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Coalition Formation with Border Carbon Adjustment*

Mark Schopf[†]

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Abstract The present paper analyzes the impact of a climate coalition's border carbon adjustment on emissions from commodity production, welfare and the coalition size. The coalition implements border carbon adjustment to reduce carbon leakage and to improve its terms of trade, while the fringe abstains from any trade policy. With symmetric countries, the optimal import tax or export subsidy is positive but smaller than the coalition's implicit emission price.

With a linear-quadratic specification, the coalition exports the commodity. Total emissions decrease with the coalition size, and total welfare increases [decreases] with the coalition size if the coalition is large [small]. Then, the reduced climate costs outweigh [are outweighed by] the increased trade distortions. The unique stable coalition consists of three or more countries, including the grand coalition, and raises the welfare of each country compared to the business-as-usual equilibrium.

If no [each] country implements a trade policy, the stable coalition consists of two [three] or less countries. Compared to the case in which only the coalition implements border carbon adjustment, the welfare of each country is reduced [if the stable coalition then consists of four or more countries]. All results are derived analytically.

Keywords Carbon Leakage · Climate Change · Environmental Policy · Nash Equilibrium · Terms of Trade

JEL Classification F13 · F18 · H23 · Q54 · Q56 · Q58

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1 Introduction

“But there is no point in only reducing greenhouse gas emissions at home, if we increase the import of CO₂ from abroad. It is not only a climate issue; it is also an issue of fairness. It is a matter of fairness towards our businesses and our workers. We will protect them from unfair competition. One way for doing so is the Carbon Border Adjustment Mechanism.” (von der Leyen 2020)

The global climate problem is one of the biggest challenges of our time. Down to the present day, its public good character has prevented the existence of an effective global climate agreement (UNEP 2019, p. 27). In the absence of such an agreement, a coordinated climate policy by a subgroup of countries can be second best. However, theoretical analyses typically find that these climate coalitions are small and ineffective. By contrast, numerical analyses show that border carbon adjustment can be used to create large and effective climate coalitions (Nordhaus 2015, Böhringer et al. 2016). This paper attempts to fill the gap between the theoretical and the numerical results by applying a tractable analytical model of coalition formation with border carbon adjustment.

Eichner & Pethig (2013) extend the basic model of the literature on international environmental agreements (Barrett 1994) by introducing international trade. Eichner & Pethig (2015*a*) modify the game structure from Stackelberg to Nash, and Eichner & Pethig (2015*b*) change the policy instrument from consumption caps to consumption taxes. Without international trade, the climate coalition always consists of four or less countries.¹ With international trade, it is either small (Nash) or ineffective (Stackelberg) in the case of consumption caps, and it either consists of three or all countries in the case of consumption taxes. These results are in contrast to the observation of relatively large or effective climate coalitions like the Paris Agreement or the Kyoto Protocol.

One reason for the small or ineffective climate coalitions in the basic trade model could be the non-consideration of trade or supply-side policies. Eichner & Pethig (2014) extend the basic trade model by considering consumption caps and production taxes. Indeed, they find a relatively large (5 out of 10 countries) and effective climate coalition. Kuhn

¹These results were derived analytically by Carraro & Siniscalco (1991, Proposition 8) for the Nash equilibrium, and by Diamantoudi & Sartzetakis (2006, Proposition 3) for the Stackelberg equilibrium with non-negative emissions.

et al. (2019) modify this model by introducing multiple goods and preferential free trade agreements, which make the climate coalition larger (7 out of 10 countries) and more effective.² Al Khourdajie & Finus (2020) develop an intra-industry trade model in which all countries use production taxes and the coalition countries use import taxes to equalize domestic producer prices for signatory and non-signatory firms. This border carbon adjustment increases the coalition size from 1-3 to 6-10 out of 10 countries. While these results suggest that the consideration of trade or supply-side policies leads to relatively large and effective climate coalitions, they rely on numerical examples. Thus, it is neither clear whether they persist with, e.g., more than ten countries nor what their underlying mechanisms are. To the best of our knowledge, only Helm & Schmidt (2015) derive analytical results on coalition formation with border carbon adjustment. They find that the unique stable coalition consists of all countries for sufficiently large export shares.³ However, their focus is on technology investments, such that they use exogenous policies for the non-signatories, assume inelastic demands and abstract from terms-of-trade effects to keep the analysis tractable.⁴

We fill these research gaps by applying an analytically tractable model with endogenous policies, elastic demands and terms-of-trade effects to derive the underlying mechanisms of coalition formation with border carbon adjustment. Our model is close to that of Eichner & Pethig (2014), but differs by the game structure (Nash instead of Stackelberg) and by the strategic behavior of the countries.⁵ In particular, we assume that the coalition accounts for its influence on the commodity price, while the fringe takes the commodity

²A sensitivity analysis conducted in Zenker (2020, Appendix B.5) reveals that the coalition size decreases with the number of countries and is smaller than or equal to five with eleven or more countries.

³Ceteris paribus, larger export shares imply higher import tax revenues and lower abatement efforts for the signatories and higher abatement efforts for the non-signatories.

⁴Furthermore, Helm & Schmidt (2015) implicitly assume that the production factors are perfectly mobile. Thus, the coalition countries must use import taxes and export subsidies that equalize producer prices for signatory and non-signatory firms in order to prevent the relocation of all signatory firms.

⁵Further differences are that we assume emissions from production instead of emissions from consumption, and that we choose production and import taxes as policy instruments. Since the general structure is the same, our model can be extended to a general equilibrium model without changing the results, and it can be microfounded as in Eichner & Pethig (2013, Footnote 10).

price as given. Then, the coalition implements border carbon adjustment to reduce carbon leakage and to improve its terms of trade, while the fringe abstains from any trade policy. The two main results of this paper are: First, large and effective climate coalitions are possible with border carbon adjustment, and they are fostered by low marginal production costs and consumption benefits. Second, the coalition's border carbon adjustment typically reduces emissions and increases welfare compared to the cases in which no or each country accounts for its influence on the commodity price. Thus, there should be a general interest to allow border carbon adjustment for climate clubs and to forbid any trade policy for non-participants.

The remainder of the paper is organized as follows: Section 2 outlines the model and characterizes the grand-coalition allocation and the business-as-usual allocation. Section 3 investigates the case in which the coalition acts strategically and the fringe does not. First it characterizes the fringe's and the coalition's optimal policy as well as the allocation and welfare for a coalition of a given size, then it derives the equilibrium coalition size. Section 4 does the same for the cases in which either no or each country acts strategically, and compares the results to those of the previous section. Section 5 concludes.

2 The model

In what follows, we apply Hoel's (1994) partial equilibrium framework extended by coalition formation.⁶ Consider a trade model with n countries. In each country $i \in N$, the respective representative household derives benefits $B_i(y_i)$ from commodity consumption y_i , where $B'_i > 0$ and $B''_i < 0$, and the respective representative firm faces costs $C_i(x_i)$ of commodity production x_i , where $C'_i, C''_i > 0$. Emissions are proportional to commodity production, and i faces climate costs $H_i(x)$ from global emissions $x := \sum_k x_k$, where $H'_i > 0$ and $H''_i \geq 0$.

In each country, there are three possible policy instruments, namely a consumption tax γ_i , a production tax χ_i and a uniform import tax τ_i .⁷ The respective tax revenue is returned to the respective representative household and to the respective representative

⁶Harstad (2012, Section 2) shows that this model can be extended to a general equilibrium model without changing the results.

⁷If i imports [exports] the commodity, then $\tau_i > 0$ is an import tax [export subsidy] and $\tau_i < 0$ is an import subsidy [export tax].

firm as a lump-sum transfer

$$T_i = \gamma_i y_i + \chi_i x_i + \tau_i \cdot (y_i - x_i) = (\gamma_i + \tau_i)y_i + (\chi_i - \tau_i)x_i = T_i^y + T_i^x. \quad (1)$$

The representative households and firms take the international commodity price p , the taxes and the lump-sum transfers as given and do not consider the climate costs. Their consumption and production decisions follow from

$$\max_{y_i} B_i(y_i) - p y_i - (\gamma_i + \tau_i)y_i + T_i^y \quad \text{and} \quad \max_{x_i} p x_i - C_i(x_i) - (\chi_i - \tau_i)x_i + T_i^x, \quad (2)$$

which implies

$$B_i'(y_i) - (\gamma_i + \tau_i) = C_i'(x_i) + (\chi_i - \tau_i) = p. \quad (3)$$

From (1) and (3), any lump-sum transfer and any commodity allocation just depends on $(\gamma_i + \tau_i)$ and $(\chi_i - \tau_i)$. To put it another way, two taxes are sufficient to control two quantities. We summarize our result in

LEMMA 1. *Any lump-sum transfer and any commodity allocation that is implemented by $\tilde{\gamma}_i$, $\tilde{\chi}_i$ and $\tilde{\tau}_i$ can also be implemented by any combination $(\gamma_i + \tau_i) = (\tilde{\gamma}_i + \tilde{\tau}_i)$ and $(\chi_i - \tau_i) = (\tilde{\chi}_i - \tilde{\tau}_i)$. In particular, it can be implemented by $\gamma_i = \tilde{\gamma}_i + \tilde{\tau}_i$, $\chi_i = \tilde{\chi}_i - \tilde{\tau}_i$, $\tau_i = 0$ or $\gamma_i = \tilde{\gamma}_i + \tilde{\chi}_i$, $\chi_i = 0$, $\tau_i = -(\tilde{\chi}_i - \tilde{\tau}_i)$ or $\gamma_i = 0$, $\chi_i = \tilde{\gamma}_i + \tilde{\chi}_i$, $\tau_i = \tilde{\gamma}_i + \tilde{\tau}_i$.*

In what follows, we make use of this result and set the consumption taxes equal to zero. However, note that any equilibrium combination of production and import taxes could be replaced by a combination of consumption and production taxes, such that the WTO rules cannot prevent effective border carbon adjustment.⁸

The respective representative household's commodity demand and the respective representative firm's commodity supply follow from (3) and are given by

$$y_i = B_i'^{-1}(p + \tau_i) =: D_i(p + \tau_i) \quad \text{and} \quad x_i = C_i'^{-1}(p - \chi_i + \tau_i) =: S_i(p - \chi_i + \tau_i). \quad (4)$$

Totally differentiating (3) and (4) and rearranging yields

$$\frac{dy_i}{d(p+\tau_i)} = D_i' = \frac{1}{B_i''} < 0 \quad \text{and} \quad \frac{dx_i}{d(p-\chi_i+\tau_i)} = S_i' = \frac{1}{C_i''} > 0. \quad (5)$$

⁸For example, an import tax that exceeds the difference in the production taxes discriminates between own and foreign commodities and does violate WTO rules (Cosbey et al. 2019), while a production subsidy combined with a consumption tax can implement the same allocation without being likely to violate WTO rules (Trachtman 2017).

Finally, the international commodity market clearing condition reads

$$\sum_k D_k(p + \tau_k) = \sum_k S_k(p + \tau_k - \chi_k). \quad (6)$$

Totally differentiating (6) and rearranging yields

$$\frac{\partial p}{\partial \chi_i} = \frac{S'_i}{S' - D'} \in (0, 1) \quad \text{and} \quad \frac{\partial p}{\partial \tau_i} = -\frac{S'_i - D'_i}{S' - D'} \in (-1, 0), \quad (7)$$

where $D' := \sum_k D'_k$ and $S' := \sum_k S'_k$.

Throughout the paper, we assume that all countries take the taxes in all other countries as given, i.e. we solve for the Nash equilibrium. Furthermore, we assume that fringe countries take the international commodity price as given, whereas coalition countries account for their influence on the international commodity price.⁹ Thus, single countries are small open economies, whereas coalitions of countries become large open economies.

As a first benchmark, the grand-coalition's optimal policy follows from maximizing the sum of all countries' welfare $W := \sum_k W_k$, i.e. their benefits from commodity consumption minus their costs of commodity production minus their climate costs $H := \sum_k H_k$ plus their trade revenues,

$$\begin{aligned} W = & \sum_k B_k[D_k(p + \tau_k)] - \sum_k C_k[S_k(p + \tau_k - \chi_k)] - \sum_k H_k \left[\sum_k S_k(p + \tau_k - \chi_k) \right] \\ & + p \underbrace{\sum_k [S_k(p + \tau_k - \chi_k) - D_k(p + \tau_k)]}_{=0}, \end{aligned} \quad (8)$$

with respect to χ_i and τ_i for all i subject to (6).¹⁰ Solving the maximization problem yields $\chi_i = H'$ and $\tau_i = \tau_j$.¹¹ The production taxes induce the representative firms to account for their global climate externality, and the import taxes are set equal to each other to avoid trade distortions. The corresponding allocation follows from substituting

⁹In Section 4, we then investigate the cases in which no or each country accounts for its influence on the commodity price. In Appendix A.1, we derive the equilibrium policies of all cases with consumption, production and import taxes and show that any equilibrium policy just depends on $(\gamma_i + \tau_i)$ and $(\chi_i - \tau_i)$. Together with Lemma 1, this implies that any equilibrium with two taxes coincides with that with three taxes, such that the import taxes could be replaced by consumption taxes without changing the results.

¹⁰In (8), the tax revenues and the lump-sum transfers cancel out.

¹¹See Lemma A.1 of Appendix A.1.

the taxes into (3). Indicating the grand-coalition equilibrium by an asterisk, we get

$$B'_i(y_i^*) = B'_j(y_j^*), \quad (9a)$$

$$C'_i(x_i^*) = C'_j(x_j^*), \quad (9b)$$

$$B'_i(y_i^*) = C'_i(x_i^*) + H'(x^*), \quad (9c)$$

for all i . Thus, marginal consumption benefits and marginal production costs are the same in all countries, and marginal consumption benefit is equal to marginal global social cost, i.e. marginal production cost plus marginal global climate cost, in each country. It can easily be verified that a social planner would choose the same allocation.

As a second benchmark, the business-as-usual policy follows from maximizing country i 's welfare, i.e. its benefits from commodity consumption minus its costs of commodity production minus its climate costs plus its trade revenues,

$$\begin{aligned} W_i = & B_i[D_i(p + \tau_i)] - C_i[S_i(p + \tau_i - \chi_i)] - H_i \left[\sum_k S_k(p + \tau_k - \chi_k) \right] \\ & + p \cdot [S_i(p + \tau_i - \chi_i) - D_i(p + \tau_i)], \end{aligned} \quad (10)$$

with respect to χ_i and τ_i for all i .¹² Solving the maximization problem yields $\chi_i = H'_i$ and $\tau_i = 0$.¹³ The production tax induces i 's representative firm to account for its climate externality on i , and the import tax is set equal to zero, because i overlooks its influence on the other countries' emissions and on its terms of trade via the commodity price. The corresponding allocation follows from substituting the taxes into (3). Indicating the business-as-usual equilibrium by a circle, we get

$$B'_i(y_i^\circ) = B'_j(y_j^\circ), \quad (11a)$$

$$C'_i(x_i^\circ) = C'_j(x_j^\circ) - H'_i(x^\circ) + H'_j(x^\circ), \quad (11b)$$

$$B'_i(y_i^\circ) = C'_i(x_i^\circ) + H'_i(x^\circ), \quad (11c)$$

for all i . Thus, marginal consumption benefits are the same in all countries, but marginal production costs may differ. Ceteris paribus, x_i° is too high compared to x_j° if $H'_i(x^\circ)$ falls short of $H'_j(x^\circ)$ and vice versa. Furthermore, y_i° is too high compared to y_i^* , because the climate externality is only partially internalized. To see this, suppose $y_i^\circ \leq y_i^*$,

¹²In (10), the tax revenue and the lump-sum transfer cancel out.

¹³See Lemma A.2 of Appendix A.1.

such that $H'_i(x^\circ) \leq H'_i(x^*)$. Then, (9c) and (11c) imply $x_i^\circ > x_i^*$, which contradicts $H'_i(x^\circ) \leq H'_i(x^*)$. With symmetric climate costs, i.e. $H'_i(x^\circ) = H'_j(x^\circ)$, $y_i^\circ > y_i^*$ then implies $x_i^\circ > x_i^*$.

3 The coalition-fringe scenario

3.1 Equilibrium policies

Now suppose that m countries represent the coalition E , whereas the remaining $n - m$ countries represent the fringe F . Then, the fringe's optimal policy is given by $\chi_i = H'_i$ and $\tau_i = 0$ for all $i \in F$, and the coalition's optimal policy follows from maximizing the sum of all coalition countries' welfare, i.e. their benefits from commodity consumption minus their costs of commodity production minus their climate costs plus their trade revenues,

$$\begin{aligned} \sum_{k \in E} W_k &= \sum_{k \in E} B_k[D_k(p + \tau_k)] - \sum_{k \in E} C_k[S_k(p + \tau_k - \chi_k)] - \sum_{k \in E} H_k \left[\sum_k S_k(p + \tau_k - \chi_k) \right] \\ &\quad + p \sum_{k \in E} [S_k(p + \tau_k - \chi_k) - D_k(p + \tau_k)], \end{aligned} \quad (12)$$

with respect to χ_i and τ_i for all $i \in E$ subject to (6).¹⁴ Solving the maximization problem yields $\chi_i = \sum_{k \in E} H'_k$ and $\tau_i = \frac{\sum_{k \in E} H'_k \sum_{k \in F} S'_k}{\sum_{k \in F} [S'_k - D'_k]} + \frac{\sum_{k \in E} (y_k - x_k)}{\sum_{k \in F} [S'_k - D'_k]}$.¹⁵ The production taxes induce the coalition's representative firms to account for their climate externality on the coalition, and the import taxes are set equal to each other to avoid trade distortions within the coalition. Furthermore, they partially induce the fringe's representative firms to account for their climate externality on the coalition, and they improve the coalition's terms of trade. The corresponding allocation follows from substituting the taxes into (3). Indicating this coalition-fringe equilibrium by a check, we get

$$B'_i(\check{y}_i) = B'_j(\check{y}_j) + \check{\tau}_i, \quad (13a)$$

$$C'_i(\check{x}_i) = C'_j(\check{x}_j) + \check{\tau}_i - \sum_{k \in E} H'_k(\check{x}) + H'_j(\check{x}), \quad (13b)$$

$$B'_i(\check{y}_i) = C'_i(\check{x}_i) + \sum_{k \in E} H'_k(\check{x}), \quad (13c)$$

$$B'_j(\check{y}_j) = C'_j(\check{x}_j) + H'_j(\check{x}), \quad (13d)$$

¹⁴In (12), the tax revenue and the lump-sum transfer cancel out.

¹⁵See Lemma A.1 of Appendix A.1.

for all $i \in E$ and $j \in F$, where $\tilde{\tau}_i = \frac{\sum_{k \in E} H'_k(\tilde{x}) \sum_{k \in F} S'_k}{\sum_{k \in F} [S'_k - D'_k]} + \frac{\sum_{k \in E} (\tilde{y}_k - \tilde{x}_k)}{\sum_{k \in F} [S'_k - D'_k]}$. Using (6), we can rewrite the coalition's optimal import tax or export subsidy as

$$\tilde{\tau}_i = \frac{\partial[\sum_{k \in E} H_k(\tilde{x})]}{\partial[\sum_{k \in F} x_k]} \cdot \frac{\partial[\sum_{k \in F} \tilde{x}_k]}{\partial p} \cdot \frac{\partial \tilde{p}}{\partial[\sum_{k \in E} (y_k - x_k)]} + \sum_{k \in E} (\tilde{y}_k - \tilde{x}_k) \cdot \frac{\partial \tilde{p}}{\partial[\sum_{k \in E} (y_k - x_k)]}. \quad (14)$$

The first part accounts for the impact of the coalition's imports via the international commodity price and via the fringe's production on the coalition's climate costs. This carbon-leakage effect is always positive. The second part accounts for the impact of the coalition's imports on its import expenditures or export revenues. This terms-of-trade effect is positive [negative] if the coalition imports [exports] the commodity. Then, it depresses [raises] the commodity price in order to reduce [increase] the coalition's import expenditures [export revenues]. Thus, if the coalition imports the commodity, then it implements an import tax, and if the coalition exports the commodity, then it implements an export subsidy [tax] if the carbon-leakage effect outweighs [is outweighed by] the terms-of-trade effect.

To gain further insights into the sign and the magnitude of the coalition's optimal import tax or export subsidy, we now consider the following symmetric specification

$$B_i(y_i) = B_j(y_j), \quad C_i(x_i) = C_j(x_j), \quad H_i(x) = H_j(x), \quad (15)$$

such that $y_i = y_e, x_i = x_e$ for all $i \in E$ and $y_i = y_f, x_i = x_f$ for all $i \in F$. In Appendix A.2, we prove

PROPOSITION 1. *Consider the symmetric specification (15).*

- *The coalition's optimal import tax or export subsidy is positive, but smaller than its optimal production tax:*

$$- \tilde{\tau}_e \in \left(0, \max \left\{ (m-1)H'_i(\tilde{x}), \frac{mH'_i(\tilde{x})S'_f}{S'_f - D'_f} \right\} \right).$$

- *Compared to the business-as-usual equilibrium, we find:*

$$\begin{aligned} & - \tilde{y}_e < y_i^\circ, \\ & - \tilde{x}_e < x_i^\circ \text{ and } H(\tilde{x}) < H(x^\circ) \text{ for } \tilde{\tau}_e \leq (m-1)H'_i(\tilde{x}), \\ & - \tilde{x}_f < x_i^\circ \text{ and } \tilde{y}_f > y_i^\circ \text{ for } \tilde{\tau}_e \geq (m-1)H'_i(\tilde{x}). \end{aligned}$$

- *Compared to the grand-coalition equilibrium, we find:*

$$\begin{aligned} & - \tilde{y}_f > y_i^*, \\ & - \tilde{x}_f > x_i^* \text{ and } H(\tilde{x}) > H(x^*) \text{ for } \tilde{\tau}_e \leq (m-1)H'_i(\tilde{x}), \\ & - \tilde{x}_e > x_i^* \text{ for } \tilde{\tau}_e \geq (m-1)H'_i(\tilde{x}). \end{aligned}$$

Since the coalition's import tax or export subsidy is positive, each coalition country's consumption is below that of each fringe country, see (13a). If the import tax or export subsidy exceeds the difference between the production taxes, then each coalition country's production is above that of each fringe country, see (13b). Then, the coalition definitely exports the commodity. By contrast, if the import tax falls short of the difference between the production taxes, then each coalition [fringe] country's consumption *and* production are below [above] those in the business-as-usual [grand-coalition] equilibrium. Then, the climate costs are between those in the business-as-usual equilibrium and those in the grand-coalition equilibrium.

3.2 Impact of a climate coalition on the allocation

To derive the impact of a larger coalition on the allocation in general and on global emissions in particular, we now consider the following linear-quadratic specification

$$B_i(y_i) = ay_i - \frac{b}{2}y_i^2, \quad C_i(x_i) = \frac{c}{2}x_i^2, \quad H_i(x) = hx, \quad (16)$$

such that $S'_i = 1/c$, $D'_i = -1/b$ and $H'_i = h$. Then, the equilibrium with arbitrary production and import taxes is characterized by

$$a - by_i - \tau_i = a - by_j - \tau_j \quad \forall i \setminus j \in N, \quad (17a)$$

$$a - by_i = cx_i + \chi_i \quad \forall i \in N, \quad (17b)$$

$$\sum_k y_k = \sum_k x_k. \quad (17c)$$

Solving for y_i and x_i yields

$$y_i = \frac{a - \chi_i}{b + c} + \frac{c}{b + c} \cdot (y_i - x_i), \quad (18a)$$

$$x_i = \frac{a - \chi_i}{b + c} - \frac{b}{b + c} \cdot (y_i - x_i), \quad (18b)$$

$$x = \frac{na - \sum_k \chi_k}{b + c}, \quad (18c)$$

where

$$(y_i - x_i) = -\frac{(b + c)\tau_i - b\chi_i}{bc} + \frac{\sum_k [(b + c)\tau_k - b\chi_k]}{nbc}. \quad (19)$$

This already implies that global emissions do not depend on the import taxes. Furthermore, since $\check{\chi}_e = mh$ and $\check{\chi}_f = h$ hold, global emissions are the lower the greater the

climate coalition.¹⁶ In equilibrium, each coalition country's imports and the coalition's import tax are given by

$$(\check{y}_e - \check{x}_e) = -\frac{(n-m)[(b+c)\check{\tau}_e - (m-1)bh]}{nbc}, \quad (20)$$

$$\check{\tau}_e = \frac{mbh}{b+c} + \frac{mbc(\check{y}_e - \check{x}_e)}{(n-m)(b+c)}. \quad (21)$$

Solving for $(\check{y}_e - \check{x}_e)$ and $\check{\tau}_e$ yields

$$(\check{y}_e - \check{x}_e) = -\frac{(n-m)h}{(n+m)c} < 0, \quad \frac{\partial(\check{y}_e - \check{x}_e)}{\partial m} = \frac{2nh}{(n+m)^2c} > 0, \quad (22)$$

$$\check{\tau}_e = \frac{(n+m-1)mhb}{(n+m)(b+c)} \in \left(0, \frac{mhb}{b+c}\right), \quad \frac{\partial \check{\tau}_e}{\partial m} = \frac{[(n+m)^2 - n]hb}{(n+m)^2(b+c)} \in \left(0, \frac{hb}{b+c}\right). \quad (23)$$

Thus, the climate coalition exports the commodity, and each coalition country's exports are strictly decreasing in m .¹⁷ This implies that the strictly increasing export subsidy, which speaks in favor of increasing exports, is outweighed by the strictly increasing wedge between the production taxes, which speaks in favor of decreasing exports. In Appendix A.2, we prove

PROPOSITION 2. *Consider the linear-quadratic specification (16).*

- *Each coalition country's production ($\check{x}_e > x_i^*$), consumption ($y_i^\circ > \check{y}_e > y_i^*$) and exports are strictly decreasing in m .*
- *Each fringe country's production ($\check{x}_f < x_i^\circ$) is strictly decreasing in m , and its consumption ($\check{y}_f > y_i^\circ > y_i^*$) and imports are strictly increasing in m .*
- *Global emissions are strictly decreasing in m , and total trade is strictly increasing [decreasing] in m for $m < [>](\sqrt{2} - 1)n$.*

The increasing production tax and the increasing export subsidy lead to decreasing production and to decreasing consumption in E , respectively. Each coalition country's exports are decreasing in m , whereas each fringe country's imports are increasing in m due to the increasing export subsidy. These increasing imports lead to decreasing production and to increasing consumption in F .¹⁸ Each country's production is decreasing in m , such that global emissions are also decreasing in m . Total trade is increasing [decreasing]

¹⁶In Appendix A.2, we show that $\frac{d\check{x}}{dm} < 0$ also holds for $H_i'' > 0$.

¹⁷In Appendix A.2, we show that $(\check{y}_e - \check{x}_e) < 0$ and $\frac{d(\check{y}_e - \check{x}_e)}{dm} > 0$ also holds for $H_i'' > 0$.

¹⁸In Appendix A.2, we show that $\frac{d\check{y}_f}{dm} > 0$ also holds for $H_i'' > 0$.

in m if the coalition is small [large]. Then, the additional exports of the new signatories outweigh [are outweighed by] the reduced exports of the existing signatories.

3.3 Impact of a climate coalition on welfare

We already know that a larger coalition implies lower global emissions. What we do not know is whether it implies greater welfare for all countries, and how the welfare of a coalition country relates to that of a fringe country. To answer the latter question, we rewrite (10) as

$$\check{W}_i = \int_0^{\check{y}_i} [B'_i(y_i) - p] dy_i + \int_0^{\check{x}_i} [p - C'_i(x_i)] dx_i - H_i(\check{x}), \quad (24)$$

where the first [second] integral is the consumer [producer] surplus and the last term are the climate costs. By symmetry, the climate costs are the same in all countries. Since consumption and production are not subsidized, i.e. $B'_i(y_i) - p = \check{\tau}_i \geq 0$ for all $y_i \in [0, \check{y}_i]$ and $p - C'_i(x_i) = \check{\chi}_i - \check{\tau}_i > 0$ for all $x_i \in [0, \check{x}_i]$, a fringe country's consumer surplus exceeds that of a coalition country ($\check{y}_f > \check{y}_e$), and a coalition's country welfare can only exceed a fringe country's welfare if its producer surplus exceeds that of a fringe country ($\check{x}_e > \check{x}_f$). This is the case if and only if the export subsidy exceeds the difference in the production taxes ($\check{\chi}_e - \check{\tau}_e < \check{\chi}_f \iff \check{\tau}_e > (m-1)H'_i(\check{x})$).

For the impact of a larger climate coalition on welfare, we differentiate (24) with respect to m and find

$$\frac{d\check{W}_i}{dm} = [B'_i(\check{y}_i) - p] \frac{d\check{y}_i}{dm} + [p - C'_i(\check{x}_i)] \frac{d\check{x}_i}{dm} - (\check{y}_i - \check{x}_i) \frac{dp}{dm} - H'_i(\check{x}) \frac{d\check{x}}{dm}, \quad (25)$$

where $\frac{dp}{dm} = B'_f \frac{d\check{y}_f}{dm} < 0$. Since $\frac{d\check{x}}{dm} < 0$ holds, the climate costs of all countries are declining in m . Furthermore, we know that $\frac{d\check{y}_e}{dm}, \frac{d\check{x}_e}{dm}, \check{y}_e - \check{x}_e < 0$ holds, such that the sum of a coalition country's consumer and producer surplus is declining in m . Substituting (3) into (25) yields

$$\frac{d\check{W}_f}{dm} - \frac{d\check{W}_e}{dm} = \left\{ -\check{y}_f \frac{dp}{dm} - \left[\check{\tau}_e \frac{d\check{y}_e}{dm} - \check{y}_e \frac{dp}{dm} \right] \right\} + \left\{ \check{\chi}_f \frac{d\check{x}_f}{dm} + \check{x}_f \frac{dp}{dm} - \left[(\check{\chi}_e - \check{\tau}_e) \frac{d\check{x}_e}{dm} + \check{x}_e \frac{dp}{dm} \right] \right\}, \quad (26)$$

where the first [second] curly-bracketed term is the development of the difference between the consumer [producer] surpluses. Since the commodity price is declining in m , a fringe country's consumer surplus increases. By contrast, a coalition country's consumer surplus

can decrease, because the export subsidy drives a wedge between the marginal benefit and the commodity price, which reduces the consumer surplus by $\tilde{\tau}_e \frac{d\check{y}_e}{dm}$. Furthermore, $\check{y}_f > \check{y}_e$ implies that the difference between the consumer surpluses is increasing in m . The producer surpluses are declining in m , because the commodity price and each country's production decrease. In Appendix A.2, we prove

PROPOSITION 3. *Consider the linear-quadratic specification (16).*

- *The welfare of a coalition country exceeds that in BAU ($\check{W}_e > W_i^\circ$).*
- *If the welfare of a fringe country exceeds that in BAU ($\check{W}_f > W_i^\circ$), then the welfare of each country and total welfare are strictly increasing in m ($\frac{\partial \check{W}_f}{\partial m}, \frac{\partial \check{W}_e}{\partial m}, \frac{\partial \check{W}}{\partial m} > 0$).*
- *The welfare difference between a fringe country and a coalition country is strictly increasing in m ($\frac{\partial \check{W}_f}{\partial m} > \frac{\partial \check{W}_e}{\partial m}$).*

Although a coalition country's welfare can fall short of a fringe country's welfare, it always exceeds that in the business-as-usual equilibrium.¹⁹ Furthermore, each country's welfare and, thus, the sum of all countries' welfare can in general be declining in m , implying that the reduced sum of the respective consumer and producer surplus exceeds the reduced climate costs. However, a larger coalition implies that the reduced climate costs become more important,²⁰ and it implies that the welfare of a fringe country eventually exceeds that of a coalition country. Then, each country's welfare and, thus, the sum of all countries' welfare is increasing in m .

One important implication of Proposition 3 is that a coalition with $\check{W}_f \leq W_i^\circ$ cannot be externally stable, because joining the coalition always increases welfare to $\check{W}_e > W_i^\circ$. Thus, in any externally stable coalition, each country's welfare exceeds that in the business-as-usual equilibrium, and each country's welfare and total welfare are increasing in the coalition size.

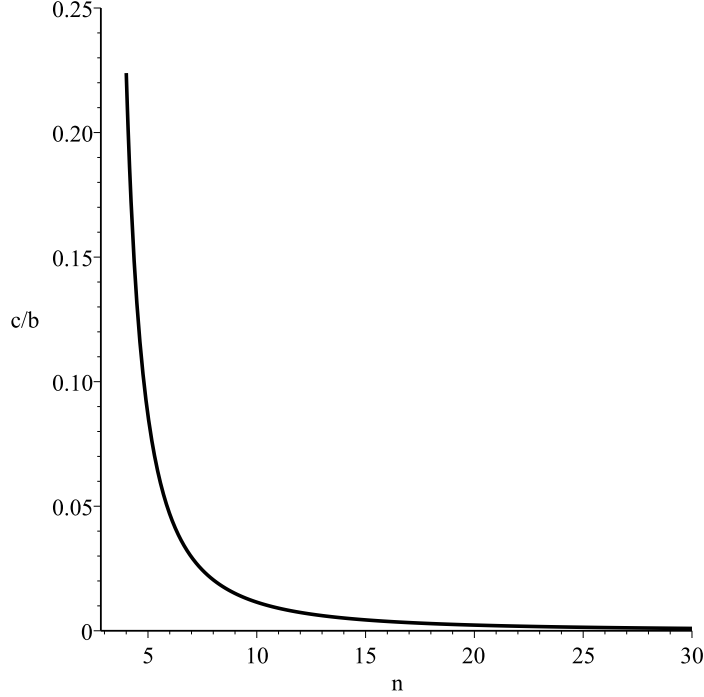


Figure 1: Maximum value of c/b dependent on n for the grand coalition to form.

3.4 Equilibrium coalition size

A coalition of a given size m is internally stable if no coalition country has an incentive to leave the coalition $\Phi(m) = W_e(m) - W_f(m-1) \geq 0$, and it is externally stable if no fringe country has an incentive to join the coalition $\Phi(m+1) = W_e(m+1) - W_f(m) \leq 0$. In Appendix A.2, we show that coalitions with three or less countries are internally stable, and that coalitions with two or less countries are externally unstable, such that some coalition of at least three countries is stable. Furthermore, we derive

$$\Phi(m) \gtrless 0 \iff \tilde{\Phi}(m, c/b, n) \gtrless 0, \quad (27)$$

where

$$\frac{\partial \tilde{\Phi}(m, c/b, n)}{\partial m} < 0, \quad \frac{\partial \tilde{\Phi}(m, c/b, n)}{\partial c/b} < 0, \quad \frac{\partial \tilde{\Phi}(m, c/b, n)}{\partial n} > 0, \quad \frac{\partial \tilde{\Phi}(u, c/b, n)}{\partial n} < 0, \quad (28)$$

and where $u = m/n$. $\frac{\partial \tilde{\Phi}(m, c/b, n)}{\partial m} < 0$ implies that the largest internally stable coalition $\Phi(\tilde{m}) > 0$ is externally stable, because $\Phi(\tilde{m} + 1) < 0$ holds, and that the second largest

¹⁹Since the coalition countries account for their influence on the commodity price and the fringe countries have dominant strategies ($\check{\chi}_f = h, \check{\tau}_f = 0$), the Nash equilibrium coincides with the Stackelberg equilibrium. In other words, if the coalition countries had suffered a welfare loss by changing their policies from $(\chi_i^\circ, \tau_i^\circ)$ to $(\check{\chi}_e, \check{\tau}_e)$, then they would not have changed their policies.

²⁰In Appendix A.2, we show that $\frac{\partial^2 \tilde{W}_i}{\partial m^2} > 0$ holds.

internally stable coalition $\Phi(\check{m} - 1) > 0$ is externally unstable, because $\Phi(\check{m}) > 0$ holds. Thus, there is a unique stable coalition consisting of three or more countries.²¹ For a coalition of a given size m , $\tilde{\Phi}(m, c/b, n)$ is positive if c/b is sufficiently small. Ceteris paribus, a coalition country's export subsidy reduces its consumer surplus, which is below that of a fringe country, but it increases its producer surplus, which can be above that of a fringe country. Thus, lower marginal costs (a small c) and lower marginal benefits (a great b) make it relatively more attractive to join the coalition. In particular, we show in Appendix A.2 that the small coalition $\check{m} = 3$ is stable if $c/b \geq 1/3$ holds, and we show that the grand coalition $\check{m} = n \geq 4$ can only be stable if $c/b \leq 11/49$ holds. In general, the grand coalition is stable if

$$\tilde{\Phi}(n, c/b, n) \geq 0 \iff \frac{c}{b} \leq \frac{3n - 1}{(n - 3)(2n - 1)^2}, \quad (29)$$

which is depicted in Figure 1. We summarize our results in

PROPOSITION 4. *Consider the linear-quadratic specification (16).*

- *The unique stable coalition consists of three or more countries.*
- *The coalition size is constant in a and h , decreases with c/b , and increases with n .
The relative coalition size decreases with n .*
- *The coalition $\check{m} = 3$ is stable if $\frac{c}{b} \geq \frac{1}{3}$ (sufficient), and the coalition $\check{m} = n \geq 4$ can only be stable if $\frac{c}{b} \leq \frac{11}{49}$ (necessary).*

4 The role of border carbon adjustment

4.1 Coalition countries take the commodity price as given

Now suppose that not only fringe countries but also coalition countries take the international commodity price as given. Then, the fringe's optimal policy is still given by $\chi_i = H'_i$ and $\tau_i = 0$ for all $i \in F$, and the coalition's optimal policy is now given by $\chi_i = \sum_{k \in E} H'_k$ and $\tau_i = 0$ for all $i \in E$.²² The production taxes induce the coalition's representative firms to account for their climate externality on the coalition, and the import taxes are set equal to zero, because the coalition overlooks its influence on the fringe

²¹If the largest internally stable coalition is characterized by $\Phi(\check{m}) = 0$, then the largest internally stable coalition and the second largest internally stable coalition are externally stable.

²²See Lemma A.1 of Appendix A.1.

countries' emissions and on its terms of trade via the commodity price. The corresponding allocation follows from substituting the taxes into (3). Indicating this coalition-fringe equilibrium without strategic action by a bar, we get

$$B'_i(\bar{y}_i) = B'_j(\bar{y}_j), \quad (30a)$$

$$C'_i(\bar{x}_i) = C'_j(\bar{x}_j) - \sum_{k \in E} H'_k(\bar{x}) + H'_j(\bar{x}), \quad (30b)$$

$$B'_i(\bar{y}_i) = C'_i(\bar{x}_i) + \sum_{k \in E} H'_k(\bar{x}), \quad (30c)$$

$$B'_j(\bar{y}_j) = C'_j(\bar{x}_j) + H'_j(\bar{x}), \quad (30d)$$

for all $i \in E$ and $j \in F$. In Appendix A.3, we prove

PROPOSITION 5. *Consider the symmetric specification (15).*

- *Each coalition country's consumption ($y_i^\circ > \bar{y}_e > y_i^*$) is strictly decreasing in m . Its production is below that in BAU ($x_i^\circ > \bar{x}_e$) and its imports are positive.*
- *Each fringe country's consumption ($y_i^\circ > \bar{y}_f > y_i^*$) is strictly decreasing in m , and its production ($\bar{x}_f > x_i^\circ$) and exports are strictly increasing in m .*
- *Global emissions are strictly decreasing in m .*

The absence of the coalition's import tax implies that consumption is the same in all countries, and the larger coalition's production tax implies that production is smaller in each coalition country than in each fringe country, such that the coalition imports the commodity. In contrast to the coalition-fringe equilibrium of Section 3, each fringe country's consumption is below that in the business-as-usual equilibrium and decreases with the coalition size, while its production is above that in the business-as-usual equilibrium and increases with the coalition size. Each country's consumption is decreasing in m , such that global emissions are also decreasing in m .

Since $\bar{y}_e = \bar{y}_f$ and $\bar{x}_e < \bar{x}_f$ hold, the consumer surplus and the climate costs are the same in all countries, while a fringe country's producer surplus exceeds that of a coalition country.²³ Since $\frac{d\bar{x}}{dm} < 0$ holds, the climate costs of all countries are declining in m . Furthermore, we know that $B'_i(\bar{y}_i) - p = \bar{\tau}_i = 0$, $p - C'_i(\bar{x}_i) = \bar{\chi}_i > 0$ and $\frac{d\bar{x}_f}{dm}, \bar{x}_f - \bar{y}_f, \frac{d\bar{p}}{dm} = B''_i(\bar{y}_i) \frac{d\bar{y}_i}{dm} > 0$ hold, such that the sum of a fringe country's consumer

²³Compare equation 24.

and producer surplus and, thus, its welfare is increasing in m .²⁴ We summarize our results in

PROPOSITION 6. *Consider the symmetric specification (15).*

- *The welfare of a fringe country exceeds that of a coalition country and that in BAU ($\bar{W}_f > \bar{W}_e, W_i^\circ$).*
- *The welfare of a fringe country is strictly increasing in m ($\frac{\partial \bar{W}_f}{\partial m} > 0$).*

One important implication of Proposition 6 is that a coalition with $\bar{W}_e \leq W_i^\circ$ cannot be internally stable, because leaving the coalition always increases welfare to $\bar{W}_f > W_i^\circ$. Thus, in any internally stable coalition, each country's welfare exceeds that in the business-as-usual equilibrium.

Now we turn to the equilibrium coalition size. In Appendix A.3, we show that no coalition $m \geq 3$ is internally stable and that the coalition $m = 2$ is externally stable. The coalition $m = 2$ is internally stable if

$$\bar{W}_e(m = 2) - \bar{W}_f(m = 1) \geq 0 \iff \frac{c}{b} \geq \frac{(n-2)(3n-2)}{n^2}. \quad (31)$$

The producer [consumer] surplus of a fringe country exceeds [is equal to] that of a coalition country. Thus, higher marginal costs (a great c) and higher marginal benefits (a small b) make it relatively more attractive to join the coalition. The right-hand side of (31) is increasing in n , equal to zero for $n = 2$ and equal to $11/9$ for $n = 3$. Thus, $\bar{m} = 2$ holds for $n = 2$, and $\bar{m} = 0$ holds for $n \geq 3$ and $c/b < 11/9$.

In Appendix A.3, we also derive how many percent of the gap between total welfare in the grand-coalition equilibrium and total welfare in the business-as-usual equilibrium can be closed. This value is depicted in Figure 2 and calculated by

$$\frac{\bar{W}(\bar{m} = 2) - W^\circ}{W^* - W^\circ} =: \Delta \bar{W}(\bar{m} = 2) \leq \frac{4n - 6}{n(n-1)^2}. \quad (32)$$

Finally, we prove in Appendix A.3 that each country's welfare and total welfare are greater if the coalition accounts for its influence on the commodity price than if it does not. We summarize our results in

PROPOSITION 7. *Consider the linear-quadratic specification (16).*

²⁴Compare equation 25.

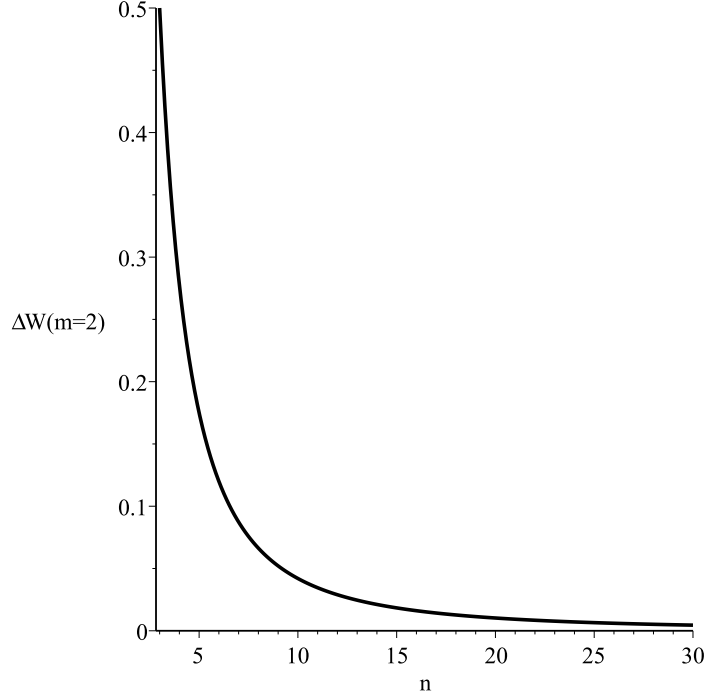


Figure 2: Maximum value of $\Delta\bar{W}(\bar{m} = 2)$ dependent on n .

- *Either the coalition consists of two countries or there is no stable coalition.*
- *Each country's welfare and total welfare are smaller than if the coalition acts strategically ($\bar{W}_i < \check{W}_i, \bar{W} < \check{W}$).*

4.2 Fringe countries account for their influence on the commodity price

Now suppose that not only coalition countries but also fringe countries account for their influence on the international commodity price. Then, the coalition's optimal policy is still given by $\chi_i = \sum_{k \in E} H'_k$ and $\tau_i = \frac{\sum_{k \in E} H'_k \sum_{k \in F} S'_k}{\sum_{k \in F} [S'_k - D'_k]} + \frac{\sum_{k \in E} (y_k - x_k)}{\sum_{k \in F} [S'_k - D'_k]}$ for all $i \in E$, and the fringe's optimal policy is now given by $\chi_i = H'_i$ and $\tau_i = \frac{H'_i \sum_{k \in i} S'_k}{\sum_{k \in i} [S'_k - D'_k]} + \frac{y_i - x_i}{\sum_{k \in i} [S'_k - D'_k]}$ for all $i \in F$.²⁵ The production tax induces i 's representative firms to account for its climate externality i , and the import tax partially induces the other countries' representative firms to account for their climate externality on i , and it improves i 's terms of trade. The corresponding allocation follows from substituting the taxes into (3). Indicating this coalition-fringe equilibrium with strategic interaction by a hat, we get

$$B'_i(\hat{y}_i) = B'_j(\hat{y}_j) + \hat{\tau}_i - \hat{\tau}_j, \quad (33a)$$

²⁵See Lemma A.2 of Appendix A.1.

$$C'_i(\hat{x}_i) = C'_j(\hat{x}_j) + \hat{\tau}_i - \hat{\tau}_j - \sum_{k \in E} H'_k(\hat{x}) + H'_j(\hat{x}), \quad (33b)$$

$$B'_i(\hat{y}_i) = C'_i(\hat{x}_i) + \sum_{k \in E} H'_k(\hat{x}), \quad (33c)$$

$$B'_j(\hat{y}_j) = C'_j(\hat{x}_j) + H'_j(\hat{x}), \quad (33d)$$

for all $i \in E$ and $j \in F$, where $\hat{\tau}_i = \frac{\sum_{k \in E} H'_k(\hat{x}) \sum_{k \in F} S'_k}{\sum_{k \in F} [S'_k - D'_k]} + \frac{\sum_{k \in E} (\hat{y}_k - \hat{x}_k)}{\sum_{k \in F} [S'_k - D'_k]}$ and $\hat{\tau}_j = \frac{H'_j(\hat{x}) \sum_{k \in j} S'_k}{\sum_{k \in j} [S'_k - D'_k]} + \frac{\hat{y}_j - \hat{x}_j}{\sum_{k \in j} [S'_k - D'_k]}$. In Appendix A.4, we prove

PROPOSITION 8. *Consider the linear-quadratic specification (16).*

- *Each coalition country's production and consumption ($x_i^\circ = y_i^\circ > \hat{x}_e = \hat{y}_e > x_i^* = y_i^*$) are strictly decreasing in m .*
- *Each fringe country's production and consumption ($x_i^\circ = y_i^\circ = \hat{x}_f = \hat{y}_f > x_i^* = y_i^*$) are constant in m .*
- *Global emissions are strictly decreasing in m .*

The presence of symmetry and linear-quadratic benefits and costs implies that the difference between the import taxes, $\hat{\tau}_e = \frac{mbH'_i(\hat{x})}{b+c} + \frac{m(\hat{y}_e - \hat{x}_e)}{(n-m)(b+c)}$ and $\hat{\tau}_f = \frac{bH'_i(\hat{x})}{b+c} + \frac{\hat{y}_f - \hat{x}_f}{(n-1)(b+c)}$, drives the same wedge between consumption and production in each country, such that there is no trade.²⁶ Compared to the coalition-fringe equilibrium of Section 3, each fringe country's consumption is decreased, while its production is increased, and both are constant in the coalition size.²⁷ Each country's consumption and production are decreasing in m , such that global emissions are also decreasing in m .²⁸

Since $\hat{x}_f = \hat{y}_f = x_i^\circ = y_i^\circ > \hat{x}_e = \hat{y}_e$ holds, the climate costs are lower than in the business-as-usual equilibrium and the same in all countries, while a fringe country's consumer and producer surplus are the same as in the business-as-usual equilibrium and exceed those of a coalition country.²⁹ Since $\frac{d\hat{x}_f}{dm} = \frac{d\hat{y}_f}{dm} = 0 > \frac{d\hat{x}_e}{dm} = \frac{d\hat{y}_e}{dm}$ holds, the climate costs of all countries are declining in m , while a fringe [coalition] country's consumer and producer surplus are constant [declining] in m .³⁰ Finally, we prove in Appendix A.4 that $\hat{W}_e - W_i^\circ, \frac{\partial \hat{W}_e}{\partial m}, \frac{\partial \hat{W}}{\partial m} > 0$ holds. We summarize our results in

²⁶In Appendix A.4, we show that $\hat{x}_f = \hat{y}_f > x_i^\circ = y_i^\circ > \hat{x}_e = \hat{y}_e$ holds for $H''_i > 0$.

²⁷In Appendix A.4, we show that $\frac{d\hat{x}_f}{dm} = \frac{d\hat{y}_f}{dm} > 0$ holds for $H''_i > 0$.

²⁸In Appendix A.4, we show that $\frac{d\hat{x}}{dm} < 0$ also holds for $H''_i > 0$.

²⁹Compare equation 24; $\hat{W}_f > \hat{W}_e, W_i^\circ$ also holds for $H''_i > 0$.

³⁰Compare equation 25; $\frac{\partial \hat{W}_f}{\partial m} > 0$ also holds for $H''_i > 0$.

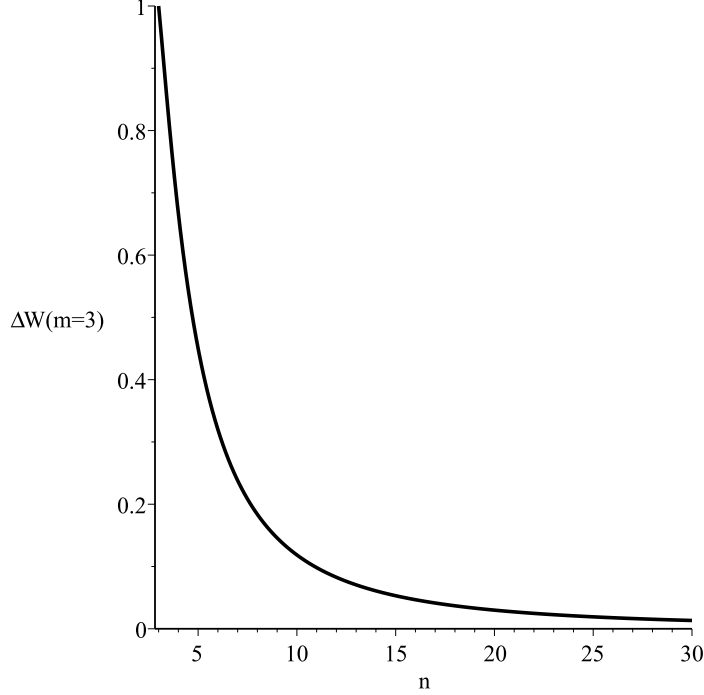


Figure 3: Value of $\Delta\hat{W}(\hat{m} = 3)$ dependent on n .

PROPOSITION 9. Consider the linear-quadratic specification (16).

- The welfare of a fringe country exceeds that of a coalition country which exceeds that in BAU ($\hat{W}_f > \hat{W}_e > W_i^\circ$).
- The welfare of each country and total welfare are strictly increasing in m ($\frac{\partial \hat{W}_f}{\partial m}, \frac{\partial \hat{W}_e}{\partial m}, \frac{\partial \hat{W}}{\partial m} > 0$).
- The welfare difference between a fringe country and a coalition country is strictly increasing in m ($\frac{\partial \hat{W}_f}{\partial m} > \frac{\partial \hat{W}_e}{\partial m}$).

Now we turn to the equilibrium coalition size. In Appendix A.4, we show that all coalitions $m \leq 3$ are internally stable and that all coalitions $m \geq 2$ are externally stable, such that either $\hat{m} = 2$ or $\hat{m} = 3$ hold.

In Appendix A.4, we also derive how many percent of the gap between total welfare in the grand-coalition equilibrium and total welfare in the business-as-usual equilibrium can be closed. For $\hat{m} = 2$, this value corresponds to the maximum value of $\bar{W}(\bar{m} = 2)$ depicted in Figure 2 and calculated in (32), and for $\bar{m} = 3$, this value is depicted in Figure 3 and calculated by

$$\frac{\hat{W}(\hat{m} = 3) - W^\circ}{W^* - W^\circ} =: \Delta\hat{W}(\hat{m} = 3) = \frac{12n - 24}{n(n - 1)^2} \in \left[2\hat{W}(\hat{m} = 2), 3\hat{W}(\hat{m} = 2) \right). \quad (34)$$

Finally, we prove in Appendix A.4 that for $\check{m} = \hat{m} = 3$ each coalition country's welfare

is smaller but each fringe country's welfare and total welfare are greater if the fringe accounts for its influence on the commodity price than if it does not.³¹ However, for $\check{m} \geq 4$ or for $\hat{m} = 2$ each country's welfare and total welfare are smaller if the fringe accounts for its influence on the commodity price than if it does not. We summarize our results in

PROPOSITION 10. *Consider the linear-quadratic specification (16).*

- *The stable coalitions consist of two and three countries.*
- *For $\check{m} = \hat{m} = 3$, each coalition country's welfare is smaller but each fringe country's welfare and total welfare are greater than if the fringe does not act strategically ($\hat{W}_e < \check{W}_e, \hat{W}_f > \check{W}_f, \hat{W} > \check{W}$). For $\check{m} \geq 4$ or for $\hat{m} = 2$, each country's welfare and total welfare are smaller than if the fringe does not act strategically ($\hat{W}_i < \check{W}_i, \hat{W} < \check{W}$).*
- *For $\hat{m} = \bar{m} = 2$, each coalition country's welfare and total welfare are greater but each fringe's country welfare is smaller than if no country acts strategically ($\hat{W}_e > \bar{W}_e, \hat{W}_f < \bar{W}_f, \hat{W} > \bar{W}$). For $\hat{m} = 3$ or for $\bar{m} = 0$, each country's welfare and total welfare are greater than if no country acts strategically ($\hat{W}_i > \bar{W}_i, \hat{W} > \bar{W}$).*

Finally, we prove in Appendix A.1 that the allocation without the possibility to trade coincides with that with the possibility to trade if there is no trade in equilibrium. Since there is no trade if all countries act strategically, Proposition 10 implies that the possibility to trade increases [decreases] each coalition country's welfare and, for $\check{m} \geq 4$ or for $\hat{m} = 2$, total welfare if only the coalition [if no country] acts strategically.

5 Conclusion

The present paper analyzes the impact of a climate coalition's border carbon adjustment on emissions from commodity production, welfare and the coalition size. We follow the literature on unilateral climate policy insofar as we assume that the coalition accounts for its influence on the commodity price, while the fringe takes the commodity price as given. In equilibrium, each coalition [fringe] country implements a production tax that matches the coalition's [country's] marginal climate damage, and each coalition country

³¹In Appendix A.4, we show that $\tau_e - \tau_f = \frac{(m-1)bH'_i(x)}{b+c} = \arg \max W$ for a coalition of a given size m .

implements an import tax or an export subsidy that shifts production from the fringe to the coalition.

With a linear-quadratic specification, each fringe country consumes more but can produce less than each coalition country, such that its consumer surplus is above but its producer surplus and, thus, its welfare can be below that of each coalition country. As a result, the coalition size increases with the marginal benefit parameter, which reduces the consumer surplus, and decreases with the marginal cost parameter, which reduces the producer surplus. In contrast to the literature on coalition formation without strategic advantage for the climate coalition, the stable coalition can be large and effective. In our model, a stable coalition of four or more countries definitely reduces emissions and increases welfare compared to the cases without strategic advantage for the climate coalition.

Throughout the paper, we assume linear-quadratic benefits and costs as well as linear climate costs. We conduct some robustness checks with convex climate costs and find that the climate coalition remains an exporter if only the coalition countries act strategically, remains an importer if no country acts strategically, and remains autarkic if all countries act strategically. Furthermore, the welfare of a fringe country still exceeds that of a coalition country if no country acts strategically or if all countries act strategically, which speaks in favor of a small coalition size. However, it is not clear whether the grand coalition can still be stable if only the coalition countries act strategically, and whether the strong welfare results remain valid with convex climate costs. Furthermore, linear climate costs imply that the fringe countries have dominant strategies if only the coalition countries act strategically, such that the Nash equilibrium and the Stackelberg equilibrium coincide. In order to check whether all results are robust to convex climate costs and in order to compare the Nash equilibrium to the Stackelberg equilibrium, it is appropriate to integrate quadratic climate costs into the model. This modification is left for future work.

A Appendix

A.1 Equilibrium policies

LEMMA A.1. *Without strategic action, the grand-coalition's optimal policy and the coalition's policy are given by $\gamma_i + \tau_i = 0$ and $\chi_i - \tau_i = H'$ and by $\gamma_i + \tau_i = 0$ and $\chi_i - \tau_i = \sum_{k \in E} H'_k$, respectively. With strategic action, they are given by $\gamma_i + \tau_i = \gamma_j + \tau_j$ and $\chi_i - \tau_i = H' - (\gamma_i + \tau_i)$ and by $\gamma_i + \tau_i = \frac{\sum_{k \in E} H'_k \sum_{k \in F} S'_k}{\sum_{k \in F} [S'_k - D'_k]} - \frac{\sum_{k \in E} [S_k - D_k]}{\sum_{k \in F} [S'_k - D'_k]}$ and $\chi_i - \tau_i = \sum_{k \in E} H'_k - (\gamma_i + \tau_i)$, respectively.*

Proof. The first-order conditions of (8) and (12) with respect to γ_i , χ_i and τ_i read

$$0 = B'_i D'_i - p D'_i + \sum_{k \in M} \left[B'_k D'_k - C'_k S'_k - H'_k S' + p \cdot [S'_k - D'_k] + S_k - D_k \right] \frac{\partial p}{\partial \gamma_i}, \quad (\text{A.1a})$$

$$0 = C'_i S'_i + H' S'_i - p S'_i + \sum_{k \in M} \left[B'_k D'_k - C'_k S'_k - H'_k S' + p \cdot [S'_k - D'_k] + S_k - D_k \right] \frac{\partial p}{\partial \chi_i}, \quad (\text{A.1b})$$

$$0 = B'_i D'_i - C'_i S'_i - H' S'_i + p \cdot [S'_i - D'_i] + \sum_{k \in M} \left[B'_k D'_k - C'_k S'_k - H'_k S' + p \cdot [S'_k - D'_k] + S_k - D_k \right] \frac{\partial p}{\partial \tau_i}. \quad (\text{A.1c})$$

where $M = N, E$, respectively. Substituting (3) into (A.1a)-(A.1c) yields

$$0 = (\gamma_i + \tau_i) D'_i + \sum_{k \in M} \left[(\gamma_k + \tau_k) D'_k - H'_k S' + (\chi_k - \tau_k) S'_k + S_k - D_k \right] \frac{\partial p}{\partial \gamma_i}, \quad (\text{A.2a})$$

$$0 = \left[\sum_{k \in M} H'_k - \chi_i + \tau_i \right] S'_i + \sum_{k \in M} \left[(\gamma_k + \tau_k) D'_k - H'_k S' + (\chi_k - \tau_k) S'_k + S_k - D_k \right] \frac{\partial p}{\partial \chi_i}, \quad (\text{A.2b})$$

$$0 = (\gamma_i + \tau_i) D'_i - \left[\sum_{k \in M} H'_k - \chi_i + \tau_i \right] S'_i + \sum_{k \in M} \left[(\gamma_k + \tau_k) D'_k - H'_k S' + (\chi_k - \tau_k) S'_k + S_k - D_k \right] \frac{\partial p}{\partial \tau_i}. \quad (\text{A.2c})$$

Since $\frac{\partial p}{\partial \tau_i} = \frac{\partial p}{\partial \gamma_i} - \frac{\partial p}{\partial \chi_i}$, the right-hand side of (A.2c) is equal to the difference between the right-hand side of (A.2a) and that of (A.2b), such that (A.2a)-(A.2c) are indeed two equations in two unknowns, $\gamma_i + \tau_i$ and $\chi_i - \tau_i$. For $\frac{\partial p}{\partial \gamma_i} = 0$ and $\frac{\partial p}{\partial \chi_i} = 0$, solving (A.2a) and (A.2b) yields $\gamma_i + \tau_i = 0$ and $\chi_i - \tau_i = \sum_{k \in M} H'_k$, which is for $M = E$ the coalition's optimal policy and for $M = N$ the grand-coalition's optimal policy without strategic action. For $\frac{\partial p}{\partial \gamma_i} = \frac{D'_i}{S' - D'}$ and $\frac{\partial p}{\partial \chi_i} = \frac{S'_i}{S' - D'}$, rearranging (A.2a) and (A.2b) yields $\gamma_i + \tau_i = \frac{\sum_{k \in M} (\gamma_k + \tau_k) [S'_k - D'_k]}{S' - D'} + \frac{\sum_{k \in M} [H'_k S' - (\chi_k + \gamma_k) S'_k]}{S' - D'} - \frac{\sum_{k \in M} [S_k - D_k]}{S' - D'}$ and $\chi_i - \tau_i = \sum_{k \in M} H'_k - (\gamma_i + \tau_i)$, which implies $\gamma_i + \tau_i = \tau_j + \gamma_j$ and $\chi_i + \gamma_i = \sum_{k \in M} H'_k$ for all $i, j \in M$. For the grand-coalition's optimal policy, where $\sum_{k \in M} H'_k = H'$, $\sum_{k \in M} S'_k = S'$ and $\sum_{k \in M} [S_k - D_k] = 0$ hold, this implies $\gamma_i + \tau_i = \tau_j + \gamma_j$ and $\chi_i - \tau_i = H' - (\gamma_i + \tau_i)$, and for the coalition's optimal policy, this implies $\tau_i + \gamma_i = \frac{\sum_{k \in E} H'_k \sum_{k \in F} S'_k}{\sum_{k \in F} [S'_k - D'_k]} - \frac{\sum_{k \in E} [S_k - D_k]}{\sum_{k \in F} [S'_k - D'_k]}$ and $\chi_i - \tau_i = \sum_{k \in E} H'_k - (\gamma_i + \tau_i)$. \square

LEMMA A.2. *Without strategic action, the business-as-usual policy and the fringe's optimal policy are given by $\gamma_i + \tau_i = 0$ and $\chi_i - \tau_i = H'_i$. With strategic action, they are given by $\gamma_i + \tau_i = \frac{H'_i \sum_{k \setminus i} S'_k}{\sum_{k \setminus i} [S'_k - D'_k]} - \frac{S_i - D_i}{\sum_{k \setminus i} [S'_k - D'_k]}$ and $\chi_i - \tau_i = H'_i - (\gamma_i + \tau_i)$. $\gamma_i + \tau_i = \gamma_j + \tau_j$ and $\chi_i - \tau_i = H'_i - (\gamma_j + \tau_j)$ is an equilibrium for the symmetric business-as-usual policy, and it is the unique equilibrium for the parametric business-as-usual policy.*

Proof. The first-order conditions of (10) with respect to γ_i , χ_i and τ_i read

$$0 = B'_i D'_i - p D'_i + \left[B'_i D'_i - C'_i S'_i - H'_i S' + p \cdot [S'_i - D'_i] + S_i - D_i \right] \frac{\partial p}{\partial \gamma_i}, \quad (\text{A.3a})$$

$$0 = C'_i S'_i + H'_i S'_i - p S'_i + \left[B'_i D'_i - C'_i S'_i - H'_i S' + p \cdot [S'_i - D'_i] + S_i - D_i \right] \frac{\partial p}{\partial \chi_i}, \quad (\text{A.3b})$$

$$0 = B'_i D'_i - C'_i S'_i - H'_i S'_i - p \cdot [D'_i - S'_i] + \left[B'_i D'_i - C'_i S'_i - H'_i S' + p \cdot [S'_i - D'_i] + S_i - D_i \right] \frac{\partial p}{\partial \tau_i}. \quad (\text{A.3c})$$

Substituting (3) into (A.3a)-(A.3c) yields

$$0 = (\gamma_i + \tau_i) D'_i + \left[(\gamma_i + \tau_i) D'_i - H'_i S' + (\chi_i - \tau_i) S'_i + S_i - D_i \right] \frac{\partial p}{\partial \gamma_i}, \quad (\text{A.4a})$$

$$0 = [H'_i - \chi_i + \tau_i] S'_i + \left[(\gamma_i + \tau_i) D'_i - H'_i S' + (\chi_i - \tau_i) S'_i + S_i - D_i \right] \frac{\partial p}{\partial \chi_i}, \quad (\text{A.4b})$$

$$0 = (\gamma_i + \tau_i) D'_i - [H'_i - \chi_i + \tau_i] S'_i + \left[(\gamma_i + \tau_i) D'_i - H'_i S' + (\chi_i - \tau_i) S'_i + S_i - D_i \right] \frac{\partial p}{\partial \tau_i}. \quad (\text{A.4c})$$

Since $\frac{\partial p}{\partial \tau_i} = \frac{\partial p}{\partial \gamma_i} - \frac{\partial p}{\partial \chi_i}$, the right-hand side of (A.4c) is equal to the difference between the right-hand side of (A.4a) and that of (A.4b), such that (A.4a)-(A.4c) are indeed two equations in two unknowns, $\gamma_i + \tau_i$ and $\chi_i - \tau_i$. For $\frac{\partial p}{\partial \gamma_i} = 0$ and $\frac{\partial p}{\partial \chi_i} = 0$, solving (A.4a) and (A.4b) yields $\gamma_i + \tau_i = 0$ and $\chi_i - \tau_i = H'_i$, which is the business-as-usual policy and the fringe's optimal policy without strategic action. For $\frac{\partial p}{\partial \gamma_i} = \frac{D'_i}{S' - D'}$ and $\frac{\partial p}{\partial \chi_i} = \frac{S'_i}{S' - D'}$, solving (A.4a) and (A.4b) yields $\gamma_i + \tau_i = \frac{H'_i \sum_{k \setminus i} S'_k}{\sum_{k \setminus i} [S'_k - D'_k]} - \frac{S_i - D_i}{\sum_{k \setminus i} [S'_k - D'_k]}$ and $\chi_i - \tau_i = H'_i - (\gamma_i + \tau_i)$, which is the business-as-usual policy and the fringe's optimal policy with strategic action. For the symmetric business-as-usual policy, $\gamma_i + \tau_i = \gamma_j + \tau_j \implies \chi_i - \tau_i = \chi_j - \tau_j$ for all $i, j \in N$ is an equilibrium: It implies $B'_i = B'_j \iff y_i = y_j$ and $C'_i = C'_j \iff x_i = x_j$, and $y_i = y_j = x_i = x_j$ then implies $\gamma_i + \tau_i = \gamma_j + \tau_j$. For the parametric business-as-usual policy, $\gamma_i + \tau_i < [>] \gamma_j + \tau_j \implies \chi_i - \tau_i > [<] \chi_j - \tau_j$ for any $i, j \in N$ cannot be an equilibrium: It implies $B'_i < [>] B'_j \iff y_i > [<] y_j$ and $C'_i < [>] C'_j \iff x_i < [>] x_j$, and $y_i - x_i > [<] y_j - x_j$ then implies $\gamma_i + \tau_i > [<] \gamma_j + \tau_j$, which contradicts $\gamma_i + \tau_i < [>] \gamma_j + \tau_j$. \square

LEMMA A.3. *Without the possibility to trade, the grand-coalition's optimal policy and the coalition's optimal policy are given by $\chi_i + \gamma_i = H'$ and by $\chi_i + \gamma_i = \sum_{k \in E} H'_k$, respectively, and the business-as-usual policy and the fringe's optimal policy are given by $\chi_i + \gamma_i = H'_i$. The*

corresponding allocations coincide with those with the possibility to trade if there is no trade in equilibrium.

Proof. Without the possibility to trade, such that $D_i(p_i + \gamma_i) = S_i(p_i - \chi_i)$, $\frac{\partial p_i}{\partial \gamma_i} = \frac{D'_i}{S'_i - D'_i} \in (-1, 0)$ and $\frac{\partial p_i}{\partial \chi_i} = \frac{S'_i}{S'_i - D'_i} \in (0, 1)$ hold, the first-order conditions of (8), (10) and (12) with respect to γ_i and χ_i read

$$0 = B'_i D'_i - p_i D'_i + \left[B'_i D'_i - C'_i S'_i - \sum_{k \in M} H'_k S'_i + p_i \cdot [S'_i - D'_i] + S_i - D_i \right] \frac{\partial p_i}{\partial \gamma_i}, \quad (\text{A.5})$$

$$0 = C'_i S'_i + \sum_{k \in M} H'_k S'_i - p_i S'_i + \left[B'_i D'_i - C'_i S'_i - \sum_{k \in M} H'_k S'_i + p_i \cdot [S'_i - D'_i] + S_i - D_i \right] \frac{\partial p_i}{\partial \chi_i}. \quad (\text{A.6})$$

where $M = N, i, E$, respectively. Substituting $B'_i - \gamma_i = C'_i + \chi_i = p_i$ and $D_i = S_i$ into (A.5) and (A.6) yields

$$0 = \gamma_i D'_i + \left[\gamma_i D'_i - \sum_{k \in M} H'_k S'_i + \chi_i S'_i \right] \frac{\partial p_i}{\partial \gamma_i}, \quad (\text{A.7})$$

$$0 = \sum_{k \in M} H'_k S'_i - \chi_i S'_i - \left[\gamma_i D'_i - \sum_{k \in M} H'_k S'_i + \chi_i S'_i \right] \frac{\partial p_i}{\partial \chi_i}, \quad (\text{A.8})$$

For $\frac{\partial p_i}{\partial \gamma_i} = 0$ and $\frac{\partial p_i}{\partial \chi_i} = 0$, solving (A.7) and (A.8) yields $\gamma_i = 0$ and $\chi_i = \sum_{k \in M} H'_k$, and for $\frac{\partial p_i}{\partial \gamma_i} = \frac{D'_i}{S'_i - D'_i}$ and $\frac{\partial p_i}{\partial \chi_i} = \frac{S'_i}{S'_i - D'_i}$, solving (A.7) and (A.8) yields $\chi_i + \gamma_i = \sum_{k \in M} H'_k$. Substituting the taxes into $B'_i - \gamma_i = C'_i + \chi_i = p_i$ yields $B'_i(y_i) = C'_i(x_i) + \sum_{k \in M} H'_k$, which corresponds to (9c) for $M = N$, to (11c) for $M = i$, and to (13c) for $M = E$. Now suppose that the climate costs are smaller [greater] without than with the possibility to trade if there is no trade in equilibrium. Then, (9c), (11c) and (13c) imply that all countries consume and produce more [less] without than with the possibility to trade, which contradicts that the climate costs are smaller [greater] without than with the possibility to trade. Thus, all countries consume and produce the same without and with the possibility to trade if there is no trade in equilibrium. \square

A.2 Proofs of Section 3

Proof of Proposition 1. Suppose $\tilde{\tau}_e \leq 0$, such that $\check{y}_e - \check{x}_e < 0$. Then, (13a) implies $\check{y}_e \geq \check{y}_f$, and (13b) implies $\check{x}_e < \check{x}_f$, which contradicts $\check{y}_e - \check{x}_e < 0$. Thus, $\tilde{\tau}_e > 0$ must hold. Suppose $\check{y}_e \geq y_i^\circ$. Then, (11a) and (13a) imply $\check{y}_f > y_i^\circ$, such that $\check{H}_i > H_i^\circ$. Then, (11c), (13c) and (13d) imply $\check{x}_e < x_i^\circ$ and $\check{x}_f < x_i^\circ$, which contradicts $\check{H}_i > H_i^\circ$. Thus, $\check{y}_e < y_i^\circ$ must hold. Suppose $\tilde{\tau}_e \leq (m-1)\check{H}'_i$ and $\check{x}_e \geq x_i^\circ$. Then, (11b) and (13b) imply $\check{x}_f \geq x_i^\circ$, such that $\check{H}_i \geq H_i^\circ$. Then, (11c) and (13d) imply $\check{y}_f \leq y_i^\circ$, which contradicts $\check{H}_i \geq H_i^\circ$. Thus, $\tilde{\tau}_e \leq (m-1)\check{H}'_i$ implies $\check{x}_e < x_i^\circ$. Suppose $\tilde{\tau}_e \leq (m-1)\check{H}'_i$ and $\check{H}_i \geq H_i^\circ$, such that $\check{x}_f > x_i^\circ$. Then, (11c) and (13d) imply $\check{y}_f < y_i^\circ$, which contradicts $\check{H}_i \geq H_i^\circ$. Thus, $\tilde{\tau}_e \leq (m-1)\check{H}'_i$ implies $\check{H}_i < H_i^\circ$. Suppose

$\check{\tau}_e \geq (m-1)\check{H}'_i$ and $\check{x}_f \geq x_i^\circ$. Then, (11b) and (13b) imply $\check{x}_e \geq x_i^\circ$, such that $\check{H}_i \geq H_i^\circ$. Then, (11c) and (13d) imply $\check{y}_f \leq y_i^\circ$, which contradicts $\check{H}_i \geq H_i^\circ$. Thus, $\check{\tau}_e \geq (m-1)\check{H}'_i$ implies $\check{x}_f < x_i^\circ$. Suppose $\check{\tau}_e \geq (m-1)\check{H}'_i$ and $\check{y}_f \leq y_i^\circ$, such that $\check{H}_i < H_i^\circ$. Then, (11c) and (13d) imply $\check{x}_f \geq x_i^\circ$, which contradicts $\check{x}_f < x_i^\circ$. Thus, $\check{\tau}_e \geq (m-1)\check{H}'_i$ implies $\check{y}_f > y_i^\circ$. Finally, $\check{y}_f > y_i^\circ$ and $\check{x}_f < x_i^\circ$ imply $\check{y}_f - \check{x}_f > y_i^\circ - x_i^\circ = 0$, such that $\check{\tau}_e \geq (m-1)\check{H}'_i$ implies $\check{\tau}_e < \frac{m\check{H}'_i\check{S}'_f}{\check{S}'_f - \check{D}'_f}$.

Suppose $\check{y}_f \leq y_i^*$. Then, (9a) and (13a) imply $\check{y}_e < y_i^*$, such that $\check{H}_i < H_i^*$. Then, (9c), (13c) and (13d) imply $\check{x}_e > x_i^*$ and $\check{x}_f > x_i^*$, which contradicts $\check{H}_i < H_i^*$. Thus, $\check{y}_f > y_i^*$ must hold. Suppose $\check{\tau}_e \leq (m-1)\check{H}'_i$ and $\check{x}_f \leq x_i^*$. Then, (9b) and (13b) imply $\check{x}_e \leq x_i^*$, such that $\check{H}_i \leq H_i^*$. Then, (9c) and (13c) imply $\check{y}_e > y_i^*$, which contradicts $\check{H}_i \leq H_i^*$. Thus, $\check{\tau}_e \leq (m-1)\check{H}'_i$ implies $\check{x}_f > x_i^*$. Suppose $\check{\tau}_e \leq (m-1)\check{H}'_i$ and $\check{H}_i \leq H_i^*$, such that $\check{x}_e < x_i^*$. Then, (9c) and (13c) imply $\check{y}_e > y_i^*$, which contradicts $\check{H}_i \leq H_i^*$. Thus, $\check{\tau}_e \leq (m-1)\check{H}'_i$ implies $\check{H}_i > H_i^*$. Suppose $\check{\tau}_e \geq (m-1)\check{H}'_i$ and $\check{x}_e \leq x_i^*$. Then, (9b) and (13b) imply $\check{x}_f \leq x_i^*$, such that $\check{H}_i \leq H_i^*$. Then, (9c) and (13c) imply $\check{y}_e > y_i^*$, which contradicts $\check{H}_i \leq H_i^*$. Thus, $\check{\tau}_e \geq (m-1)\check{H}'_i$ implies $\check{x}_e > x_i^*$. \square

Proofs of Footnotes 16, 17 and 18. Substituting $\check{\chi}_e = mH'_i(\check{x})$ and $\check{\chi}_f = H'_i(\check{x})$ into (18c) and differentiating with respect to m yields

$$\check{x} = \frac{na - (n + m^2 - m)H'_i(\check{x})}{b + c}, \quad (\text{A.9})$$

$$\frac{d\check{x}}{dm} = -\frac{(2m-1)H'_i(\check{x})}{b+c} - \frac{(n+m^2-m)H''_i(\check{x})\frac{d\check{x}}{dm}}{b+c} = -\frac{(2m-1)H'_i(\check{x})}{b+c + (n+m^2-m)H''_i(\check{x})}, \quad (\text{A.10})$$

such that $\frac{d\check{x}}{dm} < 0$. Substituting $h = H'_i(\check{x})$ into (20) and (21), solving for $\check{y}_e - \check{x}_e$ and differentiating with respect to m yields

$$\check{y}_e - \check{x}_e = -\frac{(n-m)H'_i(\check{x})}{(n+m)c} < 0, \quad (\text{A.11})$$

$$\frac{\partial(\check{y}_e - \check{x}_e)}{\partial m} = \frac{2nH'_i(\check{x})}{(n+m)^2c} - \frac{(n-m)H''_i(\check{x})\frac{d\check{x}}{dm}}{(n+m)c}, \quad (\text{A.12})$$

where $\frac{d\check{x}}{dm} < 0$ implies $\frac{\partial(\check{y}_e - \check{x}_e)}{\partial m} > 0$. Substituting $\check{\chi}_f = H'_i(\check{x})$ and $\check{y}_f - \check{x}_f = \frac{mH'_i(\check{x})}{(n+m)c}$ into (18a) and differentiating with respect to m yields

$$\check{y}_f = \frac{a - H'_i(\check{x})}{b+c} + \frac{mH'_i(\check{x})}{(n+m)(b+c)}, \quad (\text{A.13})$$

$$\frac{d\check{y}_f}{dm} = \frac{nH'_i(\check{x})}{(n+m)^2(b+c)} - \frac{nH''_i(\check{x})\frac{d\check{x}}{dm}}{(n+m)(b+c)}, \quad (\text{A.14})$$

where $\frac{d\check{x}}{dm} < 0$ implies $\frac{d\check{y}_f}{dm} > 0$. \square

Proof of Proposition 2. Substituting $\chi_i^* = nh, \tau_i^* = \tau_j^*$ and $\chi_i^\circ = h, \tau_i^\circ = 0$ into (18) and (19) yields

$$y_i^* = x_i^* = \frac{a - nh}{b+c}, \quad y_i^\circ = x_i^\circ = \frac{a - h}{b+c}. \quad (\text{A.15})$$

Substituting $\check{\chi}_e = mh$, $\check{\chi}_f = h$, $\check{\tau}_e = \frac{(n+m-1)mhb}{(n+m)(b+c)}$ and $\check{\tau}_f = 0$ into (18) and (19) and differentiating with respect to m yields

$$\check{y}_e = \frac{a-nh}{b+c} + \frac{(n-m)(n+m-1)h}{(n+m)(b+c)} > y_i^*, \quad \frac{\partial \check{y}_e}{\partial m} = -\frac{[(n+m)^2 - 2n]h}{(n+m)^2(b+c)} < 0, \quad (\text{A.16a})$$

$$\check{y}_f = \frac{a-h}{b+c} + \frac{mh}{(n+m)(b+c)} > y_i^\circ, \quad \frac{\partial \check{y}_f}{\partial m} = \frac{nh}{(n+m)^2(b+c)} > 0, \quad (\text{A.16b})$$

$$\check{x}_e = \frac{a-nh}{b+c} + \frac{(n-m)[b+(n+m)c]h}{(n+m)c(b+c)} > x_i^*, \quad \frac{\partial \check{x}_e}{\partial m} = -\frac{[(n+m)^2c + 2nb]h}{(n+m)^2c(b+c)} < 0, \quad (\text{A.16c})$$

$$\check{x}_f = \frac{a-h}{b+c} - \frac{mbh}{(n+m)c(b+c)} < x_i^\circ, \quad \frac{\partial \check{x}_f}{\partial m} = -\frac{nbh}{(n+m)^2c(b+c)} < 0, \quad (\text{A.16d})$$

$$\check{x} = \frac{na - [n + m(m-1)]h}{b+c}, \quad \frac{\partial \check{x}}{\partial m} = -\frac{(2m-1)h}{b+c} < 0, \quad (\text{A.16e})$$

$$\check{y}_e - \check{x}_e = -\frac{(n-m)h}{(n+m)c} < 0, \quad \frac{\partial(\check{y}_e - \check{x}_e)}{\partial m} = \frac{2nh}{(n+m)^2c} > 0, \quad (\text{A.16f})$$

$$\check{y}_f - \check{x}_f = \frac{mh}{(n+m)c} > 0, \quad \frac{\partial(\check{y}_f - \check{x}_f)}{\partial m} = \frac{nh}{(n+m)^2c} > 0, \quad (\text{A.16g})$$

$$m(\check{y}_e - \check{x}_e) = -\frac{m(n-m)h}{(n+m)c} < 0, \quad \frac{\partial m(\check{y}_e - \check{x}_e)}{\partial m} = -\frac{(n^2 - 2nm - m^2)h}{(n+m)^2c}. \quad (\text{A.16h})$$

From (A.15) and (A.16d), $a > \max\left\{nh, \frac{2c+b}{2c}h\right\}$ implies $y_i, x_i > 0$. \square

Proof of Proposition 3. Country i 's welfare is given by $W_i = ay_i - \frac{b}{2}y_i^2 - \frac{c}{2}x_i^2 - p(y_i - x_i) - hx_i$.

Substituting (A.15) yields

$$W_i^* = \frac{(a-nh)^2}{2(b+c)}, \quad W_i^\circ = \frac{(a-h)[a - (2n-1)h]}{2(b+c)}. \quad (\text{A.17})$$

Substituting (A.16) yields

$$\check{W}_e - W_i^\circ = \frac{(m-1)^2bh^2}{2c(b+c)} \left[\frac{c}{b} + \frac{n-m}{(m-1)^2(n+m)} \right] > 0, \quad (\text{A.18a})$$

$$\check{W}_f - W_i^\circ = \frac{m(m-1)bh^2}{c(b+c)} \left[\frac{c}{b} - \frac{2n+m}{2(m-1)(n+m)^2} \right], \quad (\text{A.18b})$$

$$\check{W}_f - \check{W}_e = \frac{(m^2-1)bh^2}{2c(b+c)} \left[\frac{c}{b} - \frac{n(n+2m)}{(m^2-1)(n+m)^2} \right], \quad (\text{A.18c})$$

$$\check{W} - W^\circ = \frac{m(m-1)(2n-m-1)bh^2}{2c(b+c)} \left[\frac{c}{b} - \frac{n(n-m)}{(m-1)(n+m)^2(2n-m-1)} \right], \quad (\text{A.18d})$$

where $\check{W}_f \geq \check{W}_e \implies \check{W}_f > W_i^\circ$ and $\check{W}_f \geq W_i^\circ \implies \check{W} > W^\circ$. Differentiating (A.18) with respect to m yields

$$\frac{\partial \check{W}_e}{\partial m} = \frac{(m-1)bh^2}{2c(b+c)} \left[\frac{c}{b} - \frac{n}{(m-1)(n+m)^2} \right], \quad \frac{\partial^2 \check{W}_e}{\partial m^2} = \frac{bh^2}{c(b+c)} \left[\frac{c}{b} + \frac{2n}{(n+m)^3} \right] > 0, \quad (\text{A.19a})$$

$$\frac{\partial \check{W}_f}{\partial m} = \frac{(2m-1)bh^2}{c(b+c)} \left[\frac{c}{b} - \frac{n^2}{(2m-1)(n+m)^3} \right], \quad \frac{\partial^2 \check{W}_f}{\partial m^2} = \frac{2bh^2}{c(b+c)} \left[\frac{c}{b} + \frac{3n^2}{(n+m)^4} \right] > 0, \quad (\text{A.19b})$$

$$\frac{\partial \check{W}_f}{\partial m} - \frac{\partial \check{W}_e}{\partial m} = \frac{mbh^2}{c(b+c)} \left[\frac{c}{b} + \frac{n}{(n+m)^3} \right] > 0, \quad (\text{A.19c})$$

$$\frac{\partial \check{W}}{\partial m} = \frac{bh^2}{2c(b+c)} \left[\frac{c}{b} [(m-1)^2 + 2(2m-1)(n-m)] + \frac{n^2(3m-n)}{(n+m)^2} \right]. \quad (\text{A.19d})$$

Since

$$\frac{2n+m}{2(m-1)(n+m)^2} - \frac{n}{(m-1)(n+m)^2} = \frac{m}{2(m-1)(n+m)^2} > 0, \quad (\text{A.20a})$$

$$\frac{n}{(m-1)(n+m)^2} - \frac{n^2}{(2m-1)(n+m)^3} = \frac{nm(n+2m-1)}{(m-1)(2m-1)(n+m)^3} > 0, \quad (\text{A.20b})$$

$$\begin{aligned} & \frac{n^2}{(2m-1)(n+m)^3} - \frac{n^2(3m-n)}{[(m-1)^2 + 2(2m-1)(n-m)](n+m)^2} \\ &= \frac{n^2[5(m-1)^2 + 6m - 4 + (2m-1)(n-m)]}{[(m-1)^2 + 2(2m-1)(n-m)](2m-1)(n+m)^3} > 0 \end{aligned} \quad (\text{A.20c})$$

hold, we have $\check{W}_f \geq W_i^\circ \implies \frac{\partial \check{W}_e}{\partial m} > 0$ and $\frac{\partial \check{W}_e}{\partial m} \geq 0 \implies \frac{\partial \check{W}_f}{\partial m} > 0$ as well as $\frac{\partial \check{W}_f}{\partial m} \geq 0 \implies \frac{\partial \check{W}}{\partial m} > 0$. \square

Proof of Proposition 4. Substituting (A.18a) and (A.18b) into $\Phi(m) = W_e(m) - W_f(m-1)$ and $\Phi(m+1) = W_e(m+1) - W_f(m)$ yields

$$\Phi(m) = \frac{bh^2}{2c(b+c)} \left[\frac{n^2 + (3m-4)n + 2(m-1)^2}{(n+m)(n+m-1)^2} n - (m-1)(m-3) \frac{c}{b} \right], \quad (\text{A.21})$$

$$\Phi(m+1) = \frac{bh^2}{2c(b+c)} \left[\frac{n^2 + (3m-1)n + 2m^2}{(n+m)^2(n+m-1)} n - m(m-2) \frac{c}{b} \right]. \quad (\text{A.22})$$

$m \leq 3 \implies \Phi(m) > 0$ and $m \leq 2 \implies \Phi(m+1) > 0$, such that some coalition of at least three countries is stable. Rearranging (A.21) for $m > 3$ yields $\Phi(m) = \frac{(m-1)(m-3)bh^2}{2c(b+c)} \tilde{\Phi}(m, c/b, n)$, where

$$\tilde{\Phi}(m, c/b, n) = \frac{n[n^2 + (3m-4)n + 2(m-1)^2]}{(m-1)(m-3)(n+m)(n+m-1)^2} - \frac{c}{b}, \quad (\text{A.23a})$$

$$\begin{aligned} \frac{\partial \tilde{\Phi}(m, c/b, n)}{\partial m} &= - \frac{2n(m-1)\{(m-1)^2[3(m-3)^2 + 10m - 24] + [9m(m-2)^2 + 7m - 13]n\}}{(m-1)^2(m-3)^2(n+m-1)^3(m+n)^2} \\ &\quad - \frac{n\{5(2m-3)[2(m-2)^2 + 7m - 13]n^2 + 2(m-2)[5(m-1)n^3 + n^4]\}}{(m-1)^2(m-3)^2(n+m-1)^3(m+n)^2}, \end{aligned} \quad (\text{A.23b})$$

$$\frac{\partial \tilde{\Phi}(m, c/b, n)}{\partial n} = \frac{2\{n^3 + m(m-1)[n^2 + (2m-3)n + (m-1)^2]\}}{(m-1)(m-3)(n+m-1)^3(m+n)^2}, \quad (\text{A.23c})$$

such that $\frac{\partial \tilde{\Phi}(m, c/b, n)}{\partial m} < 0$, $\frac{\partial \tilde{\Phi}(m, c/b, n)}{\partial c/b} < 0$ and $\frac{\partial \tilde{\Phi}(m, c/b, n)}{\partial n} > 0$ hold. Using $u = m/n$, we can rewrite (A.23a) as

$$\tilde{\Phi}(u, c/b, n) = \frac{(2u^2 + 3u + 1)n^2 - 4(u+1)n + 2}{(un-3)(un-1)(u+1)(un+n-1)^2} - \frac{c}{b}. \quad (\text{A.24})$$

Differentiating with respect to n , substituting $u = m/n$ and rearranging yields

$$\frac{\partial \tilde{\Phi}(u, c/b, n)}{\partial n} = -\frac{2m(m-2)n^3 + 2(2m-3)(2m^2-4m+1)n^2 + 2m(m-1)(5m^2-17m+15)n}{(m-1)^2(m-3)^2(n+m-1)^3(m+n)} - \frac{4m(m-1)^3(m-2)}{(m-1)^2(m-3)^2(n+m-1)^3(m+n)}, \quad (\text{A.25})$$

such that $\frac{\partial \tilde{\Phi}(u, c/b, n)}{\partial n} < 0$ holds. Substituting $m = 4$ and $m = n$ into (A.23a) yields

$$\tilde{\Phi}(4, c/b, n) = \frac{n(n^2 + 8n + 18)}{3(n+3)^2(n+4)} - \frac{c}{b}, \quad (\text{A.26a})$$

$$\frac{\partial \tilde{\Phi}(4, c/b, n)}{\partial n} = \frac{2n^3 + 24n^2 + 120n + 216}{3(n+3)^3(n+4)^2} > 0, \quad \tilde{\Phi}(4, c/b, \infty) = \frac{1}{3}, \quad (\text{A.26b})$$

$$\tilde{\Phi}(n, c/b, n) = \frac{3n-1}{(n-3)(2n-1)^2} - \frac{c}{b}, \quad (\text{A.26c})$$

$$\frac{\partial \tilde{\Phi}(n, c/b, n)}{\partial n} = -\frac{4(3n^2 - 6n + 1)}{(n-3)^2(2n-1)^3} < 0, \quad \tilde{\Phi}(n, c/b, 4) = \frac{11}{49}, \quad (\text{A.26d})$$

which proves the last part of the proposition. \square

A.3 Proofs of Section 4.1

Proof of Proposition 5. Suppose $\bar{y}_e = \bar{y}_f \geq y_i^\circ$, such that $\bar{H}_i \geq H_i^\circ$. Then, (11c), (30c) and (30d) imply $\bar{x}_e < x_i^\circ$ and $\bar{x}_f \leq x_i^\circ$, which contradicts $\bar{H}_i \geq H_i^\circ$. Thus, $\bar{y}_e = \bar{y}_f < y_i^\circ$ and $\bar{H}_i < H_i^\circ$ must hold. Suppose $\bar{x}_e \geq x_i^\circ$. Then, (30b) implies $\bar{x}_f > x_i^\circ$, which contradicts $\bar{H}_i < H_i^\circ$. Thus, $\bar{x}_e < x_i^\circ$ must hold. Suppose $\bar{x}_f \leq x_i^\circ$. Then, (30d) implies $\bar{y}_f \geq y_i^\circ$ or $\bar{H}_i \geq H_i^\circ$, which contradicts $\bar{y}_f < y_i^\circ$ or $\bar{H}_i < H_i^\circ$. Thus, $\bar{x}_f > x_i^\circ$ must hold.

Suppose $\bar{y}_e = \bar{y}_f \leq y_i^*$, such that $\bar{H}_i \leq H_i^*$. Then, (9c), (30c) and (30d) imply $\bar{x}_e > x_i^*$ and $\bar{x}_f > x_i^*$, which contradicts $\bar{H}_i \leq H_i^*$. Thus, $\bar{y}_e = \bar{y}_f > y_i^*$ and $\bar{H}_i > H_i^*$ must hold. Suppose $\bar{x}_f \leq x_i^*$. Then, (30b) implies $\bar{x}_e < x_i^*$, which contradicts $\bar{H}_i > H_i^*$. Thus, $\bar{x}_f > x_i^*$ must hold.

Suppose $\frac{d\bar{y}_e}{dm} = \frac{d\bar{y}_f}{dm} \geq 0$, such that $\frac{d\bar{H}_i}{dm} \geq 0$. Then, (30c) and (30d) imply $\frac{d\bar{x}_e}{dm} < 0$ and $\frac{d\bar{x}_f}{dm} \leq 0$, which contradicts $\frac{d\bar{H}_i}{dm} \geq 0$. Thus, $\frac{d\bar{y}_e}{dm} = \frac{d\bar{y}_f}{dm} < 0$ and $\frac{d\bar{H}_i}{dm} < 0$ must hold. Suppose $\frac{d\bar{x}_f}{dm} \leq 0$. Then, (30d) implies $\frac{d\bar{y}_f}{dm} \geq 0$ or $\frac{d\bar{H}_i}{dm} \geq 0$, which contradicts $\frac{d\bar{y}_f}{dm} < 0$ or $\frac{d\bar{H}_i}{dm} < 0$. Thus, $\frac{d\bar{x}_f}{dm} > 0$ must hold. \square

Proof of Proposition 7. Substituting $\check{\chi}_e = mh$, $\check{\chi}_f = h$ and $\check{\tau}_e = \check{\tau}_f = 0$ into (18) and (19) and differentiating with respect to m yields

$$\bar{y}_e = \bar{y}_f = \frac{a-h}{b+c} - \frac{m(m-1)h}{n(b+c)}, \quad \frac{d\bar{y}_e}{dm} = \frac{d\bar{y}_f}{dm} = -\frac{(2m-1)h}{n(b+c)} < 0, \quad (\text{A.27a})$$

$$\bar{x}_e = \frac{a-mh}{b+c} - \frac{(n-m)(m-1)bh}{nc(b+c)}, \quad \frac{d\bar{x}_e}{dm} = -\frac{[(n+1-2m)b+nc]h}{nc(b+c)}, \quad (\text{A.27b})$$

$$\bar{x}_f = \frac{a-h}{b+c} - \frac{m(m-1)bh}{nc(b+c)}, \quad \frac{d\bar{x}_f}{dm} = \frac{(2m-1)bh}{nc(b+c)} > 0, \quad (\text{A.27c})$$

$$\bar{x} = \frac{na - (n+m^2-m)h}{b+c}, \quad \frac{d\bar{x}}{dm} = -\frac{(2m-1)h}{b+c} < 0. \quad (\text{A.27d})$$

Country i 's welfare is given by $W_i = ay_i - \frac{b}{2}y_i^2 - \frac{c}{2}x_i^2 - p(y_i - x_i) - hx$. Substituting (A.27) yields

$$\bar{W}_e - W_i^\circ = \frac{(m-1)^2bh^2}{2c(b+c)} \left[\frac{c}{b} - \frac{(n-m)[n(m+1) + m(m-1)]}{n^2(m-1)} \right], \quad (\text{A.28a})$$

$$\bar{W}_f - W_i^\circ = \frac{m(m-1)bh^2}{c(b+c)} \left[\frac{c}{b} + \frac{m(m-1) + 2n}{2n^2} \right] > 0, \quad (\text{A.28b})$$

$$\bar{W}_f - \hat{W}_e = -\frac{(m^2-1)h^2}{2c} < 0, \quad (\text{A.28c})$$

$$\bar{W} - W^\circ = \frac{m(m-1)(2n-m-1)bh^2}{2c(b+c)} \left[\frac{c}{b} - \frac{(n-m)(m-1)}{n(2n-m-1)} \right], \quad (\text{A.28d})$$

where $\bar{W}_e \geq W_i^\circ \implies \bar{W} > W^\circ$. Differentiating (A.28) with respect to m yields

$$\frac{\partial \bar{W}_e}{\partial m} = \frac{(m-1)bh^2}{c(b+c)} \left[\frac{c}{b} - \frac{(n-m)[nm + (m-1)^2] - m^2(m-1)}{n^2(m-1)} \right], \quad (\text{A.29a})$$

$$\frac{\partial \bar{W}_f}{\partial m} = \frac{(2m-1)bh^2}{c(b+c)} \left[\frac{c}{b} + \frac{n + m(m-1)}{n^2} \right] > 0, \quad (\text{A.29b})$$

$$\frac{\partial \bar{W}_f}{\partial m} - \frac{\partial \bar{W}_e}{\partial m} = -\frac{mh^2}{c} < 0, \quad (\text{A.29c})$$

$$\frac{\partial \bar{W}}{\partial m} = \frac{[(m-1)^2 + 2(2m-1)(n-m)]bh^2}{2c(b+c)} \left[\frac{c}{b} - \frac{(m-1)[(3m-1)(n-m) - m(m-1)]}{n[(m-1)^2 + 2(2m-1)(n-m)]} \right]. \quad (\text{A.29d})$$

Substituting (A.28a) and (A.28b) into $\Phi(m) = W_e(m) - W_f(m-1)$ yields

$$\Phi(m) = -\frac{(m-1)bh^2}{2c(b+c)} \left[\frac{(n-m)[(m+1)(m+n) - 4] + (m-2)(m^2 - m + 2)}{n^2} + (m-3)\frac{c}{b} \right], \quad (\text{A.30})$$

$$\Phi(2) = -\frac{bh^2}{2c(b+c)} \left[\frac{(n-2)(3n+2)}{n^2} - \frac{c}{b} \right]. \quad (\text{A.31})$$

$m \geq 3 \implies \Phi(m) < 0$, such that no coalition of three or more countries is internally stable and the coalition of two countries is externally stable.

Substituting (A.17) and (A.28d) into $\Delta \bar{W}(\bar{m} = 2)$ yields

$$\Delta \bar{W}(\bar{m} = 2) = \frac{4n-6}{n(n-1)^2} \left[1 - \frac{n-2}{n(2n-3)} \frac{b}{c} \right] \leq \frac{4n-6}{n(n-1)^2}. \quad (\text{A.32})$$

Since $\check{W}_i > W^\circ$ holds, $\bar{W}_i \geq \check{W}_i$ can only hold if $\bar{m} = 2$ holds. Using (A.18) and (A.28) yields

$$\begin{aligned} \check{W}_e - \bar{W}_e(\bar{m} = 2) &= \frac{n^2(n + \check{m})\check{m}(\check{m} - 2)ch^2}{2n^2(n + \check{m})c(b+c)} \\ &+ \frac{2[(n - \check{m})(3\check{m}^2 + 3\check{m}n + 2n^2 - 4\check{m} - 2n - 2) + \check{m}(\check{m} - 2)(3\check{m} + 2)]bh^2}{2n^2(n + \check{m})c(b+c)}, \end{aligned} \quad (\text{A.33a})$$

$$\begin{aligned} \check{W}_f - \bar{W}_f(\bar{m} = 2) &= \frac{2n^2(n + \check{m})^2(\check{m} + 1)(\check{m} - 2)ch^2}{2n^2(n + \check{m})^2c(b+c)} \\ &- \frac{(2n + n\check{m} + 2\check{m})(2n^2 + 2n + n\check{m} + 2\check{m})bh^2}{2n^2(n + \check{m})^2c(b+c)}, \end{aligned} \quad (\text{A.33b})$$

$$\begin{aligned} \check{W} - \bar{W}(\bar{m} = 2) &= \frac{n(n + \check{m})^2(\check{m} - 2)[2(n - \check{m})(\check{m} + 1) + \check{m}^2 - 3]ch^2}{2n^2(n + \check{m})^2c(b + c)} \\ &\quad - \frac{[(n - \check{m})(\check{m}n^2 - 8\check{m}^2 - 6\check{m}n - 2n^2 + 12\check{m} + 4n) - 8\check{m}^2(\check{m} - 2)]bh^2}{2n^2(n + \check{m})^2c(b + c)}. \end{aligned} \quad (\text{A.33c})$$

(A.33a) is positive for $\check{m} \geq 3$, such that $\bar{W}_e < \check{W}_e$ holds. (A.33b) and (A.33c) are positive for a sufficiently great value of c . (A.31) defines a lower bound of c for $\bar{m} = 2$ to hold. Substituting this bound into (A.33b) and (A.33c) yields

$$\begin{aligned} \check{W}_f - \bar{W}_f(\bar{m} = 2) \Big|_{c=(n-2)(3n+2)b/n^2} &= \frac{bh^2}{2n^2(n + \check{m})^2c(b + c)} \\ &\left\{ (6M^2 + 30M + 24)(n - \check{m})^4 + (36M^3 + 280M^2 + 642M + 390)(n - \check{m})^3 \right. \\ &\quad + (78M^4 + 818M^3 + 3019M^2 + 4520M + 2169)(n - \check{m})^2 \\ &\quad + (72M^5 + 944M^4 + 4728M^3 + 11144M^2 + 12048M + 4536)(n - \check{m}) \\ &\quad \left. + 24M^6 + 376M^5 + 2349M^4 + 7388M^3 + 12054M^2 + 9252M + 2349 \right\}, \end{aligned} \quad (\text{A.34})$$

$$\begin{aligned} \check{W} - \bar{W}(\bar{m} = 2) \Big|_{c=(n-2)(3n+2)b/n^2} &= \frac{bh^2}{2n^2(n + \check{m})^2c(b + c)} \\ &\left\{ (6M^2 + 30M + 24)(n - \check{m})^5 + (39M^3 + 301M^2 + 679M + 417)(n - \check{m})^4 \right. \\ &\quad + (96M^4 + 994M^3 + 3589M^2 + 5284M + 2601)(n - \check{m})^3 \\ &\quad + (111M^5 + 1431M^4 + 6986M^3 + 16058M^2 + 17255M + 6903)(n - \check{m})^2 \\ &\quad + (60M^6 + 920M^5 + 5591M^4 + 17156M^3 + 27878M^2 + 22668M + 7263)(n - \check{m}) \\ &\quad \left. + 12M^7 + 212M^6 + 1528M^5 + 5800M^4 + 12476M^3 + 15204M^2 + 9792M + 2592 \right\}, \end{aligned} \quad (\text{A.35})$$

where $M = \check{m} - 3$. (A.34) and (A.35) are positive for $\check{m} \geq 3$, such that $\bar{W}_f < \check{W}_f$ and $\bar{W} < \check{W}$ hold. \square

A.4 Proofs of Section 4.2

Proof of Proposition 8. In equilibrium, each coalition country's imports and the difference between the coalition's import tax and the fringe's import tax are given by

$$\hat{y}_e - \hat{x}_e = -\frac{(n - m)[(b + c)(\hat{\tau}_e - \hat{\tau}_f) - b(\hat{\chi}_e - \hat{\chi}_f)]}{nbc}, \quad (\text{A.36})$$

$$\hat{\tau}_e - \hat{\tau}_f = \frac{(m - 1)bH'(\hat{x})}{b + c} + \frac{nm bc(\hat{y}_e - \hat{x}_e)}{(n - 1)(n - m)(b + c)}. \quad (\text{A.37})$$

Using $\hat{\chi}_e = mH'(\hat{x})$ and $\hat{\chi}_f = H'(\hat{x})$ in (A.36) and (A.37) and solving for $\hat{y}_e - \hat{x}_e$ and $\hat{\tau}_e - \hat{\tau}_f$ yields

$$\hat{y}_e - \hat{x}_e = 0, \quad (\text{A.38})$$

$$\hat{\tau}_e - \hat{\tau}_f = \frac{(m - 1)bH'(\hat{x})}{b + c} > 0. \quad (\text{A.39})$$

Substituting $\hat{\chi}_e = mH'(\hat{x})$, $\hat{\chi}_f = H'(\hat{x})$ and $\hat{y}_e - \hat{x}_e = \hat{y}_f - \hat{x}_f = 0$ into (18) and differentiating with respect to m yields

$$\hat{y}_e = \hat{x}_e = \frac{a - mH'(\hat{x})}{b + c}, \quad \frac{d\hat{y}_e}{dm} = \frac{d\hat{x}_e}{dm} = -\frac{[b + c + (n - m^2)H''_i(\hat{x})]h}{(b + c)[b + c + (n + m^2 - m)H''_i(\hat{x})]}, \quad (\text{A.40a})$$

$$\hat{y}_f = \hat{x}_f = \frac{a - H'(\hat{x})}{b + c}, \quad \frac{d\hat{y}_f}{dm} = \frac{d\hat{x}_f}{dm} = -\frac{H''(\hat{x})\frac{d\hat{x}}{dm}}{b + c} \geq 0, \quad (\text{A.40b})$$

$$\hat{x} = \frac{na - (n + m^2 - m)H'_i(\hat{x})}{b + c}, \quad \frac{d\hat{x}}{dm} = -\frac{(2m - 1)H'_i(\hat{x})}{b + c + (n + m^2 - m)H''_i(\hat{x})} < 0. \quad (\text{A.40c})$$

Since $\hat{x} < x^\circ$ and $\hat{y}_f = \hat{x}_f \geq y_e^\circ = x_f^\circ$ hold, we have $\hat{y}_e = \hat{x}_e < y_e^\circ = x_e^\circ$. \square

Proof of Footnote 31. Totally differentiating

$$B'_e(y_e) = B'_f(y_f) + \tau_e - \tau_f, \quad (\text{A.41a})$$

$$B'_e(y_e) = C'_e(x_e) + mH'_i(x), \quad (\text{A.41b})$$

$$B'_f(y_f) = C'_f(x_f) + H'_i(x), \quad (\text{A.41c})$$

$$my_e + (n - m)y_f = mx_e + (n - m)x_f, \quad (\text{A.41d})$$

for a coalition of a given size m , $B''_i = -b$ and $C''_i = c$, and solving for dy_i and dx_i yields

$$dy_e = -\frac{n - m}{nb} d(\tau_e - \tau_f), \quad dy_f = \frac{m}{nb} d(\tau_e - \tau_f), \quad dx_e = \frac{n - m}{nc} d(\tau_e - \tau_f), \quad dx_f = -\frac{m}{nc} d(\tau_e - \tau_f). \quad (\text{A.42})$$

Totally differentiating

$$W = m [B_e(y_e) - C_e(x_e)] + (n - m) [B_f(y_f) - C_f(x_f)] - nH_i(x), \quad (\text{A.43})$$

for a coalition of a given size m , and using (A.41) as well as (A.42) yields

$$\begin{aligned} dW &= m (B'_e dy_e - C'_e dx_e) + (n - m) (B'_f dy_f - C'_f dx_f) - nH'_i(x) dx \\ &= -m \frac{(n - m)(b + c)}{nbc} \left[\tau_e - \tau_f - \frac{(m - 1)bH'_i(x)}{b + c} \right] d(\tau_e - \tau_f), \end{aligned} \quad (\text{A.44})$$

such that $\tau_e - \tau_f = \frac{(m-1)bH'_i(x)}{b+c} = \arg \max W$ for a coalition of a given size m . \square

Proof of Proposition 10. Country i 's welfare is given by $W_i = ay_i - \frac{b}{2}y_i^2 - \frac{c}{2}x_i^2 - p(y_i - x_i) - hx_i$.

Substituting (A.40) for $H'(\hat{x}) = h$ and differentiating with respect to m yields

$$\hat{W}_e - W_i^\circ = \frac{(m - 1)^2 h^2}{2(b + c)} > 0, \quad \frac{\partial \hat{W}_e}{\partial m} = \frac{(m - 1)h^2}{b + c} > 0, \quad (\text{A.45a})$$

$$\hat{W}_f - W_i^\circ = \frac{m(m - 1)h^2}{b + c} > 0, \quad \frac{\partial \hat{W}_f}{\partial m} = \frac{(2m - 1)h^2}{b + c} > 0, \quad (\text{A.45b})$$

$$\hat{W}_f - \hat{W}_e = \frac{(m^2 - 1)h^2}{2(b + c)} > 0, \quad \frac{\partial \hat{W}_f}{\partial m} - \frac{\partial \hat{W}_e}{\partial m} = \frac{mh^2}{b + c} > 0 \quad (\text{A.45c})$$

$$\hat{W} - W^\circ = \frac{m(m - 1)(2n - m - 1)h^2}{2(b + c)} > 0, \quad \frac{\partial \hat{W}}{\partial m} = \frac{[2(n - m)(2m - 1) + (m - 1)^2]h^2}{2(b + c)} > 0. \quad (\text{A.45d})$$

Substituting (A.45a) and (A.45b) into $\Phi(m) = W_e(m) - W_f(m-1)$ yields

$$\Phi(m) = \frac{(m-1)(3-m)h^2}{2(b+c)}, \quad \Phi(m-1) = -\frac{m(2-m)h^2}{2(b+c)}. \quad (\text{A.46})$$

$m \leq 3 \implies \Phi(m) \geq 0$, such that all coalitions of three or less countries are internally stable, and $m \geq 2 \implies \Phi(m-1) \leq 0$, such that all coalitions of two or more countries are externally stable.

Substituting (A.17) and (A.45d) into $\Delta\check{W}(\check{m} = 2)$ and $\Delta\check{W}(\check{m} = 3)$ yields

$$\Delta\hat{W}(\hat{m} = 2) = \frac{4n-6}{n(n-1)^2}, \quad \Delta\hat{W}(\hat{m} = 3) = \frac{12n-24}{n(n-1)^2}. \quad (\text{A.47})$$

Using (A.18) and (A.45) yields

$$\check{W}_e - \hat{W}_e(\hat{m} = 2) = \frac{[\check{m}(\check{m}-2)(n+\check{m})c + b(n-\check{m})]h^2}{2(n+\check{m})c(b+c)}, \quad (\text{A.48a})$$

$$\check{W}_e - \hat{W}_e(\hat{m} = 3) = \frac{[(\check{m}+1)(\check{m}-3)(n+\check{m})c + b(n-\check{m})]h^2}{2(n+\check{m})c(b+c)}, \quad (\text{A.48b})$$

$$\check{W}_f - \hat{W}_f(\hat{m} = 2) = \frac{[2(\check{m}+1)(\check{m}-2)(n+\check{m})^2c - b\check{m}(2n+\check{m})]h^2}{2(n+\check{m})^2c(b+c)}, \quad (\text{A.48c})$$

$$\check{W}_f - \hat{W}_f(\hat{m} = 3) = \frac{[2(\check{m}+2)(\check{m}-3)(n+\check{m})^2c - b\check{m}(2n+\check{m})]h^2}{2(n+\check{m})^2c(b+c)}, \quad (\text{A.48d})$$

$$\check{W} - \hat{W}(\hat{m} = 2) = \frac{\{(\check{m}-2)(\check{m}+n)^2[2(\check{m}+1)(n-\check{m}) + \check{m}^2 - 3]c - n\check{m}(n-\check{m})b\}h^2}{2(n+\check{m})^2c(b+c)}, \quad (\text{A.48e})$$

$$\check{W} - \hat{W}(\hat{m} = 3) = \frac{\{(\check{m}-3)(\check{m}+n)^2[2(\check{m}+2)(n-\check{m}) + \check{m}^2 + \check{m} - 8]c - n\check{m}(n-\check{m})b\}h^2}{2(n+\check{m})^2c(b+c)}. \quad (\text{A.48f})$$

(A.48a) and (A.48b) are positive for $\check{m} \geq 3$, such that $\hat{W}_e < \check{W}_e$ holds. (A.48d) and (A.48f) are negative for $\check{m} = 3$, such that $\hat{W}_f(\hat{m} = 3) > \check{W}_f(\check{m} = 3)$ and $\hat{W}(\hat{m} = 3) > \check{W}(\check{m} = 3)$ hold. (A.48c) and (A.48e), for $\hat{m} \geq 3$, as well as (A.48d) and (A.48f), for $\hat{m} \geq 4$, are positive for a sufficiently great value of c . (A.22) defines a lower bound of c for $\Phi(\check{m}+1) \leq 0$ to hold. Substituting this bound into (A.48c) to (A.48f) yields

$$\begin{aligned} \check{W}_f - \hat{W}_f(\hat{m} = 2) \Big|_{c = \frac{n[n^2+(3\check{m}-1)n+2\check{m}^2]b}{\check{m}(\check{m}-2)(n+\check{m})^2(n+\check{m}-1)}} &= \frac{bh^2}{2\check{m}(n+\check{m}+1)(n+\check{m})^2c(b+c)} \\ &\left\{ (2M+8)(n-\check{m})^3 + (10M^2+70M+118)(n-\check{m})^2 \right. \\ &\quad + (15M^3+151M^2+497M+537)(n-\check{m}) \\ &\quad \left. + 6M^4+79M^3+385M^2+825M+657 \right\}, \end{aligned} \quad (\text{A.49a})$$

$$\begin{aligned} \check{W}_f - \hat{W}_f(\hat{m} = 3) \Big|_{c = \frac{n[n^2 + (3\check{m}-1)n + 2\check{m}^2]b}{\check{m}(\check{m}-2)(n+\check{m})^2(n+\check{m}-1)}} &= \frac{bh^2}{2\check{m}(\check{m}-2)(n+\check{m}+1)(n+\check{m})^2c(b+c)}. \\ \left\{ (2N^2 + 14N + 12)(n - \check{m})^3 + (10N^3 + 110N^2 + 330N + 212)(n - \check{m})^2 \right. \\ &+ (15N^4 + 226N^3 + 1148N^2 + 2200N + 1056)(n - \check{m}) \\ &\left. + 6N^5 + 115N^4 + 816N^3 + 2604N^2 + 3424N + 960 \right\}, \end{aligned} \quad (\text{A.49b})$$

$$\begin{aligned} \check{W} - \hat{W}(\hat{m} = 2) \Big|_{c = \frac{n[n^2 + (3\check{m}-1)n + 2\check{m}^2]b}{\check{m}(\check{m}-2)(n+\check{m})^2(n+\check{m}-1)}} &= \frac{nbh^2}{2\check{m}(n+\check{m}+1)(n+\check{m})^2c(b+c)}. \\ \left\{ (2M + 8)(n - \check{m})^3 + (10M^2 + 68M + 109)(n - \check{m})^2 \right. \\ &+ (15M^3 + 143M^2 + 436M + 429)(n - \check{m}) \\ &\left. + 6M^4 + 71M^3 + 297M^2 + 516M + 306 \right\}, \end{aligned} \quad (\text{A.49c})$$

$$\begin{aligned} \check{W} - \hat{W}(\hat{m} = 3) \Big|_{c = \frac{n[n^2 + (3\check{m}-1)n + 2\check{m}^2]b}{\check{m}(\check{m}-2)(n+\check{m})^2(n+\check{m}-1)}} &= \frac{nbh^2}{2\check{m}(\check{m}-2)(n+\check{m}+1)(n+\check{m})^2c(b+c)}. \\ \left\{ (2N^2 + 14N + 12)(n - \check{m})^3 + (10N^3 + 108N^2 + 315N + 208)(n - \check{m})^2 \right. \\ &+ (15N^4 + 218N^3 + 1055N^2 + 1959N + 1044)(n - \check{m}) \\ &\left. + 6N^5 + 107N^4 + 688N^3 + 1979N^2 + 2496N + 1104 \right\}, \end{aligned} \quad (\text{A.49d})$$

where $M = \check{m} - 3$ and $N = \check{m} - 4$. (A.49a) and (A.49c) are positive for $\check{m} \geq 3$, such that $\hat{W}_f(\hat{m} = 2) < \check{W}_f$ and $\hat{W}(\hat{m} = 2) < \check{W}$ hold, and (A.49b) and (A.49d) are positive for $\check{m} \geq 4$, such that $\hat{W}_f(\hat{m} = 3) < \check{W}_f(\check{m} \geq 4)$ and $\hat{W}(\hat{m} = 3) < \check{W}(\check{m} \geq 4)$ hold.

Using (A.28) and (A.45) yields

$$\hat{W}_e - \bar{W}_e(\bar{m} = 2) = \frac{[n^2m(m-2)c + (n-2)(3n+2)b]h^2}{2n^2c(b+c)}, \quad (\text{A.50a})$$

$$\hat{W}_f - \bar{W}_f(\bar{m} = 2) = \frac{[n^2(m+1)(m-2)c - 2(n+1)b]h^2}{n^2c(b+c)}, \quad (\text{A.50b})$$

$$\hat{W} - \bar{W}(\bar{m} = 2) = \frac{\{n(m-2)[2(m+1)(n-m) + m^2 - 3]c + 2(n-2)b\}h^2}{2nc(b+c)}. \quad (\text{A.50c})$$

(A.50a) and (A.50c) are positive for $\hat{m} \geq 2$, such that $\hat{W}_e > \bar{W}_e$ and $\hat{W} > \bar{W}$ hold. (A.50b) is negative for $\hat{m} \leq 2$, such that $\hat{W}_f(\hat{m} = 2) < \bar{W}_f(\bar{m} = 2)$ holds. (A.50b) is positive for a sufficiently great value of c . (A.31) defines a lower bound of c for $\bar{m} = 2$ to hold. Substituting this bound into (A.50b) yields

$$\begin{aligned} \hat{W}_f - \bar{W}_f(\bar{m} = 2) \Big|_{c = (n-2)(3n+2)b/n^2} &= \frac{bh^2}{n^2c(b+c)}. \\ \left\{ (3M^2 + 15M + 12)(n - \check{m})^2 + (6M^3 + 44M^2 + 94M + 54)(n - \check{m}) \right. \\ &\left. + 3M^4 + 29M^3 + 93M^2 + 109M + 36 \right\}, \end{aligned} \quad (\text{A.51})$$

where $M = \check{m} - 3$. (A.51) is positive for $\check{m} \geq 3$, such that $\hat{W}_f(\hat{m} = 3) > \bar{W}_f$ holds. \square

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