Kim, Kwanho

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Informational Content of Volatility Forecasts in Eurodollar Markets

Kwanho Kim

Professor, Department of Economics, Chungbuk National University, Cheongju, Republic of Korea

A B S T R A C T

The volatility of asset prices as a measure of risk in the financial market has motivated many financial economists and industry professionals, and induced the innovation in the financial markets. The paper studies how expectations of future volatility are formed, and whether or not historical or implied volatilities measures for different maturity and moneyness of options have any information to explain ex post actual volatility over the life of the options in Eurodollar futures and futures options markets. Employing the autocorrelation and heteroscedasticity consistent GMM regression test, we find that the volatilities implied in the at-the-money options tend to outperform the in-the-money or out-of-the-money implied volatilities and different definitions of historical volatilities.

Keywords: Implied Volatility; Volatility Bias; Eurodollar Futures Options; GMM Regression

Ⅰ. Introduction

Global financial markets have been growing rapidly for the past several decades and financial assets evolve to be more complicated as investors and market participants become more sophisticated. Futures and options contracts are the instruments that allow investors to capitalize the available information in the market while limiting risk to a predetermined level. Financial futures markets began in the late 1970's, about 150 years after commodity futures trading began. In spite of their late start, financial futures trading exceeded the trading of the sum of all other futures contracts combined after the first decade. The Eurodollar futures are by far the most liquid of all financial futures contracts.

This paper studies how expectations of future volatility are formed and whether or not historical or implied volatility have any information relevant to explain future realized volatility over the life of each option. The option pricing model is inverted to calculate the implied volatilities where it assumes that the volatility process is stationary and at most a deterministic function of time. Option valuation models were first developed by Black and Scholes (1973) and Merton (1973) for the European option, and the early exercise problem and stochastic volatility problem have been investigated by many researchers. Among others, Geske (1979) examines the valuation of options where stocks are considered as options on the value of a firm and the stock price follows a nonstationary random walk process. Thus, the volatility of the stock price is expected to fluctuate

† Kwanho Kim
Department of Economics, Chungbuk National University, 1 Chungdae-Ro Seowon-Gu, Cheongju, Republic of Korea
Tel.: +82-43-261-3015; https://sites.google.com/site/financekkim/
E-mail: kimk@chungbuk.ac.kr

as the stock price changes. Hull and White (1987) also investigate the model of European option pricing assuming stochastic instantaneous variance. They find that the Black and Scholes specification often overprices options. The valuation of the futures option contract is first introduced by Black (1976) for European options, and Whaley (1986) corrects Black's futures option valuation model for the early exercise of American options based on the assumption of lognormal distribution of futures prices. Bakshi, Cao and Chen (1997) derive an option pricing model that allows volatility, interest rates and jumps to be stochastic, and show that this model improves the pricing and internal consistency for S&P 500 options. Cetin, Jarrow, Protter and Warachka (2006) add the liquidity risk as a stochastic supply curve in Black Scholes model, and their empirical results show that the liquidity costs are a significant component of the option's price and increase quadratically in the number of options being hedged.

Evolving from the option valuation model, the relationship between the volatilities implied by option prices and the volatilities historically estimated from underlying security prices has long been studied in many literatures. Latane and Rendleman (1976) investigate a sample of equity option prices, and find the weighted average implied standard deviation is a better predictor of future variability than historical standard deviation. Day and Lewis (1988) examine the options on the market index, and find that the volatility implicit in option prices increases around the expiration of the option contract. More recently, Harvey and Whaley (1992) and Fleming (1998) examines the performance of the S&P 100 implied volatility and Corrado and Miller (2005) examine the forecast quality of CBOE implied volatility indexes as a forecast of future stock market volatility. They find that the implied volatility dominates the historical volatility in terms of *ex ante* forecasting power. On the other hand, Canina and Figlewski (1993) examine the S&P 100 index option market, but argue that the implied volatility is an inefficient and biased forecast of actual future volatility while the historical measure of volatility partly forecasts future volatility for S&P 100 index option. Poteshman (2000) also finds the bias in implied volatility forecast for S&P 500 index options, and suggests to employ an option pricing model to eliminate the forecasting bias.

For long-term interest rate option markets, Jarrow, Li and Zhao (2007) examine the volatility smile in interest rate caps and floors, and find that even a multifactor term structure models augmented with stochastic volatility and jumps do not fully capture the volatility smile. Deuskar, Gupta and Subrahmanyam (2008) investigate the economic determinants of interest rate volatility bias for the over-the-counter interest rate caps and floors which have up to 10-year maturity. They find the strong volatility smile patterns in the interest rate caps and floors markets for different maturities, and that the shape of the smile is positively related to the short-term interest rate and the liquidity costs, and negatively related to the slope of the term structure of interest rates especially for longer maturity options.

In this paper, we examine and compare the various measures of the historical volatility and the volatility implied in option price as predictor of future actual volatility in Eurodollar futures and futures options markets, which are by far the most popular exchange-traded short-term interest rate derivatives markets. The volatilities in different underlying asset classes and in different markets tend to exhibit different characteristics throughout different time periods. Rather than assuming a lognormal distribution for the futures index which is always below the predetermined level, this paper assumes the Eurodollar yield itself has a lognormal diffusion process, and the modified versions of Black's option pricing models are applied. Instead of examining the cross-sectional data of individual contracts, we construct the time series of futures and futures options contracts with less than one year to maturity, and investigate the volatility in the Eurodollar markets. That is, the time series of futures contracts with less than three, six, nine, and twelve months to maturity, and the time series of futures options contracts with different maturities separately for at-the-money,

in-the-money, and out-of-the-money options are formed and studied. The behaviors of actual volatility and implied volatility expected in the market tend to exhibit different patterns for different maturities and for different moneyness of options, and we test whether various volatility forecasts contain relevant information and whether Eurodollar futures and options markets are informationally efficient.

Ⅱ. Eurodollar Futures and Eurodollar Futures Option Markets

The Eurodollar market has grown steadily over the past fifty years and accounts for the largest share of the Eurocurrency market, as the U.S. dollar has become the major international currency. The Eurodollar is a U.S. dollar deposit in a foreign bank or subsidiary of a U.S. bank outside the jurisdiction of the United States. Since the Eurodollar is not subject to the Federal Reserve requirements and is subject to default risk, its yield is expected to be higher than that on domestic deposits in the United States. Yet, it still represents a low risk investment tool, and its yield is fairly stable over time.

The Eurodollar futures contract is an agreement to place or take a time deposit with a principal value of $1,000,000 Eurodollars for three months at a specific future date in the prime London Interbank market. When the contract is initiated, the futures price is the price at which the current value of the contract would be zero. The futures contracts trade on a quarterly cycle and have four contract months per year: March, June, September, and December.[1] The delivery date of the Eurodollar futures contract is the second London bank business day immediately preceding the third Wednesday of the futures contract month. On the delivery day, it calls for cash settlement instead of actual delivery of the underlying instrument. The futures price is quoted based on an IMM (International Monetary Market) index, which is the difference between 100 and the Eurodollar yield. For instance, if the yield on the Eurodollar time deposit is 1.50%, then the quoted Eurodollar futures index will be 98.50, and the dollar value of a contract is

$$\$1{,}000{,}000 \cdot (100\% - 0.25 \times 1.50\%) = \$996{,}250,$$

since the futures contract delivers the three-month Eurodollar time deposit with a principal value of $1 million. Hence, a basis point change in the IMM Index, or 0.01 percent change in the Eurodollar yield, is interpreted as a $25 change in the value of a Eurodollar futures contract.

The Eurodollar futures contract and the underlying Eurodollar time deposit have scheduled maturities unlike the stock prices or exchange rates. At maturity, the value of the three month Eurodollar time deposit must converge to a principal value of $1 million. Hence, the volatility of the futures price is expected to decline as the contract matures. The price of the Eurodollar futures contract falls when interest rates move up, and rises when interest rates move down. Therefore, investors buy the Eurodollar futures contract to protect against falling interest rates and sell to hedge against rising interest rates. If interest rates fall and the futures price rises, an investor will be able to sell the futures contract at a higher price to offset low interest rates.

A call [put] option on Eurodollar futures contract gives its buyer the right to buy [sell] an underlying Eurodollar futures contract at a specified strike price on or before the expiration date. The maturity cycle of the Eurodollar futures option is the same as the cycle of the underlying futures contract, and the option expires on the same day as the underlying futures contract. The dollar value of a Eurodollar futures option is equal to the quoted price of an option times $2,500, as in the dollar value of a futures contract. Consider, for example, an investor who holds a call option on December Eurodollar futures with an exercise price of 97.75. If the current futures index is 98.50 and if the option is exercised today, the

1) Short-maturity contracts also trade less frequently for months other than March, June, September, and December.

option holder acquires a long position in a Eurodollar futures contract to buy a Eurodollar time deposit in December at the price of 98.50. When the option is exercised, the futures position can be immediately closed out at no cost. Thus, when the futures contract is marked-to-market at the close of the day's trading, the investor receives $1,875 (= (98.50 - 97.75) ´ $2,500), an amount equal to the current futures price minus the exercise price times $2,500. If the option is quoted at 0.90, the dollar value of the option premium would be $2,250 (= 0.90 ´ $2,500), and the time value of this option would be $375.

We collect the daily settlement prices for Eurodollar futures and futures options from the Chicago Mercantile Exchange (CME) for the 25-year period from March 1985 to November 2009. To investigate the volatility of the Eurodollar market, four time series of futures and futures option contracts are formed according to their maturity. That is, we construct the daily time series of futures and futures option contracts with the first-nearby (three-month), second-nearby (six-month), third-nearby (nine-month), and fourth-nearby (twelve-month) maturity. The time series of the first-nearby maturity contract has maturity up to three months; the second-nearby maturity contract has maturity from three months to six months; the third-nearby maturity contract has maturity from six months to nine months; and the fourth-nearby maturity contract has maturity from nine months to twelve months.

As for option contracts on Eurodollar futures, three time-series are constructed for each of four maturity contracts according to the nearness of money of the underlying options. That is, we formed the time series of at-the-money, in-the-money, and out-of-the-money call and put options for the three-, six-, nine-, and twelve-month futures option contracts. The time series of the at-the-money futures option is constructed daily by finding the one nearest exercise price to the futures price with the same maturity, but with the exercise price within the range of 10 basis points from the futures prices. Thus, the in-the-money futures call [put] option has an exercise price near the futures price minus [plus] 1.00 index point prices within 10 basis point range. The out-of-the-money futures call [put] option has a nearest exercise price within 10 basis point range of the futures price plus [minus] 1.00 index point. For example, we pick the in-the-money call options if the futures price minus the exercise price is closest to one index point but is between 0.9 and 1.1 index point.

Ⅲ. The Informational Content of Implied and Historical Volatility

We calculate the historical volatility and the implied volatility of the logarithm of Eurodollar yield changes to compare with the *ex post* actual volatility. Here, the Eurodollar yield, rather than the Eurodollar futures price, is assumed to have lognormal distribution at the expiration of the underlying contract, since the lognormal distribution allows for the possibility of infinitely large prices and the Eurodollar futures price index never exceeds 100. The *ex post* actual volatility is defined as the annualized standard deviation of the logarithm of yield relatives over the exact remaining life of the underlying futures or option contract until maturity. Thus, the actual volatility at time t of an underlying asset maturing at T, $\sigma_{t,T}^{A}$, is expressed as the square root of the unbiased estimator of variance:

$$\sigma_{t,T}^{A} = \sqrt{\frac{af}{\tau - 1}\sum_{j=t}^{T}\left(R_j - \frac{1}{\tau}\sum_{i=t}^{T}R_i\right)^2}, \qquad (1)$$

where $R_t = \ln\left(y_t / y_{t-1}\right)$, y_t is the Eurodollar yield at time t, $\tau\,(\equiv T - t + 1)$ is the time to maturity of an underlying asset, and af is an annualizing factor.[2)]

2) The actual and historical volatilities are calculated for the underlying contract with at least one week to maturity to avoid the erroneous estimation of the volatility due to the small number of observations in the market. Hence, the estimation period for the actual and historical volatilities ranges from one week to one year since we are considering only the Eurodollar

The forecast of actual volatility should reflect the expected future variability over the remaining life of an underlying asset.

We define the historical volatility in three different ways. The first definition of historical volatility at time *t* of an underlying contract maturing at *T*, $\sigma_{t,T}^{H1}$, is taken from the annualized standard deviation of changes in the logarithm of the Eurodollar yield for the same length of the life of option but from the recent past period. That is,

$$\sigma_{t,T}^{H1} = \sqrt{\frac{af}{\tau - 1} \sum_{j=t-\tau}^{t-1} \left(R_j - \frac{1}{\tau} \sum_{i=t-\tau}^{t-1} R_i \right)^2} \tag{2}$$

The second historical volatility definition at time *t*, $\sigma_{t,T}^{H2}$, is taken from the matching period for the life of the immediately preceding contract maturing at *T'*. That is,

$$\sigma_{t,T}^{H2} = \sqrt{\frac{af}{\tau - 1} \sum_{j=T'-\tau+1}^{T'} \left(R_j - \frac{1}{\tau} \sum_{i=T'-\tau+1}^{T'} R_i \right)^2} \tag{3}$$

The third definition of the historical volatility at time *t*, $\sigma_{t,T}^{H3}$, is calculated from the constant length *M* of the recent past period. It can be expressed as the annualized standard deviation of the logarithm of yield changes for the past fixed length period *M*:

$$\sigma_{t,T}^{H3} = \sqrt{\frac{af}{M - 1} \sum_{j=t-M-1}^{t-1} \left(R_j - \frac{1}{M} \sum_{i=t-M-1}^{t-1} R_i \right)^2} \tag{4}$$

The *ex post* actual volatility and the first and second historical volatilities are calculated from the same length of the period as the life τ of an underlying asset, while the third definition of historical volatility is estimated from the fixed time period *M* regardless of the maturity of the underlying contract.

Based on the assumption of the lognormal distribution of futures prices, Black (1976) and Whaley (1986), respectively, derived the European and American pricing functions for the futures option. In Black's pricing formula, the option on a futures contract can be treated in the same way as the option on a security paying a continuous dividend at risk-free rate. In other words, the value of a call or put option on a futures contract can be determined by replacing the underlying spot price with the discounted futures price, $F \cdot e^{-r\tau}$. However, a Eurodollar time deposit with a principal value of $1 million matures three months after the futures and futures option expiration. Hence, an infinitely large futures price three months before the maturity is not plausible. Therefore, we modify the European option pricing model of Black to apply to the Eurodollar futures call and put options, assuming the Eurodollar yield, rather than the Eurodollar futures price, has a lognormal distribution. When the relative change in yield is assumed to have a lognormal diffusion process,

$$\frac{dy}{y} = \alpha\, dt + \sigma\, d\tilde{z},$$

the partial differential equation for the value of the futures option O is:

$$r\mathrm{O} + \mathrm{O}_{\tau} - \frac{1}{2}\sigma^2 y^2 \mathrm{O}_{yy} = 0 \tag{5}$$

with the boundary condition $C_E = \max[100 - X - y, 0]$ and $P_E = \max[y - (100 - X), 0]$ at maturity.

Then the modified version of Black's European option pricing formula for the futures call option, C_E, and futures put option, P_E, based on the lognormal yield distribution can be expressed as:

$$C_E = e^{-r\tau}\left[(100 - X) \cdot \mathrm{N}(-d_2) - (100 - F) \cdot N(-d_1)\right], \tag{6}$$

$$P_E = e^{-r\tau}\left[(100 - F) \cdot \mathrm{N}(d_1) - (100 - X) \cdot N(d_2)\right], \tag{7}$$

where $d_1 = \dfrac{\ln[(100 - F) / (100 - X)]}{\sigma\sqrt{\tau}} + \dfrac{1}{2}\sigma\sqrt{\tau}$,

futures and futures option contracts with up to twelve months to maturity. On average, there are 252 trading days per year. Hence, we assume 252 for the annualizing factor for the daily time series.

Table 1. Actual, Historical, and Implied Volatilities in Eurodollar Futures Markets

		3-month	6-month	9-month	12-month
Actual Volatility		0.1401	0.1991	0.2390	0.2633
Historical Volatility	H1	0.1880	0.2693	0.2809	0.2649
		(0.075)	(0.099)	(0.089)	(0.085)
	H2	0.1376	0.1905	0.2218	0.2382
		(0.064)	(0.073)	(0.076)	(0.092)
	H3	0.1809	0.2460	0.2778	0.2809
		(0.071)	(0.089)	(0.093)	(0.091)
Call Implied Volatility	ATM	0.1697	0.2172	0.2406	0.2576
		(0.055)	(0.062)	(0.066)	(0.079)
	ITM	0.2265	0.2131	0.2241	0.2452
		(0.125)	(0.082)	(0.079)	(0.096)
	OTM	0.2208	0.2045	0.2116	0.2222
		(0.094)	(0.070)	(0.068)	(0.071)
Put Implied Volatility	ATM	0.1731	0.2173	0.2411	0.2573
		(0.058)	(0.061)	(0.066)	(0.078)
	ITM	0.2126	0.2059	0.2053	0.2208
		(0.078)	(0.069)	(0.068)	(0.079)
	OTM	0.2458	0.2219	0.2217	0.2270
		(0.159)	(0.108)	(0.084)	(0.080)

Notes: 3-month, 6-month, 9-month, and 12-month stand for the underlying futures and futures option contracts with 0-3 months, 3-6 months, 6-9 months, and 9-12 months to maturity at each time, respectively. The volatility is reported by the annualized standard deviation of the logarithm of the Eurodollar yields for each of the Eurodollar futures time series of the 6224 trading days from 1985 to 2009. The actual volatility is calculated from the period of the underlying Eurodollar futures contract. The historical volatility (H1) is calculated for the same length of recent past period as the remaining period until maturity, the historical volatility (H2) is calculated from the previous contract's matching period that has the same length as the underlying contract's life, and the historical volatility (H3) is calculated from the past one month period. The implied volatilities are calculated from at-the-money, in-the-money, and out-of-the-money options applying the option pricing model. The mean absolute errors between actual volatility and the estimated historical or implied volatility are reported in the parentheses.

$d_2 = d_1 - \sigma\sqrt{\tau}$,

$N(\cdot)$ is a standardized normal distribution function,

F is a futures price index,

X is an exercise price, and

r is a risk-free rate.[3)]

This is analogous to Black's model except that F and X are now replaced with 100 - F and 100 - X, respectively, and the call and put option formulae are switched relative to each other.

The implied volatility can be calculated by inverting the option pricing function given the other parameters. In other words, the volatility at time t, $\sigma^I_{t,T}$, implied in the futures option price O is computed from the inverse function of the option pricing formula:

$$\sigma^I_{t,T} = f^{-1}(O, F, X, r, \tau) \tag{8}$$

where f is the futures option pricing function. Since the option pricing function is not easily invertible, we numerically approximate the volatility implied in the option price by equating the model price with the market price of the call or put option. The quasi-Newton method and a finite difference gradient are employed to the modified option pricing model for the futures options.

The volatility implied in different maturity and moneyness options tends to exhibit different behavior.

3) We use the three-month Treasury bill rate as a proxy for risk-free interest rate, which is available from the Federal Reserve Board's statistical releases and historical data.

Figure 1 shows the changes of actual and implied volatilities taken from the Eurodollar call and put options using the option pricing formula as underlying contracts matures. Each point in the graph represents the average of the actual volatility with the same number of days to maturity and the average of the implied volatilities with the same number of days to maturity for each of at-the-money, in-the-money, and out-of-the-money options.[4)]

In general, the actual volatility (dark solid line) and the volatility implied in at-the-money options (dotted line) stay stable and gradually decrease as the underlying contract matures. This is because investors in the market have more consensus on the futures price with less noise as the underlying asset matures and the uncertainty in the market declines. The implied volatilities taken from in-the-money and out-of-the-money options with less than three months to maturity increase rapidly as the underlying contracts mature. This is mainly due to noise and the lack of liquidity in the market. The behavior of the implied volatility taken from the futures put option is very similar to that from a call option.

If the implied volatility from the option price incorporates all of the information in the market including the past history of volatility, whereas the historical volatility imposes equal weight on past prices, the implied volatility is expected to informationally dominate the historical volatility. The changes in the actual, historical and implied volatilities are summarized in Table 1 for each of the three-, six-, nine-, and twelve-month contracts and for different moneyness of call and put options using the option pricing models. Each volatility in the table represents the average of annualized volatility from the logarithm of the Eurodollar yield for the 25-year period from 1985 to 2009.

The actual volatility gradually declines as the underlying futures contract matures and drops rapidly near the maturity. It declines from the mean of 26.33% for the twelve-month maturity to a mean of 14.01% for less than three months to maturity. In order to examine the predicting power of the estimates for the actual volatility, Table 1 also reports the mean absolute errors of the historical and implied volatilities in parentheses.[5)] The implied volatilities taken from in-the-money and out-of-the-money call and put options underestimate the actual volatilities for longer maturity options, but overestimate for short maturity options with the mean absolute errors much higher than those for the historical volatilities and for the implied volatilities taken from at-the-money options. At-the-money implied volatilities for both call and put options fairly resembles the *ex post* actual volatilities with smaller mean absolute errors. Historical volatilities H1 and H2 have relatively large mean absolute errors whereas those for H2 are closer to the actual volatilities with moderate mean absolute errors, but still higher than the mean absolute error of at-the-money implied volatilities.

The volatility bias implied in option prices especially for in-the-money and out-of-the-money options could be partly due to the liquidity risk in options markets as evidenced by Cetin, Jarrow, Protter and Warachka (2006) and many other researchers. The median trading volumes for the time-series of different maturity and different moneyness options are reported in Table 2. As expected, the at-the-money options are traded more frequently with larger trading volume compared with the in-the-money and out-of-the money options, implying the positive correlation between the illiquidity in the market and the implied volatility reported in Table 1. In addition, trading volume increases as the Eurodollar futures option contract matures, especially for at-the-money options.

In the next section, we investigate more closely the informational content of the volatility in the Eurodollar futures and futures option markets for

4) There exist about 106 contracts during the 25-year period of Eurodollar futures and futures options that mature four times a year. For short maturity out-of-the-money options, only a small number of observations are available due to the lack of trading.

5) Mean absolute errors are calculated as $\frac{1}{T}\sum_{j=1}^{T}\left|\sigma_j^{A}-\sigma_j^{i}\right|$ where $i = H$ for the historical volatility and $i = I$ for the implied volatility, and T is the 6224 trading days for maximum observations.

Table 2. Trading Volumes in Eurodollar Futures Options Markets

Eurodollar Futures Options		3-month	6-month	9-month	12-month
Call Volume	ATM	2975	2860	1241	700
	ITM	92	77	57	52
	OTM	147	200	360	403
Put Volume	ATM	2288	2367	1100	625
	ITM	150	178	144	109
	OTM	148	40	30	35

Notes: This table presents the median trading volumes of at-the-money, in-the-money and out-of-the money Eurodollar futures call and put options with 3-month, 6-month, 9-month, and 12-month maturities. 3-month, 6-month, 9-month, and 12-month stand for the underlying futures and futures option contracts with maturities ranging 0-3 months, 3-6 months, 6-9 months, and 9-12 months, respectively. The time series of the at-the-money futures option is constructed by finding the one nearest exercise price to the futures price with the exercise price within the range of 10 basis points. In-the-money or out-of-the-money option has a nearest exercise price within 10 basis point range of the futures price plus or minus 1.00 index point.

different maturity and nearness to money options. We perform a regression test of the implied and historical volatilities as predictors of the future actual volatility applying the autocorrelation and heteroscedasticity consistent GMM technique.

Ⅳ. GMM Regression Test Correcting for Autocorrelation and Heteroscedasticity

When a volatility forecast contains all relevant information about the future course of volatility, it should reflect the unbiased expectations of the future actual volatility over the life of the underlying asset, conditional upon the available information set, Ω_t. That is,

$$\sigma_{t,T}^{i} = E\left(\sigma_{t,T}^{A} \middle| \Omega_t\right) \tag{9}$$

where σ^i represents the market's expectation of the volatility over the life of underlying asset.

In this section, the rationality of the volatility forecast is investigated using the regression test for the relation (9), adjusting for the autocorrelation and heteroscedasticity of the forecasting error with the generalized method of moments (GMM) technique. To compare the alternative measure of volatility forecasts, we consider the relation between the actual volatility, $\sigma_{t,T}^{A}$, and either the historical volatility, $\sigma_{t,T}^{H}$, or the implied volatility, $\sigma_{t,T}^{I}$:

$$\sigma_{t,T}^{A} - \mu^{A} = \beta \cdot \left(\sigma_{t,T}^{i} - \mu^{i}\right) + \varepsilon_t \quad \text{for} \quad i = H, I, \tag{10}$$

where μ is a long-term mean. This model can be rewritten as:

$$\sigma_{t,T}^{A} = \alpha + \beta \cdot \sigma_{t,T}^{i} + \varepsilon_t, \quad \text{for} \quad i = H, I, \tag{11}$$

where $\alpha = \mu^{A} - \beta\mu^{i}$. ε_t includes the forecast error of market expectation and the measurement error due to noise in the market. If the volatility forecast is a true expectation of future actual volatility, the above regression is estimated such that the statistical sampling model is on average correct, given the available information set. That is, $E\left(\sigma_{t,T}^{A} \middle| \Omega_t\right) = \sigma_{t,T}^{i}$ with orthogonal error term, $E\left(\varepsilon_t \middle| \Omega_t\right) = 0$.

Market efficiency does not necessarily imply that *ex ante* expectations of volatility equal the *ex post* realization of actual volatility. However, the efficiency in the option market suggests that the volatility implied in option prices predicts the actual variability of returns from the underlying asset. In regression equation (11), a better forecast is expected to have a higher coefficient of determination, R^2, and a β close to one.

OLS estimation of the linear statistical model

assumes that errors are specified as homoscedastic and the sampling process for ε_t and regressor $\sigma_{t,T}^{A}$ is uncorrelated. However, the regression (11) involves the overlapping error structure defined by the maturity cycle of the underlying security. Hence, yesterday's forecast error tends to be transmitted to today's volatility forecast. In addition, since the volatility time series are calculated from a different number of price observations over different lengths of the option's life at each time, the forecasting errors for different time periods are expected to have different precisions. For example, $\sigma_{t,T}$ from the *(T-t+1)* time period of underlying asset's life is more reliable than $\sigma_{t-1,T}$ from the *(T-t)* time period. In other words, the forecasting errors are heteroscedastic, and this should be reflected in the estimation process with different weights.

While OLS estimation would generate unbiased and consistent parameter estimates as long as ε_t is independent of $\sigma_{t,T}^{i}$, the OLS covariance matrix of parameters would be inconsistent because of autocorrelation and heteroscedasticity. Hansen (1982) demonstrates that the OLS estimator converges in distribution to a normally distributed random vector in large samples. That is,

$$\sqrt{n}\cdot\left(\hat{\beta}-\beta\right)\xrightarrow{A}\mathrm{N}(0,\Theta), \tag{12}$$

where n is a sample size, and $\hat{\beta}$ is an OLS estimator of parameter β. Define x_t as the $1'k$ row vector of explanatory variables for observation t. That is, $x_t=\left(1 \quad \sigma_{t,T}^{i}\right)$ in regression (11). Then the asymptotic covariance matrix Θ can be consistently estimated by the GMM technique as:

$$\hat{\Theta}=\left(\frac{1}{n}\sum_{t=1}^{n}x_t'x_t\right)^{-1}\hat{\Psi}\left(\frac{1}{n}\sum_{t=1}^{n}x_t'x_t\right)^{-1}. \tag{13}$$

Newey and West (1987) propose a consistent and positive semi-definite estimator $\hat{\Psi}$ using the modified Bartlett weights $\omega(j,m)=1-\frac{j}{m+1}$:

$$\hat{\Psi} = \hat{C}(0)+\sum_{j=1}^{m}\omega(j,m)\left[\hat{C}(j)+\hat{C}'(j)\right], \tag{14}$$

where the sample autocovariance $\hat{C}(j)=\frac{1}{n}\sum_{t=J+1}^{n}\hat{\varepsilon}_t\hat{\varepsilon}_{t-j}\cdot x_t'x_{t-j}$, m is the bound on the number of non-zero autocorrelation, and $\hat{\varepsilon}_t$ is the OLS residual for observation t. The resulting standard error and t-statistics correct for the autocorrelation and heteroscedasticity and, hence, are consistent.[6]

To test the predictive power of historical and implied volatilities, regression equation (11) is fitted separately for samples of different maturity and moneyness options. The equation is estimated using the ordinary least squares method, and the covariance matrix is adjusted for heteroscedasticity and serial dependence in the time series of forecast errors. We are interested in testing the null hypothesis, H_0: $\beta=1$, against the alternative, H_1: $\beta\neq1$. The test statistics used for this purpose is $\sqrt{n}(\hat{\beta}-1)/\sqrt{\hat{\Theta}_{22}}$ where $\hat{\Theta}_{22}$ is the second diagonal element of the consistent covariance matrix in equation (13).[7] The results are presented in Table 3 for the historical volatility, and Table 4 and Table 5 for the implied volatility calculated from the call and put option prices, respectively. The consistent t-statistics for α

6) Application of the GMM technique may not result in asymptotically efficient estimators compared with the generalized least squares procedures. However, the GLS procedure can result in inconsistent parameter estimates and requires the complete specification of the nature of the serial correlation and heteroscedasticity, while the GMM technique implicitly permits the disturbance terms to be both serially correlated and heteroscedastic in the construction of the orthogonality conditions.

7) Most previous research reports the statistics testing β = 0, which can easily be misleading. For example, the slope coefficient, β, can be estimated positive and significantly different from zero with a small standard error. However, this does not imply that β is close to one with statistical significance and that the volatility forecast is a good predictor of the actual volatility. The small standard error may, rather, result in the rejection of the hypothesis of β = 1 even though the slope coefficient is fairly close to one.

Table 3. Regression Test of Historical Volatility as a Predictor of Actual Volatility in Eurodollar Futures and Futures Option Markets

$$\sigma^{A} = \alpha + \beta\sigma^{H} + \varepsilon$$

		3-month	6-month	9-month	12-month
Historical Volatility (H1):	a	0.0279	0.0256	0.0296	0.0328
	t-stat	(15.27)	(1.00)	(13.66)	(13.13)
	b	0.5968	0.6441	0.7456	0.8754
	t-stat	(-34.14)	(-35.09)	(-26.73)	(-11.25)
	adj R^2	0.612	0.626	0.675	0.648
	No. of obs	5805	6184	6184	6071
Historical Volatility (H2):	a	0.0422	0.0468	0.0343	0.0536
	t-stat	(20.23)	(25.46)	(14.98)	(18.67)
	b	0.7128	0.8013	0.9249	0.8822
	t-stat	(-17.67)	(-17.86)	(-6.53)	(-8.97)
	adj R^2	0.472	0.536	0.612	0.496
	No. of obs	5801	6183	6182	6183
Historical Volatility (H3):	a	0.0338	0.0485	0.0610	0.0693
	t-stat	(17.78)	(20.25)	(28.87)	(31.91)
	b	0.5881	0.6119	0.6408	0.6906
	t-stat	(-32.57)	(-33.27)	(-38.88)	(-32.83)
	adj R^2	0.623	0.622	0.648	0.658
	No. of obs	5805	6184	6184	6184

Notes: 3-month, 6-month, 9-month, and 12-month stand for the underlying futures and futures option contracts with 0-3 months, 3-6 months, 6-9 months, and 9-12 months to maturity at each time, respectively. The actual volatility, σ^{A}, is regressed on the historical volatility, σ^{H}, for each maturity group. The actual volatility is calculated over the remaining life of the underlying Eurodollar futures contract. The historical volatility (H1) is calculated for the same length of the recent past period as the remaining period until maturity; the historical volatility (H2) is calculated from the previous contract's matching period that has the same length as the underlying contract's life, and the historical volatility (H3) is calculated from the past one month period. The consistent *t*-statistics for α test for $\alpha = 0$ and the *t*-statistics for β test for the null hypothesis of $\beta = 1$.

Table 4. Regression Test of Implied Volatility from Call Option Prices as a Predictor of Actual Volatility in Eurodollar Futures and Futures Option Markets

$$\sigma^{A} = \alpha + \beta\sigma^{I} + \varepsilon$$

		3-month	6-month	9-month	12-month
ATM	a	0.0113	0.0136	0.0013	0.0007
	t-stat	(5.65)	(4.97)	(0.55)	(0.22)
	b	0.7401	0.8612	1.0096	1.0732
	t-stat	(-18.77)	(-9.23)	(0.82)	(5.31)
	adj R^2	0.743	0.701	0.696	0.682
	No. of obs	5105	5218	4981	4847
ITM	a	0.0237	-0.1143	-0.0750	-0.0030
	t-stat	(1.08)	(-12.45)	(-10.64)	(-0.51)
	b	0.3701	1.3239	1.3489	1.1650
	t-stat	(-6.37)	(6.95)	(9.96)	(6.35)
	adj R^2	0.052	0.411	0.460	0.452
	No. of obs	2207	3649	3417	3188
OTM	a	0.0494	-0.0417	-0.0893	-0.0957
	t-stat	(3.64)	(-4.4)	(-11.12)	(-13.98)
	b	0.4232	0.9645	1.3310	1.4207
	t-stat	(-9.47)	(-0.73)	(8.09)	(12.02)
	adj R^2	0.093	0.359	0.520	0.582
	No. of obs	648	2873	3870	3914

Notes: 3-month, 6-month, 9-month, and 12-month stand for the underlying futures and futures option contracts with 0-3 months, 3-6 months, 6-9 months, and 9-12 months to maturity at each time, respectively. The actual volatility, σ^{A}, is regressed on the implied volatility, σ^{I}, calculated from the option pricing model for each maturity and moneyness group. The consistent *t*-statistics for α test for $\alpha = 0$ and the *t*-statistics for β test for the null hypothesis of $\beta = 1$.

Table 5. Regression Test of Implied Volatility from Put Option Prices as a Predictor of Actual Volatility in Eurodollar Futures and Futures Option Markets

$$\sigma^A = \alpha + \beta\sigma^I + \varepsilon$$

		3-month	6-month	9-month	12-month
ATM	a	0.0099	0.0114	0.0022	0.0015
	t-stat	(5.11)	(4.23)	(0.93)	(0.5)
	b	0.7281	0.8684	1.0053	1.0690
	t-stat	(-20.5)	(-8.9)	(0.46)	(5.04)
	adj R^2	0.738	0.705	0.698	0.685
	No. of obs	5115	5209	4988	4903
ITM	a	-0.0334	-0.0570	-0.0501	-0.0064
	t-stat	(-1.91)	(-6.67)	(-9.49)	(-1.39)
	b	0.8749	1.1632	1.2635	1.1458
	t-stat	(-1.44)	(3.65)	(8.97)	(6.4)
	adj R^2	0.382	0.411	0.492	0.481
	No. of obs	796	3299	4281	4475
OTM	a	0.2026	0.0164	-0.1340	-0.1563
	t-stat	(16.31)	(1.06)	(-10.01)	(-13.82)
	b	-0.4455	0.5084	1.4217	1.5846
	t-stat	(-28.46)	(-6.73)	(6.59)	(10.43)
	adj R^2	0.111	0.095	0.470	0.583
	No. of obs	1473	2503	2552	2521

Notes: 3-month, 6-month, 9-month, and 12-month stand for the underlying futures and futures option contracts with 0-3 months, 3-6 months, 6-9 months, and 9-12 months to maturity at each time, respectively. The actual volatility, σ^A, is regressed on the implied volatility, σ^I, calculated from the option pricing model for each maturity and moneyness group. The consistent *t*-statistics for α test for $\alpha = 0$ and the *t*-statistics for β test for the null hypothesis of $\beta = 1$.

test for $\alpha = 0$, and the *t*-statistics for β test for the null hypothesis of $\beta = 1$.

As indicated in the preliminary study of mean absolute errors in Table 1, the implied volatility tends to outperform different definitions of the historical volatility. Table 3 reports the coefficient estimates and the statistics for the regression of the actual volatility on the different definitions of the historical volatility for different maturity samples. The slope coefficients are estimated as significantly different from one, rejecting the null hypothesis of $\beta = 1$ even at the one percent level for all definitions of the historical measures of volatility and for all maturities. The intercept coefficient estimates are also significantly different from zero in most cases. For example, for the first definition of the historical volatility (H1) with a three-month maturity, the intercept and slope coefficients are estimated as 0.0279 and 0.5968 with an adjusted coefficient of determination of 0.612. The slope estimates are significantly below one for most cases which imply that various measures of the historical volatility are biased and are inefficient forecasts of future variability in the Eurodollar futures market.

In Table 4 and Table 5, the regressions of the actual volatility against the implied volatility from the call and put option prices, respectively, are fitted using the ordinary least squares method and the heteroscedasticity and autocorrelation consistent estimate of the covariance matrix of Newey and West. For most implied volatilities, the slope coefficient estimates are now much closer to one especially for at-the-money options, but still significantly different from one at the one percent level. For example, 9-month maturity implied volatilities from call or put at-the-money options have the slope coefficients not different from one even at the five percent significance level. The coefficient estimates for the intercept term are now much close to zero, and about half of them are not different from zero at the one percent level. The adjusted coefficients of determination for

at-the-money options are much higher, but those for in-the-money and out-of-the-money options are much lower on average than those for the historical volatilities. However, the contracts with less than three months to maturity have slope coefficient estimates significantly different from one even at the one percent level. The implied volatility from the at-the-money call or put options dominates the various measures of historical volatilities in terms of *ex ante* forecasting power. The historical volatilities, especially for H1 and H3, are less accurate forecasts of future actual volatility than the implied volatility, and hence are not informationally efficient. The slope coefficient estimated as less than one could be interpreted as an indication that high volatilities are usually under-predicted by the alternative forecasting rules.

Overall, the empirical results here are consistent with the previous researches including Latane and Rendleman (1976), Day Lewis (1988) and Fleming (1998) in equity market which find the implied volatility dominates the historical volatility in forecasting power. However, they are contrary to the findings of Canina and Figlewske (1993) in equity market also where the choice of wider at-the-money band may lead to the bias in their research. The implied volatility bias in Figure 1 especially for short maturity options is also consistent with Jarrow, Li and Zhao (2007) and Deuskar, Gupta and Subrahmanyam (2008) which examine the patterns of volatility smile in long-term over-the-counter interest rate derivatives markets.

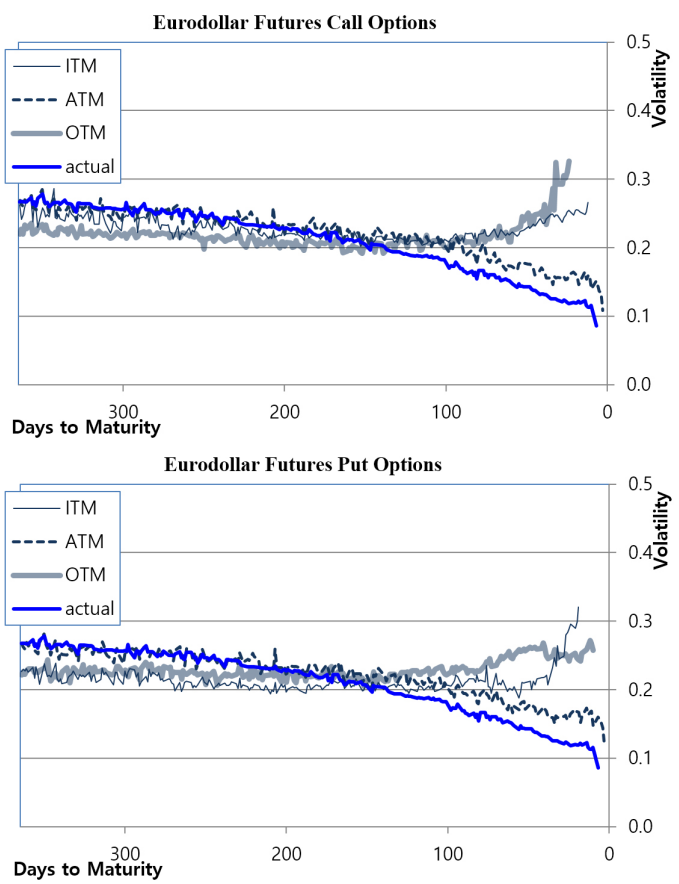

Figure 1. Actual and Implied Volatilities Using the Option Pricing Model in Eurodollar Futures and Futures Option Markets

Ⅴ. Concluding Remarks

The relationship between asset return and risk and, hence, the volatility of asset prices has been of interest to financial economists. Derivative securities can be used for hedging and mitigating risk, as well as for speculation and arbitrage which sometimes cause turbulences in the financial markets. The volatility implied in the option price reflects investors' assessments of future market volatility. If the option market is efficient, all relevant information should be contained in the option price, and the implied volatility should represent a rational forecast of future volatility when the appropriate option pricing model is employed.

This study contributes to the empirical research on market efficiency and the informational content of volatility in the Eurodollar futures and futures options markets which are considered as the largest exchange-traded short-term interest rate derivatives markets. We investigate how volatility expectations are formed and test for the predictive power of volatility estimators. We establish that both historical and implied volatility have informational content relevant to the prediction of *ex post* future realized volatility. In general, when defined properly, the implied volatility is a better estimate of the market's future volatility than the historical measure of volatility since the implied volatility contains the information available in the market, including the past history of the underlying security. However, the implied volatility taken from short maturity deep in-the-money and deep out-of-the-money options may be sensitive to the option pricing model used. In the GMM regression of the actual volatility on the rationally forecast volatility, the implied volatility tends to outperform different definitions of the historical volatility. The serial dependence and heteroscedasticity in the time series of forecast errors are adjusted using Newey and West's positive semi-definite consistent estimate of the covariance matrix. For longer maturity contracts, the expectation on Eurodollar yield and hence on Eurodollar futures price does not change much with relatively stable volatility, and the volatility in the Eurodollar futures and futures options markets tends to decrease as the underlying contract matures and the uncertainty in the market is reduced.

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