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## Social Optimum in a Model with Hierarchical Firms and Endogenous Promotion Time<sup>\*</sup>

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#### Abstract

This paper develops a search and matching model with hierarchical firms, human capital accumulation, internal promotions and on-the-job search. At the time of their market entry firms maximize present value of profits with respect to their promotion rule. Workers who are eligible for promotion but cannot be promoted because the senior position in the firm is taken start searching on-the-job. The decentralized equilibrium is then compared to the socially optimal one. The welfare analysis is conducted in two steps: in the first one fixed firm entry is assumed, while in the second firm entry is determined by a free-entry condition. Under fixed firm entry, the social planner can induce approx. 5% welfare gain by imposing earlier promotion timing compared to the one firms choose in the decentralized equilibrium. The inefficiency of the decentralized equilibrium is caused by strategic complementarity in firms' promotion choices. If a firm delays internal promotions it creates a negative externality on all other firms by reducing the pool of potential candidates to the high productivity senior jobs. However, due to strategic complementarity the competitors respond by also increasing their promotion requirement. Imposing a free-entry condition further reveals that in the decentralized equilibrium firm entry is biased downwards which exacerbates the allocative inefficiency in the economy.

Keywords: promotions, on-the-job search, human capital, efficiency

**JEL Classification**: D21, D61, D63, J63, J64

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## 1 Introduction

Human capital is viewed as one of the main drivers of economic growth. Bassanini and Scarpetta (2002) find, for example, that an additional year of education is associated with 6% boost in output. Furthermore, attainment of higher education is on the rise. For instance, 28.7% of EU citizens had some tertiary educational level in 2018, compared to 22% in 2009 (EUROSTAT data). One could argue that besides school and university, on-the-job learning and human capital accumulation that occurs outside of formal education are also an important determinant of productivity. Professional tasks often rely on experience which is not taught in formal education, so a substantial part of human capital accumulation takes place during a worker's career. Moreover, technological advancements might render some knowledge obsolete so continued learning is needed. In an imperfect labour market, however, it is not guaranteed that the "right" level of human capital will be achieved. The aim of this study is to analyse the efficiency of the amount of human capital accumulation that is required for promoting a worker to a senior position.

To answer this question, we assume that the labour market is populated with hierarchical firms. Upon meeting an inexperienced worker who is applying for the lower-level, junior job, the firm and the worker sign a fixed-length binding contract which specifies the amount of time the worker will spend on level one before being eligible for promotion. During this period the worker accumulates human capital and once they reach the human capital level, specified in the contract, they are promoted instantaneously. If, however, the senior position in the firm is already taken, the firm is obliged to provide a certificate confirming the experience level of the worker. This permits the workers to start searching on-the-job for senior positions on the external market. Under this framework, we look at firms' promotion decisions and analyse the efficiency of these decisions. The choice variable of all firms is the cutoff level of human capital required for promoting a worker from junior to senior level. The research question is then, if the decentralized equilibrium is also socially efficient. The welfare analysis is done in two steps where in the first, we fix the number of firms entering per period. We find that the decentralized equilibrium is inefficient in the sense that firms promote their junior worker to the senior level too late, compared to what would be socially optimal. A new firm chooses its promotion requirement taking into account the actions of other firms and the market conditions. The result is a Nash equilibrium characterized by a strategic complementarity of firms' promotion decisions for the given market transition rates which is the main reason for the inefficient outcome of firms' promotion choices. More specifically, higher average promotion timing of incumbent firms on the market induces later promotions from entering firms. This is an inefficient outcome, because firms neglect the negative externalities that their decisions create. The delay of internal promotions reduces the pool of potential candidates to senior positions such that it becomes more difficult for firms to recruit workers for their high productivity senior jobs. This in turn suppresses firms' profits and overall output. This is an externality which the decentralized market cannot internalize. The resulting welfare loss is then due to the fact that workers are allocated inefficiently among the hierarchical levels. Moreover, the market outcome is inefficient for any value of worker's bargaining power which determines how the worker and the firm split the the output of the match.

In the second step of the welfare analysis, the equilibrium number of firms is determined by

a free-entry condition. Here, the paper contributes to the literature on efficiency of search and matching models. In a labour market with search frictions, social optimum is not guaranteed and an intervention by a social planner could improve welfare. Hosios (1990) and later Pissarides (2000) demonstrate that in order for market entry and exit as well as match creation and destruction to be sociably desirable, the matching function has to have constant returns to scale and the sharing rule should be such that the bargaining power of the worker equals the elasticity of the matching function with respect to unemployment. Hence, there is a unique value of bargaining power parameter that internalizes the congestion externalities on both sides of the market and leads to an efficient outcome in the decentralized economy. We show that in a model with hierarchical firms and endogenous promotions, the Hosios conditions do not deliver a constrained efficient outcome. Under free-entry, the social planner chooses even earlier promotion timing compared to the case with fixed firm entry, which shows that the socially optimal equilibrium is even further away from the decentralized one. This reveals an additional inefficiency in the model, namely that firm creation is downward biased which exacerbates the allocative inefficiency in the market. High promotion requirement imply that filling a senior position is relatively difficult. This outweighs the benefit from employing a senior worker who is highly productive and in turn firms' profits are suppressed which leads to inefficiently low market entry. Next it is shown that the socially optimal and the decentralized equilibrium with free-entry could coincide if a higher fraction of the match output is obtained by junior workers while a lower fraction of the output is retained by senior workers. This reveals that firm are not adequately compensated for creating the high-productivity senior jobs.

Finally, the case when the social planner sets the promotion and the output sharing rules simultaneously is considered. Then, welfare is maximized for lower promotion requirement compared to the decentralized equilibrium and the optimal output sharing rule is such that a higher fraction of the output accrues to workers. Hence, assuming frictional labour markets and hierarchical firms with experience evaluation as a prerequisite for employment in seniorlevel jobs underlines the importance of turnover dynamics in multiple-worker firms with jobs with heterogeneous productivities in relation to the efficiency of human capital required for promotion.

Further implication of the model is connected to the perfect information which follows from the certification that workers need in order to be eligible for applying to senior jobs. The role of certification with respect to the under- or over-provision of training is debated in the literature. On the one hand, certificates reduces asymmetric information about workers' human capital. Katz and Ziderman (1990) argue that if workers' skills are easily observable to outside employers, current employers are deterred from investing in training. Acemoglu and Pischke (2000), on the other hand, argue that certification might be necessary to induce firms to sponsor training of their workers. The authors argue that the role of certificates is to provide incentive to workers to exert effort and "to balance the power between workers and firms evenly" (p. 919). Here, the focus is on the role of certification in the promotion decision of competing firms. The implication of the model is that while it removes possible uncertainty about the worker's human capital it also acts as a barrier that firms set preventing workers from advancing in their career. Also, certification reduces the threat of poaching since the workers are obliged to stay with the firm providing skill evaluation. Hence, certification gives too much power to the firms and contributes to the inefficient aggregate outcome that is found in the model.

The seminal work on human capital theory by Becker (1962) shows that in a perfectly competitive setting the socially efficient level of investment in training is achievable. Workers are willing to pay the cost of training in the form of lower wages during the training period because they claim the benefit of general training while firms never have an incentive to invest in workers' general skills. Since all returns to general training accrue to the worker, in a perfectly competitive market, there are no positive externality on future employers and there is no underinvestment in training. Some of the predictions of the competitive model of training have, however, not been supported by empirical findings. For example, Barron et al. (1999) find that employees bear a small fraction of the cost of training and that most of the training they receive can be interpreted as general human capital. Acemoglu and Pischke (1999) survey the literature on the topic of apprenticeship training in Germany and find that German firms provide general training to their apprentices at a positive cost.

This paper is related to the literature which highlights externalities that lead to inefficient human capital accumulation in markets which are not perfectly competitive. Surveys are given by Leuven (2005) and Brunello and De Paola (2004). One possibility to why workers would not receive an optimal amount of training are credit constraints and capital market imperfections which prevent workers from financing their general training (Stevens, 2001). Galor and Zeira (1993) show that under the condition of indivisibility of investment in human capital if credit markets are imperfect, then children from poor families might be unable to invest in human capital. The initial wealth distribution then determines the aggregate level of human capital investment also in the long-run. Kaas and Zink (2011) further show that in a frictional labour market poor workers who take up loans to fund their education alter their search behaviour to prefer higher paying but riskier jobs. Higher unemployment risk among these workers then suppresses their returns to human capital which has a negative effect on their educational investment.

Minimum wages could also lead to under-investment in training if firms are not allowed to offer low enough starting wage (Mincer and Leighton, 1980). Hold-up problems have also been identified to lead to under-investment in firm-specific training. If renegotiation takes place after investment, the incentive to invest ex-ante is reduced which leads to a sub-optimal outcome (Brunello and De Paola, 2004). Moral hazard problems are another reason why firms would under-investment in training (Schlicht, 1996).

Further externalities associated with inefficient worker training in imperfect labour markets involve information asymmetries such as, if the firm has better information about the training of its workers or has better information about the abilities of its employees (Leuven, 2005). Katz and Ziderman (1990) argue that the recruiting firms are less likely to know the amount or type of training a worker has obtained. The difference in information on worker's training then translates into increased information-based cost of the recruiting firms and higher risk associated with hiring the worker. As a result, a worker with general training gains less from moving to a different firm. Asymmetric information can thus give incentive to firms to invest in workers' general training. Chun and Wang (1995), on the other hand, show that adverse selection provides further rationale for firms to invest in general training.

Another externality is associated with worker turnover and poaching. Under imperfect com-

petition, training firms would have an incentive to provide suboptimal level of training if there is a positive probability that another firm would poach the worker (Stevens, 1994). This occurs because part of the returns associated with training would then be captured by future employers. Acemoglu (1997) shows that because of search frictions labour turnover creates positive externality on future employers. Even though in the initial period the worker and the firm can write a binding contract, the fact that an unknown different firm might benefit from the training the worker receives and the inability to involve this third party in the negotiations leads to underinvestment in general training. In this respect, Moen and Rosén (2004) provide conditions under which the frictional labour markets can be organized so that there is no poaching externality. This involves directed search and the use of long-term contracts or efficient bargaining.

The paper is also related to literature that studies optimal contract length. In a frictional labour market with homogeneous firms and workers Burdett and Coles (2003) show that the optimal wage-tenure contract is such that wages increase smoothly with tenure. This reduces employee's quitting probability by backloading wages. To study the relationship between optimal contract length and the provision of training Malcomson et al. (2003) take as a motivating example the German apprenticeship system. They show that the apprentice contracts are more profitable for firms than the no contract scenario and in equilibrium more workers receive training. However, the authors argue that even with apprenticeship contracts, less than the efficient level of general training is achieved because there is still a probability that future employers or the workers themselves will capture some of the returns to training. Similarly, connected to the issues of contract length, worker heterogeneity and asymmetric information, Hermalin (2002) shows that under-provision of general training occurs as a result of the preference of short term contracts which are used for screening workers' abilities. Cantor (1990), on the other hand, focusing on firm specific training and the moral hazard problem shows that contracts with intermediate length and fixed wages are preferable to career long fixed-wage contracts or continuously renegotiable wage contracts as they induce higher efforts and more efficient training.

The model developed here is closest to Bernhardt and Scoones (1993) and Bernhardt (1995) who also model firm's promotion decision and derive results with respect to its efficiency. In the context of asymmetric learning, Bernhardt and Scoones (1993) and Bernhardt (1995) find that firm's promotion rules are inefficient. In both models employers face a trade-off between placing able workers in higher positions and revealing information about their abilities to competitors. Bernhardt and Scoones (1993) consider a two period model with two firms, each of which has two occupational levels. Worker's managerial potential is revealed to their employer during the first period and conditional on the worker-firm managerial match, the firm decides whether to promote the worker in the second period and what should be the wage offer. In case the the worker-firm managerial match is high firms offer a preemptive wage to deter the competitor from bidding for the worker. However, promotions are inefficient because for some marginally matched workers with managerial potential, the firm would decide not to promote them so as to not engage in bidding and to not lose the worker's firm-specific skills acquired during the first period. In a multi period setting, Bernhardt (1995) also shows that since current employers have information about the ability of their own employees, they strategically use this knowledge and delay promotions beyond the social optimum. Competitors can observe worker's employment but not ability and employers trade-off the productivity gain associated with promotion and

the value of information on worker's ability. An implication is that promotions even for very able workers are delayed and that some workers are always inefficiently employed at the lower level even though they would be more productive as managers. The result that promotions are delayed above the social optimum is the closest to the one derived here, although the underlying mechanism is quite different.

The model proposed here takes a different approach to answering the question whether or not workers gain the optimal amount of human capital in the earlier stages of their careers and deviates from the above discussed modelling frameworks in several key aspects which renders direct comparison difficult. Firstly, a hierarchical firm structure is added to a frictional labour market similar to Dawid et al. (2019). It follows that human capital accumulation is important not only for productivity gains but is also a prerequisite for internal promotions. The longer firms let their junior worker gain experience, the higher future profits the firms can obtain after promotion. Moreover, firms' promotion choices induce feedback effects which shape the market conditions. Hence, the model focuses on the strategic interaction between firms when they optimally set the promotion timing of workers, rather than the strategic interaction between the firm and its employees.

Secondly, firms do not pay a direct cost for training their entry-level (junior) worker, but this cost can be interpreted in the sense of foregone profits that the firm bears, given that output in the second (senior) level is higher for each level of worker's human capital. Hence, firms "invest" in the training of their junior workers by letting them accumulate human capital while in the junior level. Furthermore, it is important to note that because workers need certification before applying in the senior market, there is no information asymmetry in the model, i.e. firms anticipate correctly the productivity of a senior worker hired from a different firm. There is no adverse selection in the model and all workers are identical with respect to skills.

The rest of the paper has the following structure: section 2 describes the modelling framework while section 3 illustrates the decentralized equilibrium via a numerical example. Next, the welfare analysis is presented in section 4 and section 5 concludes. Details on the derivation of the decentralized equilibrium are shown in Appendix A, while additional figures are presented in Appendix B.

## 2 Economic framework: labour market flows

Firms and workers are risk-neutral and the mass of workers is normalized to 1. The economy is populated with hierarchical firms, each of which has two positions: one junior position which does not require professional experience, and one senior position for which only experienced candidates are accepted. There is a pool  $e_0$  of inexperienced workers in simple jobs that do not provide a career advancement possibility and these workers search for their first professional job. Once young and inexperienced workers find a junior job they begin their career with no professional experience (x = 0) and starts accumulating it according to  $\dot{x} = 1$ . The experience is general and perfectly transferable between different employers. Here, the modelling framework of Dawid et al. (2019) is followed precisely. Similarly to them, it is assumed that each firm ichooses the experience level  $\bar{x}_i$  which will be required for internal promotions. Once the worker reaches the predetermined level of human capital<sup>1</sup>, s/he is promoted to the senior level, provided that the position is vacant. In the case that the senior position is occupied, the firm is obliged to provide a certificate to the junior worker which in turn makes them eligible for applying to senior positions in other firms. Moreover, it is assumed that human capital accumulation is costly, so once workers reach the promotion threshold, they do not attempt to accumulate any more human capital. This implies that while productivity of workers is increasing at the early stages of their careers, it remains constant at the senior level. More precisely, the output at the junior level produced by a worker with human capital level x is  $d_1 + c_1 e^{\gamma x}$ , while the output of a senior worker employed at a senior job is  $d_2 + c_2 e^{\gamma \bar{x}_i}$ . It is assumed that  $d_1 > d_2$  and  $c_1 < c_1$ as in Gibbons and Waldman (1999) which implies that human capital is complementary to the hierarchical level.

Let  $m_1 = m_1(e_0, d_{00} + d_{01})$  be the matching technology in the junior market. Here, the stock of available vacancies is  $d_{00} + d_{01}$ , where  $d_{00}$  is the stock of "empty" firms and  $d_{01}$  is the stock of firms with a senior worker only. It is assumed that  $m_1(.,.)$  is increasing in both arguments and exhibits constant returns to scale. The tightness of the junior labour market  $(\theta_1)$  is then  $\frac{d_{00}+d_{01}}{e_0}$ . The vacancy-filling  $(q_1)$  and job-finding  $(\lambda_1)$  rates can be expressed in terms of the market tightness such that:

$$q_1(\theta_1) = \frac{m_1(e_0, d_{00} + d_{01})}{d_{00} + d_{01}} = m_1\left(\frac{1}{\theta_1}, 1\right); \qquad \lambda_1(\theta_1) = \frac{m_1(e_0, d_{00} + d_{01})}{e_0} = \theta_1 q_1(\theta_1).$$

On the other hand, senior positions are available only for workers who have reached  $\bar{x}_i$ level of experience. This leads to two distinct labour markets – one for junior workers without professional experience and one for experienced professionals. Similarly to  $m_1$ , let  $m_2 =$  $m_2(d_{11}^S, d_{00} + d_{10})$  be the matching technology in the senior market such that it is increasing in both arguments and has constant returns to scale. The first argument  $d_{11}^S$ , denotes the stock of firms with a junior worker who has already reached  $\bar{x}_i$  experience but is not promoted because the senior position is occupied. These junior worker can thus apply for a senior position on the external market. The second argument  $d_{00} + d_{10}$  is the stock of senior vacancies on the marker, where  $d_{10}$  are firms that have one junior but no senior worker. Hence, the senior vacancy-filling rate and the senior job-finding rate are given as:

$$q_2(\theta_2) = \frac{m_2(d_{11}^S, d_{00} + d_{10})}{d_{00} + d_{10}} = m_2\left(\frac{1}{\theta_2}, 1\right); \qquad \lambda_2(\theta_2) = \frac{m_2(d_{11}^S, d_{00} + d_{10})}{d_{11}^S} = \theta_2 q_2(\theta_2)$$

where  $\theta_2 = \frac{d_{00}+d_{10}}{d_{11}^S}$  is the tightness of the senior market. Assuming a Cobb-Douglas matching functions of the form  $m_1(e_0, d_{00} + d_{01}) = \mu(e_0)^{\alpha_1}(d_{00} + d_{00})^{\alpha_2}$  $(d_{01})^{1-\alpha_1}$  and  $m_2(d_{11}^S, d_{00} + d_{10}) = \mu(d_{11}^S)^{\alpha_2}(d_{00} + d_{10})^{1-\alpha_2}$  we find the following job-filling and job-finding rates in the two sub-markets:

$$q_1 = \mu \left(\frac{e_0}{d_{00} + d_{01}}\right)^{\alpha_1} \qquad q_2 = \mu \left(\frac{d_{11}^S}{d_{00} + d_{10}}\right)^{\alpha_2} \tag{1}$$

<sup>1</sup>Here, the terms "experience" and "human capital" are used interchangeably.



Figure 1: Firm transitions

$$\lambda_1 = \mu \left(\frac{d_{00} + d_{01}}{e_0}\right)^{1-\alpha_1} \qquad \lambda_2 = \mu \left(\frac{d_{00} + d_{10}}{d_{11}^S}\right)^{1-\alpha_2} \tag{2}$$

where  $0 < \alpha_1 < 1$  and  $0 < \alpha_2 < 1$  are the elasticities of the junior and senior vacancy-filling rates  $(q_1 \text{ and } q_2)$ , respectively and  $\mu$  is the efficiency of the matching function.

### 2.1 Firm dynamics

Next, dynamic equations which govern the transitions of firms between different states are described where a schematic representation is shown in figure 1. The model builds upon the one proposed in Dawid et al. (2019) with the exception that all types of workers, regardless of their status can exit the market at some exogenous separation rate  $\rho$ . Specifically, in each period there is an inflow **n** of new, "empty"  $d_{00}$  firms into the market. At a rate  $q_1$  they find a junior worker and become of  $d_{10}$  type while at a rate  $q_2$  they find a senior worker and transition into  $d_{01}$  state. At the steady state  $\dot{d}_{00} = 0$  such that:

$$\dot{d}_{00} = n - (q_1 + q_2)d_{00} \quad \Rightarrow d_{00} = \frac{n}{q_1 + q_2}$$
(3)

Considering the stock of  $d_{10}(x)$  firms, which have a junior worker with human capital level x, the inflow is given by firms with both positions filled, where the junior worker is not yet searching on-the-job for a senior position:  $d_{11}^N(x)$ , which lose their senior worker. The outflow occurs either if the junior worker exits (at a rate  $\rho$ ) or if the firm finds a senior worker from the secondary market (at a rate  $q_2$ ). The inflow into  $d_{11}^N(x)$  state, on the other hand, comes from  $d_{10}(x)$  firms that hire a senior worker, while the outflow occurs at a rate  $2\rho$  since both workers can be exogenously separated with the firm. Hence, the system of first order linear differential

equations is given by:

$$\begin{cases} \partial d_{10}(x)/\partial x = -(\rho + q_2)d_{10}(x) + \rho d_{11}^N(x) \\ \partial d_{11}^N(x)/\partial x = q_2 d_{10}(x) - 2\rho d_{11}^N(x) \end{cases}$$

The eigenvalues of the corresponding coefficient matrix are  $-\rho$  and  $-(q_2 + 2\rho)$  which leads to the following general solution:

$$\begin{cases} d_{10}(x) = k_1 \rho e^{-\rho x} + k_2 e^{-(q_2+2\rho)x} \\ d_{11}^N(x) = k_1 q_2 e^{-\rho x} - k_2 e^{-(q_2+2\rho)x} \end{cases}$$

The total stock of  $d_{11}^N$  firms is found by integrating  $d_{11}^N(x)$  over the interval  $[0..\bar{x}]$  where  $\bar{x}$  is the market promotion cutoff:

$$d_{11}^N = \int_0^{\bar{x}} d_{11}^N(x) dx = \frac{k_1 q_2}{\rho} (1 - e^{-\rho \bar{x}}) - \frac{k_2}{q_2 + 2\rho} (1 - e^{-(q_2 + 2\rho)\bar{x}})$$
(4)

Similarly, the total stock of  $d_{10}$  firms is found by integrating  $d_{10}(x)$  over the interval  $[0..\bar{x}]$ :

$$d_{10} = \int_0^{\bar{x}} d_{10}(x) dx = k_1 (1 - e^{-\rho \bar{x}}) + \frac{k_2}{q_2 + 2\rho} (1 - e^{-(q_2 + 2\rho)\bar{x}})$$
(5)

In order to find  $k_1$  and  $k_2$ , we use the initial conditions  $q_1d_{00} = d_{10}(0)$  and  $q_1d_{01} = d_{11}^N(0)$ , which state that the stock of  $d_{10}$  firms in which the junior worker has no professional experience equals the stock of "empty" firms who just found a junior worker. The second initial condition shows similarly that the stock of  $d_{11}^N$  firms in which the junior worker has 0 experience equals the the stock of  $d_{01}$  firms who just hired a junior worker from the pool of workers in simple jobs. The equations imply that  $k_1\rho + k_2 = q_1d_{00}$  and  $k_1q_2 - k_2 = q_1d_{01}$ , respectively. Let us first consider the stock of firms which have both positions filled and the junior worker is already searching on the external market:  $d_{11}^S$ . The outflow of firms from this state is  $(\lambda_2 + 2\rho)d_{11}^S$  which is due to the junior worker moving to a different firm  $(\lambda_2)$  or due to either of the workers exiting the market  $(2\rho)$ . The inflow, on the other hand, equals all  $d_{11}^N(\bar{x})$  firms, i.e.  $d_{11}^N$  firms in which the junior worker becomes eligible for promotion. In the steady state, we get:

$$0 = \dot{d}_{11}^S = d_{11}^N(\bar{x}) - (\lambda_2 + 2\rho)d_{11}^S \quad \Rightarrow \quad d_{11}^S = \frac{d_{11}^N(\bar{x})}{\lambda_2 + 2\rho} = \frac{k_1 q_2 e^{-\rho\bar{x}} - k_2 e^{-(q_2 + 2\rho)\bar{x}}}{\lambda_2 + 2\rho} \tag{6}$$

Next, consider the stock of firms with senior workers only:  $d_{01}$ . Firms which employ a junior worker only promote this worker once s/he reaches  $\bar{x}$  experience level. Hence, there is an inflow  $d_{10}(\bar{x})$  into state  $d_{01}$ . Further, at a rate  $\lambda_2$  searching junior workers find a senior position on the external market, change firms and the  $d_{11}^S$  firms enter  $d_{01}$  state. Additionally,  $d_{11}^S$  firms could lose both workers at a rate  $\rho$  due to exit. This implies that there is inflow  $(\lambda_2 + 2\rho)d_{11}^S$  into  $d_{01}$ state. Next,  $\rho d_{11}^N$  is the outflow of  $d_{11}^N$  firms due to exit of the junior worker. These firms then transition into  $d_{01}$  state. And finally, at a rate  $q_2$  entrant firms  $d_{00}$  find a senior worker and become of type  $d_{01}$ . At a rate  $\rho$ ,  $d_{01}$  firms lose their worker and at a rate  $q_1$  they fill their junior vacancy. Hence,  $(\rho + q_1)d_{01}$  is the outflow of state  $d_{01}$ . Combining all expression, substituting

	Value		Value		Value
r	0.010	$\gamma$	0.003	ρ	0.014
$\beta$	0.500	s	0.100	$\mu$	0.069
$c_1$	0.500	$c_2$	2.000	$\alpha_1$	0.500
$d_1$	0.200	$d_2$	0.100	$\alpha_2$	0.500

Table 1: Values of exogenous parameters

in (3), (4) ,(6) and evaluating  $d_{10}$  at  $x = \bar{x}$  we find the steady state stock of  $d_{01}$  firms,  $\dot{d_{01}} = 0$ :

$$d_{01} = \frac{k_1(\rho e^{-\rho \bar{x}} + q_2)}{\rho + q_1} - \frac{\rho k_2(1 - e^{-(q_2 + 2\rho)\bar{x}})}{(\rho + q_1)(q_2 + 2\rho)} + \frac{q_2 n}{(\rho + q_1)(q_1 + q_2)}$$
(7)

The two initial conditions can be then re-written as:

$$\begin{cases} k_1\rho + k_2 &= \frac{nq_1}{q_1+q_2} \\ k_1q_2 - k_2 &= q_1 \left( \frac{k_1(\rho e^{-\rho\bar{x}} + q_2)}{\rho+q_1} - \frac{\rho k_2(1-e^{-(q_2+2\rho)\bar{x}})}{(\rho+q_1)(q_2+2\rho)} + \frac{q_2n}{(\rho+q_1)(q_1+q_2)} \right) \end{cases}$$

Solving this system for  $k_1$  and  $k_2$  we find:

$$\begin{cases} k_1 &= \frac{nq_1(q_1\rho e^{-(q_2+2\rho)\bar{x}} + (q_2+\rho)(q_1+q_2+2\rho))}{\rho(q_1+q_2)(q_1\rho_0 e^{-(q_2+2\rho)\bar{x}} - q_1(q_2+2\rho)e^{-\rho\bar{x}} + (\rho+q_2)(q_1+q_2+2\rho))} \\ k_2 &= -\frac{nq_1^2(q_2+2\rho)e^{-\rho\bar{x}}}{(q_1+q_2)(q_1\rho e^{-(q_2+2\rho)\bar{x}} - q_1(q_2+2\rho)e^{-\rho\bar{x}} + (\rho+q_2)(q_1+q_2+2\rho))} \end{cases}$$

Finally, the population is normalized to 1 such that:

$$e_0 + d_{10} + d_{01} + 2d_{11}^S + 2d_{11}^N = 1.$$
(8)

The stocks of junior and senior workers can be expressed as  $e_1 = d_{10} + d_{11}^S + d_{11}^N$  and  $e_2 = d_{01} + d_{11}^S + d_{11}^N$ , respectively. On the other hand, the steady state stock of  $e_0$  workers is:  $\frac{\rho e_1 + \rho e_2}{\lambda_1(\theta_1)}$ .

## 3 Firm's promotion choice: decentralized equilibrium

In this section, the promotion choice of firms is endogenized using a similar approach to Dawid et al. (2019). Let  $\beta$  be the fraction of output that accrues to workers and  $1 - \beta$  be the rest of the output that firms get as profit. Then,  $\pi_1(x) = (d_1 + c_1 e^{\gamma x})(1 - \beta)$  denotes the flow profit of a firm from having its junior position filled where the junior worker has experience x. Next, let  $\pi_2(y) = (d_2 + c_2 e^{\gamma y})(1 - \beta)$  be the flow profit accruing to a firm with filled senior position and a senior worker with experience y. The present value equation of a firm which employs one junior worker  $rJ_{10}$  is given by:

$$rJ_{10}(x|x_i,\bar{x}) = \pi_1(x) - s - \rho J_{10}(x|x_i,\bar{x}) + q_2(J_{11}^N(x,\bar{x}|x_i,\bar{x}) - J_{10}(x|x_i,\bar{x})) + \frac{\partial J_{10}(x|x_i,\bar{x})}{\partial x}$$
(9)

where the first two terms capture the flow profit from the match and the cost of searching for a senior worker. At a rate  $\rho$  the junior worker quits and the firm exits the market while at a rate  $q_2$  the firm is successful in finding a senior worker from the external market where applicants have

 $\bar{x}$  level of experience. That is why the present value of the firm depends not only on its choice for internal promotion  $\bar{x}_i$  but also on the market promotion cutoff  $\bar{x}$ . The last term captures the marginal change of firm's profits stemming from increasing productivity of the junior worker.

Next, the present value of a firm with both positions filled with a junior worker with experience x and a senior worker with experience y is given by:

$$rJ_{11}^N(x,y) = \pi_1(x) + \pi_2(y) - \rho(J_{11}^N(x,y) - J_{10}(x)) - \rho(J_{11}^N(x,y) - J_{01}(y)) + \frac{\partial J_{11}^N(x,y)}{\partial x}$$
(10)

where the indirect dependence on  $\{x_i, \bar{x}\}$  is suppressed for ease of notation. The first two terms are the flow profits of the firm, the third and the fourth capture the fact that at a rate  $\rho$  the firm loses its senior or its junior worker and moves to state  $J_{10}(x)$  or  $J_{01}(y)$ , respectively. The last term, is the gain from increasing output of the junior worker over time. The present value of a firm with one senior worker with experience y is:

$$rJ_{01}(y) = \pi_2(y) - s - \rho J_{01}(y) + q_1(J_{11}^N(0, y) - J_{01}(y)).$$
(11)

At a rate  $\rho$  the worker retires and the firm exits the market, while at a rate  $q_1$  the firm is successful in finding an inexperienced junior worker and transitions to state  $J_{11}^N(0, y)$ . Further, the present value of a firm that has both workers and the junior worker is already eligible for promotion  $rJ_{11}^S$  is:

$$rJ_{11}^{S}(\bar{x}_{i},y) = \pi_{1}(\bar{x}_{i}) + \pi_{2}(y) - \rho(J_{11}^{S}(x,y) - J_{01}(\bar{x}_{i})) - (\rho + \lambda_{2})(J_{11}^{S}(x,y) - J_{01}(y))$$
(12)

where  $\pi_1(\bar{x}_i) + \pi_2(y)$  is the flow profit of the firm. At a rate  $\rho$  the senior worker exits, the junior one is instantaneously promoted and the firm transitions into state  $J_{01}(\bar{x}_i)$ . At a rate  $\rho$  the junior worker exits, while at a rate  $\lambda_2$  the junior worker is finds a senior job in a different firm and quits. In both cases the firm transitions into state  $J_{01}(y)$ . Finally, the present value of a new firm which enters the market  $rJ_{00}$  is given by:

$$rJ_{00} = -2s + q_1(J_{10}(0) - J_{00}) + q_2(J_{01}(\bar{x}) - J_{00})$$
(13)

where -2s is the flow cost that the firm incurs from searching in both sub-markets. At a rate  $q_1$  it finds an inexperienced junior worker and moves into state  $J_{10}(0)$  and at a rate  $q_2$  it finds a senior worker and moves to state  $J_{01}(\bar{x})$ . An entering firm maximizes its present value with respect to the promotion timing  $\bar{x}_i$  and the optimal choice is:

$$\bar{x}_{i}^{*}(\bar{x}) = \operatorname*{argmax}_{\bar{x}_{i} \ge 0} J_{00}(\bar{x}_{i}, \bar{x}).$$
(14)

Below, a symmetric Nash equilibrium  $x_i^*(\bar{x}) = \bar{x}$  is analysed. The solution procedure for finding the decentralized equilibrium is discussed in detail in appendix A. In a nutshell, we find  $J_{10}(x)$ and  $J_{11}^N(x, \bar{x})$  from the first order linear differential equations in terms of two integration constants. The two integration constants are then found from two boundary conditions. The first one  $J_{10}(\bar{x}_i) = J_{01}(\bar{x}_i)$  states that the present value of the firm with a junior worker with experience  $\bar{x}_i$  and no senior worker is equal to the present value of the firm if the worker is immediately

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Variable	Value	Interpretation	Variable	Value	Interpretation
$q_1$	0.053	Junior vacancy-filling rate	$4(d_{10}(\bar{x}) + \rho d_{11}^S)/e_1$	0.026	Promotion rate
$q_2$	0.027	Senior vacancy-filling rate	$4\lambda_2 d_{11}^S/e_1$	0.042	Job-to-job trans. rate
$\lambda_1$	0.090	Junior job-finding rate	n	0.004	Entering firms
$\lambda_2$	0.177	Senior job-finding rate	$d_{00}$	0.049	Firm distribution
$ heta_1$	1.696	Junior market tightness	$d_{10}$	0.102	Firm distribution
$ heta_2$	6.544	Senior market tightness	$d_{01}$	0.179	Firm distribution
$e_0$	0.135	Workers in simple jobs	$d_{11}^N$	0.269	Firm distribution
$e_1$	0.394	Workers in junior jobs	$d_{11}^{S}$	0.023	Firm distribution
$e_2$	0.471	Workers in senior jobs	$\bar{x}$	40	Optimal prom. timing

Table 2: Decentralized equilibrium. Promotion and job-to-job transition rates are in annual terms.

promoted. The second boundary condition  $J_{11}^N(\bar{x}_i, \bar{x}) = J_{11}^S(\bar{x}_i, \bar{x})$  states that at the promotion cutoff of firm *i*, the present value changes from  $J_{11}^N(\bar{x}_i, \bar{x})$  to  $J_{11}^S(\bar{x}_i, \bar{x})$  and that the junior worker starts searching for a senior job. Also, in order to find  $J_{01}(y)$ , we need an expression for  $J_{01}(\bar{x})$ . In equilibrium it must be then true that  $J_{01}(\bar{x}) = J_{01}(y|_{y=\bar{x}})$ . The decentralized equilibrium is found from (3)-(8) which determine the distribution of firms and workers, (1)-(2) which define the transition rates and the first order condition of the value function of an entering firm  $J_{00}$ with respect to the promotion timing.

Because of the complexity of the best response function, an analytical analysis is not feasible. Therefore, the decentralized equilibrium is characterized numerically. The values of the exogenous parameters are summarized in table 1. Most of them are chosen to be exactly the same as in the model of Dawid et al. (2019). The exceptions are provided in the last column of table 1. The exit rate  $\rho$  is slightly lower (compared to  $\rho = 0.015$  in Dawid et al. (2019)) reflecting the fact that all workers exit the market, not only the senior ones. The next three parameters are due to the different matching technology. The efficiency of the matching function  $(\mu)$  is chosen such that  $\bar{x} = 40$  is the general equilibrium outcome of the model. This is also comparable to Dawid et al. (2019) where  $\bar{x} = 45$  is a general equilibrium. Here, the equilibrium promotion timing is lower since there is a positive probability that the junior worker will be exogenously separated from the firm which gives an incentive to the firms to speed up promotions. The model is calibrated on quarterly basis, so  $\bar{x} = 40$  corresponds to 10 years of professional experience which junior workers need before becoming eligible for promotion. Finally, recall that  $\alpha_1$  and  $\alpha_2$  are the elasticities of the junior and senior vacancy-filling rates, respectively. Their values are set =  $\beta$  such that the Hosios conditions are fulfilled in both sub-markets. Then, the values of the resulting variables are summarized in table 2.

It is evident that hiring junior workers is much easier than finding senior workers from the external market  $(q_1 > q_2)$ . This is reflected also in the job-finding rates, such that it is much easier for workers to find a senior position compared to finding their first professional job  $(\lambda_2 > \lambda_1)$ . There are fewer workers competing for  $e_2$  jobs compared to young and inexperienced workers, searching for  $e_1$  positions  $(e_0 > d_{11}^S)$ . Consequently, the senior market is approximately four times tighter than the junior one. Further, in equilibrium firms choose  $\bar{x}$  such that there is high probability that they are in  $d_{11}^N$  state. This is favourable since in that state firms operate with both positions filled, while the treat of losing a worker comes only from the exogenous separation rate. The left panel of figure 2 displays the objective function  $J_{00}$  of an entering firm



Figure 2: Left panel: Objective function of firm i and the optimal choice  $\bar{x}_i(\bar{x})$  for a fixed market promotion cutoff  $\bar{x} = 40$  and fixed transition rates. Right panel: Optimal response function  $\bar{x}_i(\bar{x})$ for different values of  $\bar{x}$  and constant transition rates.

*i*, given that the market promotion cutoff is  $\bar{x} = 40$ . The right panel of the same figure shows the optimal response function of an entering firm (black curve) for varying market promotion timing and fixed transition rates (blue curve). First of all, it is evident that  $x_i^*(\bar{x} = 40) = 40$  so that  $\bar{x} = 40$  is a symmetric general equilibrium. Secondly, the result of Dawid et al. (2019) of strategic complementarity of firms' promotion choices is preserved under the current specification of the model. This can be inferred from the positively sloped response curve of an entering firm. If the average promotion time in the market is increasing, an entering firm has an incentive to also choose a higher promotion requirement. Given that external candidates have a higher experience level, the firm prefers to delay promoting its own junior worker provided that it can find a highly qualified worker from the market.

Furthermore, due to the strategic complementarity the equilibrium is not unique. The second equilibrium is at  $\bar{x} \approx 43.55$  as can be seen in figure 16 in Appendix B. The right panel of the figure provides a close up of the optimal response function where the two equilibria can be distinguished. However,  $\bar{x} \approx 43.55$  is not a stable equilibrium, therefore in the analysis we focus on the unique stable one:  $\bar{x} = 40$ .

### 4 Welfare analysis

Previously, we have derived the optimal promotion time that an entering firm chooses in order to maximize the present value of its profits. In this section, we investigate whether the firm's choice is also socially optimal. The welfare analysis proceeds in two steps. In the first we fix the inflow of new firms per period:  $\mathbf{n}$ . In the second step, the stock of firms is determined via a free-entry condition.



Figure 3: Left panel: Welfare for varying  $\bar{x}$  and  $y_0 = 0$ . Maximum is achieved at  $\bar{x}^S \approx 18.3$ . Right panel: welfare for varying  $\bar{x}^S$  and  $y_0 = 0.7$ . Maximum is achieved at  $\bar{x}^S \approx 19$ 

#### 4.1 Fixed firm entry

Since workers are risk-neutral, the social planner maximizes total output, which is the sum of the match output generated by all workers in junior and senior jobs as well as those at level  $e_0$  net of costs:

$$\max_{\bar{x}} \quad W(\bar{x}) = \int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{10}(x) dx + \int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{11}^N(x) dx + (d_1 + c_1 e^{\gamma \bar{x}}) d_{11}^S \\ + (d_2 + c_2 e^{\gamma \bar{x}}) (d_{01} + d_{11}^N + d_{11}^S) + y_0 e_0 - s(2d_{00} + d_{10} + d_{01})$$

The social planner chooses the optimal promotion timing  $\bar{x}$  while facing the same matching constraints as firms and workers:  $\{\dot{d}_{00}, \dot{d}_{10}, \dot{d}_{01}, \dot{d}_{11}^N, \dot{d}_{11}^S, \dot{e}_0\}$ . At this stage we also impose a fixed number of entering firms per period n which ensures that the total number of firms does not vary by much for different promotion cutoffs. This restriction allows us to sequentially analyse the potential externalities that might drive an inefficient outcome in the model. The social planner's constrained maximization problem is solved numerically under the simplifying assumption that  $r \to 0$  and we proceed the analysis by comparing welfare for different steady-states arising by varying the promotion cutoff.

First of all, looking at the different terms entering the welfare function, the first two determine the total output of junior workers who are still accumulating human capital and are not eligible for promotion. Output of junior workers in  $d_{10}$  firms is then:

$$\begin{aligned} &\int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{10}(x) dx = \int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) (k_1 \rho e^{-\rho x} + k_2 e^{-(q_2 + 2\rho)x}) dx \\ &= d_1 \left[ \frac{k_1 \rho}{\rho} (1 - e^{-\rho \bar{x}}) + \frac{k_2}{2\rho + q_2} (1 - e^{-(2\rho + q_2)\bar{x}}) \right] \\ &+ c_1 \left[ \frac{k_1 \rho}{\rho - \gamma} (1 - e^{(\gamma - \rho)\bar{x}}) + \frac{k_2}{2\rho + q_2 - \gamma} (1 - e^{-(2\rho + q_2 - \gamma)\bar{x}}) \right] \end{aligned}$$

Decentralized eq.				Socially efficient eq.				
$\bar{x}$	40	$e_0$ (		$\bar{x}$	19	$e_0$	0.113	
$q_1$	0.0531	$e_1$	0.394	$q_1$	0.0436	$e_1$	0.325	
$q_2$	0.0271	$e_2$	0.471	$q_2$	0.1087	$e_2$	0.562	
$\lambda_1$	0.0901	Promotion rate	0.026	$\lambda_1$	0.1099	Promotion rate	0.033	
$\lambda_2$	0.1770	Job-to-job trans. rate	0.042	$\lambda_2$	0.0441	Job-to-job trans. rate	0.064	
$\theta_1$	1.696	$ heta_2$	6.544	$\theta_1$	2.523	$ heta_2$	0.405	
W	W 1.4541					1.5263		

Table 3: Decentralized vs. socially efficient equilibrium with fixed firm entry. Promotion and job-to-job transition rates are in yearly terms.

Similarly, the output of all junior workers employed in  $d_{11}^N$  firms is:

$$\int_{0}^{\bar{x}} (d_{1} + c_{1}e^{\gamma x}) d_{11}^{N}(x) dx = \int_{0}^{\bar{x}} (d_{1} + c_{1}e^{\gamma x}) (k_{1}q_{2}e^{-\rho x} - k_{2}e^{-(q_{2}+2\rho)x}) dx$$
$$= d_{1} \left[ \frac{k_{1}q_{2}}{\rho} (1 - e^{-\rho \bar{x}}) - \frac{k_{2}}{2\rho + q_{2}} (1 - e^{-(2\rho + q_{2})\bar{x}}) \right]$$
$$+ c_{1} \left[ \frac{k_{1}q_{2}}{\rho - \gamma} (1 - e^{(\gamma - \rho)\bar{x}}) - \frac{k_{2}}{2\rho + q_{2} - \gamma} (1 - e^{-(2\rho + q_{2} - \gamma)\bar{x}}) \right]$$

Summing the two and simplifying leads to:

$$\int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{10}(x) dx + \int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{11}^N(x) dx$$
$$= k_1 (\rho + q_2) \left( \frac{d_1 (1 - e^{-\rho \bar{x}})}{\rho} + \frac{c_1 (1 - e^{(\gamma - \rho) \bar{x}})}{\rho - \gamma} \right)$$

which is the total output of junior workers who are not yet searching for senior positions. Next, total output of those junior workers who are eligible for promotion is simply  $(d_1 + c_1 e^{\gamma \bar{x}}) d_{11}^S$ . The output of all senior worker is:  $(d_2 + c_2 e^{\gamma \bar{x}})(d_{01} + d_{11}^N + d_{11}^S)$  and  $y_0$  is the output of workers in  $e_0$  jobs. Finally, we denote the socially efficient promotion cutoff as:

$$\bar{x}^S = \operatorname*{argmax}_{\bar{x} \ge 0} W(\bar{x}).$$

Figure 3 plots the welfare function for two values of the productivity of workers in simple jobs:  $y_0$ . In the left-hand panel,  $y_0$  is set to 0, which means that workers in  $e_0$  jobs have no output, whereas in the right-hand panel it is assumed that  $y_0 = d_1 + c_1$ , i.e. their productivity is equal to that of a newly hired junior worker with zero professional experience. It is straightforward that increasing the productivity parameter  $y_0$  quantitatively increases social welfare, measured on the y-axis. Further, higher  $y_0$  induces later socially optimal promotion cutoff. Specifically, for  $y_0 = 0$ ,  $\bar{x}^S = 18.32$ , while for  $y_0 = d_1 + c_1 = 0.7$  it is  $\bar{x}^S = 19.03$ . This is due to the fact that  $y_0 = 0$  is the extreme assumption that workers in simple jobs do not contribute at all to total output. Hence, the social planner puts higher weight on minimizing the stock of such workers. Quantitatively, however, the effect on  $\bar{x}^S$  of relaxing  $y_0 = 0$  is small. For the rest of the analysis, it will be assumed that  $y_0 = d_1 + c_1$ .

From welfare perspective it is not optimal to promote junior workers to senior positions



Figure 4: Comparative statics with respect to the promotion cutoff  $\bar{x}$  and fixed firm entry. Left panel: transition rates. Right panel: firm distribution.

right away even though senior jobs are more productive, irrespective of worker's accumulated experience  $(d_1 + c_1 < d_2 + c_2)$ . This is similar to the decentralized equilibrium. However, the social planner chooses an earlier promotion cutoff than firms do:  $\bar{x}^S = 19.03$  vs.  $\bar{x} = 40$ . The two equilibria are compared side by side in table 3. At  $\bar{x}^S \approx 19$ , fewer workers are in  $e_0$ and  $e_1$  jobs while more are employed in senior positions. Furthermore, because of the lower promotion cutoff workers reach the promotion threshold faster and more of them are eligible for promotion. This substantially increases both the promotion and the job-to-job transition rates. In the decentralized equilibrium, 2.6% of junior workers are promoted internally per year while another 4.2% change employers to gain a promotion. In the socially optimal equilibrium those number increase to 3.3% and 6.4%, respectively.

Further, figure 4 displays the adjustment of transition rates (left panel) and firm types (right panel) as a function of promotion timing, with fixed inflow of new firms n. Higher promotion requirement reduces the stock of competing workers searching for senior jobs  $(d_{11}^S)$ and leads to more competing senior vacancies  $(d_{00} + d_{10})$ . Therefore, the senior job-finding rate  $\lambda_2$  increases multiple-fold in  $\bar{x}$ . Also since competition between firms for senior workers increases, the senior vacancy-filling rate  $q_2$  declines. In terms of the junior job-finding rate  $\lambda_1$ : lower stock of competing junior vacancies and more workers searching in the junior sub-market suppresses  $\lambda_1$  as  $\bar{x}$  increases. For the same reasons the junior job-filling rate  $q_1$  increases in  $\bar{x}$ . The centralized equilibrium is thus characterized by lower promotion requirement, lower firm competition for senior workers, equivalently less tight senior market and higher firm competition for junior workers and a tighter junior market.

At  $\bar{x} \approx 19$  total welfare is approximately 5% higher than at the decentralized equilibrium:  $\bar{x} = 40$ . This welfare gain can be decomposed into several parts. On the one hand, there is a change in workers' productivity resulting from the different promotion cutoffs. Particularly, earlier promotion requirement means that average productivity in the pool of junior worker will be lower. Similarly, the output of senior workers will also be lower if firms promote their junior workers at a lower human capital level. On the other hand, a change in  $\bar{x}$  induces a redistribution of firm

and worker types. While, lower  $\bar{x}$  reduces average output per worker employed in a professional job it might induce higher welfare, provided it redistributes workers across hierarchical levels in a more efficient way. Finally, a change in the promotion cutoff leads to a new equilibrium number of firms. To quantify these effects, we decompose the difference in welfare under the socially optimal promotion cutoff  $W(\bar{x}^S)$  and under the decentralized equilibrium  $W(\bar{x})$  into gain (or loss) that is due to worker productivity differences in the two sub-markets, gain (loss) that is due to the different distribution of workers across hierarchical levels and gain (loss) that is due to change in the firm stock. The difference can be expressed with the following 15 elements:

$$\begin{split} W(\bar{x}^{S}) &- W(\bar{x}) = \underbrace{(c_{1}(e^{\gamma c^{S}} - e^{\gamma c}))(p_{1}^{N}(\bar{x}))}_{\Delta(1)} + \underbrace{(d_{1} + c_{1}e^{\gamma c^{S}})(p_{1}^{N}(\bar{x}^{S}|_{nF(\bar{x})}) - p_{1}^{N}(\bar{x}))}_{\Delta(2)} \\ &+ \underbrace{(d_{1} + c_{1}e^{\gamma c^{S}})(p_{1}^{N}(\bar{x}^{S}) - p_{1}^{N}(\bar{x}^{S}|_{nF(\bar{x})}))}_{\Delta(3)} + \underbrace{(c_{1}(e^{\gamma \bar{x}^{S}} - e^{\gamma \bar{x}}))d_{11}^{S}(\bar{x})}_{\Delta(4)} \\ &+ \underbrace{(d_{1} + c_{1}e^{\gamma \bar{x}^{S}})(d_{11}^{S}(\bar{x}^{S}|_{nF(\bar{x})}) - d_{11}^{S}(\bar{x}))}_{\Delta(5)} + \underbrace{(d_{1} + c_{1}e^{\gamma \bar{x}^{S}})(d_{11}^{S}(\bar{x}^{S}) - d_{11}^{S}(\bar{x}^{S}|_{nF(\bar{x})})}_{\Delta(6)} \\ &+ \underbrace{(c_{2}(e^{\gamma \bar{x}^{S}} - e^{\gamma \bar{x}}))p_{2}(\bar{x})}_{\Delta(7)} + \underbrace{(d_{2} + c_{2}e^{\gamma \bar{x}^{S}})(p_{2}(\bar{x}^{S}|_{nF(\bar{x})}) - p_{2}(\bar{x}))}_{\Delta(8)} \\ &+ \underbrace{(d_{2} + c_{2}e^{\gamma \bar{x}^{S}})(p_{2}(\bar{x}^{S}) - p_{2}(\bar{x}^{S}|_{nF(\bar{x})}))}_{\Delta(9)} - \underbrace{s(v_{1}(\bar{x}^{S}|_{nF(\bar{x})}) - v_{1}(\bar{x}))}_{\Delta(10)} \\ &- \underbrace{s(v_{1}(\bar{x}^{S}) - v_{1}(\bar{x}^{S}|_{nF(\bar{x})}))}_{\Delta(11)} - \underbrace{s((v_{2}(\bar{x}^{S}|_{nF(\bar{x})}) - v_{2}(\bar{x})))}_{\Delta(12)} - \underbrace{s(v_{2}(\bar{x}^{S}) - v_{2}(\bar{x}^{S}|_{nF(\bar{x})}))}_{\Delta(13)} \\ &+ \underbrace{y_{0}(e_{0}(\bar{x}^{S}|_{nF(\bar{x})}) - e_{0}(\bar{x}))}_{\Delta(14)} + \underbrace{y_{0}(e_{0}(\bar{x}^{S}) - e_{0}(\bar{x}^{S}|_{nF(\bar{x})}))}_{\Delta(15)} \end{aligned}$$

$$(15)$$

where  $p_2(\bar{x}) = d_{01}(\bar{x}) + d_{11}^N(\bar{x}) + d_{11}^S(\bar{x})$  is the stock of firms that have the senior position filled, given the market promotion time:  $\bar{x}$  and  $p_2(\bar{x}^S) = d_{01}(\bar{x}^S) + d_{11}^N(\bar{x}^S) + d_{11}^S(\bar{x}^S)$  is the stock of such firms under the socially efficient promotion timing. Next, we denote with  $p_2(\bar{x}^S|_{nF(\bar{x})})$  the stock of firms with senior workers given that the total number of firms is fixed to its decentralized equilibrium value but the promotion cutoff is the socially efficient one. For all following definitions, the same distinctions hold:  $(\bar{x})$  denotes the value of the variable under the decentralized equilibrium while  $(\bar{x}^S)$  stands for its value in the socially efficient steady-state and  $(\bar{x}^S|_{nF(\bar{x})})$ evaluates the value of the variable under the socially optimal promotion but assuming the total stock of firms found in the decentralized equilibrium. Next,  $p_1^N = d_{10} + d_{11}^N$  is the stock of firms with junior workers who are not yet searching. On the other hand, the total stock of firms which have the junior position filled is  $p_1^N + d_{11}^S$ . Further,  $v_1 = d_{00} + d_{01}$  and  $v_2 = d_{00} + d_{10}$  are the stocks of vacancies in the junior and senior market, respectively.

Let us first consider the difference in total output of senior jobs under the two equilbria:  $(d_2 + c_2 e^{\gamma \bar{x}^S}) p_2(\bar{x}^S) - (d_2 + c_2 e^{\gamma \bar{x}}) p_2(\bar{x})$ . Rearranging the terms and adding and subtracting  $c_2 e^{\gamma \bar{x}^S} p_2(\bar{x})$  and  $(d_2 + c_2 e^{\gamma \bar{x}^S}) (p_2(\bar{x}^S|_{nF(\bar{x})}))$ , the difference can be re-written as parts (7), (8) and (9) of the decomposition. Component (7) is interpreted as the productivity effect contributing to the change in welfare. The next term of the decomposition, (8), fixes the productivity of workers to the one under the socially efficient equilibrium, and traces the change in worker distribution

	$\Delta(1)$	$\Delta(2)$	$\Delta(3)$	$\Delta(4)$	$\Delta(5)$	$\Delta(6)$	$\Delta(7)$	$\Delta(8)$
$W(\bar{x}^S) - W(\bar{x})_{(0, 7)}$	$\approx -0.39$	$\approx -7.98$	$\approx -0.055$	-0.05	4.5	0.23	-4.46	15.1
$\overline{W(\bar{x})}$ (70)	$\Delta(9)$	$\Delta(10)$	$\Delta(11)$	$\Delta(12)$	$\Delta(13)$	$\Delta(14)$	$\Delta(15)$	Total
	-1.3	0.5	-0.11	-0.68	-0.03	-1.27	0.23	$\approx 5\%$

Table 4: Numerical decomposition of the welfare gain

assuming that total number of firms does not alter. This can be interpreted as the worker and firm re-distribution effect resulting from the new promotion cutoff. The final component, (9), captures the welfare change which is due to firm stock change. Similarly, the difference in total output of junior workers in  $d_{11}^S$  firms:  $(d_1 + c_1 e^{\gamma \bar{x}^S}) d_{11}^S(\bar{x}^S) - (d_1 + c_1 e^{\gamma \bar{x}}) d_{11}^S(\bar{x})$  can be re-written as parts (4), (5) and (6) of the decomposition. Again, the first term is the change in welfare due to productivity differences, the second is the gain or loss that stems from worker re-distribution, while the third reveals the effect of change in the total stock of firms.

In order to decompose the welfare change that is due to difference in productivity and stock of  $e_1$  workers who are still accumulating experience, we use the mean value theorem to approximate the mean human capital level of such workers employed in  $d_{10}$  and  $d_{11}^N$  firms. Let  $e_1^N(x)$  be the stock of  $e_1$  non-searching workers with experience x. The first order linear differential equation that characterizes changes in  $e_1^N(x)$ ,  $0 \le x \le \bar{x}$  is:  $\partial e_1^N(x)/\partial x = -\rho e_1^N(x)$ . At a rate  $\rho$ such workers exit the market. The solution is then  $e_1^N(x) = N e^{-\rho x}$  where N can be found from the initial condition  $e_1^N(0) = \lambda_1 e_0$ , i.e. at a rate  $\lambda_1$  workers in  $e_0$  employment find a professional junior job and start their career with no professional experience. This implies that  $N = \lambda_1 e_0$  and the general solution to the first order linear differential equation is written as:  $e_1^N(x) = \lambda_1 e_0 e^{-\rho x}$ . This expression is equivalent to  $d_{10}(x) + d_{11}^N(x)$ , i.e. the stock of firms which have a junior worker who is still accumulating experience. By the mean value theorem since  $e_1^N(x)$  is continuous on the interval  $[0, \bar{x}]$  and differentiable on  $(0, \bar{x})$ , there exists c such that  $e_1^{N'}(c) = (e_1^N(\bar{x}) - e_1^N(0))/(\bar{x})$ . For the decentralized equilibrium, we find that  $c \approx 19.07$  while for the socially efficient equilibrium  $c^S \approx 9.3$ . We then use these numerical values to estimate the three effects of switching between the decentralized and socially optimal steady states with respect to  $e_1$  employment of non-searching workers. This results in components (1), (2) and (3) of the decomposition. Even though this is an approximation, it can give us an indication to which of the effects is quantitatively larger.

Next, we need to account for changes in the stock of vacancies and its contribution to welfare. Firstly,  $-s(v_1(\bar{x}^S|_{nF(\bar{x})}) - v_1(\bar{x}))$  shows the change in total junior vacancy cost stemming only from the new promotion cutoff. Secondly, the term  $-s(v_1(\bar{x}^S) - v_1(\bar{x}^S|_{nF(\bar{x})}))$  accounts for the firm stock adjustment. Similar calculations are done for the contribution of total senior vacancy cost (components (12) and (13)). Finally, we take into account the effect of adjustment of the number of workers in simple jobs where the term  $y_0(e_0(\bar{x}^S|_{nF(\bar{x})}) - e_0(\bar{x}))$  shows the change in output of such workers holding the number of firms fixed, while  $y_0(e_0(\bar{x}^S) - e_0(\bar{x}^S|_{nF(\bar{x})}))$  allows for changes in the firm stock.

Table 4 summarizes the value of each term in the decomposition in percentage terms. The sum of components  $(1) + (4) + (7) \approx -4.94\%$  can be interpreted as the total *productivity effect* of changing the promotion timing. Further,  $(2) + (5) + (8) + (10) + (12) + (14) \approx 10.51\%$  is the total *redistribution of workers effect* of switching between the two steady states. Finally,

 $(3) + (6) + (9) + (11) + (13) + (15) \approx -0.76\%$  is the stock of firms adjustment effect. It is evident that the welfare gain stems from the redistribution of workers across hierarchical levels and predominantly from employing more workers in senior jobs (component (8)). Even though there is lower average output per worker, in the socially efficient equilibrium, this negative effect is outweighed by the gain from having more  $e_2$  workers. Furthermore, due to higher junior job-finding rate in the socially efficient equilibrium, workers move out of  $e_0$  jobs faster, which leads to fewer workers at the  $e_0$  level. This is associated with loss of output and a corresponding welfare decline of 1.27% (component (14)). However, this effect is dominated by the fact that the difference in  $e_0$  workers has moved into professional jobs which are at least as productive as the simple jobs. Finally, there is a 0.76% welfare loss associated with smaller equilibrium number of firms (nF) under socially efficient equilibrium  $(nF(\bar{x}^S) = 0.609 \text{ vs. } nF(\bar{x}) = 0.622)$ . However, this effect is quantitatively small and is driven by the fixed firm entry. Overall, we can conclude under the decentralized equilibrium, there is loss in allocative efficiency, i.e. workers are inefficiently distributed across hierarchical levels, because firms choose a promotion requirement that is too high.

In order to identify the drivers of this inefficient outcome, note that firm's incentive to delay internal promotions is two-fold. On the one hand, firms would like to reduce turnover on level one. Turnover on level one increases with earlier promotion and with higher senior job-finding rate. So, a straightforward way for firms to retain junior workers is to delay promoting them. However, choosing higher promotion requirement induces general equilibrium effects which mitigate firm's incentive to delay internal promotions. Firstly, when a single firm decides to promote later, it imposes a negative externality on all other firms because its higher promotion choice reduces the number of potential candidates in the external market for experienced workers, or in other words, reduces the senior vacancy-filling rate  $q_2$ . Strategic complementary implies that other firms then also increase their promotion threshold in response. When all firms delay promotions the senior vacancy-filling rate is substantially suppressed which leads to many unfilled, competing senior vacancies. This externality is not internalized by firms as shown by the social planner's solution.

The second general equilibrium effect is associated with the senior job-finding rate and is the following: higher promotion requirement reduces the number of applicants on the secondary market so whenever the junior worker reaches the cutoff experience s/he finds a senior job with a different firm much faster in case promotion is not possible. This is evident by the steeply increasing senior job-finding rate  $\lambda_2$  in  $\bar{x}$  (see the left panel of figure 4). Hence, once the worker reaches  $\bar{x}$  there is a very high probability of separation which is not favourable for the firm. To illustrate this, figure 5 show comparative statics with respect to  $\lambda_2$  keeping all other rates equal to the ones under the decentralized equilibrium. The blue curve shows the benchmark scenario. The black curve is the optimal response of an individual firm assuming  $\lambda_2 > \lambda_2^{eq}$ , i.e separation probability once the worker reaches  $\bar{x}$  increases. The firm's optimal response is then to delay internal promotions even further in order to keep its worker over a longer period. The opposite is true if  $\lambda_2 < \lambda_2^{eq}$ . Firm's optimal strategy is then to choose earlier promotion cutoff since the treat of losing the worker once they are eligible for promotion is reduced.

Second incentive of firms to delay internal promotions comes from the expected profit associated with hiring a senior worker from the external market. Note that the expected output of a senior match depends on the average promotion timing on the market. Higher experience



Figure 5: Comparative statics of the effect of the senior job-finding rate on firm's promotion decision.

level of potential external candidates gives incentive to firms to delay internal promotions since the gains associated with hiring a senior worker from the market increase. Overall, even though general equilibrium effects mitigate to some extent the incentive of firms to delay promotions too much, it is evident that the decentralized market cannot internalize the negative externality associated with the strategic complementarity of firms' promotion decisions. This leads to inefficiently high equilibrium promotion cutoff under which too many senior positions are vacant which in turn suppresses total output. Thus the paper identifies a novel externality in the context of human capital accumulation that leads to an inefficient market outcome, namely in a setting with competitive firms and endogenous promotion decisions, strategic complementarity of promotion choices pushes up the promotion requirements "too high" than what would be optimal from output maximizing perspective.

This result also highlights that certification gives too much power to firms since junior workers have to stay with the employer that provides their evaluation. Because of their strategic considerations, firms exploit the power that certification gives them which leads to an inefficient aggregate outcome.

#### 4.1.1 Effects of productivity and educational parameters

Next, we explore the role of education, which is proxied by the parameters  $c_1$  and  $c_2$ . On the one hand, assuming higher  $c_1$  would correspond to higher general schooling attainment such that new workers, just starting their professional career are more productive. Similarly, higher  $c_2$  can be interpreted as higher competence among professional workers, regardless of their practical experience. In our benchmark scenario  $c_1 = 0.5$  while  $c_2 = 2$  and in this section we vary the two parameters and compare how the decentralized  $(\bar{x})$  and the socially efficient  $(\bar{x}^S)$  equilibria respond to the changes. Figure 6 displays the result.

As expected, the two parameters have an opposite effect on the optimal promotion timing. Higher  $c_1$  means that workers in  $e_1$  jobs are more productive so firms can afford to keep them longer in junior jobs. On the other hand, increasing  $c_2$  leads to earlier promotion since the



Figure 6: Left panel: comparative statics of the effect of productivity parameter  $c_1$  on firm's promotion and socially optimal promotion cutoffs for fixed n and  $y_0 = 0$  (red solid curve),  $y_0 = c_1$  (red dashed curve) and  $y_0 = d_1 + c_1$  (red dotted curve). Right panel: comparative statics of the effect of productivity parameter  $c_2$  on firm's promotion and socially optimal promotion cutoffs for fixed n and  $y_0 = 0$  (red solid curve),  $y_0 = d_1 + c_1$  (red dotted curve).

foregone profits associated with keeping the workers at level one increase. Qualitatively, the socially optimal  $\bar{x}^{S}$  responds in the same way with respect to the two parameters. However, it is evident from the left panel of figure 6 that it is crucially important how productivity of young workers in simple jobs is defined. The solid red curve plots the case where  $y_0 = 0$ , i.e. those workers have 0 output. In this case  $\bar{x}^S < \bar{x}$  for all plausible values of  $c_1^2$ . Since  $e_0$  workers do not contribute to overall output, it is optimal to put a high weight on minimizing the stock of such workers. If the promotion threshold is set too high that would mean that workers will spend a lot of time in the junior jobs, reducing the vacancies on that level. Thus, finding a professional job for  $e_0$  workers will be more difficult, which implies that high promotion requirement is not optimal. On the other hand, the dashed red curve plots the case  $y_0 = c_1$  while the dotted red curve assumes that  $y_0 = d_1 + c_1$  such that output of  $e_0$  workers is the same as the one of a junior worker who just got hired and has no professional experience. We see that depending on the assumption on  $y_0$  there are parameter settings for which it is possible that the socially optimal and decentralized equilibrium coincide. For very high values of  $c_1$  and corresponding high productivity of workers in simple jobs we could also have that case  $\bar{x}^S > \bar{x}$ , i.e. firms would promote inefficiently early. However, this would require a substantial increase in  $c_1$  compared to the benchmark scenario and effectively assuming that productivity at the two hierarchical layers is almost identical for a given level of human capital.

More specifically, if we look into the case  $c_1 = 1.7$ , we can graphically disentangle the effect of  $y_0$  on the optimal promotion timing. Figure 7 displays the welfare function  $W(\bar{x})$  for the three specifications of  $y_0$ :  $y_0 = 0$  (blue curve),  $y_0 = c_1$  (red curve) and  $y_0 = d_1 + c_1$  (black curve). It is straightforward that  $y_0 > 0$  quantitatively increases overall welfare for all considered cutoff options. In order to illustrate the effect of  $y_0$  on  $\bar{x}^S$  we can look at the four main components that enter the welfare function. On the one hand, total output of junior workers increases as

<sup>&</sup>lt;sup>2</sup>Note that  $c_1$  has to be smaller than  $c_2$ .



Figure 7: Left panel: Comparative statics of welfare with respect to productivity of workers in simple jobs:  $y_0$  and  $c_1 = 1.7$ . Blue curve:  $y_0 = 0$ :  $x^S \approx 31.8$ ; red curve:  $y_0 = c_1$ :  $x^S \approx 48.9$ ; black curve:  $y_0 = d_1 + c_1$ :  $x^S \approx 56.2$ . Right panel: Welfare decomposition into output from workers at all possible states and vacancy cost for  $c_1 = 1.7$ .

 $\bar{x}$  increases (see figure 7, right panel, blue curve). This comes from both the fact that these workers are on average more productive and also because the stock of such workers increases. On the other hand, we can see that the output of senior workers increases at first too. This is because their productivity increases in  $\bar{x}$ . However, the stock of  $e_2$  workers decreases which eventually suppresses total output of  $e_2$  jobs (see figure 7, right panel, red curve).

Further, total vacancy cost (black curve in the right panel of figure 7, measure on the right axis) initially increases in  $\bar{x}$  but the effect is quantitatively small. If  $y_0 = 0$ , then maximum sum of the three components is at  $\bar{x}^S \approx 31.8$  (red vertical line). The green dashed line plots the output of  $e_0$  workers for the case  $y_0 = d_1 + c_1$ . The stock of  $e_0$  workers slightly declines for  $\bar{x}$  close to 0 but increases after. It is evident that output of workers in simple jobs, together with the higher output of of junior workers can, under this parameter setting, compensate the loss associated with lower  $e_2$  employment, so in order to maximize total output the planner delays internal promotions until  $\bar{x}^S \approx 56.2$  (black vertical line). This implies that if jobs in the economy are fairly homogeneous with respect to their productivity, then a large increase in human capital leads to overall welfare improvements. If jobs are more heterogeneous in terms of productivity, which would be a more plausible assumption in the context of human capital accumulation and assigning workers to different hierarchical levels, then concerns in regards to the efficient allocation of workers to jobs also play a dominant role.

#### 4.1.2 Effects of workers' bargaining power

Next, we consider the effect of the parameter  $\beta$  which determines how the firm and the worker split the output of the match, where a share  $\beta$  accrues to the worker and a fraction  $1 - \beta$  is retained by the firm as profits. From output maximizing perspective, the social planer is not concerned with how the output is divided between the economic agents, so potentially, there is scope for welfare improvement with respect to  $\beta$ .



Figure 8: Optimal promotion and the bargaining power.

Recall that  $\alpha_1 = -(\partial q_1(\theta_1)/\partial \theta_1)(\theta_1/q_1(\theta_1))$  and  $\alpha_2 = -(\partial q_2(\theta_2)/\partial \theta_2)(\theta_2/q_2(\theta_2))$  are the elasticities of the vacancy-filling rates in the junior and senior market, respectively. So far we have assumed that the Hosios condition  $\beta = \alpha_i$  i = 1, 2 is satisfied in both sub-markets. The left panel of figure 8 shows comparative statics of the optimal promotion timing of firms and the bargaining power for varying  $c_1$ . As discussed in the previous section, higher  $c_1$  is associated with later promotion. This is true for all feasible values of workers' bargaining power which is evident from the upward shift of the promotion timing curves. Further,  $\partial \bar{x}/\partial \beta > 0$  such that we can conclude that lower bargaining power of workers ( $\beta < \alpha_i$ , i = 1, 2) leads to welfare improvement since it induces earlier promotion. Lower  $\beta$  means that firms earn higher profits for a given level of human capital of the workers. This implies that they can reduce their promotion requirement without sacrificing profits even though worker productivity will on average be lower. On the other hand, as  $\beta$  increases, firm profits per match decrease. In order to compensate for this effect firms require higher experience level before promotion which means that on average total output per match in both sub-markets will be higher.

However, even for  $\beta \to 0$  the welfare improvement would be only marginal since the change in  $\bar{x}$  is quantitatively small. If we consider a scenario where workers' share of the match surplus differs in the two hierarchical levels, such that  $\beta_1$  is the bargaining power of junior workers and  $\beta_2$  is the bargaining power of senior workers, a different picture is revealed. The right panel of figure 8 plots the case where one of the  $\beta_i$ , i = 1, 2 is fixed at its benchmark value, while the other is varied and the optimal promotion timing. We see that for fixed  $\beta_2 = \alpha_2$ ,  $\partial \bar{x}/\partial \beta_1 < 0$ (blue curve). With higher bargaining power of junior workers, firms speed up promotions since profits associated with having a worker on the lower hierarchical level decline. Hence, we can conclude that ( $\beta_1 > \alpha_1$ ) leads to welfare improvement. The opposite is true for the relationship between  $\bar{x}$  and  $\beta_2$ . We have that for fixed  $\beta_1 = \alpha_1$ ,  $\partial \bar{x}/\partial \beta_2 > 0$  (red curve). Firms compensate for the lower profits from senior jobs by delaying promotions. Hence, ( $\beta_2 < \alpha_2$ ) leads to welfare improvement since it induces earlier promotions. Overall, however, the numerical simulations show that for the benchmark parameter setting and fixed firm entry, there is no combination of  $\beta_1$  and  $\beta_2$  that will lead to the socially efficient promotion cutoff.



Figure 9: Left panel: Welfare for varying  $\bar{x}$  and  $y_0 = 0$ . Maximum is achieved at  $\bar{x}^S \approx 6.6$ . Right panel: welfare for varying  $\bar{x}^S$  and  $y_0 = 0.7$ . Maximum is achieved at  $\bar{x}^S \approx 7.4$ 

#### 4.2 Free-entry

In this section we let the stock of firms be determined from a free-entry condition. It is assumed that firms have to pay an entry cost K upon entering the market which can be interpreted as the cost needed for buying equipment and capital. In equilibrium this implies that the number of firms is determined at the point where  $J_{00} = K$ , i.e. firms enter until there are no positive profits to be gained by creating a new firm. The social planner then solves the following problem:

$$\max_{\bar{x},n} \mathbf{W}(\bar{x}) = \beta \int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{10}(x) dx + \beta \int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{11}^N(x) dx + \beta (d_1 + c_1 e^{\gamma \bar{x}}) d_{11}^S + \beta (d_2 + c_2 e^{\gamma \bar{x}}) (d_{01} + d_{11}^N + d_{11}^S) + y_0 e_0$$

subject to the matching frictions and the free-entry condition:  $\{\dot{d}_{00}, \dot{d}_{10}, \dot{d}_{01}, \dot{d}_{11}^N, \dot{d}_{11}^S, \dot{e}_0\}$  and  $J_{00} = K$ . Since the free-entry condition ensures that in equilibrium the profits of firms are driven down to zero, the objective of the social planner is to maximize the surplus that does not accrue to the firms. That is, the planner chooses the promotion cutoff that maximizes the steady-state wage bill together with the output of  $e_0$  workers. The first four terms of W( $\bar{x}$ ) are then the total output produced by  $e_1$  and  $e_2$  workers that is paid out as wages while the last term is the productivity of those agents who are not yet in professional employment.

Figure 9 plots the welfare function for  $y_0 = 0$  (left panel) and  $y_0 = d_1 + c_1$  (right panel). Similarly to the fixed-entry case, the quantitative difference between the two cases is very small, so subsequently it will be assumed  $y_0 = d_1 + c_1$ . Also similarly to the results from the previous section, the socially optimal promotion time  $\bar{x}^S$  is earlier than the one chosen by the firms in the decentralized equilibrium. Further, note that in the decentralized equilibrium  $J_{00}(\bar{x}) = 51$ , so the entry cost K is set to 51. Table 5 displays the comparison between the two equilibria. In the socially efficient equilibrium, the number of workers in simple jobs is almost halved, while much more workers are employed in professional jobs and particularly, in senior ones. The lower promotion requirement leads to substantially larger senior vacancy-filling rate. Moreover, the

Decentralized eq.: $\bar{x} = 40$				Socially efficient eq.: $\bar{x} = 7.36$				
$\overline{n}$	0.0039	$e_0$	0.135	n	0.0065	$e_0$	0.071	
$q_1$	0.0531	$e_1$	0.394	$q_1$	0.0261	$e_1$	0.243	
$q_2$	0.0271	$e_2$	0.471	$q_2$	0.1154	$e_2$	0.686	
$\lambda_1$	0.0901	01 Promotion rate		$\lambda_1$	0.1833	Promotion rate	0.055	
$\lambda_2$	0.1770	0.1770 Job-to-job trans. rate		$\lambda_2$	0.0414	Job-to-job trans. rate	0.104	
$\theta_1$	1.696	$ heta_2$	6.544	$\theta_1$	7.019	$ heta_2$	0.359	
W	0.793					0.872		

Table 5: Decentralized vs. socially efficient equilibrium with free-entry condition. Promotion and job-to-job transition rates are in yearly terms.

low stock of workers competing for level one jobs, means that it is much more difficult for firms to fill their junior positions which leads to lower junior vacancy-filling rate. On the other hand, both promotions and job-to-job transitions occur more often, which is again a straightforward result from the lower promotion timing.

Further, figure 10 displays the adjustment of transition rates and firm stocks under freeentry as the promotion timing increases. Qualitatively, the direction of change of the transition rates in response to increasing the promotion cutoff is the same as to the one discussed for the case of fixed firm entry (see figure 4). Notably, the magnitude of change in the junior jobfilling and finding rates:  $q_1$  and  $\lambda_1$  is much larger under firm free-entry. This is the result of labour demand effects that correspond to the change in firm stock. Figure 17 in Appendix B displays the adjustment of number of firms for varying promotion cutoff under fixed firm entry (left panel) and free-entry (right panel). We observe that for larger  $\bar{x}$  the stock of active firms under free-entry declines substantially which magnifies the effects of promotion timing on the transition rates, particularly in the junior market. Firstly, an increase in  $\bar{x}$  is associated with fewer workers in senior jobs and higher firm competition in that sub-market. Since it becomes relatively more difficult for firms to fill  $e_2$  vacancies, profits are suppressed and fewer firms stay active on the market. In terms of the junior sub-market, the decrease in firm stock, together with the higher promotion requirement means that the market tightness decreases, with high worker competition for junior jobs, a steeper decline in junior job-finding rate  $(\lambda_1)$  and corresponding steeper increase in junior job-filling rate  $(q_1)$ . Due to free-entry then, the changes in  $\lambda_1$  and  $q_1$ are much stronger. On the flip side, because there is fewer firms for large  $\bar{x}$ , workers who are eligible for promotion compete for fewer vacancies and the senior job-finding rate  $\lambda_2$  increases less compared to the fixed firm entry scenario. Finally, the firm distribution adjustment is also comparable to the fixed firm entry scenario. Notably, however, there are larger quantitative changes as  $\bar{x}$  increases.

In terms of overall welfare, we see that the socially optimal  $\bar{x}^S$  leads to approximately 10% welfare improvement compared to the decentralized equilibrium. Table 6 shows the decomposition of welfare difference between the two steady-states (see equation (15)). Note that the output from professional firms enters the respective components multiplied with  $\beta$  and that the vacancy cost does not enter the consideration of the social planner since it is captured by the free-entry condition. Looking at the three major effects, we observe that the productivity effect (1) + (4) + (7) leads to 6.89% welfare loss<sup>3</sup>. This is due to the fact that workers on both hier-

<sup>&</sup>lt;sup>3</sup>Here,  $c^S \approx 3.648$ .



Figure 10: Comparative statics with respect to the promotion cutoff  $\bar{x}$  under free-entry. Left panel: transition rates. Right panel: firm distribution.

archical levels are on average less productive. Moreover, this effect is larger compared to the fixed firm entry scenario since the promotion timing in the socially efficient equilibrium with free-entry is lower. Next, the redistribution of workers effect: (2) + (5) + (8) + (14) accounts for 10.33% increase in welfare due to higher professional employment and larger number of senior workers. This effect is quantitatively similar to the one found under the fixed firm entry case. Finally, the stock of firms adjustment effect: (3) + (6) + (9) + (15) contributes to further 6.48% welfare increase. In contrast to the fixed firm entry scenario, here the last effect is quantitatively large and positive. In the decentralized equilibrium, the total stock of firms is  $nF(\bar{x}) = 0.622$  while it increases to  $nF(\bar{x}^S) = 0.741$  in the socially efficient steady-state. This reveals that firm creation is distorted downwards compared to what would be socially optimal, which is a further source of inefficiency in the model. The reason behind this distortion will be discussed in more detail in the next sub-section.

	$\Delta(1)$	$\Delta(2)$	$\Delta(3)$	$\Delta(4)$	$\Delta(5)$	$\Delta(6)$	$\Delta(7)$	$\Delta(8)$
$W(\bar{x}^S) - W(\bar{x})_{(0Z)}$	$\approx -0.56$	$\approx -12.597$	$\approx 0.135$	-0.08	8.4	-2.63	-6.25	17.4
$W(\bar{x})$ (70)	$\Delta(9)$	$\Delta(10)$	$\Delta(11)$	$\Delta(12)$	$\Delta(13)$	$\Delta(14)$	$\Delta(15)$	Total
	11.7	-	-	-	-	-2.87	-2.73	$\approx 9.9$

Table 6: Numerical decomposition of the welfare gain with free-entry

Further, we consider the effect of the entry cost K on the market outcomes. Figure 11 displays the equilibrium promotion cutoff as a function of K (left panel) and the corresponding equilibrium stock of firms (right panel). As expected, the number of active firms declines as the entry cost increases. Considering the effect of higher K on the optimal promotion timing, then there are several effects. Firstly, higher entry cost means that the present value of an entering firm must also increase. In order to achieve that firms must earn higher profits. Assuming market conditions remain constant otherwise and under fixed match output sharing rule, this is possible only if the average output per match is increased. Hence, firms have to let their junior workers accumulate more experience and delay internal promotions.



Figure 11: Left panel: Optimal  $\bar{x}$  and  $\bar{x}^S$  as a function of entry cost. Right panel: Number of firms under decentralized and socially optimal equilibrium with free-entry.

Secondly, there is a labour demand effect coming from decreasing firm competition as the equilibrium number of firms declines. Since there are less competing vacancies in both submarkets, it is easier to fill an open vacancy and both  $q_1$  and  $q_2$  go up (see figure 18 in Appendix B). The effect of a simultaneous increase of both of those variables on  $\bar{x}$  is, however, ambiguous since they have an opposite effect on the optimal promotion timing. Higher junior vacancy-filling rate is associated with earlier promotions while higher senior vacancy-filling rate leads to later internal promotions<sup>4</sup>. Overall, the effect of lower competition in the senior market dominates in this setting and optimal promotion timing rises in response to higher entry cost.

Similarly to the decentralized equilibrium, the socially optimal  $\bar{x}^S$  increases in K. As it will be discussed below, if the condition are relatively favourable for firms, implying that many firms can stay active on the market, the social planner can maximize welfare by choosing immediate or very fast promotion and employing many workers at the high productivity senior jobs. If, however, there are few active firms because of unfavourable market conditions, such as in this case, a high entry cost, welfare is maximized by delaying promotions and increasing average match output. In the next section we explore the relationship between firm entry and the socially optimal promotion timing in more detail.

#### 4.3 Constraint efficient firm entry and the bargaining power

As it has been shown in the previous section, welfare is not maximized at the Hosios level of bargaining power:  $\beta = \alpha_i$ , i = 1, 2. In this section, we numerically explore the effect of this key model parameter. Given the objective function of the social planner under free-entry, changing  $\beta$  influences both the decentralized and the socially optimal equilibrium. Figure 12 plots the comparative statics of the promotion timing in the decentralized equilibrium:  $\bar{x}$  and the socially optimal one:  $\bar{x}^S$  with respect to different values of  $\beta$ . Similarly to the fixed-entry scenario, discussed above,  $\bar{x}$  is increasing in  $\beta$  for low values of  $\beta$ . However, as workers' bargaining power increases, this relationship is reversed which is in contrast to the firm fixed-entry case.

<sup>&</sup>lt;sup>4</sup>This result and the intuition behind it are discussed in greater detail in Dawid et al. (2019).



Figure 12: Optimal promotion and the bargaining power.

Increasing  $\beta$  has several effects on firms' promotion decisions. On the one hand, if a higher share of the output goes to workers, firms could compensate by delaying promotions and thus increasing output per worker. Also, as  $\beta$  increases, firms profits decline and fewer firms are able to stay active. Hence, many workers compete for few vacancies and the vacancy-filling rates increase. As discussed above, a simultaneous increase in both  $q_1$  and  $q_2$  has an ambiguous effect on  $\bar{x}$ . We see that for low to middling values of  $\beta$  the effect of decreasing firm competition on the senior market dominates, so firms delay promotions. For higher values of  $\beta$ , the equilibrium number of firms declines so much that the effect on promotions is reversed, i.e. increasing  $\beta$ is associated with a decrease in promotion timing. The stock of firms at  $\beta = 0.75$  is nF =0.261 which is more than twice less than in the case  $\beta = 0.5$  and for  $\beta > 0.75$ , it approaches 0, so  $\bar{x}$  cannot be computed. For such high values of workers' bargaining power, there are even fewer firms and potential vacancies and many competing searching workers in the market. Consequently, the job-filling rates increase steeply in  $\beta$  (see figure 19 in Appendix B). Also, we observe that  $q_1$  increases faster compared to  $q_2$ . This combined with the decreasing senior job-finding rate  $\lambda_2$  becomes the dominant effect and so for high values of workers' bargaining power, promotion timing in the decentralized equilibrium decreases.

Next, note that the socially efficient promotion timing also depends on  $\beta$ . The red curve in figure 12 plots  $\bar{x}^S$  for different values of  $\beta$ . If  $\beta = 0$ , then the social planner maximizes the stock of  $e_0$  workers given the matching frictions and the free-entry condition. This is the extreme case in which firms retain all of the output from the match and is not of interest for the analysis. For  $\beta \in [0.3, 0.4]$  immediate promotion is optimal. Since firms retain a larger share of the total output, many firms enter the market, there is a high firm competition in both submarkets and the vacancy-filling rates are low. By choosing immediate promotions the social planner is thus able to employ many workers at the high productivity senior positions. Even though increasing  $\bar{x}$  also translates into higher output once the workers are at the  $e_2$  level, the firm competition effect dominates here. Increasing  $\bar{x}$  in this case reduces  $q_2$  even further which suppresses the profits of firms and drives some firms out of the market. Hence, setting higher promotion requirement acts as a barrier to entry for new firms and the equilibrium stock of firms monotonically decreases.

To illustrate this figure 20 in Appendix B displays the social planner's objective function, the firm stock and the job-filling rates for varying  $\bar{x}$  and  $\beta = 0.35$ .

At  $\beta = 0.5$ , we have  $\bar{x}^S = 7.4$  which is the case depicted in figure 9. As  $\beta$  increases further, so does the socially optimal promotion timing. Figure 21 in Appendix B display the case  $\beta = 0.75$ . Comparing it to the case  $\beta = 0.35$ , here the stock of firms is much lower for all considered promotion cutoffs. This implies that firm competition is lower and the the vacancy-filling rates are higher (right panel of figure 21). Setting a higher promotion requirement then increases the average output per match which leads to higher firm profits and consequently higher entry up to a certain value of  $\bar{x}$ . Hence, here the productivity effect dominates and welfare rises as promotions increase up from  $\bar{x} = 0$ . Choosing too high promotion requirement, however, suppresses  $q_2$  drastically so it becomes difficult for firms to fill their high productivity senior jobs and again welfare starts to decline.

These considerations imply that the way firms and workers split the match output is crucial in determining the social efficiency of firm's promotion timing and entry. To explore this relationship deeper, we next assume that  $\beta_1$  is the share of output that goes to junior workers and  $\beta_2$  is the share of output that accrues to senior workers. In what follows, the aim is to answer the question whether there is a pair  $\{\beta_1, \beta_2\}$  under which the socially optimal and the decentralized equilibrium coincide. Firstly, assuming  $\beta_2 = 0.5$  is constant, then the firm's promotion choice is decreasing in  $\beta_1$ :  $\partial \bar{x}/\partial \beta_1 < 0$  (see right panel of figure 12, blue curve) which is similar to the fixed entry case. Further, for fixed  $\beta_1 = 0.5$ ,  $\partial \bar{x} / \partial \beta_2$  is non-monotone mirroring the overall relationship between  $\beta$  and  $\bar{x}$  (see right panel of figure 12, red curve). In the following step, we plot the decentralized and socially optimal values for the promotion timing against the bargaining power of the junior worker for fixed bargaining power of senior workers. The result is shown in figure 13 which plots the cases  $\beta_2 = 0.35$ ,  $\beta_2 = 0.38$   $\beta_2 = 0.4$  (top row, left to right) and  $\beta_2 = 0.5$ ,  $\beta_2 = 0.7$   $\beta_2 = 0.9$  (bottom row, left to right). The benchmark case:  $\beta_2 = 0.5$  is depicted in the left panel, bottom row of the figure. We see that there is no value of  $\beta_1$  for which the social planner's promotion timing coincides with the decentralized equilibrium if  $\beta_2 = 0.5$ . Further,  $\bar{x}$  is decreasing in the bargaining power of junior workers  $\beta_1$ , while  $\bar{x}^S$  is increasing in it for low to middling values of  $\beta_2$  (top row of the figure). Since, the social planner maximizes the total wage bill together with the output of workers in simple jobs, it follows that if the share of output that workers retain at a certain hierarchical level increases, welfare can be improved if more workers are employed in those jobs. So here, since  $\beta_2$  is fixed, increasing  $\beta_1$  leads to later promotions. Firms have, however, the opposite response, such that  $\bar{x}$  decreases in  $\beta_1$ . This is the case because for higher  $\beta_1$  it becomes less profitable to retain a worker at the junior level so firms choose earlier promotion timing.

Furthermore, increasing  $\beta_2$  we see that the  $\bar{x}$  curve shifts outwards. If senior workers receive a larger share of the match output, firms optimally slow down promotions for all plausible values of  $\beta_1$ . Qualitatively, the socially optimal promotion timing still increases in  $\beta_1$  for lower values of  $\beta_2$  (see top row of figure 13). The intercept and the slope of  $\bar{x}^S$ , however, vary greatly. For  $\beta_2 = 0.38$ , for instance, immediate promotions are socially optimal for low values of  $\beta_1$  (middle panel, top row). The intuition behind is similar to above: since both  $\beta_1$  and  $\beta_2$  are relatively low, welfare is maximized by setting immediate promotions and employing more workers in senior jobs. As  $\beta_1$  increases, however, the  $\bar{x}^S$  curve begins to increase steeply. The numerical



Figure 13: Comparative statics of decentralized and socially optimal promotion cutoffs  $\bar{x}$ ,  $\bar{x}^S$  with respect to  $\beta_1$  for fixed  $\beta_2$ . Top row: left panel:  $\beta_2 = 0.35$ , middle panel:  $\beta_2 = 0.38$ , right panel:  $\beta_2 = 0.4$ .Bottom row: left panel:  $\beta_2 = 0.5$ , middle panel:  $\beta_2 = 0.7$ , right panel:  $\beta_2 = 0.9$ 

simulations show that there exists a pair  $(\beta_1, \beta_2)$  such that  $\beta_1 > \alpha_1$  and  $\beta_2 < \alpha_2$  for which the decentralized and the socially optimal equilibrium coincide. Moreover, the pair is not unique. For instance, at  $(\beta_1, \beta_2) = (\approx 0.82, 0.35)$  and  $(\beta_1, \beta_2) = (\approx 0.92, 0.38)$ , depicted in the left and middle top row panels of figure 13, the socially optimal and decentralized equilibrium coincide. In the first case:  $(\beta_1, \beta_2) = (\approx 0.82, 0.35)$ , we have  $\bar{x} = \bar{x}^S \approx 26.5$ , while for  $(\beta_1, \beta_2) = (\approx 0.92, 0.38)$  it follows:  $\bar{x} = \bar{x}^S \approx 28$ . Out of those two, however, only the equilibrium corresponding to the pair  $(\beta_1, \beta_2) = (\approx 0.82, 0.35)$  is stable. The optimal response functions for the two cases are displayed in figure 22 in Appendix B.

For higher values of  $\beta_2$  (middle and right panel of bottom row of figure 13) the promotion timing that firms choose is still decreasing in  $\beta_1$ . The socially efficient promotion cutoff, on the other hand, exhibits a non-monotone relationship with the bargaining power of junior workers, such that it is increasing at first and starts to decline for higher values of  $\beta_1$  (bottom row, middle panel). Furthermore, the intercept of the curve increases with higher  $\beta_2$  such that immediate promotion is not efficient for any value of  $\beta_1$ . The reason is similar to the one discussed above for the case that workers on both hierarchical levels have the same bargaining power. Further, when both  $\beta_1$  and  $\beta_2$  are high,  $\bar{x}^S$  declines in  $\beta_1$  since the number of active firms approaches 0. In the case  $\beta_2 = 0.9$  we see that the slope of  $\partial \bar{x}^S / \partial \beta_1$  is negative over the whole range where the socially optimal and decentralized equilibrium could be computed. Very few firms are active in the market if  $\beta_2 = 0.9$ , so increasing  $\beta_1$  leads to decreasing promotion timing as the social planner maximizes the wage bill given the very few employment opportunities. However, this indicates that if  $\beta_2 \geq \alpha_2$ , the socially optimal and decentralized equilibrium never coincide.



Figure 14: Left panel: optimal sharing rule under fixed  $\bar{x} = 40$  and  $y_0 = d_1 + c_1$ . Maximum is achieved at  $\phi = 0.678$  Right panel: optimal sharing rule under fixed  $\bar{x} = 40$  and  $y_0 = 0$ . Maximum is achieved at  $\phi = 0.619$ 

The above discussion highlights that the share of output accruing to senior workers is above the value needed so that the socially optimal and decentralized equilibrium coincide, while the opposite is true for the share of output earned by junior workers. This contributes to firms' incentives to delay promotions inefficiently long which leads to under-entry of firms and a stock of  $e_0$  workers which is above the socially efficient level. This reveals that firms are not adequately compensated for creating the high productivity  $e_2$  jobs and therefore firm entry is biased downwards. High wages in the senior market suppress firm creation which implies that the optimal bargaining power in that market has to be below the traditional Hosios value. Moreover, the adverse effect of strategic complementary can be neutralized if the bargaining power of workers in the junior market is set above the Hosios value as to deter firms from delaying promotion inefficiently long. Under those two conditions we can find multiple equilibria for which the decentralized equilibrium is also constrained efficient. The effects on workers in different hierarchical levels, however, are diverse. If we consider the case  $(\beta_1, \beta_2) = (\approx 0.82, 0.35)$ , then the resulting vector of transition rates is:  $\{\lambda_1 = 0.17, \lambda_2 = 0.16, q_1 = 0.03, q_2 = 0.03\}$ . Comparing these values to the ones in table 5 implies that workers in simple jobs gain from having a higher job-finding rate compared to the decentralized equilibrium in the case  $(\beta_1, \beta_2) = (0.5, 0.5)$ . Workers who are searching for a senior job are in a less favourable position since their jobfinding rate has decreased slightly. Finally, the wage gain associated with being promoted, either internally or via changing firms is smaller due to the fact that  $\beta_1$  has increased and  $\beta_2$  is lower.

#### 4.4 Optimal sharing rule

In the final step we relax the assumption that the social planner is constrained by the bargaining power parameter. To do so, it is assumed that the planner can choose the fraction of output that accrues to workers:  $\phi$ . First, we fix the promotion timing to its value under the decentralized



Figure 15: Objective function of the social planner in the space  $\{\bar{x}, \phi\}$ . Maximum is achieved at  $\{\bar{x}^S, \phi^*\} = \{19.728, 0.706\}.$ 

equilibrium, assuming that it remains firm's choice. Then the optimization problem becomes:

$$\max_{\phi,n} \mathbf{W}(\phi) = \phi \int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{10}(x) dx + \phi \int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{11}^N(x) dx + \phi (d_1 + c_1 e^{\gamma \bar{x}}) d_{11}^S + \phi (d_2 + c_2 e^{\gamma \bar{x}}) (d_{01} + d_{11}^N + d_{11}^S) + y_0 e_0$$

subject to the matching frictions and the free-entry condition:  $\{\dot{d}_{00}, \dot{d}_{10}, \dot{d}_{01}, \dot{d}_{11}^N, \dot{d}_{11}^S, \dot{e}_0\}$  and  $J_{00} = K$ . The left panel of figure 14 shows the objective function of the social planner for varying values of  $\phi$  and fixed  $\bar{x} = 40$ . The maximal value is achieved at  $\phi^* \approx 0.678$ . Under a fixed promotion requirement, altering the sharing rule has several effects. Wages increase as  $\phi$  increases but also the number of entering firms and consequently the vacancy-filling and job-finding rates adjust. We see that here the social planner chooses to allocate a larger fraction of the output to the workers. Hence, the direct increase in the wage bill outweighs the loss from lower firm entry. However, it is not optimal to choose values of  $\phi$  close to 1 because the negative effect associated with the decreasing stock of firms as  $\phi$  increases becomes too large. At  $\phi^* \approx 0.678$ , there are  $nF(\phi^*) = 0.404$  firms, compared to  $nF(\phi = 0.5) = 0.622$  under the decentralized equilibrium.

Further, since under the optimal sharing rule the stock of  $e_0$  workers increases due to the smaller number of firms, we check how much the assumption that  $y_0 = d_1 + c_1$  contributes to the outcome and set  $y_0 = 0$ . The result, displayed in the right panel of figure 14, shows that setting  $y_0 = 0$  reduces the optimal value of  $\phi$  slightly to  $\phi^*|_{y_0=0} \approx 0.619$ , since the social planner puts a higher weight on minimizing the stock of  $e_0$ . However, qualitatively it remains true also under this assumption that the optimal  $\phi$  is such that the bigger fraction of output accrues to workers.

Finally, the case when the social planner sets the sharing rule  $\phi$  and promotion rule  $\bar{x}$ 

simultaneously is characterized. The problem is then:

$$\max_{\phi,\bar{x},n} W(\bar{x},\phi) = \phi \int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{10}(x) dx + \phi \int_0^{\bar{x}} (d_1 + c_1 e^{\gamma x}) d_{11}^N(x) dx + \phi (d_1 + c_1 e^{\gamma \bar{x}}) d_{11}^S + \phi (d_2 + c_2 e^{\gamma \bar{x}}) (d_{01} + d_{11}^N + d_{11}^S) + y_0 e_0$$

again subject to the matching frictions and the free-entry condition. The optimal pair of promotion timing and output sharing rule is given by:  $\{\bar{x}^S, \phi^*\} = \{19.728, 0.706\}$  and is depicted in figure 15. As expected, the social planner chooses a lower promotion requirement compared to the decentralized equilibrium so as to reduce the allocative inefficiency in the economy. Hence, the stock of firms associated with the optimal pair increases compared to the case where promotion timing was kept as firm's private decision to  $nF(\bar{x}^S, \phi^*) = 0.46$ . Similarly to the case of fixed promotion timing, also here choosing a sharing rule is such that  $\phi^* > 0.5$  is optimal.

In the previous sections it was shown that if the bargaining power of workers is relatively low, then many firms enter the market. High firm competition then implies that filling open vacancies is relatively difficult and welfare is maximized by setting immediate or very fast promotions and filling the high productivity senior jobs. However, if workers' bargaining power is relatively high, then welfare is maximized by letting workers accumulate more human capital and thus increasing average match output. Here, the results suggest that comparing the different types of allocations, the highest welfare can be achieved when workers accumulate more human capital, retain the bigger share of the match output, but the firm stock is lower compared to the cases with very high firm entry and immediate promotions.

## 5 Conclusion

This paper investigates the efficiency of the promotion choices of competing firms in a frictional labour market. Once a new firm enters the market it chooses the human capital level at which it will promote a worker from the junior to the senior level such as to maximize present value of profits. From the discussion presented in the introduction and the results of the model it is evident that different model specifications imply various possible externalities which distort human capital accumulation. Which of these are empirically relevant is still an open question. Viewing human capital accumulation from the lens of strategic interaction between firms rather than between the firm and its workers gives a different perspective to the question of over- or under-accumulation of human capital. In this paper, it has been shown that competing firms have incentive to set promotion requirements that are too high resulting in a population of workers who are inadequately distributed across hierarchical levels. The allocative inefficiency in turn suppresses overall output and welfare.

Human capital accumulation at a specific hierarchical level is one component which leads to productivity improvements. One has to also acknowledge the role of firm and worker heterogeneity, or for example, match quality. The model abstracts from those aspects of the relationship of human capital accumulation and the efficient market outcome. However, even with these limitations, it provides insight on externalities which deliver an inefficient outcome. Assuming fixed firm entry, it has been shown that the main reason for the inefficiency of the decentralized equilibrium is the strategic complementarity of firms promotion choices. Firms end up in a symmetric Nash equilibrium which is characterized by a promotion threshold that is too high compared to a socially efficient benchmark. Even though general equilibrium effects mitigate firm's incentive to delay promotions even further, the analysis shows that the negative externalities caused by the strategic complementarity of firms' promotion choices are not internalized. Comparative statics further reveal that there is no value of workers' bargaining power that can induce firms to choose the socially optimal promotion cutoff. However, welfare improvements can be achieved if workers on the senior level have a lower bargaining power while workers on the junior level have a higher bargaining power compared to the standard Hosios value.

Secondly, the model has been extended to implement a free-entry condition. As it was hinted in the fixed firm entry scenario, it has been shown that the Hosios conditions do not deliver an efficient market entry. Specifically, wages in the senior hierarchical layer are too high which suppresses profits and leads to under-entry of firms. The numerical results reveal that reducing the bargaining power of senior workers below the standard Hosios value while simultaneously increasing the bargaining power of junior workers above that value can lead to firm entry which ensures that the decentralized equilibrium is also efficient. This is the case because firms are then compensated for creating the high productivity senior jobs while their incentive to keep workers on the junior level inefficiently long is reduced. Moreover, there are multiple constrained efficient decentralized equilibria arising from different pairs of the bargaining power parameters.

Finally, allowing for simultaneous setting of the promotion timing and the output sharing rule shows that the welfare maximizing pair is such that workers retain a larger share of the output and the promotion threshold is lower than the one chosen by firms under the decentralized equilbrium but higher than the one found when the Hosios condition is satisfied. Further insight into the efficiency of firms' promotion choices can be gained by extending the analysis in several possible directions. Endogenizing the wage bargaining mechanism is one of them. Considering worker or firm heterogeneity would be a second.

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## 6 Appendix A: Calculations for the decentralized equilibrium

First, we define  $\Delta J(x, \bar{x}) = J_{11}^N(x, \bar{x}) - J_{10}(x)$ . Hence,

$$(r+2\rho+q_2)\Delta J(x,\bar{x}) = \pi_2(\bar{x}) + s + \rho J_{01}(\bar{x}) + \frac{\partial \Delta J(x,\bar{x})}{\partial x}$$

Denoting with K the constant of integration, the solution to this first order linear differential equation is:

$$\Delta J(x,\bar{x}) = \frac{\pi_2(\bar{x}) + s + \rho J_{01}(\bar{x})}{r + 2\rho + q_2} + K e^{(r+2\rho+q_2)x}$$
(16)

Substituting (16) into (9), we get:

$$(r+\rho)J_{10}(x) = \pi_1(x) - s + q_2 \frac{(\pi_2(\bar{x}) + s + \rho J_{01}(\bar{x}))}{r + 2\rho + q_2} + q_2 K e^{(r+2\rho+q_2)x} + \frac{\partial J_{10}(x)}{\partial x}$$
(17)

The solution is given by:

$$J_{10}(x) = \frac{d_1(1-\beta)-s}{r+\rho} + \frac{c_1(1-\beta)e^{\gamma x}}{r+\rho-\gamma} - \frac{q_2Ke^{(r+2\rho+q_2)x}}{\rho+q_2} + q_2\frac{(\pi_2(\bar{x})+s+\rho J_{01}(\bar{x}))}{(r+\rho)(r+2\rho+q_2)} + Ae^{(r+\rho)x}$$

where A is the constant of integration. Since,  $J_{11}^N(x, \bar{x}) = \Delta J(x, \bar{x}) + J_{10}(x)$ ,  $J_{11}^N(x, \bar{x})$  is given by the following equation:

$$J_{11}^{N}(x,\bar{x}) = \frac{(r+\rho+q_2)(\pi_2(\bar{x})+s+\rho J_{01}(\bar{x}))}{(r+\rho)(r+2\rho+q_2)} + \frac{\rho K e^{(r+2\rho+q_2)x}}{\rho+q_2} + \frac{d_1(1-\beta)-s}{r+\rho} + \frac{c_1(1-\beta)e^{\gamma x}}{r+\rho-\gamma} + A e^{(r+\rho)x}$$

Next, inserting  $J_{10}(x)$  into (10), we have:

$$(r+2\rho)J_{11}^{N}(x,y) = \pi_{1}(x) + \pi_{2}(y) + \rho J_{01}(y) + \frac{\partial J_{11}^{N}(x,y)}{\partial x} + \rho \frac{d_{1}(1-\beta) - s}{r+\rho} + \frac{\rho c_{1}(1-\beta)e^{\gamma x}}{r+\rho-\gamma} - \frac{\rho q_{2}Ke^{(r+2\rho+q_{2})x}}{\rho+q_{2}} + q_{2}\rho \frac{\pi_{2}(\bar{x}) + s + \rho J_{01}(\bar{x})}{(r+\rho)(r+2\rho+q_{2})} + \rho Ae^{(r+\rho)x}$$

Denoting with D the constant of integration, the general solution is given by:

$$J_{11}^{N}(x,y) = \frac{\pi_{2}(y) + \rho J_{01}(y)}{r + 2\rho} + \frac{d_{1}(1-\beta)}{r+\rho} - \frac{s\rho}{(r+\rho)(r+2\rho+q_{2})} + \frac{c_{1}(1-\beta)e^{\gamma x}}{r+\rho-\gamma} + \frac{\rho K}{\rho+q_{2}}e^{(r+2\rho+q_{2})x} + Ae^{(r+\rho)x} + \frac{\rho q_{2}}{(r+\rho)(r+2\rho)}\left(\frac{\pi_{2}(\bar{x}) + \rho J_{01}(\bar{x})}{r+2\rho+q_{2}}\right) + De^{(r+2\rho)x}$$

This equation, evaluated at  $y = \bar{x}$  should return  $J_{11}^N(x, \bar{x})$ . We find that D = 0, because:

$$\frac{(\pi_2(\bar{x}) + \rho J_{01}(\bar{x}))}{r + 2\rho} + \frac{\rho q_2(\pi_2(\bar{x}) + \rho J_{01}(\bar{x}))}{(r + \rho)(r + 2\rho)(r + 2\rho + q_2)} = \frac{(\pi_2(\bar{x}) + \rho J_{01}(\bar{x}))(r + \rho + q_2)}{(r + \rho)(r + 2\rho + q_2)}$$

The present value of a  $d_{01}$  firm with a manager with experience y is given as:

$$rJ_{01}(y) = \pi_2(y) - \rho J_{01}(y) - s + q_1(J_{11}^N(0, y) - J_{01}(y))$$

which can be written as:

$$J_{01}(y) = \frac{\pi_2(y) - s + q_1 J_{11}^N(0, y)}{r + \rho + q_1}$$
(18)

Next, in order to find an expression for  $J_{01}(\bar{x})$  we first evaluate  $J_{11}^N(x,y)$  at x=0 and  $y=\bar{x}$ :

$$J_{11}^N(0,\bar{x}) = \frac{(\pi_2(\bar{x}) + \rho J_{01}(\bar{x}))(r+\rho+q_2)}{(r+\rho)(r+2\rho+q_2)} + \Theta$$

where  $\Theta = \frac{d_1(1-\beta)}{r+\rho} - \frac{s\rho}{(r+\rho)(r+2\rho+q_2)} + \frac{c_1(1-\beta)}{r+\rho-\gamma} + \frac{\rho K}{\rho+q_2} + A$ . Hence, substituting this into (18) and solving for  $J_{01}(\bar{x})$ , we find:

$$J_{01}(\bar{x}) = \frac{\left[(\pi_2(\bar{x}) - s)(r+\rho)(r+2\rho+q_2) + q_1(r+\rho+q_2)\pi_2(\bar{x}) + q_1(r+\rho)(r+2\rho+q_2)\Theta\right]}{(r+\rho)(r+\rho+q_1)(r+2\rho+q_2) - q_1\rho(r+\rho+q_2)}$$

On the other hand, in order to find  $J_{01}(y)$ , we need to evaluate  $J_{11}^N(x,y)$  at x = 0, which gives:

$$J_{11}^N(0,y) = \frac{\pi_2(y) + \rho J_{01}(y)}{r + 2\rho} + \Theta + \frac{\rho q_2}{(r+\rho)(r+2\rho)} \left(\frac{\pi_2(\bar{x}) + \rho J_{01}(\bar{x})}{r+2\rho+q_2}\right)$$
(19)

Plugging this in (18) and solving for  $J_{01}(y)$ , we get:

$$J_{01}(y) = \frac{(\pi_2(y) - s)(r + 2\rho) + q_1\pi_2(y)}{(r + \rho)(r + 2\rho + q_1)} + \frac{q_1(r + 2\rho)\Theta}{(r + \rho)(r + 2\rho + q_1)} + \frac{\rho q_1 q_2(\pi_2(\bar{x}) + \rho J_{01}(\bar{x}))}{(r + \rho)(r + \rho)(r + 2\rho + q_1)(r + 2\rho + q_2)}$$

Next, the present value of a  $d_{11}^S$  firm:  $rJ_{11}^S$  can be written as:

$$rJ_{11}^S(x_i, y) = \pi_1(x_i) + \pi_2(y) - \rho(J_{11}^S(x_i, y) - J_{01}(x_i)) - (\rho + \lambda_2)(J_{11}^S(x_i, y) - J_{01}(y))$$

Evaluating for  $y = \bar{x}$ , this gives us:

$$J_{11}^S(x_i, \bar{x}) = \frac{\pi_1(x_i) + \pi_2(\bar{x}) + \rho J_{01}(x_i) + (\rho + \lambda_2) J_{01}(\bar{x})}{r + 2\rho + \lambda_2}$$

The first boundary condition  $J_{10}(x_i) = J_{01}(x_i)$  is given as:

$$J_{10}(x_i) = \frac{d_1(1-\beta)-s}{r+\rho} + \frac{c_1(1-\beta)e^{\gamma x_i}}{r+\rho-\gamma} - \frac{q_2Ke^{(r+2\rho+q_2)x_i}}{\rho+q_2} + q_2\frac{(\pi_2(\bar{x})+s+\rho J_{01}(\bar{x}))}{(r+\rho)(r+2\rho+q_2)} + Ae^{(r+\rho)x_i} = \frac{\pi_2(x_i)-s+q_1J_{11}^N(0,x_i)}{r+\rho+q_1} = J_{01}(x_i)$$

Where  $J_{11}^N(0, x_i)$  is equivalent to (19) for  $x_i = y$ .

The second boundary condition  $J_{11}^N(x_i, \bar{x}) = J_{11}^S(x_i, \bar{x})$  can be written as:

$$J_{11}^{N}(x_{i},\bar{x}) = \frac{(r+\rho+q_{2})(\pi_{2}(\bar{x})+s+\rho J_{01}(\bar{x}))}{(r+\rho)(r+2\rho+q_{2})} + \frac{\rho K e^{(r+2\rho+q_{2})x_{i}}}{\rho+q_{2}} + \frac{d_{1}(1-\beta)-s}{r+\rho} + \frac{c_{1}(1-\beta)e^{x_{i}}}{r+\rho-\gamma} + A e^{(r+\rho)x_{i}} = \frac{\pi_{1}(x_{i})+\pi_{2}(\bar{x})}{r+2\rho+\lambda_{2}} + \frac{\rho(\pi_{2}(x_{i})-s+q_{1}J_{11}^{N}(0,x_{i}))}{(r+2\rho+\lambda_{2})(r+\rho+q_{1})} + \frac{(\rho+\lambda_{2})(\pi_{2}(\bar{x})-s+q_{1}J_{11}^{N}(0,\bar{x}))}{(r+2\rho+\lambda_{2})(r+\rho+q_{1})} = J_{11}^{S}(x_{i},\bar{x})$$

## 7 Appendix B: Additional figures



Figure 16: Left panel: Objective function of firm i and the optimal choice  $\bar{x}_i(\bar{x})$  for a fixed market promotion cutoff  $\bar{x} = 43.55$  and fixed transition rates. Right panel: Optimal response function  $\bar{x}_i(\bar{x})$  for different values of  $\bar{x}$  and constant transition rates.



Figure 17: Left panel: Number of firms for fixed firm entry:  $n \approx 0.004$ . Right panel: Number of firms with free-entry.



Figure 18: Comparative statics: equilibrium transition rates as functions of the entry cost K.



Figure 19: Left panel: Job-finding rates as a function of workers' bargaining power  $\beta$ . Right panel: Job-filling rates as a function of workers' bargaining power  $\beta$ .



Figure 20: Comparative statics with respect to promotion timing  $\bar{x}$  for  $\beta = 0.35$  and free-entry. Left panel: overall welfare. Middle panel: stock of firms. Right panel: junior vacancy-filling rate (blue curve) and senior vacancy-filling rate (red curve).



Figure 21: Comparative statics with respect to promotion timing  $\bar{x}$  for  $\beta = 0.75$  and free-entry. Left panel: overall welfare. Middle panel: stock of firms. Right panel: junior vacancy-filling rate (blue curve) and senior vacancy-filling rate (red curve).



Figure 22: Left panel: best response function in the case  $(\beta_1, \beta_2) = (\approx 0.82, 0.35)$ .  $\bar{x} = \bar{x}^S \approx 26.5$  is a stable equilibrium. Right panel: best response function in the case  $(\beta_1, \beta_2) = (\approx 0.92, 0.38)$ .  $\bar{x} = \bar{x}^S \approx 28$  is *not* a stable equilibrium.