0.001
Stock FE	Yes	Yes	Yes	Yes
Day FE	No	No	Yes	Yes
Observations	14097	14097	14097	14097

Table A1. Calibrated Model Parameters

This table presents the model parameters calibrated using message-level data for two FTSE-100 stocks from the London Stock Exchange. We report mean values of all parameters for the 21 trading days of January-2015. Trades Count is the number of transactions during the daily continuous trading session. $ClockSpeed_t$ is the number of seconds between two trader arrivals. Relative Tick Size is the ratio of one tick (in GBp) and the daily opening price. Low (High) volatility λ_t^L (λ_t^H) is the average daily volatility assuming a change in the fundamental value, on average, after every 8 (1.6) trader arrivals. Empirical volatility is the daily volatility computed based on 1-minute returns. $F_{|\alpha|=8}$, $F_{|\alpha|=4}$, $F_{\alpha=0}$ is the fraction of traders with $|\alpha| = 8$, $|\alpha| = 4$, $\alpha = 0$, respectively. We report the mean values for all parameters across the 21 trading days in January 2015.

	VOD	HRGV
Trade Count	12,156	2,149
$ClockSpeed_t$ in seconds	1.3	7.7
Relative Tick Size	2bps	6bps
Low Volatility ($Volatility_t^L$)	1.2%	1.4%
High Volatility ($Volatility_t^H$)	2.7%	3.2%
Empirical Volatility	1.4%	1.8%
$F_{ \alpha =8}$	24%	14%
$F_{ \alpha =4}$	42%	38%
$F_{\alpha=0}$	34%	48%

Table A2. Decomposition of Welfare by Trader Type

This table reports the welfare defined as the average realized payoff. In addition, this table presents waiting cost and money transfer defined in Equation 5. These measures are reported for each agent type. We report all statistics for a single and fragmented market under the low volatility parameterization. In addition to reporting the welfare statistics in the single and fragmented market setting with $F_\alpha = \{0.15, 0.35, 0.65, 0.85, 1.0\}$ (baseline parameterization), we also report the welfare statistics for a fragmented market with the population of agents with $\alpha = 0$ doubled, i.e. $F_\alpha = \{0.15/1.3, 0.35/1.3, 0.65/1.3, 0.85/1.3, 1.0\}$. The last column reports the average aggregate welfare per period. All measures are reported in ticks.

	Average welfare per trader			Waiting cost per trader			Money transfer per trader			Aggr. Welfare per period			
	Abs. Private Value		Total	Abs. Private Value		Total	Abs. Private Value		Total				
	0	4	8	0	4	8	0	4	8				
Single Market	0.543	3.510	7.265	3.745	0.000	-0.350	-0.162	-0.189	0.543	-0.140	-0.572	-0.065	3.745
Frag Market	0.626	3.479	7.202	3.740	0.000	-0.355	-0.172	-0.193	0.626	-0.166	-0.626	-0.066	3.740
Frag Market $2 \times \alpha = 0$	0.485	3.312	7.137	2.890	0.000	-0.127	-0.029	-0.046	0.485	-0.561	-0.835	-0.142	3.757

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