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## Monetary Policy under Imperfect Information and Consumer Confidence

by Jan-Niklas Brenneisen



# Monetary Policy under Imperfect Information and Consumer Confidence

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#### Abstract

Although it is generally accepted that consumer confidence measures are informative signals about the state of the economy, theoretical macroeconomic models designed for the analysis of monetary policy typically do not provide a role for them. I develop a framework with asymmetric information in which the efficacy of monetary policy can be improved, when the imperfectly informed central banks include confidence measures in their information set. The beneficial welfare effects are quantitatively substantial in both a stylized New Keynesian model with optimal monetary policy and an estimated medium-scale DSGE model.

JEL classification: D82, D83, D84, E52, E58, E71

**Keywords:** Consumer confidence, Monetary policy, Asymmetric information, Imperfect Information, New Keynesian macroeconomics, DSGE models

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## 1 Introduction

In standard theoretical monetary policy settings there is no role for consumer confidence measures, although these are collected, processed, and discussed by public institutions with immense effort. I suggest an intuitive theoretical setup, in which central banks can explicitly improve the efficacy of monetary policy by considering confidence measures. I find that paying attention to confidence implies a quantitatively substantial positive welfare effect in both a stylized New Keynesian model under optimal monetary policy and an estimated medium-scale DSGE model.

From a broader perspective, I contribute to the literature on imperfect information of monetary policy makers. As shown by Orphanides (2001), monetary policy recommendations differ substantially when based on real-time data instead of ex-post revised data. The seminal paper raises awareness to the great uncertainty about the underlying state of the economy that central banks face when making their decisions.<sup>1</sup> This paper does not only confirm the adverse effects of imperfect information of the central bank, but also proposes a way to mitigate them: by paying more attention to readily available signals like confidence measures when making monetary policy decisions.

The theoretical framework revolves around the approach of Barsky and Sims (2012), who assume incomplete information about technology growth. Instead of observing the growth rate directly, households forecast it by receiving and optimally processing a private noisy signal of it. I augment this channel by making explicit assumptions about the information set of monetary policy makers. More specifically, I assume that the central bank cannot observe the noisy signal of the private sector and has, thus, to forecast the private expectations in order to conduct monetary policy. In such a setup, the central bank can accommodate the welfare losses, caused by its informational disadvantage relative to the real sector, by taking into account an endogenous confidence measure. I argue that the link between confidence and macroeconomic key variables is of high importance for monetary policy when accounting for the policymakers' informational restrictions.

<sup>&</sup>lt;sup>1</sup>Based on this finding, a number of theoretical contributions highlight the features that the assumption of imperfect information provides to macroeconomic models. Neri and Ropele (2012) develop and estimate a model with imperfect knowledge about the state of the economy which has great explanatory power for the business cycle. Lippi and Neri (2007) analyze optimal monetary policy in a DSGE model with imperfect information. Collard et al. (2009) show that models with information constraints exhibit endogenous persistence which makes them less dependent on ad hoc assumptions like price indexation, habit formation or autoregressive shock processes.

In a stylized New Keynesian model I provide solutions for optimal monetary policy with consumer confidence for both discretion and commitment with asymmetric information. Under reasonable calibrations, I find that the effect of paying attention to confidence is large. Optimal processing of confidence signals leads to a reduction of 63.7% of the welfare loss induced by informational frictions.

The beneficial welfare effects persist in an enhanced medium-scale DSGE model. However, they are somewhat smaller: the estimated welfare gain of paying attention to confidence is 21.8%. The empirical assessment suggests that the central bank attained 71.5% of the potential welfare gain between 1960 and 2018.

Barsky and Sims (2012) find that confidence has powerful predictive implications for macroe-conomic variables. They develop an estimated DSGE model, that reflects the empirical link between confidence and economic activity. One attractive feature of their approach is that it gives a role to two contradicting views, which assign a role to confidence in the macroe-conomy: the *animal spirits* view and the *news* view.

On the one hand, the animal spirits view, which dates back to Keynes (1936), claims that autonomous fluctuations in beliefs have causal effects on economic activity. For example Blanchard (1993) explains the recession in 1990-1991 with an exogenous shift in pessimism, which had a causal effect on aggregate demand. On the other hand, supporters of the news view (Cochrane; 1994; Beaudry and Portier; 2006) argue that confidence merely contains fundamental information about the economy but has no causal role. Barsky and Sims (2012) find that animal spirits shocks are not an important source of the relation between confidence innovations and macroeconomic variables. Instead, news shocks explain the major part of the observed transmission from confidence to future activity.

A recent approach that implements confidence into a theoretical macroeconomic model in a different way is provided by Angeletos, Collard and Dellas (2018). In their model endogenous confidence emerges from the existence of higher-order uncertainty and frictions in expectation coordination. The confidence shock is a disturbance to the expected bias in the expectation of others. Further influential contributions that discuss the relation of confidence about technology growth and real business cycles are Angeletos and La'O (2013) and Ilut and Schneider (2014). News and noise shocks in macroeconomic frameworks are discussed in Jaimovich and Rebelo (2009) and Lorenzoni (2009). None of these contributions, however, makes explicit assumptions about restrictions to the information set of the central bank and

could, thus, provide a role for a confidence measure as informative signal in the conduct of monetary policy.

In Section 2, I investigate whether one can draw conclusions about the value monetary policy makers ascribe to confidence measures from reading central bank publications. Focusing on the Federal Open Market Committee (FOMC), I find that confidence is not explicitly mentioned in statements or publications on forecast methodologies. However, sifting through all minutes of FOMC meetings from 1959 to 2019 and counting mentions of the keywords confidence and sentiment reveals that monetary policy makers do indeed consider confidence measures as relevant source of information in their process of decision making.

In Section 3, I implement the imperfect information framework into an otherwise standard baseline New Keynesian model. I build heavily on the numerous influential monetary policy contributions to the New Keynesian literature, e.g. Clarida et al. (2000), Woodford (2003) and Galí (2015). The setup gives rise to an endogenous confidence measure. The information set of the central bank is explicitly restricted to not contain the private technology growth rate signal. In this asymmetric information setup, I provide the optimal commitment and discretion solutions, where I benefit from the general characterization of Svensson and Woodford (2004). Monte Carlo simulations of the calibrated model reveal that paying attention to confidence can mitigate the welfare losses caused by the information set restriction by almost two thirds.

To approximate the welfare effects of processing the confidence signal in a more empirically driven environment, Section 4 contains the analysis of an estimated New Keynesian DSGE model. The model is very similar to the one of Smets and Wouters (2007), but augmented with the same informational frictions as described above. The welfare effects are unsurprisingly lower than in the stylized model, but still statistically significant and quantitatively substantial.

Section 5 concludes.

## 2 Confidence and the FOMC

In standard models designed for the analysis of monetary policy, there is typically no role for confidence (or sentiment) of private market participants. In contrast, measures of these variables are frequently discussed in contributions, of both academics and media outlets, on the current and projected state of economic activity. In the light of this discrepancy, it is not obvious whether monetary policy makers regard confidence measures as a valuable source of information in their decision making process. To shed some light on this question, I broadly examine publications of the Federal Open Market Committee (FOMC), the principal monetary policy organ of the United States.

In its "Statement on Longer-Run Goals and Monetary Policy Strategy" (Federal Open Market Committee; 2019) the FOMC elaborates on its primary goals and communication principles without the slightest hint on consideration of market sentiment, let alone explicit mentioning.

However, confidence measures might be considered in the conduct of forecasts of macroe-conomic responses to monetary policy. One prominent model, that is consulted by Federal Reserve Board members for this purpose, is the FRB/US model, a large-scale model of the U.S. economy with a neoclassical core and numerous rigidities. In a brief overview of the model structure, Brayton et al. (2014) acknowledge potential sentiment effects on firms' fixed investment, but solely as an explanation of why the latter is directly affected by current business output. The model does not envisage a role for market confidence besides confidence in the central bank's commitment to restore inflation to the target level. (Brayton et al.; 1997, p. 243)

The most straightforward way to gain insights into the relevance of confidence measures for FOMC decision making might be to sift through the minutes of its official meetings. Searching for the keywords sentiment and confidence in all minutes of FOMC meetings from January 6th 1959 and December 10th-11th 2019<sup>2</sup> reveals, that monetary policy makers indeed frequently cite household, business or finance (or sometimes simply market) sentiment to substantiate their assessment of the current state of economic activity and, thus, explain their monetary policy (in)action. After manually cleansing the data of all keyword references that refer either to confidence (about forecasts, estimates, assessments, capabilities etc.) or sentiment (for certain actions) of the FOMC meeting participants themselves, or confidence of private market participants in the capabilities or intentions of the Fed or public institutions, I observe 1252 mentions in 622 meetings, implying a mean of 2.01. However, the sample is fairly volatile: realizations fluctuate between 0 and 15, the sample standard deviation is 2.67.

 $<sup>^2</sup>$ For all meetings before 1993 I resort to the *Record of Policy Actions* as I find the format to be roughly equivalent to the *minutes* that are published to date.

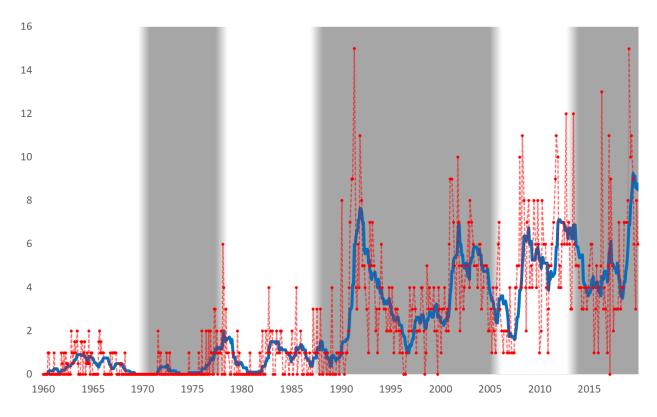


Figure 1: Mentions of the keywords 'sentiment' and 'confidence' in FOMC minutes from January 1960 to December 2019, excluding references describing 'sentiment' and 'confidence' of FOMC participants and 'confidence' in intentions and capabilities of the FOMC or other public institutions. Dashed (red) and solid (blue) line are mentions per month and its 12 months moving average, respectively. Alternating unshaded and shaded background indicate different chairs of the Board of Governors (William M. Martin, Arthur F. Burns, Paul Volcker, Alan Greenspan, Ben Bernanke, Janet Yellen), excluding the brief periods of G. William Miller (March 1978 - August 1979) and Jerome Powell (since February 2018).

In 260 minutes (41.8%) the keywords are not used at all. The time series and its 12-months moving average are shown in figure 1.

Of course, this experiment measures only imperfectly how closely the FOMC paid attention to confidence at each point in time. Nevertheless, it provides some intuition about the behavior of perceived importance of confidence for monetary policy. The following observations stand out to me. The perceived importance of confidence is positive but not constant. It fluctuates over time and exerts cyclical behavior. The perceived importance and its variance increase conspicuously in the early 1990s. While the moving average of mentions fluctuates between 0 and 2 before 1990, fluctuations thereafter occur roughly in the interval of 2 and 8.

At least some of these observations might be driven by individual or a group of FOMC members. To partially account for this, I highlight changes at the chair of the *Board of Governors* by using a shaded background for every other chair. One may notice that the keyword references are lower in frequency in the era of Burns (1970-1978) than under the chairmen Martin (-1970), Miller (1978-1979, not highlighted) and Volcker (1979-1987), while in turn the frequency is strikingly higher in the eras of Greenspan (1987-2006), Bernanke (2006-2014) and Yellen (2014-2018). However, there is no indication for changing FOMC personnel being a main driving force of perceived importance of confidence.

In summary, the results of this simple exercise are as follows. The FOMC considers confidence and sentiment measures as relevant for monetary policy making. However, the degree of attention it seems to dedicate to these variables fluctuates heavily. Does this imply that the impact of confidence on actual monetary policy decisions is positive and time-variant, too? This open question is dealt with in the empirical analysis of section 4.

## 3 Optimal Monetary Policy with Confidence and Asymmetric Information

A baseline macroeconomic model that enables analyzing the importance of considering confidence in the process of monetary policy making requires two features: nominal rigidities which provide effectiveness of monetary policy, and an endogenous confidence measure that contains information about the private sector which is otherwise not accessible for the cen-

tral bank. The former is comfortably obtained by using the well established New Keynesian framework which is at the core of most modern theoretical monetary policy analyses. For the latter, I implement into this framework the unobservable growth rate and noisy signals approach from Barsky and Sims (2012) who also establish an endogenous confidence measure based on private agent expectations. By assuming imperfect central bank information about the growth rate signal one obtains a setup in which it is beneficial for monetary policy makers to explicitly add confidence to their information set.

## 3.1 The New Keynesian model

The baseline New Keynesian model is extensively described in the seminal textbooks of Woodford (2003) and Galí (2015) who lay the foundation on which the following model is build upon. In the following, I briefly summarize its building blocks.

#### 3.1.1 Households

A representative infinitely lived household maximizes lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

subject to the period budget constraint

$$P_t C_t + Q_t B_t \le B_{t-1} + W_t N_t + D_t \tag{2}$$

where  $C_t$  is consumption,  $N_t$  denotes labor,  $P_t$  is the price level,  $W_t$  represents the nominal wage,  $B_t$  are (risk-free) government bond holdings purchased at price  $Q_t$ , and  $D_t$  denotes entrepreneurial profits. Period utility depends positively on consumption and leisure. It is defined as

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\nu}}{1+\nu}$$
(3)

#### 3.1.2 Intermediate Goods Sector

A continuum of differentiated firms with production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{4}$$

maximizes profit subject to monopolistic competition and Calvo (1983) price rigidity, i.e. only a fraction  $1 - \theta$  of firms can reset prices in any period. Technology level,  $A_t$ , is defined further below. Firms face the household demand equations

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{5}$$

where it is assumed that  $Y_t = C_t$ .

#### 3.1.3 Price setting

The dynamics of aggregate gross rate of inflation are given by

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{6}$$

where reoptimizing firms choose the optimal price,  $P_t^*$  solving

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} \frac{C_{t+k}}{P_{t+k}} \left( P_t^* - \frac{\epsilon}{\epsilon - 1} M C_{t+k|t} \right) \right\} = 0 \tag{7}$$

The marginal costs in period t + k of a firm that last reoptimized in period t,  $MC_{t+k|t}$ , are given by nominal wage divided by marginal product of labor

$$MC_{t+k|t} = \frac{W_{t+k}}{(1-\alpha)A_t N_{t+k|t}^{-\alpha}}$$
(8)

#### 3.1.4 Equilibrium

After log-linearizing around steady state values and rearranging the system, it can be expressed with two equations, the IS equation and the New Keynesian Phillips curve (NKPC).

Writing all log-linearized variables with lowercase letters, respectively, the NKPC can be expressed as

$$\pi_t = \beta E_t \left[ \pi_{t+1} \right] + \kappa x_t \tag{9}$$

where the output gap,  $x_t \equiv y_t - y_t^n$ , is defined as the percentage deviation of output from its natural level and  $\kappa \equiv \frac{(1-\alpha)(1-\theta)(1-\beta\theta)}{(1-\alpha+\alpha\epsilon)\theta} \left(\sigma + \frac{\nu+\alpha}{1-\alpha}\right)$ .

The IS equation describes the output gap dynamics and is defined as

$$x_{t} = E_{t} [x_{t+1}] - \frac{1}{\sigma} (r_{t} - E_{t} [\pi_{t+1}] - r_{t}^{n})$$

$$(10)$$

with nominal interest rate  $r_t$ . Natural output,  $y_t^n$ , and natural real interest rate,  $r_t^n$ , are defined as realizations that would occur under flexible prices ( $\theta = 0$ ). The dynamics of the latter are given by

$$r_t^n = \beta^{-1} + \sigma \psi_{ya} \left( E_t \left[ a_{t+1} \right] - a_t \right)$$
 (11)

where  $\psi_{ya} \equiv \frac{1+\nu}{\sigma(1-\alpha)+\nu+\alpha}$ .

## 3.2 Technology, News Shocks, and Animal Spirits

At this point, the so far standard New Keynesian approach is augmented with incomplete information and noisy signals about the technology growth rate. Following the assumptions of Barsky and Sims (2012), log technology follows a random walk with drift

$$a_t = a_{t-1} + g_{a,t-1} + \varepsilon_{a,t} \tag{12}$$

where the unobservable technology growth rate,  $g_{a,t}$ , follows an AR(1) process with news shock  $\varepsilon_{g_a,t}$ 

$$g_{a,t} = (1 - \rho_{g_a})g_a^* + \rho_{g_a}g_{a,t-1} + \varepsilon_{g_a,t}$$
(13)

and unconditional mean  $g_a^*$  (which can be ignored in the log-linearized system of this section). All private agents receive a noisy signal of the growth rate,  $s_t$ , with animal spirits shock  $\varepsilon_{s,t}$ 

$$s_t = g_{a,t} + \varepsilon_{s,t} \tag{14}$$

Whereas the news shock contains information about future technology dynamics, the animal spirits shock is pure noise.

Under the assumption of rational expectations, agents form optimal forecasts of the unobserved growth rate which can be computed via the *Kalman filter*, a procedure for instance described by Ljungqvist and Sargent (2018). In each period t agents receive and process two informative signals about the current technology growth rate: actual technology growth,  $a_t - a_{t-1}$ , and the additional noisy signal,  $s_t$ . Defining state vector  $\chi_t \equiv (g_{a,t}, g_{a,t-1})'$  and signal vector  $\xi_t \equiv (a_t - a_{t-1}, s_t)'$ , the state space system is given by

$$\chi_{t+1} = \underbrace{\begin{pmatrix} \rho_{g_a} & 0 \\ 1 & 0 \end{pmatrix}}_{\equiv A} \chi_t + \underbrace{\begin{pmatrix} \sigma_{g_a} & 0 \\ 0 & 0 \end{pmatrix}}_{\equiv C} w_{t+1}$$

$$(15)$$

$$\xi_t = \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\equiv G} \chi_t + \underbrace{\begin{pmatrix} \varepsilon_{a,t} \\ \varepsilon_{s,t} \end{pmatrix}}_{\sim N(0,R)} \tag{16}$$

where  $w_{t+1} \stackrel{i.i.d.}{\sim} N(0, I)$  and  $R \equiv \begin{pmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_s^2 \end{pmatrix}$ .

Optimal signal processing involves iterating on the following recursion:

$$\hat{\chi}_{t+1} = A\hat{\chi}_t + K\left(\xi_t - G\hat{\chi}_t\right) \tag{17}$$

where the Kalman gain, K, is the solution of the system

$$K = A\Sigma G' \left(G\Sigma G' + R\right)^{-1} \tag{18}$$

$$\Sigma = CC' + KRK'_t + (A - KG)\Sigma_t (A - KG)'$$
(19)

The private information processing mechanism enters the New Keynesian system via the natural rate of real interest,  $r_t^n$ , which can now be expressed as

$$r_t^n = \beta^{-1} + \psi_{ya} \hat{g}_{a,t} \tag{20}$$

with the expected technology growth rate  $\hat{g}_{a,t}$ .

## 3.3 Monetary Policy and Confidence

This section contains the core of the paper, as the New Keynesian framework with imperfect information provided by Barsky and Sims (2012) is now further augmented with explicit assumptions about the central bank's information set. The monetary policy makers are restricted regarding their ability to observe contemporary private signals of the technology growth rate. However, they may use an endogenous confidence measures to mitigate adverse effects of the informational frictions.

How can it be justified that the central bank has an informational disadvantage relative to the private sector? After all, central banks are, in general, large institutions that employ hundreds of experienced economists, statisticians and forecasters. However, the information set of the representative agent in the New Keynesian framework is the result of information aggregation. The above assumption implies by no means that individual consumers have superior information relative to monetary policy makers. Nevertheless, as Cochrane (1994) puts it, consumers have detailed information about their own prospects that correlate strongly with future aggregate income. Summing over consumers, one may obtain an aggregate private information set which contains idiosyncratic shocks that are unobservable for individual economists and consumers alike.

In the following, I analyze the impact of informational frictions on the efficacy of optimal monetary policy under both discretion and commitment. Typically, the performance of monetary policy regimes is evaluated using an approximated welfare loss function as initially proposed by Rotemberg and Woodford (1999). Following Galí (2015), the welfare loss function in the above New Keynesian setup is given by

$$W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t \tag{21}$$

with the period loss function

$$L_t = w_x x_t^2 + w_\pi \pi_t^2 (22)$$

where  $w_x \equiv \sigma + \frac{\nu + \alpha}{1 - \alpha}$  and  $w_\pi = \frac{\epsilon}{\kappa} w_x$ . For simplification, I adopt a timeless perspective when optimizing monetary policy to neglect the starting point problem.

#### 3.3.1 Optimal Monetary Policy with Asymmetric Information

To assign a role to confidence measures as informative tools for the central bank it is initially required to somewhat restrict its information set. Before considering this asymmetric information scenario, I briefly state the optimal monetary policy design of the benchmark symmetric scenario. As the set of all exogenous disturbances  $\{\varepsilon_{a,t}, \varepsilon_{g_a,t}, \varepsilon_{s,t}\}$  only affects the IS equation but not the NKPC, it is a well-known result that it is feasible for the monetary policy maker to completely stabilize output gap and inflation rate at all times.

#### **Proposition 1** (Optimal monetary policy with symmetric information)

In the case of symmetric information, i.e. when the central bank's information set contains all private signals, optimal monetary policy from a timeless perspective under both discretion and commitment involves a period welfare loss of zero at all times. In both cases the monetary policy instrument, the nominal interest rate, is required to match the natural real rate of interest:

$$r_t = r_t^n = \beta^{-1} + \psi_{ya} \hat{g}_{a,t} \tag{23}$$

In the following, I assume that the central bank does not have complete information about private technology growth rate expectations. More precisely, the noisy signal,  $s_t$ , is not contained in the central bank's information set,  $I_t^{CB}$ , although it may contain other noisy signals that are informative in the absence of  $s_t$ . This specific assumption about the design of asymmetric information ensures that the central bank information set is completely nested in the private information set, a case for which Svensson and Woodford (2004) provide a general solution. The following optimal discretion result can be derived on its basis.

#### **Proposition 2** (Optimal discretion with asymmetric information)

In the case of asymmetric information, under the assumption that the central bank's information set is fully nested in the private information set, optimal discretionary monetary policy implies that the nominal interest rate tracks the natural real rate of interest as closely as possible:

$$r_t = E\left[r_t^n | I_t^{CB}\right] = \beta^{-1} + \psi_{ya} E\left[\hat{g}_{a,t} | I_t^{CB}\right] \tag{24}$$

Proof: See Appendix A.1.

The intuition of this result is straightforward. Without perfect information about the optimal level of the nominal interest rate, i.e. the natural real rate of interest, the optimal discretion policy involves setting the nominal interest rate to the expected optimal level at all times.

If commitment is feasible for the central bank, it can further reduce the welfare loss by committing to compensate for missing its target values (of zero inflation and output gap) in the future.

#### **Proposition 3** (Optimal commitment with asymmetric information)

Under the same assumptions about asymmetric information as in Proposition 2, optimal monetary policy under commitment from a timeless perspective implies for the nominal interest rate:

$$r_t = E\left[r_t^n | I_t^{CB}\right] + \Phi \Xi_{t-1} \tag{25}$$

where  $\Xi_t$  contains the (backward-looking) shadow prices of (9) and (10), and follows the law of motion

$$\Xi_{t} = \begin{pmatrix} w_{\pi} & 0 \\ \kappa w_{\pi} & w_{x} \end{pmatrix} \begin{pmatrix} \pi_{t} \\ x_{t} \end{pmatrix} + \begin{pmatrix} 1 & \frac{1}{\beta\sigma} \\ \kappa & \frac{1}{\beta} + \frac{\kappa}{\beta\sigma} \end{pmatrix} \Xi_{t-1}$$
 (26)

with initial value

$$\Xi_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Proof: See Appendix A.2.

#### 3.3.2 Monetary policy implications of confidence signals

The conduct of optimal monetary policy under both commitment and discretion requires optimal forecasting of the natural real rate of interest which implies forecasting the perceived technology growth rate. By assuming that the central bank's information set is completely nested in the private sector's information set, this task is equivalent to forecasting the actual technology growth rate. Additionally, the assumption ensures that the central bank does not provide relevant information about the technology growth rate by setting its monetary policy instrument, the nominal interest rate.

To provide a role for a confidence measure, it is assumed that the central bank is subject to the following informational friction: when setting the nominal interest rate is has no knowledge about the private signal,  $s_t$ , which the private sector uses to form its expectations about the technology growth rate. However, the policymakers observe the realizations of all macroeconomic variables, e.g.  $x_t$ ,  $\pi_t$  and  $a_t$ , after the nominal interest rate has been set. A fully rational central bank can, thus, deduce the private signal by reverse-engineering private sector expectations from observed behavior. More precisely, at each period t,  $s_{t-1}$  is included in the central bank's information set,  $I_t^{CB}$ , but  $s_t$  is not.

To mitigate the adverse welfare consequences of the informational friction, the central bank may process a confidence measure as additional signal. It is defined as the autoregressive process

$$\tilde{C}_t = (1 - \rho_c)\tilde{C}^* + \rho_c\tilde{C}_{t-1} + u_t \tag{27}$$

where confidence innovation,  $u_t$ , is a linear combination of surprise increase in technology level and growth rate, respectively:

$$u_t = \zeta_1(a_t - a_{t-1} - \hat{g}_{a,t-1}) + \zeta_2(\hat{g}_{a,t} - \rho_{g_a}\hat{g}_{a,t-1}) + \varepsilon_{c,t}$$
(28)

The design of the endogenous confidence measure is adopted from Barsky and Sims (2012). Note that  $\hat{g}_{a,t-1} \equiv E_{t-1} [g_{a,t-1}|I_{t-1}]$ , i.e. how the private sector perceived technology growth rate one period earlier.

Figure 2 shows the perceived and actual technology growth rate responses to the four shocks for the parameterization  $\sigma_a = 0.58$ ,  $\rho_{g_a} = 0.73$ ,  $\sigma_{g_a} = 0.17$ ,  $\sigma_s = 0.13$ ,  $\rho_c = 0.94$ ,  $\zeta_1 = 1.01$ ,  $\zeta_2 = 32.76$  and  $\sigma_c = 3.78$ , as estimated by Barsky and Sims (2012). In all cases, expectations of both real sector and central bank converge rapidly to actual growth rate, as the lines can barely be distinguished only four periods after the initial shock impulse.

The real sector expectations falsely ascribe technology level and animal spirits shocks partly to an actual growth rate increase, whereas perceived growth rate underestimates actual realizations in case of news shocks. Without an additional confidence signal, the central bank expectations deviate severely from their private sector counterparts on impact, as animal spirits and news shocks affect the received signals,  $a_t - a_{t-1}$  and  $s_{t-1}$ , only with a delay of one period. Therefore, the central bank cannot react to these shocks on impact, whereas it overestimates the probability of increased growth rate in case of level shocks. When

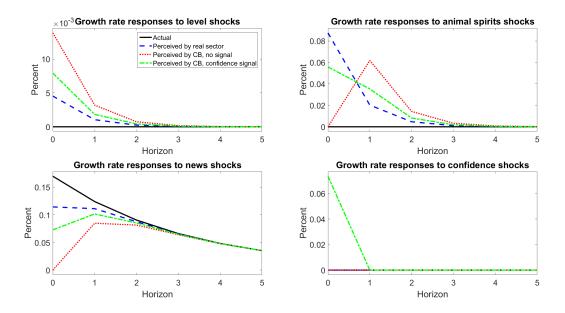


Figure 2: Technology growth rate responses to level, animal spirits, news and confidence shocks. For each shock, the figure compares the response of actual growth rate (solid, black), perceived growth rate by real sector (dashed, blue), and perceived growth rate by the central bank with (dashed-dotted, green) and without (dotted, red) an additional confidence signal.

observing an additional confidence signal, the central bank can mitigate the distance to real sector expectations by almost two thirds in all of these cases. However, this improvement comes at a cost. The central bank exposes itself to a fourth stochastic shock, the confidence shock  $\varepsilon_{c,t}$ , that may lead to deviations from private sector expectations. Under fully rational information processing via the Kalman filter, perfect knowledge about the shock standard deviations implies, however, that average forecast errors must decrease with the additional signal.

As shown in Appendix A.3, under optimal discretion the realizations of inflation and output gap depend solely on the *contemporaneous* technology growth rate forecast error of the central bank:

$$\pi_t = -\kappa \psi_{ya} \left( E \left[ \hat{g}_{a,t} | I_t^{CB} \right] - \hat{g}_{a,t} \right) \tag{29}$$

$$x_t = -\psi_{ya} \left( E \left[ \hat{g}_{a,t} | I_t^{CB} \right] - \hat{g}_{a,t} \right) \tag{30}$$

The impulse responses of the system variables  $\pi_t$ ,  $x_t$  and  $r_t$  are shown in figure 3. It is straightforward to see that, analogous to figure 2, using the confidence signal severely reduces output gap and inflation fluctuations in case of level, animal spirits and news shocks, at the cost of *one-off* deviations after confidence shocks. A notable result is the observation of

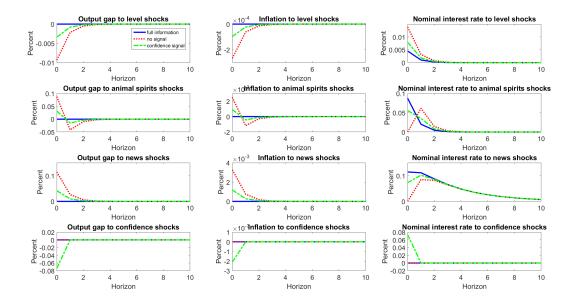


Figure 3: Output gap, inflation rate and nominal interest rate responses under optimal discretionary monetary policy to technology level, animal spirits, news and confidence shocks. For each shock, the figure compares the response under the informational regimes full information (solid, blue), imperfect information without confidence signal (dotted, red) and imperfect information with confidence signal (dashed, green).

undershooting in case of animal spirits shocks, a behavior that is typically not observed in purely-forward looking New Keynesian models under discretion.

Under optimal commitment, inflation and output gap respond to disturbances by following more complex paths (see Appendix A.4). In this case the variables depend not only on the contemporaneous forecast error of the central bank, but on the complete history of forecast errors:

$$\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \sum_{i=0}^{\infty} \tilde{\Phi}_i \left( E \left[ \hat{g}_{a,t-i} | I_{t-i}^{CB} \right] - \hat{g}_{a,t-i} \right)$$
 (31)

where  $\tilde{\Phi}_i$  is a vector of time-dependent weights. As illustrated by the impulse responses in figure 4, asymmetric information gives rise to non-monotonous dynamics. Output gap changes its sign twice in response to all shocks, while inflation dynamics show hump-shaped behavior. Once again, visual assessment suggests a fairly high contribution of confidence signal processing to stabilization.

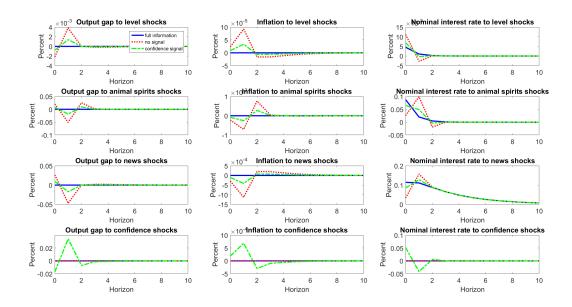


Figure 4: Output gap, inflation rate and nominal interest rate responses under optimal commitment to technology level, animal spirits, news and confidence shocks. For each shock, the figure compares the response under the informational regimes full information (solid, blue), imperfect information without confidence signal (dotted, red) and imperfect information with confidence signal (dashed, green).

To assess the welfare effects of the different informational regimes I estimate the average period loss function with Monte Carlo simulations. Under optimal discretion, the estimated loss under asymmetric information is 4.08% in the benchmark case. Processing the confidence signal helps to lower it to 1.48%. If it is feasible for policymakers to apply optimal commitment, the loss can be reduced to 0.98% and 0.36%, respectively. For both discretion and commitment, this implies a loss reduction of 63.7% caused by confidence signal processing. Independent of the underlying informational assumptions, monetary policy with optimal commitment provides a loss reduction of 75.9% in comparison to optimal discretion.

The impressive loss mitigation of almost two thirds when considering a confidence measure in the conduct of monetary policy has, of course, to be treated with caution as the underlying framework is very stylized. The exact value crucially depends on parameter calibration, most importantly on the standard deviation of confidence innovation,  $\sigma_c$ . Higher values decrease the confidence signal precision and, thus, make it less useful for welfare loss reduction. In the 95% confidence interval from the estimation of Barsky and Sims (2012) the estimated loss reduction is roughly between 80% and 45%, implying that the high degree of importance

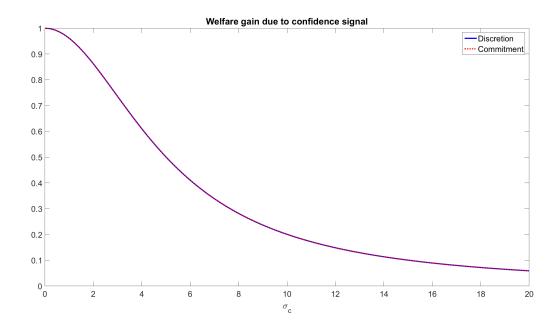


Figure 5: Estimated welfare loss mitigation due to confidence signal processing for different parameter calibrations of confidence innovation standard deviation,  $\sigma_c$ . Estimated average period loss is normalized by dividing by estimated loss without attention to confidence. Loss reduction is the same for optimal discretion (solid, blue) and commitment (dotted, red).

of the confidence signal is fairly robust to parameterization.<sup>3</sup> Even for a value five times as high as the estimated value, confidence signal processing implies a loss reduction of more than 6%. Figure 5 shows the estimated welfare loss mitigation for different values of  $\sigma_c$ . Note that the *relative* loss mitigation does not depend on the monetary policy regime while the *absolute* welfare loss reduction does.

## 4 Empirical Analysis

The analysis in the previous section suggests that confidence information is a highly important tool for monetary policy makers to achieve welfare maximization. However, the model is very stylized and lacks a number of elements that provide modern DSGE models with more realistic dynamics. In this section I seek to answer the question how well this result translates to an estimated medium-scale DSGE model. The model is enhanced to include a measure of attention the central bank has allocated to confidence measures. I analyze

<sup>&</sup>lt;sup>3</sup>Using the standard deviation that Barsky and Sims (2012) state for their estimated value of 3.81, one obtains the 95% confidence interval (2.22, 5.50).

whether its attention has shifted over time, and how much welfare was lost due to imperfect confidence signal processing.

## 4.1 Design of Monetary Policy and Informational Frictions

To describe monetary policy behavior I assume that nominal interest rates follow a rule of the type proposed by Taylor (1993):

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) \left( \delta_{\pi} E \left[ \pi_t | I_t^{CB} \right] + \delta_x E \left[ x_t | I_t^{CB} \right] + \delta_{\Delta y} E \left[ \Delta y_t | I_t^{CB} \right] \right) + \nu_t$$
 (32)

where  $\Delta y_t$  denotes output growth. By including interest rate smoothing and both output gap and growth, the monetary policy includes all components from the original Taylor rule and the policy rule proposed by Barsky and Sims (2012). Monetary policy shock,  $\nu_t$ , follows an AR(1) process.

Assuming the same informational frictions as in section 3, the central bank estimates of contemporaneous inflation and output crucially depend on its expectation about (perceived) technology growth rate,  $E\left[\hat{g}_{a,t}|I_t^{CB}\right]$ . To provide an intuitive measure on the degree of attention the central bank assigns to confidence, I assume that this expectation is a linear combination of the expected values that would be attained with and without confidence signal processing. Denoting the corresponding information sets as  $I_t^{CB,C}$  and  $I_t^{CB,0}$ , respectively, central bank growth rate expectations are given by

$$E\left[\hat{g}_{a,t}|I_t^{CB}\right] = \omega_c E\left[\hat{g}_{a,t}|I_t^{CB,C}\right] + (1 - \omega_c) E\left[\hat{g}_{a,t}|I_t^{CB,0}\right]$$
(33)

with  $0 \le \omega_c \le 1$ . Confidence weight  $\omega_c$  measures the importance the central bank assigns to confidence for monetary policy making.<sup>4</sup>

<sup>&</sup>lt;sup>4</sup>The assumption of a constant confidence weight may seem to be contradicting the evidence in section 2. Therefore, I allow for a structural break in the attention to confidence later on to address potential time variance.

## 4.2 Empirical DSGE model with asymmetric information and confidence

Apart from the informational frictions the DSGE model used for the empirical analysis is relatively standard. It is a version of the one used in Smets and Wouters (2007) and, thus, a bit richer than the one of Barsky and Sims (2012) as it contains capital utilization and price indexation, as well as additional preference, investment demand and cost shocks.

#### 4.2.1 Households

As before, households maximize a lifetime utility function of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t) \tag{34}$$

where the period utility function now accounts for external consumption habit<sup>5</sup> and stochastic preference changes:

$$U(C_t, N_t; Z_t) = Z_t \left( \log(C_t - bC_{t-1}) - \frac{N_t^{1+\nu}}{1+\nu} \right)$$
(35)

The log of  $Z_t$  is the preference shock that follows an AR(1) process.

The budget constraint is

$$C_t + I_t + B_t + T_t = W_t N_t + \frac{R_{t-1}}{\Pi_t} B_{t-1} + R_t^k U_t K_{t-1} - \Psi(U_t) K_{t-1} + D_t$$
(36)

with investment  $I_t$ , lump-sum taxation  $T_t$ , gross nominal interest and inflation rate,  $R_t$  and  $\Pi_t$ , capital  $K_t$ , rental rate of capital  $R_t^k$  and capital utilization rate  $U_t$ . Here,  $W_t$  denotes the real wage. The cost function of changing capital utilization,  $\Psi(\cdot)$  is scaled with parameter  $\psi$  according to  $U \cdot \Psi''(U)/\Psi'(U) = \psi$  (where U denotes steady-state capital utilization).

Capital accumulates according to

$$K_t = (1 - \delta)K_{t-1} + \left(1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right)I_tD_t^i \tag{37}$$

<sup>&</sup>lt;sup>5</sup>I experimented with internal habit assumptions but did not observe substantially different results.

with depreciation rate  $\delta$ . The log of  $D_t^i$ , the investment demand shock, follows an AR(1) process. Adjustment cost function  $\Phi(\cdot)$  satisfies  $\Phi'(\cdot) > 0$ ,  $\Phi''(\cdot) > 0$ ,  $\Phi(\gamma) = \Phi'(\gamma) = 0$  and  $\Phi''(\gamma) = \varphi$  where  $\gamma$  is the constant steady-state growth rate.

One obtains the first-order conditions of utility maximization

$$\Lambda_t = \frac{Z_t}{C_t - bC_{t-1}} \tag{I}$$

$$\Lambda_t W_t = Z_t N_t^{\nu} \tag{II}$$

$$\Lambda_t = \beta E_t \left[ \frac{R_t \Lambda_{t+1}}{\Pi_{t+1}} \right] \tag{III}$$

$$\Lambda_{t} = \Lambda_{t}^{k} D_{t}^{i} \left( 1 - \Phi \left( \frac{I_{t}}{I_{t-1}} \right) - \Phi' \left( \frac{I_{t}}{I_{t-1}} \right) \frac{I_{t}}{I_{t-1}} \right) + \beta E_{t} \left[ \Lambda_{t+1}^{k} D_{t+1}^{i} \Phi' \left( \frac{I_{t+1}}{I_{t}} \right) \left( \frac{I_{t+1}}{I_{t}} \right)^{2} \right]$$

$$(IV)$$

$$R_t^k = \Psi'(U_t) \tag{V}$$

$$\Lambda_t^k = \beta E_t \left[ \Lambda_{t+1} \left( R_{t+1}^k U_{t+1} - \Psi(U_{t+1}) \right) + (1 - \delta) \Lambda_{t+1}^k \right]$$
 (VI)

where  $\Lambda_t$  and  $\Lambda_t^k$  are the Lagrange multipliers of budget constraint and capital accumulation equation, respectively. Tobin's Q is defined as  $Q_t \equiv \Lambda_t^k/\Lambda_t$ .

#### 4.2.2 Goods producers and price setting

Intermediate goods production now requires physical capital as input:

$$Y_t(j) = A_t(K_t^s)^{\alpha} N_t^{1-\alpha} \tag{38}$$

 $K_t^s$  denotes the capital provided by households:

$$K_t^s = U_t K_{t-1} \tag{39}$$

First-order conditions of cost minimization are

$$W_t = (1 - \alpha)MC_t A_t (K_t^s)^{\alpha} N_t^{-\alpha}$$

$$\tag{40}$$

$$R_t^k = \alpha M C_t A_t \left( K_t^s \right)^{\alpha - 1} N_t^{1 - \alpha} \tag{41}$$

which implies

$$K_t^s = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t^k} N_t \tag{42}$$

with marginal costs given by

$$MC_t = A_t^{-1} W_t \left( K_t^s \right)^{-\alpha} N_t^{\alpha} \tag{43}$$

Assumptions about price and demand aggregation as well as Calvo pricing remain as in the baseline NKM with the caveat that the latter is subject to partial indexation measured by parameter  $\iota_p$ .

#### 4.2.3 Government expenditures and resource constraint

The log income share of government expenditures follows the AR(1) process

$$\log(G_t/Y_t) = (1 - \rho_q)\log(G/Y) + \rho_q\log(G_{t-1}/Y_{t-1}) + \varepsilon_{q,t}$$
(44)

Government expenditures are financed via lump-sum taxation

$$G_t + \frac{R_{t-1}}{\Pi_t} = T_t + B_t \tag{45}$$

The aggregate resource constraint is given by

$$Y_{t} = C_{t} + I_{t} + G_{t} + \Psi(U_{t}) K_{t-1}$$
(46)

#### 4.2.4 Log-linearized system

After detrending, log-linearizing the model equations around the steady state, and denoting log-linearized variables by lowercase letters, one obtains the system of equations

$$c_{t} = \frac{1}{1+b} E_{t} \left[ c_{t+1} \right] + \frac{b}{1+b} c_{t-1} - \frac{1-b}{1+b} \left( r_{t} - E_{t} \left[ \pi_{t+1} \right] \right) + \frac{(1-\rho_{z})(1-b)}{1+b} z_{t}$$
(47)

$$n_t = \frac{1}{\nu} w_t - \frac{1}{\nu(1-b)} \left( c_t - b c_{t-1} \right) \tag{48}$$

$$i_{t} = \frac{\beta}{1+\beta} E_{t} \left[ i_{t+1} \right] + \frac{1}{1+\beta} i_{t-1} + \frac{1}{\varphi(1+\beta)} \left( q_{t} + d_{t}^{i} \right)$$

$$(49)$$

$$r_t^k = \frac{\psi}{1 - \psi} u_t \tag{50}$$

$$q_t = \beta(1 - \delta)E_t[q_{t+1}] + (1 - \beta(1 - \delta))E_t[r_{t+1}^k] - (r_t - E_t[\pi_{t+1}])$$
(51)

$$y_t = c_y c_t + i_y i_t + g_y (g_t + y_t) + \frac{1 - \beta (1 - \delta)}{\beta \delta} i_y u_t$$
 (52)

$$y_t = \alpha k_t^s + (1 - \alpha)n_t + a_t \tag{53}$$

$$k_t^s = u_t + k_{t-1} (54)$$

$$k_t = (1 - \delta)k_{t-1} + \delta\left(i_t + d_t^i\right) \tag{55}$$

$$k_t^s = n_t + w_t - r_t^k (56)$$

$$mc_t = w_t + \alpha \left( n_t - k_t^s \right) - a_t \tag{57}$$

$$\pi_t = \frac{\beta}{1 + \beta \iota_p} E_t \left[ \pi_{t+1} \right] + \frac{\iota_p}{1 + \beta \iota_p} \pi_{t-1} + \frac{\lambda}{1 + \beta \iota_p} \left( mc_t + \varepsilon_{p,t} \right) \tag{58}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \tag{59}$$

The system is closed with the monetary policy rule (32). The solution under asymmetric information is obtained via fixed-point iteration as described in Appendix A.5, and is given by

$$X_{t} = \Gamma X_{t-1} + \tilde{\Gamma} X_{t-1}^{f} + \Gamma_{r} r_{t-1} + \Upsilon \eta_{t} + \Psi \hat{g}_{a,t} + \Psi^{CB} E \left[ \hat{g}_{a,t} | I_{t}^{CB} \right]$$
(60)

$$X_t^f = \Gamma^f X_{t-1}^f + \Upsilon^f \eta_t + \Psi^f \hat{g}_{a,t} \tag{61}$$

$$y_t^f = \gamma_u^f X_{t-1}^f + \phi_u^f \eta_t + \psi_u^f \hat{g}_{a,t} \tag{62}$$

$$\eta_t = R\eta_{t-1} + Jg_{a,t} + \epsilon_t \tag{63}$$

where  $X_t$  denotes the vector of all system variables of the sticky price model,  $X_t^f \equiv \left(c_t^f, i_t^f, k_t^f\right)'$  contains all flex-price variables with backward-looking components,  $y_t^f$  is the flex-price out-

put needed to compute the output gap in (32), and  $\eta_t$  denotes the vector of shock processes with white noise stochastic shocks  $\epsilon_t$ .

## 4.3 Econometric Methodology

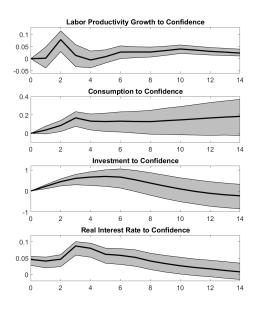
The parameters are estimated by matching the impulse response functions of empirical variables with the ones generated by model simulations. The moment matching approach used here is a Bayesian variant of the methodology in Barsky and Sims (2012). Pinning down parameter values by minimizing the distance between empirical and model-generated moments is frequently used in a variety of theoretical macroeconomic applications, e.g. Christiano, Eichenbaum and Evans (2005), Franke and Westerhoff (2012), and Altig et al. (2011). Following Kim (2002, 2014) and Christiano, Eichenbaum and Trabandt (2015), I approximate a quasi-likelihood function based on the moment distance. After reweighing with a prior density it is possible to explore the posterior parameter density as e.g. in Smets and Wouters (2007).

The data of selected key variables is used to estimate a VAR with four lags in companion form by OLS. The responses of the variables to orthogonalized innovations are computed involving a Cholesky factorization. The variables included in the VAR are:

$$V_{t} = \begin{pmatrix} \Delta(y_{t} - n_{t}) \\ \pi_{t} \\ c_{t} \\ y_{t} \\ i_{t} \\ \tilde{C}_{t} \\ r_{t} \end{pmatrix}$$

$$(64)$$

Labor productivity,  $y_t - n_t$  is measured by log of real output per hour (nonfarm business sector). Inflation,  $\pi_t$ , is measured by the growth rate of the personal consumption expenditures price index. Consumption,  $c_t$ , investment,  $i_t$ , and output  $y_t$  are measured by log of real personal consumption expenditures, private nonresidential fixed investment and GDP, respectively, each divided by civilian labor force level. Consumer confidence,  $\tilde{C}_t$ , is measured by the index E5Y which collects survey responses about expected economic conditions over the next five years. The real interest rate,  $r_t$ , is measured by the three months treasury bill rate net expected change of prices. All data is retrieved from FRED of the St. Louis



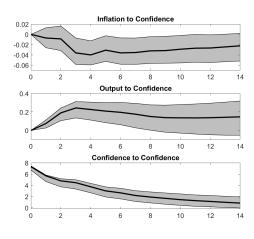


Figure 6: Empirical impulse responses to confidence innovations in an estimated VAR with three lags. The shaded area indicates 95% confidence intervals.

Fed, except the confidence measure and inflation expectations which are provided by the Michigan Surveys of Consumers. The variables are measured in quarterly frequency. The sample encompasses the interval from 1960-Q1 to 2018-Q3 which implies a sample size of T=235.

The ordering of variables is loosely based on Christiano, Eichenbaum and Trabandt (2015). As observed by Barsky and Sims (2012), only the real interest rate reacts contemporaneously to confidence innovations whereas output, consumption and inflation responses are slowly building over time. A similar dynamic behavior can be observed for investment and labor productivity. However, I follow the reasoning of Barsky and Sims (2012) that the VAR specification and structural interpretation is actually of second-order importance, as the capability of replicating empirical impulse responses is in itself a valuable feature for structural models.

The empirical responses of all variables to confidence innovations are shown in figure 6. As in Barsky and Sims (2012), consumption and output do not respond on impact to confidence innovations but increase permanently in the long run. The inflation rate responds temporarily negatively. The real interest rate increases on impact but the effect wears off in the long run. For the additionally included variables, labor productivity growth and investment, I observe temporary increases.

The empirical moments vector,  $\hat{\mathbf{m}}$ , contains the stacked responses of all variables to all orthogonalized innovations from impact to 20 lags. This provides a set of  $n_m = 1008$  moments. Under regularity conditions, it holds for the asymptotic distribution of the empirical moments that

$$\sqrt{T} \left( \hat{\mathbf{m}} - \mathbf{m}(\theta) \right) \stackrel{a}{\sim} N \left( 0, \mathbf{W}(\theta_0) \right) \tag{65}$$

where  $\theta_0$  denotes the true parameter values of the model, or equivalently

$$\hat{\mathbf{m}} \stackrel{a}{\sim} N\left(\mathbf{m}(\theta), \mathbf{V}\right) \tag{66}$$

with  $\mathbf{V} \equiv T^{-1}\mathbf{W}(\theta_0)$ .

Given these assumptions, one obtains the approximate likelihood function

$$f\left(\hat{\mathbf{m}}|\theta,V\right) = (2\pi)^{-N/2} |V|^{-1/2} \exp\left\{-\frac{1}{2} \left(\hat{\mathbf{m}} - \mathbf{m}(\theta)\right)' \mathbf{V} \left(\hat{\mathbf{m}} - \mathbf{m}(\theta)\right)\right\}$$
(67)

Denoting the prior parameter density by  $p(\theta)$ , the Bayesian posterior function can be expressed as

$$\varphi(\theta|\hat{\mathbf{m}}, \mathbf{V}) = \frac{f(\hat{\mathbf{m}}|\theta, V) p(\theta)}{\int f(\hat{\mathbf{m}}|\theta, V) p(\theta) d\theta}$$
(68)

To obtain the estimated mode of the posterior parameter function, the numerator of equation (68) is maximized directly. Afterwards, the posterior distribution is explored by running two simulations of the Metropolis-Hastings algorithm.

In the estimation procedure the matrix V is replaced by its consistent estimator,  $\bar{\mathbf{V}}$ , resulting from the bias-corrected bootstrap of Kilian (1998). Following Christiano, Trabandt and Walentin (2010), I further transform  $\bar{\mathbf{V}}$  to account for its poor small-sample properties. As in Christiano, Eichenbaum and Evans (2005) and Christiano, Eichenbaum and Trabandt (2015), I apply the extreme case of this transformation to obtain the matrix  $\hat{\mathbf{V}}$  which contains only the diagonal elements of  $\bar{\mathbf{V}}$ .

The model-generated moments,  $\mathbf{m}(\theta)$ , are obtained from averaging impulse responses of 2500 Monte Carlo simulations, each with simulation size equal to the empirical sample size T.

Parameter		Value
Discount factor	β	0.99
Capital depreciation rate	$\delta$	0.03
Steady state income share of capital	$\alpha$	0.36
Autocorrelation of government expenditure shocks	$ ho_g$	0.95
Standard deviation of government expenditure shocks	$\sigma_g$	0.25
Steady state income share of consumption	$c_y$	0.57
Steady state income share of government expenditures	$g_y$	0.20
Steady state income share of investment	$i_y$	0.23
Unconditional technology growth rate mean	$g_a^*$	0.33

Table 1: Calibrated parameter values

#### 4.4 Estimation results

Nine parameters are calibrated as stated in table 1. These are the same parameter values that Barsky and Sims (2012) calibrate in their estimation and are, in general, fairly uncontroversial. The remaining 27 parameters are estimated with the procedure described above. Assumptions about prior distributions and estimated posteriors are summarized in table 2. The acceptance ratio of the two Metropolis-Hastings simulations are 32.7% and 29.2%, respectively.

The central bank's attention on confidence,  $\omega_c$ , is estimated at 64.1%, indicating non-optimal information processing in the conduct of monetary policy making as the value is significantly lower than unity. However, confidence weight is significantly positive which suggests that consumer confidence is indeed considered when making policy decisions.

The interest rate smoothing coefficient of the monetary policy rule ( $\rho_r = 0.517$ ) is relatively low in comparison to other estimates, but is well within the confidence bounds of Barsky and Sims (2012). Inflation ( $\delta_{\pi} = 1.793$ ), output gap ( $\delta_{x} = 0.143$ ) and output growth ( $\delta_{\Delta y} = 0.244$ ) coefficients are fairly close to their counterparts in Smets and Wouters (2007) (2.03, 0.08 and 0.22, respectively), which suggests that monetary policy reacts strongly to inflation and that output growth prompts stronger responses than the output gap.

The coefficients that determine consumer confidence dynamics are similar to the original estimations of Barsky and Sims (2012). Confidence responds to the innovation in perceived

Parameter		Prior			Posterior	
		Shape	Mean	Std.	Median	90% HPDI
Monetary policy						
Attention to confidence	$\omega_c$	В	0.50	0.10	0.641	[0.476, 0.777]
Interest rate smoothing	$ ho_r$	N	0.80	0.05	0.517	[0.455,  0.580]
Inflation coefficient	$\delta_{\pi}$	N	1.50	0.125	1.793	[1.630, 1.979]
Output gap coefficient	$\delta_x$	N	0.125	0.05	0.143	[0.104,  0.190]
Output growth coefficient	$\delta_{\Delta y}$	N	0.125	0.05	0.244	[0.182,  0.310]
Persistence of monetary shock	$ ho_ u$	В	0.50	0.20	0.635	[0.522,  0.739]
Std. of monetary shock	$\sigma_{ u}$	IG	0.50	0.20	0.220	[0.185, 0.256]
Confidence						
Persistence of confidence	$ ho_c$	В	0.95	0.10	0.876	[0.847, 0.901]
Perceived technology coefficient	$\zeta_1$	N	1.00	0.25	0.796	[0.401, 1.188]
Perceived growth rate coefficient	$\zeta_2$	N	32.0	0.80	32.36	[30.88, 33.73]
Std. of confidence shock	$\sigma_c$	$\operatorname{IG}$	2.00	0.40	4.741	[3.556, 5.717]
Technology and Noisy Signals						
Std. of technology shock	$\sigma_a$	$\operatorname{IG}$	0.50	0.20	0.483	[0.454,  0.512]
Persistence of growth rate shock	$ ho_{g_a}$	В	0.50	0.10	0.552	[0.491,  0.608]
Std. of news shock	$\sigma_{g_a}$	$\operatorname{IG}$	0.50	0.20	0.172	[0.149,  0.193]
Std. of animal spirits shock	$\sigma_s$	$\operatorname{IG}$	0.50	0.20	0.030	[0.015, 0.059]
Household utility function						
Consumption habit persistence	b	В	0.70	0.10	0.329	[0.265,  0.394]
Inv. Frisch labor supply elasticity	$\nu$	N	0.50	0.05	0.423	[0.342,  0.501]
Persistence of preference shock	$ ho_z$	В	0.50	0.20	0.441	[0.312,  0.558]
Std. of preference shock	$\sigma_z$	$\operatorname{IG}$	0.50	0.20	0.597	[0.521, 0.685]
Capital and Investment						
Investment adjustment cost	$\phi$	N	4.00	1.00	4.277	[3.376, 5.311]
Capacity utilization adjustment cost	$\psi$	В	0.50	0.05	0.893	[0.863, 0.917]
Persistence of investment shock	$ ho_i$	В	0.50	0.20	0.399	[0.247,  0.546]
Std. of investment shock	$\sigma_z$	$\operatorname{IG}$	0.50	0.20	1.887	[1.418, 2.459]
Price setting						
Price stickiness	$\theta$	В	0.60	0.10	0.691	[0.646, 0.731]
Price indexation	$\iota_p$	В	0.50	0.15	0.051	[0.021,  0.094]
Persistence of cost shock	$ ho_p$	В	0.50	0.20	0.892	[0.864,  0.913]
Std. of cost shock	$\sigma_p$	$\operatorname{IG}$	0.50	0.20	1.137	[0.993,  1.300]

Table 2: Prior and (estimated) posterior distributions of the DSGE model parameters

technology ( $\zeta_1 = 0.796$ ) considerably less than to innovations in perceived growth rate ( $\zeta_2 = 32.36$ ). The standard deviation of confidence shocks ( $\sigma_c = 4.741$ ) is larger than in the original estimation (3.78) but the standard errors on  $\sigma_c$  are large in both estimations. The persistence of confidence ( $\rho_c = 0.876$ ) is somewhat lower than the original estimate of 0.94.

The standard deviations of technology level ( $\sigma_a = 0.483$ ), news ( $\sigma_{g_a} = 0.172$ ) and animal spirits shocks ( $\sigma_s = 0.030$ ) are well within the confidence bounds of Barsky and Sims (2012), although the difference between the latter two is even more pronounced. The persistence of news shocks ( $\rho_{g_a} = 0.552$ ) is fairly low in comparison to the previous result.

Inverse Frisch elasticity of labor supply ( $\nu=0.423$ ), elasticity of investment adjustment cost ( $\phi=4.277$ ) and Calvo price stickiness ( $\theta=0.691$ ) are all well within the region of typical estimates. The coefficient of consumption habit persistence (b=0.329) is relatively low in comparison – e.g. Smets and Wouters (2007) estimate the parameter at 0.71 – but very close to the estimate of Barsky and Sims (2012), 0.31. The elasticity of capital utilization adjustment costs ( $\psi=0.893$ ) is notably higher than in other estimations, e.g. Smets and Wouters (2007) and Angeletos et al. (2018). However, many authors find a posterior that barely differs from the prior and has large confidence intervals. The degree of price indexation ( $\iota_p=0.051$ ) is relatively low, the high persistence in inflation dynamics that is typically observed is instead provided by the autocorrelation coefficient of the cost shock process ( $\rho_p=0.892$ ).

Figure 7 shows the theoretical impulse responses of key variables to news, animal spirits and confidence shocks. Output reacts with a slowly building permanent increase to news shocks and increases temporarily after animal spirits shocks. The confidence shock induces a misjudgment by the central bank which has temporary negative effects on output. Inflation responds to animal spirits and confidence shocks very similar to output. In case of news shocks, inflation increases on impact, and proceeds on an non-monotonous dynamic path where inflation growth changes its sign twice. This pattern reveals an interesting interaction of the counteracting effects of increasing demand, decreasing marginal costs, and temporary forecast errors of the monetary policy maker. The real interest rate increases temporarily in response to all three shocks, with a non-monotonous path in response to news shocks similar to the one observed for inflation.

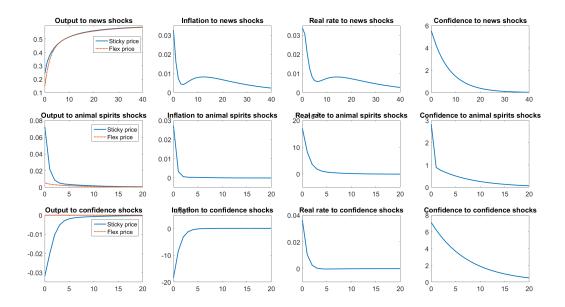


Figure 7: Theoretical impulse responses of output, inflation rate, real interest rate and consumer confidence to news, animal spirits and confidence shocks in the medium-scale DSGE model at the estimated parameter values

## 4.5 Welfare analysis

In last section's empirical analysis, the parameter that governs central bank's attention to confidence,  $\omega_c$ , is estimated at 0.641 which suggests positive yet not optimal information processing of confidence signals. In this section, I provide some insights into the welfare implications of this level of attention to confidence in the estimated DSGE model.

Analogous to section 3.3, I begin to compare the theoretical impulse responses to technology level, news, animal spirits and confidence shocks to output gap and inflation rate for three different confidence weights: no attention ( $\omega_c = 0$ ), optimal confidence signal processing ( $\omega_c = 1$ ), and estimated confidence weight. All other parameters are set to their estimated values.

In figure 8, the impulse responses to technology level and news shocks are illustrated. The output and inflation responses to level shocks are virtually indistinguishable, implying that it is almost irrelevant to consider confidence measures when attempting to mitigate adverse welfare effects from this type of shock. On the contrary, monetary policy is markedly more successful in dampening business cycle fluctuations resulting from news shocks the higher central bank's attention to confidence is.

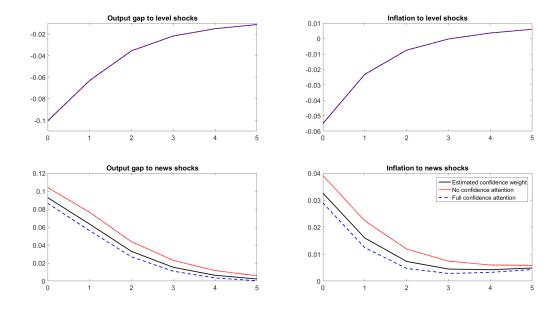


Figure 8: Theoretical impulse responses of output gap and inflation rate to technology level and news shocks in the medium-scale DSGE model for estimated central bank attention to confidence  $\omega_c = 0.641$  (black solid line), no confidence attention  $\omega_c = 0$  (red dotted line), and full confidence attention  $\omega_c = 1$  (blue dashed line).

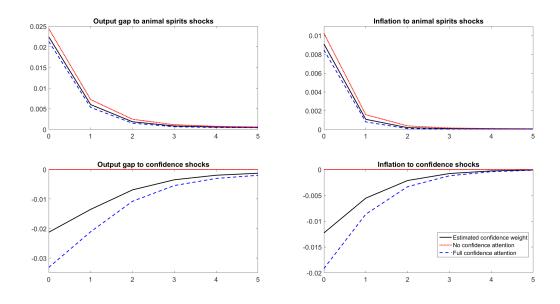


Figure 9: Theoretical impulse responses of output gap and inflation rate to animal spirits and confidence shocks in the medium-scale DSGE model for estimated central bank attention to confidence  $\omega_c = 0.641$  (black solid line), no confidence attention  $\omega_c = 0$  (red dotted line), and full confidence attention  $\omega_c = 1$  (blue dashed line).

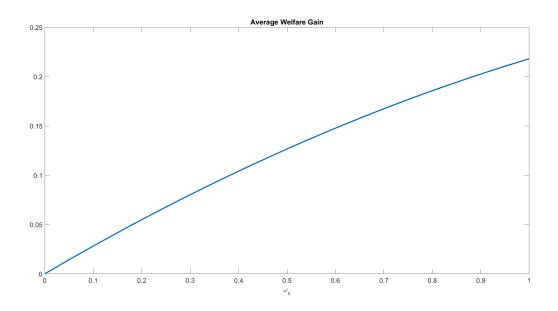


Figure 10: Monte Carlo estimated average welfare gain of paying attention to confidence in simulations of the DSGE model with technology level, news, animal spirits and confidence shock for varying confidence weights.

A similar observation is made for animal spirits shocks, as displayed in figure 9. The dynamic paths of output gap and inflation rate, differ however less strongly than in case of news shocks. Another marked effect of attention to confidence, albeit an adverse one, is observed for responses to confidence shocks. If the central bank does not process confidence signals, confidence shocks would not instigate dynamic responses of any variables less confidence. The higher the confidence weight, the more strongly decline both output gap and inflation rate to positive confidence shocks.

The visual assessment of the impulse responses can, thus, once again (c.f. figures 3 and 4) not unambiguously clarify whether or not paying attention to confidence measures implies positive welfare effects. However, since information processing with the Kalman filter is optimal, central bank's forecast accuracy must improve with higher  $\omega_c$ , such that negative welfare effects of attention to confidence would be immensely surprising.

For this purpose I conduct 10,000 Monte Carlo simulations of 1,000 periods and estimate the average of welfare loss, which is defined as the average percentage deviation of the period utility function from its flex-price counterpart. I repeat this exercise for all values of  $\omega_c$  between zero and unity. By transforming the welfare loss distribution into its negative percentage deviation from the 'no attention to confidence' ( $\omega_c = 0$ ) benchmark one obtains the welfare gain function shown in figure 10.

Sample	Estimated $\omega_c$	90% HPDI	Welfare Gain	Share of max. Gain
1960-Q1 – 2018-Q3	0.641	[0.476, 0.777]	15.58%	71.48%
1960-Q1 – 1989-Q4	0.577	[0.417,  0.727]	14.29%	65.55%
1990-Q1 - 2018-Q3	0.669	[0.524,0.791]	16.14%	74.03%

Table 3: Comparison of welfare gain associated with attention of confidence set to (1) its full sample estimate, (2) its reestimated value for subsample 1960-Q1 – 1989-Q4, and (3) its reestimated value for subsample 1990-Q1 – 2018-Q3.

The maximum welfare gain, associated with optimal confidence information processing ( $\omega_c = 1$ ), is 21.80%, which is slightly more than one third of the welfare gain obtained in the stylized New Keynesian model under optimal monetary policy. At the estimated value of  $\omega_c$ , one obtains a welfare gain of 15.58% relative to the  $\omega_c = 0$  case. This is 71.48% of the maximum welfare gain, suggesting that the central bank has, overall, done a good job in extracting relevant information from confidence measures.

#### 4.6 Variation in attention to confidence

The assessment of FOMC minutes in section 2 suggests a potential breakpoint in attention to confidence. To investigate this suspicion I reestimate confidence weight  $\omega_c$  with the same methodology as described in section 4 but divide the sample into the two subsamples 1960-Q1 – 1989-Q4 and 1990-Q1 – 2018-Q3. All other parameter values are set to their full sample estimates.

The results are summarized in table 3. In the first subsample, the estimated confidence weight,  $\omega_c = 0.577$ , is moderately lower, which is associated with a lower welfare gain (14.29%) that accounts for 65.55% of maximum welfare gain. In the reestimation procedure for the second subsample, one obtains a slightly higher degree of attention to confidence,  $\omega_c = 0.669$ . The welfare gain for this parameterization (16.14%) attains 74.03% of the welfare gain implied by perfect confidence signal processing.

The difference of the estimated values is very modest and not statistically significant. In summary, there is no convincing evidence that the FOMC has increased its consideration of confidence measures over time, although the results suggest a small improvement.

### 5 Conclusion

Are consumer confidence measures informative about the state of the economy? Given its presence in the popular press, the expenses that public institutions are willing to make for collecting and processing them, and the emergence of a strand of literature that assigns a role to confidence shocks in theoretical macroeconomic models, journalists, scientists and policymakers are likely to answer this question mostly unambiguously in the affirmative.

From this perspective, it appears puzzling that theoretical models designed for the analysis of monetary policy cannot provide a role for confidence measures as an informative signal for central banks. This paper seeks to fill this gap by providing an intuitive framework with informational frictions, in which monetary policy makers can improve their ability to achieve their targets when paying attention to confidence. The positive welfare effect is large in both a stylized New Keynesian model with optimal monetary policy and a more realistic estimated DSGE model.

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## A Appendix

# A.1 Optimal Discretion with Asymmetric Information - Proof of Proposition 2

*Proof.* Denoting  $v_t \equiv (\chi_t, \hat{\chi}_t, \xi_t)'$ , the equations (15), (16) and (17) can be rewritten as

$$\begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ -G & 0 & I \end{pmatrix} v_{t+1} = \begin{pmatrix} A & 0 & 0 \\ 0 & A - G & K \\ 0 & 0 & 0 \end{pmatrix} v_t + \left(\varepsilon_{g_a, t+1}, 0, 0, 0, \varepsilon_{a, t+1}, \varepsilon_{s, t+1}\right)'$$
(69)

$$\Leftrightarrow \qquad v_{t+1} = \tilde{A}_{11}v_t + \varepsilon_{v,t+1} \tag{70}$$

Together with equations (10) and (9), the system of equations is given by<sup>6</sup>

$$\begin{pmatrix} v_{t+1} \\ \tilde{E}E_t \left[\omega_{t+1}\right] \end{pmatrix} = \underbrace{\begin{pmatrix} \tilde{A}_{11} & 0 \\ \tilde{A}_{21} & \tilde{A}_{22} \end{pmatrix}}_{\equiv \tilde{A}} \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ \tilde{B}_2 \end{pmatrix}}_{\equiv \tilde{B}} r_t + \begin{pmatrix} \varepsilon_{v,t+1} \\ 0 \end{pmatrix}$$
(71)

and the compact form of period loss function (22)

$$L_t = \omega_t' W \omega_t \tag{72}$$

where  $\omega_t \equiv (\pi_t, x_t)'$ . Using the more compact notation for central bank expectations  $v_{t|t} = E\left[v_t|I_t^{CB}\right]$ , the solution of optimal discretion has the general form

$$r_t = F^* v_{t|t} \tag{73}$$

$$\omega_{t|t} = G^* v_{t|t} \tag{74}$$

which is verified with the method of undetermined coefficients in the following.

From (73) and the upper block of (71), one obtains

$$\omega_{t+1|t} = G^* v_{t+1|t} = G^* \tilde{A}_{11} v_{t|t} \tag{75}$$

<sup>&</sup>lt;sup>6</sup>Note that w.l.g. the constant  $\beta^{-1}$  in the natural real rate of interest equation can be neglected.

From the lower block of (71) one gets

$$\tilde{E}\omega_{t+1|t} = \tilde{A}_{21}v_{t|t} + \tilde{A}_{22}\omega_{t|t} + \tilde{B}_{2}r_{t} \tag{76}$$

Premultiplying the former with  $\tilde{E}$ , setting equal to the latter and solving for  $w_{t|t}$  provides

$$\omega_{t|t} = \underbrace{\tilde{A}_{22}^{-1} \left( \tilde{E} G^* \tilde{A}_{11} - \tilde{A}_{21} \right)}_{=A^*} v_{t|t} - \underbrace{\tilde{A}_{22}^{-1} \tilde{B}_{2}}_{\equiv B^*} r_{t}$$
(77)

With this result the expected period loss function can be rewritten as

$$L_{t|t} = \omega'_{t|t}W\omega_{t|t} = v'_{t|t}\underbrace{A^{*'}WA^{*}}_{\equiv Q^{*}}v_{t|t} + 2v'_{t|t}\underbrace{A^{*'}WB^{*}}_{\equiv U^{*}}r_{t} + r'_{t}\underbrace{B^{*'}WB^{*}}_{\equiv R}r_{t}$$

$$(78)$$

Under the assumption of (73), the optimal value to the problem has the quadratic form  $v'_{t|t}Vv_{t|t}$  such that the welfare minimization problem can be expressed by the following Bellman equation:

$$v'_{t|t}Vv_{t|t} = \min_{r_t} \left\{ L_{t|t} + \beta E \left( v_{t+1|t+1}Vv_{t+1|t+1} | I_t^{CB} \right) \right\}$$
 (79)

with first-order condition

$$r_t = \underbrace{-R^{*-1}U^{*'}}_{-E^*} v_{t|t} \tag{80}$$

and from (77)

$$\omega_{t|t} = \underbrace{A^* + B^* F^*}_{\equiv G^*} v_{t|t} \tag{81}$$

Therefore, it is verified that the optimal discretion solution has the form described in (73) and (74).

Noting that

$$F^* = -(B^{*-1}WB^*)^{-1}B^{*}WA^*$$

and using

$$\tilde{A}_{22} \equiv \begin{pmatrix} 1 & -\kappa \\ 0 & 1 \end{pmatrix}, \quad \tilde{B}_2 \equiv \begin{pmatrix} 0 \\ \frac{1}{\sigma} \end{pmatrix}, \quad W \equiv \begin{pmatrix} w_{\pi} & 0 \\ 0 & w_x \end{pmatrix}$$

implies

$$G^* = \left(\kappa^2 w_\pi + w_x\right)^{-1} \begin{pmatrix} w_x & -\kappa w_x \\ -\kappa w_\pi & \kappa^2 w_\pi \end{pmatrix} A^*$$

We have

$$\tilde{A}_{21} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\psi_{ya} & 0 & 0 & 0 \end{pmatrix}, \quad \tilde{E} \equiv \begin{pmatrix} \beta & 0 \\ \frac{1}{\sigma} & 1 \end{pmatrix}$$

such that one may note that

$$\left( \kappa^2 w_{\pi} + w_x \right)^{-1} \begin{pmatrix} w_x & -\kappa w_x \\ -\kappa w_{\pi} & \kappa^2 w_{\pi} \end{pmatrix} \tilde{A}_{22}^{-1} \tilde{A}_{21} = \mathbf{0}_{2 \times 6}$$

It follows that

$$G^* = \left(\kappa^2 w_{\pi} + w_{x}\right)^{-1} \begin{pmatrix} w_{x} & -\kappa w_{x} \\ -\kappa w_{\pi} & \kappa^2 w_{\pi} \end{pmatrix} \tilde{A}_{22}^{-1} \tilde{E} G^* \tilde{A}_{11}$$

$$= \begin{pmatrix} \frac{w_{x}}{(\kappa^2 w_{\pi} + w_{x})^{-1}} \\ \frac{-\kappa w_{\pi}}{(\kappa^2 w_{\pi} + w_{x})^{-1}} \end{pmatrix} \left(\rho_{g_{a}} (g_{11}^* + g_{16}^*) + g_{12}^* + g_{15}^* & 0 & \rho_{g_{a}} g_{13}^* & -g_{13}^* & k_{11} g_{13}^* + k_{21} g_{14}^* & k_{12} g_{13}^* + k_{22} g_{14}^* \right)$$

where  $g_{ij}^*$  is the jth element of row i of matrix  $G^*$ , respectively. The unique solution for this equation is

$$G^* = \mathbf{0}_{2 \times 6}$$

Using this result gives

$$A^* = \tilde{A}_{22}^{-1} \tilde{A}_{21}$$

which implies

$$F^* = \begin{pmatrix} 0, & 0, & \sigma \psi_{ya}, & 0, & 0, & 0 \end{pmatrix}$$

Plugging this result into (80) confirms that the nominal interest rate tracks the natural real rate of interest under optimal discretion.

# A.2 Optimal Commitment with Asymmetric Information - Proof of Proposition 3

*Proof.* Following (Svensson and Woodford; 2004, Ch. 3), the Lagrangian of the optimal commitment problem can be expressed in compact form as

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ 0.5\omega_t' W \omega_t + \tilde{\Xi}_t' \tilde{A}_{11} v_t + \Xi_t' \left( \tilde{A}_{21} v_t + \tilde{A}_{22} \omega_t + \tilde{B}_2 r_t \right) - \beta^{-1} \left( \tilde{\Xi}_{t-1}' v_t + \Xi_{t-1}' \tilde{E} \omega_t \right) \right] \right\}$$

$$(82)$$

where  $\Xi'_t \equiv (\varphi'_{1,t}, \varphi'_{2,t})$  is the vector of Lagrangian multipliers. The first-order condition w.r.t.  $r_t$  is given by

$$B_2'\Xi_{t|t} = 0 \tag{83}$$

which implies

$$\varphi_{2,t|t} = 0 \tag{84}$$

The first-order condition w.r.t.  $\omega_t$  is

$$W\omega_t = -\tilde{A}'_{22}\Xi_t + \beta^{-1}\tilde{E}'\Xi_{t-1} \tag{85}$$

Note that the general setup is, thus, equivalent to the one in Aoki (2006) who contributes most of the following solution procedure.

The two rows of CB expectations of equation (85) are given by

$$\pi_{t|t} = \frac{1}{w_{\pi}} \varphi_{2,t|t} - \frac{1}{\beta \sigma w_{\pi}} \varphi_{1,t-1} - \frac{1}{w_{\pi}} \varphi_{2,t-1}$$
(86)

$$x_{t|t} = -\frac{\kappa}{w_r} \varphi_{2,t|t} - \frac{1}{\beta w_r} \varphi_{1,t-1} \tag{87}$$

Equation (86) implies

$$\pi_{t+1|t} = \frac{1}{w_{\pi}} \varphi_{2,t+1|t} - \frac{1}{w_{\pi}} \varphi_{2,t|t}$$

From (9) one has

$$\pi_{t+1|t} = \frac{1}{\beta} \pi_{t|t} - \frac{\kappa}{\beta} x_{t|t}$$

Plugging in (86) and (87), and setting the two expressions equal yields

$$\varphi_{2,t+1|t} - \left(1 + \beta^{-1} + \frac{\kappa^2 w_{\pi}}{\beta w_x}\right) \varphi_{2,t|t} + \left(\frac{1}{\beta^2 \sigma} - \frac{\kappa w_{\pi}}{\beta^2 w_x}\right) \varphi_{1,t-1} + \beta^{-1} \varphi_{2,t-1} = 0$$
 (88)

Iterating forward one period gives

$$\varphi_{2,t+2|t} - \left(1 + \beta^{-1} + \frac{\kappa^2 w_{\pi}}{\beta w_x}\right) \varphi_{2,t+1|t} + \beta^{-1} \varphi_{2,t|t} = 0$$
(89)

The characteristic equation associated with (89) is

$$f(\lambda) = \lambda^2 - \left(1 + \beta^{-1} + \frac{\kappa^2 w_\pi}{\beta w_x}\right) \lambda + \beta^{-1}$$
(90)

Since it holds for the discriminant that

$$\mathcal{D} = \left(1 + \beta^{-1} + \frac{\kappa^2 w_{\pi}}{\beta w_x}\right)^2 - 4\beta^{-1} > 0$$

and  $f(1) = -\beta^{-1} + \frac{\kappa^2 w_{\pi}}{\beta w_x} < 0$ ,  $f(0) = \beta^{-1} > 0$ , the roots of the characteristic equation,  $\lambda_1$  and  $\lambda_2$ , are real and satisfy

$$0 < \lambda_1 < 1 < \lambda_2$$

Thus, one obtains the unique stable solution

$$\varphi_{2,t+1|t} = \lambda_1 \varphi_{2,t|t} \tag{91}$$

Plugging this solution into (88) gives after a number of rearrangements

$$\varphi_{2,t|t} = \lambda_1 \left( \left( \frac{1}{\beta \sigma} + \frac{\kappa w_{\pi}}{\beta w_x} \right) \varphi_{1,t-1} + \varphi_{2,t-1} \right)$$
(92)

such that one can write

$$\Xi_{t|t} = \lambda_1 \begin{pmatrix} 0 & 0 \\ \frac{1}{\beta\sigma} + \frac{\kappa w_{\pi}}{\beta w_{x}} & 1 \end{pmatrix} \Xi_{t-1} \equiv \tilde{\Sigma} \Xi_{t-1}$$
(93)

Plugging the solution of  $\Xi_{t|t}$  into (85) after taking expectations gives

$$\omega_t = W^{-1} \left( -\tilde{A}'_{22} \tilde{\Sigma} + \beta^{-1} \tilde{E}' \right) \Xi_{t-1} \equiv \tilde{\Gamma} \Xi_{t-1}$$
(94)

Taking expectations of (10) and solving for  $r_t$  gives

$$r_{t} = E\left[r_{t}^{n}|I_{t}^{CB}\right] + \sigma\left(\left(\frac{1}{\sigma}, 1\right)\omega_{t+1|t} - \left(0, 1\right)\omega_{t|t}\right)$$

$$= E\left[r_{t}^{n}|I_{t}^{CB}\right] + \sigma\left(\left(\frac{1}{\sigma}, 1\right)\tilde{\Gamma}\Xi_{t|t} - \left(0, 1\right)\tilde{\Gamma}\Xi_{t-1}\right)$$

$$= E\left[r_{t}^{n}|I_{t}^{CB}\right] + \sigma\left(\left(\frac{1}{\sigma}, 1\right)\tilde{\Gamma}\tilde{\Sigma} - \left(0, 1\right)\tilde{\Gamma}\right)\Xi_{t-1}$$

$$\equiv E\left[r_{t}^{n}|I_{t}^{CB}\right] + \Phi\Xi_{t-1}$$
(95)

which completes the proof of proposition 3.3.

#### A.3 Solution of the NKM under Discretion

Using the notation of Appendix A.1, one has under optimal discretion

$$\omega_{t} = \tilde{A}_{22}^{-1} \tilde{E} E_{t} \left[ \omega_{t+1} \right] + \tilde{A}_{22}^{-1} \tilde{B}_{2} \sigma \psi_{ya} \left( \hat{g}_{a,t} - E \left[ \hat{g}_{a,t} | I_{t}^{CB} \right] \right)$$

$$= \sum_{i=0}^{\infty} \left( \tilde{A}_{22}^{-1} \tilde{E} \right)^{i} \tilde{A}_{22}^{-1} \tilde{B}_{2} \sigma \psi_{ya} E_{t} \left[ \hat{g}_{a,t+i} - E \left[ \hat{g}_{a,t+i} | I_{t+i}^{CB} \right] \right]$$
(96)

As private agents are aware that the central bank will have all current information in every future period, it holds that

$$E_t \left[ E \left[ \hat{g}_{a,t+i} | I_{t+i}^{CB} \right] \right] = E_t \left[ \hat{g}_{a,t+i} \right] \quad \forall \ i > 0$$

$$(97)$$

such that

$$\omega_{t} = \tilde{A}_{22}^{-1} \tilde{B}_{2} \sigma \psi_{ya} \left( \hat{g}_{a,t} - E \left[ \hat{g}_{a,t} | I_{t}^{CB} \right] \right)$$

$$= {\kappa \choose 1} \psi_{ya} \left( \hat{g}_{a,t} - E \left[ \hat{g}_{a,t} | I_{t}^{CB} \right] \right)$$

$$(98)$$

### A.4 Solution of the NKM under Commitment

Together with (85), the system under optimal commitment is given by

$$\begin{pmatrix} \tilde{E} & \mathbf{0} \\ \mathbf{0} & \tilde{A}'_{22} \end{pmatrix} \begin{pmatrix} E_t \omega_{t+1} \\ \Xi_t \end{pmatrix} = \begin{pmatrix} \tilde{A}_{22} & \tilde{B}_2 \Phi \\ -W & \beta^{-1} \tilde{E}' \end{pmatrix} \begin{pmatrix} \omega_t \\ \Xi_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{B}_2 \sigma \psi_{ya} \\ \mathbf{0} \end{pmatrix} \begin{pmatrix} E \left[ \hat{g}_{a,t} | I_t^{CB} \right] - \hat{g}_{a,t} \end{pmatrix}$$

$$\tag{99}$$

which implies

$$\begin{pmatrix}
E_{t}\omega_{t+1} \\
\Xi_{t}
\end{pmatrix} = \underbrace{\begin{pmatrix}
\tilde{E}^{-1}\tilde{A}_{22} & \tilde{E}^{-1}\tilde{B}_{2}\Phi \\
-\left(\tilde{A}'_{22}\right)^{-1}W & \beta^{-1}\left(\tilde{E}\tilde{A}_{22}^{-1}\right)'
\end{pmatrix}}_{\equiv \hat{A}} \begin{pmatrix}
\omega_{t} \\
\Xi_{t-1}
\end{pmatrix}$$

$$+ \begin{pmatrix}
\tilde{E}^{-1}\tilde{B}_{2}\sigma\psi_{ya} \\
\mathbf{0}
\end{pmatrix} \left(E\left[\hat{g}_{a,t}|I_{t}^{CB}\right] - \hat{g}_{a,t}\right) \tag{100}$$

where it is assumed that saddlepath stability holds, i.e. that exactly two (of the four) eigenvalues of  $\hat{A}$  are larger than unity in modulus, respectively. Defining the matrix with the eigenvalues on its diagonal as

$$\Lambda \equiv \begin{pmatrix} \Lambda_1 & \mathbf{0} \\ \mathbf{0} & \Lambda_2 \end{pmatrix} \tag{101}$$

where  $\Lambda_1$  contains all eigenvalues that are larger than unity in modulus, and the corresponding matrix of eigenvectors as

$$H \equiv \begin{pmatrix} H_1 & H_2 \end{pmatrix} \tag{102}$$

one can rewrite system (100) with the Jordan decomposition method as

$$\begin{pmatrix} E_t \omega_{t+1} \\ \Xi_t \end{pmatrix} = H \Lambda H^{-1} \begin{pmatrix} \omega_t \\ \Xi_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{E}^{-1} \tilde{B}_2 \sigma \psi_{ya} \\ \mathbf{0} \end{pmatrix} \left( E \left[ \hat{g}_{a,t} | I_t^{CB} \right] - \hat{g}_{a,t} \right)$$
(103)

After defining auxiliary variables

$$\begin{pmatrix} E_t \tilde{\omega}_{t+1} \\ \tilde{\Xi}_t \end{pmatrix} \equiv H^{-1} \begin{pmatrix} E_t \omega_{t+1} \\ \Xi_t \end{pmatrix} \tag{104}$$

and

$$\hat{B} = \begin{pmatrix} \hat{B}_1 \\ \hat{B}_2 \end{pmatrix} \equiv H^{-1} \begin{pmatrix} \tilde{E}^{-1} \tilde{B}_2 \sigma \psi_{ya} \\ \mathbf{0} \end{pmatrix} \tag{105}$$

the system obtained after rearrangements

$$\begin{pmatrix} E_t \tilde{\omega}_{t+1} \\ \tilde{\Xi}_t \end{pmatrix} = \Lambda \begin{pmatrix} \tilde{\omega}_t \\ \tilde{\Xi}_{t-1} \end{pmatrix} + \hat{B} \left( E \left[ \hat{g}_{a,t} | I_t^{CB} \right] - \hat{g}_{a,t} \right)$$
(106)

can be solved blockwise: the upper subsystem via forward iteration and the lower one via backward iteration. Using (97), one obtains as the solution of the former

$$\tilde{\omega}_{t} = \Lambda_{1}^{-1} E_{t} \tilde{\omega}_{t+1} - \Lambda_{1}^{-1} \hat{B}_{1} \left( E \left[ \hat{g}_{a,t} | I_{t}^{CB} \right] - \hat{g}_{a,t} \right)$$

$$= -\sum_{i=0}^{\infty} \Lambda_{1}^{-i-1} \hat{B}_{1} E_{t} \left[ \left( E \left[ \hat{g}_{a,t+i} | I_{t+i}^{CB} \right] - \hat{g}_{a,t+i} \right) \right]$$

$$= -\Lambda_{1}^{-1} \hat{B}_{1} \left( E \left[ \hat{g}_{a,t} | I_{t}^{CB} \right] - \hat{g}_{a,t} \right)$$
(107)

The solution of the latter is given by

$$\tilde{\Xi}_{t} = \Lambda_{2} + \hat{B}_{2} \left( E \left[ \hat{g}_{a,t} | I_{t}^{CB} \right] - \hat{g}_{a,t} \right)$$

$$= \sum_{i=0}^{\infty} \Lambda_{2}^{i} \hat{B}_{2} \left( E \left[ \hat{g}_{a,t-i} | I_{t-i}^{CB} \right] - \hat{g}_{a,t-i} \right)$$
(108)

Using (104), the solution for  $\omega_t$  is

$$\omega_t = \sum_{i=0}^{\infty} \tilde{\Phi}_i \left( E \left[ \hat{g}_{a,t-i} | I_{t-i}^{CB} \right] - \hat{g}_{a,t-i} \right)$$

$$\tag{109}$$

where

$$\tilde{\Phi}_0 \equiv -\Lambda_1^{-1} \hat{B}_1 \tag{110}$$

and

$$\tilde{\Phi}_i \equiv \Lambda_2^i \hat{B}_2 \quad \forall \ i > 0 \tag{111}$$

#### A.5 Solution of the DSGE model via Fixed-Point Iteration

The sticky-price model described in section 4 has the form

$$AE_t X_{t+1} + BX_t + CX_{t-1} + D\eta_t + Fr_t = 0 (112)$$

where  $X_t$  is the vector collecting all endogenous variables and  $\eta_t$  denotes the vector containing all exogenous shock processes for which it holds

$$\eta_t = R\eta_{t-1} + Jg_{a,t} + \varepsilon_t \tag{113}$$

The monetary policy rule has the general form

$$r_{t} = \rho_{r} r_{t-1} + PE \left[ X_{t} | I_{t}^{CB} \right] + P_{l} X_{t-1} + P^{f} E \left[ y_{t}^{f} | I_{t}^{CB} \right] + Q \eta_{t}$$
(114)

where  $y_t^f$  denotes output that would be obtained in a hypothetical flex-price economy.

The solution for all variables of the flex-price economy with backward-looking elements,  $X_t^f$ , is given by  $^7$ 

$$X_t^f = \Gamma^f X_{t-1}^f + \Upsilon^f \eta_t + \Psi^f \hat{g}_{a,t} \tag{115}$$

The flex-price output (and all other variables without backward-looking elements) can be expressed as

$$y_t^f = \gamma_y^f X_{t-1}^f + \phi_y^f \eta_t + \psi_y^f \hat{g}_{a,t}$$
 (116)

 $<sup>^{7}</sup>$ The solution is fairly simply obtained with the same fixed-point iteration methodology described here and is, thus, left out.

It will be shown in the following via guess-and-verify that the solution has the form

$$X_{t} = \Gamma X_{t-1} + \tilde{\Gamma} X_{t-1}^{f} + \Gamma_{r} r_{t-1} + \Upsilon \eta_{t} + \Psi \hat{g}_{a,t} + \Psi^{CB} E \left[ \hat{g}_{a,t} | I_{t}^{CB} \right]$$
(117)

which implies for central bank expectations

$$E\left[X_{t}|I_{t}^{CB}\right] = X_{t} + \Psi\left(E\left[\hat{g}_{a,t}|I_{t}^{CB}\right] - \hat{g}_{a,t}\right)$$

$$\tag{118}$$

Plugging (116) and (118) into the monetary policy rule (114) yields

$$r_{t} = PX_{t} + P_{t}X_{t-1} + P^{f}\gamma_{y}^{f}X_{t-1}^{f} + \rho_{r}r_{t-1} + \left(P^{f}\phi_{y}^{f} + Q\right)\eta_{t} - P\Psi\hat{g}_{a,t} + \left(P\Psi + P^{f}\psi_{y}^{f}\right)E\left[\hat{g}_{a,t}|I_{t}^{CB}\right]$$
(119)

Using (113), (117) and  $E_t[\hat{g}_{a,t+1}] = E_t[E[\hat{g}_{a,t+1}|I_{t+1}^{CB}]] = \rho_{g_a}\hat{g}_{a,t}$  one obtains for the real sector expectations

$$E_t [X_{t+1}] = \Gamma X_t + \tilde{\Gamma} X_t^f + \Gamma_r r_t + \Upsilon R \eta_t + (\Upsilon J + (\Psi + \Psi^{CB}) \rho_{g_a}) \hat{g}_{a,t}$$
(120)

Finally, plugging (119) and (120) into (112) yields

$$-(A\Gamma + (A\Gamma_r + F)P + B)X_t = ((A\Gamma_r + F)P_l + C)X_{t-1}$$

$$+ (A\Gamma_r + F)P^f\gamma_y^f + A\tilde{\Gamma}\Gamma^f)X_{t-1}^f$$

$$+ (A\Gamma_r + F)\rho_r r_{t-1}$$

$$+ (A\Gamma_r + F)(P^f\phi_y^f + Q) + A\tilde{\Gamma}\Upsilon^f + A\Upsilon R + D)\eta_t$$

$$+ (-(A\Gamma_r + F)P\Psi + A\tilde{\Gamma}\Psi^f + A(\Upsilon J + (\Psi + \Psi^{CB})\rho_{ga}))\hat{g}_{a,t}$$

$$+ (A\Gamma_r + F)(P\Psi + P^f\psi_y^f)E[\hat{g}_{a,t}|I_t^{CB}]$$
(121)

which is indeed of the same form as guess (117) with coefficient matrices

$$\Gamma = -\left(A\Gamma + (A\Gamma_r + F)P + B\right)^{-1} \left((A\Gamma_r + F)P_l + C\right)$$

$$\tilde{\Gamma} = -\left(A\Gamma + (A\Gamma_r + F)P + B\right)^{-1} \left((A\Gamma_r + F)P^f \gamma_y^f + A\tilde{\Gamma}\Gamma^f\right)$$

$$\Gamma_r = -\left(A\Gamma + (A\Gamma_r + F)P + B\right)^{-1} \left(A\Gamma_r + F\right)\rho_r$$

$$\Upsilon = -\left(A\Gamma + (A\Gamma_r + F)P + B\right)^{-1} \left((A\Gamma_r + F)\left(P^f \phi_y^f + Q\right) + A\tilde{\Gamma}\Upsilon^f + A\Upsilon R + D\right)$$

$$\Psi = -\left(A\Gamma + (A\Gamma_r + F)P + B\right)^{-1} \left(-(A\Gamma_r + F)P\Psi + A\tilde{\Gamma}\Psi^f + A\left(\Upsilon J + \left(\Psi + \Psi^{CB}\right)\rho_{g_a}\right)\right)$$

$$\Psi^{CB} = -\left(A\Gamma + (A\Gamma_r + F)P + B\right)^{-1} \left(A\Gamma_r + F\right)\left(P\Psi + P^f \psi_y^f\right)$$

The above coefficient equations are solved numerically.