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Estimating the Length of the Excess Earnings Period

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ABSTRACT

This paper presents several approaches to the problem of measuring the length of the excess earnings period. Each of these models is based upon the assumption that the return on equity must inevitably decline over time and return to the opportunity cost of capital. One of these models is a simplified partial retention model that approximates the standard dividend discount model. We demonstrate that this simplified model is capable of producing a robust approximation of the standard dividend discount model, while facilitating a much simpler 3-stage pricing model. Further, we provide evidence that the excess earnings period is relatively short, even when we consider the market power of the Dow Thirty and even when we restrict all post-horizon returns to the opportunity cost of equity.

Keywords: Excess Earnings Period, Simplified Growth Model, Supranormal Growth Model, Multistage Growth Model

I. Introduction

This study addresses the problem of continuing valuation beyond the identifiable horizon: a problem which presents itself in most models of equity valuation or enterprise valuation. Damodaran (2006), Reilly and Brown (2009), and Koller, Goedhart and Wessels (2005), which we will subsequently refer to as the McKinsey study, all presume a perpetuity of earnings in excess of the opportunity cost of capital. Of course, that was the problem with the original discounted cash flow model of Gordon (1962); and while the multi-stage models presented in these subsequent studies have succeeded in reducing the impact of that presumption, they have not entirely resolved the problem. In reality, most applications of these models produce

results where the majority of value is derived from horizon value, as opposed to the preceding cash flows. Thus, the presumption of some arbitrary growth rate or return rate which is presumed to persist in perpetuity undermines the objectivity of the exercise in much the same way as did the presumption of the growth rate in the original Gordon discounted dividend model.

Recent studies such as Feltham and Ohlson (1995), and Palepu, Bernard and Healy (2000) have addressed that problem and have concluded that long run return on equity should eventually converge to the opportunity cost of equity. Soffer (2003) expresses some caveats to that assumption but those caveats are beyond the scope of this study. We believe that this study will prove valuable in simplifying the valuation process, restoring some objectivity to the computation of horizon value, and in providing a clearly defined set of elasticities by which to measure the impact of key assumptions.

We compare simplified growth models in Section II

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and provide empirical evidence in Section III. Section IV shows an application to the 3-stage growth duration model. Then, we conclude in Section V.

II. Simplified Growth Models

Incorporating this fundamental assumption into the standard dividend discount model, we make the assumption that return on equity R is only possible for n periods beyond the current year (i.e., from the end of year 1 to the end of year $n+1$), after which the return on equity is restricted to the opportunity cost of equity r . In the model given below, we designate the dividend as D_t and the earnings per share as E_t .

$$P_0 = \frac{P_n}{(1+r)^n} + \sum_{k=1}^n \frac{D_k}{(1+r)^k} \quad (1)$$

Letting $D_t = E_t(1-\lambda)$, and $E_t = E_{t-1}(1+R\lambda)$ for $t = 2 \dots n+1$,

$$P_0 = \left(\frac{1+R\lambda}{1+r} \right)^n \frac{E_1}{r} + \sum_{k=1}^n \frac{E_1(1+R\lambda)^{k-1}(1-\lambda)}{(1+r)^k} \quad (2)$$

Thus, we can derive a closed form expression (See Appendix A) for the valuation of the stock with a retention ratio of λ and a payout ratio of $(1-\lambda)$.

$$P_0 = \frac{E_1(1-\lambda)}{(r-R\lambda)} - \frac{E_1\lambda(R-r)}{r(r-R\lambda)} \left(\frac{1+R\lambda}{1+r} \right)^n \quad (3)$$

We are able to demonstrate that the sign of this derivative with respect to retention λ is positive at all points, except when $(r-R\lambda) = 0$. This model also gives us a method of estimating the excess return period.

$$n = \frac{\ln \left(\frac{E_1(1-\lambda)r - P_0 r(r-R\lambda)}{E_1\lambda(R-r)} \right)}{\ln \left(\frac{1+R\lambda}{1+r} \right)} \quad (4)$$

an expression which is defined for all positive values of R , r , and λ , except where $E_1(1-\lambda)r - P_0 r(r-R\lambda) < 0$.

A. Simplified Full Retention Two Stage Model

The first of our simplified supernormal growth models is based upon the assumption that the firm is able to reinvest the entirety of its cash flows at the excess rate of R . When that is possible, the firm follows an optimal dividend and retention policy of retaining all earnings during the supernormal earnings period. Such a model results in a current price of P_0^* , where

$$\lim_{\lambda \rightarrow 1} P_0 = P_0^* = \left(\frac{1+R}{1+r} \right)^n \frac{E_1}{r} \quad (5)$$

$$P_0 \leq P_0^* \quad (6)$$

Since the simplified model P_0^* is always larger than or equal to the standard dividend discount model P_0 , except when $\lambda = 1$, we can think of it as an upper bound price which can be applied to all firms, whether they reinvest all cash flows or not. In addition to providing this upper bound to price, our full reinvestment model provides a simplified approach to measurement of the implied excess earnings period.

$$\lim_{\lambda \rightarrow 1} n = n^* = \frac{\ln \left(\frac{P_0^* r}{E_1} \right)}{\ln \left(\frac{1+R}{1+r} \right)} \quad (7)$$

B. Simplified Partial Retention Two Stage Model

In cases where the retention rate λ is less than unity, we propose a second simplified pricing model, P_0^{**} , under the intuitive assumption that the firm will reinvest earnings at the rate R for a period of λn periods and that the firm will reinvest earnings at the opportunity cost of equity r for $(1-\lambda)n$ periods. Thus,

$$P_0^{**} = \frac{(1+R)^{\lambda n} (1+r)^{(1-\lambda)n} E_1}{(1+r)^n r} \quad \text{reduces to} \quad (8)$$

$$P_0^{**} = \left(\frac{1+R}{1+r} \right)^{\lambda n} \frac{E_1}{r} \quad (9)$$

That model results in the following measurement of the implied excess earnings period.

$$n^{**} = \frac{\ln \left(\frac{P_0^{**} r}{E_1} \right)}{\lambda \ln \left(\frac{1+R}{1+r} \right)} \quad (10)$$

C. Comparison of the Three Models

With respect to the relative pricing under the three models, we can easily demonstrate that

$$\lim_{\lambda \rightarrow 0} P_0 = P_0^{**} = \frac{E_1}{r}, \quad (11a)$$

$$\lim_{\lambda \rightarrow 1} P_0 = P_0^{**} = P_0^* = \left(\frac{1+R}{1+r} \right)^n \frac{E_1}{r}, \quad (11b)$$

$$\lim_{z \rightarrow 0} P_0 = P_0^{**} = P_0^* = \frac{E_1}{r}. \quad (12)$$

Table 1. Implied Excess Earnings Period (n) of the Dow Thirty from Raw Data as of May 9, 2014

	Price	Book	EPS	ROE	r	λ	n	n^{**}	P^{**}
MMM	140.120	27.400	7.645	0.279	0.098	0.504	8.390	7.680	147.97
AXP	86.930	18.870	5.566	0.295	0.094	0.818	2.758	2.769	86.80
T	35.630	17.410	3.602	0.207	0.065	0.464	-6.394	-7.586	38.17
BA	129.940	19.870	6.432	0.324	0.091	0.497	6.677	6.400	133.44
CAT	105.010	32.400	6.468	0.200	0.112	0.592	17.048	13.462	123.25
CVX	124.720	77.920	11.755	0.151	0.093	0.639	-0.455	-0.470	124.78
CSCO	22.940	10.800	1.672	0.155	0.099	0.500	16.056	12.205	25.25
DD	66.840	17.590	5.698	0.324	0.114	0.653	2.579	2.593	66.73
XOM	102.010	40.140	8.107	0.202	0.088	0.658	1.454	1.470	101.90
GE	26.680	13.120	1.328	0.101	0.097	0.278	xxx	xxx	xxx
GS	158.880	152.490	16.698	0.110	0.124	0.855	-13.428	-15.019	156.17
HD	79.400	9.070	4.356	0.480	0.086	0.500	2.780	2.915	77.75
INTC	26.410	11.670	2.079	0.178	0.088	0.524	2.560	2.536	26.44
IBM	191.440	16.440	15.947	0.970	0.079	0.740	-0.106	-0.115	192.15
JNJ	99.310	26.250	5.590	0.213	0.078	0.495	5.880	5.612	100.88
JPM	55.580	54.020	4.323	0.080	0.122	0.604	-14.400	-19.599	49.38
MCD	101.430	16.170	5.940	0.367	0.067	0.411	1.271	1.359	100.53
MRK	58.220	17.000	1.596	0.094	0.074	-0.158	xxx	xxx	xxx
MSFT	39.690	10.580	2.937	0.278	0.079	0.581	0.660	0.687	39.59
NKE	72.990	12.640	3.330	0.263	0.078	0.672	5.157	5.091	73.50
PFE	30.750	11.920	3.350	0.281	0.082	0.674	-2.305	-2.467	31.32
PG	81.920	25.170	4.018	0.160	0.069	0.309	18.116	13.573	91.84
KO	40.950	7.440	1.972	0.265	0.067	0.348	5.814	5.647	41.36
TRV	91.000	73.060	11.178	0.153	0.095	0.807	-5.792	-6.186	92.51
UTX	116.520	34.760	6.925	0.199	0.095	0.622	9.156	8.356	121.90
UNH	75.030	32.680	6.039	0.185	0.077	0.794	-0.507	-0.517	75.09
VZ	47.120	13.570	4.268	0.315	0.056	0.470	-4.076	-4.664	50.06
V	204.42	42.610	9.216	0.216	0.076	0.796	5.418	5.351	205.78
WMT	79.120	23.590	5.270	0.223	0.069	0.607	0.416	0.430	79.03
DIS	80.310	26.070	4.153	0.159	0.101	0.764	19.574	16.901	89.26
Mean	85.505	30.705	5.950	0.246	0.087	0.564	3.182	2.222	90.795

r represents the opportunity cost of equity capital, λ is the retention rate, n is the implied length of the excess earnings period computed under the standard dividend discount model, n^{**} is the length of the excess earning period computed under the simplified partial retention model, and P^{**} is the implied price under the SPRM, using the excess earnings period computed under the standard dividend discount model.

In addition to the conditions where n is undefined, which are enumerated above, there are a number of conditions under which n will take on a negative value; and we need to identify the causes of such negative values in order to interpret those results. For purposes of illustration, we will begin by addressing the causes of negative values in the simplified partial retention model, since its insights are more transparent.

In that model, we will compute negative values for n is whenever

$$P_0^{**} < \frac{E_1}{r} \quad (13)$$

In other words, when either the current price is so low that it implies returns below the current level, or when current earnings are so unusually high that we do not expect them to persist. A second condition which would lead to a negative n value is when

$$R < r \quad (14)$$

And a third condition which would lead to negative n value is when

$$\lambda < 0 \quad (15)$$

Table 2. Implied Excess Earnings Periods (n) of the Dow Thirty from Normalized Data as of May 9, 2014

	Price	Book	EPS	ROE	r	λ	n	n^{**}	P^{**}
MMM	140.12	27.4	7.645	0.279	0.098	0.504	8.39	7.68	147.97
AXP	86.93	18.87	5.566	0.295	0.094	0.818	2.758	2.769	86.8
T	35.63	17.41	2.32	0.133	0.065	0.464	0	0	35.63
BA	129.94	19.87	6.432	0.324	0.091	0.497	6.677	6.4	133.44
CAT	105.01	32.4	6.468	0.2	0.112	0.592	17.048	13.462	123.25
CVX	124.72	77.92	11.6	0.149	0.093	0.639	0	0	124.72
CSCO	22.94	10.8	1.672	0.155	0.099	0.5	16.056	12.205	25.25
DD	66.84	17.59	5.698	0.324	0.114	0.653	2.579	2.593	66.73
XOM	102.01	40.14	8.107	0.202	0.088	0.658	1.454	1.47	101.9
GE	26.68	13.12	2.12	0.162	0.097	0.55	6.84	6.23	27.19
GS	158.88	152.49	19.7	0.129	0.124	0.855	0	0	158.88
HD	79.4	9.07	4.356	0.48	0.086	0.5	2.78	2.915	77.75
INTC	26.41	11.67	2.079	0.178	0.088	0.524	2.56	2.536	26.44
IBM	191.44	16.44	15.124	0.97	0.079	0.74	0	0	191.44
JNJ	99.31	26.25	5.59	0.213	0.078	0.495	5.88	5.612	100.88
JPM	55.58	54.02	6.78	0.126	0.122	0.604	0	0	55.58
MCD	101.43	16.17	5.94	0.367	0.067	0.411	1.271	1.359	100.53
MRK	58.22	17	2.7	0.159	0.074	0.32	32.66	19.21	80.46
MSFT	39.69	10.58	2.937	0.278	0.079	0.581	0.66	0.687	39.59
NKE	72.99	12.64	3.33	0.263	0.078	0.672	5.157	5.091	73.5
PFE	30.75	11.92	2.52	0.211	0.082	0.674	0	0	30.75
PG	81.92	25.17	4.018	0.16	0.069	0.309	18.116	13.573	91.84
KO	40.95	7.44	1.972	0.265	0.067	0.348	5.814	5.647	41.36
TRV	91	73.06	8.65	0.118	0.095	0.807	0	0	91
UTX	116.52	34.76	6.925	0.199	0.095	0.622	9.156	8.356	121.9
UNH	75.03	32.68	5.78	0.177	0.077	0.794	0	0	75.03
VZ	47.12	13.57	2.64	0.195	0.056	0.47	0	0	47.12
V	204.42	42.61	9.216	0.216	0.076	0.796	5.418	5.351	205.78
WMT	79.12	23.59	5.27	0.223	0.069	0.607	0.416	0.43	79.03
DIS	80.31	26.07	4.153	0.159	0.101	0.764	19.574	16.901	89.26
Mean	85.71	29.76	5.91	0.244	0.087	0.592	5.709	4.683	88.37

r represents the opportunity cost of equity capital, λ is the retention rate, n is the implied length of the excess earnings period computed under the standard dividend discount model, n^{**} is the length of the excess earning period computed under the simplified partial retention model, and P^{**} is the implied price under the SPRM, using the excess earnings period computed under the standard dividend discount model.

III. Evidence

In this section, we attempt to empirically measure the implied excess earnings period: a value which we cannot observe directly, but which we can only estimate based upon the observed price of the share, observed return on equity, observed retention ratio, and the presumed opportunity cost of equity as computed under the CAPM. As stated before, we approach this problem under the stipulation that post-horizon returns are restricted to the opportunity cost of equity.

In Table 1, we extracted information from the Dow Thirty stocks, as of May 9, 2014. That information included current dividends and earnings, as well as book values. We computed return on equity by dividing forward projected EPS by the published book value per share. In computing forward projected EPS, we used last year's EPS adjusted for growth. In computing retention rates, we computed forward dividends by using last year's dividends and by increment by published growth rates. For the opportunity cost of equity, we used adjusted beta values and the CAPM, with a presumed risk free rate of 2 percent and a market risk premium of 7 percent. As you can see from table one, there are a number of negative values to n , indicating either poor expectations regarding future earnings, or alternatively above average earnings in the current period.

In table 2, we used normalized values in our computation of n where n was either negative on the first pass, or where n was undefined. Where the computed value of n was negative on the first pass, as was the case with T, CVX, GS, IBM, JPM, PFE, TRV, UNH, and VZ, we normalized the data in order to satisfy the inequality $P_0 \geq E_1 / r$. Where $P_0 < E_1 / r$, we adjusted earnings per share so that $P_0 \cdot r = E_1$. Where n was undefined, as in the case of, GE, MRK because of the condition described in (7a) we normalized values following Miller and Rock (1985) and others. That is to say we utilized the observed value of dividends, D_t , the observed growth rate of dividends, g , and the book value per share, B_{t-1} , in order to improve our estimate the "true" return on equity \bar{R} , the "true" retention rate $\bar{\lambda}$, and the "true," or sustainable earnings \bar{E}_t .

Thus, we normalized GE and MRK as follows:

$$\text{letting} \quad \bar{R} = \frac{D_t}{B_{t-1}} + g, \quad (16a)$$

$$\bar{\lambda} = \frac{g B_{t-1}}{D_t + g B_{t-1}}, \quad (16b)$$

$$\text{and} \quad \bar{E}_t = D_t + g B_{t-1}. \quad (16c)$$

And the results of our computations based upon normalized data is included in Table 2. Our analysis of the Dow Thirty in Table 2 demonstrates that, our simplified partial retention model provides a robust estimator for stock pricing. Under ideal conditions where retention rates approaching zero or unity, we have demonstrated that it produces identical results as the standard dividend discount model; but even with the Dow Thirty, with retention rates in the middle of that range, it produced an average price of 88.37, as compared with an average price of 85.71 under the standard dividend discount model.

Further, we estimate that the average length of the excess earnings period for the Dow Thirty is 5.709 years with a standard deviation of 7.641 years. (Under the simplified partial retention model we compute 4.683 years with a standard deviation of 5.391 years.) Thus, we must conclude that even when we consider natural monopolies and established brand loyalties of market leaders, and even when we restrict post-horizon returns to the opportunity cost of equity, the implied period of supernormal returns is generally quite short.

Of course, our conclusions are presented with one caveat. Since the average ROE reported for the Dow Thirty as of that date is .244, one might challenge our conclusions on the basis that such a high current ROE might lead to a lower estimation of the excess earnings period than might otherwise be recorded. Based upon the simplified partial retention model, we calculate that there is a significant negative relationship between ROE and estimated length of the excess earnings period.

$$\frac{\partial n}{\partial R} = \frac{-n}{(1+R) \ln \left(\frac{1+R}{1+r} \right)} \quad (17a)$$

Letting $n = 5.709$, $R = .244$ and $r = .087$, that results in a partial derivative of

$$\frac{\partial n}{\partial R} = -34.02 \quad (17b)$$

so that a reduction of .10 in the reported ROE would result in an added 3.402 in the estimate of the length of the excess earnings period. Unfortunately, that is one problem we cannot overcome entirely. By its very nature, our models force us to estimate ROE and n simultaneously. The next section of this paper presents what we consider to be a partial solution to that problem.

IV. An Application to the 3-Stage Growth Duration Model.

One of the most critical problems with a 2-stage growth duration model is that it forces us to simultaneously estimate ROE and the length of the excess earnings period. Exacerbating this problem is the fact that a 2-stage growth duration model forces us to presume a fixed rate of return on equity during the entire growth stage. Thus, a higher rate of presumed ROE would imply a shorter excess earnings period, and vice versa. For firms with excess earning periods exceeding 5 years, this could pose a serious problem.

In this section, we intend to address this problem. In this section, we propose to use the insights gained from our study in order to build a better 3-stage growth duration model. We begin with existing multiple stage growth models, such as the growth duration model of Holt (1962) discussed in Reilly and Brown (2009). While retaining all of the essential insights of the original Holt model, we can now address some of its limitations. First of all, it assumes a comparison of between firms A and B which share the same risk characteristics. And second, it does not allow us to factor in a horizon where return on equity returns to the opportunity cost of equity. We can propose an alternative model which overcomes these problems.

For purposes of discussion, let us assume that we have two firms, A and B , with different risk characteristics and with different retention policies. Further, assume that firm B is projected to earn return on equity of R_B for k periods, retaining of its earnings, after which it is projected to earn only the opportunity cost of equity, when

retention policy becomes truly irrelevant. And we assume that firm A is expected to earn a rate of return of R_A for n periods, followed by R_B for k periods, followed by the opportunity cost of equity in all post-horizon periods, where $0 < r < R_B \leq R_A$.

Our model addresses two of the limitations of the original Holt (1962) model, while preserving its basic insights. First, it does not require that we assume that we will be earning R_B in perpetuity, following our initial phase. Second, it does not require that two firms have identical risk characteristics. It only requires the assumption that we will be acquiring firm B 's risk characteristics and that we adopt firm B 's retention policy upon reaching the second stage: a relatively innocuous assumption. Under our simplified partial retention model

$$P_0^A = \left(\frac{1+R_A}{1+r_A} \right)^{\lambda n} \left(\frac{1+R_B}{1+r_B} \right)^{\mu k} \frac{E_1^A}{r_B} \quad (18)$$

$$\text{and} \quad P_0^B = \left(\frac{1+R_B}{1+r_B} \right)^{\mu k} \frac{E_1^B}{r_B} \quad (19)$$

When we compare PE ratios of the two firms, we can estimate the length of time we expect to earn return R_A .

$$\frac{PE_A}{PE_B} = \left(\frac{1+R_A}{1+r_A} \right)^{\lambda n} \quad (20)$$

$$\text{and} \quad n = \frac{\ln \left(\frac{PE_A}{PE_B} \right)}{\lambda \ln \left(\frac{1+R_A}{1+r_A} \right)} \quad (21)$$

Although $n+k$ represents the full excess earnings period, we can think of the first stage, or n years, as the period of technological advantage.

V. Conclusion

Starting out with the assumption that the return on

equity must eventually return to the opportunity cost of equity, we developed three separate valuation models: the standard dividend discount model and two simplified models. We have demonstrated that our simplified pricing models produce the same result as does the standard dividend discount model when the retention rate approaches zero or when it approaches one. When retention rate takes on intermediate values, as in the Dow Thirty, we have demonstrated that our simplified model is robust, and continues to produce results which closely track the standard dividend discount model. Having demonstrated that our simplified models are capable of producing a robust approximation of the standard dividend discount model, we then are able to derive a simple 3-stage pricing model, which enables us to measure the period of technological advantage. Finally, we provide evidence that the excess earnings period is relatively short, even when we consider the market power of the Dow Thirty and even when we restrict all post-horizon returns to the opportunity cost of equity.

We further suggest that future studies adopt this framework to measure the length of the excess earnings period. We realize that there is some uncertainty regarding the expected length of the excess earnings period and suggest studies such as Fogelberg (2012) where the length of the excess earnings period is treated as a “rolling window” where all that is known is the constant stopping rate.

References

- Damodaran, Aswath 2006. *Damodaran on Valuation*, 2nd ed. New York, Wiley.
- Feltham, G. and J. Ohlson. 1995. "Valuation and clean surplus accounting for operating and financial activities." *Contemporary Accounting Research*.
- Fogelberg, Lawrence 2012. *Rational Expectations versus Perfect Foresight Asset Pricing*, Troy University, Unpublished manuscript.
- Gordon, Myron J. 1962. *The Investment Financing and Valuation of the Corporation*, Irwin, Homewood, IL.
- Holt, Charles C. 1962 "The Influence of Growth Duration on Share Prices, *Journal of Finance* 7, no. 3, September 465-475.
- Koller, Tim, Marc Goedhart, and David Wessels (2005). *Valuation, Measuring and Managing the Value of Companies* 4e, Wiley and Sons.
- Miller, M., and K. Rock, 1985, Dividend policy under asymmetric information, *Journal of Finance* 40, 1031-1051
- Palepu, K., P. Healy and V. Bernard. 2000. *Business Analysis and Valuation: Using Financial Statements*. Cincinnati, OH: South-Western Publishing Co.
- Reilly, Frank K. and Keith C. Brown 2009, *Investment Analysis and Portfolio Management*, 9 ed. Southwestern.
- Soffer, Leonard C, 2003, Expected long-run return on equity in a residual income valuation model, *Review of Accounting & Finance*, Vol 2, No.1, pp.59-72

Appendix A

Deriving a closed form expression for the standard discounted dividend model

Estimating the length of the excess return period with the standard discounted dividend model.

$$P_0 = \frac{P_n}{(1+r)^n} + \sum_{k=1}^n \frac{D_k}{(1+r)^k} . \quad (\text{A.1})$$

$$P_0 = \left(\frac{1+R\lambda}{1+r} \right)^n \frac{E_1}{r} + \sum_{t=1}^n \frac{E_1(1+R\lambda)^{t-1}(1-\lambda)}{(1+r)^t} \quad (\text{A.2})$$

Letting $D_t = E_t(1-\lambda)$, and $E_t = E_{t-1}(1+R\lambda)$ for $t = 2 \dots n+1$,

$$P_0 = \left(\frac{1+R\lambda}{1+r} \right)^n \frac{E_1}{r} + \frac{(1-\lambda)E_1}{(1+R\lambda)} \sum_{t=1}^n a^t \quad (\text{A.3})$$

$$a = \left(\frac{1+R\lambda}{1+r} \right) \text{ and } \frac{a}{1-a} = \left(\frac{1+R\lambda}{r-R\lambda} \right). \quad (\text{A.4})$$

$$P_0 = \left(\frac{1+R\lambda}{1+r} \right)^n \frac{E_1}{r} + \frac{(1-\lambda)E_1}{(1+R\lambda)} \left\{ \frac{(1-a^n)a}{1-a} \right\} \quad (\text{A.5})$$

$$P_0 = \left(\frac{1+R\lambda}{1+r} \right)^n \frac{E_1}{r} + \frac{(1-\lambda)E_1}{(1+R\lambda)} \left\{ \left(\frac{1+R\lambda}{r-R\lambda} \right) - \left(\frac{1+R\lambda}{1+r} \right)^n \left(\frac{1+R\lambda}{r-R\lambda} \right) \right\} \quad (\text{A.6})$$

$$P_0 = \left(\frac{1+R\lambda}{1+r} \right)^n \frac{E_1}{r} + \frac{E_1(1-\lambda)}{(r-R\lambda)} - \frac{E_1(1-\lambda)}{(r-R\lambda)} \left(\frac{1+R\lambda}{1+r} \right)^n \quad (\text{A.7})$$

$$P_0 = \frac{E_1(1-\lambda)}{(r-R\lambda)} - \left\{ \frac{E_1(1-\lambda)}{(r-R\lambda)} - \frac{E_1}{r} \right\} \left(\frac{1+R\lambda}{1+r} \right)^n \quad (\text{A.8})$$

$$P_0 = \frac{E_1(1-\lambda)}{(r-R\lambda)} - \left\{ \frac{E_1(1-\lambda)r - E_1(r-R\lambda)}{r(r-R\lambda)} \right\} \left(\frac{1+R\lambda}{1+r} \right)^n \quad (\text{A.9})$$

$$P_0 = \frac{E_1(1-\lambda)}{(r-R\lambda)} - \left\{ \frac{E_1r - E_1\lambda r - E_1r + E_1R\lambda}{r(r-R\lambda)} \right\} \left(\frac{1+R\lambda}{1+r} \right)^n \quad (\text{A.10})$$

$$P_0 = \frac{E_1(1-\lambda)}{(r-R\lambda)} - \frac{E_1\lambda(R-r)}{r(r-R\lambda)} \left(\frac{1+R\lambda}{1+r} \right)^n \quad (\text{A.11})$$

$$P_0 \frac{r(r-R\lambda)}{E_1\lambda(R-r)} = \frac{r(r-R\lambda)}{E_1\lambda(R-r)} \frac{E_1(1-\lambda)}{(r-R\lambda)} - \left(\frac{1+R\lambda}{1+r} \right)^n \quad (\text{A.12})$$

$$\frac{E_1(1-\lambda)}{(r-R\lambda)} \frac{r(r-R\lambda)}{E_1\lambda(R-r)} - P_0 \frac{r(r-R\lambda)}{E_1\lambda(R-r)} = \left(\frac{1+R\lambda}{1+r} \right)^n . \quad (\text{A.13})$$

$$\frac{E_1(1-\lambda)r}{E_1\lambda(R-r)} - \frac{P_0 r(r-R\lambda)}{E_1\lambda(R-r)} = \left(\frac{1+R\lambda}{1+r} \right)^n . \quad (\text{A.14})$$

$$\ln \left(\frac{E_1(1-\lambda)r - P_0 r(r-R\lambda)}{E_1\lambda(R-r)} \right) = n \ln \left(\frac{1+R\lambda}{1+r} \right) . \quad (\text{A.15})$$

$$\frac{\ln \left(\frac{E_1(1-\lambda)r - P_0 r(r-R\lambda)}{E_1\lambda(R-r)} \right)}{\ln \left(\frac{1+R\lambda}{1+r} \right)} = n . \quad (\text{A.16})$$

Appendix B

Three stage growth duration model

In order to demonstrate a three stage model, we assume that firm A is the market leader and that firm B is its closest competitor. We assume that firm A will be able to generate return on equity of R_A for n years, where

$R_A > R_B$, the return of its competitor. Further we assume that firm A will follow an optimal retention policy of λ during the first stage, after which it will have all of firm B 's characteristics including return on equity and opportunity cost of equity. Thus, we assume that firm A will also adopt firm B 's optimal retention policy of λ upon reaching stage 2.

Note, this is an entirely general model, and we need

$$P_0^B = \left(\frac{1+R_B}{1+r_B} \right)^{\mu k} \frac{E_1^B}{r_B} , \quad (\text{B.1})$$

and firm A would have the following price

$$P_0^A = \frac{1}{(1+r_A)^n} P_n^A , \quad (\text{B.2})$$

where

$$P_n^A = \left(\frac{1+R_B}{1+r_B} \right)^{\mu k} \frac{E_{n+1}^A}{r_B} , \quad (\text{B.3})$$

and where

$$E_{n+1}^A = (1+R_A)^{\lambda n} (1+r_A)^{(1-\lambda)n} E_1^A . \quad (\text{B.4})$$

Thus

$$P_0^A = \left(\frac{1+R_A}{1+r_A} \right)^{\lambda n} \left(\frac{1+R_B}{1+r_B} \right)^{\mu k} \frac{E_1^A}{r_B} . \quad (\text{B.5})$$

$$\frac{PE_A}{PE_B} = \left(\frac{1+R_A}{1+r_A} \right)^{\lambda n} , \quad (B.6)$$

and

$$n = \frac{\ln \left(\frac{PE_A}{PE_B} \right)}{\lambda \ln \left(\frac{1+R_A}{1+r_A} \right)} . \quad (B.7)$$

not assume that firms A and B have the same risk characteristics. In fact, we have good reason to believe that firm A would have a slightly higher opportunity cost of equity capital $r_A > r_B$.

Adopting our simplified partial retention growth model, firm B , would have the following price

When we compare PE ratios of the two firms, we can estimate the length of the first period during which firm A is expected to earn R_A .

Normalization of earnings.

In the case of negative values of n , we can normalized

earnings per share, in order to satisfy the inequality $P_0 \geq E_1 / r$. Where $P_0 < E_1 / r$, we adjusted earnings per share so that $P_0 \cdot r = E_1$.

Full normalization of earnings, ROE and retention rate.

Consider the case where the firm wishes to retain a fixed proportion of earnings λ , and has a fixed return on equity R and has a fixed growth rate g . In that case, we can normalize E_t , R , and λ in terms of D_t , g and BV_{t-1} .

$$\bar{E}_t = D_t + \bar{g} \cdot BV_{t-1} . \quad (B.8)$$

$$\bar{R} = \frac{D_t}{BV_{t-1}} + \bar{g} . \quad (B.9)$$

$$\bar{\lambda} = \frac{\bar{g} \cdot BV_{t-1}}{D_t + \bar{g} \cdot BV_{t-1}} . \quad (B.10)$$