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PAPER

A stock-flow consistent input–output model with applications to energy price shocks, interest rates, and heat emissions

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Abstract

By synthesizing stock-flow consistent models, input–output models, and aspects of ecological macroeconomics, a method is developed to simultaneously model monetary flows through the financial system, flows of produced goods and services through the real economy, and flows of physical materials through the natural environment. This paper highlights the linkages between the physical environment and the economic system by emphasizing the role of the energy industry. A conceptual model is developed in general form with an arbitrary number of sectors, while emphasizing connections with the agent-based, econophysics, and complexity economics literature. First, we use the model to challenge claims that 0% interest rates are a necessary condition for a stationary economy and conduct a stability analysis within the parameter space of interest rates and consumption parameters of an economy in stock-flow equilibrium. Second, we analyze the role of energy price shocks in contributing to recessions, incorporating several propagation and amplification mechanisms. Third, implied heat emissions from energy conversion and the effect of anthropogenic heat flux on climate change are considered in light of a minimal single-layer atmosphere climate model, although the model is only implicitly, not explicitly, linked to the economic model.

1. Introduction

One of the key issues faced by modern society is navigating the transformation towards a sustainable economy that respects 'planetary boundaries' [1]. This transformation could be facilitated by a macroeconomic theory capable of robustly dealing with issues of energy use and emissions, but much of economic theory underemphasizes the importance of energy and natural resources [2]. In addition, many general equilibrium models abstract from institutional details of money creation and monetary flows which play a central role in real-world macroeconomic dynamics. This paper suggests one alternative route to an ecological macroeconomic model with Keynesian features, in which finance plays a central role.

One effort to explicitly represent the dynamics of debt, finance, and other monetary factors has been the post-Keynesian stock-flow consistent (SFC) approach. At the same time, input–output (IO) models have been widely used to investigate sectoral interdependencies within the real economy, while environmentally extended IO models have been used to analyze the relationship between the economy and ecological subsystems. However, the role of monetary dynamics has been left relatively unexplored in IO models [3]. This paper proposes a synthesis of elements from both SFC and IO models with insights from ecological economics to provide an avenue for investigating the interrelations between the monetary economy and the physical environment.

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The first section provides an introduction to the three fields and highlights common ground in section 1.4, showing that there is no serious theoretical impediment to integrating the three approaches. This section also underlines links to econophysics, complexity economics, and agent-based models (ABM). In section 2, we present a conceptual model combining aspects of both SFC and IO approaches and apply it to study the impact of energy price shocks and to contribute to the discussion of whether a stationary economy is compatible with positive interest rates in section 3. In section 4, implied heat emissions from energy conversion are considered in light of a minimal single-layer atmosphere climate model showing the impact of anthropogenic heat flux on climate change. We suggest that coupling well-developed large scale macroeconomic and climate models could enable fruitful analysis of the interlinkages between the economy and the physical environment, which would be relevant to addressing the issue of global climate change. A brief conclusion assesses the relevance of the contribution and potential extensions.

1.1. Stock-flow consistent (SFC) models

SFC models are a class of structural macroeconomic models grounded by a detailed and careful articulation of accounting relationships. The basis of this methodology can be found in the work of Copeland and Stone, who advocated for macroeconomic models developed with the use of social accounting matrices (SAMs) that tabulate stocks and flows of funds within the national accounts [4], and is also similar to Stützel's 'Saldenmechanik' ('balance sheet mechanics') [5]. Godley, Lavoie, and a number of other authors expanded this approach into a family of applied macroeconomic models that respect accounting identities and are closed with behavioral assumptions based on post-Keynesian theory [6–8]. Of course, models possessing neoclassical features such as intertemporal optimization and rational expectations can also be closed with accounting identities, but stock-flow consistency has been less emphasized in neoclassical models than in post-Keynesian models.

SFC models are constructed by tabulating individual sector budget constraints implied by a modified SAM, which is in the literature referred to as a transaction flow matrix. Each row of the matrix represents a transaction, while each column represents a different sector. In a manner consistent with flow of funds (FF) accounts, sources and uses of funds are represented, respectively, by positive and negative signs. All rows and columns sum to zero for financial transactions, since every financial asset has a corresponding financial liability. Because all flows necessarily accumulate to stocks, the flows of funds represented on the transaction flow matrix directly imply stock and balance sheet adjustments, which creates additional constraints. This accounting verification process is designed to ensure the model's internal logical consistency by removing potential 'black holes' [9, p 7], and by respecting a 'fundamental law of macroeconomics analogous to the principle of conservation of energy in physics' [10, p 18].

Accounting identities remove degrees of freedom from potential macroeconomic outcomes, forming a skeletal structure that can be closed by a number of competing theoretical arguments. James Tobin's Yale portfolio models introduced one such possible closure, directly inspiring the approach to portfolio allocation dominant in the SFC approach. However, despite bearing some similarity with post-Keynesian SFC models, Tobin's models relied upon loanable funds theory and an exogenous money supply. Because post-Keynesian SFC models are closed differently from Tobin's models, SFC models behave differently, and abstract less from the institutional reality of modern monetary systems [11].

Of course, causal relationships between accounting identities can also be specified via neoclassical behavioral closures, such as rational expectations and optimizing representative agents. However, with some notable exceptions [12], banks are typically modelled as reserve-constrained intermediaries with loans driven by savings and deposits in a loanable funds market. This contrasts with a substantial body of work in post-Keynesian monetary theory, along with the statements of central bankers who say that credit money is created endogenously via loan origination [13, 14]. Banks do not lend reserves, but rather make loans by simultaneously expanding both sides of their balance sheets, creating an asset of the bank (a loan) and a liability of the bank (a deposit) [15]. Furthermore, reserve requirements do not limit bank lending, because the central bank must accommodate demand for reserves by banks in order to maintain par clearing and in order to peg its target short term interest rate [16, p 283] [17, 18].

Banks create credit *ex nihilo* by creating a liability with a corresponding asset, and balance sheets must always expand or contract equally according to a symmetry principle inherent in double entry bookkeeping. Braun depicts this process with a limited analogy to physics, in which asset units are considered as money and liability units as 'antimoney' [19]. This symmetry principle, which allows the expansion and contraction of the money stock to occur endogenously, differs from many models in econophysics, in which the quantity of money as an exogenously given stock is conserved, and the focus is placed squarely upon the *exchange* of wealth [20–22]. This common approach ignores the role of production and credit creation in economic activity and disregards standard definitions of economic concepts such as transactions and income [19, 23, 24]. Money can be modeled

more realistically as a non-conservative entity, and economists have begun to build SFC ABM that may be of interest to econophysicists [25, 26].

Perhaps the single most important advantage of the SFC approach is that it enables the modeler to easily create scalable representations of institutional structures with an explicit monetary dimension. The central importance of attention to financial detail was illustrated by the failure of the macroeconomics profession to anticipate the 2007–2008 Global Financial Crisis, which was predicted nearly exclusively by those who deployed implicit or explicit macro-accounting frameworks [27–29]. Some recent dynamic stochastic general equilibrium (DSGE) models have added financial frictions, and have been able to replicate time series data from the 2007–2008 financial crisis and its aftermath [30]. Many of these models utilize the financial accelerator as a mechanism through which financial shocks can be amplified and propagated to impact the real economy [31]. Though an encouraging development, the pseudo-out-of-sample ability of such recent DSGE models to match historical data does not necessarily entail future capability of predicting novel facts. By contrast, economists who paid close attention to accounting fundamentals have achieved a notable degree of predictive success, in particular anticipating the macroeconomic dangers of rising household debt, predicting that the institutional structure of the Euro was incompatible with long-term economic stability, and that quantitative easing policies would prove ineffective [8, 32–36]. For a more comprehensive review of 'Stock-Flow Consistent Modelling Through the Ages' see [37].

1.2. IO models

IO models provide a detailed treatment of production and of the flow of real goods and services through the economy, and are commonly applied to analyze interactions and feedback effects between mutually interdependent industrial sectors. As mentioned, SFC models focus primarily on explicating flows of financial funds, and therefore often underemphasize real production. Indeed, most SFC models only include a single productive sector, and nearly all multisectoral models abstract from intermediate production. Aspects of the IO approach, which provides a detailed picture of a complex multisectoral economy, can therefore be used to import a more refined analysis of the real economy into a SFC framework. The IO approach can be traced back to classical authors [38]. The first modern IO model was created by Alfred Kähler [39], and developed by Wassily Leontief [40] into the sort of large scale empirical model now routinely produced by statistical agencies in countries across the globe.

IO tables provide a static snapshot view of the economy, assuming constant returns to scale. IO tables are displayed in matrix notation ('Leontief matrix'), where each column represents inputs to a specific sector, while each row shows the output from a given sector to the rest of the economy. For an economy with n sectors, a $n \times n$ matrix $\mathbf{a}$ is used, where $a_{ij} \geqslant 0$ is a flow of inputs produced by sector i to sector j in order to produce one unit of output j. To produce the gross outputs of the different sectors, which are displayed as the elements of a vector $\boldsymbol{x}$, a different vector $\mathbf{a}\boldsymbol{x}$ is required as intermediate inputs. Therefore, in every time period, gross output and final demand $\boldsymbol{d}$ (also referred to as net output or gross domestic product (GDP)) are coupled by:

$$\boldsymbol{x} = \mathbf{a}\boldsymbol{x} + \boldsymbol{d}, \tag{1}$$

$$\boldsymbol{x} = (\mathbf{1} - \mathbf{a})^{-1}\boldsymbol{d}. \tag{2}$$

To obtain a unique and positive solution, $(\mathbf{1} - \mathbf{a})$ has to be invertible and the principal minors have to be positive, known as the Hawkins–Simon conditions [41, pp 58ff][42], in order to guarantee that each subsystem is 'productive' such that it requires less input than it produces in terms of output.

Dynamic input–output (DIO) models incorporate a feedback effect of investment on future production which is ignored by most IO models: when industries invest, the flow of investment adds to their capital stock. A capital coefficients matrix can then be calculated, and the capital coefficients matrix can be used to adjust the DIO models Leontief technical coefficient matrix $\mathbf{a}$ from the previous time period [41, pp 639–642]. In general, environmental impacts can be deduced from a combined environmental and economic accounting [43]. To track the flow of income and its distribution among sectors, SAMs have been attached to IO models [41, pp 499–542], and IO models which incorporate SAMs are not dissimilar in general form and purpose to the model as presented later. Note that SAMs, which served as a basis for the development of SFC models, are a linkage between IO and SFC models. However, effective demand is often left exogenous in DIO models and is not determined by a logically consistent system of SFC FF equations.

While most macroeconomic models focus on net output (GDP), it is necessary to consider gross output (including intermediate production) in order to analyze the physical and environmental consequences of economic activity, since intermediate production produces waste, heat emissions, and greenhouse gas emissions just as does final production. For ecological applications such as understanding the sources and causes of heat and greenhouse gas emissions, flows of physical materials must be explicitly modeled [44]. Environmental input–output (EIO) models have been widely used to model raw material use in the production of gross output.

By expanding the SAM entries within an EEIO framework, one can analyze the impact of economic activity on a wide range of environmental variables such as energy use, land use, material use, water use, and greenhouse gas emissions [45].

1.3. The impact of energy and ecological macroeconomics

Some ecological economists have criticized approaches such as SFC models and IO models on the grounds that they focus on the circular flow of exchange value (i.e. money), rather than on the physical throughput of natural resources from which all goods and services are ultimately derived [46–50], disregarding the association between the growth of wealth and the expansion of energy services over the last several centuries [51, 52]. Some pre-classical, physiocratic, and early nineteenth century classical economists focused on the physical side of economic activity, but it has generally been downplayed in most modern theories [2, 53, 54]: most economists interpret energy services as enhanced labor or capital productivity [2, p 52] associated with technological progress, which is considered as an amorphous force that increases productive power without limit [55, p 206].

Some economists have considered energy E as a factor of production, sometimes in combination with materials M, but have underestimated the importance of those factors. The responsiveness of output to a marginal change of one production factor in the neoclassical approach is given by its output elasticity $E_{y,x}$, the point elasticity of output y of an entity with respect to a production factor x:

$$E_{y,x} = \frac{x}{y}\frac{\partial y}{\partial x}. \tag{3}$$

The theory assumes that in equilibrium, this should be identical to the cost share of the production factor. Energy costs represent about five percent of production costs; consequently, the output elasticity of energy has been estimated to be 0.05. As this is low compared to labor with 0.7 or capital with 0.25 during recent decades in OECD countries, energy has been left out of most economic models [55, p 207] [56], see [2, pp 180–212] for a longer discussion. Based on a general equilibrium framework extended by incorporating energy as a production factor, Kümmel uses nonlinear optimization with generalized shadow prices on real data to calculate time-averaged output elasticities of 0.37 for capital, 0.11 for routine labor, and 0.52 for energy, while the remaining 0.12 is ascribed to creativity [2, pp 180, 212]. Similar values are found by Ayres *et al* [57, 58]. They show that cost share and output elasticity are not equal once a third factor is added that is not independent of the other two [58, p 16]. This is the case here, since 'capital in the absence of energy is functionally inert', and technical engineering constraints limit substitution [2, p 195]. Using these elasticities, energy accounts for most of the growth attributed to technological progress [2, p 221]. This shows that the assumption that factor costs and output elasticities are identical is flawed, and the neglect of energy is without solid foundation.

The significance of these findings is underlined by the International Monetary Fund, which investigates the impact of lower oil supply in its World Economic Outlook, stating that 'if the contribution of oil to output proved much larger than its cost share, the effects could be dramatic, suggesting a need for urgent policy action' [59, p 109]. A declining capacity to extract energy has sometimes been an important trigger of societal collapse [60, p 36] [61, pp 91–122]. This not only has historical implications, but could also potentially impact theoretical accounts of modern business cycles. Hamilton [62, 63] and Murphy and Hall [64] present evidence that every US recession since World War II was accompanied by rising energy prices. In section 3.2, we suggest that this could have been caused by a drop in effective demand due to higher energy prices. Given the naturally constrained supply of fossil fuels, the connection between energy and the economy must be understood in order to avert potential challenges to the modern global industrial system, which currently depends categorically on fossil fuels and other non-renewable energy sources [65]. In a purely physical model, Dale *et al* found that future energy conversion and net energy available for industrial production is not limited by the availability of energy *as such*, but rather by the *capacity to extract renewable resources* due to the fact that the buildup of the capital stock requires energy input [66]. Other studies have underlined the contemporary significance of energy in terms of the 'energy return on investment', which is the usable energy acquired divided by the amount of energy expended to extract and process that energy resource [67].

In order for economic activity to be environmentally sustainable, it must be the case that the ecosystem can absorb waste and recycle the inputs which are required for physical production [68, p 186]. Therefore, the physical and environmental sustainability of the economy can best be analyzed by considering the economy as an open subsystem of the larger but finite physical ecosystem. Energy usage entails heat and particle emissions and increases entropy [2]. While energy conservation may provide a partial solution to this problem, there are inescapable thermodynamic limits to energy conservation which may limit decoupling of resource use and economic growth. Furthermore, rebound effects (first discovered by the economist William Stanley Jevons [69]) mean that demand for energy may not necessarily decline even if energy conservation measures render such a decline technically feasible [2, 70].

For these reasons, some ecological economists argue that the necessity of adapting to planetary boundaries and resource extraction limits may decrease energy supply, and the constraint of this main driver of economic growth may render a stationary economy or economic degrowth unavoidable, see Pueyo [71], Jackson [72], and Kallis *et al* [73] for a longer discussion.

1.4. Common ground

At least two authors, Gowdy [74] and Kronenberg [75], have explicitly argued that post-Keynesian economics and ecological economics share substantial common ground, and are ripe for a synthesis. Similarities have been recognized in terms of consumption, production theory, cumulative causation (path dependency), and the irreversibility of historical time [75–77]. Both post-Keynesian and ecological economists emphasize the significance of fundamental 'Knightian' uncertainty, as opposed to computable probabilistic risk [6, 78–80]. Ellsberg also discusses this as the distinction between ambiguity and risk, which has been confirmed experimentally in decision theory [81, 82]. However, post-Keynesians have heretofore tended to neglect the ecological dimension of the economy, which has led Mearman to conclude that 'post-Keynesians need to embrace the environment' in order to underline the relevance of their work [75, 83].

Keynesian macroeconomic theory places great emphasis on the determination of a level of effective demand commensurate with key economic policy goals, but the ecological implications of those economic policy goals have often been neglected. Despite the need for new analytical tools to explore this relationship, relatively little concrete work to that end has been thus far completed [84]. Notable exceptions include the work of Kemp-Benedict [85], Kronenberg [86], the work in progress by Dafermos *et al* [87], and the WWWforEurope project [88]. However, some previous attempts to integrate post-Keynesian and ecological economics are not SFC.

Though most ecological economists abstract from the influences of the monetary side of the economy, some analysis of the monetary dimension of ecological questions has been conducted by Tokic [89], Binswanger [90] and Wenzlaff *et al* [91]. But outside this work, some misunderstandings appear, such as a common claim that a zero interest rate is a stability condition for a stationary economy [92, 93]. We will review this argument in section 3.1. Issues such as monetary policy and interest rates can be most fruitfully discussed within a framework of ecological macroeconomics which is cognizant of the implications of financial flows of funds for the economy [88].

In the famous Cambridge Capital Controversies [94], post-Keynesian authors discovered logical problems with the aggregate neoclassical production function, such as reswitching and reverse capital-deepening. Post-Keynesians therefore rejected the neoclassical aggregate capital stock and the neoclassical production function [95], and do not posit an identity between output elasticity and cost share [53, p 29]. The neoclassical school largely deflected these theoretical critiques by asserting that the properties of production functions had empirical validity, even if the aggregate production function was logically inconsistent. However, an extensive literature demonstrates that many regressions upon deflated monetary data simply measure distributional variables in the national accounts and do not convey meaningful information about parameter values of the production function or technological relationships [95, 96]. This is to say that the regressions are simply estimating accounting identities, which are true by definition but concern a different question. In contrast, post-Keynesian authors view production as a discrete and sequential technically determined process with limited possibility for immediate substitution, as in fixed-coefficient IO models, and as is compatible with ecological economics [75]. Accordingly, the model of production in this paper uses disaggregation to avoid running afoul of some of the serious aggregation problems highlighted by the Cambridge Capital Controversies.

The issue of aggregation has mostly been ignored in macroeconomics, mostly by assuming representative agents [97], but this has recently begun to change with the advent of ABM. In order to implement locality and search costs, bounded rationality and heterogeneity among consumers, the possibility for coordination failures [98], and defaults and network effects [99], ABMs have been proposed and especially applied in econophysics in order to explain distributions with fat tails and volatility clustering, thus enabling the analysis of emergent disequilibrium dynamics created by the interactions of heterogenous agents [100, 101]. Some of these properties are directly linked to the way different time scales are incorporated in the models [102]. ABMs have also been recently applied to explain emergent network structures in interbank markets [103]. The underlying network structure of interaction governs economic evolution [97] in a manner similar to cooperation and defection [104]. As was pioneered by Bergmann [105], ABMs can also integrate a SFC description of monetary stocks and flows, recently rediscovered including endogenous credit creation [25, 26, 106–108]. Consequently, while the model presented in this paper is not an ABM, it is clear that ABMs offer SFC models a potential method to incorporate a greater degree of heterogeneity. Likewise, the SFC framework offers ABMs a way to implement financial macro constraints, which may help ABMs avert the common criticism that their results are driven too much by the choice of particular parameter values. These innovations may help to transform the SFC

perspective from a 'top-down' approach into an agent-based or 'fully-scalable' mode of macroeconomic modeling [25, 26].

If IO models are also incorporated into the analysis, it would be possible (at least in theory) to trace the implications of the behavior of heterogeneous agents in financial markets on flows of physical materials through the economy as well as through the natural world. Agent based input–output models have recently been used to study the effects of carbon credit trading on deforestation in the Amazon rain forest, and have also been applied to studying interdependencies in the vulnerability of critical infrastructure to cascading failure, given inoperability that results due to lack of the technological capacity to smoothly and instantaneously substitute technologically specific infrastructure-capital goods [109, 110]. It should be noted that in terms of network theory, these sorts of physical cascading failures are similar to cascading bank failure processes [103]. Until now, ABM have tended to disregard physical resource flows and energy and therefore miss the 'minimum complexity of endogenous growth models,' as claimed by Ayres [111]. Studying the flows and stocks of physical resources and money between agents within the framework of SFC accounting buttressed by IO data could form the basis of a fruitful research program for complexity economics [97], ecological macroeconomics [88], and ecological econophysics [71], and could be useful in helping to manage a transition towards an environmentally sustainable society.

In sum, by combining SFC models and IO models, financial flows of funds can be integrated with flows of real goods and services. Lawrence Klein, who developed large scale macroeconometric models typified by the famous FRB-MIT-Penn model, has noted the natural synergies between the National Income and Product accounts, the IO accounts, and the FF accounts [112]. The approach of combining both SFC and IO models with ecological macroeconomics affords one method to unite those accounts, as suggested by Klein, and to simultaneously model monetary flows through the financial system, flows of produced goods and services through the real economy, and flows of physical materials through the natural environment. It is hoped that models of this type may provide additional tools to aid macroeconomists, ecological economists, and physicists in the task of understanding the economy and the physical environment as one united and complexly interrelated system, rather than as a colloidal agglomeration of artificially separated analytical domains. It is precisely these modes of analysis that are required to study pressing problems such as climate change, which are neither purely economic, nor purely environmental, nor purely physical, but rather are all of the above [84].

2. SFCIO models—methodology

This section introduces a conceptual baseline model that could serve as a point of common ground between the SFC, IO, and ecological macroeconomics approaches. A SFC model of a closed economy is coupled with an IO model. This approach is similar to the work in progress by the project WWWforEurope, where researchers are also developing a large multi-sectoral model connected to an explicit articulation of financial flows [88]. The model developed in the present paper is represented in discrete time t and includes multiple (n) industry sectors, a household sector, and a government/banking system sector. However, the household sector and the government and banking system sector are both consolidated. This keeps the exposition relatively simple and tractable, and allows focus to remain squarely on the chief aim of integrating elements of an IO treatment of production into a SFC framework and on showing how the model can be applied to ecological macroeconomics. However, in a more complicated and more realistic version of the model, both the household sector and the government/banking system sector (hereafter referred to as simply 'government sector') would be deconsolidated, and heterogeneity in sectors other than the multiple industry sectors could be explicitly modeled. However, creating fully scalable models which articulate that degree of heterogeneity would likely call for an agent-based approach.

The model simultaneously tracks the values of all flows of goods and services through the economy in both nominal terms (measured in terms of money-values) and in real terms (measured in terms of physical units of the heterogeneous real physical output of industry i). In order to more easily identify which variables are in real terms and which are in nominal terms, all nominal variables are written in capital letters. Note that the subscript $_{(t-1)}$, as in $M_{h\,(t-1)}$, indicates the value of the stock at the end of period $t-1$, whereas the subscript $_{(t)}$ refers to the value of the stock at the end of period t.

The simplified model with two sectors used in section 3 is designed to facilitate an easier understanding of the core issues raised in the process of synthesizing SFC models, IO models, and ecological macroeconomics. As shown in the flow diagram in figure 1, a variety of financial flows and physical flows are included in even a simple model with two sectors. All monetary payments (solid lines) flow from one sector to another and accumulate to the corresponding stock, providing consistency between stocks and flows.

Money flows from households to the government in the form of taxes T. Money flows from both the production sector and the energy industry to households in the form of wages $\mathit{Ш}_{p/e}$ and distributed profits $\Pi_{p/e}$.

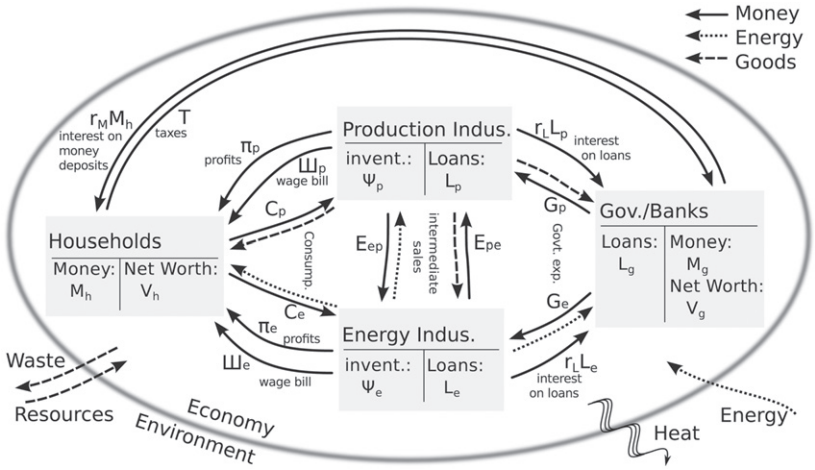

Figure 1. Stocks of sectors and flow chart of money, energy, and materials, *h*: households, *p*: production sector, *e*: energy sector, *g*: government/banking system sector. For each sector, a balance sheet is shown in the form of a T-account. Stocks: M_h, money stock of households; M_h, money issued by government/banks; $L_{p/e}$, loans of production sector/energy sector; L_g, loans made by government/banks; $\psi_{p/e}$, physical inventories of industry sectors. Money flows: $C_{p/e}$, consumption of households; $G_{p/e}$, consumption of government; E_{ep}, money paid by production sector for energy; E_{pe}, money paid by energy sector for intermediate goods; $\mathrm{III}_{p/e}$, wage bill paid by production/energy sector; $\Pi_{p/e}$, distribution of profits; T, taxes; $r_M M_h$, interest payment to households; $r_L L_{p/e}$, interest payments by industry sector. Energy/material flows: energy, energy extracted from the environment; heat, heat emissions; resources, extracted from the environment; waste, emitted to the environment; not treated explicitely in the model, but implied.

In turn, households spend their money on both production goods and energy goods, which creates flows of money $C_{p/e}$ back to the production and energy sectors and corresponding flows of real goods and services back to the households. The government likewise buys both production goods and energy goods, which creates similar flows of both real goods and services and of money $G_{p/e}$ between the government and both of the two industries. The production industry buys energy goods as intermediate inputs, which creates flows of energy goods from the energy industry to the production industry and a corresponding flow of money E_{ep} from the production industry to the energy industry. The inverse is true for purchases of production goods as intermediate inputs by the energy industry E_{pe}. Finally, as physical raw materials are used in production, and as some raw materials are expended as waste, there are flows of physical materials between the human economy and the natural environment. Likewise, energy flows into the economy from the natural world, while heat is emitted by the economy into the natural environment. These economy-nature interactions are not *explicitly* considered in the model, but rather are simply implied. If more than two industry sectors are included, one must incorporate additional interlinkages in the diagram, but the same principles continue to apply. Differently from figure 1, which only includes two industries, the mathematical formulation of the model is shown in general form for an economy with *n* sectors.

The flow diagram in figure 1 also shows the balance sheets of each of the four sectors (the households sector, the government sector, the production goods sector, and the energy goods sector) in the form of T-accounts. Assets are shown on the left side of the T-account, while liabilities and net worth are shown on the right side of the T-account. In accounting, a fundamental equation known as the *balance sheet equation* states that:

$$\text{Assets} = \text{Liabilities} + \text{Net worth}. \tag{4}$$

This means that the left side of the T-account is by definition always equal to the right side of the T-account. This is a symmetry principle, and is why balance sheets are called 'balance' sheets. We distinguish two types of stocks of financial assets: money deposits and loans. Loans appear on the asset side of the government/banking system sector's balance sheet and on the liability side of industry *i*'s balance sheet. Money deposits, on the other hand, appear on the liability side of the government/banking system sector's balance sheet, and on the asset side of the household sector's balance sheet. In the balance sheet perspective, the government sector holds assets of loans L_g on the left side of its T-Account, while it has liabilities of money deposits M_g on the right side. The difference between its assets and liabilities determines its net worth V_g, also shown on the right side of the T-account.

The money stock held by households is designated as M_h, which is always equal to M_g, because the consistent accounting in the model ensures that this will be the case, without the need for an explicit equilibrium condition equation specifying that the money 'supplied' by the government sector is equal to the money 'demanded' by the household sector.

Table 1. Balance sheet matrix in nominal terms. The money deposits of households M_h are equivalent to the money issued by the government M_g, because we assume that the industry sector does not hold money deposits. The loans L_i of each sector i are equal to the unit cost of inventories Ψ_i. They sum up to the total outstanding loans of the government/banking sector L_g. Since financial assets and liabilities within the economy sum up to zero, the net worth of the system as a whole is equal to the value of inventories (the only real asset). All sums over i are proceeded over the n industry sectors.

	Households	Industry $i \in \{1, \ldots, n\}$	Government	$\sum$
Money deposits	$+M_h$		$-M_g$	0
Loans		$-L_i$	$+L_g$	0
Inventories		$+\Psi_i$		$+\sum_i \Psi_i$
Net worth	$-V_h$	0	$-V_g$	$-V_h - V_g$
$\sum$	0	0	0	0

In addition to stocks of financial assets (money deposits and loans), stocks of real assets also appear on balance sheets. A heterogeneous vector of inventories consisting of all the unsold output of each industry i at the end of each period constitutes the real assets of the model economy. These inventories are denoted by ψ_i, each held on the balance sheet of the corresponding industry sector, and are valued at unit costs. The monetary value of the stock of inventories at unit costs is signified by Ψ_i. The production goods industry holds assets of production good inventories Ψ_p on the left, counterbalanced by loans L_p on the right. The energy goods industry similarly holds assets of energy good inventories Ψ_e on the left, counterbalanced by loans L_e on the right. It is assumed as a simplification that industries do not hold stockpiles of cash, and instead distribute all excess cash holdings at the end of each period to their owners in the household sector, keeping their net worth at zero. Since real assets can change in value, maintaining the symmetry principle requires that loans adjust in response to a change in the value of a real asset. Since for every financial asset in the economy there is a corresponding financial liability, the net worth of the model as a whole consists only of the monetary values of real assets (inventories), because all financial assets and financial liabilities must necessarily sum to zero.

The very existence of stocks introduces historical time and a certain path dependence into the model. Even though the model may asymptotically converge to a steady state if all exogenous parameters are undisturbed, the model will follow a different traverse path for every possible set of stocks. Moreover, not all conceivable sets of stocks are in fact possible; only some sets of stocks are consistent with the model's accounting. Thus, depending upon the set of stocks with which the model economy has been endowed by the past, the model will follow a different trajectory forwards into the future.

The accounting identities in table 1 for the stocks hold for all time periods t, where the sum of i is calculated over all n industry sectors:

$$V_h = M_h, \tag{5}$$

$$V_g = L_g - M_g, \tag{6}$$

$$M_h = M_g, \tag{7}$$

$$L_g = \sum_i L_i, \tag{8}$$

$$L_i = \Psi_i, \tag{9}$$

$$V_h + V_g = \sum_i \Psi_i. \tag{10}$$

The flows shown in figure 1 are also displayed in a more general representation with n sectors in the transaction matrix in table 2. Adherence to the accounting constraints imposed by the balance sheet in table 1 and the transaction-flow matrix in table 2 guarantees the consistency of the model, and can be verified by checking that all the columns and rows of the matrices sum to the appropriate values, which in the case of financial assets sum to zero [6, p 27]. All parameters are summarized in table 3. In the following, all matrices are displayed as bold roman letters, vectors in bold italic characters. $\mathbf{diag}(x_i)$ indicates a diagonal matrix with x_i on the diagonal.

2.1. Banking sector

Unlike most SFC models, the role of the banking sector (which is consolidated as a simplification into the government sector along with the central bank) is very limited here. Banks provide loans to industries whenever requested and can increase the money stock via credit creation. Though in truth banks may ration credit to industries, loans in this simple model are provided to the industry sectors on demand [17, 113]. There are two

Table 2. The transaction matrix tabulates all flows of funds within one time period. The fact that the columns sum to zero represents a sector's budget constraints, while the fact that rows sum to zero represents the fact that each financial transaction has a counterparty. Positive values indicate inflows, while negative values indicate outflows. For example, taxes paid by households to the government are an outflow from households and an inflow to the government, so $-T$ appears in the household sector column and $+T$ appears in the government sector column. In the flow of funds accounts, outflows are referred to as *uses* of funds, whereas inflows are referred to as *sources* of funds. Note, however, that if households increase their holdings of money (a change in a stock), this is a *use* of funds even though there is no outflow. So for changes in stocks, the fact that sources of funds are denoted with a plus sign and uses of funds are denoted with a negative sign can seem counterintuitive. To clarify this, consider that a positive increase in money balances constitutes a use of funds, not a source. For example, if the household sector increases its money balances, it is using its funds to accumulate a stock of money, as opposed to using its funds for another purpose such as consumption. The current account includes receipts and outlays of firms, whereas the capital account includes capital expenditures (investment) [6].

	Households	Industry i		Government	$\sum$
		Current account	Capital account		
Government spending		$+G_i$		$-\sum_i G_i$	0
Taxes	$-T$			$+T$	0
Consumption	$-\sum_i C_i$	$+C_i$			0
Wage bill	$+\sum_i \mathit{III}_i$	$-\mathit{III}_i$			0
Intermediate purchases		$\sum_i E_{ij} - \sum_j E_{ij}$			0
Profits	$+\sum_i \Pi_i$	$-\Pi_i$			0
Interest on money deposits	$+r_M M_{h(t-1)}$			$-r_M M_{g(t-1)}$	0
Interest on loans		$-r_L L_{i(t-1)}$		$+\sum_i r_L L_{i(t-1)}$	0
ΔMoney deposits	$-\Delta M_h$			$+\Delta M_g$	0
ΔLoans			$+\Delta L_i$	$-\sum_i \Delta L_i$	0
ΔInventory value		$+\Delta \Psi_i$	$-\Delta \Psi_i$		0
$\sum$	0	0	0	0	0

Table 3. Parameters in the general model and the values used in section 3. For simplicity, ω_i and λ_i were merged into one single parameter.

Parameter name	General model	Model presented in section 3
Household consumption parameters	α_1, α_2	$\alpha_1 = 0.8, \alpha_2 = 0.2$
Input–output matrix	$\mathbf{a} = (a_{ij})$	$\mathbf{a} = \begin{bmatrix} 0.48 & 0.60 \\ 0.02 & 0.15 \end{bmatrix}$
Price matrix	$\mathbf{P} = \mathbf{diag}(P_i)$	$\mathbf{P} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Partial adjustment accelerators	β, γ	$\beta = 0.75, \gamma = 0.5$
Government spending	$\boldsymbol{G}$	$G_p = 46.6, G_e = 0$
Consumption	$\boldsymbol{C^0}$	$C_p^0 = 0.961, C_e^0 = 0.039$
Individual markups	$\boldsymbol{\phi}$	$\phi_p = 0.3333, \phi_e = 0.1364$
Interest rates	r_M, r_L	$r_M = 0.04, r_L = 0.05$
Tax rate	θ	$\theta = 0.48$
Inventory to expected sales ratio	$\sigma^\top$	$\sigma^\top = 0.5$
Labor demand per output unit	$\boldsymbol{\lambda}$	$\omega_p \lambda_p = 0.25; \omega_e \lambda_e = 0.13$
Wages per labor unit	$\boldsymbol{\omega}$	

interest rates in the model. First, there is an interest rate on loans r_L paid by each industry sector to the government on the stock of loans from the previous period $L_{i(t-1)}$. Second, there is an interest rate on money deposits r_M paid by the government to households on the stock of money deposits from the previous period $M_{h(t-1)}$.

2.2. Household sector

Households are treated as an aggregated sector, which hold only one type of financial asset: money deposits. The only behavioural decision of households in this model is consumption. We assume that in the aggregate, a certain fraction α_1 of the wage bill after taxes $(1 - \theta)\mathit{III}$ (with θ: tax rate), and a smaller fraction α_2 of wealth $M_{h(t-1)}$ is consumed, see [6, p 66] for a justification of a similar consumption function. Interest payments and distributed profits are not used as sources of finance for consumption within the period, but rather are added to wealth in the current period. In subsequent periods, a portion of this accumulated stock of financial wealth will be used to finance consumption. In this manner, a smaller propensity to consume from capital income compared to wages income is guaranteed. Total consumption C is allocated to consumption goods produced by each individual industry sector by an exogenous vector $\boldsymbol{C^0}$:

$$C = \alpha_1(1-\theta)Ш + \alpha_2 M_{h(t-1)}, \tag{11}$$

$$C_i = CC_i^0 \leqslant C \qquad \text{with} \sum_j C_j^0 = 1. \tag{12}$$

Once prices P_i are set by the industry sectors, the real physical demand c_i of the households can be calculated as:

$$c_i = \frac{C_i}{P_i} \qquad \forall i. \tag{13}$$

The money stock M_h held by households is increased by incoming flows Y consisting of the wage bill $\sum_i Ш_i$ paid to households by the industry sectors, the profits $\sum_i \Pi_i$ distributed to households by the industry sectors, and interest on money deposits paid to households by the banking system $r_M M_{h(t-1)}$. The money stock M_h is decreased by consumption of goods and services from the industry sectors $\sum_i C_i$ and by taxes T that are levied as a constant share $\theta < 1$ of income Y.

$$Y = \sum_i Ш_i + \sum_i \Pi_i + r_M M_{h(t-1)}, \tag{14}$$

$$M_{h(t)} = M_{h(t-1)} + \sum_i Ш_i + \sum_i \Pi_i - \sum_i C_i - T + r_M M_{h(t-1)}, \tag{15}$$

$$T = \theta \cdot Y. \tag{16}$$

2.3. Government sector

We assume that nominal government spending on the output of each sector is exogenously given by G_i, with total expenditure $G = \sum_i G_i$. The real physical demand of the government can be calculated as:

$$g_i = \frac{G_i}{P_i} \qquad \forall i. \tag{17}$$

The money stock M_g issued by the government is increased by outflows of government spending $\sum_i G_i$, by the increase $\sum_i \Delta L_i$ in the outstanding stock of loans, and by interest payments on money deposits $r_M M_{g(t-1)}$. It is decreased by inflows of tax payments T and of interest payments on loans $\sum_i r_L L_{i(t-1)}$.

$$M_{g(t)} = M_{g(t-1)} + \sum_i G_i + \sum_i \Delta L_i + r_M M_{g(t-1)} - r_L \sum_i L_i - T. \tag{18}$$

As mentioned in section 2, the money stock M_g issued by the government is always equal to the money stock M_h held by households. However, decisions by households about how much money to hold are made separately from decisions by the government about how much money to issue. Note the differences between the equation for M_g and the equation for M_h, and note that both equations can be derived from separate columns of the transactions flow matrix in table 2. The equality of M_g and M_h is logically implied by accounting consistency spelled out in the transactions flow matrix, and this is what closes the model.

2.4. Industry sectors

In contrast to most SFC models, in which the production sector is highly aggregated, we articulate the interlinkages between sectors using an IO model. An economy with n sectors is described by an $n \times n$ IO matrix of technical coefficients $\mathbf{a} = (a_{ij})$ given in physical terms. If $a_{ij} > 0$, the production of good j by industry j requires a physical flow of inputs of good i from industry i to industry j, such that a_{ij} units of input i are required to produce each unit of output j. The prices of the goods are contained in the diagonal matrix $\mathbf{P}$. In general, real magnitudes can be converted to nominal magnitudes by multiplying by current prices.

$$\mathbf{a} = (a_{ij}), \tag{19}$$

$$\mathbf{P} = \mathbf{diag}\left(P_i\right) \qquad \Leftrightarrow \qquad P_{ij} = P_i \delta_{ij} \tag{20}$$

with δ_{ij} being the Kronecker delta that is 1 iff $i = j$ and 0 otherwise.

The IO matrix can also be viewed in monetary terms by multiplying $\mathbf{P}$ by $\mathbf{a}$, yielding matrix $\mathbf{A}$. Note that this illustrates our convention that upper case letters refer to nominal monetary values, while lower case letters refer to real values.

$$\mathbf{A} = \mathbf{P} \cdot \mathbf{a} \qquad \Leftrightarrow \qquad A_{ij} = a_{ij} P_i. \tag{21}$$

The total real quantity of goods sold $\boldsymbol{s}_{(t)}$ will consist of sales to the households $\boldsymbol{c}$, sales of intermediate inputs to other industries $\boldsymbol{\xi}$, and sales to the government $\boldsymbol{g}$. Note that because sales of intermediate inputs (which in

value-added terms net to zero) are included in the definition of sales, these are not *net* sales, but rather are *gross* sales

$$s_{(t)} = c + \xi + g. \tag{22}$$

However, the markets in the model economy are not price-clearing auction markets. As in the real world, realized sales are not yet known at the time when production decisions are made, so each industry i must estimate its *expected* sales s^X with a partial adjustment model, adjusting expectations partially ($0 < \beta < 1$) from each period to the next:

$$s^X_{(t)} = \beta s_{(t-1)} + \left(1 - \beta\right) s^X_{(t-1)}. \tag{23}$$

As a buffer against unexpected changes in sales, firms build up stocks of inventories ψ_i to serve as a quantity buffer stock to absorb unexpected fluctuations in demand, much as do retail stores in the real world [6, 114–116]. The inventory target $\boldsymbol{\psi}^\top$ is considered as a fraction of expected sales s^X, and is also updated with a partial accelerator function in light of the experiences of the previous period (with $0 < \gamma < 1$). $\sigma^\top > 0$ is the targeted ratio of inventories to expected sales. This leads to a demand for inventories $\boldsymbol{\Delta\psi}^\top$

$$\boldsymbol{\psi}^\top = \sigma^\top s^X_{(t)}, \tag{24}$$

$$\boldsymbol{\Delta\psi}^\top = \gamma\left[\boldsymbol{\psi}^\top - \boldsymbol{\psi}_{(t-1)}\right]. \tag{25}$$

The industry sectors produce a total gross output $\boldsymbol{x}$ in physical terms, which is equal to the total amount of expected sales plus the targeted change in inventory stocks by the industry itself. Note that expected *gross* sales includes expected sales of intermediate inputs as well as expected sales for final demand

$$\boldsymbol{x} = s^X_{(t)} + \boldsymbol{\Delta\psi}^\top. \tag{26}$$

This production of output requires a labor force $\boldsymbol{l}$. We assume that in each sector, the required labor force is proportional to gross production x_i of the sector:

$$l_i = \lambda_i x_i, \tag{27}$$

where the vector $\boldsymbol{\lambda}$ contains the labor forces λ_i required for production in a specific sector. Given the wages per labor unit ω_i, the wage bills paid per sector $\mathit{Ш}_i = \omega_i \lambda_i x_i$ add up to:

$$\mathit{Ш} = \sum_i \mathit{Ш}_i = \sum_i \omega_i \lambda_i x_i. \tag{28}$$

Because intermediate goods are consumed in production, gross product always exceeds net product bound for final delivery. Since we have already solved for gross output $\boldsymbol{x}$, we can solve for net output $\boldsymbol{d}$. Though we are solving for net output using a given gross output, rather than solving for gross output using a given net output, this does not mean that effective demand is unimportant. Indeed, *expectations* of net output (effective demand) play a crucial causal role in driving the model. Given expectations of net output, or for that matter expectations of gross output, entrepreneurs decide how much gross output to actually produce. Given this *realized* gross output, one can derive the realized net output implied by this decision. So, realized net output $\boldsymbol{d}$ is simply:

$$\boldsymbol{d} = (1 - \mathrm{a})\boldsymbol{x}. \tag{29}$$

Using standard IO analysis, the sales of intermediate inputs (in real terms) to other industries can be calculated as:

$$\boldsymbol{\xi} = \mathrm{a} \cdot \boldsymbol{x}. \tag{30}$$

The fact that industries require goods produced by other industries as inputs causes prices to be interdependent, because prices of other goods alter production costs. We assume that the price for each sector is set as a markup ϕ_i on unit costs from the previous time period. In reality, firms must use costing procedures to estimate their costs, and firms will often set a markup over some form of estimated normal costs, but the difficulties of cost estimation are abstracted from here. Post-Keynesian economists have often assumed constant prime or direct costs [117, p 1305]. However, there is empirical evidence that in reality average direct costs may increase or decrease as output increases, and that cost structures vary across different industries [118]. Costs in this model include wages and the costs of intermediate inputs

$$P_{i(t)} = \left(1 + \phi_{i(t)}\right)\left[\omega_{i(t-1)}\lambda_{i(t-1)} + \sum_k P_{k(t-1)} a_{ki(t-1)}\right]. \tag{31}$$

Given the gross production vector $\boldsymbol{x}$, the monetary flows for intermediate inputs can be calculated as a matrix E, where E_{ij} means a flow of money from industry j to industry i.

$$\mathbf{E} = \mathbf{P}\ \mathbf{a}\ \mathbf{diag}(x_i) \qquad \Leftrightarrow \qquad E_{ij} = a_{ij} P_i x_j. \tag{32}$$

Total realized sales are simply the sum of the purchases by households $\boldsymbol{c}$, by other industries $\boldsymbol{\xi}$, and by the government $\boldsymbol{g}$:

$$\boldsymbol{s}_{(t)} = \boldsymbol{c} + \boldsymbol{\xi} + \boldsymbol{g}. \tag{33}$$

The realized inventories $\boldsymbol{\psi}_{(t)}$ at the end of the period then depend on these realized sales $\boldsymbol{s}_{(t)}$. Because there can be a discrepancy between expected sales $\boldsymbol{s}^X_{(t)}$ and realized sales $\boldsymbol{s}_{(t)}$, the realized amount of inventories $\boldsymbol{\psi}_{(t)}$ may differ from the expected. The monetary value of inventories $\boldsymbol{\Psi}_{(t)}$ is equal to the physical quantity of inventory units multiplied by the unit cost of inventories, which includes wage costs and intermediate input costs.

$$\boldsymbol{\psi}_{(t)} = \boldsymbol{\psi}_{(t-1)} + \boldsymbol{x}_{(t)} - \boldsymbol{s}_{(t)}, \tag{34}$$

$$L_{i(t)} = \Psi_{i(t)} = \psi_{i(t)} \left[\omega_{i(t-1)} \lambda_{i(t-1)} + \sum_k P_{k(t-1)} a_{ki(t-1)} \right]. \tag{35}$$

The net profit of each industry $\boldsymbol{\Pi}$ has two components: the monetary profit of industry i is the sum of households' consumption expenditures, purchases by government, and intermediate investment E_{ij} by the other sectors of the economy minus intermediate purchases E_{ji} of sector i and minus the wage bill Π_i; additionally, interest payments $r_L L_{i(t-1)}$ must be subtracted from profits. The second component contributing to profits is any change of the value of the inventories $\Delta\Psi_i = \Psi_{i(t)} - \Psi_{i(t-1)}$ valued at current unit costs. Note that profits are a residual determined by accounting constraints; the equation for profits can simply be read off of the industry i Current Account column on the transaction flow matrix in table 2.

$$\Pi_i = C_i + G_i - \Pi_i + \sum_j E_{ij} - \sum_j E_{ji} - r_L L_{i(t-1)} + \Delta\Psi_i. \tag{36}$$

We assume that the industrial sectors distribute all profits to the household sector.

2.5. Solving the model

Normally, SFC models contain implicit functions and are typically solved numerically by iterative techniques [6, 37], but in this case, we can offer an explicit solution for the time step evolution, though because of the number of variables, the calculations will be performed numerically. All relevant parameters are put together in figure 3.

From the last period, the stocks of money $M_{h(t-1)}$ and loans $L_{i(t-1)}$ and the corresponding values for the government, the inventories $\boldsymbol{\psi}_{(t-1)}$, the prices $P_{i(t-1)}$ and the expected and realized sales $\boldsymbol{s}^X_{(t-1)}$ and $\boldsymbol{s}_{(t-1)}$ are known, together with the IO matrix $\mathbf{a}$. The prices will be updated using (31), the expected sales and targeted inventory adjustments will be calculated using (23) and (25) and the total production using (26). Then, the wage bill can be calculated using (28), and the physical demand of the households and government can be calculated using equations (11)–(13) and (17). This being known, the realized inventories $\psi_{i(t)}$ are given by (34), the monetary value of inventories at unit costs $\Psi_{i(t)}$ and therefore the loans to finance them by (35). Additionally, the realized intermediate sales $\boldsymbol{\xi}$ (30) and gross sales $\boldsymbol{s}_{(t)}$ (33) can be determined. The payments within the sector can be calculated using (32), the distributed profits using (36), and taxes using (16), the monetary stocks at the end of the period using (15) for the households, (18) for the government. In this way, all new stocks can be calculated straightforwardly, without any iterative procedure.

3. Energy in a SFC IO model

As explained in section 1.3, energy plays a crucial role in the economic process. We apply our general framework to a model with two goods: a multi-purpose consumer/industry good and energy. The consumer/industry good is sold by the production sector, and the energy good is sold by the energy sector. In order to produce the consumer/industry good, the production sector uses energy as well as its own good as inputs, while in order to provide energy, the energy sector uses the consumer/industry good and the energy good as inputs. This specification ensures that the two sectors are mutually interdependent, and that the model incorporates physical aspects and the dynamics of a monetary production economy. A representation of the flows of money and energy is given in figure 1.

The 'physical quantities' of the IO matrix $\mathbf{a}$ (19) are defined such that the prices are 1 monetary unit for all goods in the first period, but prices of these quantities may vary ofter time. The parameters are matched to the situation of Germany around 2010. The IO parameters, the markups, the wage bill, and the consumption vectors are estimated from [119]: for each unit sold, the consumer/industry sector requires an input of 0.48 from its own sector and 0.02 from the energy sector and pays 0.25 units of wages. The energy sector requires 0.60 units from

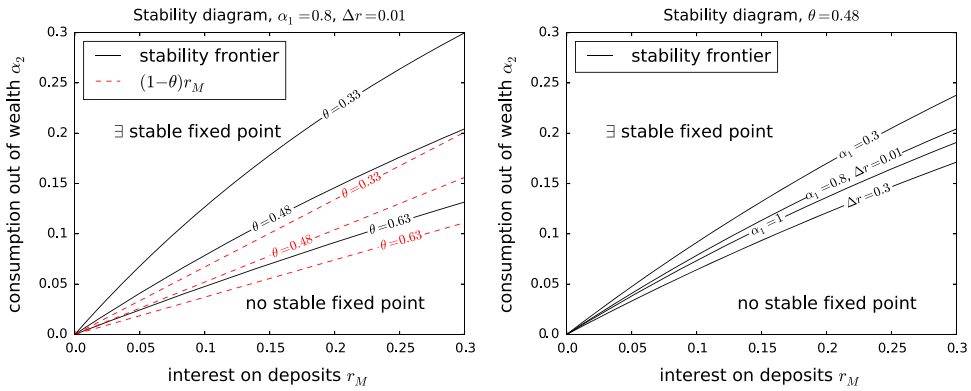

Figure 2. Stability analysis of the model. Left plot: for different tax rates, we check whether a stable stock-flow equilibrium exists. For a given interest rate r_M, there exists a minimum consumption out of wealth α_2 for which the model is stable. An increase in the tax rate reduces this threshold. If consumption out of wealth is smaller than interest income after taxes (as indicated by the red dashed lines), the fixed point will definitely be unstable, as inflows to households are always bigger than outflows for $\alpha_1 < 1$. Right plot: the impact of the interest rate spread $\Delta r = r_L - r_M$ and the consumption out of wages α_1 is depicted, $\alpha_1 = 0.8$ and $\Delta r = 0.01$ serves as a benchmark; only changes of these parameters are indicated. A higher interest rate spread shifts the stability lines down slightly. A higher consumption out of wages α_1 increases the size of the stable region.

the industry sector and 0.15 units from the energy sector itself and pays 0.13 units of wages. Therefore, the markups on costs can be calculated as $\phi_p = 0.3333$ and $\phi_e = 0.1364$. The tax rate of 0.48 is taken from [120], the interest rates, accelerators, inventory to sales ratio, and consumption parameters are set as rough estimates. All parameters used are displayed in table 3.

3.1. Stability analysis of a stationary economy with positive interest rates

Within ecological economics, several authors propose a non-growing economy as a solution to environmental problems [71–73, 121]. In recent publications, it has been claimed that this is incompatible with positive interest rates [92, 93]. It is argued that positive interest rates imply that in a non-growing economy, the stock of debts will rise, and it is argued that such an increase would be unsustainable. Using our model, we show that an equilibrium state of a stationary economy is possible, even with positive interest rates. The economy reaches a general stationary stock-flow equilibrium if all stocks and all flows remain constant over time, and therefore inflows equal outflows.

Assuming constant parameters and prices, the time evolution is defined by a non-homogeneous first-order matrix difference equation, see appendix A.2. The unique stock-flow equilibrium is a stable fixed point if the absolute values of all eigenvalues of the mapping matrix $\mathcal{M}$ are smaller than 1. The stability of the stock-flow equilibrium is graphed in the parameter space of interest rates $r_{M/L}$, consumption parameters $\alpha_{1/2}$, and for different tax rates θ. The stability frontiers are depicted in figure 2. Rising interest on deposits r_M, lower consumption parameters α_1, α_2, and a lower interest spread $\Delta r = r_L - r_M$ decrease the size of the stable region within the parameter space. If no stable fixed point exists, we see an exponential increase of private money deposits and a corresponding growth in public debt, illustrating the accounting principle that all financial assets have symmetrical financial liabilities. Flows of interest payments from the government accumulate and increase the money stock M_h held by the household sector. But if consumption out of wealth α_2 is high enough to counteract the interest and profit payments, households increase their consumption as their stock of wealth increases. The fixed point is stable which enables the economy to remain in stock-flow equilibrium, even though interest rates are positive. It is then not the case that the interest payments drive down government net worth. This shows that the stability of a non-growing economy is indeed a question of the interplay of interest payments and the propensity to save, as suggested by Wenzlaff *et al* [91].

For $\alpha_2 = 0.2$, $r_M = 0.05$, $r_L = 0.04$ and $\theta = 0.48$ as in table 3 and for a nominal GDP of $d_p P_p + d_e P_e = 100$, this is realized with $M_h \approx 162.9$, $V_g \approx -86.1$, $L_p \approx 73.7$ and $L_e \approx 3.1$. In this state, the industry sectors realize positive profits ($\Pi_p \approx 45.4$, $\Pi_e \approx 0.7$ per period) which are distributed to the households, the tax income ($T \approx 49.3$) and interest income ($r_L L_g \approx 3.8$) of the government equals the government expenditures ($G \approx 46.6$) and interest costs ($r_M M_h \approx 6.5$), and the total income of the households ($Y \approx 102.7$) equals taxes and consumption ($C_p \approx 51.3$, $C_e \approx 2.1$). Once equilibrium is reached, no sector accumulates any additional stocks, and all income is consumed or distributed, which allows for a stationary economy.

Though our model shows that positive interest rates do not necessarily imply exponential growth of government liabilities, this result crucially depends on consumption decisions by households. Additional

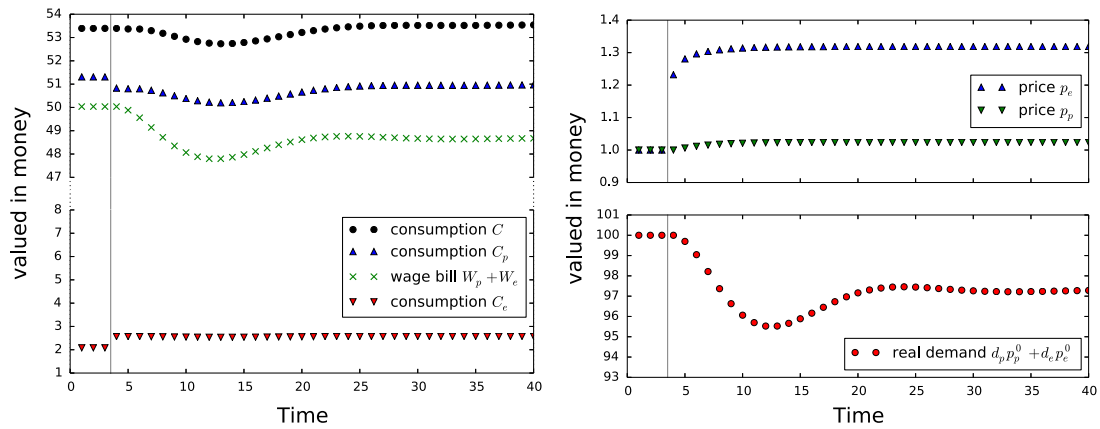

Figure 3. Impact of an increase in energy markup ϕ_e: we initiate the model at the fixed point calculated in section 3.1, and the vertical line indicates the time when the markup on energy is exogenously increased. The left plot shows the decrease of consumption that causes a reduction of demand, which drives down wages and further reduces consumption. The right plot indicates the increase in prices caused by the higher energy markup, and the time evolution of real demand, showing a significant drop.

research should be conducted on this question. It should be pointed out as well that a stable stationary economy in monetary terms does not imply an equilibrium state with the environment.

3.2. The macroeconomic response to energy price shocks

Hamilton [62, 63] and Murphy and Hall [64] present evidence that every US recession since World War II was accompanied by rising energy prices. We now use the model to examine the impact of an increase in the energy industry's markup ϕ_e. IO models have been criticized for keeping the matrix elements fixed [122], but since energy sources such as oil have very low price elasticities [123] and since we are considering the short term impact of changes in the economy, changes in **a** matrix coefficients would be small.

We assume that the markup on energy is increased from $\phi_e = 0.1364$ to 0.4. In order to incorporate the effect of the low price elasticity, households react to an energy price shock by devoting a higher portion of their consumption spending to energy services, so that in the following period, they consume the same amount of energy. This means that C_e^0 changes from 0.039 to 0.048 and remains there. The impact of the energy price shock is depicted in figure 3. The increased markup leads to a higher price of energy, which will be buffered by households via an increased energy consumption factor C_e^0. This immediately drives down consumption of other goods, which reduces the wage in the next period and the expected sales s_p^X of the production sector, leading to a reduction in inventory investment. Additionally, the rise in the price of energy drives up unit costs and therefore also drives up prices in the production sector via the IO interlinkages. Together, this leads to a serious drop in real final demand, which is calculated in prices of the 0th period. However, production goes up again once inventories are reduced, and after the rising profits of the energy sector are distributed to households, leads households to increase their consumption out of wealth. The new equilibrium settles at a reduction of real demand of 2.7%. It should be pointed out that this is due to the fact that government expenditures are kept constant in nominal terms. An increase in government expenditures due to higher prices could potentially compensate for that decline in real demand. It should be noted that if prices rise at different rates in different sectors, this would lead to a change in the distribution of income.

It is worth noting that the drop of real demand is not easily explained by many neoclassical models which abstract from finance and monetary production, in which the reduction of aggregate production in equilibrium would be caused by a reduction of utilization of the energy input, multiplied by the output elasticity (3). As pointed out above, neoclassical authors assumed that output-elasticity should correspond to the cost share, which is around 4% in our model. The reduction of energy consumption by 5% should therefore reduce final demand by 0.2%, which is barely visible. In the simulations, final demand drops by 2.7%, which is one order of magnitude bigger. Interestingly, this order of magnitude difference has also been realized in the case of historic oil crises [2]. Within post-Keynesian economics, this impact can be explained by traditional Keynesian multiplier effects [85]. So following this interpretation, which is consistent with our model, it was not the reduced supply of oil (no shortage occurs in the model), but the decreased real expenditures that triggered the recession. The drastic increase in energy prices before 2008 may have contributed to the 2007–2008 financial crisis, as reduced growth or lower expected growth may destabilize the economy and the financial system [89, 91].

Multiplier effects play a prominent role in our model and can amplify the negative impact of recessions caused by energy price shocks. An increase in the energy price markup may have the effect of decreasing the real wage, which decreases real consumption [85]. First of all, this decline in consumption is amplified by the simple IO output multiplier [41, pp 245–247]: if expected final demand is decreased by one unit, total production declines by more than one unit due to intersectoral interlinkages. This multiplier has an immediate effect in our model within each time period. Secondly, a decrease in production leads to a lower labor demand and therefore decreases the wage bill. Households are assumed to immediately spend a fraction of their wages on consumption (11), and so a lower wage bill decreases consumption. Induced effects of changes in wages which alter consumption of goods across different sectors are referred to in IO models as 'Total Output Multipliers' [41, pp 247–248]. In our model, recessions are also propagated via inventory cycles [124]. Decreased consumption because of lower wages will simply lead to an increase of inventory stocks. This occurs because firms do not decrease their production immediately, since buffering unexpected shocks is the essential role of inventories. In the next period, firms will decrease their expected sales and attempt to reduce their inventory stocks, which will decrease output even more. In principle, all of these propagation and amplification mechanisms can mutually support and strengthen each other, and yield an alternative explanation for the macroeconomic response to energy price shocks that have been observed in the past.

4. A simple climate model with anthropogenic heat emissions

Until now, the model we have considered has been solely an economic model, and although we have depicted material and energy flows crossing the boundaries between the economic system and the ecosystem in figure 1 and discussed the implicit effects of those flows, they were not explicitly incorporated into the model. A broad literature deals with the interconnection between the environment and the economy, in particular the impact of material waste and natural resource scarcity. Emission of heat resulting from thermodynamic principles, however, remains largely neglected, and we conceptualize an integration of heat emissions from economic activities into climate models.

As energy is consumed, the economic process transforms energy into unusable heat [2, p 114]. Except for some renewable energy sources such as wind, where heat dissipation would have happened anyway, this adds an anthropogenic heat flux whose impact on climate has been discussed e.g. by [125–127] or in 'The Limits to Growth' [128, pp 73 f]. Today, world average heat emission can be estimated by total primary energy consumption to be around 0.025 Wm^{-2}, which is about 1% of total radiative forcing in 2011 from anthropogenic climate change [129, p 14]. Globally, this may be negligible today [130], but is of importance for regional climate models [131]. If energy conversion continues to rise over the course of the century, this may become relevant, especially if new technologies such as nuclear fusion or energy harvesting by satellites [2, pp 76–91] are eventually implemented.

A minimal model can be introduced to get a coarse idea of the impact of human heat flux. We consider a standard model [132, 133] (see figure 4) in which the Earth is considered to act as a black body in the infrared, while the albedo for sunlight is considered to be $\alpha = 0.3$. The Earth is considered to be at uniform equilibrium temperature T_{eq}. The solar constant is S=1370 Wm^{-2}, leading to a mean insolation of $S/4$, since the surface of a sphere is four times its cross section. The atmosphere is considered as a single layer perfectly transparent for sunlight and with $\epsilon = 0.78$ being the absorptivity and emissivity of the atmosphere in the infrared spectrum. The absorbed radiation is emitted evenly up and down, such that $A\uparrow = A\downarrow = 0.5\epsilon\sigma T_{\mathrm{eq}}^4$. As a variation to the standard models, we add a layer of 'human heating' P_{hum} at the Earth's crust. The radiative balance of Earth is given using the Stefan–Boltzman law by

$$0.25 \cdot S(1-\alpha) + P_{\mathrm{hum}} + A\downarrow = \sigma T_{\mathrm{eq}}^4, \tag{37}$$

and the equilibrium temperature T_{eq} can be calculated as

$$T_{\mathrm{eq}} = \left(\frac{0.25 \cdot S \cdot \left(1-\alpha\right) + P_{\mathrm{hum}}}{\sigma \cdot (1 - 0.5 \cdot \epsilon)}\right)^{\frac{1}{4}}. \tag{38}$$

Note that this equation has to be adapted only slightly for solar energy, since efficient harvesting of sunlight requires low reflection, which would lead to an effective albedo of $\alpha_{\mathrm{eff}} = \alpha(1 - \frac{\mathrm{P_{hum}}}{0.25\,\mathrm{S}})$. This means that $\mathrm{P_{hum}}$ in the equation must be replaced by $\alpha\mathrm{P_{hum}}$ if all thermal power plants would be replaced by solar power stations.

Today's energy conversion in Germany accounts for 1.26 Wm^{-2}. If this same degree of energy conversion were to be realized on the whole landmass of planet Earth (29.3% of total surface), the temperature increase would already be 0.12 K.

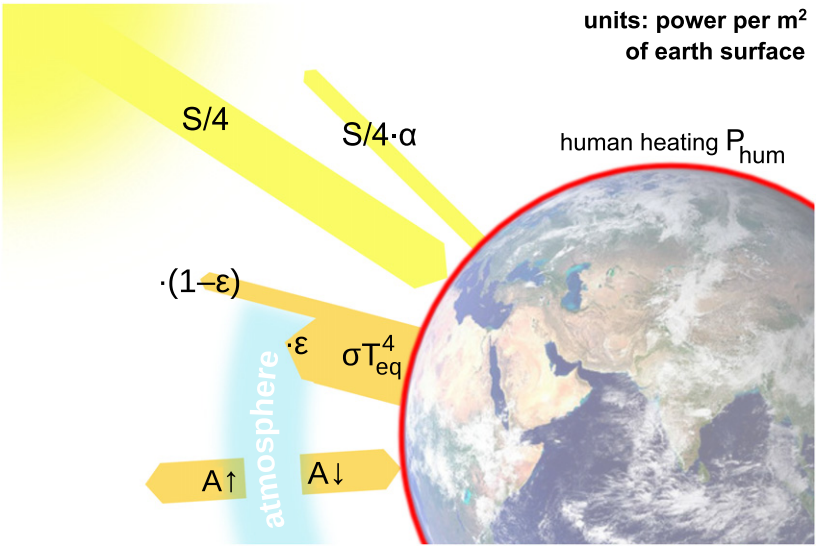

Figure 4. Single layer atmosphere with human heating P_{hum}. The albedo α indicates the fraction of incoming sunlight that will be reflected immediately. At the Earth's crust, an additional layer of human heat emissions is considered, indicated by the red sphere. The infrared emissions of the Earth are due to black body radiation, and a fraction ϵ is absorbed in the atmosphere that radiates the absorbed energy evenly in both directions.

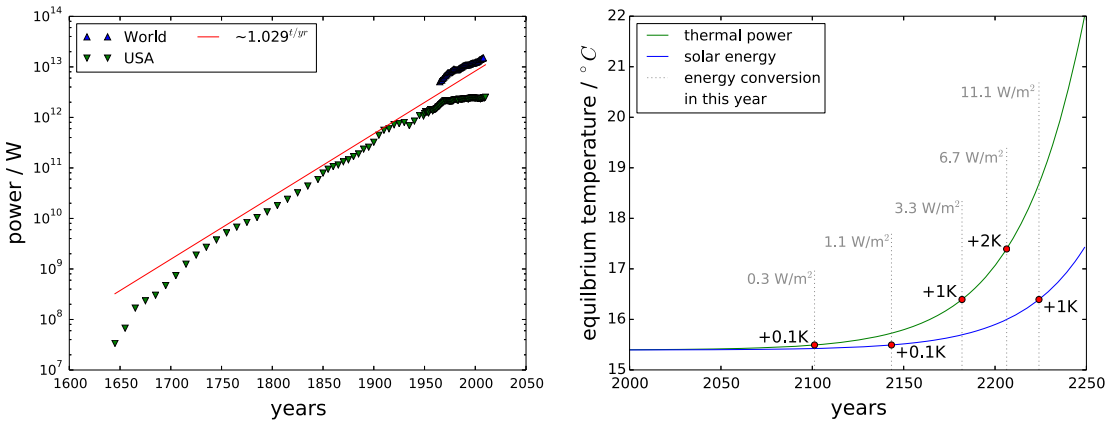

Figure 5. Left: energy conversion in the USA and the world, data from [136–138]. The red line indicates an exponentially growing curve with a yearly growth rate of 2.9%. Right: equilibrium temperature of planet Earth calculated by (38), assuming continued exponential growth of energy conversion.

In the past, global energy conversion has increased nearly exponentially with a growth rate of around 2.9%, see the left plot in figure 5. If we project this trend into the future, the impact of anthropogenic heat flux could become very relevant over the next several centuries, as it would contribute significantly to an increase of average temperature on Earth, see the right plot in figure 5. The temperature rise is smaller for solar energy than for thermal power plants. This demonstrates that the radiation balance of 'Spaceship Earth' [134] would be significantly affected by a steady increase in energy conversion. A hypothetical continuation of this 2.9% growth rate could break all reasonable limits within centuries, though such an extrapolation would exceed the model's scope. If humans were to discover a cheap, inexhaustible, and environmentally benign source of energy, one might at first glance consider it a clear boon to humanity. However, if its discovery were to lead to increased energy use, heat emissions could potentially have a serious environmental impact. The implementation of new energy technologies could potentially facilitate an explosion of the global population and an increase of consumption, possibly beyond the Earth's sustainable carrying capacity [135].

Although this integration of heat into a climate model is very basic, the results underline that the heat emissions of economic activity should be taken into account in climate modeling once long-term scenarios are examined.

5. Summary and outlook

Using a simple baseline model, this paper examines the interactions of financial assets, real physical goods and services, and the physical environment. The analysis suggests an additional method of simultaneously modeling the mutually interacting subsystems of both real flows (which correspond to IO data) and financial flows (which correspond to FF data). Though this prototypical model does not capture the rich behavior possible from either approach, the methodology is designed to enable scaling to an arbitrary number of industries. In addition, the extensive SFC and IO literatures hold the potential to inform the further development of more realistic applied models.

The role of the energy sector for the economic process and growth in the past was emphasized. The model was used to analyze a simple economy with a household sector and a consolidated government and banking system sector, along with two industrial sectors: an energy sector and a production goods sector. Contrary to some of the literature in ecological economics, we demonstrated that a stationary economy can in principle be associated with positive interest rates. The stability analysis reveals that this depends on the interplay of interest rates and consumption parameters. Next, the impact of energy price shocks on the economy was examined, in particular how rising energy prices can depress real wages, lower demand, and therefore trigger recessions. As the energy sector is one of the key linkages connecting the natural environment with the economy, we studied the environmental impact of energy conversion. Specifically, implied heat emissions from energy conversion and the effect of anthropogenic heat flux on climate change were considered in light of a minimal single-layer atmosphere climate model. The economic model was only linked implicitly to the climate model, because the climate model is used to estimate the equilibrium temperature of the Earth over long periods of time, whereas the economic model is focused on much shorter-term changes to the structure and size of the economy. But integrating well-developed larger scale macroeconomic and climate models could potentially enable fruitful analysis of the interlinkages between the economy and the physical environment, which may be relevant to addressing the issue of global climate change and numerous other environmental issues.

The primary virtue of this paper is that it suggests a path towards realizing the project of integrating diverse strands of literature, while drawing linkages between the economy and the physical world. First, the paper offers both econophysicists and post-Keynesians a possible way to more explicitly incorporate production into their models. Second, the paper offers to econophysicists in particular a method to incorporate the symmetry between financial assets and liabilities as an alternative to treating money as a conserved quantity. Third, the paper offers ecological economists an avenue to study the economy and the environment as a unified macroeconomic-ecological system.

Though the baseline model proposed here does not capture the rich behavior possible from either approach, the methodology is designed to enable scaling to an arbitrary number of industries, and also to allow the incorporation of more realistic elements from other already existing IO models, SFC models and ABM. Possible extensions may include, but are by no means limited to, the implementation of ecological and capacity constraints, a more sophisticated treatment of investment, a more fully developed financial system through which firms and households make more complex financing decisions, and the explicit consideration of interacting heterogeneous agents. There exist rich literatures in all areas, which constitute fertile ground for the synthesis of disparate insights which have heretofore been developed largely in isolation, to create disaggregated multisectoral financial macroeconomic models which are also capable of drawing explicit links between economic activity and the physical world.

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Appendix A

Equation list of model in section 2

All matrices are displayed as bold roman letters, vectors in bold italic characters. $\mathbf{diag}(x_i)$ indicates a diagonal matrix with x_i on the diagonal.

$$V_h = M_h, \tag{A.1}$$

$$V_g = L_g - M_g, \tag{A.2}$$

$$M_h = M_g, \tag{A.3}$$

$$L_g = \sum_i L_i, \tag{A.4}$$

$$L_i = \Psi_i, \tag{A.5}$$

$$V_h + V_g = \sum_i \Psi_i, \tag{A.6}$$

$$C = \alpha_1 \left(1 - \theta\right) Ш + \alpha_2 M_{h(t-1)}, \tag{A.7}$$

$$C_i = CC_i^0 \leqslant C \qquad \text{with} \sum_j C_j^0 = 1, \tag{A.8}$$

$$c_i = \frac{C_i}{P_i} \qquad \forall i, \tag{A.9}$$

$$Y = \sum_i Ш_i + \sum_i \Pi_i + r_M M_{h(t-1)}, \tag{A.10}$$

$$M_{h(t)} = M_{h(t-1)} + \sum_i Ш_i + \sum_i \Pi_i - \sum_i C_i - T + r_M M_{h(t-1)}, \tag{A.11}$$

$$T = \theta \cdot Y, \tag{A.12}$$

$$g_i = \frac{G_i}{P_i} \qquad \forall i, \tag{A.13}$$

$$M_{g(t)} = M_{g(t-1)} + \sum_i G_i + \sum_i \Delta L_i + r_M M_{g(t-1)} - r_L \sum_i L_i - T, \tag{A.14}$$

$$\mathbf{a} = (a_{ij}), \tag{A.15}$$

$$\mathbf{P} = \mathbf{diag}\left(P_i\right) \qquad \Leftrightarrow \qquad P_{ij} = P_i \delta_{ij}, \tag{A.16}$$

$$\mathbf{A} = \mathbf{P} \cdot \mathbf{a} \qquad \Leftrightarrow \qquad A_{ij} = a_{ij} P_i, \tag{A.17}$$

$$s_{(t)} = \boldsymbol{c} + \boldsymbol{\xi} + \boldsymbol{g}, \tag{A.18}$$

$$s_{(t)}^X = \beta s_{(t-1)} + \left(1 - \beta\right) s_{(t-1)}^X, \tag{A.19}$$

$$\boldsymbol{\psi}^\top = \sigma^\top s_{(t)}^X, \tag{A.20}$$

$$\boldsymbol{\Delta\psi}^\top = \gamma \left[\boldsymbol{\psi}^\top - \boldsymbol{\psi}_{(t-1)}\right], \tag{A.21}$$

$$\boldsymbol{x} = s_{(t)}^X + \boldsymbol{\Delta\psi}^\top, \tag{A.22}$$

$$l_i = \lambda_i x_i, \tag{A.23}$$

$$Ш = \sum_i Ш_i = \sum_i \omega_i \lambda_i x_i, \tag{A.24}$$

$$\boldsymbol{d} = (\mathbf{1} - \mathbf{a})\boldsymbol{x}, \tag{A.25}$$

$$\boldsymbol{\xi} = \mathbf{a} \cdot \boldsymbol{x}, \tag{A.26}$$

$$P_{i(t)} = \left(1 + \phi_{i(t)}\right)\left[\omega_{i(t-1)} \lambda_{i(t-1)} + \sum_k P_{k(t-1)} a_{ki(t-1)}\right], \tag{A.27}$$

$$\mathbf{E} = \mathbf{P}\ \mathbf{a}\ \mathbf{diag}\left(x_i\right) \qquad \Leftrightarrow \qquad E_{ij} = a_{ij} P_i x_j, \tag{A.28}$$

$$s_{(t)} = \boldsymbol{c} + \boldsymbol{\xi} + \boldsymbol{g}, \tag{A.29}$$

$$\boldsymbol{\psi}_{(t)} = \boldsymbol{\psi}_{(t-1)} + \boldsymbol{x}_{(t)} - s_{(t)}, \tag{A.30}$$

$$L_{i(t)} = \Psi_{i(t)} = \psi_{i(t)}\left[\omega_{i(t-1)} \lambda_{i(t-1)} + \sum_k P_{k(t-1)} a_{ki(t-1)}\right], \tag{A.31}$$

$$\Pi_i = C_i + G_i - Ш_i + \sum_j E_{ij} - \sum_j E_{ji} - r_L L_{i(t-1)} + \Delta\Psi_i. \tag{A.32}$$

Appendix B

Equation list of stability analysis in section 3.1

If prices are set to unity ($P_p = P_e = 1$), the time evolution can be written as a non-homogeneous first-order matrix difference equation:

$$X_{(t)} = \begin{pmatrix} s^X_{p(t)} \\ s^X_{e(t)} \\ s_{p(t)} \\ s_{e(t)} \\ \psi_{p(t)} \\ \psi_{e(t)} \\ M_{h(t)} \end{pmatrix} = \mathcal{M} \cdot X_{(t-1)} + \begin{pmatrix} 0 \\ 0 \\ +G_p \\ +G_e \\ -G_p \\ -G_e \\ 0 \end{pmatrix}, \tag{B.1}$$

$$\text{with } \mathcal{M} = \begin{bmatrix} 1-\beta & 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & 1-\beta & 0 & \beta & 0 & 0 & 0 \\ Z_{pp}\Gamma\left(1-\beta\right) & Z_{pe}\Gamma\left(1-\beta\right) & Z_{pp}\Gamma\beta & Z_{pe}\Gamma\beta & -Z_{pp}\gamma & -Z_{pe}\gamma & \alpha_2 C^0_p \\ Z_{ep}\Gamma\left(1-\beta\right) & Z_{ee}\Gamma\left(1-\beta\right) & Z_{ep}\Gamma\beta & Z_{ee}\Gamma\beta & -Z_{ep}\gamma & -Z_{ee}\gamma & \alpha_2 C^0_e \\ \left(1-Z_{pp}\right)\Gamma\left(1-\beta\right) & -Z_{pe}\Gamma\left(1-\beta\right) & \left(1-Z_{pp}\right)\Gamma\beta & -Z_{pe}\Gamma\beta & 1-\left(1-Z_{pp}\right)\gamma & Z_{pe}\gamma & -\alpha_2 C^0_p \\ -Z_{ep}\Gamma\left(1-\beta\right) & \left(1-Z_{ee}\right)\Gamma\left(1-\beta\right) & -Z_{ep}\Gamma\beta & \left(1-Z_{ee}\right)\Gamma\beta & Z_{ep}\gamma & 1-\left(1-Z_{ee}\right)\gamma & -\alpha_2 C^0_e \\ Z_p\Gamma\left(1-\beta\right) & Z_e\Gamma\left(1-\beta\right) & Z_p\Gamma\beta & Z_e\Gamma\beta & r_L\dfrac{\theta-1}{1+\phi_p} - Z_p\gamma & r_L\dfrac{\theta-1}{1+\phi_e} - Z_e\gamma & 1+r_M(1-\theta)-\alpha_2 \end{bmatrix}$$

using the following definitions:

$$Z_{ij} = a_{ij} + \alpha_1(1-\theta)\omega_j\lambda_j C^0_i \qquad \text{with} \qquad i, j \in p, e \tag{B.2}$$

$$Z_p = (1-\theta)\left(1 - a_{pp} - a_{ep} - \alpha_1\omega_p\lambda_p\right), \tag{B.3}$$

$$Z_e = (1-\theta)\left(1 - a_{pe} - a_{ee} - \alpha_1\omega_e\lambda_e\right), \tag{B.4}$$

$$\Gamma = \left(1 + \gamma\sigma^{\top}\right). \tag{B.5}$$

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