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**Article**

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## An Asynchronous Regime Switching GO GARCH Model for Optimal Futures Hedging

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### ABSTRACT

In this paper, an asynchronous Markov regime switching generalized orthogonal GARCH (ARSGO) model for optimal futures hedging is proposed. The proposed ARSGO is a regime switching GO GARCH such that different financial variables are governed by different state variables with the dependence of switching captured by a synchronization factor. Different from the conventional single-state-variable regime switching GO GARCH (RSGO), the multiple-state-variable ARSGO is more flexible in capturing the time-varying state-dependent correlation between spot and futures returns. ARSGO is applied to TAIEX futures to cross hedge the spot exposure of Taiwan stock sector indices. The empirical results reveal that the hedging effectiveness of ARSGO is superior to its nested models including the state-independent generalized orthogonal GARCH (GO) and the conventional single-state-variable RSGO models in terms of variance reductions and utility gains.

*Keywords: Asynchronous Markov switching; GO GARCH; Minimum variance hedge ratio; Index futures*

### 1. Introduction

Risk management on extreme price movement has received substantial attention in the literature. Derivative securities serve as an important instrument for dealing volatility risk (Hsu and Lee; Kim, 2017; Lien et al., 2018; Lai et al., 2017; Thiyagarajan and Naresh, 2015). A considerable body of literature has investigated the futures hedging performance by applying a variety of GARCH models (Baillie & Myers, 1991; Carbonez et al., 2011; Chang et al., 2011; Choudhry, 2009; Cifarell & Paladino, 2015; Fernandez, 2008; Haigh

& Holt, 2000; Lee, 2009b; Lee & Yoder, 2007a; Lien et al., 2018; Lien & Yang, 2008; Moschini & Myers, 2002; Pan et al., 2014; Park & Shi, 2017; Sephton, 1993). The basic idea is to apply GARCH models to estimate the time-varying minimum variance hedge ratio (MVHR) by estimating the conditional second moments of spot and futures returns. Although GARCH models capture the time-varying properties of the joint distribution of spot and futures returns, it does not take account of the effect of changing market states.

In a series papers of Samo and Valente (2000, 2005a, 2005b), the relationship between spot and futures returns is found to be characterized by regime shifts. This means that the regime-switching property of the joint distribution of spot and futures returns must be considered in estimating the MVHR. A number of regime-switching GARCH models have been

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applied to estimate the regime-switching time-varying MVHR (Alizadeh et al., 2008; Alizadeh et al., 2015; Dark, 2015; Hsu & Lee, 2018; Lai et al., 2017; Lee, 2009a, 2009b, 2010; Lee & Yoder, 2007a, 2007b; Lien et al., 2018; Pan et al., 2014; Sheu & Lee, 2014). A general finding is that regime-switching GARCH models are superior to their state-independent counterparts in implementing futures hedging strategies.

Most of the regime-switching GARCH hedging models, however, apply a single-state-variable regime-switching model for estimating the state-dependent time-varying covariance dynamics of spot and futures returns. Single-state-variable regime-switching models assume a common switching dynamic for both the spot and futures returns. This excludes a priori the possibility that the spot and futures returns might follow different switching dynamics governed by different state variables. Sheu and Lee (2014) release this assumption and suggest a multi-chain Markov regime switching GARCH (MCSG) model for estimating the state-dependent time-varying minimum variance hedge ratios. MCSG model, however, still assume a constant correlation within each regime. In this paper, we propose an asynchronous Markov regime switching generalized orthogonal GARCH (ARSGO) model for optimal futures hedging. ARSGO is a regime switching GO GARCH such that different financial variables are governed by different state variables with the dependence of switching captured by a synchronization factor. In addition, the correlation dynamic in each regime is time-varying in ARSGO which is more flexible than the correlation dynamic in MCSG. The proposed ARSGO is applied to TAIEX futures to cross hedge the spot exposure of Taiwan stock sector indices.

The contribution of this paper is twofold: first, an asynchronous Markov regime switching generalized orthogonal GARCH (ARSGO) model is proposed which incorporates the framework of asynchronous switching properties (Bengoechea et al., 2006; Camacho and Perez-Quiros, 2006) with generalized orthogonal GARCH (GO) model (van der Weide, 2002). ARSGO is an extension of the regime switching generalized orthogonal GARCH model proposed by Lee (2009a)

such that different data series are governed by different state variables and the correlation dynamic in each regime is time-varying. Second, ARSGO is applied to TAIEX futures to cross hedge the spot exposure of Taiwan stock sector indices and the empirical results reveal that the hedging effectiveness of the more flexible ARSGO is superior to its nested models including the state-independent generalized orthogonal GARCH (GO) and the conventional single-state-variable RSGO.

The remainder of the paper is organized as follows: The specification of the asynchronous Markov regime switching generalized orthogonal GARCH (ARSGO) is presented in Section II. Section III provides the recombining procedure and regime-switching filtering algorithm for ARSGO. This is followed by measuring hedging performance, data description, and empirical results in Section IV. The paper concludes with a summarization of the findings in Section V.

## II. Asynchronous Markov Regime-switching Generalized Orthogonal GARCH (ARSGO)

The proposed asynchronous Markov regime switching generalized orthogonal GARCH (ARSGO) model which incorporates the framework of asynchronous switching properties with GO GARCH such that different data series are governed by different state variables and the dependence of switching is captured by a synchronization factor.<sup>1)</sup> The specifications of

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1) ARSGO is different from Otranto's (2005) multi-chain Markov switching (MCMS) model and Sheu and Lee's (2014) multi-chain Markov regime switching GARCH (MCSG) model in two aspects: First, MCMS and MCSG capture the switching dependence with lagged transition variables entering into the transition probability. ARSGO, however, captures the dependence of switching with a synchronization factor which measures explicitly the weight of synchronous regimes. Second, ARSGO embedded van der Weide's GO GARCH (2002) with an asynchronous switching framework and capture the state-dependent time-varying correlation dynamic with a state-dependent mapping matrix. MCMS and MCSG, however, estimate models with a constant correlation

the ARSGO are as follows:

Let  $\mathbf{R}_t = \begin{bmatrix} r_{c,t} \\ r_{f,t} \end{bmatrix}$  denotes a  $2 \times 1$  vector of returns with  $r_{c,t}$  and  $r_{f,t}$  being the stock sector index returns and stock index futures returns, respectively with the conditional mean equations given by

$$r_{c,t} = \mu_{c,s_{c,t}} + e_{c,s_{c,t}}, \quad (1)$$

$$r_{f,t} = \mu_{f,s_{f,t}} + e_{f,s_{f,t}}, \quad (2)$$

where  $s_{c,t}$  and  $s_{f,t}$  are respectively the state variables governing the regime-shifting dynamics of stock sector index returns and stock index futures returns.

To complete the dynamic specification of the process, a new state variable  $\mathbf{S}_t$  is defined and is given by

$$\mathbf{S}_t = \begin{cases} 1 & \text{if } s_{c,t} = 1 \text{ and } s_{f,t} = 1 \\ 2 & \text{if } s_{c,t} = 1 \text{ and } s_{f,t} = 2 \\ 3 & \text{if } s_{c,t} = 2 \text{ and } s_{f,t} = 1 \\ 4 & \text{if } s_{c,t} = 2 \text{ and } s_{f,t} = 2. \end{cases} \quad (3)$$

The four-state matrix of transition probability is defined as

$$\mathbf{P}_s = \begin{bmatrix} P_{S,11} & P_{S,21} & P_{S,31} & P_{S,41} \\ P_{S,12} & P_{S,22} & P_{S,32} & P_{S,42} \\ P_{S,13} & P_{S,23} & P_{S,33} & P_{S,43} \\ P_{S,14} & P_{S,24} & P_{S,34} & P_{S,44} \end{bmatrix}, \quad (4)$$

whose columns sum to unity and  $P_{S,ij} = P(\mathbf{S}_t = j | \mathbf{S}_{t-1} = i)$  stands for transition probability of being in state  $i$  at time  $t-1$  and in state  $j$  at time  $t$ . With the definition above, the matrix form of the mean equation is given by

$$\mathbf{R}_t = \boldsymbol{\mu}_s + \mathbf{e}_{t,s_t}, \quad (5)$$

where  $\boldsymbol{\mu}_s = [\mu_{c,s_{c,t}} \quad \mu_{f,s_{f,t}}]^T$  is a  $2 \times 1$  vector of state-dependent conditional means and

$$\mathbf{e}_{t,s_t} = \begin{bmatrix} e_{c,s_{c,t}} & e_{f,s_{f,t}} \end{bmatrix}' = \mathbf{Z}(\theta_{s_t}) \boldsymbol{\varepsilon}_{t,s_t} \quad (6)$$

is a state-dependent residual vector. The state-dependent linear mapping matrix  $\mathbf{Z}(\theta_{s_t})$  is given by

$$\mathbf{Z}(\theta_{s_t}) = \begin{pmatrix} 1 & 0 \\ \cos \theta_{s_t} & \sin \theta_{s_t} \end{pmatrix}, \quad (7)$$

where  $\theta_{s_t}$  is a state-dependent rotation angle capturing the correlation dynamic of stock sector index returns and stock index futures returns and is governed by both state variables of  $s_{c,t}$  and  $s_{f,t}$ .

The state-dependent residual vector  $\boldsymbol{\varepsilon}_{t,s_t}$  is assumed to be normally distributed

$$\boldsymbol{\varepsilon}_{t,s_t} | \psi_{t-1} \sim BN(\mathbf{0}, \mathbf{H}_{t,s_t}), \quad (8)$$

where  $BN$  stands for bivariate normal,  $\psi_{t-1}$  is information set available at time  $t-1$  and the time-varying state-dependent covariance matrix  $\mathbf{H}_{t,s_t}$  is specified as

$$\mathbf{H}_{t,s_t} = \text{diag}(h_{c,t,s_{c,t}}, h_{f,t,s_{f,t}}). \quad (9)$$

The volatility dynamics of spot and futures returns are assumed to follow a regime switching GARCH process given by

$$h_{c,t,s_{c,t}} = \gamma_{c,s_{c,t}} + \alpha_{c,s_{c,t}} e_{c,t-1}^2 + \beta_{c,s_{c,t}} h_{c,t-1}, \quad (10)$$

$$h_{f,t,s_{f,t}} = \gamma_{f,s_{f,t}} + \alpha_{f,s_{f,t}} e_{f,t-1}^2 + \beta_{f,s_{f,t}} h_{f,t-1,s_{f,t}}. \quad (11)$$

With the above definition, the vector  $\mathbf{R}_t$  is normally distributed with conditional mean  $\boldsymbol{\mu}_s$  and conditional covariance matrix  $\boldsymbol{\Omega}_{t,s_t}$ ,

$$\mathbf{R}_t | \psi_{t-1} \sim N(\boldsymbol{\mu}_s, \boldsymbol{\Omega}_{t,s_t}), \quad (12)$$

where

$$\boldsymbol{\Omega}_{t,s_t} = \mathbf{Z}(\theta_{s_t}) \mathbf{H}_{t,s_t} \mathbf{Z}(\theta_{s_t})'. \quad (13)$$

The state-dependent correlation coefficient  $\rho_{t,s_t}$  can be shown as

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within each regime.

$$\rho_{t,s_i} = \frac{\cos(\theta_{s_i}) h_{c,t,s_{c,j}}}{\sqrt{h_{c,t,s_{c,j}} [\cos^2(\theta_{s_i}) h_{c,t,s_{c,j}} + \sin^2(\theta_{s_i}) h_{f,t,s_{f,j}}]}} \quad (14)$$

The contemporaneous correlation between stock sector index returns and stock index futures returns  $\rho_{t,s_i}$  depends on both the state variables  $s_{c,t}$  and  $s_{f,t}$ . As a consequence, there are four possible time-varying correlation dynamics under high/high, high/low, low/high and low/low volatility regimes. With *ARSGO*, we can investigate the contemporaneous time-varying correlations between stock sector index returns and stock index futures returns under different regime combinations.

### III. Recombining Procedure and Regime-switching Filtering Algorithm

To solve the well-known path-dependency problem (Cai, 1994; Hamilton and Susmel, 1994; Gray, 1996) due to the recursive nature and regime switching property, Gray's recombining method (Gray, 1996) is applied for residuals and volatilities:

$$e_{c,t} = r_{c,t} - E[r_{c,t} | \psi_{t-1}] = r_{c,t} - \{p_{c,t} \mu_{c,s_{c,j}=1} + (1-p_{c,t}) \mu_{c,s_{c,j}=2}\}, \quad (15)$$

$$e_{f,t} = r_{f,t} - E[r_{f,t} | \psi_{t-1}] = r_{f,t} - \{p_{f,t} \mu_{f,s_{f,j}=1} + (1-p_{f,t}) \mu_{f,s_{f,j}=2}\}, \quad (16)$$

where  $p_{c,t} = P(s_{c,t} = 1 | \psi_{t-1})$  and  $p_{f,t} = P(s_{f,t} = 1 | \psi_{t-1})$  are regime probabilities of being in state 1 at time  $t$  for state variables  $s_{c,t}$  and  $s_{f,t}$ , respectively. The recombining process for the variance of stock sector returns is given by

$$\begin{aligned} Var(r_{c,t}) = & h_{c,t} = P(s_{c,t} = 1 | \psi_{t-1}) (\mu_{c,s_{c,t}=1}^2 + h_{c,t,s_{c,t}=1}) \\ & + (1 - P(s_{c,t} = 1 | \psi_{t-1})) (\mu_{c,s_{c,t}=2}^2 + h_{c,t,s_{c,t}=2}) \\ & - [P(s_{c,t} = 1 | \psi_{t-1}) \mu_{c,s_{c,t}=1} + (1 - P(s_{c,t} = 1 | \psi_{t-1})) \mu_{c,s_{c,t}=2}]^2. \quad (17) \end{aligned}$$

The recombining process for the variance of stock index futures returns is given by

$$\begin{aligned} Var(r_{f,t}) = & \sum_{i=1}^4 (\mu_{c,s_{c,j}=1}^2 + \cos^2(\theta_{s_{c,i}}) h_{c,t,s_{c,j}} + \sin^2(\theta_{s_{c,i}}) h_{f,t,s_{f,j}}) \times P(S_t = i | \psi_{t-1}) \\ & - [P(s_{f,t} = 1 | \psi_{t-1}) \mu_{f,s_{f,j}=1} + (1 - P(s_{f,t} = 1 | \psi_{t-1})) \mu_{f,s_{f,j}=2}]^2 \quad (18) \end{aligned}$$

Since the variance of stock index futures returns is equal to  $\cos^2(\theta) h_{c,t} + \sin^2(\theta) h_{f,t}$ , it follows that

$$h_{f,t} = \frac{Var(r_{f,t}) - \cos^2(\theta) h_{c,t}}{\sin^2(\theta)}, \quad (19)$$

where the rotation coefficients  $\theta$  is equal to the weighted average of  $\theta_{11}$ ,  $\theta_{12}$ ,  $\theta_{21}$  and  $\theta_{22}$ , weighted by the conditional state probability  $P(S_t = i | \psi_{t-1})$ .

The state-dependent volatilities  $h_{c,t,s_{c,j}}$  and  $h_{f,t,s_{f,j}}$  are driven by different state variables  $s_{c,t}$  and  $s_{f,t}$ , respectively and the state-dependent dynamic conditional correlation is governed by both state variables of  $s_{c,t}$  and  $s_{f,t}$ . In Lee's regime switching *GO GARCH* (2009a), all data series are governed by a single state variable, namely,  $s_{c,t} = s_{f,t}$ . Although the switching dynamic between stock sector index returns and stock index futures returns is normally positively correlated, the correlation is less than perfect. Following Bengoechea et al. (2006) and Camacho and Perez-Quiros (2006), in *ARSGO*, the comovement between stock sector index returns and stock index futures returns is assumed to be  $\delta_{cf}$  times the case of independent and  $(1-\delta_{cf})$  times the case of perfect dependent, where  $0 \leq \delta_{cf} \leq 1$ . The weight  $\delta_{cf}$  as a consequence measures the extent of desynchronization between stock sector index returns and stock index futures returns.

To incorporate the asynchronous factor  $\delta_{cf}$  into *ARSGO*, the following filtering algorithm is applied. If we collect all state probabilities in the vectors and define

$$\xi_{t/t-1} = \begin{bmatrix} P(S_t = 1 | \psi_{t-1}; \Theta) \\ P(S_t = 2 | \psi_{t-1}; \Theta) \\ P(S_t = 3 | \psi_{t-1}; \Theta) \\ P(S_t = 4 | \psi_{t-1}; \Theta) \end{bmatrix}, \quad \xi_{t/t} = \begin{bmatrix} P(S_t = 1 | \psi_t; \Theta) \\ P(S_t = 2 | \psi_t; \Theta) \\ P(S_t = 3 | \psi_t; \Theta) \\ P(S_t = 4 | \psi_t; \Theta) \end{bmatrix}, \quad (20)$$

where  $\Theta$  is a vector of system parameters to be estimated. The updated probability vector is given by

$$\hat{\xi}_{t|t-1} = \mathbf{P}_s \hat{\xi}_{t-1|t-1}, \quad (21)$$

where  $\mathbf{P}_s$  is the transition matrix. Because it has  $\delta_{cf}$  times for the case of independent and  $(1-\delta_{cf})$  times for the case of perfect dependent,  $\mathbf{P}_s$  is calculated as

$$\mathbf{P}_s = \delta_{cf} \mathbf{P}^I + (1-\delta_{cf}) \mathbf{P}^D, \quad (22)$$

Where  $\mathbf{P}^I$  and  $\mathbf{P}^D$  are the transition matrices in cases of independent and perfect synchronous, respectively. Substituting equation (22) into equation (21), we have

$$\begin{aligned} \hat{\xi}_{t|t-1} &= [\delta_{cf} \mathbf{P}^I + (1-\delta_{cf}) \mathbf{P}^D] \hat{\xi}_{t-1|t-1} \\ &= \delta_{cf} \hat{\xi}_{t|t-1}^I + (1-\delta_{cf}) \hat{\xi}_{t|t-1}^D, \end{aligned} \quad (23)$$

where  $\hat{\xi}_{t|t-1}^I = \mathbf{P}^I \hat{\xi}_{t-1|t-1}$  and  $\hat{\xi}_{t|t-1}^D = \mathbf{P}^D \hat{\xi}_{t-1|t-1}$  are the predicted probability vector for the cases of independent and perfect synchronous, respectively.

In the case when the switching dynamics of spot and futures returns are mutually independent, elements in the transition matrix  $\mathbf{P}_s$  are calibrated as a product of those for the independent chains governing  $s_{c,t}$  and  $s_{f,t}$ . For example,

$$P(\mathbf{S}_t = 1 | \mathbf{S}_{t-1} = 3) = P(s_{c,t} = 1 | s_{c,t-1} = 2) \times P(s_{f,t} = 1 | s_{f,t-1} = 1). \quad (24)$$

The forecasted probability vector in the case of mutually independent denoted as  $\xi_{t|t-1}^I$  is then given by:

$$\xi_{t|t-1}^I = \begin{bmatrix} P(s_{c,t} = 1 | \psi_{t-1}; \Theta) \times P(s_{f,t} = 1 | \psi_{t-1}; \Theta) \\ P(s_{c,t} = 1 | \psi_{t-1}; \Theta) \times P(s_{f,t} = 2 | \psi_{t-1}; \Theta) \\ P(s_{c,t} = 2 | \psi_{t-1}; \Theta) \times P(s_{f,t} = 1 | \psi_{t-1}; \Theta) \\ P(s_{c,t} = 2 | \psi_{t-1}; \Theta) \times P(s_{f,t} = 2 | \psi_{t-1}; \Theta) \end{bmatrix}, \quad (25)$$

As in the case when both spot and futures returns share a common pattern of regime switches, state 2 and 3 for  $\mathbf{S}_t$  are excluded. In this case, the transition matrix is reduced to a  $2 \times 2$  matrix and the forecasted probability vector in the case of perfect synchronization denoted as  $\xi_{t|t-1}^D$  is given by:

$$\xi_{t|t-1}^D = \begin{bmatrix} P(s_t = 1 | \psi_{t-1}; \Theta) \\ 0 \\ 0 \\ P(s_t = 2 | \psi_{t-1}; \Theta) \end{bmatrix}, \quad (26)$$

where  $s_t = s_{c,t} = s_{f,t}$ . To estimate the transition probability matrix, the following logistic functions are applied to calculate the transition probabilities for spot and futures returns.

$$P(s_{c,t} = 1 | s_{c,t-1} = 1) = \frac{\exp(p_{c,0})}{1 + \exp(p_{c,0})}, \quad P(s_{c,t} = 2 | s_{c,t-1} = 2) = \frac{\exp(q_{c,0})}{1 + \exp(q_{c,0})}, \quad (27)$$

$$P(s_{f,t} = 1 | s_{f,t-1} = 1) = \frac{\exp(p_{f,0})}{1 + \exp(p_{f,0})}, \quad P(s_{f,t} = 2 | s_{f,t-1} = 2) = \frac{\exp(q_{f,0})}{1 + \exp(q_{f,0})}, \quad (28)$$

where  $p_{c,0}$ ,  $q_{c,0}$ ,  $p_{f,0}$ ,  $q_{f,0}$  are estimated along with system parameters. The filtered probabilities are updated according to the following formula:

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t-1} \circ \boldsymbol{\eta}_t}{\mathbf{1}'(\hat{\xi}_{t|t-1} \circ \boldsymbol{\eta}_t)}, \quad (29)$$

where  $\mathbf{1}$  is a  $2 \times 1$  vector of ones,  $\circ$  denotes elements-by-elements and  $\boldsymbol{\eta}_t$  is a vector of conditional density given by

$$\boldsymbol{\eta}_t = \begin{bmatrix} f(\mathbf{R}_t | \mathbf{S}_t = 1) \\ f(\mathbf{R}_t | \mathbf{S}_t = 2) \\ f(\mathbf{R}_t | \mathbf{S}_t = 3) \\ f(\mathbf{R}_t | \mathbf{S}_t = 4) \end{bmatrix}. \quad (30)$$

The density of spot and futures returns conditional on past observations and being in regime  $\mathbf{S}_t$  at time  $t$  is denoted as

$$\begin{aligned} f(\mathbf{R}_t | \mathbf{S}_t = j, \psi_{t-1}; \Theta) &= \frac{1}{(2\pi) \left| \mathbf{Z}(\theta_s) \mathbf{H}_{t-1, \mathbf{S}_t} \mathbf{Z}(\theta_s) \right|^{1/2}} \times \\ &\exp \left\{ -\frac{1}{2} (\mathbf{R}_t - \boldsymbol{\mu}')' \mathbf{Z}(\theta_s) \mathbf{H}_{t-1, \mathbf{S}_t} \mathbf{Z}(\theta_s)^{-1} (\mathbf{R}_t - \boldsymbol{\mu}') \right\} \end{aligned} \quad (31)$$

The mixture likelihood is the weighted average of conditional densities given by

$$f(\mathbf{R}_t | \psi_{t-1}; \Theta) = \sum_{i=1}^4 f(\mathbf{R}_t | \mathbf{S}_t = j, \psi_{t-1}; \Theta) \times P(\mathbf{S}_t = j | \psi_{t-1}; \Theta). \quad (32)$$

The unknown parameters in ARSGO are  $\Theta = \{\delta_{cf}, P_{c,0}, q_{c,0}, P_{f,0}, q_{f,0}, \mu_{c,s_{c,t}}, \mu_{f,s_{f,t}}, \gamma_{c,s_{c,t}}, \alpha_{c,s_{c,t}}, \beta_{c,s_{c,t}}, \gamma_{f,s_{f,t}}, \alpha_{f,s_{f,t}}, \beta_{f,s_{f,t}}, \theta_{s_t}\}$  for  $s_{c,t} \in \{1,2\}$ ,  $s_{f,t} \in \{1,2\}$  and  $\mathbf{S}_t \in \{1,2,3,4\}$ , which can be estimated by maximizing the following log-likelihood function

$$LL = \sum_{t=1}^T \log[f(\mathbf{R}_t | \psi_{t-1}; \Theta)], \quad (33)$$

where  $f(\mathbf{R}_t | \psi_{t-1}; \Theta)$  is defined in equation (32).

#### IV. Measuring Hedging Performance, Data Description, and Empirical Results

Let  $r_{p,t} = r_{c,t} - \hat{\chi}_{t|t-1} r_{f,t}$  be the hedged portfolio return constructed by shorting  $\hat{\chi}_{t|t-1}$  units of futures for each unit of spot exposure, where  $\hat{\chi}_{t|t-1}$  is the conditional hedge ratio estimated at time  $t-1$  and to be held over the time period  $[t-1, t]$ . A hedger chooses a hedging strategy to minimize the variance of the hedged portfolio return or equivalently maximize the percentage variance reduction compared with the unhedged position. The hedging effectiveness (HE) is defined as

$$HE = \frac{Var(r_{c,t}) - Var(r_{p,t})}{Var(r_{c,t})} \times 100, \quad (34)$$

The hedge ratios  $\hat{\chi}_{t|t-1}$  is given by

$$\hat{\chi}_{t|t-1} = \frac{Cov(r_{c,t}, r_{f,t} | \psi_{t-1})}{Var(r_{f,t} | \psi_{t-1})}, \quad (35)$$

where  $Cov(r_{c,t}, r_{f,t} | \psi_{t-1})$  is the conditional covariance of spot and futures returns and  $Var(r_{f,t} | \psi_{t-1})$  is the

conditional variance of futures return estimated with GARCH models. Suppose that a hedger has the following mean-variance expected utility function:

$$E[U(r_{p,t} | \psi_{t-1})] = E[r_{p,t} | \psi_{t-1}] - \kappa Var(r_{p,t} | \psi_{t-1}), \quad (36)$$

where  $\kappa$  stands for the degree of risk aversion. From a utility-maximizing standpoint, a hedger chooses a hedging strategy that maximizes the expected utility.

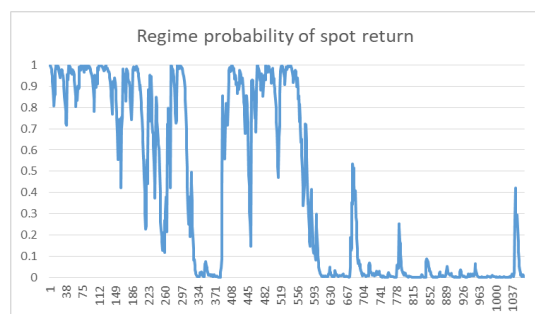
The proposed asynchronous Markov regime switching generalized orthogonal GARCH (ARSGO) model is applied to nearby contracts of TAIEX futures to hedge the spot exposure of Taiwan stock sector indices including food, textiles, electric & machinery, automobile, and glass products. All data series are from July 1998 to December 2018 obtained from Datastream. The weekly returns of each price series are computed as the changes in the natural logarithms of prices multiplied by 100 using Wednesday closing prices. The estimation of all the models are conducted using data from July 1998 to December 2017, and the data from January 2018 to December 2018 are used for out-of-sample analysis. Summary statistics for spot and futures returns are shown in Table 1. All unconditional mean returns are small and less than 0.1%. The largest mean return is equal to 0.06% for the case of food sector and the smallest mean return is equal to -0.065% for the case of glass products sector. Glass products sector has the largest return volatility with a standard deviation of 4.349 and Electric & Machinery sector has the lowest return volatility with a standard deviation of 3.159. According to the skewness, leptokurtosis, and significant Jarque-Bera statistics, the unconditional distribution of spot and futures returns are all asymmetric, fat tailed, and non-Gaussian.

Table 2 shows the estimates of unknown parameters of ARSGO. In the transition probability equation,  $P_{c,0}$  and  $q_{c,0}$  are parameters of the logistic functions in equations (27) for spot return and  $P_{f,0}$  and  $q_{f,0}$  are parameters of the logistic functions in equations (28) for futures return. Most of the parameters in the transition probability equation are significant showing that it is important to model spot and futures returns

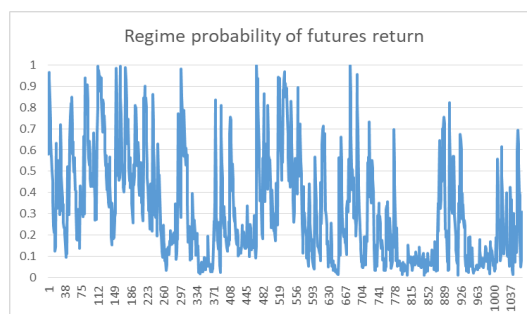
**Table 1.** Summary statistics of weekly returns (In percentage)

	Food	Textiles	Electric & Machinery	Automobile
Mean	0.060	0.034	0.026	0.009
Maximum	17.406	18.011	14.053	24.236
Minimum	-15.295	-16.402	-14.219	-16.175
Std. Dev.	3.540	3.840	3.159	3.907
Skewness	-0.134	-0.113	-0.424	0.296
Kurtosis	5.911	5.169	5.357	6.660
Jarque-Bera	379.549***	211.225***	278.662***	610.569***
	Glass Products	Transportation	TAIFEX Futures	
Mean	-0.065	-0.019	0.017	
Maximum	14.005	18.543	14.987	
Minimum	-18.507	-15.558	-14.211	
Std. Dev.	4.349	3.861	3.323	
Skewness	-0.312	0.091	-0.268	
Kurtosis	4.601	5.460	5.536	
Jarque-Bera	131.078***	-0.019***	298.290***	

Note: \*\*\* indicates significance at the 1% level and returns (in percentage) are calculated as the differences in the logarithm of prices multiplied by 100.



**Figure 1.** Regime probability of being in the high volatility state estimated with ARSGO for food sector index



**Figure 2.** Regime probability of being in the high volatility state estimated with ARSGO for TAIEX futures

with multiple state variables. Figures 1 and 2 show the regime probabilities of being in the high volatility state estimated with ARSGO for food sector index and TAIEX futures, respectively.

In the volatility equation, for a given  $\alpha + \beta$ , higher  $\gamma$  implies a higher volatility. Take glass products sector for instance,  $\gamma$  is respectively equal to 5.289 and 0.015 in state 1 and state 2 for spot return and is respectively equal to 1.580 and 0.011 in state 1 and state 2 for futures return. State 1 has higher  $\gamma$  and is the higher volatility state. The persistence in

volatility is measured with  $\alpha_{j,s_t} + \beta_{j,s_t}$ ,  $j \in \{c, f\}$ , and  $s_t \in \{1, 2\}$ . Take automobile sector for instance,  $\alpha_{c,1} + \beta_{c,1}$  and  $\alpha_{c,2} + \beta_{c,2}$  are respectively equal to 1.000 and 0.490 and  $\alpha_{f,1} + \beta_{f,1}$  and  $\alpha_{f,2} + \beta_{f,2}$  are respectively equal to 1.000 and 0.822. Because state 1 is the high-volatility state, automobile sector has higher volatility persistence in the high volatility state. For most of the cases, high-volatility state exhibits higher volatility persistence. Figures 3 and 4 respectively illustrate the state-dependent volatilities of food sector index and TAIEX futures



**Table 2.** Estimates of the unknown parameters of ARSGO

	Food	Textiles	Electric & Machinery		Food	Textiles	Electric & Machinery
	Transition probability Equation				Variance Equation		
$P_{c,0}$	5.746 (0.628)*** <sup>1</sup>	6.057 (1.380)***	1.046 (0.774)*	$\gamma_{c1}$	5.334 (1.958)***	6.308 (5.813)	9.951 (13.222)
$q_{c,0}$	6.722 (0.857)***	6.203 (1.140)***	3.043 (0.992)***	$\gamma_{c2}$	0.563 (0.240)***	0.253 (0.253)	0.000 (0.021)
$P_{f,0}$	2.164 (1.255)**	1.641 (0.984)**	-1.765 (6.064)	$\gamma_{f1}$	7.617 (3.483)**	11.493 (23.835)	5.432 (5.104)
$q_{f,0}$	3.291 (0.759)***	4.655 (1.598)***	-0.334 (0.756)	$\gamma_{f2}$	0.965 (0.501)**	0.735 (0.992)	0.000 (0.029)
	Mean Equation						
$\mu_{c1}^2$	-0.268 (0.213)	-0.086 (0.247)	-1.321 (1.015)*	$\alpha_{c1}$	0.054 (0.033)**	0.124 (0.089)*	0.000 (0.017)
$\mu_{c2}^2$	0.108 (0.086)	0.056 (0.069)	0.293 (0.117)***	$\alpha_{c2}$	0.024 (0.016)**	0.000 (0.025)	0.000 (0.024)
$\mu_{f1}$	-0.762 (0.450)**	-0.560 (0.806)	-1.277 (0.821)*	$\alpha_{f1}$	0.035 (0.056)	0.075 (0.178)	0.082 (0.135)
$\mu_{f2}$	0.275 (0.070)***	0.136 (0.081)**	0.347 (0.164)**	$\alpha_{f2}$	0.000 (0.016)	0.022 (0.050)	0.032 (0.058)
	Correlation Equation						
$\theta_{11}$	0.880 (0.106)***	0.682 (0.525)*	0.764 (0.078)***	$\beta_{c1}$	0.707 (0.103)***	0.669 (0.245)***	1.000 (0.953)
$\theta_{12}$	1.194 (0.083)***	0.972 (0.063)***	0.856 (0.280)***	$\beta_{c2}$	0.836 (0.050)***	0.915 (0.092)***	0.771 (0.063)***
$\theta_{21}$	0.504 (0.085)***	0.429 (0.210)**	0.603 (0.202)***	$\beta_{f1}$	0.965 (0.340)***	0.875 (0.582)*	0.918 (0.633)*
$\theta_{22}$	0.907 (0.048)***	0.969 (0.042)***	0.744 (0.051)***	$\beta_{f2}$	0.536 (0.110)***	0.643 (0.385)**X	0.809 (0.107)***
$\delta_{cf}$	0.950 (0.029)***	0.889 (0.044)***	0.140 (0.094)*				
$LL^3$	-4865.04	-4779.83	-4590.17				

1. Figures in parentheses are standard errors and \*, \*\* and \*\*\* indicate significance at the 10% level, 5% level and 1% level, respectively.  
 2. The subscripts 1 and 2 stand for state 1 and 2, respectively. State 1 is the higher volatility state.  
 3. LL stands for log likelihood.

estimated with ARSGO. The mean volatilities of food sector index and TAIEX futures are equal to 11.327 and 5.995, respectively, in the low volatility state and are equal to 14.845 and 17.065, respectively, in the

high-volatility state.

In the correlation equation, there are four possible correlation dynamics. Taking food for instance, when both spot and futures are in the high volatility regime,

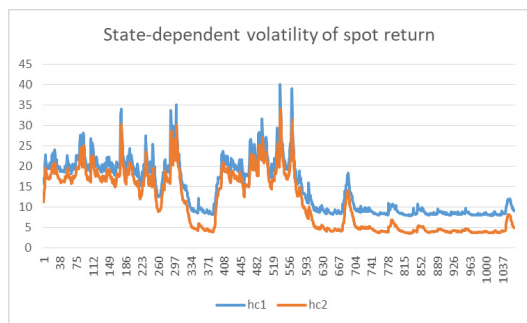
Table 2. Continued

	Automobile	Glass Products	Transportation		Automobile	Glass Products	Transportation
	Transition probability Equation				Variance Equation		
$p_0$	1.273 (0.256)*** <sup>1</sup>	1.341 (0.219)***	2.225 (1.044)**	$\gamma_{c1}$	8.252 (15.321)	5.289 (1.248)***	1.434 (1.469)
$q_0$	1.460 (0.295)***	0.324 (0.078)***	1.574 (0.524)***	$\gamma_{c2}$	0.000 (0.055)	0.015 (0.008)**	0.000 (0.033)
$p_0$	-3.247 (13.097)	-3.168 (0.981)***	-2.915 (4.175)	$\gamma_{f1}$	1.642 (2.498)	1.580 (0.550)***	0.928 (0.387)***
$q_0$	-1.859 (2.002)	-1.318 (0.197)***	-0.363 (1.155)	$\gamma_{f2}$	0.000 (0.113)	0.011 (0.003)***	0.056 (0.145)
Mean Equation							
$\mu_{c1}^2$	-0.165 (0.572)	0.409 (0.014)***	0.141 (0.537)	$\alpha_{c1}$	0.066 (0.085)	0.037 (0.020)**	0.000 (0.033)
$\mu_{c2}^2$	0.093 (0.586)	-0.324 (0.059)***	-0.163 (0.405)	$\alpha_{c2}$	0.016 (0.083)	0.250 (0.196)	0.350 (0.121)***
$\mu_{f1}$	-0.617 (0.305)**	-0.039 (0.059)	-0.161 (0.461)	$\alpha_{f1}$	0.137 (0.063)**	0.130 (0.031)***	0.086 (0.038)**
$\mu_{f2}$	0.491 (0.217)**	0.407 (0.015)***	0.368 (0.181)**	$\alpha_{f2}$	0.000 (0.144)	0.133 (0.020)***	0.112 (0.091)
Correlation Equation							
$\rho_{11}$	1.145 (0.093)***	1.178 (0.022)***	1.084 (0.067)***	$\beta_{c1}$	0.934 (0.818)	0.963 (0.215)***	1.000 (0.079)***
$\rho_{12}$	0.403 (0.909)	0.033 (0.022)*	0.489 (0.239)**	$\beta_{c2}$	0.474 (0.073)***	0.205 (0.029)***	0.531 (0.127)***
$\rho_{21}$	0.949 (2.394)	0.001 (0.000)**	0.188 (0.179)	$\beta_{f1}$	0.863 (0.373)**	0.870 (0.041)***	0.914 (0.189)***
$\rho_{22}$	0.896 (0.089)***	0.956 (0.093)***	0.832 (0.130)***	$\beta_{f2}$	0.822 (0.390)**	0.486 (0.195)***	0.592 (0.266)**
$\delta_{cf}$	0.042 (0.365)	0.038 (0.022)**	0.156 (0.207)				
$LL^3$	-4979.22	-5122.78	-4895.30				

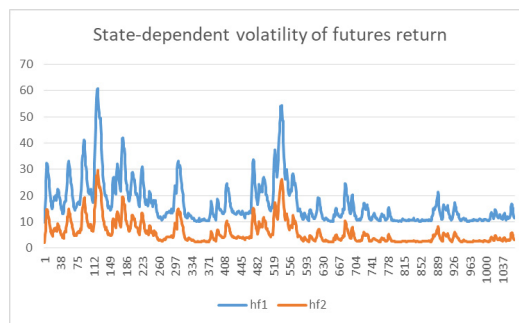
1. Figures in parentheses are standard errors and \*, \*\* and \*\*\* indicate significance at the 10% level, 5% level and 1% level, respectively.
2. The subscripts 1 and 2 stand for state 1 and 2, respectively. State 1 is the higher volatility state.
3.  $LL$  stands for log likelihood.

the mean correlation is equal to 0.609 and when both of them are in the low volatility regime, the mean correlation is equal to 0.723. When the spot return is in the high volatility regime and futures return

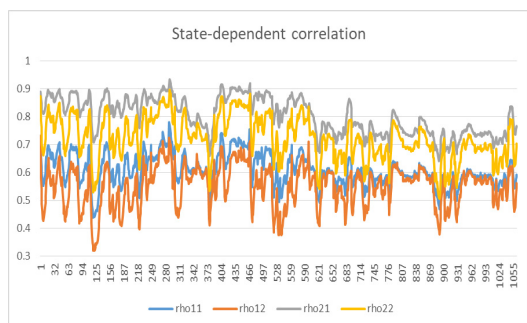
is in the low volatility regime, the mean correlation is equal to 0.550 and when the spot return is in the low volatility regime and futures return is in the high volatility regime, the mean correlation is equal to 0.802.



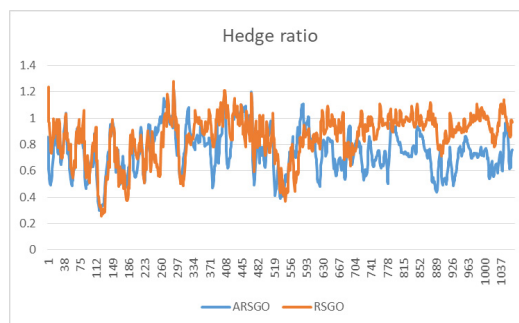
**Figure 3.** State-dependent volatility of food sector index estimated with ARSGO



**Figure 4.** State-dependent volatility of TAIEX futures return estimated with ARSGO



**Figure 5.** State dependent correlations of food sector index and TAIEX futures estimated with ARSGO



**Figure 6.** Hedge ratio of TAIEX futures for cross hedging food sector index estimated with ARSGO

Figure 5 illustrates the state-dependent correlations of food sector index and TAIEX futures estimated with ARSGO.

In this paper, ARSGO is applied to TAIEX futures to cross hedge the spot exposure of Taiwan stock sector indices including food, textiles, electric & machinery, automobile, glass products, and transportation sectors. The performance of ARSGO is compared with the state-independent generalized orthogonal GARCH (GO) and the conventional single-state-variable regime switching GO GARCH (RSGO) models. Figure 6 shows the hedge ratio of TAIEX futures for cross hedging food sector index estimated with ARSGO and RSGO models. Table 3 reports the out-of-sample hedging effectiveness. Overall, the proposed ARSGO model exhibits superior out-of-sample hedging performance. Taking food as an example, the variance of the unhedged portfolio is equal to 4.149. If a hedger applies the GO model for hedging, the variance is reduced to 3.063 or a

26.16% variance reduction. If a hedger applies the RSGO model for hedging, the variance of the hedged portfolio is equal to 3.282 or a 20.89% variance reduction. For the case of food industry, single-state-variable regime switching model does not improve the hedging performance compared with the state-independent GO model. If a hedger applies ARSGO hedging, the variance of the hedged portfolio is equal to 2.873 or a 30.74% variance reduction. The marginal improvements of ARSGO over RSGO and GO are equal to 4.57% and 9.85%, respectively. Overall, multiple-state-variable ARSGO model outperforms the conventional single-state-variable RSGO model for all of the data considered.

A hedger chooses a hedging strategy that minimizes the variance or maximizes the utility of the hedged portfolio return. The expected utility gains are reported in Table 3. The hedger is assumed to have an expected utility function given by equation (36) with the coefficient

**Table 3.** Out-of-sample hedging effectiveness of alternative models

	Variance of Hedged Portfolio Return	Percentage Variance Reduction <sup>1</sup>	Improvement of ARSGO over Other Models <sup>2</sup>	Hedged Portfolio Return	Expected Weekly Utility <sup>3</sup>	Utility Gain of ARSGO over Other Models <sup>4</sup>
Food						
Unhedged	4.149					
GO	3.063	26.16%	4.57%	0.138	-12.115	0.838
RSGO	3.282	20.89%	9.85%	0.180	-12.948	1.670
ARSGO	2.873	30.74%		0.216	-11.277	
Textiles						
Unhedged	5.027					
GO	3.376	32.84%	1.89%	0.224	-13.281	0.391
RSGO	3.288	34.60%	0.13%	0.224	-12.927	0.038
ARSGO	3.281	34.73%		0.236	-12.890	
Electric & Machinery						
Unhedged	9.986					
GO	5.219	47.73%	-2.27%	-0.187	-21.064	-0.955
RSGO	5.502	44.91%	0.55%	-0.244	-22.250	0.231
ARSGO	5.446	45.46%		-0.235	-22.020	
Automobile						
Unhedged	6.909					
GO	3.365	51.29%	6.97%	-0.290	-13.751	1.956
RSGO	3.320	51.95%	6.32%	-0.320	-13.600	1.805
ARSGO	2.884	58.27%		-0.261	-11.795	
Glass Products						
Unhedged	12.948					
GO	7.476	42.26%	2.28%	-0.625	-30.528	1.233
RSGO	7.577	41.48%	3.07%	-0.697	-31.006	1.711
ARSGO	7.180	44.55%		-0.575	-29.295	
Transportation						
Unhedged	3.503					
GO	1.691	51.72%	-0.97%	0.003	-6.763	-0.080
RSGO	1.758	49.81%	0.93%	0.064	-6.969	0.125
ARSGO	1.725	50.74%		0.059	-6.843	

1. The percentage variance reductions are calculated as the differences of the variance of unhedged position and the estimated variances of alternative models over the variance of unhedged position multiplied by 100.
2. The improvement of ARSGO over other hedging strategies is defined as the differences of the percentage variance reduction of ARSGO and the percentage variance reduction of alternative models.
3. The expected weekly utility is calculated based on equation (36)
4. The utility gains of ARSGO over other hedging strategies are defined as the differences of the expected utility of ARSGO and the expected utilities of alternative models.

of absolute risk aversion  $\kappa$  equal to 4 (Lee 2010; Sheu & Lee, 2014; Lien et. al., 2018). If the hedger uses RSGO hedging, the average weekly utility is  $U_{RSGO} = 0.180 - 4(3.282) \approx -12.948$ . With ARSGO hedging,

the average weekly utility is  $U_{ARSGO} = 0.216 - 4(2.873) \approx -11.277$ . The hedger's net benefit from using ARSGO hedging over RSGO hedging is equal to  $U_{ARSGO} - U_{RSGO} = 1.670$ . A meanvariance expected

utility-maximizing hedger should thus adopt ARSGO hedging because of the positive utility gain. As presented in Table 3, ARSGO model outperforms the conventional single-state-variable RSGO model for all of the data considered in terms of utility gains.

## V. Conclusions

Most of the existing regime switching GARCH hedging models assume a common state variable for both the spot and futures returns. In this paper, we suggest an asynchronous Markov regime switching generalized orthogonal GARCH (ARSGO) model such that different financial variables are governed by different state variables with the dependence of switching captured by a synchronization factor. ARSGO is different from Sheu and Lee's (2014) multi-chain Markov regime switching GARCH (MCSG) model which restricts the correlation to be a constant within each regime. The multiple-state-variable ARSGO is more flexible in capturing the time-varying state-dependent correlation between spot and futures returns.

ARSGO is applied to TAIEX futures to cross hedge the spot exposure of Taiwan stock sector indices including food, textiles, electric & machinery, automobile, glass products, and transportation sectors. The empirical results reveal that the hedging effectiveness of multiplestate- variable ARSGO is superior to the conventional single- state-variable RSGO in terms of both variance reduction and utility gain. The improvement of ARSGO over RSGO on variance reduction could be as large as 9.85% for the case of food industry and the improvement of ARSGO over GO on variance reduction could be as large as 6.97% for the case of automobile industry. Most of the expected utility gains are positive showing the economic significance of the superiority of ARSGO over RSGO and GO. This shows the importance of modelling the spot and futures returns with multiple-state-variable regime-switching time-varying correlation GARCH model in optimal futures hedging. A mean-

variance expected utility-maximizing hedger should adopt ARSGO hedging in order to minimize his hedged portfolio risk for a given level of expected return.

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## Appendix A. Summary of estimation procedure for *ARSGO*

To complete the likelihood function in (33) for the proposed asynchronous regime switching generalized orthogonal GARCH model (*ARSGO*), the filtering algorithm is summarized below:

- (i) Given the filtered probabilities  $\hat{\xi}_{t-|t-1}$ , projects the state probabilities

$$\hat{\xi}_{t|t-1} = [\delta_{tR} \mathbf{P}' + (1 - \delta_{tR}) \mathbf{P}^D] \hat{\xi}_{t-|t-1}, \quad (23)$$

where  $\mathbf{P}'$  and  $\mathbf{P}^D$  are respectively the transition variable in cases of independent and perfect synchronous.

- (ii) Evaluate the regime dependent likelihood

$$f(\mathbf{R}_t | \mathbf{S}_t = j, \psi_{t-1}; \Theta) = \frac{1}{(2\pi)^{1/2} |\mathbf{Z}(\theta_s) \mathbf{H}_{t-1, \mathbf{S}_t} \mathbf{Z}(\theta_s)|^{1/2}} \times \exp\left\{-\frac{1}{2}(\mathbf{R}_t - \boldsymbol{\mu}')' \mathbf{Z}(\theta_s) \mathbf{H}_{t-1, \mathbf{S}_t} \mathbf{Z}(\theta_s)^{-1} (\mathbf{R}_t - \boldsymbol{\mu}')\right\} \quad (31)$$

where the mapping matrix  $\mathbf{Z}(\theta_s)$  and volatility elements  $\mathbf{H}_{t-1, \mathbf{S}_t}$  are defined in equations (7), (9) and (10).

- (iii) Evaluate the mixture likelihood

$$f(\mathbf{R}_t | \psi_{t-1}; \Theta) = \sum_{j=1}^4 f(\mathbf{R}_t | \mathbf{S}_t = j, \psi_{t-1}; \Theta) \times P(\mathbf{S}_t = j | \psi_{t-1}; \Theta). \quad (32)$$

where the projected probabilities  $P(\mathbf{S}_t = j | \psi_{t-1}; \Theta)$  are estimated in step (i).

- (iv) Update the joint probabilities

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t-1} \circ \boldsymbol{\eta}_t}{\mathbf{1}'(\hat{\xi}_{t|t-1} \circ \boldsymbol{\eta}_t)} \quad (29)$$

Where  $\mathbf{1}$  is a  $2 \times 1$  vector of ones,  $\circ$  denotes elements-by-elements and  $\boldsymbol{\eta}_t$  is a vector of conditional density defined in equation (30).

- (v) Recombining

Apply equations (15)-(19) to recombine the statedependent residuals and volatilities. The recombined rotation coefficients  $\theta$  is a weighted average of  $\theta_{11}$ ,  $\theta_{12}$ ,  $\theta_{21}$  and  $\theta_{22}$  using projected state probabilities.

- (vi) Iterate (i) to (v) until the end of the sample and the likelihood is obtained as a by-product of this filter given by

$$L(\Theta) = \sum_{t=1}^T \log(\mathbf{1}'(\hat{\xi}_{t|t-1} \circ \boldsymbol{\eta}_t)) \quad (33)$$

The vector of steady state probabilities are used as initial regime probabilities to initialize the filtering algorithm which is given by

$$\boldsymbol{\pi} = \begin{bmatrix} P(\mathbf{S}_0 = 1) \\ P(\mathbf{S}_0 = 2) \\ P(\mathbf{S}_0 = 3) \\ P(\mathbf{S}_0 = 4) \end{bmatrix},$$

which is the solution of the system of equations  $\mathbf{P}\boldsymbol{\pi} = \boldsymbol{\pi}$  and  $\mathbf{1}'\boldsymbol{\pi} = \mathbf{1}$ . The solution can be derived as  $\boldsymbol{\pi} = (\mathbf{A}'\mathbf{A})^{-1} \mathbf{A}'\mathbf{v}_{2^n+1}$ , where  $\mathbf{A} = \begin{bmatrix} \mathbf{I}_{2^n} - \mathbf{P} \\ \mathbf{1}' \end{bmatrix}$ ,

$\mathbf{v}_{2^n+1} = \begin{bmatrix} \mathbf{0}_{2^n} \\ 1 \end{bmatrix}$ ,  $n$  is the number of state variables,

$\mathbf{I}_{2^n}$  is a  $2^n \times 2^n$  identity matrix and  $\mathbf{0}_{2^n}$  is a  $2^n \times 1$  zero vector.