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Banking Crises under a Central Bank Digital Currency (CBDC)

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Abstract

One of the main concerns when considering Central Bank Digital Currency (CBDC) is the disintermediating effect on the banking sector in normal times, and even more the risk of a bank run in times of crisis. This paper extends the bank run model of [Gertler and Kiyotaki \(2015\)](#) by analyzing the impact of a CBDC. A CBDC is an additional type of liability to the central bank which, by accounting identity, must be accompanied by respective accommodations on the asset side. The model compares the effects of two different asset side policies with each other and to the economy without a CBDC. I find that a CBDC reduces net worth in the banking sector in normal times but mitigates the risk of a bank run in times of crisis. The prevailing concerns about the risk of a bank run turn out to be partial equilibrium considerations disregarding the asset side effects of a CBDC.

Keywords: Central Bank Digital Currency (CBDC), Digital Currency, Central Banking, Financial Intermediation, Bank Runs, Lender of Last Resort

JEL: E42, E58, G01, G21

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1 Introduction

Throughout history, money has adapted to the needs of people and its form has changed substantially – from cowrie shells and commodity money, to commodity backed money, to fiat money. Digital transformation has and will further change our payment and monetary system. It is therefore crucial for central banks to carefully analyze these developments and evaluate how this affects their goal of providing a resilient and price stable payment system and the transmission channels of monetary policy. That is why recently, central bankers and academics have started to discuss the potential of introducing a Central Bank Digital Currency (CBDC)¹. Among others, a CBDC would ensure that the public retains access to central bank money – but the motivation for considering CBDC are manifold and the relevance of the individual arguments also depends on the economic context². Nevertheless, a CBDC would constitute a major change of the monetary architecture associated with substantial risks and uncertainties. What however is certain is that this topic raises numerous important and interesting questions for research³. There is no uniform definition of a CBDC. Instead, several definitions, forms and designs have been proposed and discussed⁴. This paper will define a (retail) CBDC as a potentially interest bearing, centrally issued, account based, digital type of central bank liability that is accessible to the general public. What are the distinguishing features of a CBDC compared to other forms of money? Table 1 differentiates the key features of CBDC from cash, government bonds and deposits. A CBDC is similar to a government bond as both are generally considered to have no default risk and bear interest. However, a government bond is not a medium of exchange and cannot be used to settle transactions. Deposits are interest bearing and settle transactions, but yet (despite deposit insurance) are not inherently safe. This difference will be a crucial feature in the following analysis. Cash is used to make payments and cannot default but it does not bear (nominal) interest.

¹According to surveys by the BIS, around 80% of the central banks are studying CBDCs in some form (Boar et al. (2020), Barontini and Holden (2019)).

²A compact yet comprehensive overview over benefits associated with CBDC can be found in the paper of Bindseil (2020). Moreover, Boar et al. (2020), Barontini and Holden (2019) survey the motivations of central banks for potentially issuing CBDC, whereas Auer et al. (2020) invest economic and institutional drivers for the exploration of CBDC.

³There are a number of papers providing a general overview and discussing the potential design features and characteristics of a CBDC, its impact, risks and benefits such as Berentsen and Schär (2018), Mancini-Griffoli et al. (2018), Bank for International Settlements (2018), Engert and Fung (2017), Bech and Garratt (2017) Bordo and Levin (2017), Barrdear and Kumhof (2016) or Meaning et al. (2018).

⁴Most distinctions are between (i) a retail and a wholesale CBDC, (ii) whether a CBDC should be interest bearing, (iii) about the accessibility, (iv) the convertibility to other means of payment, (v) whether it is account or token based (vi) whether credit/overdraft should be permitted and (vii) its administration.

The comparison shows that the digital characteristic is not a crucial and distinguishing feature of CBDC. On that account, the name CBDC may sometimes be misleading but more precise names such as “Reserves for All” (Niepelt (2018)) or “Digital Base Money” (Mersch (2017)) did not prevail.

	Gov. Bonds	Deposits	Cash	CBDC
No Default Risk	✓	✗	✓	✓
Interest-bearing	✓	✓	✗	✓
Medium of Exchange	✗	✓	✓	✓

Table 1: Characteristics of different money & asset types

One of the most crucial and critical aspects is the impact of a CBDC on the banking sector - especially in times of crises. The concern is that a CBDC may increase the risk of an aggregate bank run in any situation⁵. The argument is that it reduces the costs of a bank run by providing an attractive alternative to bank deposits. In times with low confidence in the financial system, depositors compare costs and advantages of different possibilities. Because a CBDC provides a less costly but still completely safe alternative than cash (or e.g. tax accounts), the fear is that runs may take place under a CBDC when depositors otherwise would not have run. As a further aspect, runs could unroll much faster due to (potentially unlimited) 24/7 access to CBDC and in addition costs and frictions of running would be lower.

The aim of this paper is to study the impact of a CBDC on an aggregate banking crisis and how it affects the possibility of aggregate bank runs⁶. Analyzing this issue in a formal model, helps to gain more thorough insights into what is considered one of the most serious potential drawbacks of a CBDC. The study thereby contributes to the literature of evaluating the potential effects of introducing a CBDC and the possible and optimal design features of a CBDC.

A CBDC is a liability on the central bank balance sheet. However, the question whether to introduce a CBDC necessarily carries along the question how a CBDC will

⁵The necessity for more thorough inquiries into this topic has been acknowledged by many academics and policy makers alike, among others Lagarde (2018), Bank for International Settlements (2018), Carney (2018) or Meaning et al. (2018).

⁶My analysis focuses only on aggregate crises and runs. However, there is a consensus in the literature that a CBDC would not have a major impact in the case of *individual* bank runs (e.g. see Juks (2018), Mancini-Griffoli et al. (2018) or Barrdear and Kumhof (2016)). A CBDC would not change the situation of an individual bank experiencing a run, it would just provide an additional channel how to run on the bank. Moreover, a CBDC may even improve the way how individual bank runs are dealt with and thereby might even decrease the risk of such runs (Kumhof and Noone (2018), Mancini-Griffoli et al. (2018)) and their amplification (Meaning et al. (2018)).

be accommodated on the asset side. This “asset side question” has only received little attention so far, however is an important factor influencing the effect a CBDC has on the economy. The operations accommodating a CBDC in the central bank’s balance sheet can be grouped into four stylized categories: (1) Loans to banks (LOB), (2) Corporate Asset Purchases (CAP), (3) Government Bond Purchases and (4) Loans to households which are more thoroughly discussed in section 3. The subsequent model analysis investigates incorporates the two policy options (1) LOB and (2) CAP and compares the effect of these two policies with each other and to the situation without a CBDC. Analyzing options (1) and (2), will also implicitly consider option (3) as argued in section 3. Although option (4) is theoretically feasible, practically it is currently not a viable option and therefore will be excluded from the analysis for now.

The model extends the framework of [Gertler and Kiyotaki \(2015\)](#) to incorporate a CBDC. The real, infinite horizon, endowment economy consists of households, bankers and a central bank. Goods are distinguished into a non-durable consumption good and a durable capital good. Capital is used as a proxy for the firm sector and modeled as a “productive technology” yielding a return each period which is subject to aggregate shocks. Households can place their savings in CBDC, deposits or capital. However, the bank is more efficiently investing into capital. It therefore takes deposits, and under *LOB policy* loans and purchases capital. The central bank issues CBDC and in the *LOB policy* scenario accommodates it by providing credit to banks, whereas it purchases capital itself in the *CAP policy* scenario. During normal times, the model features a unique equilibrium in which financial intermediation works smoothly. However, if a sufficiently large shock hits the economy, a second equilibrium emerges which is characterized by a bank run to the aggregate banking system. If households believe that other agents will withdraw deposits, they will revoke their funds, causing a bank run triggered by self-fulfilling beliefs.

The model is illustrated on a numerical example which offers four main insights: First, in Steady State, a CBDC does not change aggregate output and prices. However, it affects the composition of household savings, bank funding and capital investment, leading to a reduction in bank profits. Second, in a crisis both CBDC policy options have a stabilizing effect on the economy, mitigating the impact of the shock by reducing it more to its “fundamental” part. It cushions the decline in bank capital and net worth but also shifts the relative weight of the welfare loss more towards households. Third, both CBDC policy scenarios improve financial stability by deferring the emergence of a bank run

equilibrium to larger shocks. This delays the possibility of a default due to illiquidity and a default due to insolvency. Fourth, in a bank run under *CAP policy* the loss in output is attenuated through the stabilizing effect of CBDC on capital prices. Thus, I find that a CBDC strains the banking sector in normal times by reducing deposits and net worth. Yet, contrary to prevailing concerns, CBDC improves financial stability in times of crisis by impeding the emergence of bank runs. The discrepancy between concerns and results stems from partial equilibrium considerations disregarding the “asset side issue” of CBDC. The following section 2 sets this paper into the context of the related literature. Section 3 discusses the “asset side question” after which in section 4 the model economy will be described in normal times and in a bank run scenario, followed by a simulation of a numerical example in section 5. Section 6 finally concludes.

2 Related Literature

There are four papers thematically most similar to this paper who investigate the impact of a CBDC on financial stability and bank runs in different conceptual frameworks. The model by [Fernández-Villaverde et al. \(2020a\)](#) studies the effect of a CBDC on financial intermediation and bank runs on commercial banks but also on the central bank in the seminal framework of [Diamond and Dybvig \(1983\)](#). In their model, the central bank gives out CBDC and, similar to banks, can either store the funds from the CBDC or indirectly invest it into a long-term project. However, unlike banks, the central bank cannot abort these projects prematurely yielding only a fraction of the initial investment. This is the key difference, opening the door for bank runs on commercial banks and preventing bank runs on the central bank. In case of a bank run on the central bank, it is not possible to pay the promised return to all depositors. The central bank would therefore either punish withdrawing depositors or suspend payment until the next period which would prevent the bank run from unrolling in the first place. Most likely, the measures preventing a bank run would trigger a great controversy – even if just raised as a discouraging threat.

Second, [Fernández-Villaverde et al. \(2020b\)](#) extend the above framework such that depositors hold CBDC contracts in nominal terms. In the nominal world, the central bank can now manipulate the price level and impede real runs on the central bank at the cost of giving up its inflation target. In this setting, runs on the central bank can unroll either as real run or as “run on the price level” which both lead to comparable real consequences. The central bank can however prevent runs by threatening depositors with

an inflationary policy. Even though the model can prevent bank runs on the central bank, it leads to a trade-off between price stability and financial stability in which credibility concerns are also intertwined.

In the third paper by [Skeie \(2019\)](#) similar trade-offs between financial and price stability arise as the central bank's function as a lender of last resort requires inflationary policy. The model framework builds on a nominal OLG economy in which depositors have preferences to consume early or late in the period in the spirit of [Diamond and Dybvig \(1983\)](#). In the economy there is a competition between private and public digital currency. Banks can take deposits both in form of private and public digital currency. The central bank can only prevent runs by acting as a lender of last resort on bank deposits held in public digital currency, whereas the private digital currency is subject to a default risk. However, the central bank cannot (credibly) commit to a money supply rule and can therefore create distortionary inflation.

The fourth closely related analysis by [Kim and Kwon \(2019\)](#) remains in the context of an OLG model in which deposits and CBDC intermediate assets between households born with an endowment and households who only receive an endowment in the second period of their lifetime. In their framework, banks work locally and, in contrast to CBDC, claims on deposits can only be verified in the region of operation. In the baseline scenario in which CBDC is accommodated via government bonds, CBDC decreases financial stability. Banks are in less cases of the state-space able to pay out depositors that move to a different location. However, when the central bank channels the funds back to the banks in form of credit, it increases financial stability. [Kim and Kwon \(2019\)](#) look at banking “panics” describing a situation of financial distress in which the claims of a fraction of depositors that need to withdraw money because they relocate, cannot be fully repaid. This contrasts to the definition of a bank run on which this paper is based where depositors want to withdraw money at a time at which they otherwise would not have withdrawn more classically representing a “panic”.

In this paper, in contrast to [Fernández-Villaverde et al. \(2020b\)](#) and [Fernández-Villaverde et al. \(2020a\)](#) bank runs are studied in an infinite horizon and more dynamic but real setting. The sequential service constraint is replaced by the timelier assumption – especially for digital bank runs – of an equal haircut for depositors. Bank runs are not possible anytime, but the existence of a bank run equilibrium depends on economic fundamentals emerging only in a crisis which further exhibits a financial accelerator.

[Skeie \(2019\)](#) assumes that the central bank can only operate under discretion, whereas this model analysis applies a rule driven framework. By investigating CBDC under two stylized accommodation policies, this paper shares policy features of all four papers: In [Fernández-Villaverde et al. \(2020a\)](#) and [Fernández-Villaverde et al. \(2020b\)](#) the central bank invests into capital itself or via an investment bank which resembles the *CAP policy* scenario in this paper. Conversely, in [Skeie \(2019\)](#) and [Kim and Kwon \(2019\)](#) the central bank provides credit to banks which reflects the *LOB policy* scenario.

Besides projects focusing on banking crises under a CBDC, there is a number of papers studying the *general* impact of CBDC on the banking sector. The papers come to differing results and conclusions, majorly depending on the form of competition that is assumed in the banking sector. [Siciliani \(2018\)](#) studies competition between CBDC and banking in a 2-sided platform network analysis. He concludes that in the most optimistic scenario, CBDC at least increases the costs of funding for banks and decreases the net interest margin, but in a more pessimistic scenario may lead to a substantial outflow of deposits and, most radically, would force banks to effectively turn their business into a narrow bank with uninsured lending. However, the presence of demand side frictions due to brand preferences and the interoperability between both CBDC and bank platforms can mitigate the effects of deposit disintermediation. [Keister and Sanches \(2019\)](#) analyze the competition between CBDC and bank deposits within a perfectly competitive but constrained banking sector in a New Monetarist model in which CBDC affects the liquidity premium. They find that a CBDC increases payment efficiency, however, also crowds out bank deposits and increases bank funding costs. In contrast to the papers above, the subsequent papers come to more optimistic conclusions. [Chiu et al. \(2019\)](#) extend the model of [Keister and Sanches \(2019\)](#) by introducing an imperfectly competitive banking sector into the model. They show that a CBDC can increase investment improving financial intermediation, as CBDC interest rates are in an appropriate intermediate range. The impact of CBDC on the banking sector has also been studied in an OLG framework by [Andolfatto \(2018\)](#). In his model the banking sector is also under monopolistic competition. Similar to [Chiu et al. \(2019\)](#), he finds that a CBDC does not necessarily disintermediate banking services and could even increase bank deposits, as well as decrease monopoly profits and promote financial inclusion.

3 CBDC and the Asset Side Question: Two sides of the same coin

The idea of opening the central bank balance sheet to the public is not new. Quite contrary, it was a common central bank practice until the mid of the 20th Century. As elaborated in [Fernández-Villaverde et al. \(2020a\)](#), historically many major central banks not only allowed deposits but also granted loans to private firms and individuals, which can partly be explained by the origins of central banking from public and private banks ⁷. This practice of enabling deposits by the public was prominently brought back on stage and into the electronic sphere by [Tobin \(1985, 1987\)](#) who proposed a “deposited currency” from which the current discussion about CBDC mostly picks up from.

However, (re)introducing a Central Bank Digital Currency would create a new type of liability on the central bank balance sheet. This would lead to an increase in the size of the central bank balance sheet, unless the demand for CBDC is fully offset by a decline in cash⁸. An increase on the liability side needs, by accounting identity, to be offset by respective accommodations on the asset side. Therefore, the question whether to introduce a CBDC necessarily carries along the question how a CBDC will be accommodated on the asset side. But the way how the central bank invests funds from CBDC, has substantially different implications for market participants and thus it is necessary to carefully study the impact on households and banks. Likewise, [Barker and Thomas \(2017\)](#) argue that “although there has been a lot of discussion about how central bank digital currency could radically change payment systems – and even the financial sector as a whole – the implication for the assets on central bank balance sheets could be just as critical”. However, this issue has so far been neglected in most of the discussion about CBDC, although it crucially drives results⁹.

There are several possibilities how to accommodate CBDC on the asset side of the balance sheet, boiling down to the question to which group of market participants the funds from CBDC are channeled. Table 2 shows the stylized balance sheet of a central bank with the extension of a CBDC, following the representation of central bank balance sheet data of the International Financial Statistics by the IMF. The asset side is struc-

⁷ see [Bindseil \(2019\)](#) or [Roberds and Velde \(2014\)](#) for a comprehensive review of early public banks and the origins of central banking.

⁸This seems unrealistic as it would imply that there would be no substitution effect from deposits.

⁹To my knowledge, besides [Barker and Thomas \(2017\)](#) this issue so far has only been raised by [Kaminska \(2017\)](#) and [Niepelt \(2018\)](#).

Stylized Central Bank Balance Sheet with a CBDC

Assets	Liabilities
	Banknotes
	CBDC
Claims on non-residents	Liabilities to non-residents
Claims on financial institutions	Liabilities to financial institutions
Claims on government	Liabilities to government
Claims on private sector	

Table 2: Stylized Central Bank Balance Sheet with a CBDC. Own representation following the counterparty structure of the standardized IFS data

tured by counterparties to the central bank, holding claims on financial institutions , the corporate sector, the government and foreigners. Although, as further option, granting loans to households is not a current central banking practice, it is theoretically an possibility and has been used historically (e.g. see [Fernández-Villaverde et al. \(2020a\)](#)) and therefore will be touched upon in this chapter. Analyzing the IFS data in a comparative study, [Pattipeilohy \(2016\)](#) highlight the heterogeneity in the balance sheet composition of central banks across countries – especially between emerging and advanced economies – and across time.

It is to be expected that central banks in practice would use a mix of those asset side operations to offset CBDC movements. However, for analytical purposes, looking at the impact of these measures in isolation can provide valuable insights. The following will examine the different options and translate them into the model framework for subsequent analysis. The stylized model features households, banks and the central bank as active agents in the economy investing in capital as a proxy for the firm sector. Table 3 illustrates the “asset side question” in the balance sheet representation of the model economy. As a simplification, the central bank only issues CBDC on the liability side and can either (1) grant loans to banks (LOB policy), (2) buy capital (equivalent to corporate asset purchases - CAP Policy), (3) buy government bonds (GOP policy) or (4) grant loans to households (LOH policy). I consider a closed economy and therefore abstract from foreign exchange holdings by the central bank.

Option 1: Loans to Banks (LOB policy)

Firstly, the central bank can invest the funds obtained from CBDC as loans to the financial sector. Thereby, it would channel the lost deposit funding through CBDC back to the banks in form of credit. While credit to financial institutions already is a substantial

Households		Banks		Central Bank	
A	L	A	L	A	L
Capital	Equity	Capital	Deposits	???	CBDC
Deposits	(endowment)		Equity		
CBDC			(net worth)		

Table 3: Balance sheet representations of the stylized agents in the model

component of central bank assets in normal times, it becomes especially relevant in times of crisis. The paradigm of the central bank as lender of last resort, dating back to [Bagehot \(1873\)](#), is also documented by [Pattipeilohy \(2016\)](#) who shows a clear shift towards private sector lending by the Federal Reserve, Bank of England and the Eurosystem in the financial crisis from 2007 onwards. Providing credit to banks from the funds obtained from CBDC is therefore a most relevant and evident option, in normal times and even more in a financial crisis which is the focus of this analysis. Therefore, in the following this policy option will be carefully analyzed and abbreviated with “LOB policy” which is illustrated by table 4. Furthermore, it is also the basis for the equivalence result of “public and private money” of [Brunnermeier and Niepelt \(2019\)](#). They show that under the assumption that CBDC is fully redistributed as credit (to the same conditions) to private banks, equilibrium allocations stay unaltered by CBDC and only the counterparty changes. This accommodation policy is also used in [Skeie \(2019\)](#) and [Kim and Kwon \(2019\)](#).

Households		Banks		Central Bank	
A	L	A	L	A	L
Capital	Equity	Capital	Deposits	Loan B	CBDC
Deposits	(endowment)		Loan B		
CBDC			Equity		

Table 4: Option LOB Balance sheet representations of the stylized agents in the model

Option 2: Corporate Asset Purchases (CAP Policy)

Secondly, the central bank could purchase corporate assets such as corporate bonds. Although still an unconventional monetary policy tool, corporate bond purchases have gained importance since the financial crisis and found increasing application mitigating the economic effects of the covid pandemic. Translated into the stylized nature of the model framework, this policy option would be equivalent to direct capital purchases by the central bank as visualized by table 5. In the following, this option will be dubbed as “CAP policy”. This policy has been modeled indirectly by [Fernández-Villaverde et al.](#)

(2020a) and directly by [Fernández-Villaverde et al. \(2020b\)](#) extending the seminal paper of [Diamond and Dybvig \(1983\)](#) in which the central bank invests into long-term projects (through an investment bank).

Households		Banks		Central Bank	
A	L	A	L	A	L
Capital	Equity	Capital	Deposits	Capital	CBDC
Deposits	(endowment)		Equity		
CBDC					

Table 5: Option CAP Balance sheet representations of the stylized agents in the model

Option 3: Government Bond Purchases (GOP policy)

As a third option, the central bank could buy government bonds in exchange for CBDC. For most central banks, government bonds make up the largest part of the asset side of the balance sheet. This is vividly illustrated by [Pattipeilohy \(2016\)](#) who develop a classification scheme of central banks by their balance sheet composition in which “Treasury holder” is one type. Most of the major central banks classify for to this category in 2015. To be implemented into the model economy, the government would have to be introduced as further player as shown in table 6. It would issue government bonds that would be held by banks and the central bank. The question that arises thereof is: For what will the government debt be used? As before, the options can be grouped into different categories, boiling down to the question at which group of market participants the spending is targeted. Thus, the “asset side question” is just shifted up one entity further. Therefore, in the model this will not be included as a separate policy option because the option of purchasing government bonds is already embedded in other options and implicitly analyzed. This can be made even more explicit by regarding the monetary and fiscal authority as a consolidated entity¹⁰. In the framework, it would also not make a difference, whether the central bank would buy government bonds in the primary market (from the government directly) or on the secondary market (from banks). This option has been applied in the literature e.g. by [Barrdear and Kumhof \(2016\)](#) and [Kumhof and Noone \(2018\)](#) who even argue that CBDC should only be issued against government bonds and has also been used by [Kim and Kwon \(2019\)](#).

Option 4: Loans to households (LOH policy)

A fourth possibility could be that the central bank does not only open the liability but also the asset side to the broad public and would also grant loans. However, in this way

¹⁰I would like to thank Frank Smets for raising this point.

Households		Banks		Central Bank		Government	
A	L	A	L	A	L	A	L
Capital	Equity	Capital	Deposits	Gov.	CBDC	???	Gov.
Deposits	(endowment)	Gov.	Equity	Bonds			Bonds
CBDC		Bonds					

Table 6: Option GOB Balance sheet representations of the stylized agents in the model

the central bank would come even closer to the business model of traditional banks. While the advantages of this option remain questionable, it would require huge investments to build up the capabilities and expertise to screen and monitor household credit within the central bank. Furthermore, the possibility of “negative CBDC holdings” i.e CBDC credit to the broad public is not considered in the current debate and also has been categorically excluded by some (e.g. see [Sveriges Riskbank \(2017\)](#) or [Barrdear and Kumhof \(2016\)](#)). That is why, for now, this option will not be further considered.

4 Model

4.1 Basic Setup

I introduce a central bank digital currency into the real bank run model of [Gertler and Kiyotaki \(2015\)](#) (henceforth: GK15). The compact and stylized model follows the spirit of [Diamond and Dybvig \(1983\)](#), yet improves the seminal bank run paper along many dimensions¹¹. Closely following the model of GK15 and using the same calibration provides an important benchmark that allows me to compare the results under a CBDC to the results of GK15 without a CBDC. The model description below applies to both CBDC policy scenarios: *LOB* and *CAP* accommodation policy. There are some differences that will be highlighted at the respective passage.

The goods in the economy consist of two types: a non-durable consumption good C_t and a durable capital good K_t . The amount of capital in the economy is of fixed supply, normalized to unity and does not depreciate. A unit of capital good at time t , K_t , yields Z_{t+1} units of consumption good the next period and can then be sold at the price Q_{t+1} . The productivity of capital Z_{t+1} is subject to a multiplicative aggregate shock to productivity. Thus, capital is a productive technology that stands as proxy for the firm

¹¹The infinite horizon model by GK15 incorporates the financial accelerator and bank runs depend on economic fundamentals, although ultimately still triggered by sunspot shocks. Furthermore the liquidation price of capital is determined endogenously and the bank run equilibrium does not depend on a sequential service constraint, but depositors receive an equal hair cut in the event of a run – a much more timely assumption, especially for “digital bank runs”.

sector. The process for the evolution of aggregate productivity is given by:

$$Z_t = Ze^{z_t} \quad \text{with} \quad z_t = \rho z_{t-1} + \epsilon_z \quad (1)$$

where Z is aggregate productivity in Steady State and ϵ_z with $E(\epsilon_z) = 0$ is a shock to productivity.

Two types of representative agents live in the economy: households and bankers, each with a unity mass of one. Households consume and save for which they have three options: (i) invest directly in capital, (ii) hold bank deposits or (iii) acquire CBDC. Banks are more efficient managing the productive asset but are subject to a leverage constraint. They maximize profits by investing funds from households and the central bank (under *LOB policy*) into productive assets. Furthermore, there is a monetary authority giving out CBDC to households and reinvest the liabilities by either granting loans to the banks (under *LOB policy*) or investing it directly in the productive asset (under *CAP policy*).

4.1.1 Households

Each period, households receive an endowment of $z_t W^h$ units of the consumption good, thereby abstracting from labor choices. In this way, household income is also affected by the multiplicative aggregate shock to productivity z_t . Households, maximize lifetime utility through the choice of consumption and saving:

$$U_t = E_t \left(\sum_{i=0}^{\infty} \beta^i \ln C_{t+i}^h \right)$$

Households have three options to save. They can:

(i) hold deposits D_t at a bank which promises to pay a non-contingent gross rate of return R_{t+1} in the next period in the absence of a bank run. In the case of a bank run, households receive only a fraction of the promised payment:

$$R_{t+1} = \begin{cases} \bar{R}_{t+1}, & \text{under no bankruptcy} \\ x_{t+1} \bar{R}_{t+1}, & \text{under bankruptcy} \end{cases}$$

A bankruptcy can be caused either by a sufficiently large shock (default due to insolvency) or by a bank run (default due to illiquidity). However, so far, households do not anticipate a bankruptcy and therefore expect a zero default probability of their deposits.

(ii) **directly invest in productive capital** K_t^h . When buying capital, households face an increasing and convex “management cost” for each unit of capital K_t^h they purchase. This reflects the characteristic that household are less efficient investors lacking expertise compared to the banks. Following GK15, the management costs take the form: $f(K_t^h) = \frac{\alpha}{2} (K_t^h)^2$. In the subsequent period, households receive a payoff of $Z_{t+1}K_t^h$ on the invested capital.

(iii) **hold CBDC** M_t which pays a non-contingent rate of R_{t+1}^m set by the central bank. Furthermore, households receive a transfer T_t from the central bank paying out its seigniorage. However, if the central bank incurs losses ($T_t < 0$), the transfer turns into lump-sum taxation. The households budget constraint is thus given by:

$$C_t + D_t + Q_t K_t^h + \frac{\alpha}{2} (K_t^h)^2 + M_t^h = Z_t W^h + R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h + R_t^m M_{t-1}^h + T_t$$

Maximizing expected utility subject to the budget constraint yields the optimality conditions that in order to save through all three options, the return must be equal to the inverse of the stochastic discount factor $\Lambda_{t,t+i} \equiv \beta^i \frac{C_t^h}{C_{t+i}^h}$:

$$R_{t+1}^m = R_{t+1} = E_t(R_{t+1}^h) = E_t\left(\frac{C_{t+1}^h}{C_t^h} \beta^{-1}\right) = \frac{1}{\Lambda_{t,t+1}} \quad (2)$$

$$\text{where } R_{t+1}^h \equiv \frac{Z_{t+1} + Q_{t+1}}{Q_t - \alpha K_t^h}$$

.

4.1.2 Bankers

There exists a unit mass of risk-neutral bankers, each managing their own financial intermediary. Every period, there is an i.i.d. probability of $0 < 1 - \sigma < 1$ that the banker goes out of business resulting in an expected lifetime of $\frac{1}{1-\sigma}$. Bankers only enjoy consumption upon exit and consume their accumulated net worth in the period they exit. Each period, the same amount of $1 - \sigma$ new bankers enters with a starting capital of w^b , such that the number of banks remains constant¹². Hence, while in business, bankers aim to maximize their net worth n_t . They do so by collecting deposits from households d_t and loans l_t (only in the *LOB policy* scenario) and invest it, along with their net worth, into the productive technology k_t^h at price Q_t . Bankers are considered to have a relative efficiency advantage

¹²The limited lifetime approach is a common modeling feature that ensures that bankers hold leverage in equilibrium and do not save up equity over time to evade the financial constraint.

in screening and monitoring projects and therefore do not face the “management costs” of capital that households have to pay¹³. This way bankers hold capital on the asset side and household deposits, central bank credit and equity/net worth on the liability side of their balance sheet:

$$Q_t k_t^b = d_t + n_t + l_t. \quad (3)$$

The difference between the earnings from capital and the cost of deposits as well as central bank loans yields the net worth of the bank:

$$n_t = (Z_t + Q_t)k_{t-1}^b - R_t d_{t-1} - R_t^L l_{t-1}. \quad (4)$$

The franchise value of the bank is then the sum of the discounted probability to exit in period i and consume the accumulated net worth ($n_t = c_t^b$ upon exit):

$$V_t = E_t \left[\sum_{i=1}^{\infty} \beta^i (1 - \sigma) \sigma^{i-1} n_{t+i}^b \right] = E_t \left[\sum_{i=1}^{\infty} \beta^i (1 - \sigma) \sigma^{i-1} c_{t+i}^b \right] \quad (5)$$

and can be expressed recursively as the discounted expected probability to exit in $t + 1$ plus the discounted probability weighted continuation value of the financial intermediary:

$$V_t = E_t \left[\beta (1 - \sigma) n_{t+1}^b + \beta \sigma V_{t+1} \right]. \quad (6)$$

In the absence of financial frictions, it would be optimal that all funds are intermediated by the bank. However, bankers face an endogenous capital constraint microfounded by a moral hazard problem: Bankers can divert a certain fraction $0 < \theta < 1$ of assets for personal use. Deciding whether to “cheat” or operate honestly, bankers compare the value of both options. Knowingly, depositors and the central bank will only fund the bank to an amount such that the incentive to divert funds does not arise. This results in the constraint that the franchise value of the bank always has to be larger or equal than the gains from diverting assets. However, it is assumed that the central bank is to a lesser extent subject to the moral hazard risk which the households face. Due to its supervisory power, its legal authority and additional measures such as collateral requirements, the central bank is better at enforcing repayment from bankers as it is similarly modeled e.g. in [Gertler and Kiyotaki \(2010\)](#), [Gertler and Karadi \(2013\)](#) or [Gertler et al.](#)

¹³This modeling feature can also be seen in a way that bankers do face a positive management cost but the cost is normalized to zero.

(2016). Thus, for any unit of central bank credit supplied, a borrowing bank can only divert $\theta(1 - \omega)$ with $0 < \omega < 1$. The moral hazard problem boils down to the following incentive constraint:

$$V_t \geq \theta (Q_t k_t^b - \omega l_t) \quad (7)$$

General maximization problem of the banker

The banker chooses k_t^b, d_t, l_t each period to maximize the franchise value of the bank (6) subject to the incentive constraint (7), the evolution of net worth (4) and the balance sheet constraint (3):

$$\begin{aligned} V_t &= \max_{k_t^b, d_t, l_t} E_t [\beta^i (1 - \sigma) n_{t+1} + \beta \sigma V_{t+1}] \\ \text{s.t. } V_t &\geq \theta (Q_t k_t^b - \omega l_t) \\ n_{t+1} &= (Z_{t+1} + Q_{t+1}) k_t^b - R_{t+1} d_t - R_{t+1}^l l_t \\ Q_t k_t^b &= d_t + n_t + l_t \end{aligned} \quad (8)$$

The objective as well as the constraints of the bank are constant returns to scale and thus can be made independent of the total level of net worth by expressing the problem in terms of per unit of net worth. A detailed description of the reformulation and solution to the maximization problem can be found in the Appendix 7.1. In the optimum, the bank chooses its leverage as high as possible such that the incentive constraint is always binding. This yields the following optimality conditions:

$$(1 - \omega) E_t [(R_{t+1}^b - R_{t+1})] = E_t [(R_{t+1}^b - R_{t+1}^l)] \quad (9)$$

where $R_{t+1}^b \equiv \frac{Z_{t+1} + Q_{t+1}}{Q_t}$ is the return of a unit of capital invested by banks. Equation 9 determines the allocation of deposit funding and credit by the central bank. In order for the bank to be indifferent between the two funding sources, the expected net interest margin of using deposit financing and central bank credit, adjusted for the relative tightness of the leverage constraint, must be equal. Due to the relatively tighter limit on deposits captured by $(1 - \omega)$, this implies that the central bank lending rate must be higher than the deposit rate. The lower deposit rate compensates for its stricter limit on leverage and makes the banker indifferent between deposits and central bank credit. Following the argumentation of Gertler and Kiyotaki (2010), central bank lending can be

interpreted to be done at a penalty rate. The rationale for this, dating back to [Bagehot \(1873\)](#), is to prevent inefficient use of central bank credit.

Optimization problem under CAP policy

The above elaboration is the more general and applies for the *LOB policy*, as well as the *CAP policy*. However, under *CAP policy*, the situation of the banker is simplified as the central bank does not grant loans to the bank and thus $l_t = 0$. In this case, the bank's optimization problem reduces to the case in GK15 as there is only one type of leverage (deposits). Therefore, the bank simply chooses deposits per unit of net worth $\frac{d_t}{n_t}$ such that the incentive constraint is binding.

4.1.3 Central Bank

Liability Side

How is the issuance of CBDC determined? In principle, the monetary authority can either determine CBDC supply or the interest rate. Under a CBDC supply rule, the amount of CBDC would be determined by the central bank. This is a viable possibility that should be examined further. However, studying how variation in CBDC affect the banking sector in course of a crisis under a CBDC supply rule does not seem expedient as the answer would be given by the rule itself. That is why, the following will adopt an interest rate rule to determine the issuance of CBDC. This is also closer to the current implementation of monetary policy.

How should the CBDC interest rate rule be specified¹⁴? Considering the monetary policy goals, the CBDC interest rate rule would react to changes in the aggregate price level as well as the output gap. As the underlying model is a real model, the only price in the economy is the relative price between the capital and consumption good. As it is not the goal of monetary policy to stabilize relative prices, the CBDC policy rule in this model will only react to fluctuations in the output gap.

Moreover, concerns have been voiced about the ability to manage CBDC demand, especially in times of stress (e.g. see [Kumhof and Noone \(2018\)](#), [Juks \(2018\)](#) or [Bank for International Settlements \(2018\)](#)). As a possible solution, [Bindseil \(2020\)](#) suggests a tiered

¹⁴And should there be an non-zero interest rate on CBDC at all? It is a relevant discussion (e.g. see [Sveriges Riskbank \(2018\)](#) or [Camera \(2017\)](#)) whether CBDC should have a non-zero interest rate at all and there are good reasons to be cautious. However, an interest rate not restricted to zero is the more general case and it further enables the central bank to manage CBDC and have a stabilizing influence on the economy.

remuneration approach as common in central banking practice. The proposal entails that up to a certain amount, CBDC yields an attractive interest rate. However, the amount of CBDC that surpasses the limit is remunerated with a less attractive interest rate. In this way, the demand for CBDC could be effectively managed. To stabilize CBDC demand, the spirit of the tiering proposal of [Bindseil \(2020\)](#) will be incorporated in the policy rule. For modeling purposes, the rule will be transformed into a continuous setting discouraging to take on more CBDC when the amount of CBDC in circulation is already high.

Therefore, the CBDC rule will react to the output gap as well as on CBDC demand in the simple form of:

$$R_t^m = \delta_0 + \delta_1 \left(\frac{Y - Y_{t-1}}{Y} \right) - \delta_2 M_{t-1} \quad (10)$$

where Y is the output in Steady State. The negative response of the CBDC interest rate to the output gap might sound counterintuitive at first. However, it is important to be aware that this is not an interest rate influencing credit conditions. In this setting, the CBDC interest rate is just an interest rate for savers, granting households an additional opportunity for savings. Further it is to be noted that the stylized framework abstracts from all other policy instruments of the central bank.

Asset Side: LOB Policy

The rule allocating credit to the banks is straightforward. By assumption, this policy option accommodates CBDC by providing credit to banks. Therefore, all CBDC funding is directly channeled back to the banks:

$$L_t = M_t \quad (11)$$

The central bank “reinvests” CBDC into the banking sector in a 1:1 fashion and grants loans L_t to banks equivalent to the amount of CBDC. The interest rate on central bank credit R_t^l is determined endogenously.

Following, the central bank flow of funds is given by:

$$M_t + R_t^l L_{t-1} = R_t^m M_{t-1} + L_t + T_t \quad (12)$$

where each period the central bank issues CBDC M_t and receives the loan repayments plus interest of the last period. It uses the funds to repay the claims and interest on CBDC from the previous period and additionally gives out new loans. The remainder is distributed as transfers T_t to the households (or levied through taxation in case of a

negative residue). In the no-run equilibrium, the interest rate on credit is higher than the CBDC interest rate and therefore the transfers, although small, are positive.

Asset Side: CAP Policy

Contrary, we now assume that the central bank does not accommodate CBDC with credit but buys capital itself. However, in doing so the central bank is less efficient than the bank but more efficient than private households. Therefore, if the central bank acquires capital it faces – similarly to households but at a lower level – an increasing and convex cost $f(K_t^{cb}) = \frac{\alpha_{cb}}{2} (K_t^{cb})^2$ with $\alpha_{cb} < \alpha$.

As under *LOB policy*, the central bank completely offsets CBDC by investments in capital and therefore buys as much capital such that the gross investment equals the amount of CBDC supplied to households:

$$Q_t K_t^{CB} + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 = M_t \quad (13)$$

The flow of funds of the central bank evolves similarly to 12. The funds from the CBDC and returns of capital from the last period are used to pay back CBDC claims from the last period and for investments into new capital plus management costs:

$$M_t + (Z_t + Q_t) K_{t-1}^{cb} = R_t^M M_{t-1} + Q_t K_t^{cb} + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 + T_t \quad (14)$$

4.1.4 Aggregation, Timing and Equilibrium (no bank run)

Summing up the individual banks (denoted in small letters) leads to the banking sector aggregate (denoted in capital letters). As there is a constant number of symmetric banks normalized to unity, but banks exit and new banks enter each period, aggregate net worth evolves according to:

$$N_t = \sigma [(Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} - R_t^l L_{t-1}] + W^b \quad \text{with} \quad (1 - \sigma) w^b = W^b \quad (15)$$

Capital is of fixed supply and normalized to unity, therefore capital holding of households K_t^h , banks K_t^b and the central bank K_t^{cb} add up to:

$$K_t^h + K_t^b + K_t^{cb} = 1 \quad (16)$$

Under the *CAP policy* $K_t^{cb} \geq 0$, however under *LOB policy* $K_t^{cb} = 0$ as the central bank does not invest into the productive technology itself.

Gross output adds up to:

$$C_t^h + \frac{\alpha}{2} (K_t^h)^2 + \frac{1-\sigma}{\sigma} (N_t - W^b) + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 = Z_t + Z_t W^h + W^b \quad (17)$$

The sequence of events is the following: At the beginning of period t , the productivity of capital Z_t is realized. The new allocation of deposits, CBDC, loans and capital investments is then determined. Deposits and credit are issued such that the moral hazard constraint is satisfied and the banker does not have the incentive to divert assets subsequently. The next period start with the realization of return on capital invested the previous period.

Under *LOB policy*, the equilibrium is given by the vector of real prices Q_t, R_t, R_t^m, R_t^l and a vector of quantities $Z_t, K_t^b, K_t^h, D_t, M_t, L_t, N_t, \frac{V_t}{N_t}, C_t^h, T_t$ which is determined by equations 1, 2, 3, 4, 6, 7, 9¹⁵, 10, 11, 12, 16 and 17 constitute an equilibrium.

Under *CAP policy* the equilibrium is determined by the slightly different vector of real prices Q_t, R_t, R_t^M and a vector of quantities $Z_t, K_t^b, K_t^h, K_t^{cb}, D_t, M_t, N_t, \frac{V_t}{N_t}, C_t^h, T_t$ established by the equivalent set of equations as under *LOB policy*, in which equations 11 and 12 are replaced by 13 and 14 and equation 9 falls out¹⁶. A compilation of the equilibrium equation under both policies can be found in the Appendix 7.2 and 7.3.

4.2 Banking Crises and Bank Runs

Under certain conditions, a bank run equilibrium can emerge, alongside the “normal” equilibrium set out in the previous section. The following will look at the conditions under which a bank run equilibrium exists, outline how it unfolds under the two policy scenarios and describe the regions in which the equilibria can occur¹⁷.

Conditions for a Bank Runs Equilibrium

Each period, households decide whether to roll over their deposits to the next period. The individual decision to roll over deposits crucially depends on the expectation of other households. If a household expects that other households will not renew deposits and this

¹⁵3, 4, 6, 7 and 9 on the level of banking sector aggregates

¹⁶The attentive reader might have noticed, that the system is described by more unknowns than equations. However, equation 2 are the three optimality conditions for the households expressed condensed into one equation.

¹⁷It is important to note that so far, households do not expect a bank run to happen with a positive probability and the possibility of a run does not influence the decisions of agents.

would leave the bank with zero assets resulting in a bankruptcy, it is individual rational not to roll over deposits as well. If it comes to a bank run, forced to liquidate their assets, banks need to sell all their capital at the fire sale price Q_t^* . With the revenue they pay back the claims to households. The redemption value to the households determines whether a bank run equilibrium is possible: A run is only possible if the funds from the sold capital are less than the deposit claims such that depositors have to accept an equal haircut

$$x_t = \frac{(Q_t^* + Z_t)K_{t-1}^b}{R_t D_{t-1}} < 1 \quad (18)$$

on their claims $R_t D_{t-1}$. The existence of a bank run equilibrium is thus determined by economic fundamentals. However, whether a bank run is triggered is ultimately determined by the beliefs of households, induced in the model by a sunspot shock. Due to the strategic complementarity in behavior, the expectation that a bank run will happen is a self-fulfilling prophecy, given that the run would lead to default.

How does CBDC affect the existence of a bank run equilibrium and how does a bank run unfold under CBDC? As a reference, it is useful to first look what happens in the baseline scenario of GK15 without a CBDC: A large shock realizes and households withdraw their deposits. They have no other option than to invest in capital themselves. Because households now hold all the capital and they have increasing marginal costs of managing capital, they will only buy capital to a lower price. This makes the “fire sale price” of assets very low.

Bank Runs under *LOB Policy*

What changes if the central bank gives out CBDC and accommodates it with credit to banks? Generally, it becomes more difficult for a bank run equilibrium to emerge for two reasons: First, deposits make up a smaller share of the balance sheet of the bank because it also comprises loans from the central bank. Assuming that the central bank does not participate in the run and keeps the credit at the bank, the fire sale price of capital would now have to be much lower such that the value of the asset side does not cover the deposit claims anymore. In this way, central bank credit could be regarded as an additional “buffer” besides equity protecting against runs. Second and most importantly, households don’t have to absorb all capital in a bank run, as a considerable part of the lost deposit funding will be channeled back to the banks in form of central bank credit. The amount of credit provided to the bank depends on the share of withdrawn deposits that households hold in CBDC and capital. The implication that households only absorb

a part of capital in a run situation increases the fire sale price of capital which makes that the condition for a run harder to satisfy.

What happens if, against all odds, a run would take place in this scenario? Households withdraw their deposits, taking on a haircut and leave the bank with no assets but outstanding debt to the central bank. Without intervention the bank would go bankrupt and default on its claims to the central bank. Instead, I will assume that the monetary authority, acting as lender of last resort, further grants credit to the bank to be able to continue to operate but is under close supervision of the central bank¹⁸. In the period of the bank run the bank, the bank only invests the central bank credit into capital as it has zero net worth and zero deposits. Due to the outstanding claims, the capital return in the period of the run is fully paid out to the monetary authority. Following, the share of σ banks survive the bank run period but enter the period after the bank run with zero net worth. At the time of the bank run, the monetary authority must absorb the loss of the defaulted credit which it has to distribute via lump sum taxes (the transfers become negative) to households. However, in the period after the bank run, the central bank collects the full profit of the invested capital by the bank which it can distribute to households¹⁹.

Bank Runs under *CAP Policy*

How is the bank run equilibrium affected if a CBDC is introduced but the funds are invested into capital by the central bank itself? Compared to the baseline scenario of GK15, the condition for a run is harder to realize but less restrictive than under the *LOB policy*. The argument of the additional buffer on the bank's balance sheet dissolves because the central bank does not grant loans to the bank. What remains is the influence on the liquidation price: If the bank fire sells its capital both, households and the central bank, will buy the capital. Because the central bank faces lower capital management costs than households and capital holdings are divided, the price of capital will not drop as much as in the baseline case. The second key difference is that if it comes to an aggregate bank run, the central bank will not step in as a lender of last resort and the banks will go bankrupt, just as in GK15. The set of the condensed equilibrium condition at the time of the bank run under CAP Policy can also be found in the Appendix 7.5.

¹⁸Note that under this approach moral hazard concerns can arise which could be mitigated by close supervision.

¹⁹The set of the equilibrium conditions at the time of the bank run can be found in the Appendix 7.4.

4.2.1 Equilibrium Regions

In order to answer the question how a CBDC affects financial stability, it is crucial to also study how the different CBDC policies affect the equilibrium regions in the framework. With the existence of a second equilibrium, three equilibrium regions exist: In the first region there is a unique equilibrium with functioning financial intermediation – let’s call it the “banking equilibrium”. In the second region, the banking and the bank run equilibrium coexist. Here, the condition for a bank run equilibrium requires that the fire sale price of capital leaves the bank with zero net worth. Conversely, under non-firesale prices, the bank would have sufficient funds to repay its debtors. The scenario is therefore considered as a bankruptcy due to illiquidity and requires:

$$(Q_t^* + Z_t)K_{t-1}^b < R_t D_{t-1} \quad (19)$$

However, if a sufficiently large shock hits the economy and the liabilities of the bank exceed the capital value and return even without a withdrawal of fund from depositors, the economy enters the third equilibrium region in which only the bank run equilibrium prevails. This can be considered a bankruptcy due to insolvency given by:

$$\begin{aligned} (Q_t + Z_t)K_{t-1}^b &< R_t D_{t-1} \quad (\text{CAP policy \& GK15}) \\ (Q_t + Z_t)K_{t-1}^b &< R_t D_{t-1} + R_t^l L_{t-1} \quad (\text{LOB policy}) \end{aligned} \quad (20)$$

The conditions for a default due to illiquidity and default due to insolvency thus only differ in the price of capital. Figure 1 illustrates the thresholds for the different equilibrium conditions.

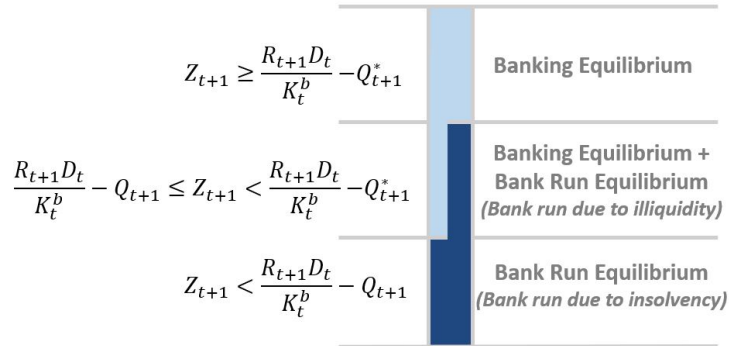


Figure 1: Equilibrium Ranges

5 Numerical Example

The following numerical example simulates the presented economy with a CBDC under *LOB policy* and *CAP policy*. The impact of the policies will be analyzed 5.2 in the Steady State , 5.3 after a 5% shock to capital productivity (no bank run) 5.4 determining the effect on the equilibrium regions 5.5 and after a 10% shock triggering a bank run under *CAP policy*.

5.1 Calibration and Computation

Calibration

To be able to compare the results under a CBDC with the baseline case of GK15 without a CBDC, all their parameter values are adopted. In this way, if the effect of a CBDC is muted, the results of GK15 are replicated. There are a few more parameters that are not in the specification of GK15 – most importantly the parameters for the CBDC interest rate rule. Table 7 lists all the parameter choices. It is not apparent how to set the parameters of the interest rate rule because there is no empirical counterpart. The value of the parameters $\delta_0 = 1.01948$ and $\delta_2 = 0.1$ targets a CBDC share of 10% of total household saving in the Steady State. Further, with $\delta_2 = 0.1$ an increase in the demand of CBDC has a distinct but moderate effect on the CBDC interest rate. The choice of $\delta_1 = 0.13$ is set such the reaction to output fluctuation is 30% stronger than the reaction to fluctuation in the CBDC demand.

Parameter	Value	Source	Description
α	0.008	GK15	Marginal management costs households
α_{cb}	0.004	new	Marginal management costs central bank (CAP)
β	0.99	GK15	Discount rate
σ	0.95	GK15	Bankers survival probability
θ	0.19	GK15	Moral hazard seizure rate
ω	0.5	GK13	Advantage in seizure rate for central bank credit (LOB)
ρ	0.95	GK15	Serial Correlation of productivity shock
W^h	0.045	GK15	Household endowment
W^b	0.0011	GK15	Bankers endowment
δ_0	1.0195	new	CBDC base parameter
δ_1	0.13	new	CBDC policy response to the price of capital
δ_2	0.1	new	CBDC policy response to CBDC demand

Table 7: Parameter Calibration;
GK15=Gertler and Kiyotaki (2015), *GK13=Gertler and Karadi (2013)*

Computation

As this analysis extends the model of GK15, it also follows the computational procedures as outlined in more detail in the [Online Appendix](#) of [Gertler and Kiyotaki \(2015\)](#)²⁰. The response to a shock without a bank run is computed as shock to a nonlinear perfect foresight model. To simulate a shock followed by a bank run, the equilibrium path is split into three components, calculated backwards. First, the path of the economy starting one period after the bank run back to the Steady State is calculated. This is possible, because the terminal values i.e. Steady State values are known but also the initial values of the backward looking variables. Those values can be used to calculate the forward-looking bank run equilibrium. Finally, until the period in which the bank run takes place, the equilibrium path is the same as under the shock without a bank run.

5.2 Steady State

Variable	LOB Policy	CAP Policy	GK15
Price of Capital Q_t	1	1	1
Output Y_t	0.0583	0.0583	0.0583
Capital Bank K_t^b	0.6899	0.5975	0.6895
Capital Household K_t^h	0.3101	0.3087	0.3105
Capital Central Bank K_t^{cb}	/	0.0938	/
Household Consumption C_t^h	0.0550	0.0551	0.0548
Banker Consumption C_t^b	0.0034	0.0032	0.0036
Bank Net Worth N_t	0.0650	0.0621	0.0689
Deposits D_t	0.5311	0.5354	0.6268
CBDC M_t	0.0938	0.0938	/
Loans to banks L_t	0.0938	/	/
Bank Leverage ϕ_t	10.6186	9.6150	10
Deposit Rate R_t	1.0404	1.0404	1.0404
CBDC Rate R_t^M	1.0404	1.0404	/
Loan Rate R_t^L	1.0452	/	/

Table 8: Comparison of Steady State Results, GK15=[Gertler and Kiyotaki \(2015\)](#)

The Steady State values of aggregate quantities and prices are the same in all three scenarios, but the CBDC affects the allocations. Table 8 lists the Steady State values of the model under *LOB policy*, *CAP policy* and baseline scenario of GK15 without a CBDC. Steady State output is at the same level in all three cases, as well as the price of

²⁰The code for the subsequent analysis has been built on the basis of the replication code of [Gertler and Kiyotaki \(2015\)](#).

capital and the deposit. Welfare, defined as aggregate consumption, is in all scenarios at the same level in the Steady State. However, welfare is below the first best due to the financial frictions that cause less efficient households and the central bank (under *CAP policy*) to hold capital. The first best is defined by the scenario without financial frictions in which only bank invests into capital²¹.

Generally, household consumption slightly profit and the banking sector – especially capital holdings and net worth – is strained by CBDC in the Steady State. The increase in household consumption is only marginally differing by less than 1%. What changes more substantially for households is the allocation of savings as depicted in figure 2. The share of capital investment remains constant at a level of one third under CBDC. Thus, the flows into CBDC originate entirely from deposits. Per calibration target, CBDC holdings make up a share of 10% of savings, reducing the percentage of deposits from two thirds to 57%.

Even though consumption of bankers makes up only about 6% of total consumption, it drops more substantially in percentage terms. The reason for this is the decline in net worth. The diminished profitability is caused by increased funding costs due to a higher interest rate of central bank credit under the *LOB policy* and a reduced balance sheet under *CAP policy*. These results are in line with the findings of [Keister and Sanches \(2019\)](#) and [Siciliani \(2018\)](#). The change in the funding situation is visualized in figure 3. The composition of liabilities and equity remains the same with a equity ratio of 10% under *CAP policy* and 9% under *LOB policy*. The lower equity ration under *LOB policy* can be credited to the more lenient leverage constraint. Under *LOB policy*, the size of the balance sheet remains constant and the lost deposits are replaced by more expensive central bank credit. Under *CAP policy*, the decrease in deposits lead to a decrease in the size of the balance sheet by 14%. Finally, under *CAP policy* also the allocation of capital changes and the central bank takes over 10% of the capital investments which were previously invested by banks as depicted by figure 4.

Summing up, both CBDC policy options do not change Steady State aggregate output and prices but only affect the distributions between agents such as households savings, capital holdings and bank funding.

²¹Without financial frictions, the Steady State results would resemble the equivalence theorem of public and private money of [Brunnermeier and Niepelt \(2019\)](#) as well the results of [Fernández-Villaverde et al. \(2020a,b\)](#) that the central bank can implement the socially optimal allocations under a CBDC.

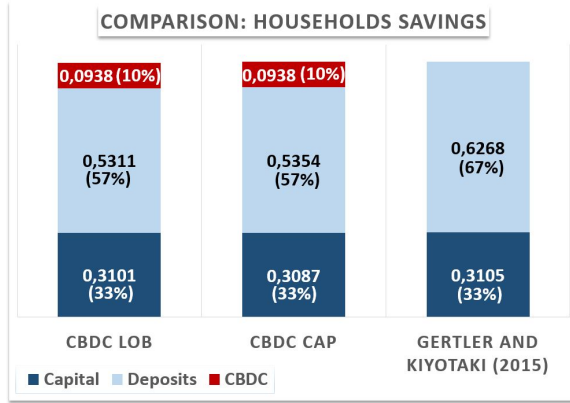


Figure 2: Comparison of Steady State Results: Households Savings

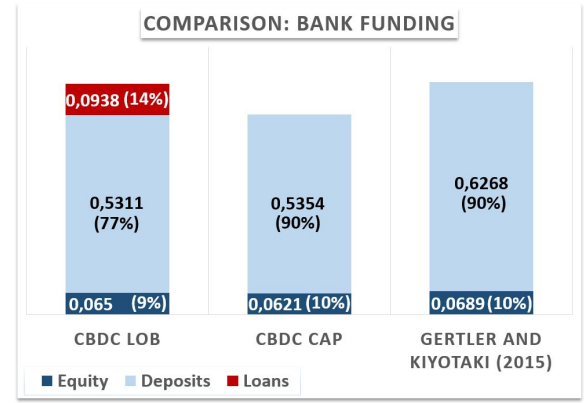


Figure 3: Comparison of Steady State Results: Bank Funding

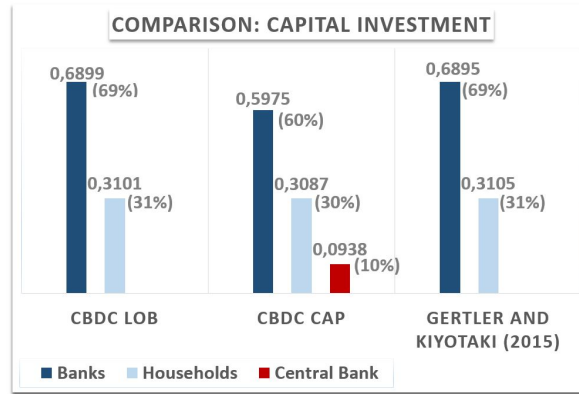


Figure 4: Comparison of Steady State Results: Capital Allocation

5.3 Banking crisis after a 5% shock to productivity (No Bank Run)

The following section looks at the impact of a 5% shock to capital productivity, again comparing both policy scenarios and the results of GK15. Figure 5 shows the reaction of key variables to the shock. CBDC mitigates some of the losses in output which decreases by 5% under CBDC compared to 6% in the baseline case. The presence of CBDC reduces the drop in output and prices more to its “fundamental” part i.e. to the size of the capital shock. This can be explained through the effect of CBDC on capital prices by mitigating the increase in capital management costs. Under *LOB policy* this is achieved by refinancing bank funding with loans and under *CAP policy* central bank alleviates the capital increase from households by holding capital itself.

The shock has a substantial effect on bank net worth which falls by about 40% under CBDC and 50 % without CBDC, tightening the leverage constraint. Under the *CAP*

policy, the drop in net worth is the lowest, due to the lower original size of bank's capital investments in the Steady State. The tighter leverage constraint reduces deposits and forces banks to sell about 20% of their capital under *CAP policy*. Households redirect the funding by increasing capital holdings by 20% and CBDC holdings by 60%. As defined by the *CAP policy* rule, central bank capital holdings rise accordingly. This contrasts to the baseline case in which households need to absorb all the capital themselves and hence increase household capital by 60% in GK15.

Under *LOB policy*, the recession similarly leads to an increase in CBDC by about 60%. However, the funding is channeled back as loans such that bank capital only drops by 15%. Furthermore, the optimal funding structure of banks changes towards central bank credit because the increase in the interest margin is higher for credit than for deposits. That also explains why the decline in deposits is steeper than in the other scenarios also resulting in a higher increase in household capital.

Under both policy options, the relative weight of the welfare losses is shifted from banks more towards households compared to the baseline scenario of GK15. Thus, under CBDC households suffer from a relatively larger drop in consumption and the decline in bank net worth (and also bank consumption) is the less. However, the later also originates in the already lower level of net worth in the Steady State.

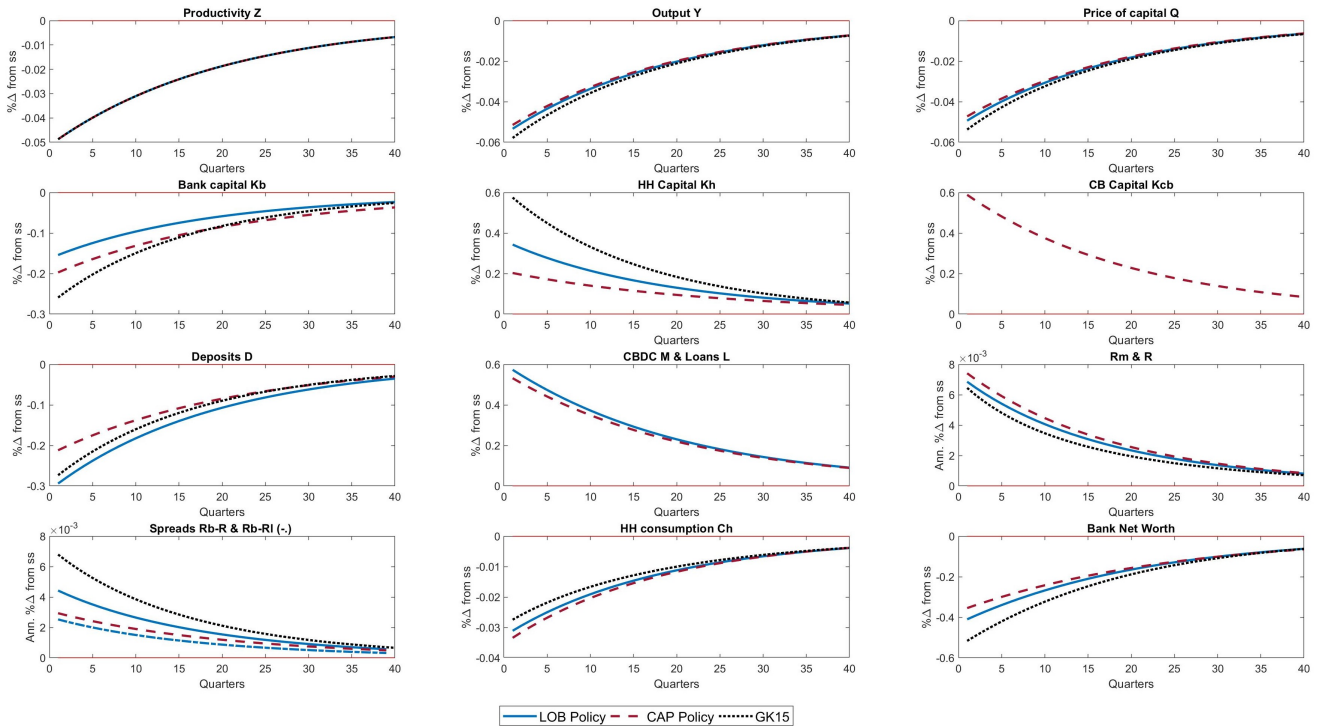


Figure 5: Recession after a 5% shock to productivity under *LOB* & *CAP policy* (no bank run)

Taking stock, a CBDC reduces the drop in output and capital prices more to its fundamental part, cushioning the decline in bank capital and net worth. In this way the relative weight of the welfare loss is shifted more towards households.

5.4 Equilibrium Regions

In section 4.2.1 the different equilibrium regions were defined in general. The following will analyze the equilibrium regions in the numerical example. How large does a shock to the Steady State levels has to be (a) for the bank run equilibrium to emerge causing a default due to illiquidity and (b) for leaving the bank with zero net worth leading to a default due to insolvency? Figure 6 provides answers to these questions depicting the equilibrium regions for the different policies for the numerical example²².

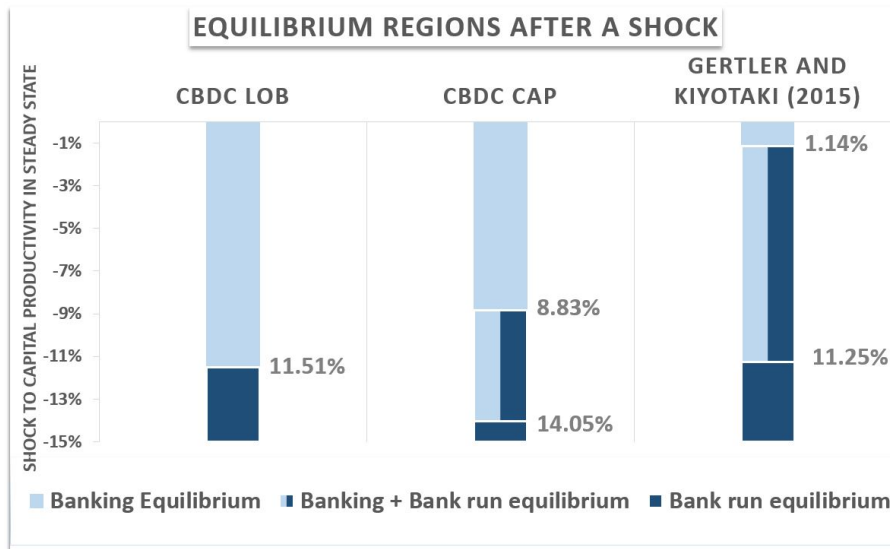


Figure 6: Shock sizes to Steady State levels that are needed for a bank run to emerge leading to default due to illiquidity and default due to insolvency

Under *LOB policy* there is no region in which the banking and the bank run equilibrium coexist. Default due to illiquidity is not possible. If a bank run would occur, the removed deposits would (partially) be replaced by central bank credit and therefore the run would not be triggered in the first place. Furthermore, the central bank loans serve as an additional buffer on the banks balance sheet as explained in sections 4.2. However, if the shock is sufficiently large, in this case 11.51%, the whole net worth of the bank is eradicated and the bank defaults due to insolvency.

Under *CAP policy* it is also more difficult for a bank run equilibrium to arise requiring

²²Note that the higher the simulated shock, the more unstable the simulations becomes.

a shock of 8.83%. The central bank does not step in a lender of last resort but purchases capital itself and thereby stabilizes the liquidation price which makes the emergence of a bank run equilibrium more difficult but not impossible. Because the banks have a smaller balance sheet and buy less capital, the absolute loss they suffer if a shock hits the economy is smaller than in the other scenarios. Therefore, a default due to insolvency requires a shock of 14.05% or more.

Without a CBDC, a bank run equilibrium is already possible after a shock of 1.14% to capital productivity and banks default due to insolvency after a shock of 11.25%.

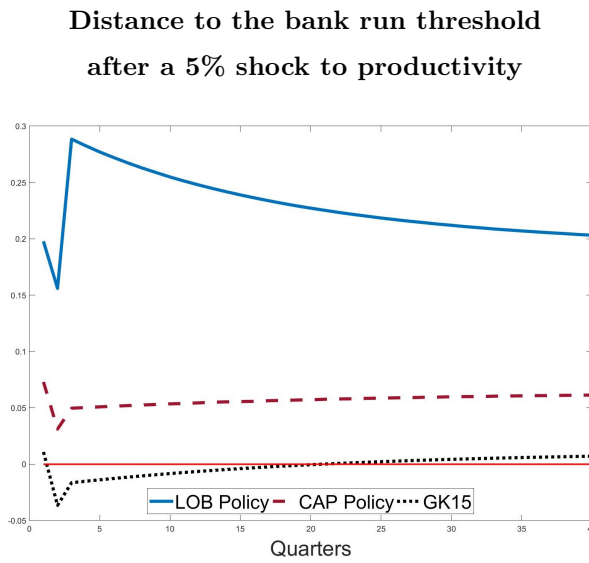


Figure 7: Difference between liquidation price and the bank run price threshold

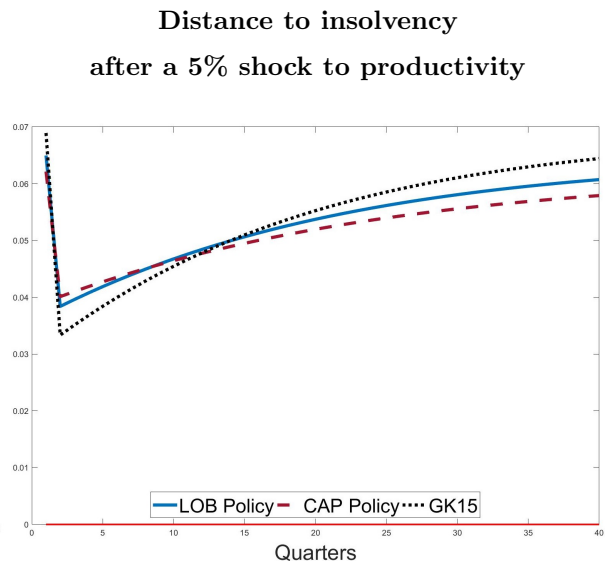


Figure 8: Evolution of bank net worth (absolute)

Figure 7 and 8 illustrate the evolution of the distance to thresholds of the equilibrium regions after a 5% shock to capital productivity. Figure 7 plots the difference between the fire sale price of capital and the price that is required for a bank run equilibrium to emerge. As mentioned above, only in the baseline scenario of GK15 the fire sale price drops below the threshold price and a bank run equilibrium is possible. The evolution of the distance to the bank run equilibrium under *CAP policy* is very similar to GK15, just at a higher level. At the time of the shock, the difference between the prices drops sharply but then slowly diverges again. Under *LOB policy*, the distance to the bank run region also reduces at the time of the shock but then recovers sharply and even overshoots. It overshoots because in response to the shock, households reduce deposits and increase CBDC holdings leading to a higher credit from the central bank further cushioning the “credit buffer” on the bank’s balance sheet.

Summarizing, under both CBDC policy scenarios, CBDC affects the equilibrium regions in a way that stabilizes financial stability. Under *CAP policy* CBDC defers the emergence of the bank run equilibrium substantially, whereas under *LOB policy* the bank run equilibrium does not even arise before insolvency. Under *LOB policy* the insolvency threshold is at a similar level to GK15, whereas *CAP policy* further detains insolvency.

5.5 Bank Run under CAP Policy after a 10% shock to productivity

The above analyzed shock of 5% is not enough to open up the possibility for a bank run under CBDC although GK15 asses the magnitude of the shock to be characteristic for a major recession. However, under CBDC (for both policy options) the shock has a considerable impact on the economy but it is not enough to reduce the liquidation price of assets below the threshold that opens up the possibility of a bank run. Therefore the following will analyze a bank run under *CAP policy* after a 10% shock to capital productivity. A default due to illiquidity under *LOB policy* is not possible in this numerical example and therefore it will not be considered here.

Figure 9 visualizes the impact of a 10% shock to productivity with (dashed red lines) and without (blue solid lines) a run in the period of the shock. In the first period, a 10% shock to productivity hits the economy. The shock creates substantial losses on the bank's balance sheet and the economy under CBDC *CAP policy* is catapulted into the region where the bank run and the banking equilibrium coexist. Still, the bank run is only possible in the period of the shock. The self-fulfilling beliefs of agents that the shock leads to a withdrawal of deposits, triggers an aggregate bank run. As a response, the bank has to liquidate all its capital and depositors have to accept a haircut of $x_t = 0.9895$ on their deposit claims. The net worth of the bank of which 83% was already destroyed by the initial shock is fully eradicated and the bank goes bankrupt.

Depositors compensate the withdrawal of all their deposits by increasing their capital and CBDC holdings. Household capital holdings increase by almost 150%, whereas CBDC increases by 120% (only 8% more than in the run scenario). Thus, households still absorb most of the liquidated capital holding. Households hold 76% of capital in the period of the run, whereas only 24% of capital is held by the central bank. Why is this the case? The economy suffers a large loss to output and as a response, the CBDC interest rate increases. However, the higher the CBDC demand, the lower the CBDC interest rate which moderate the inflows into CBDC. Furthermore, households know that the prices

and productivity will recover expecting excess returns for the subsequent period despite the high management costs of capital. These countervailing effects lead to CBDC holdings and a CBDC interest rate R^m that is only little higher than in the no bank run case.

The shift of capital to less efficient households and the central bank decreases the price of capital by 12% (instead of 10%) and output drops by 12% (instead of 11%). The additional decline in output compared to the no-run scenario is remarkably low. This is caused by the relatively low increase in capital management costs. For one, the central bank has lower capital management costs. Additionally, households and central bank split up the capital holdings which further mitigates the costs due to its convexity. In the period of the run, households suffer of a large drop in consumption, originating from two sources. First, their own increased management cost and loss in the value of capital eat up part of consumption. But second, they also have to bear the increased management costs and loss of value of capital of the central bank which is distributed as lump-sum transfers – and in this case taxes – to households.

New banks can enter in the period after the bank run. Endowed with little net worth, they take up a small amount of deposits but which stabilizes the price of capital yet also reduces the excess return on capital. This leads to a smaller interest rate on CBDC R^m and deposits R . The still very large output gap enables a flow into CBDC peaking in the period after the bank run at 0.21 which is a increase of 159% compared to its Steady State value.

In the third period (second period after the bank run), the banks slowly recapitalize and can further increase their deposits. The interest rates spike up again, reducing demand in CBDC. Thereafter, capital prices continue to rise and CBDC demand falls, removing the countervailing forces on the interest rate which then adjust back to its Steady State level. CBDC and thus central bank capital almost fully adjust to the case as if no bank run had happened converging back to Steady State.

Concluding, a bank run under *CAP policy* deteriorates welfare compared to the banking equilibrium. Yet, the additional decline in output is remarkably small, originating from the stabilization of the liquidation price by reducing total management costs.

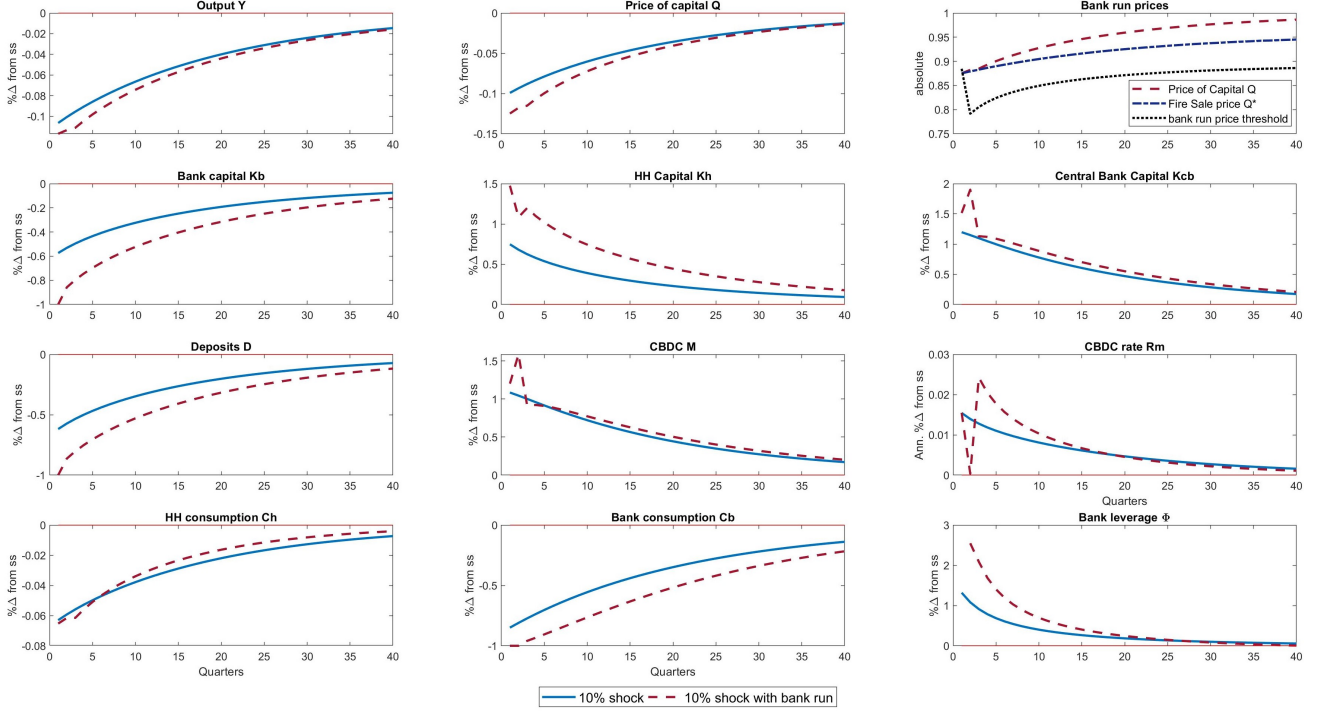


Figure 9: *CAP Policy*: 10% Shock to productivity with and without a bank run in the period of the shock

6 Conclusion

One of the main concerns when considering Central Bank Digital Currency is the disintermediating effect on the banking sector in normal times, and even more the increased risk of a bank run in times of crisis. This paper analyzes the impact of a Central Bank Digital Currency as an extension to the bank run model by [Gertler and Kiyotaki \(2015\)](#). A CBDC is a liability on the asset side of the central bank balance sheet. But how will the CBDC be accommodated on the asset side of the balance? So far, this “asset side question” has received only little attention, but considerably influences the impact of a CBDC on the economy. Therefore, the model compares two different asset side policies with each other and to the situation without a CBDC. There are four main results: First, in Steady State, a CBDC does not change aggregate output and prices. However, it affects the composition of household savings, bank funding and capital investment and leads to a reduction in bank profits. Second, in a crisis both CBDC policy options have a stabilizing effect on the economy, mitigating the impact of the shock by reducing it more to its “fundamental” part. Third, both CBDC policy scenarios improve financial stability by deferring the emergence of a bank run equilibrium to larger shocks. Fourth, in a bank run

under *CAP policy* the loss in output is attenuated through the stabilizing effect of CBDC on capital prices. Concluding, I find that a CBDC strains the banking sector in normal times by reducing deposits and net worth. Yet, contrary to prevailing concerns, CBDC improves financial stability in times of crisis by impeding the emergence of bank runs. The discrepancy between concerns and results stems from partial equilibrium considerations disregarding the “asset side issue” of CBDC.

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7 Appendix

7.1 Reformulating and Solution to the optimization problem of the banker in terms of per unit of net worth

To make the maximization problem independent from its initial conditions, the equations in 8 need to be expressed in terms of per unit of net worth. For this, the evolution of net worth (4) is combined with the flow of funds constraint (3) and reformulated such that

$$\begin{aligned}
n_{t+1} &= R_{t+1}^b Q_t k_t^b - R_{t+1} d_t - R_{t+1}^L l_t \quad \text{with} \quad R_{t+1}^b \equiv \frac{Z_{t+1} + Q_{t+1}}{Q_t} \\
\Rightarrow n_{t+1} &= R_{t+1}^b n_t + (R_{t+1}^b - R_{t+1}) d_t + (R_{t+1}^b - R_{t+1}^L) l_t \\
\Rightarrow \frac{n_{t+1}}{n_t} &= R_{t+1}^b + (R_{t+1}^b - R_{t+1}) \tilde{d}_t + (R_{t+1}^b - R_{t+1}^L) \tilde{l}_t \\
&\quad \text{with} \quad \tilde{d}_t \equiv \frac{d_t}{n_t} \quad \text{and} \quad \tilde{l}_t \equiv \frac{l_t}{n_t}
\end{aligned}$$

where R_{t+1}^b is the return of a unit of capital invested by banks, \tilde{d}_t are deposits per unit of net worth, \tilde{l}_t is central bank credit per unit of net worth and $\frac{n_{t+1}}{n_t}$ is the growth rate of net worth.

Likewise, we can then express the value function as:

$$\begin{aligned}
V_t &= E_t [\beta^i (1 - \sigma) n_{t+1} + \beta \sigma V_{t+1}] = E_t [\Omega_{t+1} n_{t+1}] \quad \text{with} \quad \Omega_{t+1} \equiv \beta \left(1 - \sigma + \sigma \frac{V_{t+1}}{n_{t+1}} \right) \\
\Rightarrow \Psi_t \equiv \frac{V_t}{n_t} &= E_t \left[\Omega_{t+1} \frac{n_{t+1}}{n_t} \right] \\
&= E_t \left[\Omega_{t+1} \left(R_{t+1}^b + (R_{t+1}^b - R_{t+1}) \tilde{d}_t + (R_{t+1}^b - R_{t+1}^L) \tilde{l}_t \right) \right] \\
&= \nu_t + \mu_t^d \tilde{d}_t + \mu_t^l \tilde{l}_t
\end{aligned} \tag{21}$$

$$\text{with: } \nu_t \equiv \Omega_{t+1} R_{t+1}^b, \quad \mu_t^d \equiv \Omega_{t+1} (R_{t+1}^b - R_{t+1}), \quad \mu_t^l \equiv \Omega_{t+1} (R_{t+1}^b - R_{t+1}^L)$$

where Ω_{t+1} is the weighted average of the discounted marginal value of net worth to exiting and to continuing bankers. Ψ is the franchise value of the bank per unit of asset and can thus be interpreted as Tobin's q ratio. Similarly, the flow of funds constraint (3) can be plugged into the incentive constraint (7) and expressed in terms of per unit of net worth:

$$\begin{aligned}
\frac{V_t}{n_t} &\geq \theta (d_t + n_t + l_t - \omega l_t) / n_t \\
\Rightarrow \Psi_t &\geq \theta \left(1 + \tilde{d}_t + (1 - \omega) \tilde{l}_t \right)
\end{aligned} \tag{22}$$

Finally, the reformulated optimization problem is choosing leverage i.e. deposits and

credit per unit of net worth (\tilde{d}_t, \tilde{l}_t) each period to maximize the franchise value per unit of net worth which can also be regarded as Tobin's q ratio:

$$\Psi_t = \nu_t + \mu_t^d \tilde{d}_t + \mu_t^l \tilde{l}_t \quad \text{s.t.} \quad \Psi_t \geq \theta \left(1 + \tilde{d}_t + (1 - \omega) \tilde{l}_t \right) \quad (23)$$

This leads to the Lagrangian function

$$L_t = \nu_t + \mu_t^d \tilde{d}_t + \mu_t^l \tilde{l}_t + \lambda_t \left[\nu_t + \mu_t^d \tilde{d}_t + \mu_t^l \tilde{l}_t - \theta \left(1 + \tilde{d}_t + (1 - \omega) \tilde{l}_t \right) \right] \quad (24)$$

with the Kuhn Tucker conditions

$$\frac{\partial L}{\partial \tilde{d}_t} = \mu_t^d + \lambda_t \mu_t^d - \lambda_t \theta \leq 0 \quad \tilde{d}_t \geq 0 \quad \text{and} \quad \tilde{d}_t \frac{\partial L}{\partial \tilde{d}_t} = 0 \quad (25)$$

$$\frac{\partial L}{\partial \tilde{l}_t} = \mu_t^l + \lambda_t \mu_t^l - \lambda_t \theta (1 - \omega) \leq 0 \quad \tilde{l}_t \geq 0 \quad \text{and} \quad \tilde{l}_t \frac{\partial L}{\partial \tilde{l}_t} = 0 \quad (26)$$

$$\frac{\partial L}{\partial \lambda_t} = \nu_t + \mu_t^d \tilde{d}_t + \mu_t^l \tilde{l}_t - \theta \left(1 + \tilde{d}_t + (1 - \omega) \tilde{l}_t \right) \geq 0 \quad \lambda_t \geq 0 \quad \text{and} \quad \lambda_t \frac{\partial L}{\partial \lambda_t} = 0 \quad (27)$$

To show that incentive constraint has to be binding, assume oppositely that the constraint is slack and thus $\lambda_t = 0$. This would imply for equations 25 $\mu_t^d \leq 0$ and equation 26 $\mu_t^l \leq 0$. These are the expected discounted excess returns on capital financed by deposits (μ_t^d) and central bank credit (μ_t^l). However, to be economically meaningful, the excess returns need to be greater than zero. Therefore in any relevant economic scenario $\mu_t^d > 0$ and $\mu_t^l > 0$ and thus $\lambda_t \neq 0$ and the incentive constraint is binding with $\lambda_t > 0$. This result can be seen intuitively by looking at the objective function which is strictly increasing in \tilde{d}_t and \tilde{l}_t and thus for the bank it would optimal to choose these variable as high as possible. However, the incentive constraint restricts the size of leverage, being linear in \tilde{d}_t and \tilde{l}_t . Therefore, in the optimum, the bank chooses its leverage as high as possible such that the incentive constraint is always binding. The Kuhn-Tucker conditions collapse to

$$\frac{\partial L}{\partial \tilde{d}_t} = \mu_t^d + \lambda_t \mu_t^d - \lambda_t \theta = 0 \quad (28)$$

$$\frac{\partial L}{\partial \tilde{l}_t} = \mu_t^l + \lambda_t \mu_t^l - \lambda_t \theta (1 - \omega) = 0 \quad (29)$$

$$\frac{\partial L}{\partial \lambda_t} = \nu_t + \mu_t^d \tilde{d}_t + \mu_t^l \tilde{l}_t - \theta \left(1 + \tilde{d}_t + (1 - \omega) \tilde{l}_t \right) = 0 \quad (30)$$

yielding the following optimality conditions:

$$\begin{aligned} \frac{\theta}{\mu_t^d} &= \frac{\theta(1 - \omega)}{\mu_t^l} \Leftrightarrow (1 - \omega) \mu_t^d = \mu_t^l \\ \Rightarrow (1 - \omega) E_t [(R_{t+1}^b - R_{t+1})] &= E_t [(R_{t+1}^b - R_{t+1}^L)] \end{aligned} \quad (31)$$

7.2 Equilibrium Equations for LOB Policy

Household

Optimality condition for bank deposits:

$$\beta \frac{C_t^h}{C_{t+1}^h} R_{t+1} = 1 \quad (I)$$

Optimality condition for CBDC:

$$\beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^m = 1 \quad (II)$$

Optimality condition for capital:

$$\beta \frac{C_t^h}{C_{t+1}^h} \left(\frac{Z_{t+1} + Q_{t+1}}{Q_t + \alpha K_t^h} \right) = \beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^h = 1 \quad (III)$$

Aggregate Banking sector

Aggregate Balance Sheet:

$$Q_t K_t^b = D_t + N_t + L_t \quad (IV)$$

Aggregate evolution of net worth:

$$N_t = \sigma \left((Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} - R_t^l L_{t-1} \right) + W^b \quad (V)$$

Maximisation problem:

$$\begin{aligned} \Psi_t &= E_t \left\{ \beta (1 - \sigma + \sigma \Psi_{t+1}) \left[R_{t+1}^b + (R_{t+1}^b - R_{t+1}) \frac{D_t}{N_t} + (R_{t+1}^b - R_{t+1}^l) \frac{L_t}{N_t} \right] \right\} \\ \Leftrightarrow \Psi_t &= E_t \left\{ \beta (1 - \sigma + \sigma \Psi_{t+1}) \left[\frac{Z_{t+1} + Q_{t+1}}{Q_t} + \left(\frac{Z_{t+1} + Q_{t+1}}{Q_t} - R_{t+1} \right) \frac{D_t}{N_t} + \left(\frac{Z_{t+1} + Q_{t+1}}{Q_t} - R_{t+1}^l \right) \frac{L_t}{N_t} \right] \right\} \end{aligned} \quad (\text{VI})$$

Incentive constraint:

$$\Psi_t = \theta \left(1 + \frac{D_t}{N_t} + (1 - \omega) \frac{L_t}{N_t} \right) \quad (\text{VII})$$

Combined Optimality condition for $\frac{L_t}{N_t} = \tilde{l}_t$ & $\frac{D_t}{N_t} = \tilde{d}_t$:

$$\begin{aligned} \left(\frac{Z_{t+1} + Q_{t+1}}{Q_t} - R_{t+1}^l \right) &= (1 - \omega) \left(\frac{Z_{t+1} + Q_{t+1}}{Q_t} - R_{t+1} \right) \\ \Leftrightarrow (R_{t+1}^b - R_{t+1}^l) &= (1 - \omega) (R_{t+1}^b - R_{t+1}) \end{aligned} \quad (\text{VIII})$$

Central Bank

Central Bank Balance Sheet:

$$M_t + R_t^l L_{t-1} = R_t^m M_{t-1} + L_t + T_t \quad (\text{IX})$$

CBDC interest rate rule:

$$R_{t+1}^m = \delta_0 + \delta_1 \left(\frac{Y - Y_{t-1}}{Y} \right) - \delta_2 M_t \quad (\text{X})$$

Central Bank credit supply & credit market clearing:

$$L_t = M_t \quad (\text{XI})$$

Aggregation and market clearing

Total capital:

$$K_t^b + K_t^h = 1 \quad (\text{XII})$$

Total output/ consumption:

$$C_t^h + \frac{\alpha}{2} (K_t^h)^2 + \frac{1 - \sigma}{\sigma} (N_t - W^b) = Z_t + Z_t W^h + W^b \quad (\text{XIII})$$

Evolution of aggregate productivity after date t after shock:

$$\ln Z_{t+i} - \ln Z = \rho(\ln Z_{t+i-1} - \ln Z) \quad \text{for all } i = 1, 2, \dots \quad (\text{XIV})$$

7.3 Equilibrium Equations for CAP Policy

Household

Optimality condition for bank deposits:

$$\beta \frac{C_t^h}{C_{t+1}^h} R_{t+1} = 1 \quad (\text{I})$$

Optimality condition for CBDC:

$$\beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^m = 1 \quad (\text{II})$$

Optimality condition for capital:

$$\beta \frac{C_t^h}{C_{t+1}^h} \left(\frac{Z_{t+1} + Q_{t+1}}{Q_t + \alpha K_t^h} \right) = \beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^h = 1 \quad (\text{III})$$

Aggregate Banking sector - same problem as in GK15

Aggregate Balance Sheet:

$$Q_t K_t^b = D_t + N_t \quad (\text{IV})$$

Aggregate evolution of net worth:

$$N_t = \sigma \left((Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right) + W^b \quad (\text{V})$$

Maximisation problem:

$$\begin{aligned} \Psi_t &= E_t \left\{ \beta (1 - \sigma + \sigma \Psi_{t+1}) \left[R_{t+1}^b + (R_{t+1}^b - R_{t+1}) \frac{D_t}{N_t} \right] \right\} \\ \Leftrightarrow \Psi_t &= E_t \left\{ \beta (1 - \sigma + \sigma \Psi_{t+1}) \left[\frac{Z_{t+1} + Q_{t+1}}{Q_t} + \left(\frac{Z_{t+1} + Q_{t+1}}{Q_t} - R_{t+1} \right) \frac{D_t}{N_t} \right] \right\} \end{aligned} \quad (\text{VI})$$

Incentive constraint:

$$\Psi_t = \theta \left(1 + \frac{D_t}{N_t} \right) \quad (\text{VII})$$

Central Bank

Central Bank Balance Sheet:

$$M_t + (Z_t + Q_t)K_{t-1}^{cb} = R_t^m M_{t-1} + Q_t K_t^{cb} + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 + T_t \quad (\text{VIII})$$

CBDC interest rate rule:

$$R_{t+1}^m = \delta_0 + \delta_1 \left(\frac{Y - Y_{t-1}}{Y} \right) - \delta_2 M_t \quad (\text{IX})$$

Central Bank capital demand:

$$Q_t K_t^{cb} + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 = M_t \quad (\text{X})$$

Aggregation and market clearing

Total capital:

$$K_t^b + K_t^h + K_t^{cb} = 1 \quad (\text{XI})$$

Total output/ consumption:

$$C_t^h + \frac{\alpha}{2} (K_t^h)^2 + \frac{1-\sigma}{\sigma} (N_t - W^b) + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 = Z_t + Z_t W^h + W^b \quad (\text{XII})$$

Evolution of aggregate productivity after date t/ after shock:

$$\ln Z_{t+i} - \ln Z = \rho (\ln Z_{t+i-1} - \ln Z) \quad \text{for all } i = 1, 2, \dots \quad (\text{XIII})$$

7.4 Bank Run Equilibrium Equations for LOB Policy

Aggregate Balance Sheet:

$$Q_t^* K_t^b = L_t \quad (\text{I})$$

Central Bank

Central Bank Balance Sheet:

$$M_t + R_t^l L_{t-1} = R_t^m M_{t-1} + L_t + T_t \quad (\text{II})$$

CBDC interest rate rule:

$$R_{t+1}^m = \delta_0 + \delta_1(1 - Q_t^*) - \delta_2 M_t \quad (\text{III})$$

Central Bank credit supply & credit market clearing:

$$L_t = M_t \quad (\text{IV})$$

Aggregation and market clearing

Total capital (*same as GK15, determines K_t^h*):

$$K_t^b + K_t^h = 1 \quad (\text{V})$$

Total output/ consumption:

$$C_t^h + \frac{\alpha}{2} (K_t^h)^2 = Z_t + Z_t W^h \quad (\text{VI})$$

7.5 Bank Run Equilibrium Equations for CAP Policy

Household

Optimality condition for CBDC:

$$\beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^m = 1 \quad (\text{I})$$

Optimality condition for capital:

$$\beta \frac{C_t^h}{C_{t+1}^h} \left(\frac{Z_{t+1} + Q_{t+1}}{Q_t + \alpha K_t^h} \right) = \beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^h = 1 \quad (\text{II})$$

Central Bank

Central Bank Balance Sheet:

$$M_t + (Z_t + Q_t) K_{t-1}^{cb} = R_t^m M_{t-1} + Q_t K_t^{cb} + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 + T_t \quad (\text{III})$$

CBDC interest rate rule:

$$R_{t+1}^m = \delta_0 + \delta_1 \left(\frac{Y - Y_{t-1}}{Y} \right) - \delta_2 M_t \quad (\text{IV})$$

Central Bank capital demand:

$$Q_t K_t^{cb} + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 = M_t \quad (\text{V})$$

Aggregation and market clearing

Total capital:

$$K_t^b + K_t^h = 1 \quad (\text{VI})$$

Total output/ consumption:

$$C_t^h + \frac{\alpha}{2} (K_t^h)^2 + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 = Z_t + z_t W^h \quad (\text{VII})$$