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## Business cycles and production networks

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*Business Cycles and Production Networks*

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# *Business Cycles and Production Networks*

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# Business Cycles and Production Networks <sup>\*</sup>

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## Abstract

Where do business cycles originate? The traditional view is that a business cycle is the result of shocks correlated across sectors. This view is complemented by a recently emerging literature showing that idiosyncratic shocks to large or highly interconnected sectors contribute to aggregate variation. This paper addresses the relative empirical importance of these two channels of business cycle variation. Results indicate that up to one-third of the business cycle is driven by idiosyncratic productivity variation together with network amplifications.

**Keywords:** Production Networks, Micro to Macro, Aggregate Volatility, Sectoral Distortions

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# Introduction

Where do business cycles originate? The classical view in real business cycle and New Keynesian models is that business cycles are driven by economy-wide (aggregate or correlated) shocks giving rise to variations in aggregate outcomes. The classical view has been complemented by a recent and rapidly expanding literature on idiosyncratic variations at the firm (or sector) level together with asymmetric importance of firms (or sectors) as a potential additional source of business cycle fluctuations. This paper addresses the relative empirical importance of these two views.

Formally, there are two irrelevance arguments that will be addressed in this paper and are commonly discussed in the emerging literature. The first is the *irrelevance of idiosyncratic shocks*. This argument is based on the law of large numbers, where an idiosyncratic shock to one sector is offset by a contracting shock to another sector. Gabaix (2011) has proven this argument false in the case of weights (determining the importance of firms, or sectors, in the aggregate outcomes) that are sufficiently fat-tail distributed with a right tail that follows a power-law distribution.<sup>1</sup> Empirical work by, for example, Axtell (2001) and Acemoglu et al. (2012) has confirmed that the size distribution of firms, and the importance of sectors as input-suppliers to other sectors, meets this distributional requirement in general. The second argument is the *irrelevance of networks* for estimating the aggregate impact of a shock. This argument concerns the accurate measure of weights. Ultimately, the importance (weight) of each firm should be based on the total effect that the firm has on all other firms, directly and indirectly through trade in intermediates, and final consumers. However, accounting for all channels is difficult both because of the general lack of available data on firm-to-firm linkages and because accurate measures of linkages between all firms in the economy result in a gigantic and complicated matrix. Thus, in order to facilitate, for example, stabilization policy, a more readily available and sufficient measure for capturing the aggregate effects of firm- (or sector-) level shocks is valuable. One feasible, and at a first glance obvious, measure of importance is the firm's share of aggregate value added. However, such a measure implies either abstraction from intermediate goods or that value-added accumulation is *the* important part of the production process, an assumption that

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<sup>1</sup>The distributional requirements are a fat right tail, where the probability of observing a value  $X$  larger than  $x$  is inversely proportional to  $x$ ,  $Pr(X > x) = x^{-\zeta}$  for  $\zeta \in (1, 2)$ . For  $\zeta = 1$  the distribution follows Zips law. If  $\zeta$  is larger than two, the distribution is thin-tailed and the law of large numbers applies, see Gabaix (2011) for proofs.

is misleading for firms that do not produce a lot of value added but are key players in other firms' production, e.g. an importer or a sales-facilitating firm with important flows but no value-added transformation. An alternative and theory-based measure that captures the flow is sales weights in aggregate value added, so-called Domar weights. This goes back to the Hulten (1978) theorem, which shows that Domar weights are a sufficient statistic for the aggregation of productivity shocks; a result later extended by Acemoglu et al. (2012) and Gabaix (2011). These papers show that network linkages provide a micro-foundation for Domar weights to be the appropriate aggregation weights, but do not play an independent role in the aggregation itself. However, Hulten's theorem is not robust to accounting for higher-order terms and non-linearities (see the theoretical contributions by Baqaee and Farhi, 2018*b*) or to adding frictions (markups) as in Bigio and La'O (2016), Lou (2018) and Grassi (2017). The violation of the theorem implies first-order allocative effects following a productivity shock, which is not captured correctly by Domar weights. In this case, the appropriate aggregation weight must be constructed from measures of network linkages. Despite the superiority of aggregation weights based on network linkages, data is scarce, complicate to compute and rarely available at the firm level. But do we need to care about frictions, and thereby the network structure, when evaluating the aggregate effects of idiosyncratic shocks? Or, can we continue to use the more readily available statistic of sales share? This paper sheds light on this issue by evaluating the importance of frictions on aggregation weights and in understanding business cycle fluctuations. Both of the irrelevance arguments are important for stabilization policy, first, to know what type of signals that are of systemic importance, and second, to facilitate measurement under the complex nature of direct and indirect linkages.

To set ideas, this paper first outlines a theoretical motivation for a specific weighting structure, largely built on Bigio and La'O (2016), where connections between industries are accounted for. Using this framework, the role for sector-specific frictions is introduced, as are the conditions under which the Hulten's theorem is violated. This leaves room for linkages between sectors to play a role in addition to the size distribution when there are important (and asymmetric) sectoral frictions. The theoretical section is followed by an empirical section that starts out by creating sector-specific weights using either sales shares in aggregate value added or an influence measure based on linkages. This section presents the difference between the two weights, i) sales weights (Domar), which are sufficient in the frictionless case (i.e. when Hulten's theorem holds), and ii) a more complicated weighting scheme accounting for

linkages, which is appropriate when there are frictions. This is followed by a decomposition of sector-level productivity into a sector-specific (idiosyncratic) component and a common component. Finally, counterfactual variation in aggregate productivity, shutting down first the common and then the idiosyncratic component, is calculated under the two weighting schemes and related to the aggregate productivity shock. In addition to the results for aggregate productivity (which follow the predictions from Hulten, 1978, about how idiosyncratic productivity moves the whole production possibility frontier of an economy), implications for movements in observed real GDP growth are estimated.

The results show, first of all, that the proposed weighting scheme is asymmetric. Thus, sectors are not equally important, which gives rise to the potential for idiosyncratic shocks to important sectors to propagate into business cycle variations. Secondly, idiosyncratic variations can explain up to two-thirds of the variance of the aggregate shock. The variance, however, is even further explained by common components. For the business cycle, idiosyncratic productivity growth and common productivity growth have similar contributions, accounting for one-third of the variation in real GDP growth using the network weight for aggregation. Thirdly, using Domar weights does not change the predictive power of the idiosyncratic component by any magnitude. Hence, the results suggest, first of all, that idiosyncratic variation matters for the business cycle, i.e. the *irrelevance of idiosyncratic shocks* is not supported in data. In addition, the results suggest that there may exist important sector-level frictions, but that they do not further skew the inter-connectedness between sectors. This implies that Domar weights are a sufficient statistic for estimating macro effects of micro distortions, i.e. the *irrelevance of networks* does not fall in data. Moreover, the evidence indicates that the driver behind the skewness in firm size is mainly asymmetries in final sales and only partly the inter-sectoral network per se.

This paper is closely related to the granularity and micro-to-macro literature where Acemoglu et al. (2012), Atalay (2017), Gabaix (2011), Carvalho and Gabaix (2013) and others have shown that idiosyncratic shocks can matter for aggregate variation via network linkages and asymmetrically large sectors in sales to aggregate value added terms. For example, Gabaix (2011) found that one-third of output growth variations can be explained by idiosyncratic labor productivity growth in the largest 100 firms in the United States. This result is supported by Bruyne et al. (2016) using networks at the firm level in Belgium. In addition, Bruyne et al. (2016) found

that both the size distribution and network connections contribute to aggregate variations. In Bruyne et al. (2016), the relative strength of the channels depends on the share of intermediate goods in the production. This paper complements the findings in Bruyne et al. (2016) at the sector level, with a focus on frictions, using a different empirical approach that includes decomposition of factors and looking at the effects on the aggregate shock and GDP. Bruyne et al. (2016) looks solely at GDP effects. However, the results support Bruyne et al. (2016) in many dimensions. The contribution of this paper is to shed light on the role of networks and frictions in generating macro volatilities from micro disturbances at the sector level from an empirical perspective.

The structure of this paper is as follows. The first section gives the theoretical framework with key equations from micro shocks to macro implications together with the role for frictions. The empirical approach is outlined in the second section. The third section presents the main results from the theory-implied weights before turning to the empirical relevance of frictions evaluated by contrasting the empirical results with results derived from Domar weights. The fourth section elaborates on the different heterogeneous elements to disentangle the results. The final section concludes.



# 1 Theoretical Motivation

This section presents the theoretical framework. The first part outlines the model to describe the economic environment and derives the key equations, which are largely built on Bigio and La'O (2016). The second part presents the business cycle implications and conditions for amplifications of idiosyncratic variations. The last part of this section elaborates on the aggregation weights and implications of frictions.

## 1.1 Economic Environment

The model is a simplified version of Bigio and La'O (2016), where I rely on the CES limit and abstract from dividends to households.<sup>2</sup> The framework is sectors, but I will frequently use the term *firm*. The model abstracts from international trade and capital accumulation. In the model economy, households cannot save, so their wealth is equal to labor income. Aggregate GDP is then given by aggregate consumption, which equals household wealth/labor income. Moreover, all goods are demanded both as inputs and final goods. Firm  $i$  produces according to the production function  $Q_i$ , using labor  $l_i$  and intermediate inputs  $x_{ij}$ .

$$Q_i = z_i l_i^{\alpha_i} \left( \prod_{j=1}^N x_{ij}^{w_{ij}} \right)^{(1-\alpha_i)}, \quad (1)$$

where  $z_i$  is productivity and  $x_{ij}$  is intermediate inputs from firm  $j$  used by firm  $i$ . Firms have heterogeneous labor shares represented by  $\alpha_i$ .  $w_{ij}$  is the share of intermediate inputs from firm  $j$  in  $i$ 's intermediate requirements for production,  $w_{ij} \in (0, 1)$ . There is no entry or exit in the model. Frictions are firm-specific and modeled as any wedge (denoted by  $\phi_i$ ) between the firm's revenues  $g_i$  and expenditures  $u_i$ ,

$$g_i \phi_i = u_i. \quad (2)$$

These wedges can arise from imperfect competition, taxes, subsidies or financial frictions (the key argument stressed in Bigio and La'O, 2016). Lower  $\phi_i$  implies larger distortions, and a unit value of  $\phi_i$  implies no distortions. Without loss of generality, the wage rate  $h_i$  is normalized to 1 for all  $i$ . Assume that there are no dividends to households, i.e. any profit from the distortion is perfectly lump-sum taxed.

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<sup>2</sup>Adding dividends to the model and the regression equations does not change the results in any meaningful way for how it is used in this paper, but complicates the prediction.

Define total cost  $u_i \equiv l_i + \sum_j p_j x_{ij}$ , and total revenue  $g_i = p_i Q_i$ . From the budget constraint, equation (2) implies that  $Q_i p_i \phi_i = h_i l_i + \sum_{j=1}^N p_j x_{ij}$ . From Cobb Douglas production, it follows that, for a cost-minimizing firm, firm  $i$ 's cost of the production factors will be in proportion to the share of total cost in that factor,

$$p_j x_{ij} = (1 - \alpha_i) w_{ij} u_i, \quad (3)$$

$$l_i = \alpha_i u_i. \quad (4)$$

This in turn implies that the input-output network structure between firms is pinned down by the technology. The identical households, indexed by  $k$ , have CES utility, and  $v_i \in (0, 1)$  is the expenditure share on good  $i$ ,  $\sum_{i=1}^N v_i = 1$ . Households get utility from consumption and disutility from working. Consumption of each household is equal to the expenditure-weighted consumption basket of goods  $i$ ,

$$c_k = \prod_i c_i^{v_i}. \quad (5)$$

This implies that the final demand structure is pinned down by preferences that then close the full input-output structure in the model. The ideal price index for households is

$$\bar{p} = \prod_i \left( \frac{p_i}{v_i} \right)^{v_i}. \quad (6)$$

Each household faces a budget constraint that is assumed to be binding,

$$l_k = \bar{p} c_k, \quad (7)$$

taking prices and wages as given and maximizing utility by choosing consumption and labor. Assume that the wage is equal across firms, normalized to 1, and the labor market clears,

$$l = \sum_k l_k = \sum_i l_i. \quad (8)$$

From market clearing, the output of each firm  $i$  is either used as final consumption by households or sold as intermediates into the production of other firms,

$$Q_i = \sum_{j=1}^N x_{ji} + c_i. \quad (9)$$

Equation (9) implies that, given final consumption, firms with large output  $Q_i$  are

large intermediate suppliers to other firms, implying that large firms are more interconnected, *ceteris paribus*. In the absence of frictions ( $\phi_i = 1 \forall i$ ), this is the micro foundation of the sales weights in Acemoglu et al. (2012). However, given that we have frictions that distort prices and allocations together with heterogeneous final demand, large output does not necessarily imply a large interconnection (importance), which will be shown below.

The dependence of connected firms on revenues is visible in how other firms' revenues enter into the firm's revenue function. Similar to equation (9), revenues  $g_i$  for each firm can be decomposed into revenues from final sales and intermediate sales,

$$g_i = p_i Q_i = p_i \sum_{j=1}^n x_{ji} + p_i c_i. \quad (10)$$

By substituting factor demand, equation (3), for firm  $j$ 's demand of goods produced by firm  $i$ ,

$$g_i = \sum_{j=1}^n (1 - \alpha_j) w_{ji} u_j + p_i c_i, \quad (11)$$

and using  $u_j = \phi_j g_j$ , we have an expression where revenues of firm  $i$  depend on the revenues of other firms, which in turn depends on inputs, prices and frictions,

$$g_i = \sum_{j=1}^n (1 - \alpha_j) w_{ji} \phi_j g_j + p_i c_i. \quad (12)$$

In contrast to equation (9), equation (12) implies that sales depend on sales and frictions in the whole network even if we assume homogeneous final consumption.

Use the household optimality condition (expenditure on a good is proportional to total expenditure)

$$p_i c_i = v_i \bar{p} c, \quad (13)$$

and let  $\bar{p} c = u_0$  denote household wealth, which in a world without dividends is equal to income, and, under normalized wage equals labor supply  $u_0 = l$ ,

$$g_i = \sum_{j=1}^n (1 - \alpha_j) w_{ji} \phi_j g_j + v_i l. \quad (14)$$

Stacking equations on top of each other into matrices and solving for  $\mathbf{g}$ , we arrive at equilibrium sales

$$\mathbf{g} = [\mathbf{I} - (\mathbf{1} - \boldsymbol{\alpha})' \odot \mathbf{W}' \odot (\boldsymbol{\phi}' \cdot \mathbf{1}')]^{-1} (\mathbf{v} \odot \mathbf{1}), \quad (15)$$

where  $\odot$  is the notation for the entry-wise product.  $\mathbf{g}$  is a vector of  $(N * 1)$ , consumption shares is  $\mathbf{v} = (N * 1)$ ,  $\mathbf{I}$  is the diagonal identity matrix  $(N * N)$ ,  $(\mathbf{1} - \boldsymbol{\alpha})$  is  $(N * N)$  of  $(\mathbf{1} - \boldsymbol{\alpha}_i)$ ,  $\mathbf{W} = \{w_{ij}\}$  is the symmetric input-output table  $(N * N)$ , and  $\mathbf{1}$  is a column vector of ones to transform  $\boldsymbol{\phi}$  to a symmetric matrix.

From the literature, the proposed alternative weighting structure, which does not require knowledge about the network and is sufficient when Hulten's theorem holds, is Domar weights.<sup>3</sup> This weight is defined as the firms' sales weight in aggregate value added,

$$D_i = \frac{\text{sales of industry } i}{\text{nominal GDP}}. \quad (16)$$

Note that Domar weights are equivalent to the sales representation under the model, equation (15), after some rearranging,

$$\mathbf{g}' \odot \mathbf{l}'^{-1} = \mathbf{v}' [\mathbf{I} - (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W} \odot (\boldsymbol{\phi} \cdot \mathbf{1})]^{-1}, \quad (17)$$

where  $l$  is household income (wealth) that in equilibrium is equal to the value of consumption; thus,  $l$  is equal to nominal GDP,

$$\mathbf{D} \equiv \mathbf{pQ} \odot \bar{\mathbf{p}}\mathbf{GDP}^{-1} = \mathbf{g}' \odot \mathbf{l}'^{-1} = \mathbf{v}' [\mathbf{I} - (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W} \odot (\boldsymbol{\phi} \cdot \mathbf{1})]^{-1}. \quad (18)$$

Equation (18) is readily available to measure since it requires only the information in equation (16). This is one of the key equations and one of the weights that will be contrasted in this paper and discussed in detail in Section 1.3.

To close the model and derive the expression for aggregate GDP, I continue with the derivation for prices. Starting off by combining the production function, equation (1), with the definition of sales,

$$g_i = p_i Q_i \quad (19)$$

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<sup>3</sup>See e.g. Gabaix (2011)

$$g_i = p_i z_i l_i^{\alpha_i} \left( \prod_j x_{ij}^{w_{ij}} \right)^{(1-\alpha_i)}, \quad (20)$$

and the f.o.c. for expenditure shares, equations (3) and (4),

$$g_i = p_i z_i (\alpha_i u_i)^{\alpha_i} \left( \prod_j \left( (1 - \alpha_i) \frac{w_{ij}}{p_j} u_i \right)^{w_{ij}} \right)^{(1-\alpha_i)}. \quad (21)$$

Rearrange  $u_i$  and  $(1 - \alpha_i)$  from the product that runs over  $j$ ,

$$g_i = p_i z_i \alpha_i^{\alpha_i} u_i^{\alpha_i} \left[ (1 - \alpha_i) u_i \prod_j \left( \frac{w_{ij}}{p_j} \right)^{w_{ij}} \right]^{1-\alpha_i}, \quad (22)$$

and simplify

$$g_i = p_i u_i z_i \alpha_i^{\alpha_i} \left[ (1 - \alpha_i) \prod_j \left( \frac{w_{ij}}{p_j} \right)^{w_{ij}} \right]^{1-\alpha_i}. \quad (23)$$

In log form we then have,

$$\begin{aligned} \log g_i = \log p_i + \log z_i + \log u_i + \alpha_i \log \alpha_i + (1 - \alpha_i) \log(1 - \alpha_i) + \\ (1 - \alpha_i) \sum_j w_{ij} (\log w_{ij} - \log p_j), \end{aligned} \quad (24)$$

and define a constant  $k_i \equiv \alpha_i \log \alpha_i + (1 - \alpha_i) \log(1 - \alpha_i) + (1 - \alpha_i) \sum w_{ij} \log w_{ij}$  and use matrix notations,

$$\log \mathbf{g} = \log \mathbf{p} + \log \mathbf{z} + \log \mathbf{u} + \mathbf{k} - (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W} \log \mathbf{p}. \quad (25)$$

Solve for  $\mathbf{p}$ , using  $\mathbf{u} = \boldsymbol{\phi} \mathbf{g}$

$$\log \mathbf{p} = -[\mathbf{I} - (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W}]^{-1} (\log \mathbf{z} + \log \boldsymbol{\phi} + \mathbf{k}). \quad (26)$$

We then have a vector of sectoral price indices that depend on productivity, sectoral distortions and constants. To close the model, take the ideal (clearing) price index from the household, which is price-weighted by the household's final consumption shares, equation (6). In matrix notation and log, it is equivalent to

$$\log \bar{\mathbf{p}} = \mathbf{v}' \log \mathbf{p} - \mathbf{v}' \log \mathbf{v}. \quad (27)$$

As mentioned in the beginning of this section, this economy abstracts from exports and imports; hence, real GDP is equal to the value of household consumption. In the market-clearing equilibrium, households consume all of their wealth (income).

Hence, real GDP is equivalent to household income over the consumption-weighted price indices:

$$\mathbf{GDP} = \mathbf{c} = \frac{\mathbf{u}_0}{\bar{\mathbf{p}}} = \frac{\mathbf{l}}{\bar{\mathbf{p}}}. \quad (28)$$

Note that the wage is normalized to unity, so the real wage is  $\frac{1}{\bar{p}}$ . Take logs and combine them with equations (26) and (27),

$$\log \mathbf{GDP} = \log \mathbf{l} + \mathbf{v}'[\mathbf{I} - (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W}]^{-1}(\log \mathbf{z} + \log \boldsymbol{\phi} + \mathbf{k}) + \mathbf{v}' \log \mathbf{v}. \quad (29)$$

We then have aggregate real GDP depending on aggregate productivity, distortions ( $\log \boldsymbol{\phi}$  captures the reallocation wedge from frictions), input shares, labor and consumption.<sup>4</sup> A more simplified notation is

$$\log \mathbf{GDP} = \mathbf{q}(\log \mathbf{z} + \log \boldsymbol{\phi} + \mathbf{k}) + \log \mathbf{l} + \mathbf{v}' \log \mathbf{v}, \quad (30)$$

where

$$\mathbf{q} \equiv \mathbf{v}'[\mathbf{I} - (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W}]^{-1}. \quad (31)$$

Equation (30) can be viewed as the aggregate production function of the economy, and  $\mathbf{q}$  is the expansion of the production possibility frontier following a productivity shock,  $\Delta \log \mathbf{z}$ .  $\mathbf{q}$  is called the influence vector, is of dimension  $(N \times 1)$ , and shows the aggregate effect of a shock in sector  $i$  through all linkages in the network. Expanding the influence vector shows all the linkages and higher-order terms; first its direct effect, then on its first connections, on their connector's connections, and so forth:  $\mathbf{q} = \mathbf{v}'[\mathbf{I} - (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W}]^{-1} = \mathbf{v}'[\mathbf{I} + (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W} + ((\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W})^2 + ((\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W})^3 + ((\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W})^4 + \dots]$ . The influence vector allows for heterogeneity in the linkages between sectors, demand from final consumption and use of intermediate inputs in production.<sup>5</sup> If there are no linkages between sectors (no intermediate production),  $\alpha_i = 1 \forall i$ , the effect of a productivity shock is  $\mathbf{v}$ . Moreover, note that frictions enter additively in the GDP expression; hence, they do not affect the expansion of production possibilities following a productivity shock.

<sup>4</sup>In the Cobb-Douglas production function, shares are constant over time. All variables are time constants except for productivity.

<sup>5</sup>If we assume intermediate input shares to be constant between firms,  $(1 - \alpha_i) = (1 - \bar{\alpha})$ , homogeneous final demand  $\mathbf{v} = 1/N$  and Harrod-neutral productivity shocks (in the production function  $z$  is raised to  $\alpha$ ), equation (31) is identical to the influence vector in Acemoglu et al. (2012).

## 1.2 (Ir)relevance of Idiosyncratic Shocks?

According to the classical view, the interesting component of productivity,  $\mathbf{z}$ , is economy-wide (aggregate or correlated) shocks that, following the theory, propagate to the business cycle by  $\mathbf{q}$ . Under this view, idiosyncratic shocks will have negligible effects on the business cycle because these shocks are specific by nature. It follows that a negative shock to one sector is accompanied by a positive shock to another sector. If sectors are equally important, the shocks will cancel each other out, i.e. the sum of orthogonal shocks with equal weights retain the orthogonality and thus the irrelevance of idiosyncratic shocks. The phrase *equally important* corresponds to symmetric weights, such that  $q_i = q_j \forall i, j$ . For example, the star network looks fairly complicated with all sectors as direct suppliers to each other. However, if all links are equally strong, the network is symmetric with weights equal to  $1/N$  for all sectors. In this network, all sectors ultimately affect each other, and the impact on aggregate variations of a shock to one sector is equal to  $1/\sqrt{N}$ , i.e., for large  $N$ , specific shocks washes out.<sup>6</sup> Hence, if sectors are equally important, the business cycle is driven by common shocks (aggregate or correlated), and idiosyncratic variations are unimportant sector variations without aggregate implications.

Complementing the classical view is the recent and rapidly expanding literature on granularity and networks showing that idiosyncratic shocks can contribute to aggregate variations. Elegantly proved in Gabaix (2011) and Acemoglu et al. (2012), idiosyncratic volatility does not wash out in the aggregate if the relevant aggregation weights are sufficiently fat-tailed with a power-law distribution in the right tail. If sectors are asymmetrically important as suppliers to the economy (to final consumption or other sectors), this implies that the weight of a few sectors will be larger than the others. A productivity shock to sectors with larger weights will not be offset by a contracting shock to other sectors, enabling propagating effects that are reflected in the aggregate variation. Thus, asymmetry of the weight is the key to break the *irrelevance of idiosyncratic shocks*. The condition for this distribution is that the size distribution between sectors (firms) must be fat-tailed or there must be a sufficiently skewed network structure of production due to non-easily substituted inputs and/or asymmetric frictions and monopolistic competition. In either case, the business cycle is driven in parts by idiosyncratic shocks and dependence between sectors.

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<sup>6</sup>See Gabaix (2011) for all derivations of the large  $N$  properties and rate of decay of a shock.

### 1.3 (Ir)relevance of Networks?

The second irrelevance argument addressed in this paper is the *irrelevance of networks*. Despite the label of this hypothesis, it does not mean that networks are not an important feature of the economy, but rather that we do not have to know the network structure to evaluate aggregate effects of idiosyncratic shocks. This argument is based on Hulten’s theorem. From the theorem, the aggregate effects of a sector productivity shock are summarized in their Domar weight (the ratio of a sector’s share of sales to aggregate value added), equation (16). Thus, it is sufficient to know the Domar weight to know the aggregate effects of shocks. While the influence measure is always the right measure, under the assumptions of Hulten’s theorem, Domar weights enable the use of much more readily available data and, in particular, the use of detailed firm-level data, a level where researchers in general lack good data on linkages. As mentioned, in an economy with intermediate goods and linkages between firms, the relevant statistic is the influence weight of the model, equation (31), showing how the production possibility frontier responds to productivity shocks. Domar weights, on the other hand, are the equilibrium ratio of sales to nominal GDP, which depends on prices. In fact, the influence weight comes from the optimal production, whereas Domar weights are based on the equilibrium production. The element that breaks the equality between influence (31) and Domar (18) is  $\phi$ . When there are sectoral frictions,  $\phi_i \neq 1$  for some  $i$ , prices are distorted. Therefore, following an idiosyncratic productivity shock, there will be first-order allocative effects (since equilibrium production is not equal to the optimum). Domar weights do not account for these allocative effects; prices do not only reflect technology, and will therefore be misleading under the presence of frictions. The influence measure accounts for allocations and shows the effect on the production possibilities, making it the preferred weight (see Bigio and La’O, 2016, and Jones, 2013). In contrast, under the frictionless economy  $\phi_i \rightarrow 1 \forall i$ , there are no first-order allocative effects, and prices are undistorted and reflect technology, implying that the equilibrium is optimal.<sup>7</sup> This implies that Domar weights (18) are equivalent to the influence vector (31) and Hulten’s theorem holds.<sup>8</sup>

How much error is associated with the Domar weights short-cut?<sup>9</sup> The answer to

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<sup>7</sup>See Baqaee and Farhi (2018a) for proofs, discussions and higher-order terms.

<sup>8</sup>Under a frictionless model, Acemoglu et al. (2012) proves that the influence vector and the Domar weights are equivalent under large N properties. Thus, centrality in a network is the micro foundation for the size distribution if there are no important frictions.

<sup>9</sup>Note that network linkages are important for answering other questions, for instance up- and down-stream effects, while the application of Hulten’s theorem is mainly to questions related to aggregate implications.



this question depends on whether there are sector-level frictions and, in particular, how frictions are distributed. If frictions are asymmetrically distributed,  $\phi_i \neq 1$  and  $\phi_i \neq \phi_j$  for some  $i, j$ , Domar weights will have a different distribution than influence weights. In addition, under Domar aggregation, the aggregate response will depend on where frictions occur, if they add to or reduce the asymmetry, and how influential that sector is as a direct and indirect supplier to the economy. Therefore, the approximation of Domar weights for network centrality is misleading under asymmetric frictions. On the other hand, symmetric frictions,  $\phi_i \neq 1$  and  $\phi_i = \phi_j \forall i, j$ , shift the mean of Domar weights but do not change the aggregate behavior from micro to macro.<sup>10</sup> Symmetric frictions induce an aggregate labor wedge but not an efficiency wedge because “the route for inputs” is fixed. Therefore, the production possibilities do not change, and Domar weights are proportionally smaller; but in terms of variations in the efficiency measure, there are no important differences between the two weights.

It follows that the wedge between the two measures will depend on the whole system of equations and how frictions interact with all elements in the weighting matrix. Regardless of how frictions are distributed, there will be a labor wedge from the standard dead-weight loss from monopoly (or other frictions that induce less than optimal production, leading to a wedge between the marginal product of labor and the real wage). If frictions are distributed symmetrically to a network, they will not give rise to misallocation. On the other hand, if frictions are distributed asymmetrically, they can alter the route for inputs, giving rise to misallocation of inputs in addition to the efficiency loss of aggregate production.<sup>11</sup> This paper cannot separate the effects that productivity has on misallocation and the labor wedge; thus, the

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<sup>10</sup>Under symmetric frictions, the predictions for the wedge are that weights calculated from sales shares in GDP will be smaller than the influence weight, compare equations (18) and (31). Note, large frictions translate into smaller  $\phi$ . The difference between the two weights grows with the friction

$$\frac{q}{D} = \frac{[I - (1 - \alpha) \odot W]^{-1}}{[I - (1 - \alpha) \odot W \odot (\phi \cdot 1)]^{-1}} \geq 1, \quad \text{for } \phi_i = \phi \in [0, 1]. \quad (32)$$

Frictions have a direct negative effect on equilibrium sales but not on the influence vector, which is evident from taking the partial derivative of the weights with respect to  $\phi$ . Without matrix notation,  $\partial D / \partial \phi = -v(I - (1 - \alpha)\phi W)^{-2}(-(1 - \alpha)W) \geq 1$ . Note that this derivative is the response of the aggregate Domar weight to a uniform increase in  $\phi$ .

<sup>11</sup>Resource misallocation affects the aggregate outcome, see e.g. Jones, 2013, Epifani and Gancia, 2011, and Opp, Parlour and Walden, 2014. The latter studies intra-industry oligopolistic competition, which leads to markup distortions across industries and affects the equilibrium consumption bundle of households. Heterogeneous markups distort relative prices compared to the optimum, implying misallocation of resources and lower aggregate consumption. Hence, if an idiosyncratic shock affects how resources are distributed in the economy, the aggregate effects can be large.

estimated effects are a combination of the two.

To summarize this theoretical section, when the economy is characterized by a sufficiently skewed aggregation weight, micro disturbances can lead to macro variations. In addition, the preferred aggregation weight is always the influence measure, but when there are no important and asymmetrically distributed frictions, the Domar weight is a good approximation and a useful short-cut that enables the use of available firm-level data for evaluating aggregate propagations, e.g. for stabilization policy.

## 2 Empirical Strategy

This section begins by introducing the data and describes the elements of the input-output matrix. This is followed by a decomposition of sector-level productivity into common and idiosyncratic components. The empirical outline for the implied aggregate shock, counterfactual series and aggregation weights is then presented. This section ends with the regression equations for the contribution of idiosyncratic and common productivity growth to the aggregate shock and production.

### 2.1 Data

Input-output tables are from the OECD while production and productivity data comes from the KLEMS database for Sweden, which covers the years 1970-2007.<sup>12</sup> Productivity is the TFP series and production is real value added, which is calculated from value added deflated by the price index of value added. Growth of each series is taken as the log difference of the series, and the series are linearly detrended. Productivity (TFP) is from the KLEMS database and is available from 1994, which will be the lower bound for the data used in this paper (the upper bound is 2007 for consistent data classifications). The highest level of disaggregation with consistent data and linkages between industries (not products) is 29 industries. A more disaggregated dataset would be desirable, ultimately on firm-to-firm linkages, but at the moment this is the best available data with consistent classifications.

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<sup>12</sup>KLEMS version 09I. The OECD and KLEMS data are structures using the same industry classification (a few classifications has been merged when they were reported together in KLEMS and separate in OECD). Tables from OECD are domestic (excluding imports and exports). Ideally, the international markets should be part of the analysis since Sweden is a small open economy, highly dependent on international trade. A complete international analysis is left for future research.

The input-output table (IO) is a description of the flows, or links, between industries. IO matrices are symmetric with elements containing how many inputs (expenditures) from one sector are used in the production of other sectors.<sup>13</sup> I adjust all elements by total intermediate expenditure of the using sector, that is  $\sum_j w_{ij} = 1$ .<sup>14</sup> The corresponding sum over  $i$  is not restricted and shows how many inputs a sector supplies to the rest of the economy. In shares this is called the weighted out-degree of an industry. Motivated by the theory, I assume that industry linkages are constant over time and use the IO matrix from 1995.<sup>15</sup>

## 2.2 Productivity Growth and Counterfactual Series

The productivity shock considered in this paper is TFP growth, which consists of a common and sector-specific component. While productivity, and production, in one sector is linked to production in connected sectors, productivity is most likely not directly affected by productivity in other sectors. The intuition follows that higher productivity in one sector, in general, implies that production increases for given inputs and the sector can charge a lower price for its goods, which leads to a reduction in costs for their customers, who can increase their production, but the decrease in costs will not necessarily make the connected sector more productive. Therefore, TFP growth can be decomposed into a common component and an idiosyncratic component by regressing the series on year-dummies, i.e. removing the cross-section mean in every period.<sup>16</sup>

$$\Delta \log z_{i,t} = \eta_t + \epsilon_{i,t} \quad \text{and} \quad \eta \perp \epsilon_i. \quad (33)$$

The distribution of fitted values (*common*) and residuals (*idiosyncratic*) from OLS regressions of equation (33) is depicted in Figure 1, winsorized at  $\pm 0.2$ . The distribution of the common component is denser than the idiosyncratic component. The difference in density is expected since the common component is shared across sectors, while the idiosyncratic component is not. Reassuringly, both components have

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<sup>13</sup>I transpose the matrix to get the dimension (*use, produce*).

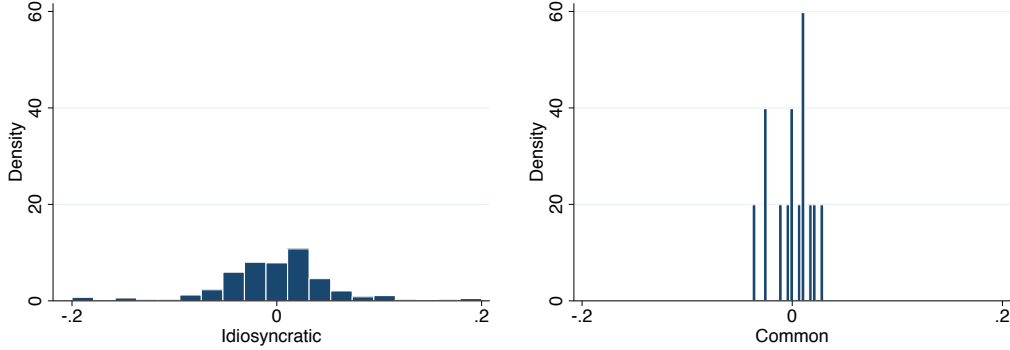
<sup>14</sup>The CES assumption implies that this sum is to equal one. Acemoglu et al. (2012), Bigio and La'O (2016) and Bruyne et al. (2016) make this adjustment. The adjustment is also required in order to make cells correspond to directed coefficients (shares) instead of monetary values.

<sup>15</sup>This assumption is equivalent to Acemoglu et al. (2012) and Bigio and La'O (2016), who use the 1992 IO matrix for the U.S.

<sup>16</sup>However, this is not true if we were to consider value added or labor productivity. If idiosyncratic shocks induce movements in the aggregate, in the world of linkages and non-negligible transmission of idiosyncratic shocks between sectors, then idiosyncratic variations cannot be defined as deviations from the aggregate means since they are not orthogonal. An idiosyncratic shock in connected sectors, together with the full characterization of the network, is required for decomposition of value added.

a mean equal to zero from working with detrended series.

Figure 1: Idiosyncratic and Common Component of Sector-Level TFP Growth



The purpose of decomposing sector-level productivity is to calculate the counterfactual (implied) aggregate variation from only idiosyncratic or common productivity variations and relate them to the business cycle. Following equation (30), the aggregate shock is sector-level TFP growth aggregated with the weight

$$\text{Shock}_t = \sum_{i=1}^N \text{weight}_i \Delta \log z_{it}. \quad (34)$$

The counterfactual common series is the weighted predicted values from (33), and the counterfactual idiosyncratic series is the weighted sum of the residuals,

$$\text{Counterfactual common}_t = \sum_{i=1}^N \text{weight}_i \hat{\eta}_t \quad (35)$$

$$\text{Counterfactual idiosyncratic}_t = \sum_{i=1}^N \text{weight}_i \hat{\epsilon}_{it}. \quad (36)$$

In other words, the weighted series are counterfactual productivity growth, shutting down idiosyncratic variations in the former ( $\hat{\epsilon}_{it} = 0$ ) and common variations in the latter ( $\hat{\eta}_t = 0$ ). It follows that the aggregate volatility in TFP coming from only idiosyncratic productivity variations,  $\epsilon_i$ , is

$$\sigma_{\Delta TFP} = \sqrt{\sum_i \text{weight}_i^2 \text{Var}(\epsilon_i)}. \quad (37)$$

Although there may be more factors that affect GDP variations than productivity, following equation (30), the part of the GDP fluctuations that come from idiosyncratic productivity variations has a corresponding equation to (37). Thus, from theory, the effect of counterfactual productivity variations on the shock and GDP

is equivalent, but this is not necessarily true in data. In this paper, I first structurally decompose the variation in the aggregate productivity shock that comes from idiosyncratic and common productivity variations, creating counterfactual productivity series, to answer what aggregate shocks consist of. In a second step, the analysis is extended to the business cycle in terms of GDP variations, which is the real series about which we ultimately want to gain knowledge.<sup>17</sup> The next subsection describes the weights that are used for calculating the counterfactual series.

## 2.3 Weights

Weights are constructed in two different ways to highlight the role of frictions and the implications of Bigio and La’O (2016) on the empirical relevance for Hulten’s theorem. The first set of weights is calculated as the influence vector from IO and the second set directly from sales shares in aggregate value added. Note that neither the influence nor sales weight necessarily sum to one, a feature that is common among the so-called Domar weights and sometimes loosely interpreted as the *macroeconomic return to scale parameter*.<sup>18</sup> Implied by the Cobb-Douglas model, expenditure shares, input requirements and linkages are constant. Therefore, weights are calculated using data from 1995, i.e. all weights are time-invariant.

### 2.3.1 Influence (Network Weights)

To construct the influence vector corresponding to equation (31), repeated below for convenience, I start by weighting all elements of the IO matrix  $\mathbf{W}$  by total intermediate expenditure of the using sector. Then, I estimate the vector of final consumption shares  $\mathbf{v}$  with elements  $v_i = \frac{\text{consumption}_i}{\sum_{i=1}^N \text{consumption}_i}$ . By construction, the  $v_i$ ’s sum to one. The last argument that needs to be computed before putting the weight together is  $(\mathbf{1} - \boldsymbol{\alpha})$ , which is a repeated vector (to matrix dimensions) with elements  $(1 - \alpha_i)$ , where  $\alpha_i$  is the sector labor share in production.<sup>19</sup> The influence vector is

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<sup>17</sup>While many papers in the micro-to-macro literature speak about the link between idiosyncratic productivity and GDP, the original theorem by Hulten (1978) discusses how productivity translates into aggregate productivity (TFP<sub>*i*</sub> to TFP). Bruyne et al. (2016) is about the former while Gabaix (2011) proves the theoretical link between productivities and estimates the part of GDP that can be explained by idiosyncratic TFP (under assumptions of input elasticities). Bigio and La’O (2016) calibrate how frictional shocks transmit to aggregate TFP movements. This paper follows Hulten (1978) and Bigio and La’O (2016) and estimates the effect of idiosyncratic productivity on the implied aggregate productivity shock before extending the analysis to observed GDP.

<sup>18</sup>It measures the percentage change in the aggregate from a uniform shock to productivity. See Baqaee and Farhi (2018*b*) for interpretation, discussion and analytical results.

<sup>19</sup>Labor shares are computed as  $\alpha_i = (\text{labor expenses})_i / (\text{total expenses})_i$  using KLEMS data. Note that capital requirements are neglected in the model setup; hence, total expenses are the sum of labor costs and intermediate expenses. This follows from the model’s first-order conditions,

then combined with element-by-element multiplication of elements in the inner sum, inverted and left-multiplied with the consumption vector. The resulting influence vector is identical to the expression in Bigio and La'O (2016):

$$\mathbf{q} = \mathbf{v}'[\mathbf{I} - (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W}]^{-1}. \quad (38)$$

### 2.3.2 Domar (Sales Weights)

The Domar weights are constructed along the lines of Gabaix (2011) as the sectors' nominal gross sales,  $p_i Q_i$ , divided by the sum of industry-wide value added  $p_i y_i$  (i.e. GDP),

$$D_i = \frac{p_i Q_i}{\sum_{i=1}^N p_i y_i}. \quad (39)$$

## 2.4 Contribution of Variance

The counterfactual common and idiosyncratic series are described by equations (35) and (36). The empirical exercise is to calculate the contribution of idiosyncratic variation to aggregate variation in the shock, (34) and production. While sector-level idiosyncratic and common terms are orthogonal (equation 33), this is not necessarily true for their aggregated counterparts. In order to account for possible covariance between the counterfactual common and idiosyncratic productivity growth series, I set up two scenarios for the contribution of idiosyncratic and common productivity shocks to the aggregate variable. To do so, I first regress the idiosyncratic component on the aggregate variable. The resulting R2 (given by  $(SS - RSS)/SS$ , where  $SS$  is the sum of squares and  $RSS$  is the residual sum of squares) is the contribution of idiosyncratic variation when the common component is not controlled for; I call this scenario (i). The second scenario, (ii), accounts for possible covariance. In this case, I calculate the contribution by first regressing the counterfactual common series on the aggregate followed by a regression using the counterfactual series. Finally, the contribution is given by the decrease in the residual sum of squares from adding the idiosyncratic component to the common component in proportion to the sum of squares. Thus, the contribution of each component is determined by the explained variance when the covariance (i) is or (ii) is not accounted for. All components are calculated first using influence weights and in Section 3.3 using Domar weights. Results are compared between the two weighting schemes to attain implications for frictions and the relevance of Hulten's theorem.

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equation (4).

### 3 Results

Before turning to the results for the decomposition of productivity and the contribution of variance, the first part of this section presents the relative importance of sectors and heterogeneous elements of the input-output economy. The last part of this section contrasts the results to Domar aggregation.

#### 3.1 The Network Weights

In the cross-section, according to IO data, 62 percent of production is bought by other sectors as intermediates for production and 38 percent goes to final consumption. The relatively large share of intermediate production implies a large interconnection of firms on average. Reassuringly, intermediate production corresponds well to intermediate shares used in production,  $(1 - \alpha_i)$ , which has a mean of 0.65, see the third column of Table A1 for the distribution across sectors. Final consumption shares  $v_i$  is heterogeneous with a mean of 0.035 and a standard deviation of 0.051.

Summary statistics for the calculated weight are presented in Table 1. The weights for each sector are in Table A1 in the Appendix. According to the measure, the most important sectors are the public sectors, construction, financial intermediation, real estate, transport and storage, and wholesale and retail trade.

Table 1: Summary Statistics Influence Weights and Intermediate Share

	Influence	$(1 - \alpha_i)$
Mean	0.120	0.653
Median	0.096	0.710
Standard deviation	0.058	0.191
Total	3.488	

The weight sum,  $\sum_{i=1}^N \text{weight}_i$ , last row, is larger than one. This is as expected because higher productivity in a sector creates more final products and intermediates. The increase in intermediates (or lowering of the price of intermediates) affects sectors that are using that intermediate good in that they increase productivity and output, generate more final and intermediate goods for their customers, and so forth. The increase in productivity in one sector leads to a shift in the production possibility frontier of connected sectors and the aggregate. Thus, the *macroeconomics of scale* is larger than one, which indicates multiplicative (aggregate) effects of a uniform shock.

### 3.2 Contribution of Variance in the Aggregate

The counterfactual series, together with the implied shock and aggregate production, is depicted in Figure 2a, and the standard deviation of the series is in Table 2. The series corresponds to equations (35) and (36) with the theory-implied influence weights. The analysis is on variations; hence, sign coefficients are irrelevant. Therefore, Figure 2b plots the negative of the idiosyncratic series together with production (detrended real value added growth). This reveals an interesting pattern, where the negative of counterfactual idiosyncratic productivity and the production series follow one another. This pattern suggests that idiosyncratic growth can be a predictor of the business cycle.

Figure 2: Decomposed Time Series Growth Rates



Note: Shock is the weighted sum of productivity growth, idiosyncratic is the weighted sum of idiosyncratic productivity growth, and common is the weighted sum of common productivity growth. (a) shows the series and (b) plots the negative of the idiosyncratic series together with real production growth.

As indicated in Figure 2a, the standard deviation of the common component is larger than the standard deviation of both production growth and the idiosyncratic component, as shown in Table 2. The standard deviation of real value added growth is about half that of the idiosyncratic component, (columns (1) and (3)), while the implied shock and common series have about four and six times larger standard deviations, respectively, than production, (columns (2) and (4)). The counterfactual productivity series (weighted idiosyncratic series and weighted common series) is highly negatively correlated and has a negative covariance, see the second row.

Given the high covariance, it is useful to construct the contribution under two scenarios, which are outlined in Section 2.4 and presented in Table 3. The first column shows the contribution to the aggregate shock, equation (34). The second and third



Table 2: Standard Deviation and Covariance 1994-2007

	Influence			
	Production	Shock	$\sum q_i \epsilon_i$	$\sum q_i \eta$
St.d.	0.0121	0.0495	0.0214	0.0682
Cov.				-0.0013
Corr.				-0.9112

Note: Standard deviation of real GDP growth and implied time series growth.  $\epsilon_i$  is idiosyncratic productivity growth and  $\eta$  is common productivity growth. Weights,  $q_i$ , are the influence weight. Covariance and correlation are between the weighted idiosyncratic and weighted common productivity series.

are the corresponding numbers for aggregate production from data, with alternatives on lags. The results show that the contribution of idiosyncratic variation to the shock is 68 percent under the first scenario and close to 3 when accounting for the covariance with the common component. The last two columns of Table 3 show the estimated interval for production, i.e. real GDP growth, contemporaneous variation in column (2) and contemporaneous with the one lag in column (3).<sup>20</sup> Surprisingly, the contribution for the idiosyncratic and common components are similar within and across weights. Thirty-five percent of the contemporaneous variation in production is explained by idiosyncratic variation without accounting for the covariance, and 2 percent when including the covariance. Including the first lag further increases the explained variance to 42 under the first scenario and 29 under the second (obviously, a part of the increase is mechanical from more regressors). The common component explains 97 percent of the variation in the shock under the first scenario and 32 under the second. Hence, a large part of the variation in column (1) can likely be attributed to the common component. This is in contrast to production where, interestingly, the counterfactual productivity series has similar contributions. Moreover, adding both components to the regression gives an explanatory power between 36-40 percent for production, and, by construction, all variance of the aggregate shock is jointly explained by the two factors.<sup>21</sup>

To sum up, most of the variation in the productivity shock is from common variation, and up to 67 percent can be explained by idiosyncratic shocks aggregated with influence weights. Up to 35 percent of variation in value added growth is explained

<sup>20</sup>Note that the weight is time-invariant.

<sup>21</sup>For reference, the regression outcome from the implied aggregate shock (before decomposition to common and idiosyncratic) on production is included in the Appendix.

Table 3: Contribution of Variance

			Aggregate		
			Shock	Production	
			(1)	(2)	(3)
Influence	Scenario (i)	Idiosyncratic	0.6782	0.3500	0.4260
		Common	0.9684	0.3461	0.3740
	Scenario (ii)	Idiosyncratic	0.0316	0.0182	0.2884
		Common	0.3218	0.0143	0.2365
Total Variance Contribution			1.0000	0.3643	0.6624
Lag			no	no	yes

Note: Scenario (i) is R2 where idiosyncratic (common) is the only regressor. Scenario (ii) is the change in R2 from adding idiosyncratic (common) to a regression on the common (idiosyncratic) component. Total contribution is R2 from regression with both components. Columns (1) and (2) are contemporaneous, and column (3) includes both the contemporaneous and lagged productivity variation.

by contemporaneous and lagged idiosyncratic productivity. Thus, the results show that the *irrelevance of idiosyncratic shocks* does not hold for the shock and production. All results should be interpreted with some caution due to the few number of observations (yearly data with consistent classifications, 29 sectors and the years 1993-2007).

### 3.3 Is Knowledge of the Network Important?

This sub-section explores the shortcut of Domar weights, including a discussion about the empirical relevance of frictions in the input-output network.

The summary statistics for all weights are presented in Table 5 and across sectors in Table A1. Domar weights are on average lower than influence weights, and the weight sum is approximately 60 percent lower; thus, the *macroeconomics of scale* is larger under influence aggregation.<sup>22</sup> The correlation between the weights is 0.647,

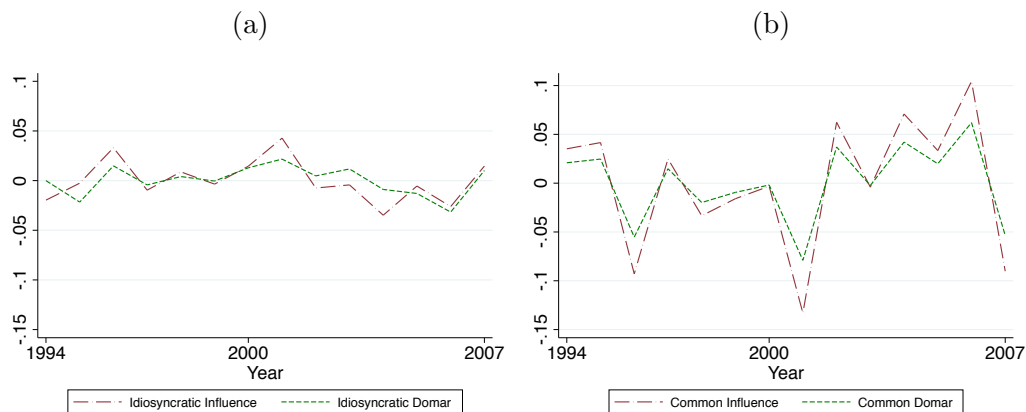
<sup>22</sup>From Section 1.3, the network effect of a friction is the wedge between weights calculated from observed sales shares in value added and the influence measure:

$$\mathbf{q} - \mathbf{D} = \mathbf{v}' [\mathbf{I} - (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W}]^{-1} - \mathbf{v}' [\mathbf{I} - (\mathbf{1} - \boldsymbol{\alpha}) \odot \mathbf{W} \odot (\boldsymbol{\phi} \cdot \mathbf{1})]^{-1}. \quad (40)$$

The key for this difference is the friction  $\boldsymbol{\phi}$ . If all elements were scalars, the solution for  $\boldsymbol{\phi}$  is unique. In contrast to the scalar case, the system of matrices makes the friction depend on the whole system and relative coefficients. The solution for the friction will not be unique since it does not have to be symmetric; therefore, this paper does not calculate the implied friction. Note also

and the ranking of sectors according to the weight is preserved only three times between the two weighting schemes.<sup>23</sup> From these indications of underlying differences in the weights, potentially aroused by frictions, the implied and counterfactual series are calculated under Domar weighting. The series is depicted in Figure 3, supporting a higher standard deviation under influence aggregation, which is in line with on average larger *macroeconomics of scale*, and negative covariance between the weighted idiosyncratic series and weighted common series under both weights (Table 6).

Figure 3: Counterfactual Time Series under Influence and Domar Weights



Note: (a) shows the weighted sum of idiosyncratic productivity growth, and (b) plots the weighted sum of common productivity growth, under influence and Domar aggregation.

The contribution of the series is calculated following the same structure as under the theory-implied measure. The result is presented in Table 4. The contribution for the shock is somewhat lower under Domar aggregation, 33 vs. 68 percent under the first scenario and 3 vs. 9 percent when accounting for the covariance. Interestingly, there are substantial similarities in the contribution to production between the weighting structures. The numbers for contemporaneous variation are 37 and 5 under Domar and 35 and 2 percent under influence aggregation. The main difference in the contribution of factors between the two weights is when including the first lag and accounting for the covariance, 5 percent under Domar and 29 percent under influence. Taken together, although the comparison depends on the preferred scenario, it does not seem to be of major importance which weight is used to assess the importance of idiosyncratic productivity for the business cycle. Thus, the result does not provide a clear dismissal of the *irrelevance of network* hypothesis.

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that equation (40) gives the prediction that in order for the friction to be in the theoretical zero-one interval the difference should be positive. But, under interconnection of sectors and asymmetries of the friction, the condition is not sufficient.

<sup>23</sup>Preserved ranking for sector: textiles, private households and real estate.

Table 4: Contribution of Variance using Domar Weights

		Aggregate			
		Shock	Production		
		(1)	(2)	(3)	
Domar	Scenario (i)	Idiosyncratic	0.3326	0.3685	0.3850
		Common	0.9100	0.3461	0.3740
	Scenario (ii)	Idiosyncratic	0.0900	0.0527	0.0463
		Common	0.6674	0.0303	0.0353
Total Variance Contribution		1.0000	0.3988	0.4204	
Lag		no	no	yes	

Note: Scenario (i) is R2 where idiosyncratic (common) is the only regressor. Scenario (ii) is the change in R2 from adding idiosyncratic (common) to a regression on the common (idiosyncratic) component. Total contribution is R2 from regression with both components. Columns (1) and (2) are contemporaneous, and column (3) includes both the contemporaneous and lagged productivity variation.

Without more details on the friction, this paper cannot with certainty answer if the difference in weights comes from frictions or other factors not included in the theoretical framework. Importantly, this paper does not find any large differences in the macro effects of micro distortions under Domar compared to influence weights, which is the preferred weighting scheme when there are frictions, i.e. when Hulten’s theorem does not hold. In addition, as indicated in the summary statistics in Table 5, the weights have similar standard deviations, supporting that frictions mainly shift the mean of the weight. The results thus suggest that there are sector-level frictions, but that they are not distributed such that the interconnectedness of sectors is more skewed than the sales weight. Hence, the results suggest that Hulten’s theorem holds in data.

What is driving the relevance of idiosyncratic shocks and irrelevance of networks? One explanation is heterogeneity of at least one of the elements in the weight. Section 4 elaborates on these possibilities.

## 4 Heterogeneity of the Weighting Structure?

Following that weights depend on relative coefficients, what drives the result is an empirical question. This section will disentangle the different dimensions of the results by rotations on the heterogeneous elements of the influence weight.

## 4.1 Shutting Down Heterogeneous Elements

There are three elements of the influence weight that can drive the asymmetry giving rise to the potential idiosyncratic propagation. In this section, I will explore these elements, starting with the adjustment of the influence vector to homogeneous consumption shares,  $v_i = v = 1/N$ , and then to homogeneous intermediate input share equal to the mean,  $(1 - \alpha_i) = (1 - \bar{\alpha})$ . Finally, I adjust the influence vector to homogeneous intermediate input requirements, that is  $w_{ij} = \frac{1}{N} \forall i, j$ . In other words, the last adjustment is equivalent to making the inter-sectoral network symmetric. These rotations are similar to Bruyne et al. (2016) but with slightly different theoretical assumptions. Table 5 shows in the upper panel descriptive statistics for all weights while the lower panel gives the correlation. The first column of the lower panel shows the correlation between influence weights and the three other weights. The last row and column of both panels give the statistics for Domar weights, as discussed in Section 3.3. What stands out from the results is the importance of heterogeneous consumption shares that have a low and negative correlation with the other measures. Surprisingly, neither homogeneous labor shares nor a symmetric inter-sectoral network change the correlation by any magnitude.

Table 5: Weight Descriptives and Correlation of Weights

	Influence	Homogeneous			Domar
		v	$\alpha$	W	
Mean	0.120	0.101	0.126	0.092	0.071
Max.	0.266	0.121	0.345	0.235	0.205
Min.	0.000	0.034	0.000	0.057	0.000
St.d.	0.058	0.016	0.072	0.050	0.052
Sum.	3.488	2.924	3.664	2.659	2.072
Influence	1				
Homogeneous $v$	-0.0479	1			
Homogeneous $\alpha$	0.972***	-0.187***	1		
Homogeneous $W$	0.897***	-0.338***	0.947***	1	
Domar	0.650***	-0.0261	0.634***	0.664***	1

Note: Summary statistics in the upper panel and correlations in the lower panel. Influence is the theory-implied weight. Homogenous  $v$ ,  $\alpha$ ,  $W$  adjust the indicated variable of the influence weight to homogeneity. Domar is the proposed alternative weight under Hulten's theorem.

Counterfactual (implied) aggregate standard deviation under the different weights is presented in Table 6, with the idiosyncratic series in the first row and the common series in the second row. The result for homogeneous final consumption shares,

again, is what stands out with a reduction of the implied standard deviation from the idiosyncratic component of 0.0028, which corresponds to 23 percent of the standard deviation of production (0.0121, see Table 2). Note also that the covariance of the weighted series turns positive.

Table 6: Standard Deviation and Covariance of Counterfactual Series

	Influence	Homogeneous			Domar
		$v$	$\alpha$	$W$	
St.d. $\sum \text{weight}_i \epsilon_i$	0.0214	0.0028	0.0249	0.0163	0.0149
St.d. $\sum \text{weight}_i \eta$	0.0682	0.0565	0.0716	0.0508	0.0405
Cov. ( $\sum \text{weight}_i \epsilon_i, \sum \text{weight}_i \eta$ )	-0.0013	0.0001	-0.0016	-0.0008	-0.0005

Note: Columns indicate the weights used for aggregation of the idiosyncratic,  $\epsilon_i$ , and the common,  $\eta$ , series. Influence is the theory-implied weight. Homogenous  $v$ ,  $\alpha$ ,  $W$  adjust the indicated variable of the influence weight to homogeneity. Domar is the proposed alternative weight under Hulten's theorem.

The contribution of idiosyncratic variation under both scenarios is presented in Panel A in Table 7. For each rotation on the heterogeneous elements, the implied aggregate shock and the counterfactual idiosyncratic and common series are calculated, equations (34) to (36), respectively. The contribution to variation in production is presented in columns (2) and (3), where the latter includes the first lag of idiosyncratic productivity. The evidence points toward an important role for heterogeneous consumption shares in the propagation from micro to macro; compare the second to the first row by scenario. Muting this channel substantially lowers the contribution. Surprisingly, homogeneous labor shares or homogeneous networks do not qualitatively change the theory-implied variation for the shock. However, they do reduce the contribution on production close to zero when accounting for the covariance.<sup>24</sup>

To further test the strength of final demand, I calculate the contributions using heterogeneous final consumption shares while setting the labor share and the intersectoral network as their homogeneous counterparts, Panel B of Table 7. The contributions are highly similar to the homogeneous network (the difference is that under this rotation final consumption shares are the only heterogeneous variable, while under the homogeneous network rotation, above, both final consumption shares and labor shares are heterogeneous). In Scenario (i), the estimated contribution under influence aggregation is almost completely approximated by only allowing for heterogeneous final demand. However, when accounting for the covariance, Scenario (ii), the contribution to production growth requires both the network and

<sup>24</sup>The result is not sensitive to excluding the public sector, not shown.

Table 7: Contribution of Idiosyncratic Productivity with Homogeneous Adjustments

			Aggregate		
			Shock	Production	
			(1)	(2)	(3)
Panel A	Scenario (i)	Influence	0.6782	0.3500	0.4260
		Homogeneous $v$	0.5466	0.1202	0.1500
		Homogeneous $\alpha$	0.6611	0.3158	0.3710
		Homogeneous $W$	0.6785	0.3126	0.3277
	Scenario (ii)	Influence	0.0316	0.0182	0.2884
		Homogeneous $v$	0.0011	0.0115	0.0265
		Homogeneous $\alpha$	0.0410	0.0036	0.1688
		Homogeneous $W$	0.0330	0.0029	0.0689
Panel B	Scenario (i)	Homogeneous $\alpha$ and $W$	0.6926	0.3126	0.3277
	Scenario (ii)	Homogeneous $\alpha$ and $W$	0.0278	0.0029	0.0689
Panel C	Scenario (i)	$(1 - \alpha) = 0$	0.0086	0.1079	0.2137
	Scenario (ii)	$(1 - \alpha) = 0$	0.2792	0.0012	0.1019
Lag			no	no	yes

Note: Scenario (i) is R2 where idiosyncratic is the only regressor. Scenario (ii) is the change in R2 from adding the idiosyncratic component to a regression on the common component. Columns (1) and (2) are contemporaneous and column (3) includes both the contemporaneous and lagged productivity variation. Influence is the theory-implied weight. Homogenous  $v$ ,  $\alpha$ ,  $W$  adjust the indicated variable of the influence weight to homogeneity. Panel A rotates one variable at a time. Panel B adjusts two variables. Panel C sets intermediate trade to zero, effectively removing the inter-sectoral network and leaving the weights equal to  $v$ .

final demand as heterogeneous elements to approximate the contribution under the theory-implied (influence) aggregator. However, this suggests that final demand is the most important (last column of Panel B relative to Panel A).

As a final rotation, Panel C, I set intermediate shares  $(1 - \alpha_i)$  to zero for all  $i$ , which results in a collapse of the aggregation weight to  $\mathbf{v}$ . Note that this gives a weight that by construction sums to unity. While the former rotation, in theory, suggests that heterogeneity of value added is important and likely more important than the flow of intermediates between firms, the latter would mean that there is no intermediate trade and therefore value added is equal to final sales. Using this weight, idiosyncratic variations do not drive the business cycle by the same magni-

tude as the theory-implied estimate, and in particular the shock cannot be driven by idiosyncratic variations in Scenario (i). This confirms that intermediate goods are important to get the theory-implied propagation effects, and that even though heterogeneous final demand is important in the influence weight, it is not sufficient to explain the business cycle variations in isolation. In other words, for evaluating the effects from micro disturbances to macro volatilities, it is not sufficient to use final consumption shares as weights without taking into account the flow of intermediate goods between sectors.

Taken together, these results imply that one of the most interesting elements of the influence weight is final demand; thus, a prospect for future research is to endogenize consumption shares in final consumption,  $v$ , and to learn more about their role in the amplification.

## 5 Conclusion

This paper uses a theoretical framework from the recent and rapidly growing literature on how micro disturbances can propagate into macro volatilities. While most previous studies rely on frictionless markets, this paper follows Bigio and La'O (2016) and allows for sectoral frictions while estimating the contribution of idiosyncratic and common productivity variations in aggregate variation using a theoretically motivated influence (network) measure. The effect is compared with aggregation using Domar (sales to aggregate value added) weights. While the first part asks if idiosyncratic shocks can drive the business cycle, the latter part studies the importance of the weighting structure for the results and the relative empirical importance of frictions.

The result indicates that the business cycle is driven by both idiosyncratic and common variations. While the common component is the most important part of the aggregate shock, idiosyncratic variation can explain up to two-thirds of the variation in the aggregate shock and one-third of production variations using the influence weight for aggregation. Thus, the *irrelevance of idiosyncratic shocks* does not hold in the data. Elaborations on the heterogeneous elements of the weight indicate that heterogeneity in final sales, and only partly the network per se, is the most important component in generating macro movements from micro variations. Moreover, using Domar weights gives similar predictions as the theory-implied influence weight. The difference between the weighting structures, but a lack of an important



difference in the contribution to the aggregate under the two weights, suggests that there are sector-level frictions, but that they are not distributed in such a way that they further skew the interconnectedness between sectors. This result implies that the *irrelevance of network* holds in the data, suggesting that, for questions regarding macro effects of micro distortions, we can continue using the Domar weights as a good approximation of aggregate influence from sector-level shocks.

Going back to the initial question, the business cycle is partly driven by common variation, but a non-negligible part is driven by idiosyncratic variation amplified by, in particular, asymmetries in final consumption.

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# A Appendix

## A.1 Weights and Contribution under Influence and Domar

Table A1: Weights and Intermediate Share by Sector, Year 1995

Sector	Influence	Domar	$(1 - \alpha_i)$
Agriculture, hunting, forestry and fishing	0.124	0.046	0.699
Basic metals and fabricated metal	0.068	0.082	0.767
Chemicals and chemical	0.080	0.048	0.774
Coke, refined petroleum and nuclear fuel	0.093	0.016	0.954
Computer, electronic and optical equipment	0.076	0.069	0.760
Construction	0.163	0.098	0.574
Education	0.188	0.082	0.362
Electricity, gas and water supply	0.141	0.050	0.718
Financial intermediation	0.164	0.080	0.550
Food, beverages and tobacco	0.168	0.078	0.832
Health and social work	0.258	0.137	0.313
Hotels and restaurants	0.099	0.033	0.654
Machinery, nec	0.078	0.071	0.726
Manufacturing nec; recycling	0.087	0.017	0.625
Mining and quarrying	0.085	0.007	0.710
Motor vehicles, trailers and semi-trailers	0.084	0.085	0.805
Other community, social and personal services	0.171	0.071	0.537
Other non-metallic mineral	0.096	0.012	0.709
Post and telecommunications	0.100	0.043	0.649
Private households with employed persons	0.000	0.000	0.000
Public admin and defence; compulsory social security	0.219	0.121	0.541
Pulp, paper, paper, printing and publishing	0.088	0.095	0.766
Real estate activities	0.266	0.205	0.879
Renting of m&eq and other business activities	0.090	0.143	0.580
Rubber and plastics	0.075	0.016	0.717
Textiles, textile, leather and footwear	0.074	0.009	0.717
Transport and storage	0.122	0.157	0.753
Wholesale and retail trade	0.138	0.169	0.462
Wood and of wood and cork production	0.093	0.032	0.806
Mean	0.120	0.071	0.653
Median	0.096	0.071	0.710
Standard deviation	0.058	0.052	0.191
Total	3.488	2.072	

## A.2 Aggregate TFP Regressions

Table A2 shows the regression outcome of the implied aggregate shock, without decomposition, to production. Shock Domar corresponds to  $\sum D_i z_i$  and Shock Influence to  $\sum q_i z_i$ . Under both weights, the shock significantly increases production. Adding lags does not change the results by any magnitude.

Table A2: Regression: Shock on Production

VARIABLES	(1) Production	(2) Production	(3) Production	(4) Production
Shock Domar	0.197* (0.101)	0.216* (0.117)		
Shock Influence			0.135** (0.058)	0.146* (0.069)
Lag	no	yes	no	yes
Observations	14	13	14	13
R-squared	0.242	0.253	0.308	0.319

Standard errors in parentheses  
 \*\*\* p<0.01, \*\* p<0.05, \* p<0.1