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Parametric vs. Nonparametric Estimation of an Equilibrium Search Model with Employer Heterogeneity

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Würzburg Economic Papers

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Parametric vs. Nonparametric Estimation of an Equilibrium Search Model with Employer Heterogeneity

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Abstract

We demonstrate that non-monotone behaviour of the right tail of earnings density may lead to the fact that nonparametric estimation of the Bontemps et al. (2000) equilibrium search model with employer heterogeneity does not yield consistent estimates. We propose an easy way of checking whether the Bontemps et al. (2000) estimation procedure can be applied and review the robustness of the alternative strategy once the Bontemps et al. (2000) procedure is inapplicable.

1 Introduction

In the empirical literature on equilibrium search models with identical workers and heterogeneous firms there exist two key estimation methods that differ in their assumption about the form of employers' productivity dispersion. Bontemps et al. (2000) assume that the firms' productivity distribution is continuous and formulate an easy-to-implement nonparametric 3-step estimation procedure. The alternative method of

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Bowlus et al. (2001), in contrast, relies on the assumption of discrete productivity dispersion. A finite number of productivity types in the economy leads to a conceptually different, and relatively harder, parametric estimation method. Additionally, unlike in the approach of Bontemps et al. (2000), the right tail of the predicted theoretical earnings density in Bowlus et al. (2001) is locally increasing, which consequently implies *a priori* worse fit to the data.

Though, presenting their nonparametric method, Bontemps et al. (2000) notice that it guarantees consistent estimation of the parameters only if the model predicts a proper (nondecreasing) productivity distribution function. Since it is not possible to rule out negative values of the implied productivity density theoretically, Bontemps et al. (2000) suggest constrained MLE subject to non-negativity restriction that assures positive-valued estimated density. In this note we demonstrate that there may exist no positive arrival rate of wage offer that satisfies the above mentioned restriction and avoids nonnegativity of the productivity density. In this case the procedure of Bontemps et al. (2000) cannot be applied. We also derive a simple data-driven condition which can tell about the applicability of the nonparametric method in advance. Finally we point out that the alternative approach of Bowlus et al. (2001), since relying on the discrete productivity dispersion, does not suffer from specification failure of the considered kind. This makes it a substitute for the inapplicable first best procedure.

The note is structured as follows. In Section 2 we describe the structural econometric model and demonstrate when the nonparametric approach of Bontemps et al. (2000) becomes inapplicable. Then we review the parametric approach. In Section 3, using the data from the German Socio-Economic Panel (GSOEP), we provide an illustration of the failure of the Bontemps et al. (2000) nonparametric method and briefly discuss the performance of the alternative parametric procedure. Section 4 concludes.

2 Structural Econometric Model

Here we outline the econometric model and review both approaches.

2.1 The Likelihood Function

Provided that workers are identical the form of the likelihood function is the same irrespective of the assumptions about the distribution of firms' productivity.

Let λ_0 and λ_1 be Poisson arrival rates of wage offer to unemployed and employed worker respectively and let δ be the match separation rate. Furthermore let $F(w)$ and $f(w)$ denote the wage offer distribution and density functions and $G(w)$ and $g(w)$ denote the actual earnings distribution and density functions. Defining $\kappa_0 = \lambda_0/\delta$, $\kappa_1 = \lambda_1/\delta$ and $\bar{F}(w) = 1 - F(w)$ we get the following contributions of unemployed (ℓ_u) and employed (ℓ_e) individuals to the likelihood function:

$$\ell_u = \frac{1}{1 + \kappa_0} [\delta \kappa_0]^{2-d_r-d_l} \exp\{-\delta \kappa_0 [t_e + t_r]\} [f(w)]^{1-d_r}, \quad (1)$$

$$\begin{aligned} \ell_e = \frac{\kappa_0 g(w)}{1 + \kappa_0} & [\delta (1 + \kappa_1 \bar{F}(w))]^{1-d_l} \exp\{-\delta (1 + \kappa_1 \bar{F}(w)) [t_e + t_r]\} \times \\ & \left[[\delta \kappa_1 \bar{F}(w)]^{d_t} \delta^{1-d_t} \right]^{1-d_r}. \end{aligned} \quad (2)$$

In (1) and (2) $d_l = 1$, if a spell is left-censored, 0 otherwise; $d_r = 1$, if a spell is right-censored, 0 otherwise; $d_t = 1$ if there is a job-to-job transition, 0 otherwise.

2.2 Nonparametric Estimation and Its Limitations

Using the steady state identities of the theoretical Burdett-Mortensen model

$$F(w) = \frac{1 + \kappa_1}{1 + \kappa_1 G(w)} G(w), \quad (3a)$$

$$f(w) = \frac{1 + \kappa_1}{[1 + \kappa_1 G(w)]^2} g(w). \quad (3b)$$

Bontemps et al. (2000) propose the following 3-step estimation procedure. On the first step $g(w)$ and $G(w)$ in (3a)-(3b) are estimated nonparametrically. On the second step expressions in (3a)-(3b) are substituted into (1) and (2) and the likelihood function is maximized with respect to $\{\kappa_0, \kappa_1, \delta\}$. On the third step the equilibrium productivity levels

$$p = K^{-1}(w) = w + \frac{1 + \kappa_1 G(w)}{2\kappa_1 g(w)} \quad (4)$$

and productivity density

$$\gamma(p) = \frac{2\kappa_1(1 + \kappa_1)g(w)^3}{3\kappa_1 g(w)^2[1 + \kappa_1 G(w)]^2 - g'(w)[1 + \kappa_1 G(w)]^3} \quad (5)$$

are calculated.

Bontemps et al. (2000) notice that the third step is possible only if the model is well specified with respect to the equilibrium productivity distribution, i.e., if $3\kappa_1 g(w)^2 - g'(w)[1 + \kappa_1 G(w)] > 0$. Once this is not the case the authors suggest performing the second step of the procedure under this theoretically implied constraint, which can be conveniently rewritten as

$$\kappa_1 [3g(w)^2 - g'(w)G(w)] > g'(w) \quad \{w : g'(w) \geq 0\}. \quad (6)$$

(also notice that if $g'(w) < 0$ productivity density $\gamma(p)$ is always positive).

In the applications of the proposed methodology so far (Bontemps et al., 2000) the constraint in (6) was never violated. Attempting to estimate the model with GSOEP data, however, we face the opposite case. Therefore, following the suggestion of Bontemps et al. (2000) on the second step we try maximizing the likelihood function with respect to (6).

Though, the constrained maximization may not always be feasible. To see this notice that for some values of w the term $3g(w)^2 - g'(w)G(w)$ on the l.h.s. of (6) can be negative. This happens when we observe clusters of high-wage earners in the data (moreover, in survey data on top two earnings deciles, clusters can frequently appear due to respondents providing heaped earnings information). Such clustering is reflected by a bump far on the right tail of the estimated kernel density. Whenever the bump obtains, $g'(w)$ is greater than zero and at the same time $G(w) \rightarrow 1$ and $g(w) \rightarrow 0$. So the value of $g(w)$ may be too small to make the term $3g(w)^2 - g'(w)G(w)$ positive. In this case the constraint (6) yields

$$\kappa_1 < \min_{\{w\}} \frac{g'(w)}{3g(w)^2 - g'(w)G(w)} < 0 \quad \{w : g'(w) \geq 0\}. \quad (7)$$

As a result there is no κ_1 that can satisfy (6), since κ_1 is always positive. We call this result “*constraint inconsistency*”.

In the opposite case, i.e. when $3g(w)^2 - g'(w)G(w) > 0$, the constraint becomes

$$\kappa_1 > \max_{\{w\}} \frac{g'(w)}{3g(w)^2 - g'(w)G(w)} > 0 \quad \{w : g'(w) \geq 0\} \quad (8)$$

and constrained maximization on the second step indeed returns an appropriate estimate of κ_1 . A typical example for the opposite case could be the region of the left tail of the earnings density, where $g(w)$ increases, but its’ values are high enough to ensure

that $3g(w)^2 - g'(w)G(w) > 0$ holds when $g'(w) \geq 0$.¹

As we find that constraint inconsistency is purely the earnings data property, the

$$\text{sign} \left[3\hat{g}(w)^2 - \hat{g}'(w)\hat{G}(w) \right] \quad (9)$$

on $\{w : \hat{g}'(w) \geq 0\}$ becomes a simple criterion that would allow checking in advance whether the nonparametric 3-step procedure is applicable. Negative values of (9) imply non-applicability.

Finally, we also warn from oversmoothing the kernel density estimate in order to achieve a “consistent” constraint. By oversmoothing one can indeed get a strictly decreasing right tail with minor changes of the curvature of the rest of the estimated density. However, from (8) it can be seen that by manipulating the magnitude of the bandwidth one arbitrarily fixes the value of the lower bound of the constraint. This inevitably biases the estimated κ_1 .

2.3 Parametric Estimation of the Model

Constraint inconsistency forces using alternative parametric procedures. Parametric assumptions can be imposed on either the earnings or the productivity distribution.

Parametric Assumptions on the Earnings Distribution The easiest way to avoid an inconsistent constraint is to assume for $g(w)$ in (1)-(2) some parametric form with a strictly decreasing right tail, instead of using its’ nonparametric estimate. The most natural suggestion is that $g(w)$ is a log-normal density. We estimate the model under this assumption and find that (6) is actually always satisfied. However, calculating (4) we discover that it violates the requirement that the offered wage is a monotone increasing function of productivity.² This generates an improper estimated productivity density and implies the necessity of imposing parametric assumptions on the productivity distribution directly.

¹Though still, constraint inconsistency on the left tail may also be possible. This can happen when minimum wages are not binding and there is a dispersion of reservation wages. If there exists a cluster of workers with very low reservation wage we get $g'(w) > 0$ and $g(w) \rightarrow 0$, which points towards inconsistent constraint. However, here the danger is much smaller than at the rightmost of the support, because in this situation $G(w) \rightarrow 0$.

²Monotonicity of offered wages as a function of productivity follows from Proposition 10 of Bon-temps et al. (1997), which is a generalization of Burdett and Mortensen (1998) finding that more productive firms pay higher wages. It can also be deducted from (4).

Parametric Assumptions on the Productivity Distribution This approach differs from the one above by the fact that now the productivity parameter p appears in the likelihood function explicitly. The reason is that instead of nonparametric estimates of $\{f(w), F(w)\}$ and $\{g(w), G(w)\}$ the *theoretically implied* functional forms of both distributions are invoked. These constitute a part of the equilibrium solution of the model and depend on both search intensity parameters and firm productivity p .

Mortensen (1990) demonstrates that for the productivity distribution with Q points of support, $j = 1, \dots, Q$, the theoretical offer distribution has a form

$$F(w) = \frac{1 + \kappa_1}{\kappa_1} \left[1 - \frac{1 + \kappa_1(1 - \gamma_{j-1})}{1 + \kappa_1} \sqrt{\frac{p_j - w}{p_j - w_{H_{j-1}}}} \right], \quad (10)$$

where $w \in (w_{L_j}, w_{H_j}]$, with w_{L_j} and w_{H_j} being the lowest and the highest wages offered by p_{j-1} -type employer, $w_{H_{j-1}} = w_{L_j}$, $F(w_{L_1}) = 0$, $F(w_{H_Q}) = 1$ and γ_j standing for the fraction of employers with productivity level less than or equal to p_j . Moreover Mortensen (1990) shows that

$$F(w_{H_j}) = \gamma_j, \quad \forall j = 1, \dots, Q. \quad (11)$$

Bowlus et al. (2001) pursue this approach maximizing the likelihood function with $F(w)$ as in (10). Using (3a)-(3b) and differentiating with respect to w they also obtain the theoretical wage offer and earnings densities which closes the model in (1)-(2).

Since there are kinks in (10), the offer and earnings densities, the likelihood function is discontinuous. For this reason the procedure of Bowlus et al. (2001) is stepwise, switching between simulated annealing and smooth optimization steps. This makes its implementation more difficult than the method of Bontemps et al. (2000). However, from both the fact that the offer distribution in (10) is a positive-valued increasing function of w and (11) it follows that the estimation procedure of Bowlus et al. (2001) never encounters negative probability mass values for the distribution of the productivity types. This makes the approach of Bowlus et al. (2001) robust to the non-monotone behaviour at the right tail of the actual earnings density and ensures estimability of the model in case the alternative procedure of Bontemps et al. (2000) is not feasible.

For completeness we also notice that yet another approach to estimating the model with continuous productivity dispersion exists in the literature. It takes the version of (10) with identical firms and imposes an *ad hoc* assumption on the probability

distribution of p (see Koning et al., 1995). Even though this approach is also robust, it does not capture the demonstrated later by Bontemps et al. (2000) result that in equilibrium

$$F(w) = \Gamma(p), \quad (12)$$

where $\Gamma(p)$ stands for the assumed continuous productivity distribution.³ Therefore the structural model in this case will be incompletely specified.

3 The Data and Numerical Illustration

We estimate the model using two data samples from GSOEP. The sampling points are taken to be the waves of 1986 and 1995 after which the duration and earnings information was retrieved in the standard way (see Koning et al., 1995).

Kernel plots of the densities of net monthly earnings are provided in Figure 1. The “wave” behaviour of the right tail, that indicates a danger of an inconsistent constraint, can be seen already above DM 5000. Next we check the consistency condition (9). Figure 2 plots the term within the sign function of (9) against the wage. Remember that negativity of this term for some w implies that the nonparametric method of Bontemps et al. (2000) is not applicable. Figure 2 clearly shows that this is the case for a set of earnings intervals at the right tail of the earnings distribution.

Once we discover that the nonparametric procedure cannot be applied the model is estimated using the method of Bowlus et al. (2001). We treat the number of productivity types as unknown and when estimating the model we start from a homogeneous case ($Q = 1$) adding productivity levels one by one. To judge on the ultimate magnitude of Q Bowlus et al. (2001) propose a quasi-LRT test $V = -2(\log L_{j-1} - \log L_j) < \chi^2(1)$. However, since the exact distribution of the likelihood ratio in this particular case is unknown, we heuristically make our choice of the number of mass points on the basis of information criteria (consistent AIC and SBC).

The estimation procedure terminates at $Q = 8$ for the sample of 1986 and $Q = 10$ for the data of 1995. The estimates of the search frictions and the productivity distribution are presented in Table A.1 of the Appendix.

³The identity in (12) is a generalization of the original result of Mortensen (1990) for the discrete productivity distribution (11).

Figure 1: Earnings Densities

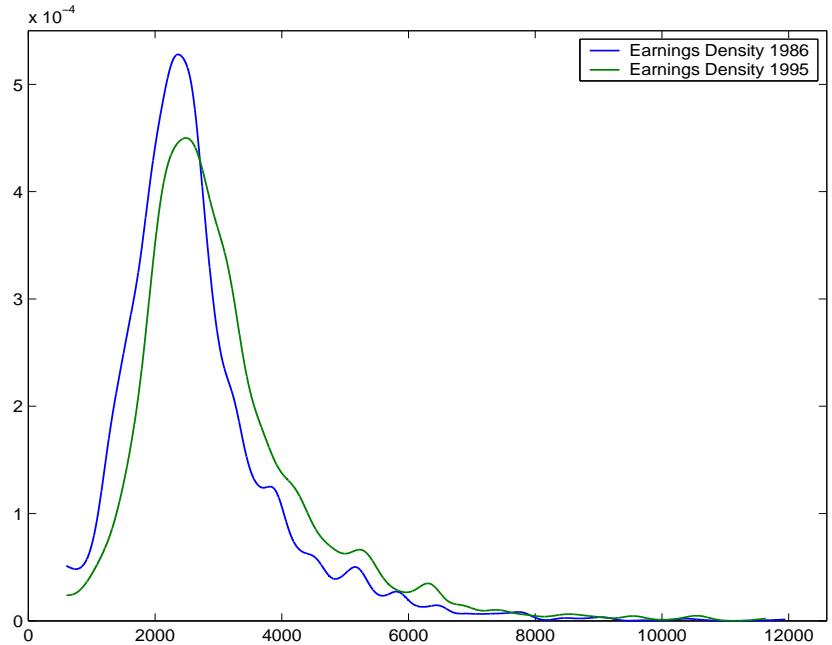
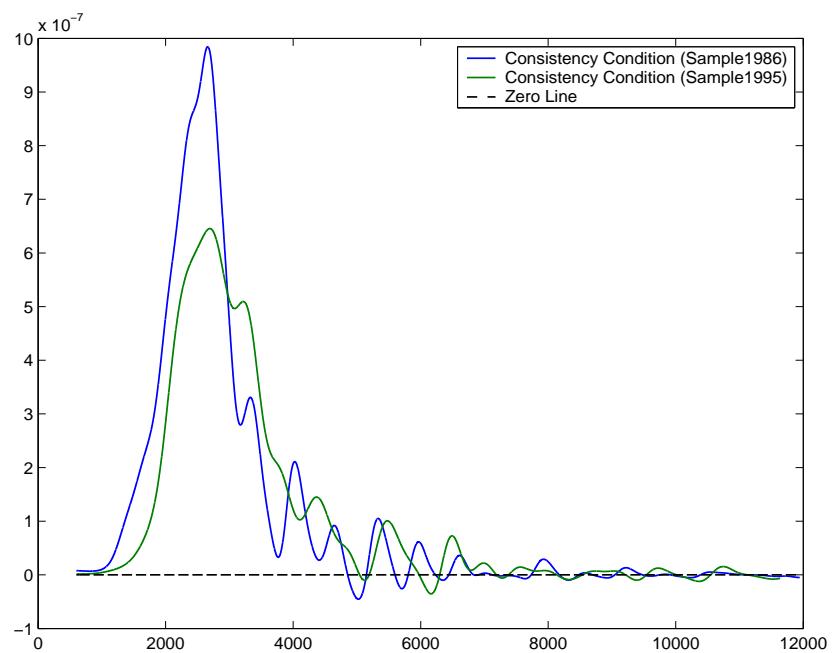


Figure 2: Consistency Condition



The most appealing criterion of the goodness of fit is the discrepancy between the predicted theoretical earnings distribution and the nonparametric estimate of the earnings distribution obtained from wage data. From Figures A.1-2 of the Appendix we can see that for both 1986 and 1995 samples this fit is very close, which should ensure sound inference from the obtained estimation results. Furthermore, the fit can be improved to an arbitrarily high degree by simply adding points to the support of the productivity distribution. This, however, does not significantly change the estimated parameters and does not improve the model any further in terms of information criteria.

4 Conclusion

In this note we describe the case in which the nonparametric estimation of search equilibrium model with employer heterogeneity formulated by Bontemps et al. (2000) becomes unfeasible. We also offer a simple data-driven condition that provides information about the applicability of the nonparametric method beforehand. Since the nonparametric procedure of Bontemps et al. (2000) is *a priori* superior over the alternatives in terms of the ease of implementation and fit to the data, our discussion provides useful a hint for choosing the appropriate specification in the applied work.

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Appendix

Table A.1: Estimation Results

Sample 1986			Sample 1995		
	Coefficients	(Std.Errors)		Coefficients	(Std.Errors)
κ_0	12.7914	(0.4767)	κ_0	9.3459	(0.3428)
κ_1	4.8014	(0.1157)	κ_1	4.0133	(0.1126)
δ	0.0036	($6.3 \cdot 10^{-5}$)	δ	0.0041	($8.0 \cdot 10^{-5}$)
Estimated Productivity Distribution:			Estimated Productivity Distribution:		
$j :$	p_j	γ_j	$j :$	p_j	γ_j
1	2304.6	0.65561	1	2758.0	0.62421
2	2726.6	0.81784	2	3120.4	0.79455
3	3289.8	0.90804	3	3845.8	0.88384
4	4601.5	0.95306	4	4738.5	0.92208
5	7997.2	0.98269	5	6147.2	0.94792
6	18630.5	0.99529	6	8673.3	0.97320
7	62728.1	0.99897	7	13906.5	0.98731
8	437143.1	1	8	24442.1	0.99331
			9	53593.8	0.99769
			10	232585.7	1
Log(Likelihood):			Log(Likelihood):		
	-74245.072			-61075.378	

Figure A.1: Estimated Theoretical Offer and Earnings Distributions (Sample 1986)

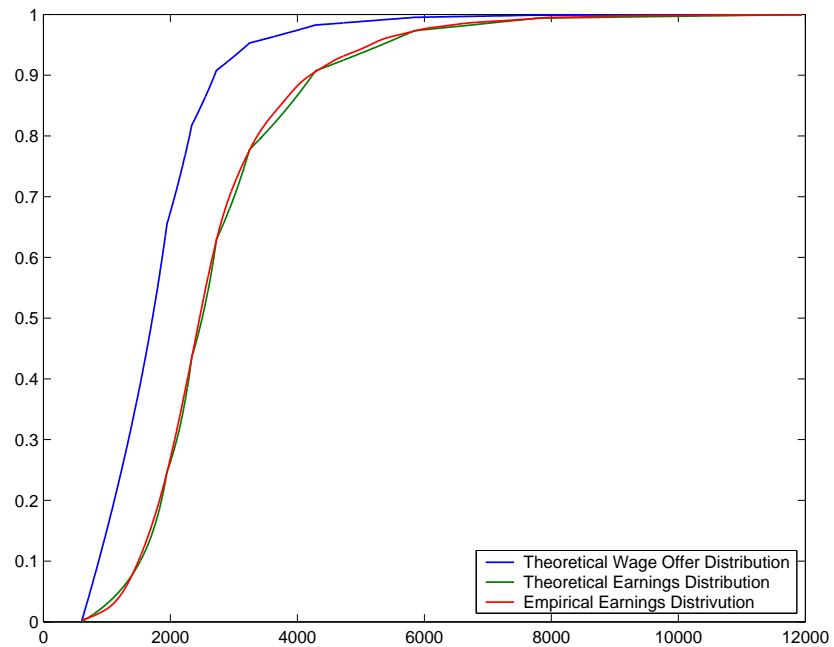
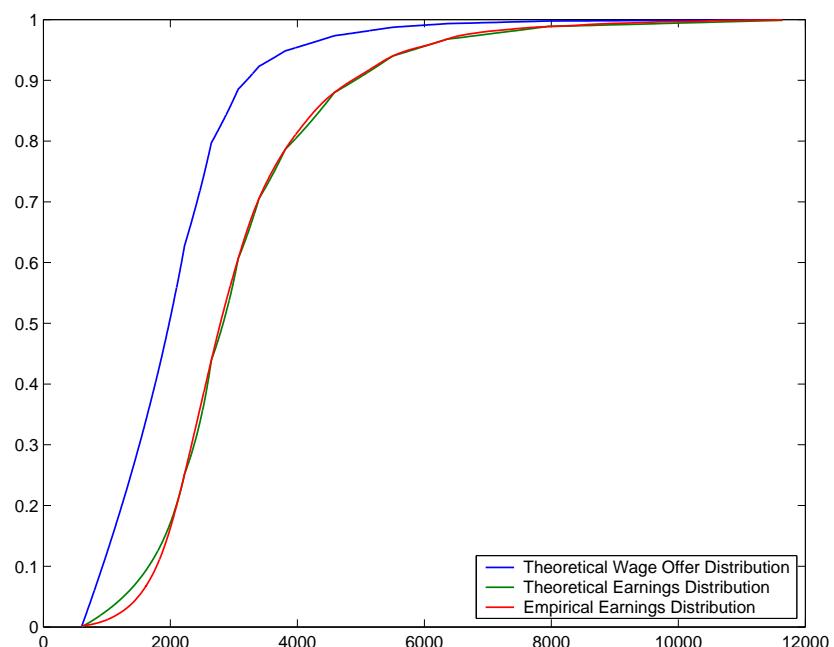


Figure A.2: Estimated Theoretical Offer and Earnings Distributions (Sample 1995)



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