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## Article

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## Variance Bounds Test of Volatility Expectations in Eurodollar Futures Options Markets

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### ABSTRACT

The subject of market efficiency has long been investigated in the area of financial economics and drawn much attention from investors in financial markets. This paper is about the variance bounds test of market efficiency through the option valuation model in the Eurodollar futures and options markets. In equity market, most empirical research found that fluctuations in observed stock prices seem to be too large to be explained by the changes in underlying economic fundamentals. We test empirically whether the volatility expectations are too volatile to be explained by the changes in fundamental value in Eurodollar futures options markets. The tests of the variance bounds inequality suggest that the rationally forecast implied volatility over the life of option fluctuates less than the *ex post* actual volatility and conveys available information efficiently about future volatility in the market of the underlying security. The bootstrap method is applied for the statistical significance of the test without any distributional assumptions.

*Keywords: Eurodollar futures options, implied volatility, variance bound test, bootstrap method, market efficiency*

### 1. Introduction

Risk and return have been the main themes in academics as well as in practical areas of finance. The volatility as a measure of risk and uncertainty in the financial market has motivated many financial economists, and induced the innovation of the financial market. Eurodollar market has been growing rapidly for the last several volatile decades, as the financial market has expanded and the national markets have become integrated. Eurodollar futures

and options markets are the most active interest rate derivative security markets on short-term instruments.

Futures and options are the instrument that allows investors to capitalize the available information in the market while limiting risk to a predetermined level. Investors in option markets have a rational expectation of the volatility and the usefulness of the option valuation model depends to a great extent upon the good forecast of future volatility. Option valuation model was developed by the ground-breaking works of Black and Scholes (1973) and Merton (1973), and has later been extended for the early exercise and stochastic volatility problems by many other researchers. The valuation of futures option contract was first introduced by Black (1976) for

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European option, and Whaley (1986) extended Black's futures option for the early exercise of American option based on the assumption of lognormal distribution of futures prices.

Comparisons of different measure of volatilities in options markets have long been studied in many literatures, with mixed empirical results. These studies investigate the informational contents of the historical volatility estimated from the movements of past prices and the volatilities implied in current option prices. For example in equity markets, Latane and Rendleman (1976), Chiras and Manaster (1978), Harvey and Whaley (1992), and Corrado and Miller (2005) find that the implied volatility is a better predictor of *ex post* realized volatility than the historical volatility. On the other hand, Canina and Figlewski (1993) report that the implied volatility is an inefficient and biased forecast of actual future volatility, while a historical volatility measure partly forecasts future volatility. More recently, the volatility forecast in interest rate markets are examined by Jarrow, Li and Zhao (2007), Deuskar, Gupta and Subrahmanyam (2008).

The rational expectations present value model of stock price describes the stock price as a discounted value of rationally expected future dividend stream, and predicts that the fluctuations in observed stock prices should be less than the changes in economic fundamentals. However, most empirical studies found that the price in equity market which is the expectation of the future dividends seems to be too volatile to be explained by the changes in the fundamental value. Shiller (1981) and Leroy and Porter (1981) provide significant evidence that stock market volatility in general cannot be explained by movements in the rational expectation of future dividends and interest rates. Lo and MacKinlay (1988) also test and reject the random walk hypothesis in equity market by comparing variance estimators derived from different frequency data. They claim that the rejections cannot be attributed to the effects of infrequent trading or time-varying volatilities.

A similar relation for the variance of volatility can be derived and tested for option markets. Traders in options markets have rational forecast of the volatility of the underlying asset when they trade options, and

the market's perception of the future volatility is reflected in option prices. In this paper, we derive an inequality which suggests that the market's forecast of volatility rationally expected over the life of an option should fluctuate less than the actual volatility realized from the market prices. Hence, we investigate whether the rationally forecasted volatility is too volatile to be explained by the changes fundamental value in Eurodollar futures and options markets. That is, we test the variance bounds inequality to examine that the rational expectation of volatility fluctuates less than the *ex post* actual volatility realized in the market and conveys available information about the future volatility. Our results suggest that implied volatility often supports our variance bounds tests while historical volatility does not support our variance inequality.

The organization of the paper is as follows. In the next section, the details of Eurodollar futures and futures option markets are explained. Then the measures of historical and implied volatilities are explained in section 3. In section 4, the volatility test is performed in Eurodollar market as a joint test of the efficiency of Eurodollar market and the option pricing model. Section 5 applies the bootstrapping method to reinforce the variance ratio test. Section 6 summarizes and concludes the paper.

## II. Eurodollar Futures and Options Database

Eurodollars are time deposits with a specified maturity denominated in U.S. dollars and held at financial institutions outside the jurisdiction U.S. Hence, they are subject to lower level of regulation and maintain higher level of risks and higher interest rates than Treasury rates. Eurodollar market is one of the largest short-term money market in the world, and its interest rate is often used as a reference rate and benchmark for corporate funding. Futures contract on Eurodollar is an agreement to place or take a three-month time deposit with a principal value of \$1 million Eurodollars

at a specific future date. Futures price is quoted based on an IMM (International Monetary Market) index, which is the difference between 100 and 3-month London Interbank offered Rate. Option on Eurodollar futures contract is a right to buy or sell an underlying Eurodollar futures contract at a specified strike price on or before the expiration date. The Quarterly cycles of maturity for the Eurodollar futures options are the same as those for the underlying futures contracts with the same expiration dates. These instruments usually offer effective means of managing the interest rate risk of fixed income portfolios.

Figure 1 plots and compare the time series of the 3-month Eurodollar yield, TED spread and 10-year minus 3-month Treasury yield spread. Eurodollar yield remains at higher level in 1980s, medium level in 1990s and lower level in 2000s but increasing before the financial crisis period. TED spread, which is the difference

between the 3-month Eurodollar yield and 3-month Treasury bill rate, reflects the credit risk of interbank loans, and is often higher during recession periods. The Treasury yield spread, which is the difference between the long-term and short-term Treasury rates, indicates the likelihood of a recession or recovery, showing the inverse pattern of Eurodollar yield, and remains low when Eurodollar yield decreases, and vice versa. Figure 2 shows the movements of the logarithm of the 3-month Eurodollar yield changes. This graph shows that the futures markets on Eurodollar were volatile during the 1980s, relatively less volatile during mid- to late- 1990s and volatile again in early 2000s recession and 2007-2008 financial crisis period.

Eurodollar futures and futures option markets started trading in early 1980s in the Chicago Mercantile Exchange (CME). In recent years, as the Eurodollar market activity increased, Eurodollar futures and options have become

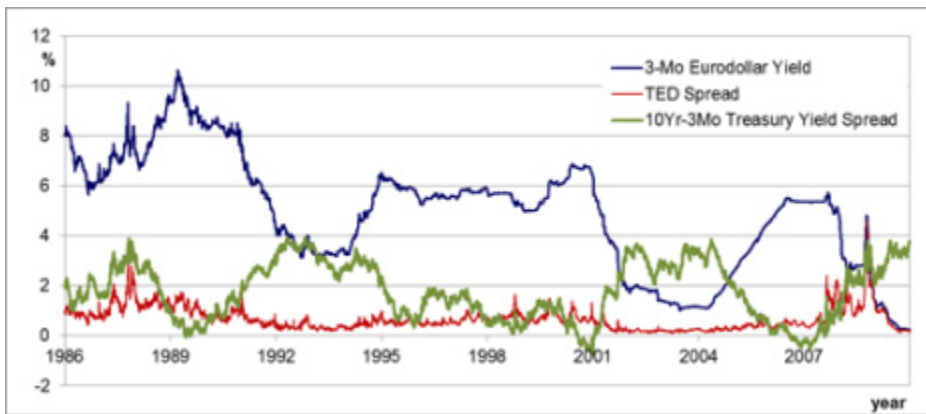


Figure 1. 3-month Eurodollar Yield against TED and 10year-3month Treasury Yield Spread

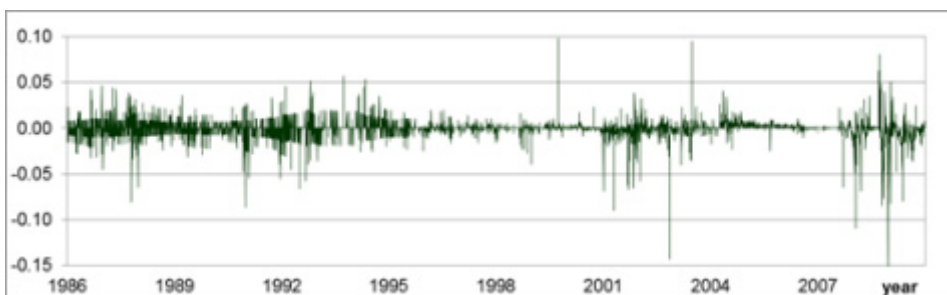


Figure 2. Logarithm of Eurodollar Yield Changes in Eurodollar Futures Market

among the most widely traded exchange-traded short-term money market derivatives in the world. We collected the daily settlement prices for Eurodollar futures and futures options with less than one-year maturity from the CME for the 25-year period from March 1985 to October 2009. There are 108 Eurodollar futures contracts, 2566 call and 2819 put option contracts with different maturity dates and different striking prices for the sample period. We use the 3-month Treasury rate is used as a proxy for the risk-free rate.

To investigate the volatility in the Eurodollar market, we form the time series of futures and options contracts according to their maturity into the first-nearby, second-nearby, third-nearby, and fourth-nearby maturity, where the first-nearby maturity contract has maturity up to 3 months; the second-nearby maturity contract has maturity from 3 months to 6 months; the third-nearby maturity contract has maturity from 6 months to 9 months; and the fourth-nearby maturity contract has maturity from 9 months to 12 months. We name those contracts as the *three-month*, *six-month*, *nine-month*, and *twelve-month* contracts, respectively.

Descriptive statistics of the daily Eurodollar yields for different maturity futures contracts are reported in Table 1. For the entire 25-year sample period, the means of the Eurodollar yields are 5.074%, 5.183%, 5.346%, and 5.533%, and the standard deviations are 2.2881%, 2.2852%, 2.264%, and 2.231% for the three-, six-, nine-, and twelve-month futures contracts, respectively. The Eurodollar yields in 1980's exhibit higher level with short-maturity having lower volatility than longer-maturity as the uncertainty of the futures decreases as the contract matures. 1990's Eurodollar rates exhibit medium level of interest rates around 5 or 6%. Eurodollar yields in 2000's exhibit lowest level of interest rate around 3%, but short maturities fluctuate more than longer maturity interest rates. For the entire sample period from 1985 to 2009, Eurodollar interest rates show upward sloping term-structure of interest rate where the standard deviation for different maturities are similar up to 12-month maturity.

For option contracts, we form the time series of at-the-money, in-the-money, and out-of-the-money call and put options in addition to each of the four

nearby maturity contracts. For example, the time series of the at-the-money option is constructed daily by finding the one nearest exercise price to the futures price with the same maturity, but with the exercise price within the range of 10 basis points from the futures prices. More specifically, with the futures price  $F$  and exercise price  $X$ , we form the time series of the at-the-money call option contracts by selecting the option with the exercise price  $X$  closest to futures price  $F$ , but within the boundary  $|F - X| < 10$  bp. Likewise, the in-the-money call option contracts are formed by selecting the option with the exercise price  $X$  closest to  $F$  minus 1 index point but satisfying the boundary  $|F - X - 1.0| < 10$  bp, and out-of-the-money call options by selecting the option with the exercise price  $X$  closest to  $F$  plus 1 index point but within the boundary  $|F - X + 1.0| < 10$  bp. The time series of different moneyness put options are constructed in the similar way.

### III. Volatility Forecasts: Historical and Implied Volatility

We compute the series of historical volatility and implied volatility of the logarithm of Eurodollar yield changes to compare with the *ex post* actual volatility. Rather than assuming the futures price itself, which is bounded to a maximum of 100, has lognormal distribution, we take the logarithm of Eurodollar yield changes as having lognormal distribution. The *ex post* actual volatility is defined as the annualized standard deviation of the logarithm of yield changes over the remaining life of the underlying options contract until maturity. That is, the actual volatility at time  $t$  of an underlying asset maturing at time  $T$ , is expressed as:

$$\sigma_{t,T}^A = \sqrt{\frac{af}{\tau-1} \sum_{j=t}^T \left( R_j - \frac{1}{\tau} \sum_{i=t}^T R_i \right)^2} \quad (1)$$

**Table 1.** Yearly Sample Statistics for the Eurodollar Yield in Futures Markets

year	3-month		6-month		9-month		12-month	
	mean	std dev	mean	std dev	mean	std dev	mean	std dev
1985	8.290	0.561	8.671	0.704	9.061	0.763	9.423	0.778
1986	6.684	0.714	6.707	0.730	6.850	0.745	7.083	0.742
1987	7.270	0.726	7.516	0.859	7.735	0.962	7.933	0.995
1988	8.104	0.779	8.292	0.685	8.462	0.614	8.661	0.550
1989	9.125	0.780	8.967	0.999	8.903	1.038	8.897	0.925
1990	8.185	0.334	8.142	0.460	8.219	0.513	8.381	0.505
1991	6.005	0.752	6.140	0.838	6.371	0.872	6.689	0.862
1992	3.852	0.404	4.090	0.494	4.469	0.642	4.876	0.661
1993	3.378	0.113	3.585	0.183	3.840	0.200	4.128	0.280
1994	5.052	1.007	5.564	1.053	5.979	1.114	6.301	1.083
1995	6.005	0.379	6.021	0.662	6.106	0.816	6.202	0.886
1996	5.538	0.180	5.638	0.370	5.771	0.479	5.916	0.521
1997	5.787	0.128	5.929	0.165	6.061	0.225	6.177	0.237
1998	5.473	0.297	5.360	0.449	5.342	0.489	5.372	0.481
1999	5.453	0.438	5.575	0.447	5.738	0.456	5.906	0.461
2000	6.608	0.267	6.715	0.353	6.782	0.455	6.836	0.489
2001	3.611	1.118	3.626	0.984	3.844	0.856	4.162	0.744
2002	1.836	0.284	2.070	0.481	2.451	0.654	2.917	0.785
2003	1.195	0.083	1.254	0.119	1.444	0.208	1.732	0.302
2004	1.801	0.546	2.105	0.600	2.396	0.600	2.727	0.585
2005	3.793	0.566	4.040	0.509	4.201	0.441	4.287	0.379
2006	5.268	0.239	5.275	0.204	5.209	0.216	5.115	0.238
2007	5.172	0.296	4.938	0.440	4.762	0.515	4.652	0.533
2008	2.677	0.530	2.578	0.546	2.623	0.563	2.724	0.588
2009	0.769	0.343	0.887	0.274	1.104	0.217	1.359	0.176
1985-1990	7.925	1.036	8.024	1.072	8.173	1.087	8.359	1.061
1991-2000	5.311	1.058	5.460	1.061	5.648	1.053	5.842	1.022
2001-2009	2.936	1.634	3.010	1.559	3.151	1.451	3.332	1.334
1985-2009	5.074	2.288	5.183	2.285	5.346	2.264	5.533	2.231

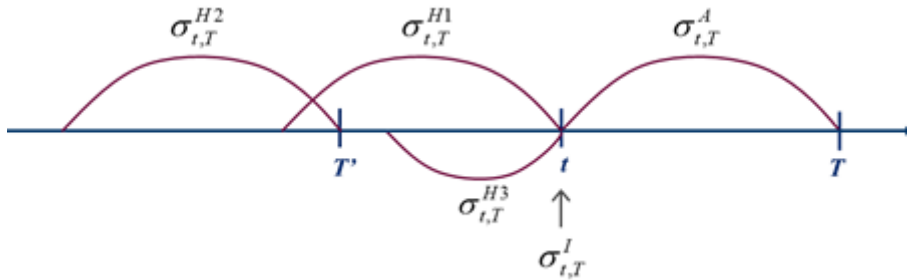
Notes: The table reports the descriptive statistics for daily Eurodollar yields with less than one year to maturity. For Eurodollar futures and futures option contracts, 3-month, 6-month, 9-month, and 12-month maturity contracts are selected for the underlying futures and futures option contracts with 0-3 months, 3-6 months, 6-9 months, and 9-12 months to maturity at each time, respectively. For 25-year period from March 1985 to October 2009, 24,754 Eurodollar yields are observed excluding missing observations. Mean and std dev represent the average and standard deviation for the annualized Eurodollar yield implied in futures price for each period.

where  $R_t = \ln(y_t / y_{t-1})$ ,  $y_t$  is the Eurodollar yield at time  $t$ ,  $\tau (\equiv T - t + 1)$  is the time to maturity of an underlying asset, and  $af$  is an annualizing factor.<sup>1)</sup>

1) On average, Eurodollar market has about 252 trading days per year, and we assume 252 for the annualizing factor for the daily time series. The volatility estimates are calculated for the underlying contract with at least one week to maturity to avoid the erroneous estimation of the volatility due to the small

Historical volatilities are computed similarly, but defined in three different ways depending on the estimation period. The first historical volatility at time  $t$  of an underlying contract maturing at  $T$ ,  $\sigma_{t,T}^{H1}$ ,

number of observations in the market. Hence, the estimation period for the volatilities ranges from one week to one year since we are considering only the Eurodollar futures and futures option contracts with up to twelve months to maturity.



**Figure 3.** Diagram for the Various Definitions of the Historical Volatility and Implied Volatility as Predictors of *Ex Post* Actual Volatility

is taken from the annualized standard deviation of changes in the logarithm of the Eurodollar yield for the same length of the life of an underlying asset but from the recent past period from  $t - \tau$  to  $t - 1$ . The second historical volatility at time  $t$ ,  $\sigma_{t,T}^{H2}$ , is taken from the matching period for the life of the immediately preceding contract maturing at time  $T'$  so that the standard deviation is estimated from  $T' - \tau + 1$  to  $T'$ . The third historical volatility at time  $t$ ,  $\sigma_{t,T}^{H3}$ , is calculated from the constant length of the recent past period. That is, the *ex post* actual volatility and the first and second historical volatilities are calculated from the same length of the period as the life  $\tau$  of an underlying asset, while the third definition of historical volatility is estimated from the fixed time period regardless of the maturity of the underlying contract.<sup>2)</sup> Figure 3 illustrates estimation time period for the actual, historical and implied volatility at time  $t$  of an underlying asset maturing at time  $T$ .

Based on the assumption of the lognormal distribution of futures price, Black (1976) and Whaley (1986), respectively, derived the European and American pricing functions for the futures option. In Black's pricing formula, the option on a futures contract can be treated in the same way as the option on a security paying a continuous dividend at risk-free rate. We modify the European option pricing model of Black (1976) to apply to the Eurodollar futures call and

put options, assuming the Eurodollar yield has a lognormal distribution. It is more realistic to assume that Eurodollar yield rather than futures price itself is lognormal since Eurodollar future price is always below the predetermined level.<sup>3)</sup> The volatility at time  $t$ ,  $\sigma_{t,T}^I$ , implied in the call or put option price maturing at  $T$  can be calculated by inverting the option pricing function given the other parameters. Since the option pricing function is not easily invertible, we numerically approximate the volatility implied in the option price by equating the model price with the market price of the call or put option. The quasi-Newton method and a finite difference gradient are employed to the modified option pricing models for the futures options.

In the next section, we investigate the efficiency of volatility expectation in the Eurodollar futures and futures option markets for different maturity and nearness to money options. The variance bounce test can be applied by deriving the variance bounds inequality from the volatility expectation equation to examine the behavior of the historical and implied volatilities.

2) Here, we take the past one-month period for the calculation of the historical volatility H3. Changing the estimation period for H3 doesn't change much for our main empirical findings.

3) Refer to Kim (2017) for the exact derivations of the modified version of the option pricing model of Black (1976) and Whaley (1986) for the futures option contracts where the Eurodollar yield rather than futures price itself is assumed to be lognormal.

#### IV. Variance Bounds Tests in Eurodollar Futures and Options Markets

Tests on volatility have been performed in both the stock and bond markets examining the joint validity of the present value relation and the efficiency of the markets. The rational expectations present value model of stock prices and dividends describes the stock price as a discounted sum of the rationally expected or optimally forecasted future dividend stream when investors maximize their lifetime expected utility by choosing stochastic consumption and investment plans. That is, the market price of stock,  $P_t$ , at time  $t$  is the expectation of the fundamental value,  $P_t^*$ , that can be expressed as the sum of discounted future dividends given the information set  $\Omega_t$  available at time  $t$ :

$$P_t = E(P_t^* | \Omega_t), \quad \text{or} \quad P_t^* = P_t + e_t. \quad (2)$$

In other words, the fundamental value of a stock is the sum of the current price, which reflects the market expectation, and the forecasting error at time  $t$ ,  $e_t$ , which is uncorrelated with the market price,  $P_t$ , and is orthogonal to the information set,  $\Omega_t$ .

Instead of directly comparing given prices with the fundamental values, Shiller (1981) and LeRoy and Porter (1981) test the volatility of stock indices to find the excess volatility of the market price from the variance bounds test. The variance bounds inequality can easily be derived from the above present value relation as:

$$\text{var}(P_t) \leq \text{var}(P_t^*) \quad (3)$$

since  $\text{var}[E(x|y)] = \text{var}(x) - E[\text{var}(x|y)] \leq \text{var}(x)$  for any random variables  $x$  and  $y$ . This inequality implies that sudden movements in prices are attributed to new information about future dividends and the market price, which is the expectation of the future dividend stream, should be less volatile than the fundamental value. However, most empirical evidences, including

Shiller (1981), Leroy and Porter (1981) and Lo and MacKinlay (1988), indicate that fluctuations in observed stock prices seem to be too large to be explained by the changes in underlying economic fundamentals in various equity markets and in different time periods.

Traders in option markets desire rational forecast of the volatility of the underlying securities when they trade options. The option price reflects the market's perception of the future volatility of the underlying security, and there is one-to-one relationship between option price and the volatility of underlying asset. If the market expects high volatility, the option price will be high, and similarly, a low option price implies that low future volatility is anticipated by the market. This is because call or put option buyers are willing to pay larger premiums for greater protection against adverse price changes when the market becomes volatile, and option sellers are willing to accept smaller premiums when the market is less volatile.

When a volatility forecast contains all relevant information about the future course of volatility, it should reflect the unbiased expectations of the future actual volatility over the life of the underlying asset, conditional upon the available information set,  $\Omega_t$ . That is,

$$\sigma_{i,t}^j = E(\sigma_{i,t}^A | \Omega_t), \quad (4)$$

where  $\sigma^j$  represents the volatility forecast or the market's expectation of the future volatility, and  $\sigma^A$  represents the actual volatility that would be realized over the life of the option in the market, *ex post*. The errors between  $\sigma_{i,t}^j$  and  $\sigma_{i,t}^A$  represent the forecast errors of market expectations on volatility. The variance bounds inequality for the volatility forecast and actual volatility in the futures and options markets can then be expressed as:

$$\text{var}(\sigma^i) \leq \text{var}(\sigma^A), \quad \text{for } i = H, I, \quad (5)$$

where the volatility forecast  $\sigma^i$  can be estimated by either the historical volatility,  $\sigma^H$ , or the implied



volatility,  $\sigma^I$ , in option prices.<sup>4)</sup> This inequality suggests that the market's forecast of volatility rationally expected over the life of option should fluctuate less than does the actual volatility.

From the volatility expectation (4), the orthogonality condition can also be derived. If we multiply the equation (4) by any random variable  $z_t$  observed at time  $t$ , and take expectations, then it can be expressed as:

$$E(z_t \cdot \sigma_{i,T}^i) = E(z_t \cdot \sigma_{i,T}^A), \quad i = H, I.$$

When  $z_t = \sigma_{i,T}^i - E(\sigma_{i,T}^i)$ , the above equation can be rewritten as:

$$\text{var}(\sigma_{i,T}^i) = \text{cov}(\sigma_{i,T}^i, \sigma_{i,T}^A) \quad (6)$$

In other words, the rationally predicted volatility fluctuates as much as the covariance with the future course of volatility when the volatility forecast reflects all the relevant information in the market, and hence it represents the unbiased expectation of future volatility. This orthogonality equality (6) is equivalent to the null hypothesis of slope being one in the regression test of the implied and historical volatilities as predictors of the future actual volatility. Hence, the variance bounds test is equivalent to the regression test of volatility expectations and, hence, is expected

4) When the implied volatility from the option price is used for the rationally forecast volatility, equation (4) involves additional errors from the misspecification of the option valuation model. Let  $\sigma^i$  be the true implied volatility without any specification error from an option pricing model. Then the observed implied volatility,  $\sigma^I$ , from the specific option valuation model with given parameters may involve the errors from the misspecification of the option model. That is,  $\sigma^I = \sigma^i + \eta$ , where  $\eta$  is the specification error from the option valuation model. Then the variance of  $\sigma^I$  will be greater than or equal to the variance of  $\sigma^i$ :  $\text{var}(\sigma^I) \geq \text{var}(\sigma^i)$ . Therefore, when  $\sigma^I$  is used for the rationally forecast volatility, the violation of variance bounds inequality (5) directly implies neither market inefficiency nor invalidity of the option model. However, the observation of inequality (5), when  $\sigma^I$  is applied in place of  $\sigma^i$ , supports the joint hypothesis of market efficiency and the model validity in the Eurodollar futures and futures option markets.

to generate consistent results.

Table 2 reports the variances for the time series of the actual volatility, historical volatility and the implied volatility taken from at-the-money, in-the-money, and out-of-the-money call and put options for different terms to maturities. The variance bounds inequality is tested for the null hypothesis:

$$H_0: \text{var}(\sigma^A) = \text{var}(\sigma^i), \quad \text{for } i = H, I, \quad (7)$$

against the alternative hypothesis

$$H_1: \text{var}(\sigma^A) > \text{var}(\sigma^i), \quad \text{for } i = H, I. \quad (8)$$

When the time series of  $\sigma^A$  and  $\sigma^i$  have independent normal distributions with unknown means and the numbers of observations are  $n$  and  $m$ , respectively, the unbiased estimates for the ratio of variances,  $\hat{\text{var}}(\sigma^A) / \hat{\text{var}}(\sigma^i)$ , has an  $F$  distribution with  $n-1$  and  $m-1$  degrees of freedom when  $H_0$  is true. Hence, the null hypothesis is rejected in favor of the variance bounds inequality (5) at significance level  $\alpha$  if

$$\frac{\hat{\text{var}}(\sigma^A)}{\hat{\text{var}}(\sigma^i)} > F_\alpha(n-1, m-1) \quad (9)$$

where  $F_\alpha$  is the critical value of the  $F$  distribution at significance level  $\alpha$ . The  $p$  values are reported in parentheses to test the differences in variances given the  $F$  distribution of variance ratios for the appropriate degrees of freedom. The  $p$  value is the probability, under the null hypothesis, of the variance ratio that is as extreme as the observed variance ratio. If this  $p$  value is small, we tend to reject the null hypothesis,  $\text{var}(\sigma^A) = \text{var}(\sigma^i)$ , in favor of the variance bounds inequality.

The volatility is generally lower for shorter maturity than longer maturity contracts since the uncertainty decreases as the contract matures. As shown in the table, the variance of the actual volatility is also lower for short maturity contracts than that for long maturity. The variances for the actual volatility are

0.0165, 0.0253, 0.0325, and 0.0376 for the 3-month, 6-month, 9-month, and 12-month contracts, respectively. In general, the variation of the implied volatility rationally expected in the Eurodollar market is

significantly smaller than that of the *ex post* actual volatility, and the null hypothesis of equal variance is strongly rejected in favor of the variance bounds inequality at statistically significant levels except for

**Table 2.** Variance Bounds Tests in Eurodollar Futures and Options Markets

		3-month	6-month	9-month	12-month
<b>Actual Volatility:</b>					
	Var( $\sigma^A$ )	0.0165	0.0253	0.0325	0.0376
	No. of obs	5805	6184	6184	6184
<b>Historical Volatility (H1):</b>					
	Var( $\sigma^{H1}$ )	0.0283	0.0382	0.0394	0.0323
	p value	(1.0000)	(1.0000)	(1.0000)	(0.0000)
	No. of obs	5805	6184	6184	6071
<b>Historical Volatility (H2):</b>					
	Var( $\sigma^{H2}$ )	0.0154	0.0212	0.0233	0.0241
	p value	(0.0034)	(0.0000)	(0.0000)	(0.0000)
	No. of obs	5804	6185	6184	6185
<b>Historical Volatility (H3):</b>					
	Var( $\sigma^{H3}$ )	0.0297	0.0420	0.0512	0.0519
	p value	(1.0000)	(1.0000)	(1.0000)	(1.0000)
	No. of obs	5805	6184	6184	6184
<b>Implied Volatility in Call Options:</b>					
<b>In-the-Money</b>	Var( $\sigma^I$ )	0.0020	0.0033	0.0065	0.0124
	p value	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	No. of obs	2207	3649	3417	3188
<b>At-the-Money</b>	Var( $\sigma^I$ )	0.0220	0.0251	0.0235	0.0261
	p value	(1.0000)	(0.3621)	(0.0000)	(0.0000)
	No. of obs	5389	5218	4981	4849
<b>Out-of-the-Money</b>	Var( $\sigma^I$ )	0.0025	0.0031	0.0052	0.0067
	p value	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	No. of obs	648	2873	3870	3914
<b>Implied Volatility in Put Options:</b>					
<b>In-the-Money</b>	Var( $\sigma^I$ )	0.0037	0.0043	0.0069	0.0112
	p value	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	No. of obs	796	3299	4281	4475
<b>At-the-Money</b>	Var( $\sigma^I$ )	0.0223	0.0246	0.0238	0.0261
	p value	(1.0000)	(0.1626)	(0.0000)	(0.0000)
	No. of obs	5427	5209	4988	4905
<b>Out-of-the-Money</b>	Var( $\sigma^I$ )	0.0020	0.0025	0.0047	0.0058
	p value	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	No. of obs	1473	2503	2552	2521

Notes: This table reports the variances for the actual, historical and implied volatilities from the option pricing models. *p* values are reported in the parentheses to test the differences between the variance of actual volatility and the variance of volatility forecast, assuming *F* distribution of variance ratio. 3-month, 6-month, 9-month, and 12-month represent the underlying futures and futures option contracts with 0-3 months, 3-6 months, 6-9 months, and 9-12 months to maturity, respectively.

short maturity at-the-money options. This supports the fact that the rationally expected implied volatility fluctuates less than the *ex post* actual volatility and conveys available information about future volatility in the market. The variance bounds inequality is still violated for the implied volatilities taken from short maturity at-the-money call and put options, nevertheless the variances of implied volatilities are more stable across different maturities. If the option market is efficient, all relevant information including historical volatility should be reflected in the option price, and a high or low option price directly indicates the high or low volatility in the market.

On the other hand, the various measures of the historical volatility fluctuate more than the *ex post* actual volatility in all maturity options. For most maturity contracts, the  $p$  values are large for the null hypothesis testing the variance of the historical volatility.<sup>5)</sup> This is due to large estimation errors since the historical volatility does not efficiently reflect all the relevant information in the market of the underlying security, even when the historical volatility is estimated for the same length of the option's life. Hence, if we employ the historical volatility as the market's expectation of the volatility over the life of the option, the variance bounds inequality (5) is likely to be violated for most time series of historical volatility.

## V. Bootstrapping Variance Ratios: A Monte Carlo Simulation

In the previous section, we assumed that the variance ratio has an  $F$  distribution applying large sample theory. However, the test on variances may be sensitive to the model assumption that the parent

population is normally distributed. In this section, we estimate the statistical significance of the variance bounds inequality of volatilities in the Eurodollar market applying the bootstrap method of Efron (1979) without relying on any distributional assumption. Theoretical and empirical results in the literature suggest that the bootstrap method provide better estimates of sampling distribution in many applications than the conventional normal approximations.

The bootstrap method is a technique for estimating standard errors by approximating the theoretical distribution of the parameters of interest by the empirical distribution. The idea is to apply the Monte Carlo simulation based on a nonparametric estimate of the underlying distribution so that the original observations are resampled in a suitable way. We estimate the empirical standard errors for the random variance ratios in the following steps:

- Step 1: Construct the sample probability distribution of volatility time series, putting mass  $\frac{1}{N}$  at each  $\sigma_1, \sigma_2, \dots, \sigma_N$ , for each of the previously calculated actual, historical and implied volatilities.
- Step 2: Generate the pseudo data by drawing a random sample  $\sigma^* = (\sigma_1^*, \sigma_2^*, \dots, \sigma_N^*)'$  of size  $N$  with replacement from the fixed sample distribution of  $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_N)'$ .
- Step 3: Estimate the variance ratio  $\text{var}^*(\sigma^A) / \text{var}^*(\sigma^I)$  from the pseudo data  $\sigma^*$  for  $i = H$  or  $I$ .
- Step 4: Perform a Monte Carlo approximation of the sampling distribution of  $\text{var}(\sigma^A) / \text{var}(\sigma^I)$  by repeating steps 2 and 3 a large number of times to generate the bootstrap distribution of  $\text{var}^*(\sigma^A) / \text{var}^*(\sigma^I)$ .

The resulting bootstrap is distribution-free and develops the appropriate finite sample behavior for the estimates. Hence, it can be applied to a wide range of econometric models such as the problems of simultaneity, correlated errors, and heteroscedasticity.

The variance ratios of *ex post* actual volatility to either the historical or implied volatility are reported in Table 3 with the bootstrap standard errors reported

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5) The second definition of historical volatility theoretically has the same variance as the actual volatility since it represents the same time series of volatility as the actual volatility except that the second historical volatility is taken from the immediately previous contract. Hence, the variance bounds test for the second definition of the historical volatility does not convey much information.

within the parentheses. In order to approximate the sampling distribution of the variance ratios, the bootstrap procedure is repeated a thousand times to generate the pseudo time series of  $\sigma^*$  using Monte Carlo simulation, which gives a thousand values of variance ratios. The resulting statistics should be fairly accurate for a sample size. The bootstrap results in

Table 3 are consistent with the variance bounds test in Table 2 based on the assumption of an  $F$  distribution of the variance ratio. The five percent statistical significance rejecting the null hypothesis of equal variances of the actual volatility,  $\sigma^A$ , and the volatility forecast,  $\sigma^I$ , is denoted by asterisks.

The variance ratios of the actual volatility to the

**Table 3.** Bootstrapping the Variance Ratios of Actual Volatility and Volatility Forecasts

	3-month	6-month	9-month	12-month
Historical Volatility (H1):				
Var( $\sigma^A$ )/Var( $\sigma^{H1}$ )	0.5825	0.6632	0.8248	1.1835*
std.err	(0.0200)	(0.0191)	(0.0171)	(0.0242)
Historical Volatility (H2):				
Var( $\sigma^A$ )/Var( $\sigma^{H2}$ )	1.0778*	1.1995*	1.3994*	1.5685*
std.err	(0.0435)	(0.0459)	(0.0367)	(0.0366)
Historical Volatility (H3):				
Var( $\sigma^A$ )/Var( $\sigma^{H3}$ )	0.5549	0.6023	0.6341	0.7250
std.err	(0.0202)	(0.0200)	(0.0196)	(0.0195)
Implied Volatility in Call Options				
In-the-Money:				
Var( $\sigma^A$ )/Var( $\sigma^I$ )	2.6860*	4.2703*	3.9638*	3.0069*
std.err	(0.1829)	(0.1960)	(0.1894)	(0.1117)
At-the-Money:				
Var( $\sigma^A$ )/Var( $\sigma^I$ )	0.7398	1.0593*	1.4684*	1.6917*
std.err	(0.0244)	(0.0351)	(0.0425)	(0.0451)
Out-of-the-Money:				
Var( $\sigma^A$ )/Var( $\sigma^I$ )	1.9235*	2.5619*	3.4034*	3.4784*
std.err	(0.2386)	(0.1471)	(0.1482)	(0.1212)
Implied Volatility in Put Options				
In-the-Money				
Var( $\sigma^A$ )/Var( $\sigma^I$ )	2.0293*	3.2881*	3.2466*	2.7291*
std.err	(0.1881)	(0.1600)	(0.1445)	(0.0993)
At-the-Money				
Var( $\sigma^A$ )/Var( $\sigma^I$ )	0.7208	1.0743*	1.4492*	1.6706*
std.err	(0.0234)	(0.0353)	(0.0418)	(0.0445)
Out-of-the-Money				
Var( $\sigma^A$ )/Var( $\sigma^I$ )	1.7871*	2.6662*	4.2886*	4.3107*
std.err	(0.1413)	(0.2204)	(0.2552)	(0.2002)

Notes: The table reports the ratios of variances for the actual volatility to the variance for either the historical volatility or the implied volatility using the option pricing models for each maturity contract. The standard errors are derived from the empirical distribution of the bootstrap method and are reported in the parentheses. The variance ratios  $\text{var}(\sigma^A) / \text{var}(\sigma^I)$  greater than one at the five percent significance level are denoted by an asterisk. 3-month, 6-month, 9-month, and 12-month represent the underlying futures and futures option contracts with 0-3 months, 3-6 months, 6-9 months, and 9-12 months to maturity, respectively.

historical volatility are less than one in most cases, and the null hypothesis is not rejected, violating the variance bounds inequality. The ratios of the variance of the actual volatility to the variance of the implied volatility are greater than one most of the time, strongly rejecting the null hypothesis of equal variances, except for the implied volatilities taken from the short maturity at-the-money call and put options. The bootstrap method provides more robust results supporting the variance bounds inequality for the implied volatility especially for the medium term call or put option contracts. For example, the variance ratio for the 3-month at-the money call option is 0.7398, violating the variance bound inequality, but those for the 6-month, 9-month and 12-month at-the money call options are 1.0593, 1.4684 and 1.6917, respectively, supporting the fact that the rationally expected implied volatility fluctuates less than the *ex post* actual volatility and conveys available information about future volatility in the market.

The variance bounds inequality, in general, is well observed for most implied volatilities predicting actual volatilities over the life of underlying asset. Given the appropriate option valuation model, the rationally expected implied volatility reflects the relevant information in the futures and futures option markets when those markets are efficient, and a high or low option price directly indicates the high or low volatility in the market. Hence, according to the variance bounds inequality, the variance of the implied volatility is greater than that of the *ex post* actual volatility most of the time whereas the historical volatility quite often violates the variance bounds inequality.

## VI. Concluding Remarks

One of the earliest but still most important subject for financial economists is whether financial asset prices are forecastable and markets are efficient. In equity market, many empirical researches have been

performed to test the hypothesis of efficient market. One stream of this test investigates the variance ratio of asset prices, and most empirical evidences indicate that fluctuations in observed stock prices seem to be too large to be explained by the changes in underlying economic fundamentals.

The volatility implied in the option price reflects investors' assessments of the market volatility prevailing during the option's life. If the option market is efficient, all relevant information should be contained in option price, and the implied volatility should represent a rational forecast of future volatility when appropriate option pricing model is employed. In options market, historical volatility or implied volatility is often considered as a predictor of future volatility that will be realized in the market. In this paper, we test the joint hypothesis of market efficiency and the validity of the option pricing model. That is, we test whether the volatility expectation in Eurodollar futures options markets is too volatile, applying the variance bounds inequality condition satisfied by the volatility estimators. This inequality suggests that the market's forecast of volatility rationally expected over the life of an option should fluctuate less than the *ex post* actual volatility.

We construct the time series of 3-month, 6-month, 9-month, and 12-month maturity contracts, and of at-the-money, in-the-money, and out-of-the-money option contracts to compare the different behavior of volatility for different maturity and for different nearness to money of the underlying contracts. It is shown that volatilities for different maturity and different moneyness options exhibit different characteristics. We examine the variance ratios of *ex post* actual volatility vs. historical or implied volatility forecast measures, and test the statistical significance with distributional assumption or using the bootstrap method without any distributional assumption. Our empirical results show that the implied volatilities taken from option prices fluctuate less than the *ex post* actual volatility, supporting the variance bounds inequality, while the historical volatility does not support the inequality. The rationally expected implied volatility conveys available information about future volatility more

efficiently in the Eurodollar futures and options markets than do the various definitions of historical volatility.

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