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Do Exit Options Increase the Value-For-Money of Public-Private Partnerships?

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2030 Agenda

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Summary

We study the effects of granting an exit option that enables the private party to early terminate a PPP project if it turns out to be financially loss-making. In a continuous-time setting with hidden information about operating profits, we show that an exit option, acting as a risk-sharing device, can soften agency problems and, in so doing, accelerate investment and increase the government's expected payoff, even while taking into account the costs that the public sector will have to meet in the future to take direct responsibility on service provision.

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JEL Classification: D81, D82, D86, H54

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Do Exit Options Increase the Value-For-Money of Public-Private Partnerships?

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March, 2020

Abstract

We study the effects of granting an exit option that enables the private party to early terminate a PPP project if it turns out to be financially loss-making. In a continuous-time setting with hidden information about operating profits, we show that an exit option, acting as a risk-sharing device, can soften agency problems and, in so doing, accelerate investment and increase the government's expected payoff, even while taking into account the costs that the public sector will have to meet in the future to take direct responsibility on service provision.

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1 Introduction

Partnerships between private and government entities for the provision of public services are not exclusively a contemporary phenomenon. For instance, examples can be found during the Roman Empire, where postal stations were constructed and managed by private providers under contracts, sometimes also including maintenance of associated road segments, awarded by municipalities through competitive bidding (PPIAF, 2009). Recent decades, however, have witnessed an increasing outsourcing of traditionally public sector activities for several reasons, including the presumed inherent efficiency superiority of private management, the need of leveraging scarce public funds, in the face of increasing demands for infrastructure services, and, sometimes, the attempt “to shift public investment off budget (and out of public eye)” (Sadka, 2006:20).

Nowadays, the term Public-Private Partnership (PPP) covers a wide range of contractual arrangements, which, however, tend to share some common features that make them different from other forms of cooperation between the public and the private sector. For instance, compared with conventional procurement methods, a distinctive ingredient of PPPs is that the private party must take a substantial proportion of risk, insofar as they generally involve responsibility over several project functions (*e.g.* construction as well as operation and maintenance of public infrastructures) and remuneration is closely tied to performance (World Bank, 2017).

In principle, tasks bundling and the direct link between rewards and performance can prove beneficial both in terms of service quality for the public and value-for-money for the taxpayer (Hart, 2003). However, the long duration of PPPs, needed to attract private funding and secure investment, can lead to several problems due to changing circumstances that may occur throughout the term of the partnership (Saussier and de Brux, 2018). In fact, “contracts [can] suffer from being signed in contexts with pervasive uncertainty over future demands and costs” (Iossa and Martimort, 2015:8) and around the world there are several examples of PPPs which have encountered problems because of unrealistic demand

expectations, cost inflation, changes in user preferences or changes in policies affecting the use of the facilities (Engel et al. 2014). As public authorities generally retain the ultimate responsibility for service delivery (Forrer et al., 2010; Yang and Zhang, 2010), governments have often been forced to undertake costly renegotiations or simply to resume operations (Guash, 2014; Zhang and Xiong, 2015).

Based on these evidences, scholars, as well as PPP stakeholders, have often called for injecting more “flexibility” in PPP deals, that is, contracts ought to be drafted so as to proactively anticipate and rapidly react to changing circumstances throughout the term of the partnership (Demirel et al., 2017).¹ Such *ex ante* provisions may take on several forms, including for instance minimum revenue, minimum income or loan guarantees (see, *e.g.*, Iossa et al, 2007; Takashima et al, 2010; Cruz and Marques, 2013; Adkins and Paxsons, 2017; Jin et al., 2019). However, while presumably increasing the chances of attracting private capital, these mechanisms can impose uncertain public costs (World Bank, 2019) and, perhaps even more importantly, require continuous and precise monitoring and assessment of project performance, in order to avoid opportunistic behaviour by private partners and unjustified escalations of public costs.

Another provision, we will focus on in this paper, could consist in drafting the contract so as to allow the private partner to early terminate the service agreement if operation becomes financially unviable.² As a matter of fact, even though such an opportunity is rarely spelled out in PPP contracts -arguably because of the public concern about service continuity- “exit options” are often embedded in PPPs, in the form of termination provisions for non-compliance with the stipulated contract term.

Our main objective is to examine whether an (explicit or implicit) early termination

¹As pointed out by Beuve et al. (2018), unlike private contracts, where the parties tend to adapt ad hoc when faced with unexpected circumstances, public-to-private contracts, being subject to public scrutiny and accountability, typically require *ex-ante* specified rules for amendments and adaptations.

²Other papers (see, *e.g.*, Alonso-Conde et al., 2007; Iossa et al., 2007) have instead focused on the government’s decision to early terminate the contract unilaterally, even if the partner has performed satisfactorily. Unlike the situation considered in our model, the government will typically take-over a (beyond expectations) financially profitable venture and, therefore, according to the “market value principle” will have to indemnify the partner for the loss of expected earnings (European PPP Expertise centre, 2013).

option could contribute to accelerate private investment and, more generally, under what circumstances the granting of an exit option could yield improved value-for-money to the public sector with respect to “lock-in” contracts.

To this purpose, we develop a model where a private investor is untrusted with the responsibility for building, at the time foreseen by the government, an infrastructure project of public interest, and then operate it in exchange of a partial contribution to capital expenditures and the right to collect user charges. Besides the capital grant, the government’s offer also includes an exit option which can be exercised, at any time, by the contractor by paying a termination fee fixed upon contract award. Moreover, we shall assume that in the event of exit by the partner, the government, acting as a provider of last resort, will resume the project by taking over direct responsibility for service provision.

The model is developed in continuous time, by incorporating into a real options framework a principal-agent problem where the contractor holds private information on operating profits. In so doing, we intend to restrict our attention on situations where regulators, being unable to implement state-contingent risk-sharing provisions, have to resort on stationary incentives. Our main result is that the blending of direct investment subsidies and exit options can contribute to soften agency problems and, in so doing, provide a higher value for money to the government, by accelerating investment at a lower public cost.

The remainder is organised as follows. In Section 2 we position our work within the literature. In Section 3 we present the model. In Section 4 we describe the impact of the exit option on the project’s private value. In Section 5 we derive the optimal mix of front-loaded and option incentives for a revenue-maximizing government, by first solving for the benchmark case of symmetric information and then turning on the case of private information on the operating profits. Section 6 concludes. The proofs are presented in the appendices.

2 Related literature

This paper can be positioned at the intersection of two literatures. The first is the one using principal-agent frameworks to examine how contract design and incentive mechanisms can shape behaviour and PPP performance (see, *e.g.*, Hart, 2003; Martimort and Pouyet, 2008; Auriol and Picard, 2013; Hoppe and Schmitz, 2013; Iossa and Martimort, 2015). Within this literature, relatively few studies, using a dynamic approach, have explored the effects of endogenous (Engel et al., 2001) or state-contingent (Danau and Vinella, 2017) service duration.

The second body of literature has focused instead on the value of real options embedded into public-private contracts (see, *e.g.*, Huang and Chou, 2006; Alonso-Conde et al., 2007; Brandao and Saraiva, 2008; Martins et al., 2015; Blank et al., 2016) without, however, providing much guidance on how to efficiently design option-like incentives.

Some papers have tried to bridge the gap between the two literatures by incorporating into a real option framework a contract design problem. A common feature of these models is the attention paid to the timing of service delivery which, rather than being taken as exogenously given, is modelled as a decision variable. For instance, Takashima et al. (2010) study the interaction between a government and a private firm when they time an investment decision, while Scandizzo and Ventura (2010) consider a concession contract for developing a publicly-owned natural resource where the private party is required to pay a price to compensate the government for the loss of amenities.

Broer and Zwart (2013) and Soumare and Lai (2016) depart from these works by introducing asymmetric information. In particular, Broer and Zwart (2013) examine the optimal regulation of an investment undertaken by a monopolist with private information on capital costs, while Soumare and Lai (2016) compare, within a model with hidden information, different forms of public support (loan guarantee *vs* direct investment) in PPPs. Di Corato et al. (2018), for their part, study how early-exit options, resulting from the government's inability to enforce sufficiently strong penalties for breach of contracts, can affect bidding

behaviour in multidimensional auctions for the provision of long-term environmental services.

Within this mixed literature, our specific contribution is twofold. First, in a PPP setting with private information on the project's cash flows, we derive the optimal government's decision regarding the degree of exit flexibility granted to the contractor. Second, we examine the effects of flexibility upon the timing of investment and the government's overall payoff, by taking into account also the potential financial costs deriving from taking charge of the project in the case of termination by the private partner.

Regarding the methodology, we follow the dynamic mechanism design approach developed by Baron and Besanko (1984), Battaglini (2005), Esö and Szentes (2007), Pavan et. al. (2014) and others. Specifically, we build on the approach suggested by Kruse and Strack (2015; 2019) who show, in a continuous time setting, that an incentive-compatible allocation mechanism can be obtained without continuous monitoring of the state variable. Our specific contribution consists of extending their methodology to the case where, besides direct incentives (cash payments), the government also relies on option-like mechanisms, namely, on an early termination fee for breach of contract.

3 Set up

A government entity (such as a city) intends to rely on a private firm for building and operating a public infrastructure facility which, besides the benefits realized by direct users, is expected to generate valuable externalities for the community as a whole. One may think, as an example, of a new by-pass planned to ease gridlocks in a heavily congested urban area and to curb local pollution by increased traffic fluidity.

The project, whose technical and functional features are clearly defined and verifiable, requires a sunk investment of I and then a fixed O&M cost per unit of time (say, per annum) denoted as c . For technical convenience, we assume that construction can be instantaneously carried out, that the infrastructure has an infinite life and that, once implemented, the

project will provide a perpetuity of annual public non-financial benefits, valued at b , above and beyond those accruing to direct users.³

We shall assume that the government (she) is able to commit at time $t = 0$ to a take-it-or-leave-it contract offer to a firm (he), having no outside option, including the following terms and conditions.⁴

First, the contract gives the firm the responsibility for building the facility at the time foreseen by the government and then maintaining and operating it all along the contract period which is assumed to be long enough to be approximated as infinite.

Second, the contract entitles the firm to receive upon investment a fixed non-repayable capital grant $0 \leq S \leq I$ and, afterwards, to collect user charges. Since the issue of the optimal pricing policy lies beyond the scope of the paper, we simply assume that the firm will be allowed to extract all users' surplus.

Third, the agreement legally binds the firm to pay a fixed sum of money, denoted by $L \geq 0$, in the event of premature abandonment of operation. Throughout the paper, L will be referred to as the early termination (or exit) fee. We deliberately avoid using terms like "penalty" or "liquidated damages", whose scope and legal consequences vary across different jurisdictions, namely, civil and common-law countries (Di Matteo, 2001; Marin Garcia, 2012). For our purpose, it suffices to think of L as a sort of "strike price" to be paid out if the contract is actually broken by the firm.

We shall assume that, in case of early termination by the firm, the government will resume the project by incurring the same cash flows (revenues and operating costs) that would have been incurred by the firm. As on termination the project will hold a negative financial value,

³In many of the applications we have in mind, external benefits may not be constant but, like cash-flows, also evolve over time. For instance, in toll roads, positive externalities generally increase with the traffic diverted from the existing roads, which, in turn, influences the amount of toll revenues. However, as long as the process governing positive spillovers is public information, we can generalize our model to allow for this.

⁴The assumption that the firm's reservation utility is zero can be justified on the ground that the provision of public services is normally subject to governmental authorization or licencing. The same reasoning applies in the case of projects involving the production of marketable goods (*e.g.*, renewable energy) which require as input specific assets which are fully controlled by the government, such as publicly-owned land or state buildings.

this implies that the resumption cost will be simply given by the negative project value (net of the exit fee received from the firm).⁵ However in the Appendix E we show that the main results presented in the main text hold even when the government needs to afford additional costs beyond those the firm himself would have incurred absent the termination.

The project's cash flows, before fixed O&M costs c , are denoted by $x_t \in (0, \infty)$ and are assumed to evolve stochastically according to the following trendless Geometric Brownian process:⁶

$$\frac{dx_t}{x_t} = \sigma dz_t \quad x_0 = x \quad (1)$$

where $\sigma > 0$ is the constant instantaneous volatility and $z_t \sim N(0, t)$ is a standard Wiener process having normal distribution with zero mean and variance t .⁷ As the solution of the differential equation (1) is given by $x_t = x \exp\left(-\frac{\sigma^2}{2}t + \sigma z_t\right)$, the cash flows at time t depend on the initial value x at $t = 0$ and the contemporaneous shock z_t at time t .

The uncertainty parameter σ is assumed to be common knowledge between the government and the firm; however, an agency problem exists because the latter privately observes the true cash flows x_t ($t \geq 0$). The initial value x , reflecting potentially different abilities to seize opportunities coming from the project, is assumed to be distributed on $[x^l, x^h]$, according to the cumulative distribution function $G(x)$, with density $g(x)$ and $g(x^l), g(x^h) > 0$, which is common knowledge.⁸

⁵This requires assuming that, over time, the government will be able (*e.g.*, by gathering information through monitoring the partner's activity) to narrow the productive efficiency gap which might had provided a motivation for using a PPP instead of direct public provision of services (see, *e.g.*, Auriol and Picard, 2013).

⁶The assumption of a trendless random walk allows us to focus on the pure effect of uncertainty, namely, on the effect of σ on both the optimal timing of investment and the firm's optimal timing of withdrawing service provision. However, by the Markov property of (1), our results would not be qualitatively altered by using a non-zero trend for x_t .

⁷Notice that (1) is consistent with the case where the project's cash-flows are uncertain because variable operating costs are uncertain, whereas the revenues are set as part of the contract. For instance, the instantaneous profit maximization of the operating profits under a Cobb-Douglas production technology $h(n) = n^\alpha$ with $\alpha \in (0, 1)$ and n a scalar input, gives the input demand function $n = (\alpha/w)^{1/(1-\alpha)}$ where w is the input price. If the uncertainty comes from changing input prices, the profit flow would be: $x_t = Nw_t^{\alpha/(\alpha-1)}$, where $N = (1-\alpha)(\alpha)^{\alpha/(1-\alpha)}$. By the Ito's Lemma, if w_t is driven by a stochastic process as (1), then also x_t is a Geometric Brownian process.

⁸As in Arve and Zwart (2014) and Skrzypacz and Toikka (2015), this is equivalent to assuming that the

The function $G(x)$ is such that $\phi(x) = \frac{1-G(x)}{g(x)x}$ is monotone and decreasing, with $g(x^l) \geq 1/x^l$. Notice that this condition is strictly weaker than the standard increasing hazard rate assumption (see, *e.g.*, Guesnerie and Laffont, 1984; Jullien, 2000).

Finally, as standard in the real option approach to investment under uncertainty, we shall assume that all parties are risk-neutral. In fact, within our framework, introducing risk aversion, while complicating the analysis, would essentially lead to an erosion of the project value and an increase of the option value of waiting to invest (Hugonnier and Morellec, 2013), without qualitatively altering our main findings.

4 The project's private value

Before looking at the government's optimal offer, it is worth first examining the project value to the firm after works completion. Denoting with $t > 0$ the time of investment and working backward, the private value is given by:

$$\begin{aligned} V(x_t, x_T) &\equiv E_t \left[\int_t^T e^{-r(s-t)} (x_s - c) ds - e^{-r(T-t)} L \right] \\ &= \frac{x_t - c}{r} - E_t \left\{ e^{-r(T-t)} \left[\int_T^\infty e^{-r(s-T)} (x_s - c) ds + L \right] \right\} \end{aligned} \quad (2)$$

where r is the discount rate and T is the (unknown) future time the firm will (eventually) find it convenient to breach the contract.

Using the law of iterated expectations, Eq. (2) can be expanded as follows:

$$\begin{aligned} V(x_t, x_T) &= \frac{x_t - c}{r} - E_t(e^{-r(T-t)}) E_T \left[\int_T^\infty e^{-r(s-T)} (x_s - c) ds + L \right] \\ &= \frac{x_t - c}{r} - \left(\frac{x_t}{x_T} \right)^{\beta_2} \left(\frac{x_T - c}{r} + L \right) \end{aligned} \quad (3)$$

firm's private information is represented by two stochastic processes where the one representing the initial value is constant after time zero, but influences the transitions of the second one.

where $\beta_2 < 0$ is the negative root of the characteristic equation $\Psi(\beta) = (\sigma^2/2)\beta(\beta - 1) - r = 0$, while $E_t(e^{-r(T-t)}) = \left(\frac{x_t}{x_T}\right)^{\beta_2}$ is the “expected discount factor” that allows transforming tomorrow’s uncertain payoff into present (time t) value (Dixit and Pindick, 1994).

The first term on the RHS of (3) measures the expected present value of total profits if the infrastructure was operated forever, whereas the second term measures the exit-option value to the firm.

Eq. (3) implies that, in order to maximize the total value, the firm should cut back the option value by identifying the optimal “exit trigger” that is, the cash flow level x_T at which it will become no longer advisable to continue operation.

Defining $x_T = x^E$ as the optimal exit trigger, this is given by:⁹

$$x^E = \frac{\beta_2}{\beta_2 - 1} (c - rL) \quad (4)$$

Eq. (4) shows the relationship between x^E and the termination fee L . Intuitively, other things being equal, the higher is L , the lower is x^E . For instance, if the government “locked in” the firm, by foreseeing a relatively high “strike price” for breaching the contract (i.e., $L \geq \frac{c}{r}$), then $x^E = 0$, that is, the firm will never find it optimal to quit the project. At the opposite extreme, if $L = 0$, then $x^E = \frac{\beta_2}{\beta_2 - 1} c > 0$.

Since $0 < \frac{\beta_2}{\beta_2 - 1} < 1$ and, thus, $x^E < c - rL$, Eq. (4) implies that, because of uncertainty and because the decision to quit is irreversible, the firm will tend to postpone the exercise of the exit option, even while losing money, in the hope of recovering previous losses.

By substituting (4) into (3), we get:

$$V(x_t) = \frac{x_t - c}{r} + O(x_t) \quad \text{for } x_t \geq x^E \quad (5)$$

where $O(x_t) = -\left(\frac{x_t}{x^E}\right)^{\beta_2} \frac{x^E}{\beta_2 r} > 0$ represents the value of the exit option.

Notice that $\frac{\partial O}{\partial x_t} = -\left(\frac{x_t}{x^E}\right)^{\beta_2 - 1} / r < 0$. In other words, the higher are the cash flows at

⁹Consistently with Eq. (4), the firm’s optimal exit time is described by a first passage time T of the state variable x_t by a constant threshold x^E . Formally $T = \inf(t > 0, \text{ such that } x_t = x^E)$.

the time of investment, the lower will be the value of the exit option. The simple intuition is that, if the firm was allowed to invest only when cash-flows are sufficiently strong enough, the probability of reaching the lower boundary x^E will decline and, thus, also the option value.

Finally, since $\frac{\partial O}{\partial x^E} = -\frac{1-\beta_2}{\beta_2} \left(\frac{x_t}{x^E}\right)^{\beta_2} / r > 0$, the lower is L and, thus, the higher is x^E , the higher is the value of the exit option.

5 Revenue maximization

Armed with these insights, we now turn to the government's optimization problem which consists of identifying, on the one hand, the timing of investment and, on the other hand, the combination of capital grant S and exit fee L which maximize the expected revenue, defined as the difference between the present value of public benefits ($\frac{b}{r}$) and the present value of public costs. As for the latter, the granting of an exit option implies that ex-ante cost assessment requires adding to the direct cost S the indirect cost of (eventually) resuming a financially loss-making project.¹⁰ Stated differently, the value of the option granted to the firm can be regarded as the government's implicit cost for injecting flexibility into the contract.

We proceed by first deriving the government's optimal offer under symmetric information and then turning to the case of private information on the project's cash flows.

5.1 Symmetric information

The optimization problem can be broken down into two parts. The first step consists of identifying the optimal timing of investment, specifically, the level of cash flows x_t at which

¹⁰Notice that, because of the negative project value, the firm, in accordance to the "market value principle", will not receive any compensation for the asset transferred to the government. In fact, according to this principle, developed in the UK in the late 1990s under the Private Finance Initiative (PFI), "[...] any compensation to the defaulting Private Partner should be determined by reference to the market value of the PPP contract at the date of termination" (European PPP Expertise Center, 2013: 41).

it will become publicly optimal to implement the project (“the investment trigger value”). The second step consists of determining the minimal grant S needed to enforce the optimal investment trigger as a function of the exit fee L .

Denoting with τ the stochastic time of investment, the objective function to be maximized at $t = 0$ is given by:

$$\begin{aligned} R(x, x_\tau) &= E_0(e^{-r\tau}) \left[\int_\tau^\infty e^{-r(t-\tau)} b dt - S \right] + E_0 \left\{ e^{-rT} \left[\int_T^\infty e^{-r(t-T)} (x_t - c) dt + L \right] \right\} \\ &= E_0(e^{-r\tau}) \left[\left(\frac{b}{r} - S \right) + E_\tau(e^{-r(T-\tau)}) \left(\frac{x^E - c}{r} + L \right) \right] \end{aligned} \quad (6)$$

Recalling that, by (3) and (5), $E_\tau(e^{-r(T-\tau)}) \left[\frac{x^E - c}{r} + L \right] = -O(x_\tau)$, where x_τ is the level of cash flows at the moment of investment, the objective function reduces to:

$$\begin{aligned} R(x, x_\tau) &= E_0(e^{-r\tau}) \left[\frac{b}{r} - (S + O(x_\tau)) \right] \\ &= \left(\frac{x}{x_\tau} \right)^{\beta_1} \left[\frac{b}{r} - (S + O(x_\tau)) \right] \end{aligned} \quad (7)$$

where $\beta_1 > 1$ is the positive root of the characteristic equation $\Psi(\beta)$, and $S + O(x_\tau)$ is the total (direct and indirect) public cost.

Rearranging (7), the government’s expected payoff can be rewritten as follows:

$$R(x, x_\tau) = W(x, x_\tau) - F(x, x_\tau) \quad (8)$$

where:

$$W(x, x_\tau) \equiv \left(\frac{x}{x_\tau} \right)^{\beta_1} \left(\frac{b}{r} + \frac{x_\tau - c}{r} - I \right) \quad (9)$$

represents the total economic value, that is, the benefits for all parties net of capital and

operating costs, and:

$$\begin{aligned} F(x, x_\tau) &\equiv \left(\frac{x}{x_\tau}\right)^{\beta_1} [V(x_\tau) - I + S] \\ &= \left(\frac{x}{x_\tau}\right)^{\beta_1} \left[\frac{x_\tau - c}{r} - (I - S - O(x_\tau)) \right] \end{aligned} \quad (10)$$

is the value to the firm, with the second term in squared brackets measuring the private cost of undertaking the project, given by the capital outlay I net of the public grant and the exit option value.

Eq. (8) implies that, for any given exit fee and, thus, option value $O(x_\tau)$, the government's payoff is maximized when the total private value is brought down to zero. In practice, this requires identifying the investment trigger which maximizes (9) and, thus, a transfer S such that $F(x, x_\tau) = 0$.

Denoting with $x_\tau = x_{\tau w}$ the welfare-maximizing (first-best) investment trigger value, this is given by:¹¹

$$x_{\tau w} = \frac{\beta_1}{\beta_1 - 1} (c - b + rI) \quad (11)$$

The multiplier $\frac{\beta_1}{\beta_1 - 1} > 1$, which accounts for uncertainty and investment irreversibility, implies that the first-best trigger is higher than the long-run average entry cost, i.e., $c - b + rI > 0$. In other words, even while taking into account the external benefits b , there is an efficiency (and not just financial) rationale for delaying the project beyond the point where the economic NPV becomes positive.

By (10) and (11) we get that, for any given exit fee L , the government can always bring the private project value down to zero by the following transfer:

¹¹We assume that the initial value x ($< x_{\tau w}$) is low enough to guarantee that immediate exercise does not happen.

$$\begin{aligned}
S^W &= I - O(x_{\tau^W}, L) - \frac{x_{\tau^W} - c}{r} \\
&= \frac{b}{r} - \frac{1}{\beta_1 - 1} \left(\frac{c - b}{r} + I \right) - O(x_{\tau^W})
\end{aligned} \tag{12}$$

It is worth notice that $S^W < \frac{b}{r}$, i.e., the optimal subsidy is lower than the standard Pigouvian “compensation” for external benefits. The reason is twofold. First, since there is an economic rationale for delaying the project, it would not be suitable to fully internalize the external benefits, in that this would lead the the firm to inefficiently accelerate investment. Second, the subsidy needed to spur investment is reduced by the exit option, which lowers the private cost of committing a capital outlay.

Working on (12), it is easy to show that an increase of the exit option value, via a reduction of L , would not bring by itself any additional benefit to the government ($\frac{\partial R}{\partial L} = 0$). The reason is that the marginal gains in terms of up-front savings ($\frac{\partial S^W}{\partial L} = -\frac{\partial O(x_{\tau^W})}{\partial L} > 0$) would be offset by the lower project’s market value at the time the option will be exercised by the firm ($\frac{\partial V}{\partial L} = \frac{\partial O(x_{\tau^W})}{\partial L} < 0$).

Summarizing, the main findings are as follows. First, not surprisingly, under symmetric information revenue-maximization (rather than welfare-maximization) does not imply a distortion with respect to the economically efficient timing of investment. In other words, a government focused on her own maximum payoff will find it optimal to implement the project at the same time that would be chosen by a benevolent social planner.

Second, revenue-maximization requires calibrating direct subsidies on capital costs against the private value of the exit option which, in turn, depends on the price at which the option itself will be exercised. Stated differently, the decision about the public contribution to capital expenditures should not be divorced from the termination fee set out in the contract. For instance, if the government decided to lock-in the firm, by setting $L \geq \frac{c}{r}$, the grant required to optimally trigger investment would amount to $\frac{b}{r} - \frac{1}{\beta_1 - 1} \left(\frac{c - b}{r} + I \right)$. At the opposite

extreme, if $L = 0$, the capital subsidy ought to be reduced by $O(x_\tau w, L = 0)$.

Having said this, under symmetric information, that is, when the government and the firm share the same information on the project's cash flows, the granting of an exit option, as a complement to direct support of capital expenditures, would not bring, per se, any particular financial benefit to the government.

5.2 Asymmetric information

We now turn to the case where the initial value x as well as the future realizations of the process (1) are not observable by the government.

According to the standard direct-revelation mechanism approach, the government should offer at $t = 0$ a menu of contracts $(\tau, S(\tau))$ that specifies, as a function of the investment time τ foreseen by the government, a payment $S(\tau)$ such that the firm will find it optimal to invest at τ . The mechanism should be incentive-compatible, in the sense that, whatever is his initial type $x \in [x^l, x^h]$, the firm maximizes his expected utility by truthfully reporting, at each time $t \geq 0$, the realization of x_t .

Unfortunately, since the space of communication strategies between the parties (i.e., the times of investment among which the firm can choose) could be very rich, the standard incentive-compatible mechanism is in general hard to be implemented (Board, 2007; Pavan et al., 2014; Bergemann and Valimaki, 2018).

An alternative approach has been developed by Kruse and Strack (2015; 2019), who show that if the principal wishes to implement a certain timing τ (i.e., an investment trigger x_τ) she could rely on a much simpler direct revelation mechanism that does not require exchange of information between the parties about the realizations of the state variable, with the exception of the initial value x .

Under some regularity conditions, Kruse and Strack show that, for any given x_τ , there exists a transfer S , independent of the future realizations of x_t , which specifies the payment due at the time of investment, such that it becomes optimal for the firm to invest when x_τ is

hitted for the first time. Furthermore, by interpreting x_τ as a reflecting barrier¹², they show that the transfer S admits a closed-form solution, given by the expected present value of all future cash flows that the firm, once x_τ is reached, would loose if x_t was kept below x_τ forever.

Since S is independent of x_t , the transfer happens to be ex-post incentive-compatible. In other words, if the firm has not invested up to t , he will never find it optimal to invest before x_τ and there is no reason why he should change strategy in the future, i.e.:

$$x_\tau = \arg \max F(x_t; S) \quad \text{for all } t < \tau \quad (13)$$

where $F(x_t; S)$ is given by (10) and $\tau = \inf(t > 0; x_t = x_\tau)$.

Using these arguments, the firm will invest the first time cash flows hit x_τ , with the latter given by (see Appendix A):¹³

$$\frac{\beta_1 - 1}{\beta_1} x_\tau + r \left[S - \frac{x_\tau}{\beta_1} \frac{\partial O}{\partial x_\tau} \right] = c + r(I - O(x_\tau)) \quad (14)$$

Condition (14) simply says that the firm will invest when the expected marginal cost (i.e., the RHS) is equal to the expected marginal benefit (the LHS). In other words, the government can govern the timing of investment either directly, by subsidizing capital costs, or indirectly, by increasing the exit-option value (i.e., by lowering the termination fee).

Hence, working backwards, a sufficient condition for a contract to be ex-ante incentive-compatible is when the firm announces the true initial value x . In the specific, by substituting (14) into (10), the value to invest for a firm of type x is given by:

¹²A reflected process is like a process that has the same dynamics as the original process, but is required to stay below a given barrier whenever the original process tends to exceed it. See Harrison (2013) for a formal definition of these processes.

¹³Kruse and Strack show that a transfer S exists even when the principal's benefits (i.e., in our framework, b) are time-dependent (see footnote 1), in which case the optimal investment time will also be time-dependent. However, in general, a closed-form solution for $x_\tau(t)$ (and, thus, for $S(t)$), does not exist, and numerical methods are required.

$$\begin{aligned}
F(x) &= \left(\frac{x}{x_\tau}\right)^{\beta_1} \frac{x_\tau}{\beta_1 r} \left[1 + r \frac{\partial O}{\partial x_\tau}\right] \\
&= \left(\frac{x}{x_\tau}\right)^{\beta_1} \frac{x_\tau}{\beta_1 r} \left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\right] \quad \text{for } x^E < x < x_\tau
\end{aligned} \tag{15}$$

where the term $\frac{x_\tau}{\beta_1 r} \left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\right]$ is the NPV of the project at the moment of investment.

Compared with other incentive-compatible methods, the approach envisaged by Kruse and Strack has several attractive features. First, since the transfer is independent of the future realizations of the state variable, it can be paid even when cash flows are not observable, that is, when alternative mechanisms, such as contingent pay schemes, are not implementable. Second, since the transfer is only contingent to the observable investment decision, there is no need of any further transmission of information between the parties. In other words, the government only needs to know that the investment was made, rather than the reasons why it has happened (Board, 2007). Finally, since, once the contract has been awarded (at $t = 0$), the firm has no further incentives to misreport, the government is able to learn the true value of x_t at the time of exit by the firm (i.e., x^E). This let us to extend the Kruse and Strack's methodology to the case where (unlike their model) the firm holds also an exit option.

The following Lemma allows us to apply the results of Kruse and Strack to our setting.

Lemma 1 Letting x_τ an arbitrary investment trigger and $\tilde{x}_t$ a version of the process x_t reflected at x_τ , the stopping time $\tau = \inf(t > 0 / x_t = x_\tau)$ can be implemented by the following incentive-compatible transfer:

$$\begin{aligned}
S &= I - O(x_\tau) - \frac{x_\tau - c}{r} + E_\tau \left\{ \int_\tau^\infty e^{-r(t-\tau)} \left| \frac{\partial O}{\partial x_t}(x_\tau) \right| d(x_t - \tilde{x}_t) \right\} \\
&= I - O(x_\tau) - \frac{x_\tau - c}{r} + \frac{x_\tau}{\beta_1 r} \left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\right]
\end{aligned} \tag{16}$$

Proof: See Appendix B.

Comparison between Eq. (16) and Eq. (12) shows that, as in the case of symmetric information, the transfer S should cover the difference between the capital outlay I and the expected present value of operating profits, net of the exit option value. However, under asymmetric information the transfer must be increased by an additional payment such that the firm will not find suitable to delay the investment beyond the government's desired entry trigger x_τ .

Specifically, since for $t \geq \tau$ the firm's marginal incentive to delay is $d(x_t - \tilde{x}_t)$, the first line in (16) shows that the information rents can be determined by calculating the expected present value of all future cash flows the firm would lose by keeping x_t below x_τ . However, since the firm also benefits of an exit option, the lost revenues are remunerated at a lower rate than the interest rate r , namely, at $|\frac{\partial O}{\partial x_t}| < 1/r$.¹⁴

Moreover, the second line in (16) shows that the integral admits a closed-form solution and that the information rents, represented by the last term of the RHS of Eq. (16), are nothing but the net present value to invest evaluated at x_τ .

Armed with these insights, let's return to the government's revenue-maximization problem. Since an investment time $\tau = \inf(t > 0; x_t = x_\tau)$ can be implemented by the transfer (16), by the standard mechanism design approach (Laffont and Martimort, 2002) we can confine the analysis on menus of ex-ante incentive-compatible contracts that induce the firm to reveal his initial type $x \in [x^l, x^h]$.

Hence, the government's problem reduces to choosing the investment trigger $x_\tau(x)$ that maximizes the following objective function:

¹⁴In Appendix B we show that the term $\frac{\partial O}{\partial x_t} d(\tilde{x}_t - x_t) > 0$ represents the cost per unit of the distance through which x_t is reflected to keep x_t at x_τ .

$$\begin{aligned}
\mathcal{R}(x, x_\tau(x)) &\equiv \int_{x_l}^{x_h} R(x, x_\tau(x)) g(x) dx \\
&= \int_{x_l}^{x_h} [W(x, x_\tau(x)) - F(x, x_\tau(x))] g(x) dx
\end{aligned} \tag{17}$$

where $F(\cdot)$ is given by (15).

In order to ensure that the contract duration is always positive and to avoid bunching, we introduce the following assumption which guarantees that the second order condition is satisfied and that the optimal trigger $x_{\tau R}(x)$ is decreasing in $x \in [x^l, x^h]$.¹⁵

Assumption 1. $\frac{x^E}{x_{\tau W}} < K \min \left[\frac{g(x^l)x^l-1}{\beta_1-1}, 1 \right]$, where $K = \left(\frac{\beta_1-1}{\beta_1-\beta_2} \right)^{1/1-\beta_2} < 1$.

The solution is summarized in the following proposition.

Proposition 1 Under Assumption 1, for any given exit fee $L < \frac{c}{r}$:

a) the government's revenue is maximized by the investment time $\tau^R(x) = \inf(t > 0 / x_t = x_{\tau R}(x))$, where the investment trigger $x_{\tau R}(x)$ is defined by:

$$x_{\tau R}(x) = x_{\tau W} + [x_{\tau W} - r(\beta_1 - \beta_2)O(x_{\tau R}(x))] \frac{\phi(x)}{1 - \phi(x)} \tag{18.1}$$

with $\frac{\partial x_{\tau R}(x)}{\partial x} < 0$;

b) the transfer that implements $\tau^R(x)$ is:

$$S^R(x) = \frac{b}{r} - \frac{\beta_1 - \beta_2}{\beta_1} O(x_{\tau R}(x)) - \left(\frac{\beta_1 - 1}{\beta_1} \right) \frac{x_{\tau R}(x) - x_{\tau W}}{r} \tag{18.2}$$

with $\frac{\partial S^R(x)}{\partial x} > 0$.

Proof: See Appendix C.

¹⁵Although it is not the main focus of our work, in Appendix B we briefly discuss the consequences of violating Assumption 1. We show that when the exit trigger x^E is relatively high (i.e., when the exit fee is low), the optimal contract can involve a bunching interval for the most efficient types.

Eq. (18.1) shows that, except for the highest-type contractor ($x = x^h$), the government has an incentive to delay the investment compared with the first-best solution, that is, $x_{\tau R}(x) > x_{\tau W}$.¹⁶ Moreover, comparison between (18.2) and (12) shows that (except for $x = x^h$) the optimal subsidy is lower than the one paid under full information: $S^R(x) < S^W$. Thus, as usual in a principal-agent problem, the government faces a rent-efficiency trade-off.

Eq. (18.1) also confirms the result underlined by condition (14): the government can always reduce the rents left to all inframarginal types by lowering L , in order to increase the exit option value and, in so doing, accelerate investment. Furthermore, the time distortion, used to squeeze information rents, allows the government to save on capital subsidies.

By substituting (18.1) into (17) we get that the government's expected revenue depends only on the optimal trigger $x_{\tau R}(x)$ and equals the expected welfare when the latter is replaced by the “virtual welfare” (Myerson, 1981):

$$\mathcal{R}(x, x_{\tau R}(x)) = \int_{x_l}^{x_h} \hat{W}(x, x_{\tau R}(x)) g(x) dx \quad (19)$$

where:

$$\hat{W}(x, x_{\tau R}(x)) \equiv W(x, x_{\tau R}(x)) - \frac{1 - G(x)}{g(x)} \frac{1}{\beta_1} F_x(x, x_{\tau R}(x)) \quad (20)$$

Moreover, by using (18.2), we are also able to generalize the Myersonian equivalence between the expected revenue and the expected virtual surplus:

$$\hat{W}(x, x_{\tau R}(x)) = \left[F(x, x_{\tau R}(x)) - \frac{1 - G(x)}{g(x)} \frac{1}{\beta_1} F_x(x, x_{\tau R}(x)) \right] + \hat{O}(x_{\tau R}(x)) \quad (21)$$

where $\hat{O}(x_{\tau R}(x)) = -E_0(e^{-r\tau R(x)})\beta_2 \left[\frac{1}{\beta_1} - \phi(x) \right] O(x_{\tau R}(x))$.

The squared term on the RHS of (21) indicates the standard “virtual value”, while the second term measures what the government expects to earn by increasing the value of the exit-option value.

¹⁶When $x = x^h$, then $\phi(x^h) = 0$ and $x_{\tau R}(x^h) = x_{\tau W}$.

By substituting (21) in (19) and taking the derivative with respect to L , we get (see Appendix D):

$$\frac{\partial \mathcal{R}}{\partial L} = \int_{x_l}^{x_h} \frac{\partial \hat{O}(x_{\tau R}(x))}{\partial L} g(x) dx = \frac{\beta_2 - 1}{\frac{c}{r} - L} \int_{x_l}^{x_h} \hat{O}(x_{\tau R}(x)) g(x) dx \quad (22)$$

Since $\frac{\beta_2 - 1}{\frac{c}{r} - L} < 0$, the sign of (22) depends on the sign of $\hat{O}(x_{\tau R}(x))$ which, in turn, depends on the sign of $\left[\frac{1}{\beta_1} - \phi(x)\right]$. Specifically, since $\phi'(x) < 0$ and $\beta_1 > 1$, the term $\left[\frac{1}{\beta_1} - \phi(x)\right]$ is positive for all values of x when the distribution of types is such that $g(x^l)x^l > \beta_1$. In this case a reduction of the exit fee always contributes to increasing the government's payoff, i.e., $\frac{\partial \mathcal{R}}{\partial L} < 0$.¹⁷ On the contrary, when $1 < g(x^l)x^l < \beta_1$, the effect of a reduction of L remains positive only if the firm happens to be relatively efficient (i.e., high values of x). Moreover, since β_1 is decreasing in σ , the term $\left[\frac{1}{\beta_1} - \phi(x)\right]$ is more likely to be positive if the volatility of cash flows (i.e., the riskiness of the project) is particularly high.

The above results are summarized in the following proposition.

Proposition 2 The government maximizes the expected revenue by maximizing the value of the exit option (i.e., by setting $L = 0$) if:

- a) given the level of uncertainty σ , the distribution of types is such that $g(x^l)x^l > \beta_1$,
- b) or if, given the distribution of types $\phi(x)$, the level of uncertainty is high.

Proof: See Appendix D.

The first part of the proposition becomes clear if we look more in detail at the investment trigger for a contractor of type x^l (the lowest possible efficiency level):

$$x_{\tau R}(x^l) = x_{\tau W} + [x_{\tau W} - r(\beta_1 - \beta_2)O(x_{\tau R}(x^l))] \frac{1}{g(x^l)x^l - 1} \quad (23)$$

The second term on the RHS of Eq. (23) is always positive. However, since $\beta_1 > 1$, if $g(x^l)x^l > \beta_1$, then $\frac{1}{g(x^l)x^l - 1} < 1$, that is, the term tends to become negligible, so that

¹⁷For example, if we consider an uniform distribution with $x \in [2, 3]$, then $\phi(x) = \frac{3}{x} - 1$. Moreover if $r = 0.05$ and $\sigma = 0.25$, such that $\beta_1 = 1.86$, we obtain $\frac{1}{\beta_1} - \phi(x) = 1.54 - \frac{3}{x} > 0$ for all x .

$x_{\tau R}(x^l) \simeq x_{\tau W}$, meaning that the least and the most efficient type will invest more or less at the same time.

As for the second part of the proposition, the simple intuition is that, as $\frac{\partial \beta_1}{\partial \sigma} < 0$, the higher is σ , the less likely is that $1 < g(x^l)x^l < \beta_1$. In other words, a high level of uncertainty about future cash flows could make also an inefficient contractor more willing to accept a lower capital subsidy in exchange of more flexibility in service duration.

Summarizing, in the case of asymmetric information the granting of an exit option can increase the government's net revenue, even while taking into account the potential cost deriving from the need of resuming a financially loss-making project. The reason is that, in order to squeeze information rents, the government does not need to delay project implementation as much as it would occur if the private investment was only sustained by cash payments. Stated differently, an exit option allows the government to increase the value for money of public spending, by accelerating investment at a lower (expected) total (i.e., direct and indirect) financial cost.¹⁸

Notice that this result is essentially similar to that of Arve and Martimort (2016) who, using a two-period model with uncorrelated shocks, show (Proposition 2, p. 3254) that when the agent is risk-averse in the second period, the regulator can relax the first-period incentive-compatibility constraint by offering the agent higher profits in the second period. The reason is that the income effect, induced by risk aversion, reduces the production distortion in the first period and, thus, the government does not need to distort production as much as under risk neutrality. Here we obtain a similar result, by embedding an exit option within a continuous-time model with risk-neutral agents.

Before concluding, it is worth looking at the effect of exit flexibility on the firm's expected payoff. By totally differentiating (17) with respect to the exit fee, we get:

¹⁸As shown in Appendix E this result holds even when the government will have to afford additional operating costs, beyond those the firm himself would have incurred absent the termination, unless additional costs are "very high", in which case it would be optimal for the government to lock in the firm by setting $L = \frac{c}{r}$.

$$\begin{aligned}
\frac{\partial F(x)}{\partial L} &= \left[\frac{\partial F(x)}{\partial x_{\tau R}} \frac{\partial x_{\tau R}}{\partial x^E} + \frac{\partial F(x)}{\partial x^E} \right] \frac{\partial x^E}{\partial L} \\
&\propto \left[(\beta_1 - 1) \left(1 - \left(\frac{x_{\tau R}}{x^E} \right)^{\beta_2 - 1} \right) \frac{\partial x_{\tau R}}{\partial x^E} - (\beta_2 - 1) \left(\frac{x_{\tau R}}{x^E} \right)^{\beta_2 - 1} \left(\frac{\partial x_{\tau R}}{\partial x^E} - 1 \right) \right] \frac{\partial x^E}{\partial L} > 0
\end{aligned} \tag{24}$$

Eq. (24) shows that the lower is the exit fee, the lower will be the project's total value to the firm. This apparently counterintuitive result can be explained as follows.

Within our model, the incentives used by the government to spur investment can be thought as a compound option since, on the one hand, the firm is entitled to receive a fixed subsidy upon investment (a call option) and, on the other, he is allowed to terminate the contract at a strike price (a put option). While efficient firms attach a higher value to the possibility of quitting the project (because they invest earlier), the reverse applies to less efficient firms, which attribute a greater value to the call option. This negative correlation thus reduces the variance of the project value across different types and, in so doing, allows the government to squeeze information rents and to save on public funds.¹⁹

6 Final remarks

Public-private partnerships for the provision of public infrastructures and services have gained increased interest over the past decades. While many PPPs have been success stories, others have largely failed to meet expectations. For instance, the frequency of early terminations and renegotiations has raised concern about the real benefits of PPPs over direct provision of public services or more conventional procurement methods, and has stimulated a debate on how to prevent or mitigate the effects of breach of contracts.

¹⁹A similar result can be found in Board (2007) and Dosi and Moretto (2015) where, however, rent extraction comes through the competitive bid pressure generated by a put option.

Our paper contributes to this debate by arguing that the project’s abandonment by the private party may not constitute a problem per se as long as the risk of termination is properly (“proactively”) accounted for by the government at the time of contract formation. Specifically, we have shown that the granting of an exit option can contribute to accelerate private investment, and thus, provision of public benefits, at a relatively lower cost compared with “lock-in” contracts.

In particular, the blending of direct subsidies and option incentives could prove particularly useful in the case of risky projects, when an exit option, by softening agency problems, can contribute to increase the government’s net revenue, even while taking into account the potential costs that the public sector will have to meet in the future to take direct responsibility over a financially loss-making activity.

A Equations 14 and 15.

For every transfer S , the privately optimal investment strategy can be represented by a cut-off function x_τ such that it is optimal to invest the first time that x_t hits x_τ . Let's suppose that a transfer S independent of x_t exists. Hence, in the range where $x^E < x_t < x_\tau$, $F(x_t)$ satisfies the first order condition:

$$\left(\frac{x_t}{x_\tau}\right)^{\beta_1} \left\{ -\beta_1 \left(\frac{1}{x_\tau}\right) \left[\frac{x_\tau - c}{r} - (I - O(x_\tau)) + S \right] + \left[\frac{1}{r} + \frac{\partial O(x_\tau)}{\partial x_\tau} \right] \right\} = 0 \quad (\text{A.1})$$

while the second order condition is always satisfied:

$$\left(\frac{x_t}{x_\tau}\right)^{\beta_1} \left\{ \beta_1 \left(\frac{1}{x_\tau}\right) \left[\left(\frac{1}{x_\tau}\right) \left(\frac{x_\tau - c}{r} - (I - O(x_\tau)) + S \right) - \right. \right. \quad (\text{A.2})$$

$$\left. \left. \frac{1}{r} - \frac{\partial O(x_\tau)}{\partial x_\tau} \right] + \frac{\partial^2 O(x_\tau)}{\partial x_\tau^2} \right\} < 0$$

Substituting (A.1) in (15) we get:

$$F(x_t) = \left(\frac{x_t}{x_\tau}\right)^{\beta_1} \frac{x_\tau}{\beta_1 r} \left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1} \right] \quad \text{for } x^E < x_t < x_\tau \quad (\text{A.3})$$

Now, let's define the firm's continuation value as:

$$\begin{aligned} u(x_t) &= F(x_\tau) + S - F(x_t) \\ &= \frac{x_\tau}{\beta_1 r} \left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1} \right] + S - \left(\frac{x_t}{x_\tau}\right)^{\beta_1} \frac{x_\tau}{\beta_1 r} \left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1} \right] \end{aligned} \quad (\text{A.4})$$

Eq. (A.4) indicates the firm's willingness to pay for holding the option to delay the investment beyond the current value of cash flow x_t . By definition, the continuation value is

non-negative, i.e. $u(x_t) - S \geq 0$. If $u(x_t) - S > 0$, the firm will continue to hold the option alive, whereas if $u(x_t) - S = 0$ there will be no gain to go forward. Further, since $\beta > 1$, $u(x_t)$ is decreasing in x_t :²⁰

$$\frac{\partial u(x_t)}{\partial x_t} = -\frac{1}{r} \left(\frac{x_t}{x_\tau} \right)^{\beta-1} \left[1 - \left(\frac{x_\tau}{x^E} \right)^{\beta-1} \right] < 0 \quad \text{for } x^E < x_t < x_\tau \quad (\text{A.5})$$

Since, at every point in time, the firm can decide whether to invest or to postpone the decision, if it is optimal to invest with x_t , then it is also optimal to invest for $x'_t > x_t$. Thus, as the marginal incentive to keep the option alive decreases as x_t increases, there exists, for any transfer S , a level of cash flow x_τ where is optimal to invest.

B Proof of Lemma 1.

We prove the Lemma in two steps by adapting the procedure proposed by Kruse and Strack (2015, Theorem 1) and Kruse and Strack (2019, Theorem 11).

First step

For any time-autonomous investment time τ (i.e., x_τ), the associated transfer S that implements it can be calculated as the expected discounted value of future cash flows the firm would lose if the process x_t cannot exceed (is reflected to) x_τ once that it is reached.

For a given barrier a , a reflected process is a process that has the same dynamics as the original one but it is required to stay below a whenever the original process tends to exceed it. Defining $\tilde{x}_t$ the reflected process, it can be represented as (Harrison 2013):

$$\tilde{x}_t \equiv x_t / D_t, \quad \text{for } \tilde{x}_t \in (0, a], \quad (\text{B.1})$$

where:

²⁰Note that the fact that Eq. (A.4) is decreasing in x_t follows from what Kruse and Strack (2015), call “dynamic single crossing” condition. Arve and Zwart (2014) refer to Eq. (A.5) as the ex-post incentive compatible condition.

- *i*) x_t is a geometric Brownian motion, with stochastic differential as in (1);
- *ii*) D_t is an increasing and continuous process, with $D_0 = 1$ if $x_0 \leq a$, and $D_0 = x_0/a$ if $x_0 > a$, so that $\tilde{x}_0 = a$;
- *iii*) D_t increases only when $\tilde{x}_t = a$.

By applying Ito's lemma to (B.1), we get:

$$d\tilde{x}_t = \sigma\tilde{x}_t dz_t - d\tilde{D}_t, \quad \tilde{x}_t \in (0, a] \quad (\text{B.2})$$

where $d\tilde{D}_t = \tilde{x}_t \frac{dD_t}{D_t}$ indicates the infinitesimally small level of “regulation” exerted to let x_t stay at a . By (B.2), until x_t hits for the first time a the two processes coincide, i.e. $\tilde{x}_t = x_t$, and after that we get $\tilde{x}_t < x_t$. In particular, when $\tilde{x}_t = a$, we get $d\tilde{x}_t = 0$ and the rate of variation of D_t is equal to the one required to keep $\tilde{x}_t$ constant.

Now, defining the difference $\tilde{x}_t - x_t \equiv U_t = (D_t - 1)x_t$ as the cumulative amount of cash-flows lost up to t to keep the process below a , we are able to calculate the expected future values of cash flows evaluated at the process reflected at a . In the specific, generalizing for any arbitrary initial value x_t , $t > 0$, we get:

$$v(x_t, a) = E_t \left\{ \int_t^\infty e^{-r(s-t)} [(\tilde{x}_s - c)ds + \varrho dU_s] \right\} \quad (\text{B.3})$$

where $\varrho > 0$ is the marginal reflection cost (i.e., the value attributed to each unit of cash flows) and dU_s is the reduction of cash flows, if any, in the interval $(s, s + ds)$. For all $x_t < a$, the function $v(x_t; a)$ is the unique solution of the following partial differential equation:

$$\frac{1}{2}\sigma^2 x_t^2 \frac{\partial^2 v(x_t, a)}{\partial x_t^2} - rv(x_t, a) + \frac{x_t}{r} = 0 \quad (\text{B.4})$$

with the boundary conditions:

$$\frac{\partial v(x_t, a)}{\partial x_t} \Big|_{x_t=a} = \varrho \quad (\text{B.5})$$

The general solution of (B.4) is:

$$v(x_t, a) = \frac{x_t - c}{r} + Ax_t^{\beta_1} \quad (\text{B.6})$$

where A is a constant to be determined.

Imposing the boundary condition (B.5), it is easy to show that:

$$A = \frac{a}{\beta_1 r} [r\varrho - 1] a^{-\beta_1} \quad (\text{B.7})$$

which is negative if $\varrho < 1/r$.

By (B.6) and (B.7) the expected discounted value of lost cash flows is given by:

$$Ax_t^{\beta_1} = E_t \left\{ \int_t^\infty e^{-r(s-t)} [\varrho dU_s] \right\} = \left(\frac{x_t}{a} \right)^{\beta_1} \frac{a}{\beta_1} \left[\varrho - \frac{1}{r} \right] \quad (\text{B.8})$$

Setting $a = x_\tau$ and the reflection cost equals to the marginal value of the exit option evaluated at $a = x_\tau$, i.e. $\varrho = -\frac{\partial O}{\partial x}(x_\tau) = \frac{1}{r} \left(\frac{x_\tau}{x^E} \right)^{\beta_2-1}$, we get:

$$v(x_t, x_\tau) = \frac{x_t - c}{r} - \left(\frac{x_t}{x_\tau} \right)^{\beta_1} \frac{x_\tau}{\beta_1 r} \left[1 - \left(\frac{x_\tau}{x^E} \right)^{\beta_2-1} \right] \quad (\text{B.9})$$

where we express ϱ in term of present value, i.e., the reflection cost has the dimension of the present value of the marginal cost of one unit of cash flows lost forever. Notice that the second term on the R.H.S. of (B.9) is indeed the value to invest (A.3).

Second step

Finally, direct inspection of (A.4) and (B.9) shows that, provided that $v(x_t, x_\tau) < I - O(x_t)$, a transfer $S(x_\tau) = (I - O(x_t) - v(x_t, x_\tau))$ compensates the firm at each time t for the loss of value due to the reflecting barrier, i.e. $u(x_t, x_\tau) - S(x_\tau) > 0$. When $x_t = x_\tau$, we get $u(x_\tau, x_\tau) - S(x_\tau) = 0$ and the firm is indifferent whether to invest or to postpone the

decision. Hence, the transfer is:

$$\begin{aligned} S(x_\tau) &= (I - O(x_\tau)) - v(x_\tau, x_\tau) \\ &= I + \frac{c}{r} - \frac{\beta_1 - 1}{\beta_1} \frac{x_\tau}{r} - \frac{\beta_1 - \beta_2}{\beta_1} O(x_\tau) \end{aligned} \tag{B.10}$$

C Proof of Proposition 1

For every initial value x there exists a non-increasing function $\tau(x) = \inf(t > 0 / x_t = x_\tau(x))$ that maximizes (17). Further, the allocation mechanism $\tau(x)$ can be implemented by using simple transfers that only depend on the initial value x and the realized time of investment. Specifically, along with the condition (A.5), with the definition that $u(x) = u(x, x_\tau(x))$, an incentive compatible contract requires the monotonicity of the optimal investment trigger, i.e.:

$$\frac{dx_\tau(x)}{dx} < 0 \tag{C.1}$$

Conditions (A.5) and (C.1) are the first and the second order incentive-compatibility constraints to induce the firm to reveal his type, i.e., x . The standard approach is to ignore, for the moment, the monotonicity constraint (C.1) and to solve the relaxed problem. For any choice of $x_\tau = x_\tau(\hat{x})$, by applying the envelope theorem where the firm maximizes over both the report $\hat{x}$ and the investment time $x_\tau(\hat{x})$, we get the ex-ante value of the firm's option to invest as:

$$\begin{aligned} u(x) - u(x_l) &= - \int_{x_l}^x \frac{1}{r} \left(\frac{y}{x_\tau} \right)^{\beta_1 - 1} \left[1 - \left(\frac{x_\tau}{x^E} \right)^{\beta_2 - 1} \right] dy \\ F(x) &= F(x_l, x_\tau(x_l)) + \int_{x_l}^x \frac{1}{r} \left(\frac{y}{x_\tau} \right)^{\beta_1 - 1} \left[1 - \left(\frac{x_\tau}{x^E} \right)^{\beta_2 - 1} \right] dy \end{aligned} \tag{C.2}$$

where by the Revelation Principle, the optimal choice of $\hat{x}$ is x .²¹ Since $F(x)$ is increasing

²¹Eq (C.2) follows from the application of the envelope theorem. For the integral form of the envelope

in x , it is optimal for the government to set the transfer in such a way so that the value of the lowest type is zero, i.e. $F(x_l, x_\tau(x_l)) = 0$. Substituting (C.2) in (19), the government's objective function becomes:

$$\mathcal{R}(x, x_\tau) = \int_{x_l}^{x_h} \left[W(x, x_\tau) - \frac{1}{r} \int_{x_l}^x \left(\frac{y}{x_\tau} \right)^{\beta_1-1} \left(1 - \left(\frac{x_\tau}{x^E} \right)^{\beta_2-1} \right) dy \right] g(x) dx \quad (\text{C.3})$$

Integrating by parts the second term on r.h.s. of (C.3), yields:

$$\begin{aligned} & \int_{x_l}^{x_h} \int_{x_l}^x \left(\frac{y}{x_\tau} \right)^{\beta_1-1} \left(1 - \left(\frac{x_\tau}{x^E} \right)^{\beta_2-1} \right) dy g(x) dx \\ &= \int_{x_l}^{x_h} \left(\frac{x}{x_\tau} \right)^{\beta_1-1} \left(1 - \left(\frac{x_\tau}{x^E} \right)^{\beta_2-1} \right) (1 - G(x)) dx \end{aligned} \quad (\text{C.4})$$

Substituting (C.4) in (C.3), $\mathcal{R}(x, x_\tau)$ reduces to:

$$\mathcal{R}(x, x_\tau) = \int_{x_l}^{x_h} \left(\frac{x}{x_\tau} \right)^{\beta_1} \left\{ \frac{x_\tau}{r} \left[1 - \left(1 - \left(\frac{x_\tau}{x^E} \right)^{\beta_2-1} \right) \phi(x) \right] - \left(I + \frac{c}{r} - \frac{b}{r} \right) \right\} g(x) dx \quad (\text{C.5})$$

where $\phi(x) = \frac{1-G(x)}{g(x)x}$, with $\phi'(x) < 0$, $\phi(x^h) = 0$ and $\phi(x^l) = \frac{1}{g(x^l)x^l} < 1$.

By maximizing (C.5) with respect x_τ the first order condition is:

$$x_{\tau w} - x_\tau(1 - \phi(x)) - \frac{(\beta_1 - \beta_2)}{(\beta_1 - 1)} \left(\frac{x_\tau}{x^E} \right)^{\beta_2-1} x_\tau \phi(x) = 0 \quad (\text{C.6})$$

from which we obtain the expression in the text with $x_\tau(x^h) = x_{\tau w}$.

Equation (C.6) may admit two solutions. Let's define $f(x_\tau, x_{\tau w}) \equiv x_{\tau w} - (1 - \phi(x))x_\tau - \left(\frac{\beta_1 - \beta_2}{\beta_1 - 1} \right) \left(\frac{x_\tau}{x^E} \right)^{\beta_2} x^E \phi(x)$. It is easy to show that $f(x_\tau, x_{\tau w})$ is concave in x_τ , i.e. :

$$f'(x_\tau, x_{\tau w}) = -(1 - \phi(x)) + (\beta_1 - \beta_2) \left(\frac{x_\tau}{x^E} \right)^{\beta_2-1} \phi(x)$$

theorem, see Milgrom (2004).

and

$$f''(x_\tau, x_{\tau w}) = (\beta_1 - \beta_2)(\beta_2 - 1) \left(\frac{x_\tau}{x^E}\right)^{\beta_2-2} \frac{1}{x^E} \phi(x) < 0$$

Moreover, since $\lim_{x_\tau \rightarrow \infty} f(x_\tau, x_{\tau w}) = -\infty$ and $\lim_{x_\tau \rightarrow 0} f(x_\tau, x_{\tau w}) = -\infty$, the maximum is given by:

$$(\beta_1 - \beta_2) \left(\frac{x_\tau^{\max}}{x^E}\right)^{\beta_2-1} \frac{\phi(x)}{1 - \phi(x)} = 1$$

Substituting $x_\tau^{\max}$ into $f(x_\tau, x_{\tau w})$ we obtain:

$$f(x_\tau^{\max}, x_{\tau w}) = x_{\tau w} - (\beta_1 - \beta_2) \left(\frac{x_\tau^{\max}}{x^E}\right)^{\beta_2} x^E \phi(x) \left(\frac{\beta_1}{\beta_1 - 1}\right)$$

Hence, if $f(x_\tau^{\max}, x_{\tau w}) > 0$, the first order condition (C.6) admits two solutions and the optimal one satisfies $x_\tau > x_\tau^{\max}$.

Let's now prove the monotonicity. By totally differentiating (C.6), we obtain:

$$\frac{dx_\tau}{dx} \left[(1 - \phi(x)) - (\beta_1 - \beta_2) \left(\frac{x_\tau}{x^E}\right)^{\beta_2-1} \phi(x) \right] = x_\tau \phi'(x) \left[1 - \frac{(\beta_1 - \beta_2)}{(\beta_1 - 1)} \left(\frac{x_\tau}{x^E}\right)^{\beta_2-1} \right] \quad (\text{C.7})$$

Defining $\Omega(x_\tau) \equiv \frac{(\beta_1 - \beta_2)}{(\beta_1 - 1)} \left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}$, it is easy to show that $\frac{dx_\tau}{dx} < 0$ if:

$$\Omega(x_\tau) < \frac{1}{\beta_1 - 1} \frac{1 - \phi(x)}{\phi(x)} = \frac{1}{\beta_1 - 1} \frac{(x - \frac{1-G(x)}{g(x)})}{\frac{1-G(x)}{g(x)}} \quad (\text{C.8.1})$$

and

$$\Omega(x_\tau) < 1 \quad (\text{C.8.2})$$

hold simultaneously, where (C.8.1) is the second order condition of the maximization, while (C.8.2) implies $x_\tau > x_{\tau w}$, i.e.:

$$\begin{aligned}
1 - \Omega(x_\tau) &= \left[1 - \frac{(\beta_1 - \beta_2)}{(\beta_1 - 1)} \left(\frac{x_\tau}{x^E} \right)^{\beta_2 - 1} \right] \\
&= \left[\frac{1}{\phi(x)} - \frac{1}{x_\tau \phi(x)} \frac{\beta_1}{(\beta_1 - 1)} r \left(I + \frac{c}{r} - \frac{b}{r} \right) \right] \\
&= \frac{1}{\phi(x)} \frac{1}{x_\tau} [x_\tau - x_{\tau w}]
\end{aligned}$$

Since $\frac{1-\phi(x)}{\phi(x)}$ is increasing in x , a sufficient condition for the second order condition (C.8.1) to hold is $\Omega(x_\tau) < \frac{g(x^l)x^l-1}{\beta_1-1}$. Thus, we need to distinguish two cases:

1. If $g(x^l)x^l \in (1, \beta_1]$, then (C.8.2) is satisfied if and only if (C.8.1) is also satisfied;
2. If $g(x^l)x^l > \beta_1$, then (C.8.1) is satisfied if and only if (C.8.2) is also satisfied.

Let's consider the first case where $g(x^l)x^l \in (1, \beta_1]$. This implies that $\frac{g(x^l)x^l-1}{\beta_1-1} < 1$ and, since $x_\tau(x^h) = x_{\tau w}$, we can reduce (C.8.1) to:

$$\left(\frac{x^E}{x_{\tau w}} \right) < K \frac{g(x^l)x^l - 1}{\beta_1 - 1}$$

where $K = \left(\frac{\beta_1-1}{\beta_1-\beta_2} \right)^{1/1-\beta_2} < 1$.

As for the second case we get $\frac{g(x^l)x^l-1}{\beta_1-1} > 1$. Thus, condition (C.8.2) is satisfied by simply setting $\left(\frac{x^E}{x_{\tau w}} \right) < K$. Therefore, Assumption 1 guarantees that both (C.8.1) and (C.8.2) are satisfied.

Suppose now that $\frac{x^E}{x_{\tau w}}$ is such that Assumption 1 is not satisfied. Then, for higher value of x , conditions (C.8.1) and (C.8.2) may not hold. In this case we have an interval $[x', x''] \subset [x^l, x^h]$ where a constant trigger (bunching) applies such that $x_\tau(x') = x_\tau(x'') = \bar{x}_\tau > x_\tau(x^h) = x_{\tau w}$. Hence, from (C.7), we should have $\phi'(x') = \phi'(x'')$ and from (C.6):

$$-(1 - \phi(x'))\bar{x}_\tau - \left(\frac{\beta_1 - \beta_2}{\beta_1 - 1}\right) \left(\frac{\bar{x}_\tau}{x^E}\right)^{\beta_2} x^E \phi(x') = -(1 - \phi(x''))\bar{x}_\tau - \left(\frac{\beta_1 - \beta_2}{\beta_1 - 1}\right) \left(\frac{\bar{x}_\tau}{x^E}\right)^{\beta_2} x^E \phi(x'') \quad (\text{C.9})$$

$$(\phi(x') - \phi(x'')) \left[1 - \left(\frac{\beta_1 - \beta_2}{\beta_1 - 1}\right) \left(\frac{\bar{x}_\tau}{x^E}\right)^{\beta_2 - 1} \right] \bar{x}_\tau = 0$$

$$(\phi(x') - \phi(x'')) [1 - \Omega(\bar{x}_\tau)] \bar{x}_\tau = 0$$

This leads to a contradiction as $x' > x''$ and $1 - \Omega(\bar{x}_\tau) \neq 0$ except when $\bar{x}_\tau = x_{\tau W}$. Hence we cannot have $x'' < x^h$ and if bunching is optimal it occurs at the top of the interval. However since there is no distortion at the top, the optimal solution is $x_\tau(x^h) = x_{\tau W}$ for an interval $[\hat{x}, x^h] \subset [x^l, x^h]$ for some $\hat{x}$.

The transfer is given by Lemma 1. By substituting (C.6) into (B.10):

$$S(x_\tau) = I + \frac{c}{r} - \frac{\beta_1 - 1}{\beta_1} \frac{x_\tau}{r} - \frac{\beta_1 - \beta_2}{\beta_1} O(x_\tau, x^E) \quad (\text{C.10})$$

$$= \frac{b}{r} - \frac{\beta_1 - \beta_2}{\beta_1} O(x_\tau, x^E) - \frac{\beta_1 - 1}{\beta_1} \left[\frac{x_\tau^W}{r} - (\beta_1 - \beta_2) O(x_\tau; x^E) \right] \frac{\phi(x)}{1 - \phi(x)}$$

with $S(x^h) = \frac{b}{r} - \frac{\beta_1 - \beta_2}{\beta_1} O(x_{\tau W}, x^E) = S^W$. By the monotonicity of x_τ , it is easy to show that:

$$\frac{\partial S}{\partial x} = -\frac{\beta_1 - 1}{\beta_1} \frac{1}{r} \frac{\partial x_\tau}{\partial x} + \frac{\beta_1 - \beta_2}{\beta_1} \frac{1}{r} \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1} \frac{\partial x_\tau}{\partial x} \quad (\text{C.11})$$

$$= \frac{1}{\beta_1 r} \frac{\partial x_\tau}{\partial x} \frac{1}{(\beta_1 - 1)} \left(-1 + \frac{(\beta_1 - \beta_2)}{(\beta_1 - 1)} \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1} \right) > 0$$

Finally, taking the derivative of (C.6) with respect to x^E we get:

$$\frac{\partial x_\tau(x)}{\partial x^E} = -\frac{(\beta_2 - 1) \frac{(\beta_1 - \beta_2)}{(\beta_1 - 1)} \left(\frac{x_\tau}{x^E}\right)^{\beta_2} \phi(x)}{SOC} < 0 \quad (\text{C.12})$$

where the SOC is satisfied by Assumptions 1, and the derivative of (C.10) gives:

$$\frac{\partial S}{\partial x^E} = \frac{(\beta_1 - \beta_2)}{\beta_1} \frac{\partial O(x_\tau, x^E)}{\partial x^E} \left[\frac{\phi(x)}{1 - \phi(x)} (\beta_1 - 1) - 1 \right] \quad (\text{C.13})$$

which is negative for all x if $g(x^l)x^l > \beta_1$. Otherwise $\frac{\partial S}{\partial x^E} > 0$ for low values of x and $\frac{\partial S}{\partial x^E}$ for high values of x if $1 < g(x^l)x^l < \beta_1$.

D Proof of Proposition 2

By substituting (C.6) into (C.5) we get:

$$\begin{aligned} \mathcal{R}(x, x_\tau) &= \int_{x_l}^{x_h} \left(\frac{x}{x_\tau} \right)^{\beta_1} \left\{ \frac{x_\tau}{r} \left[1 - \left(1 - \left(\frac{x_\tau}{x^E} \right)^{\beta_2 - 1} \right) \phi(x) \right] - \left(I + \frac{c}{r} - \frac{b}{r} \right) \right\} g(x) dx + \\ &= \int_{x_l}^{x_h} \left\{ W(x, x_\tau) - \frac{1 - G(x)}{g(x)} \frac{1}{\beta_1} F_x(x, x_\tau) \right\} g(x) dx \end{aligned} \quad (\text{D.1})$$

where $W(x, x_\tau) - \frac{1 - G(x)}{g(x)} \frac{1}{\beta_1} F_x(x, x_\tau)$ indicates the virtual welfare. Further, by substituting (C.10) into (D.1) we get:

$$\begin{aligned} \mathcal{R}(x, x_\tau) &= \int_{x_l}^{x_h} \left(\frac{x}{x_\tau} \right)^{\beta_1} \left\{ \frac{x_\tau}{\beta_1 r} \left[(1 - \phi(x)) + \beta_2 \left(\frac{x_\tau}{x^E} \right)^{\beta_2 - 1} \phi(x) \right] \right\} g(x) dx \\ &= \int_{x_l}^{x_h} \left\{ (1 - \phi(x)) F(x, x_\tau) - E_0(e^{-r\tau}) \beta_2 \left[\frac{1}{\beta_1} - \phi(x) \right] O(x_\tau, x^E) \right\} g(x) dx \end{aligned}$$

To disentangle the effect of L on the government's revenue, we need to determine the sign of $\frac{\partial \mathcal{R}}{\partial L}$. By defining:

$$\Psi(x, x_\tau, x^E) = (1 - \phi(x)) F(x, x_\tau) - E_0(e^{-r\tau}) \beta_2 \left[\frac{1}{\beta_1} - \phi(x) \right] O(x_\tau, x^E), \quad (\text{D.2})$$

the sign of $\frac{\partial \mathcal{R}}{\partial L}$ is given by:

$$\frac{\partial \mathcal{R}}{\partial L} = \int_{x_l}^{x_h} \left[\frac{\partial \Psi(x, x_\tau, x^E)}{\partial x_\tau} \frac{\partial x_\tau}{\partial L} + \frac{\partial \Psi(x, x_\tau, x^E)}{\partial L} \right] g(x) dx.$$

Since x_τ is the optimum, the first term is equal to zero. Thus the derivative simplifies to:

$$\begin{aligned} \frac{\partial \mathcal{R}}{\partial L} &= \int_{x_l}^{x_h} -E_0(e^{-r\tau})\beta_2 \left[\frac{1}{\beta_1} - \phi(x) \right] \frac{\partial O(x_\tau, x^E)}{\partial x^E} \frac{\partial x^E}{\partial L} g(x) dx. \\ &= \int_{x_l}^{x_h} -E_0(e^{-r\tau})\beta_2 \left[\frac{1}{\beta_1} - \phi(x) \right] \left[-\frac{1-\beta_2}{\beta_2 r} \left(\frac{x_\tau}{x^E} \right)^{\beta_2} \right] \left[-\frac{\beta_2}{\beta_2 - 1} r \right] g(x) dx \\ &= \int_{x_l}^{x_h} E_0(e^{-r\tau})\beta_2 \left[\frac{1}{\beta_1} - \phi(x) \right] \left(\frac{x_\tau}{x^E} \right)^{\beta_2} g(x) dx. \\ &= r\beta_2 \frac{1}{x^E} \int_{x_l}^{x_h} -E_0(e^{-r\tau})\beta_2 \left[\frac{1}{\beta_1} - \phi(x) \right] O(x_\tau, x^E) g(x) dx \\ &= r\beta_2 \frac{1}{x^E} \int_{x_l}^{x_h} \hat{O}(x_\tau, x^E) g(x) dx \end{aligned} \tag{D.3}$$

where $\hat{O}(x_{\tau R}(x)) = -E_0(e^{-r\tau R(x)})\beta_2 \left[\frac{1}{\beta_1} - \phi(x) \right] O(x_{\tau R}(x))$.

E Additional resumption costs

In the main text we have assumed that the government will take-over the project by incurring the same cash flows that would have been incurred by the firm, which implies that the resumption cost to the government will be simply given by the (negative) project's market value. However, our main results would not qualitatively change if the government were called to bear additional costs beyond those the firm himself would have incurred absent the termination.

For instance, suppose that the government needs to afford an additional (one-time, sunk) cost $Z \geq 0$. In this case, Eq. (D.1) becomes:

$$\mathcal{R}(x, x_\tau) = \int_{x_l}^{x_h} \{ \Psi(x, x_\tau, x^E) - E_0(e^{-rT})Z \} g(x) dx \tag{E.1}$$

The first derivative is given by:

$$\frac{\partial \mathcal{R}}{\partial L} = \beta_2 \int_{x_l}^{x_h} \left[E_0(e^{-r\tau}) \left[\frac{1}{\beta_1} - \phi(x) \right] \left(\frac{x_\tau}{x^E} \right)^{\beta_2} - \left(\frac{x}{x^E} \right)^{\beta_2} \frac{rZ}{(c - rL)} \right] g(x) dx \quad (\text{E.2})$$

while the second derivative is always positive, i.e.:

$$\frac{\partial^2 \mathcal{R}}{\partial L^2} = -\beta_2 r \left(\frac{rZ}{(c - rL)} \right)^2 \left(\frac{x}{x^E} \right)^{\beta_2} > 0$$

Thus, under the results of Proposition 2, the net revenue R is U-shaped and admits a minimum in the range $(0, \frac{c}{r})$ given by:

$$L^{\min} = \frac{c}{r} - Z \frac{\int_{x_l}^{x_h} \left(\frac{x}{x_\tau} \right)^{\beta_2} g(x) dx}{\int_{x_l}^{x_h} \left(\frac{x}{x_\tau} \right)^{\beta_1} \left[\frac{1}{\beta_1} - \phi(x) \right] g(x) dx} \quad (\text{E.4})$$

If $L^{\min} \leq 0$, then it would be optimal to choose $L = \frac{c}{r}$ (i.e., to lock in the firm). However, more generally, by the mean value theorem, since $\mathcal{R}$ is a continuous function on the closed interval $[0, \frac{c}{r}]$ and is differentiable on the open interval $(0, \frac{c}{r})$, then there exists a point $\hat{L}$ in $(0, \frac{c}{r})$ such that:

$$\mathcal{R}(L = \frac{c}{r}) - \mathcal{R}(L = 0) = \mathcal{R}'(\hat{L}) \frac{c}{r} \quad (\text{E.5})$$

where $\mathcal{R}(\frac{c}{r})$ and $\mathcal{R}(0)$ represent the government's net revenue without and with the exit option respectively. Therefore, we can state that the government will still increase the expected net revenue by maximizing the value of the exit option (i.e., by setting $L = 0$) if additional costs are not too high relative to the benefits arising from injecting flexibility into the contract, i.e. :

$$Z < \left(\frac{c}{r} - \hat{L} \right) \frac{\int_{x_l}^{x_h} \left(\frac{x}{x_{\tau R}(x)} \right)^{\beta_1} \left[\frac{1}{\beta_1} - \phi(x) \right] g(x) dx}{\int_{x_l}^{x_h} \left(\frac{x}{x_{\tau R}(x)} \right)^{\beta_2} g(x) dx} \quad (\text{E.6})$$

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