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Optimal Unemployment Benefits in the Pandemic

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DISCUSSION PAPER SERIES

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JUNE 2020

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ABSTRACT

Optimal Unemployment Benefits in the Pandemic

How should unemployment benefits vary in response to the economic crisis induced by the COVID-19 pandemic? We answer this question by computing the optimal unemployment insurance response to the COVID-induced recession. We compare the optimal policy to the provisions under the CARES Act—which substantially expanded unemployment insurance and sparked an ongoing debate over further increases—and several alternative scenarios. We find that it is optimal first to raise unemployment benefits but then to begin lowering them as the economy starts to reopen — despite unemployment remaining high. We also find that the \$600 UI supplement payment implemented under CARES was close to the optimal policy. Extending this UI supplement for another six months would hamper the recovery and reduce welfare. On the other hand, a UI extension combined with a re-employment bonus would further increase welfare compared to CARES alone, with only minimal effects on unemployment.

JEL Classification: J65, E6, H1

Keywords: COVID-19, epidemic, unemployment insurance, optimal policy

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1 Introduction

The Coronavirus Aid, Relief, and Economic Security Act (CARES) Act, passed in response to the COVID-19 pandemic and the ensuing economic crisis, included an aggressive expansion of unemployment insurance (UI). Specifically, the Federal Pandemic Unemployment Compensation added \$600 to the weekly benefit amount of all UI recipients through the end of July 2020. As documented by Ganong et al. (2020), this resulted in more than a 100% replacement rate of lost earnings for many job losers. This expansion of UI generosity paralleled an unprecedented increase in jobless claims starting in March 2020—due to the pandemic and the subsequent economic lockdown. The efficacy of this UI expansion — and the desirability of extending it beyond July 2020 — is the subject of an ongoing heated debate in the U.S. Congress. Proponents are pinpointing the dire need for an additional safety net, and opponents argue that it provides work disincentives in an already recovering economy. In this paper, we ask what the optimal UI response is to the economic crisis and how it compares to the current implementation under the CARES act and alternative policy proposals.

We answer this question quantitatively in a search model in which unemployment insurance may be contingent on labor market conditions. We model the COVID-19 crisis as the destruction of job matches coupled with a deterioration of workers’ search efficiency. The latter consists of a sequence of adverse shocks to search efficiency, which leads the job-finding rate to initially halt and then gradually recover. This adverse shock can be interpreted as the combination of reduced labor demand, reduced ability to search due to shelter-in-place restrictions, increased cost of search due to infection risk, or reallocation costs associated with sector-specific effects of the epidemic.¹

We find that the optimal policy calls for raising the replacement rate of unemployment benefits dramatically in response to the fall in search efficiency, and then lowering them once search efficiency starts to recover. Importantly, this means that the rise and fall in the optimal UI replacement rate closely track the shock to search efficiency, not the unemployment rate. This distinction is significant because the unemployment rate is a slow-moving variable that remains persistently high, even as the economy is reopening. Indexing UI to the unemployment rate would (sub-optimally) keep benefits high for longer than our optimal policy implies, thereby impeding the economic recovery and reducing consumer welfare. As a by-product, our results show that the policy implemented under the CARES Act—with

¹We abstract from heterogeneity in re-employment probabilities amongst those separated. However, see Gregory et al. (2020) for evidence that the separation shocks induced by the pandemic may have disproportionately affects workers that take significantly longer to find stable jobs in the future.

its expiration set for July 31, 2020—is close to the optimal policy. We also conduct counterfactual exercises to experiment with alternative policy proposals following July 2020. A policy of extending the elevated replacement rate for an additional six months would lead to more protracted high unemployment and lower welfare. On the other hand, a re-employment bonus providing \$450 a week to both re-employed and unemployed would deliver higher welfare than the current UI supplement alone, despite leading to a slightly slower recovery of unemployment.

2 Model

Time is discrete, and the time horizon is infinite. The economy is populated by a continuum of infinitely-lived risk-averse workers, with utility

$$\mathcal{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(x_t) - \frac{1}{\zeta_t} c(S_t) \right] \quad (1)$$

where x_t denotes period- t consumption, and S_t denotes period- t search effort, incurred only when unemployed and restricted to be between 0 and 1. The economy is subject to aggregate shocks to ζ_t ; a lower ζ_t implies a higher cost of finding a job, which can be interpreted as reduced ability to search due to shelter-in-place restrictions, increased cost of search due to infection risk, or reallocation costs associated with sector-specific effects of the epidemic. We assume that ζ_t follows an AR(1) process

$$\ln \zeta_t = \rho_\zeta \ln \zeta_{t-1} + \sigma_\zeta \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, 1) \quad (2)$$

and denote the history of of ζ -shocks up to period t as $\mathcal{Z}_t = \{\zeta_1, \dots, \zeta_t\}$. The cost function $c(S)$ is strictly increasing, strictly convex, and satisfies $c'(0) = 0$, $c'(1) = \infty$. In the numerical analysis below, we will assume the functional form adopted in Mitman and Rabinovich (2015)

$$c(S) = A \left[\frac{(1-S)^{-(1+\psi)} - 1}{1+\psi} - S \right]. \quad (3)$$

Workers can be either employed or unemployed. When employed, they separate from their job the next period with an exogenous probability δ , and when unemployed, they find a job the next period with the endogenous probability S_t . The law of motion for aggregate

employment, denoted l_t , is then

$$l_t = (1 - \delta) l_{t-1} + S_t (1 - l_{t-1}) \quad (4)$$

When employed, workers receive exogenous income w and pay a tax τ ; when unemployed, they receive $h + b_t$, where h is an exogenous value of home production and b is the government-provided unemployment benefit, which is the policy choice of interest. This unemployment benefit b_t can potentially be contingent on the entire past history of shocks, \mathcal{Z}_t .²

Unemployed workers choose S_t at each point in time to maximize expected utility, taking as given the government policy $b_t(\mathcal{Z}_t)$. We show in Appendix A.1 that the worker's optimal search behavior leads to the Euler equation for search intensity,

$$\frac{1}{\zeta_t} c'(S_t) = \ln(w - \tau) - \ln(h + b_t) + \beta \mathbb{E}_t \left[\frac{1}{\zeta_{t+1}} c(S_{t+1}) + (1 - \delta - S_{t+1}) \frac{1}{\zeta_{t+1}} c'(S_{t+1}) \right]. \quad (5)$$

The Euler equation equates the marginal cost of additional search to the marginal benefit; the latter is the combination of the consumption gain from becoming employed and the benefit of economizing on search costs in the future. Given a policy path $b_t(\mathcal{Z}_t)$, the equilibrium is fully characterized by law of motion (4) and Euler equation (5).

3 Optimal policy

In the optimal policy analysis, we consider the optimal path of history-contingent b_t , l_t and S_t chosen by a benevolent, utilitarian government with commitment power. Such a government maximizes the expected value of

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[l_t \ln(w - \tau) + (1 - l_t) \ln(h + b_t) - \frac{1}{\zeta_t} (1 - l_{t-1}) c(S_t) \right] \quad (6)$$

We assume that the government budget needs to be balanced in expectation, so that

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [l_t \tau - (1 - l_t) b_t] = 0 \quad (7)$$

In other words, the expected present value of unemployment benefits cannot exceed the expected present value of tax receipts. The maximization of (6) is therefore subject to the budget constraint (7), the law of motion for employment (4), and the optimal search behavior

²For tractability, we abstract from policies that can depend on individual worker histories.

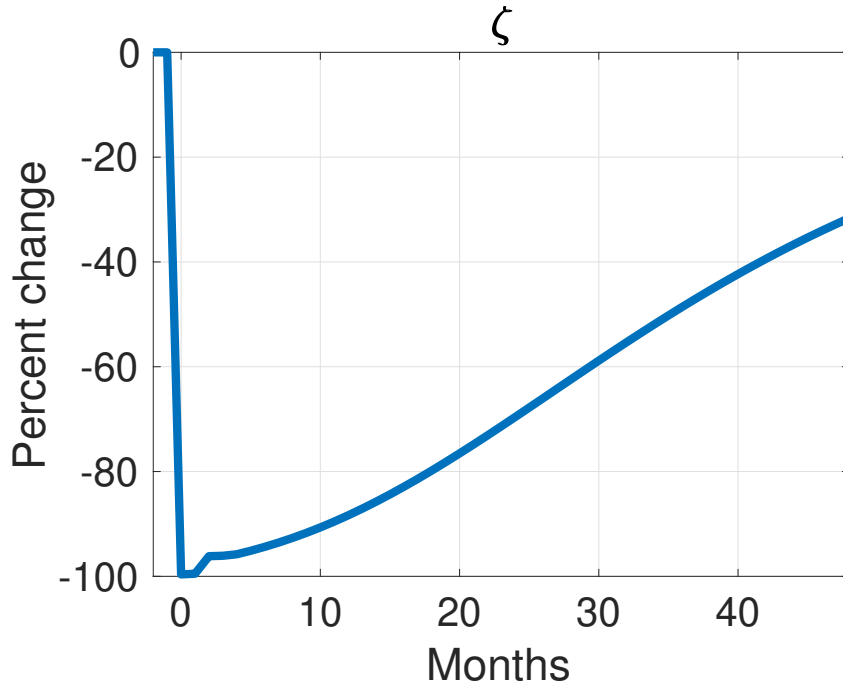


Figure 1: Simulated path for ζ_t that mimics the COVID-19 shock to the economy.

of workers, (5).

4 Quantitative Analysis

We calibrate the model to match salient features of the U.S. labor market prior to the onset COVID-19 pandemic. The model period is one week. We set the discount factor equal to $\beta = 0.99^{\frac{1}{2}}$ to match a 4% annual discount rate. We set $\delta = 0.0081$ to match the weekly job separation rate. We jointly estimate the disutility parameters in the search cost function $A = 3$ and $\psi = 1.9$ so that the model is consistent with the average unemployment rate and empirical estimates of the elasticity of unemployment duration with respect to unemployment benefits from Meyer (1990).³

We treat the COVID pandemic in the model as an unexpected shock to the economy. Starting from a steady state at $t = 0$, the economy is hit by a one-time increase in the separation rate, $\delta_0 > \delta$, combined with a sequence of negative ζ_t shocks. Agents have perfect foresight of the entire future path of ζ_t - making this effectively an "MIT shock" to the

³We note that there is an ongoing and active debate regarding the effects of unemployment benefits (levels and duration) on worker search effort (micro effects) and firm vacancy creation (macro effects). In innovative work using administrative data from Missouri, Johnston and Mas (2018) find significant affects of potential benefit duration on worker search effort, as measured through exits into employment.

economy (Boppart et al. (2018)). We think of ζ_t as encompassing policy responses and the decline in economic activity resulting from the spread of the virus. For example, it reflects NPI's, such as orders to limit restaurants to take-out only and stay-at-home orders, as well as reluctance or inability to search due to the fear of becoming infected (consistent with the evidence provided by Wiczer et al. that observable measures of search intensity declined during this period, along with posted job openings).

We choose the size of the separation shock to generate a 15% drop in employment by the end of April 2020. We calibrate the path for ζ , shown in Figure 1, to match the evolution of NPIs. For the first two months, we set ζ such that it's roughly 200 times more costly to find a job than pre-COVID. The fact that many sectors were effectively closed by policy justifies this extreme increase in the cost (moreover, there was a substantial drop in job vacancies, see e.g. Kahn et al. (2020)). For the next two months, we assume that the cost falls by one order of magnitude, to reflect the reversal of NPIs. After that, ζ mean reverts to its pre-COVID level with a monthly persistence of 0.96. The persistence of ζ is calibrated to be relatively high, to match the slow increase in visit to establishments and hours worked (Bognanni et al., 2020) even after NPI's are lifted. For research that explicitly models the interaction between employment and the spread of the virus see Kapicka and Rupert (2020).

We then perform a series of computational experiments. In 4.1, we simulate the response of unemployment in response to the actual policy implemented under the CARES Act. In 4.2, we compute the optimal policy response to the shocks, which is the solution to the problem described in Section 3. In 4.3, we perform other counterfactual experiments, to assess the effects of policy alternatives being discussed.

4.1 Baseline: CARES Act

Figure 2 shows path of the UI replacement rate and the response of employment under the implemented \$600 weekly CARES UI supplement through July 2020. Consistent with the data, the COVID shock combined with the UI extension generates a large and protracted fall in employment.

4.2 Optimal policy response

Next, in Figure 3, we plot the path of the UI replacement rate prescribed by the optimal policy, as characterized above in Section 3. There are important differences as well as similarities between the optimal policy and the one implemented under CARES. First, while the optimal policy still calls for a significant rise in the replacement rate (60%) on impact in

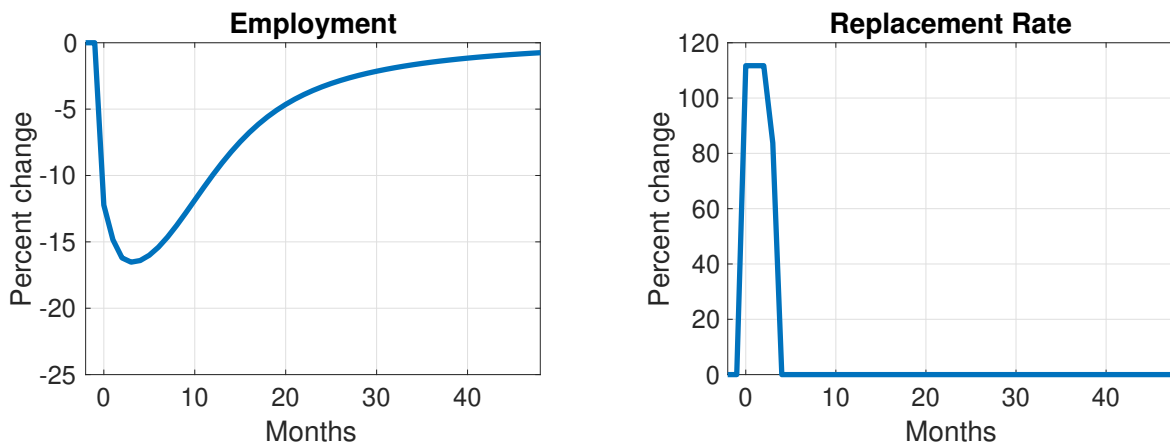


Figure 2: Employment and Benefits under the CARES Act.

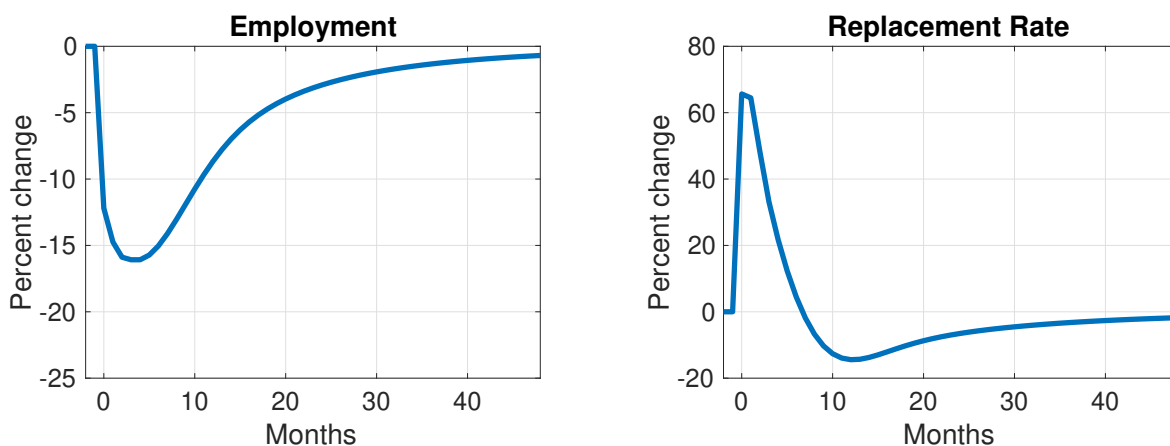


Figure 3: Employment and Benefits under the Optimal Policy.

response to the shock, this is lower than the rise in the replacement rate under CARES. Second, the optimal policy features a rapid fall in benefits around the point where ζ_t (see Figure 1) starts to recover. Importantly, this drop-off in benefits precedes the substantial recovery in employment. Overall, the CARES policy turns out to be close to optimal, largely because of the timing of the UI benefit decline. We find that the two policies are similar in terms of the employment recovery, though it is somewhat faster under the optimal policy. The welfare gain from implementing the optimal policy rather than CARES, in consumption-equivalent variation terms, is 0.1% of lifetime consumption.

4.3 Alternative policies

Next, we consider two alternative policy proposals. First, we consider the proposal (e.g. as included in a provision of the HEROES Act) to extend the \$600 weekly UI supplement beyond

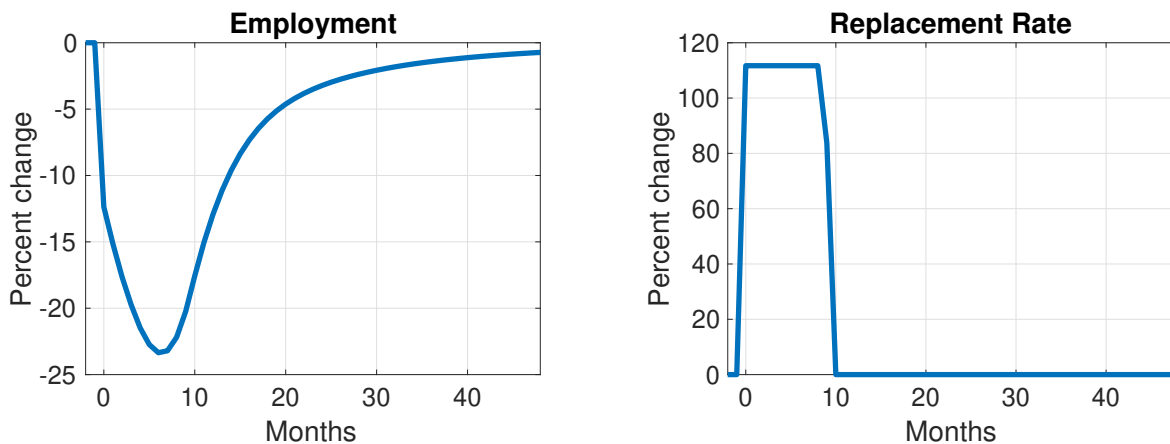


Figure 4: Employment and Benefits under an extension of the \$600 weekly CARES UI supplement for all unemployed through December 31, 2020.

July 2020. Figure 4 displays the corresponding path of the replacement rate and the implied employment trajectory. Extending the \$600 weekly UI supplement through December 2020 substantially delays the recovery of employment. Further, it generates a deeper trough in the employment drop relative to both the optimal policy and the original CARES act. With regard to the normative implications, we find that the extended CARES policy entails a 0.1% welfare loss in lifetime consumption-equivalent terms relative to the optimal policy.

Second, we consider a recent proposal to extend the weekly UI supplement, but to have it be implemented additionally as a re-employment bonus that newly hired workers could keep. The motivation behind the policy is to remove the moral hazard distortion from the high effective replacement rates under the CARES supplement. Following the bonus proposal being considered, we assume that from August 1, 2020 through December 31, 2020 unemployed individuals receive a weekly \$450 supplement. Newly hired workers during this time period keep receiving the \$450 supplement in addition to their weekly wage. The dynamics of employment and the benefits/bonus policy are plotted in Figure 5. Employment falls slightly more than under the optimal and CARES scenarios, but by significantly less than in the scenario where the \$600 CARES UI supplement is extended through December 31, 2020. The bonus program can therefore effectively overcome the majority of the moral hazard distortion induced by the higher benefit replacement rate. In terms of normative implications, we find that the CARES Bonus program delivers roughly the same welfare (in CEV terms) as the optimal policy, despite leading to a slightly slower recovery of unemployment.⁴

Figure 6 plots the relative sizes of the unemployment rate increase and recovery under the

⁴The slower recovery occurs because, under risk aversion, a lump-sum payment lowers search effort even when the payment accrues to both employed and unemployed, i.e. the flow surplus from being employed is $\ln(w - \tau + \Delta) - \ln(h + b_t + \Delta) < \ln(w - \tau) - \ln(h + b_t)$ for $\Delta > 0$.

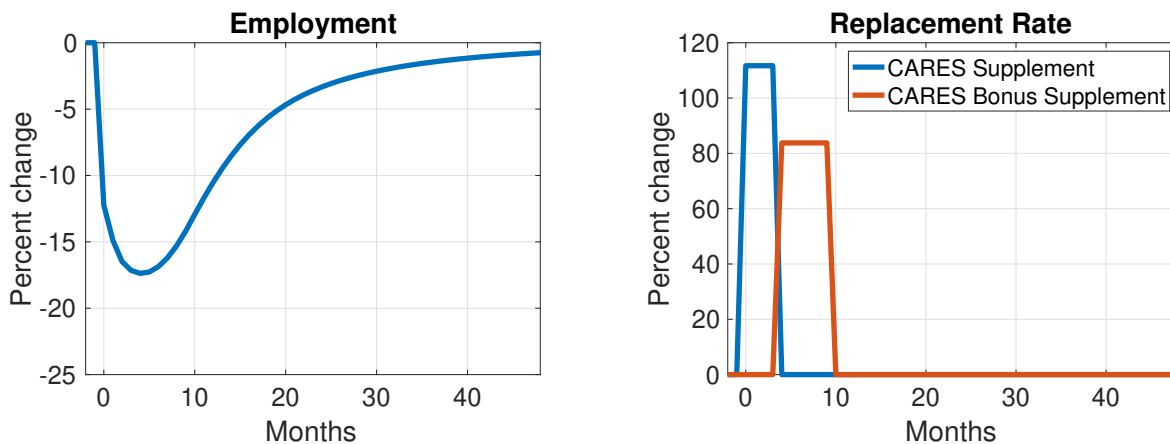


Figure 5: Employment and Benefits under a \$450 CARES UI supplement/re-employment bonus for all unemployed and newly hired through December 31, 2020.

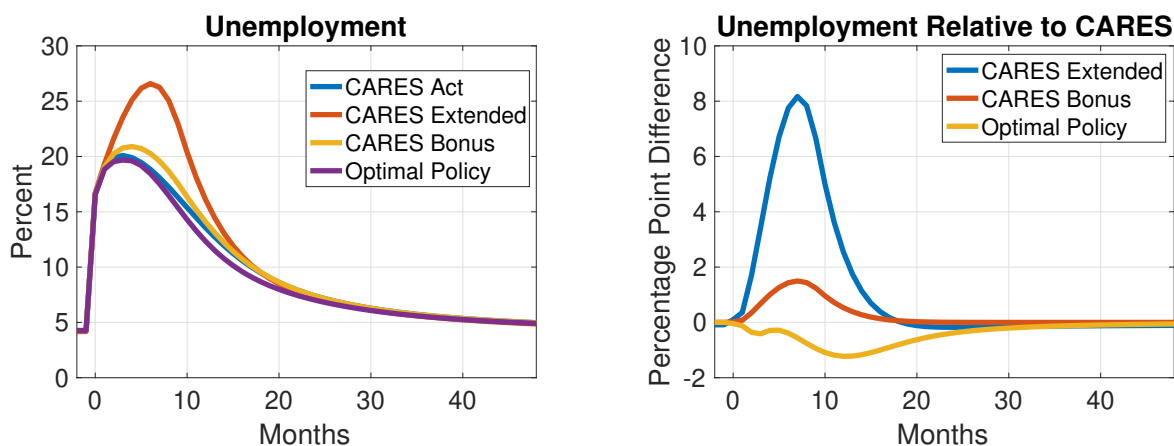


Figure 6: Unemployment rate under the four different scenarios: baseline CARES Act, the optimal policy, an extension of CARES \$600 weekly payment through December 31, 2020, and a CARES "Bonus" program through December 31, 2020.

four scenarios considered. The right panel illustrates the relative size of the unemployment rate, under the optimal policy, the extended CARES policy, and the re-employment bonus, as compared to the baseline CARES policy.

4.4 A more optimistic recovery scenario

For robustness, we also consider a more optimistic recovery scenario. We lower the persistence of the ζ_t process from 0.99 to 0.9 on a weekly basis. The path of ζ_t is illustrated in Figure 7. With this lower persistence, the cost of finding of job is essentially back to the steady state level within one year of the onset of the pandemic. Figure 8 illustrates the optimal policy response in this case, and Figure 9 compares the unemployment trajectory across the

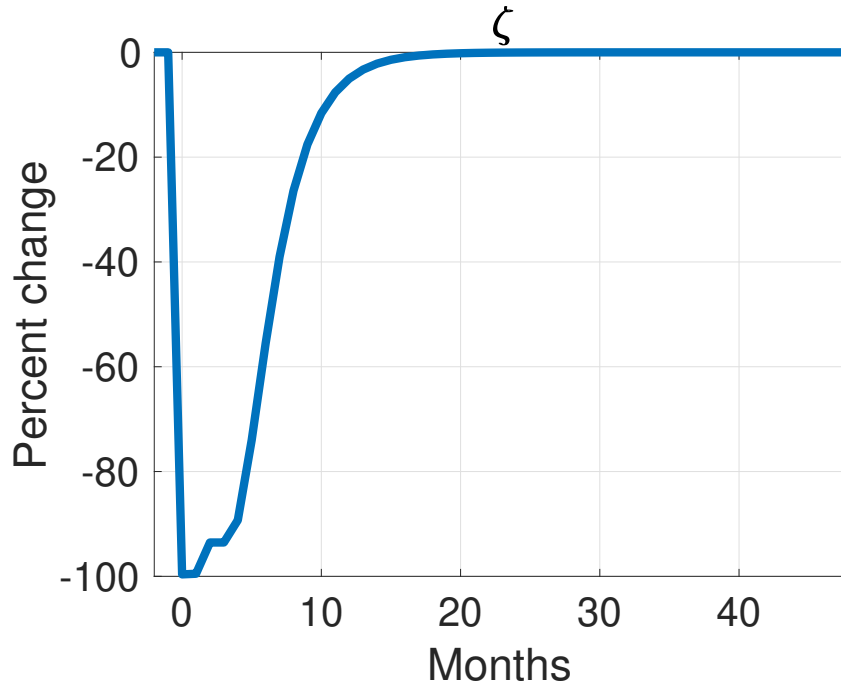


Figure 7: Simulated path for ζ_t : optimistic recovery scenario.

different policy alternatives. Not surprisingly, we find significantly faster recoveries under all policy alternatives — unemployment is back to steady state levels within 18 months of the shock. Interestingly, we find that the moral hazard distortion of the extended CARES policy is much *stronger* under this more optimistic scenario, as can be seen by comparing Figures 6 and 9. The easy availability of jobs associated with the faster recovery worsens the moral hazard distortions. If agents know that the cost of finding a job will be very low after the UI supplement runs out, the cost of delaying search (and collecting the high UI supplement) is low. On the other hand, in the more pessimistic baseline scenario, the more sluggish recovery makes households more willing to accept jobs even if the replacement rate is higher than the wage, because they are afraid of being unemployed after the supplement runs out, when it's still costly to find a job. The weak recovery in our baseline case thus served as a discipline device mitigating the moral hazard distortion.

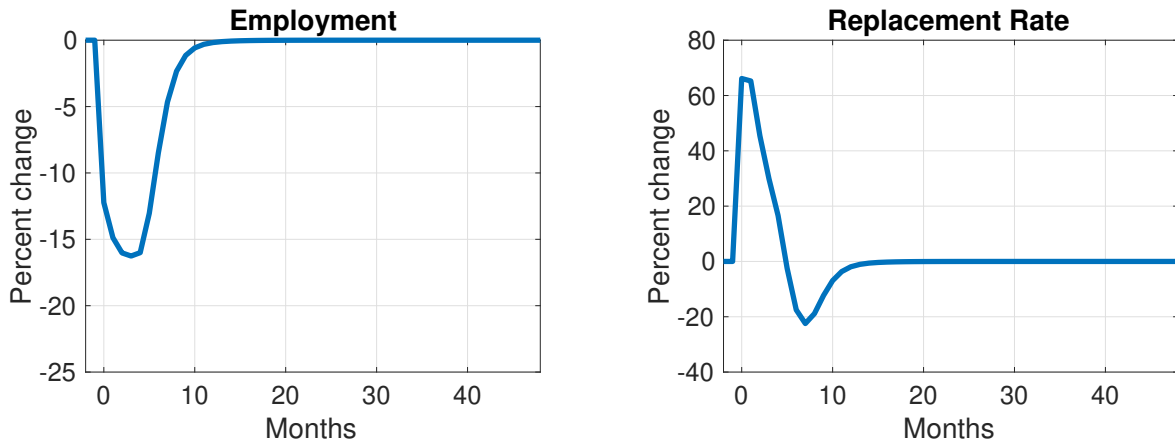


Figure 8: Employment and benefits under the optimal policy: optimistic scenario.

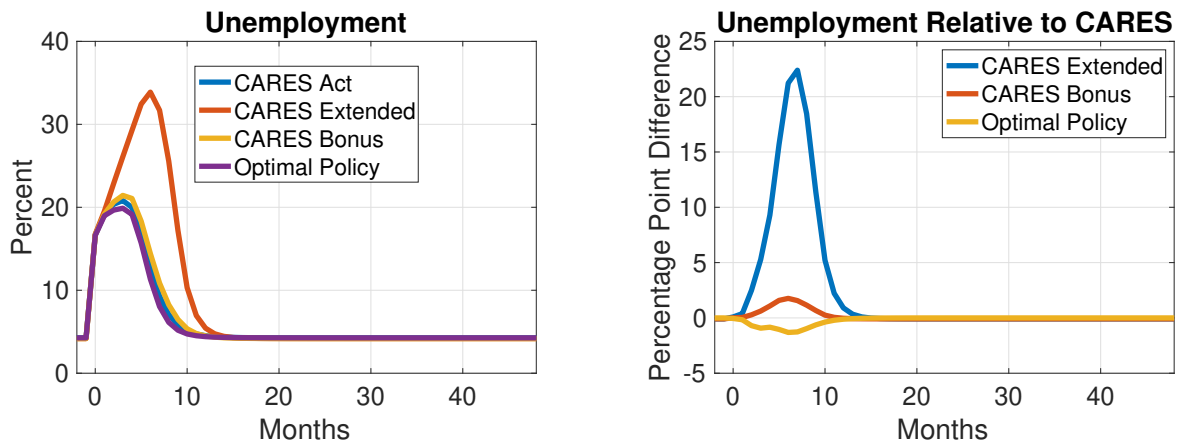


Figure 9: Comparison across policies under optimistic scenario: baseline CARES Act, the optimal policy, an extension of CARES \$600 weekly payment through December 31, 2020, and a CARES "Bonus" program through December 31, 2020.

5 Discussion

We assessed the optimal UI policy and compared it to the one currently implemented, as well as the most prominent alternative proposals. We found that the UI supplement applied under the baseline CARES Act policy performs quite well. The UI supplement combined with a re-employment bonus would perform even better, despite a somewhat slower employment recovery. On the other hand, a blanket extension of the UI supplement for another six months would substantially hamper the recovery and reduce welfare. The broad lesson is that expectations matter. The optimal policy starts lowering the UI payment when the economy begins to reopen - *before* the recovery of employment. A policymaker that indexes UI benefits to the level of unemployment would keep them high for too long, generating hysteresis (see, e.g., Mitman and Rabinovich (2019)). Furthermore, expectations of weak labor market conditions in the future mitigate the moral hazard problem today, as we showed by comparing alternate recovery scenarios. High future costs of search make it easier to incentivize current search effort, creating a further reason for a temporary UI expansion.

We have focused on the amount and timing of unemployment benefits, and thus abstracted from two other important aspects of the current crisis: the distinction between temporary and permanent separations, as examined in Gregory et al. (2020) and Birinci et al. (2020); and the epidemiological side of the discussion, as applied to a search model by e.g. Kapicka and Rupert (2020). Combining these unique features of the recession with our analysis of unemployment insurance is an important extension. We have also abstracted from two general equilibrium feedback mechanisms. First, we have ignored potential aggregate demand effects induced by providing transfers to the unemployed that could speed the recovery (Kekre (2019); Ravn and Sterk (2016); Den Haan et al. (2018)). Our view is that the COVID-19 pandemic (and ensuing policy response with lockdown orders) represents a supply shock and thus that normal demand channels will be muted (see Guerrieri et al. (2020) for an alternative view). Second, we have abstracted from firm labor demand and the response of wages and labor force participation to benefit policy (see, e.g., Hagedorn et al. (2013, 2015)). We leave these for future work.

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A Supplementary derivations

A.1 Details on the worker problem

Throughout, let $\mathcal{Z}_t = \{\zeta_1, \dots, \zeta_t\}$ denote the history of shocks. Let $W_t = W_t(\mathcal{Z}_t)$ be the value of a worker entering period t employed, and $U_t = U_t(\mathcal{Z}_t)$ the value of a worker entering period t unemployed. These values satisfy the Bellman equations

$$W_t = (1 - \delta) [\ln(w - \tau) + \beta \mathbb{E}_t W_{t+1}] + \delta [\ln(h + b_t) + \beta \mathbb{E}_t U_{t+1}] \quad (8)$$

$$U_t = \max_S -\frac{1}{\zeta_t} c(S) + S [\ln(w - \tau) + \beta \mathbb{E}_t W_{t+1}] + (1 - S) [\ln(h + b_t) + \beta \mathbb{E}_t U_{t+1}] \quad (9)$$

where the period- t expectation is taken with respect to ζ_{t+1} and dependence on \mathcal{Z}_t is suppressed for notational convenience. From (9), the first-order necessary condition for the optimal $S = S_t$ is

$$\frac{1}{\zeta_t} c'(S_t) = \ln(w - \tau) - \ln(h + b_t) + \beta \mathbb{E}_t [W_{t+1} - U_{t+1}] \quad (10)$$

Subtracting (9) from (8) also gives

$$W_t - U_t = \frac{1}{\zeta_t} c(S_t) + (1 - \delta - S_t) \{ \ln(w - \tau) - \ln(h + b_t) + \beta \mathbb{E}_t [W_{t+1} - U_{t+1}] \} \quad (11)$$

Combining (10) with (11) gives (5).