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**Working Paper**

## Incentives, Globalization, and Redistribution

CESifo Working Paper, No. 8450

**Provided in Cooperation with:**

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

*Suggested Citation:* Haufler, Andreas; Perroni, Carlo (2020) : Incentives, Globalization, and Redistribution, CESifo Working Paper, No. 8450, Center for Economic Studies and Ifo Institute (CESifo), Munich

This Version is available at:

<https://hdl.handle.net/10419/223522>

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# Incentives, Globalization, and Redistribution

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## **Impressum:**

CESifo Working Papers

ISSN 2364-1428 (electronic version)

Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH

The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute

Poschingerstr. 5, 81679 Munich, Germany

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Editor: Clemens Fuest

<https://www.cesifo.org/en/wp>

An electronic version of the paper may be downloaded

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# Incentives, Globalization, and Redistribution

## Abstract

We offer a new explanation for why taxes have become less progressive in many countries in parallel with an increase in income inequality. When performance-based compensation differentials are needed to incentivize effort, redistribution through progressive income taxes becomes less precisely targeted. Taxation reduces after-tax income inequality but undermines incentive contracts, lowering effort and raising pre-tax income differentials. Market integration can widen the spread of project returns and make contract choices more responsive to changes in the level of taxation, resulting in a lower optimum income tax rate even when individuals are not inter-jurisdictionally mobile.

JEL-Codes: H210, F150, D630.

Keywords: redistributive taxation, performance-based contracts, market integration.

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This version: 15 July 2020

This paper was presented at conferences and workshops in Leipzig, Louvain, Munich and Osaka. We thank David Agrawal, Philippe de Donder, Jean Hindriks, Michael Kramm, Yukihiro Nishimura, Andreas Peichl, Pierre Pestieau, Dominik Sachs, Klaus Schmidt and Jonas von Wangenheim for helpful comments and suggestions, and Lucas Coronel for excellent research assistance. This paper was started when Andreas Haufler visited the University of Warwick and continued when Carlo Perroni visited the Center for Economic Studies in Munich. We wish to thank both institutions for their hospitality. Financial support by the German Research Foundation (DFG) through CRC TRR 190 is gratefully acknowledged.

# 1 Introduction

Starting from the 1980s, the distribution of income of many developed economies has become progressively more unequal and more concentrated.<sup>1</sup> And yet, tax-transfer systems have not become more redistributive—on the contrary, in a number of countries they have actually become less so. Egger et al. (2019) have recently shown that, since the mid-1990s, economic globalization has resulted in a higher labor tax burden on the middle classes of OECD countries and a reduced labor tax burden for the top one percent of earners.<sup>2</sup>

The observation of greater inequality going hand-in-hand with less redistribution through taxation poses a puzzle, and one that is of major policy importance. Standard theories of optimal taxation would predict an increase in tax progressivity in response to an increase in inequality (Saez, 2001; Slemrod and Bakija, 2001). A combination of higher levels of inequality with less progressive income taxes can only be reconciled with those theories if the increase in inequality is accompanied by an increase in the elasticity of the tax base. Accordingly, one prominent argument in the literature is that the tax base has become more elastic at the upper end of the income distribution because high income earners have become more inter-jurisdictionally mobile (e.g. Lehmann et al., 2014). But for a significant fraction of individuals at the top of the income distribution, international mobility remains limited (Battisti et al., 2018; Kleven et al., 2020).<sup>3</sup>

In this paper we advance an alternative, and complementary, explanation for the concurrent observation of higher income inequality and lower tax progressivity. Our argument focuses on how optimal redistributive policies are affected by the internationalization of *product markets*—rather than labor markets.

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<sup>1</sup>In many countries this has mainly happened at the top of the income distribution (Atkinson et al., 2011). But in some countries, like the United States, it has been shown to apply to the entire distribution (Piketty et al., 2018). Detailed international evidence on the development of various measures of income inequality are collected by the OECD (<http://www.oecd.org/social/income-distribution-database.htm>) and in the World Wealth and Income Database (Facundo et al., 2017).

<sup>2</sup>According to Egger et al., (2019), this reverses the pattern from the 1980s and early 1990s, when globalization led to increased progressivity in income taxation. Similar results have been reported by Immervoll and Richardson (2011) in a comprehensive study of redistribution policy among OECD countries that considers both taxes and benefits. Bargain et al. (2015) decompose the increase in U.S. income inequality since the 1980s and find a non-negligible, positive effect of tax policy changes on income inequality.

<sup>3</sup>One likely reason for this limited mobility is that high-earning occupations, such as the provision of legal services, often involve jurisdiction-specific human capital investment.

Our starting point is the observation that labor markets are fundamentally shaped by incentive contracts. This is in line with evidence that a large share of all jobs in developed countries involve some element of performance pay (Lemieux et al., 2009).<sup>4</sup> With incentive contracts, income inequality stems from two different sources: differences in individual abilities and outcome-dependent wage differentials for individuals of identical abilities. Redistributive taxation can address the first source of wage inequality but not the second, as the latter arises as a second-best market solution to a moral hazard problem. Performance pay thus limits the redistributive role of tax policy for any empirically observed earnings distribution.

The second building block in our argument is the observation that product market integration is associated with higher individual income risk (Rodrik, 1997, 1998) and steeper pay incentives (Cuñat and Guadalupe, 2009). Globalization makes product markets potentially more profitable, but also more competitive.<sup>5</sup> The stronger competition raises income risks for firms and their workers, which translates into sharper (higher-powered) incentives in performance-based contracts. These sharper incentives, in turn, reduce the effectiveness of income taxation as a redistributive tool, resulting in lower optimal rates of taxation.

To develop our arguments, we describe a model of second-best contracting where incomplete insurance is required to elicit effort in the presence of moral hazard. Risk-averse workers with heterogeneous abilities choose between an incentive contract and a less efficient fixed-wage contract that does not induce effort. In equilibrium, higher productivity workers select into performance-based contracts, whereas less productive workers choose fixed-wage contracts. Economic globalization increases the variance of returns within incentive contracts (while possibly also increasing workers' productivity), which in turn raises the critical productivity level above which a worker will select into a performance-based contract.

We then ask how these changes feed into the choice of an optimal indirectly-progressive tax rate. The wider spread in market returns that follows from globalization causes the choice of higher-productivity, performance-based contracts over lower-productivity, fixed-wage contracts to become more responsive to tax changes, making tax revenues more tax-elastic and thus raising the efficiency cost of redistribution. The redistributive value of the tax also rises, however, as the destruction of incentive contracts reduces the income of all individuals from redistributed tax revenues.

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<sup>4</sup>Lemieux et al. (2009) present evidence for the United States. The prevalence of incentive contracts in the labor markets of other developed countries is detailed in Section 2 below.

<sup>5</sup>This is also consistent with the emergence of 'superstar firms' that dominate markets worldwide (Autor et al., 2017).

When the distribution of ability types is uniform, the trade-off between the higher efficiency costs and the higher distributive value of the tax does not change along the ability distribution. This makes the uniform case a natural benchmark case for our analysis. At the same time, a uniform distribution of *ability types* is fully capable of generating a positively skewed distribution of *earnings realizations* that is consistent with empirically observed distributions. In the uniform case, we show that the effect of globalization on the redistributive value of the tax is only an indirect effect—which cannot dominate the direct effect on efficiency costs of the tax. Therefore, economic globalization is always associated with a lower optimal tax rate, under both a revenue-maximization (Rawlsian) and a utilitarian government objective. We also derive conditions under which our results carry over to alternative type distributions.

Our study is related to several strands of literature. In a domestic context, it relates to the large literature on the optimal progressivity of wage income taxes (see Diamond and Saez, 2011, for an overview), and on redistributive income taxation in the presence of earnings risk (see Boadway and Sato, 2015, for a recent synthesis). Most of this literature considers earnings volatility as exogenous, driven by luck (Varian, 1980). The implications of endogenous earnings risk are considered in a small literature strand that focuses on the ‘crowding out’ of private insurance by social insurance or redistributive taxation (Golosov and Tsyvinski, 2007; Chetty and Saez, 2010; Krueger and Perri, 2011). Most closely related to our basic setting is Doligalski et al. (2020), who analyze redistributive taxation in a model of performance pay contracts. Their focus, however, is on the role of non-linear tax reforms and labor supply responses, rather than on the effects of international market integration as in our analysis.

A number of recent studies (Lemieux et al., 2009; Abraham et al., 2017) have emphasized the contribution of performance-based pay to rising wage inequality, while a number of older studies (Schmidt, 1997; Raith, 2003) have stressed the role of market competition in the determination of incentive pay. Our analysis connects these findings to the optimal taxation literature. The effects of economic globalization on optimal income taxation have so far been studied mainly for environments where high-income earners are internationally mobile (Simula and Trannoy, 2010; Bierbrauer et al., 2013; Lehmann et al., 2014; Tóbias, 2016).<sup>6</sup> These models generally find that economic integration leads to inefficiently low redistributive income taxes. Here we focus instead on the global competition in product markets and its effects on performance-based contracts as an alternative channel through which economic globalization influences

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<sup>6</sup>Empirical contributions to this literature have studied the migration responses of high-income earners both at the international level (Kleven et al., 2014) and at the national level (e.g., Agrawal and Foremny (2019). The literature is summarized in Kleven et al. (2020).

tax policy choices. As we will show, this has implications that are akin to those of introducing ‘superstar effects’ into a model of optimal taxation (Scheuer and Werning, 2017; Scheuer and Slemrod, 2019).

Our analysis proceeds as follows. Section 2 summarizes available empirical evidence on the prevalence of incentive contracts in labor markets. In Section 3, we describe the model and study the sorting of heterogeneous worker types into performance-based versus fixed-wage contracts. Section 4 analyzes how the equilibrium structure of contracts is affected by higher taxes and by a globalization-induced increase in the variance of output realizations. Section 5 turns to redistributive taxation and analyzes the effects of market integration on the optimal redistributive tax rate. Section 6 summarizes and discusses our results. Section 7 concludes.

## 2 The pervasiveness of performance-related incentives

In the most narrow sense, performance-based contracts can be distinguished by an explicit bonus element that is paid on the basis of measured performance. Such bonus contracts play an important role in all OECD countries, particularly at the top of the income distribution. For the UK example, Bell and Van Reenen (2014) show that bonus income represents more than 10% of the total salary for the top 10% of earners employed in financial services, and for the top 5% of earners in other sectors. This share rises steeply for top salary earners: for the top percentile, bonuses make up 44% of total pay in financial services, and 35% of total pay in other sectors.<sup>7</sup> More generally, Lemieux et al. (2009) find for a panel of more than 3,000 employees in the United States that almost 40% of workers receive some form of performance pay. In other OECD countries, the share of incentive contracts is somewhat lower, but still above 20% (Bryson et al., 2012).

In a broader interpretation, many employed individuals who do not explicitly receive bonuses still face implicit performance-related incentives: Fama (1980) and Holmstrom (1999) have emphasized the role of implicit incentive contracts in a dynamic setting where satisfactory performance of a worker today is rewarded by promotions and a higher (nominally fixed) pay tomorrow. The importance of such implicit incentives has been empirically confirmed with both field and experimental data (Frederiksen, 2013; Sliwka and Werner, 2017).<sup>8</sup>

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<sup>7</sup>There is an active discussion of whether these bonus contracts reflect “rent extraction” by CEOs (Piketty et al., 2014), or are compatible with modern shareholder value theories (Edmans and Gabaix, 2016). Our discussion restricts attention to the productivity enhancing effects of incentive contracts.

<sup>8</sup>In a still broader interpretation, every employment contract can be regarded as a performance-



Incentives that are analogous to those in performance-based contracts are also present for the self-employed. In the EU, this group accounts for 15% of all employed individuals; the corresponding figure for the US is 6.5% (OECD).<sup>9</sup> Models of occupational choice (Lucas, 1978; Boadway et al., 1991) interpret this group as high-ability individuals who efficiently self-select into self-employment and in many ways face similar incentives as top-earning employees within incentive contracts.

There are several indications that the importance of (broadly-defined) performance-related incentives has increased in recent decades. Lemieux et al. (2009) find that the number of incentive pay contracts in the United States has increased over the period from the mid-1970s to the late 1990s, emphasizing that the increase has primarily occurred at the top of the income distribution. Similarly, Bryson et al. (2012) document an increase in the share of performance-related pay in both Europe and the United States in the 2000s, primarily in higher-earning jobs. Moreover, rising intra-occupational wage inequality, for example in the United States (Heathcote et al., 2010) and in Germany (Bayer and Kuhn, 2019), can be seen as indirect evidence for a rising importance of implicit incentive contracts in a dynamic career perspective.

The pervasiveness of a performance-based component in individuals' income streams implies that the distribution of income is, at least in part, shaped by the stochastic structure of contractual arrangements rather than just by ability differentials.<sup>10</sup> As we will show, this has direct implications for how income-based redistributive policies should be assessed, and for how redistributive policies respond to changes in the structure of incentives.

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based contract when job insecurity and unemployment are taken into account. In recent decades, job protection has generally been reduced, both by changes in labour market laws and through reductions in support payments for the unemployed, in particular by cutting the duration of unemployment benefits. From an individual worker's perspective, these changes translate into an increased income gap between a 'successful outcome' where (s)he stays in the job, and an 'unsuccessful' outcome, where (s)he is fired from the job and has to rely on social support payments. The widespread use of customer ratings to measure performance for low-earning service providers in the gig economy is also consistent with this interpretation.

<sup>9</sup><https://data.oecd.org/emp/self-employment-rate.htm>. Eurofound (2017) gives a detailed breakdown of the self-employed in the EU and finds that about half of the self-employed are professionals or business owners with high incomes – a group that also tends to be immobile internationally (cf. footnote 3). See also Hamilton (2000) for one of the few empirical studies on the returns to self-employment in the United States.

<sup>10</sup>Abraham et al. (2017) find that incentive contracts contribute about 10% of the observed US wage inequality.

### 3 A model of incomplete contracting

#### 3.1 Preferences, technologies, and incentives

We consider an economy populated by a unit mass of individuals who are ex-ante identical (at time  $\tau = 0$ ) and are risk averse, with von Neumann-Morgenstern utility,  $u(x)$ , from consumption,  $x$ , satisfying  $u'(x) > 0$  and  $u''(x) < 0$ . For tractability, the rest of our analysis will focus on the logarithmic case  $u(x) = \ln x$ , which exhibits falling absolute risk aversion and constant relative risk aversion.

At time  $\tau = 1$  each individual is assigned a productivity type,  $\alpha$ , drawn from a continuous distribution with positive support  $[0, \bar{\alpha}]$  and cumulative density function  $F(\alpha)$ . As the number of individuals is large,  $F(\alpha)$  also coincides with the c.d.f. of the ex-post distribution of productivity types in the population. Without loss of generality, we assume  $\bar{\alpha} = 1$ .

Production takes place through risky projects that are run by risk-neutral firms operating under conditions of perfect competition. Each project involves a single individual, hired by a firm as a worker. A project that employs an individual of productivity type  $\alpha$  yields output

$$\frac{\alpha}{\pi}, \quad \pi \in (0, 1), \quad (1)$$

if successful and yields zero output if unsuccessful (irrespective of the individual's productivity type). The probability,  $\gamma(e)$ , of the project being successful depends on the individual's chosen level of effort,  $e \in \{0, 1\}$ , as follows:

$$\gamma(e) = e\pi + (1 - e)\eta\pi, \quad (2)$$

with  $\eta \in (0, 1)$ . Hence, the probability of success is  $\pi$  with positive effort but it is only  $\eta\pi < \pi$  if no effort is exerted. From (1), the expected output from a worker of productivity type  $\alpha$  (gross of effort costs) is then

$$\gamma(e) \frac{\alpha}{\pi} = e\alpha + (1 - e)\eta\alpha; \quad (3)$$

i.e. it is equal to the individual productivity type,  $\alpha$ , if the individual exerts positive effort, but is reduced to  $\eta\alpha < \alpha$  if the individual exerts no effort.

Effort is costly: the individual incurs a private utility cost  $ce$ ,  $c > 0$ , irrespective of her productivity type. For the remainder of our discussion, we shall assume that, if the individual could secure an income equal to her expected output in all states, then exerting positive effort would be always preferred to exerting no effort. This implies  $\ln \alpha - c > \ln \eta\alpha$ , which in turn requires

$$c < -\ln \eta \equiv \bar{c}. \quad (4)$$

A worker's productivity type is publicly observable, and so is output. Effort, however, is not observable (or, if it is observable, it is not verifiable), which implies that wage payments cannot be directly conditioned on effort. Contracting between firms and workers thus runs against a fundamental moral hazard problem, which can only be (partially) addressed by an incentive-compatible contract that induces workers to exert effort by conditioning the wage payment on output. This implies a *performance-based* contract prescribing two different gross-of-tax wage levels: a higher wage,  $w_H$ , paid if the project is successful, and a lower wage,  $w_L$ , if the project is unsuccessful. The lower wage level can be thought of as a base wage, with the higher wage level adding a bonus,  $w_H - w_L$ , to the base wage contingent on success.

Income is taxed at rate  $t$ , and revenues from the tax are distributed equally amongst the population in lump-sum fashion, i.e. through a uniform transfer that individuals take as exogenous. As the population has unit mass, the uniform transfer equals total tax revenue,  $g$ . This is an *indirectly* progressive, linear income tax scheme that collects comparatively more tax revenues from higher earners while redistributing all proceeds on an equal per-capita basis. It is also an anonymous scheme, in the sense that the tax an individual pays and the transfer she receives are not conditioned on her type (whether or not that can be observed by the tax planner) but only depends on her income.<sup>11</sup>

Given this tax/transfer scheme, if an individual receives a gross wage income equal to  $w$ , her net-of-tax wage income is  $w^N = (1 - t)w$  and her total disposable income is  $(1 - t)w + g$ , which must equal consumption,  $x$ . A worker's expected utility from such a contract, if she chooses to exert positive effort, is thus

$$\pi \ln((1 - t)w_H + g) + (1 - \pi) \ln((1 - t)w_L + g) - c \equiv EU^E. \quad (5a)$$

If zero effort is chosen, expected utility from the incentive contract is instead<sup>12</sup>

$$\eta\pi \ln((1 - t)w_H + g) + (1 - \eta\pi) \ln((1 - t)w_L + g) \equiv EU^N. \quad (5b)$$

In order to induce positive effort from workers, performance-based contracts must satisfy the incentive-compatibility constraint  $EU^E \geq EU^N$ . Competitive firms will choose the contract that involves the lowest expected wage cost to them and still induces workers to exert effort. Hence,  $EU^E = EU^N$  must hold in a competitive equilib-

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<sup>11</sup>The focus on linear income taxation differentiates our model from related work that assumes a non-linear income tax schedule (e.g. Lehmann et al., 2014; Doligalski et al., 2020).

<sup>12</sup>Note that workers' utility in this no-effort incentive contract will generally differ from utility in the (no-effort) fixed wage contract, as given in (11) below.

rium. Equating expected utilities in (5a) and (5b) gives

$$\frac{(1-t)w_H + g}{(1-t)w_L + g} = e^{\frac{c}{\pi(1-\eta)}} \equiv \Gamma > 1. \quad (6)$$

Condition (6) determines the equilibrium spread between the high wage,  $w_H$ , and the low wage,  $w_L$ , that just induces effort. The expected level of worker remuneration is then pinned down by the free entry and exit of risk-neutral firms under perfect competition. This implies that expected profits for a firm offering a performance-based contract to a worker of productivity type  $\alpha$  (who will exert positive effort in equilibrium) must be zero:

$$\alpha - (\pi w_H + (1-\pi)w_L) \equiv E\Pi^E = 0. \quad (7)$$

The zero-profit condition (7) and the incentive-compatibility constraint (6) jointly determine the equilibrium levels of gross-of-tax wages in each of the two states, as functions of the productivity type,  $\alpha$ , and of the tax/transfer scheme  $(t, g)$ :

$$w_L = \frac{\alpha - \pi(\Gamma - 1)g/(1-t)}{1 + \pi(\Gamma - 1)}, \quad (8a)$$

$$w_H = \frac{\alpha\Gamma + (1-\pi)(\Gamma - 1)g/(1-t)}{1 + \pi(\Gamma - 1)}, \quad (8b)$$

where  $\Gamma$  has been introduced in (6). From (8a) and (8b) we see that both state-contingent wage levels are rising in the worker's productivity level,  $\alpha$ .

By (7), in a performance-based contract each worker must receive her full productivity,  $\alpha$ , as her *expected* gross wage. However, in order to induce effort, this needs to be delivered in the form of a lottery that leaves the worker exposed to income risk: since  $\Gamma > 1$ , the high wage,  $w_H$ , is always above the expected productivity,  $\alpha$ , of a worker who exerts effort, whereas the low wage,  $w_L$ , is below the worker's expected productivity. Given that workers are risk-averse and firms are risk-neutral (and therefore capable, in principle, to insure workers at no cost), such an outcome is inefficient.

Firms can, alternatively, offer workers fixed-wage contracts that fully insure the worker, paying the same wage level,  $w_F$ , in all contingencies, whether or not the project is successful.<sup>13</sup> But then the firm must fully anticipate that, absent any incentive for workers to exert positive effort, workers of any productivity type will choose

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<sup>13</sup>The choice between performance-based and fixed-wage contracts in the model is a stylized representation of what would in reality be a choice within a continuum of possible contract structures all featuring some performance-based element to a greater or lesser extent. Here we just restrict the contract choice to the two endpoints of this distribution. From this perspective, fixed-wage employment contracts that do not provide full job security would correspond to some combination between these two extremes rather than to what we call here a "fixed-wage contract".

to exert zero effort. From (3), expected output will thus only be  $\eta\alpha$ . Under conditions of perfect competition, expected firm profits under this contract must also be zero:

$$\eta\alpha - w_F \equiv E\Pi^F = 0, \quad (9)$$

which implies a fixed pre-tax wage rate

$$w_F = \eta\alpha. \quad (10)$$

Expected utility for a worker of type  $\alpha$  in a fixed-wage contract will thus equal

$$\ln((1-t)\eta\alpha + g) \equiv EU^F. \quad (11)$$

By (4), exerting positive effort is productively efficient. So, a fixed-wage contract removes the welfare cost associated with income risk but replaces it with a welfare cost that comes from inefficient production choices.

### 3.2 Contract choice and the distribution of income

A performance-based contract will only be selected if it yields a higher expected utility than a fixed-wage contract does, i.e. if  $EU^E > EU^F$ . Since  $EU^E$  must equal  $EU^N$  in equilibrium, we can directly compare  $EU^N$  to  $EU^F$ .

Replacing the expression for  $w_L$  from (8a) into (5b), using (11), and differentiating  $EU^N - EU^F$  with respect to  $\alpha$ , we obtain

$$\frac{\partial(EU^N - EU^F)}{\partial\alpha} = \frac{(1-t)(1-\eta)g}{((1-t)\alpha + g)((1-t)\eta\alpha + g)} > 0. \quad (12)$$

Therefore, if  $EU^N \geq EU^F$  for a productivity type  $\alpha'$ , this must also be true for productivity types  $\alpha \geq \alpha'$ ; and if  $EU^N < EU^F$  for a productivity type  $\alpha''$ , this must also be true for productivity types  $\alpha < \alpha''$ . Equating  $EU^N$  with  $EU^F$ , and solving for  $\alpha$ , we obtain

$$\alpha = \frac{g}{(1-t)K} \equiv \hat{\alpha}, \quad K \equiv -\frac{\eta(1+\pi(\Gamma-1)) - \Gamma^{\eta\pi}}{1+\pi(\Gamma-1) - \Gamma^{\eta\pi}}, \quad (13)$$

where  $K$  is positive for  $\eta < 1$  and  $c$  sufficiently low.<sup>14</sup> Because of (12), the critical value  $\hat{\alpha}$  partitions the range of productivity types into two intervals: all workers with a productivity level  $\alpha < \hat{\alpha}$  will choose the fixed-wage contract and exert zero effort, and workers with a productivity level  $\alpha \geq \hat{\alpha}$  will choose a performance-based contract and exert positive effort.

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<sup>14</sup>See the proof of Proposition 1.

A relevant constraint for performance-based contracts is that the low wage,  $w_L$ , cannot become negative. This constraint is always satisfied in our model (Lemma 1 in Appendix B.1).

We summarize the properties of the equilibrium contract structure in the following result:

**Proposition 1.** *For any given combination of tax and transfer, and for effort cost,  $c$ , lying below a positive threshold  $\bar{c} < \bar{c}$ , there exists a productivity level,  $\hat{\alpha} \in (0, 1)$  such that:*

- *individuals with productivity types  $\alpha \geq \hat{\alpha}$  will select into performance-based contracts with state-contingent wage levels,  $w_H$  and  $w_L$ , as given in (8a)-(8b), and will exert positive effort;*
- *individuals of productivity types  $\alpha < \hat{\alpha}$  will receive a fixed wage  $w_F = \eta\alpha$  and will exert no effort.*

PROOF: See Appendix B.2.

Our result that performance-based contracts are concentrated among high-ability individuals is aligned with the empirical evidence. Bell and Van Reenen (2014, Figure 3) document a strong and positive relationship between the percentile of earners in the U.K. wage distribution, and their bonus share in the total pay. Similarly, Lemieux et al. (2009, Table 1) show that hourly earnings of U.S. employees were 30% higher in performance-pay jobs, as compared to non-performance-pay jobs.

In equilibrium, realized ex-post, pre-tax wages will reflect both the heterogeneous abilities of individuals and, for those in an incentive contract, the stochastic nature of the production process. For each productivity type  $\alpha < \hat{\alpha}$ , all individuals of that type will receive a wage  $w_F$ , whereas for each productivity type  $\alpha \geq \hat{\alpha}$ , a fraction  $\pi$  of individuals of that type will receive a wage  $w_H$  and a fraction  $1 - \pi$  of individuals of that type will receive a wage  $w_L$ . Let  $w_F^{-1}(w)$ ,  $w_L^{-1}(w)$  and  $w_H^{-1}(w)$  denote the inverse mappings from wage realizations to productivity types,  $\alpha$ , corresponding respectively to (10), (8a) and (8b):

$$w_F^{-1}(w) = w/\eta; \tag{14}$$

$$w_L^{-1}(w) = w + \pi(\Gamma - 1)(w + g/(1 - t)); \tag{15}$$

$$w_H^{-1}(w) = \frac{w_L^{-1}(w) - (\Gamma - 1)g/(1 - t)}{\Gamma}. \tag{16}$$

We can then describe the entire distribution of realized, pre-tax wage incomes in the economy as a mixture of three component distributions:

- (i) a distribution with c.d.f.  $F(w_F^{-1}(w))/F(\hat{\alpha})$  over the support  $[0, \hat{\alpha}/\eta]$  and mixture weight  $F(\hat{\alpha})$ ;
- (ii) a distribution with c.d.f.  $F(w_L^{-1}(w))/(1 - F(\hat{\alpha}))$  over the support  $(w_L(\hat{\alpha}), w_L(1)]$  and mixture weight  $(1 - \pi)(1 - F(\hat{\alpha}))$ .
- (iii) a distribution with c.d.f.  $F(w_H^{-1}(w))/(1 - F(\hat{\alpha}))$  over the support  $(w_H(\hat{\alpha}), w_H(1)]$  and mixture weight  $\pi(1 - F(\hat{\alpha}))$ .

Figures 1 and 2 provide an illustrative example, assuming a uniform distribution of productivity types. Figure 1 gives the wage realizations by productivity type. For the given parameters, the critical value,  $\hat{\alpha}$ , is approximately 0.32. Individuals just below this type receive a fixed wage of about 0.25, whereas individuals marginally above  $\hat{\alpha}$  receive a high wage of 0.6 with probability 0.4, and a low wage of 0.12 with probability 0.6. Figure 2 plots the distribution of pre-tax wages for the same example. Note that even though the distribution of productivity types is uniform, the resulting distribution of pre-tax wages is not, with a large mass of low earners at the bottom of the distribution and a fat tail of high earners.

## 4 Taxation, market integration, and the equilibrium structure of contracts

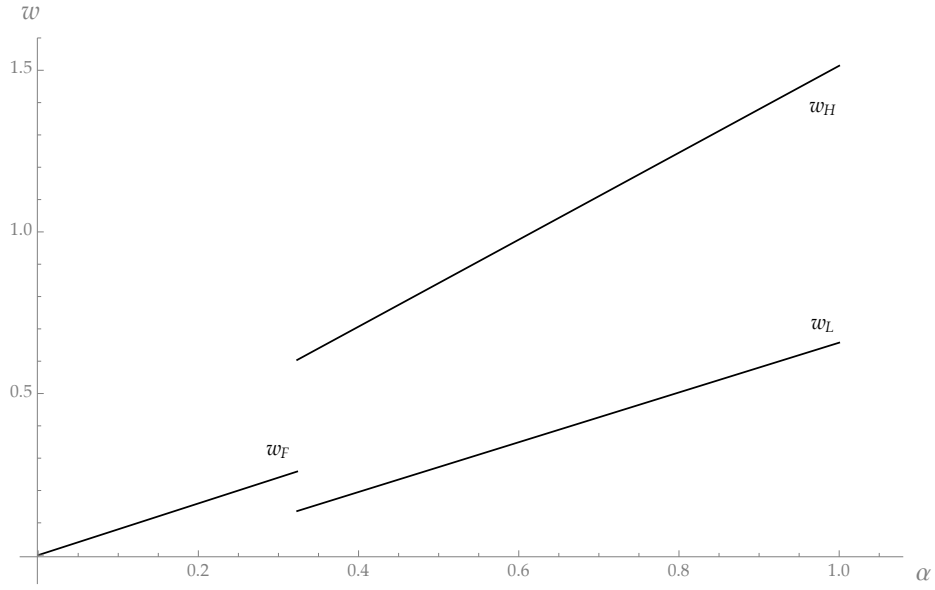
We next examine how changes in the economic environment that are exogenous to agents' decisions, namely a change in the tax/redistribution scheme and a change in output volatility caused by market integration, are reflected in the equilibrium structure of contracts and the distribution of pre-tax wages. In the next section we will build on these results to examine how the choice of an optimal redistributive tax is affected by an increase in output risk.

### 4.1 Taxation and contract choice

We first analyze the effects of changing the wage income tax rate on the cutoff productivity level  $\hat{\alpha}$  in (13). With  $K > 0$ , it is straightforward to establish that

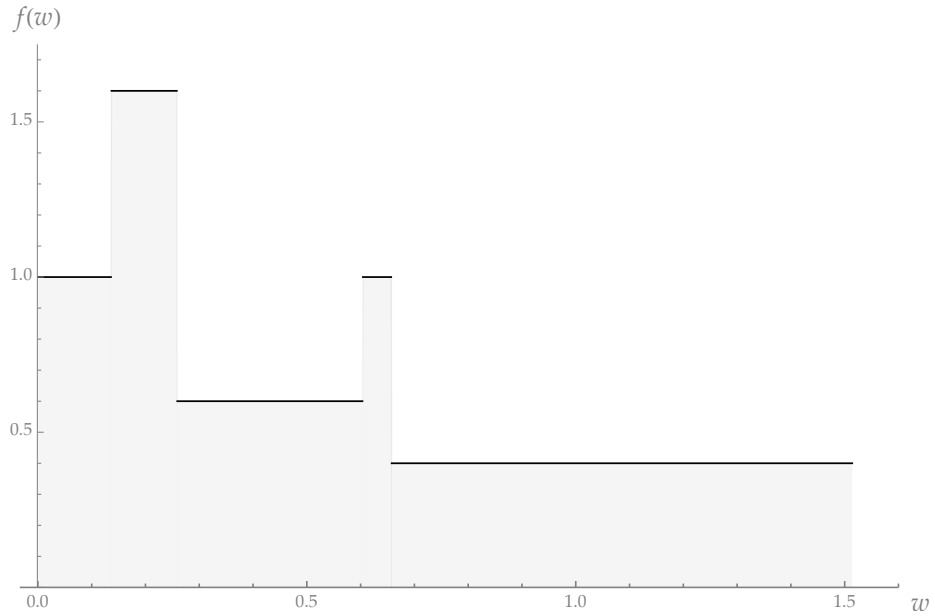
$$\frac{\partial \hat{\alpha}}{\partial t} = \frac{g}{(1-t)^2 K} = \frac{\hat{\alpha}}{1-t} > 0, \quad \frac{\partial \hat{\alpha}}{\partial g} = \frac{1}{(1-t)K} = \frac{\hat{\alpha}}{g} > 0. \quad (17)$$

An increase in the tax,  $t$ , for  $g > 0$ , raises the critical productivity level  $\hat{\alpha}$  below which workers select low-output, fixed-wage contracts. This result corresponds to the standard labor supply distortion in models with continuous effort choice (Doligalski et al.,



With  $\pi = 2/5$ ,  $\eta = 4/5$ ,  $c = \bar{c}/5$ ,  $t = 0.5$ ,  $g \approx 0.24$  (from the government budget constraint, assuming a uniform distribution of productivity types,  $F(\alpha) = \alpha$ )

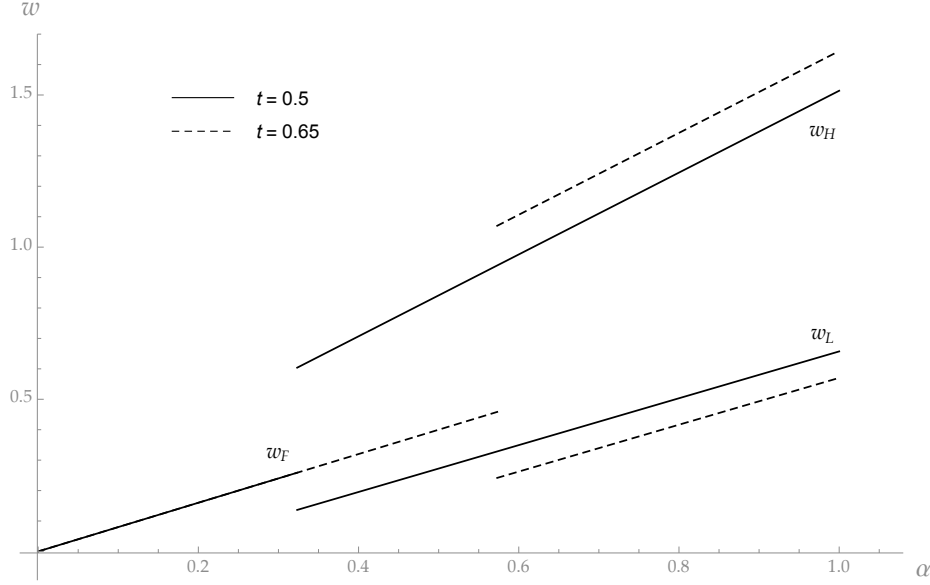
Figure 1: Wage realizations by productivity type



With  $\pi = 2/5$ ,  $\eta = 4/5$ ,  $c = \bar{c}/5$ ,  $t = 0.5$ ,  $g \approx 0.24$ ,  $F(\alpha) = \alpha$

Figure 2: Distribution of pre-tax wages





With  $\pi = 2/5, \eta = 4/5, c = \bar{c}/5, F(\alpha) = \alpha$

Figure 3: Contract responses to tax changes

2020), as the fixed-wage contract substitutes leisure in exchange for a higher expected wage. An increase in the transfer,  $g$ , also raises  $\hat{\alpha}$ , because it reduces the marginal utility of wage income.

The transfer,  $g$ , and the tax rate,  $t$ , are linked through the government budget constraint:

$$g = tQ, \quad Q = \eta \int_0^{\hat{\alpha}(t,g)} \alpha dF(\alpha) + \int_{\hat{\alpha}(t,g)}^1 \alpha dF(\alpha), \quad (18)$$

where we have written  $\hat{\alpha}(t, g)$  to highlight the dependence of  $\hat{\alpha}$  on  $t$  and  $g$ . The tax base,  $Q$ , consists of the expected output of all workers, which are employed either under a fixed-wage contract without effort (the first term in  $Q$ ), or under a performance-based contract with effort (the second term).

From (17),  $\hat{\alpha}$  is increasing in both  $t$  and  $g$  and the analysis of a change in  $t$  must incorporate the induced change in the transfer  $g$ . It can never be optimal, however, to select a tax rate at a level such that the total derivative of  $g$  with respect to  $t$ ,  $dg/dt$  is negative. Then, in the relevant range of rationalizable tax rate choices,  $t$  and  $g$  must be positively related. Therefore, an increase in the tax will unambiguously raise  $\hat{\alpha}$ , i.e.  $d\hat{\alpha}/dt > 0$ , and thus lower the total income tax base in (18). This is summarized in:

**Proposition 2.** *Consider two tax levels  $t'$  and  $t'' > t'$ , with  $\hat{\alpha}'' \in (0, 1)$  denoting a productivity type that is indifferent between the two forms of contracts under  $t = t''$ . Then the range*

*of productivity types that select into performance-based contracts is narrower under  $t''$  than under  $t'$ , and the tax base is also smaller.*

To illustrate, Figure 3 incorporates the same parameterization as in Figures 1 and 2, but with a higher tax rate,  $t = 0.65$ , up from  $t = 0.5$ .<sup>15</sup> There are now more productivity types selecting into fixed-wage contracts (about 57% of them, up from 32%), resulting in a reduction in the tax base (by an amount  $f(\alpha)(1 - \eta)\alpha$  for each productivity type that switches from one contract form to the other). Those productivity types that do select into performance-based contracts now face a larger variation in pre-tax income. This is because contracts respond to offset the tax change by increasing the pre-tax wage differential,  $w_H - w_L$ , in order to still induce effort under the higher tax rate.

## 4.2 Income volatility, contracts, and the distribution of income

Consider next the effects of an increase in the spread of output realizations. In our formalization, output in the favourable state for a project employing an individual of productivity  $\alpha$  and exerting positive effort is  $\alpha/\pi$  whereas output in the unfavourable state is zero. A decrease in the parameter  $\pi$  thus amounts to a mean-preserving spread in the returns to effort, which leaves expected output unchanged at  $\alpha$  while raising output risk.

Such a change can be interpreted as being associated with economic globalization. Appendix A presents a fully specified model that rationalizes this interpretation. In short, if ex-ante identical firms compete in any given market and face idiosyncratic cost shocks, there will be realizations in which the most productive firm will be able to capture the entire market, while other firms are left with a zero market share. Market integration gives each firm access to a larger combined market with more competitors. This increases the amount of revenues a firm can obtain if it manages to capture the larger market, but it makes such an event comparatively less likely because of the larger number of competitors. The question we want to address then is how greater product market integration, which we model here simply as a decrease in  $\pi$ , affects the equilibrium choice of contracts.

From (13), the critical value  $\hat{\alpha}$  is decreasing in  $K$ , which in turn can be shown to be increasing in  $\pi$  (see (B.1) in the appendix). Intuitively, a lower success probability  $\pi$  raises the risk involved in a performance-based contract, making this contract less attractive to risk-averse workers, in comparison to a fixed-wage contract. From (8b),

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<sup>15</sup>Note that in both cases, the maximum average tax rate is much lower, peaking at about 34% for the highest-ability individual ( $\alpha = 1$ ) with  $t = 0.5$  and at about 46% with  $t = 0.65$ .

we can also see that the high wage level,  $w_H$ , in a performance-based contract is rising when  $\pi$  falls. It immediately follows from this that a decrease in  $\pi$  must unambiguously increase the fraction of total income that accrues to a subset of high earners in favorable realizations:

**Proposition 3.** *For a given  $t$ , consider a reduction in the success probability of all projects from  $\pi'$  to  $\pi'' < \pi'$ , with  $\hat{\alpha}'' \in (0, 1)$  denoting a productivity type that is indifferent between the two forms of contracts under  $\pi = \pi''$ . Then:*

- i. *the range of productivity types that select into performance based contracts is narrower under  $\pi''$  than under  $\pi'$  ( $\hat{\alpha}'' > \hat{\alpha}'$ ), and the tax base is also smaller;*
- ii. *the fraction of pre-tax income accruing to income recipients above a certain percentile threshold,  $p''$ , in the distribution of ex-post, pre-tax income is greater under  $\pi''$  than under  $\pi'$ .*

PROOF: See Appendix B.3.

In qualitative terms, the effect of a mean-preserving spread in the returns to effort associated with greater product market integration (a fall in  $\pi$ ), will be analogous to that of an increase in  $t$ : it will reduce the range of productivity types that select into incentive contracts and raise the dispersion of pre-tax wages within incentive contracts. This model implication is consistent with the empirical evidence that economic globalization increases wage inequality (Goldberg and Pavcnik, 2007; Dreher and Gaston, 2008).<sup>16</sup> It is also consistent with evidence showing that stronger import competition in global markets increases the sensitivity of performance-based contracts and leads to a shift in the pay structure away from fixed pay and towards performance-related pay (Cuñat and Guadalupe, 2009).

Figure 4 incorporates the same parameterization as in Figures 1 and 2, but with a lower probability of success,  $\pi = 0.3$ , down from  $\pi = 0.4$ . The critical productivity level below which individuals select into a fixed-wage contract has risen to about 0.49. Figure 5 shows how the distribution of pre-tax wages has become more concentrated. For  $\pi = 0.3$ , bonus recipients are those above the 85th percentile of the income distribution ( $\hat{\alpha} \approx 0.49 \Rightarrow \pi(1 - F(\hat{\alpha})) \approx 0.15$ ). The fraction of total pre-tax income accruing to them is approximately 46%. The corresponding figure for earners above

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<sup>16</sup>Dorn et al. (2018) have recently re-examined the link between globalization and income inequality for 140 countries over the period 1970-2014. They find a robust positive relationship between globalization and wage inequality for most countries, including China and Eastern Europe, though the effect is not significant for the most advanced economies.

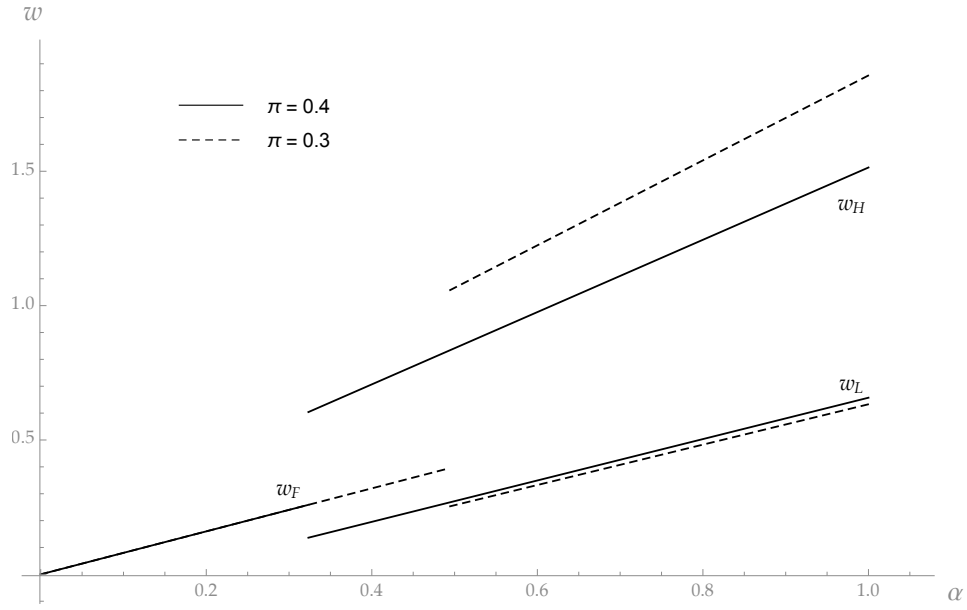


Figure 4: Contract responses to output volatility

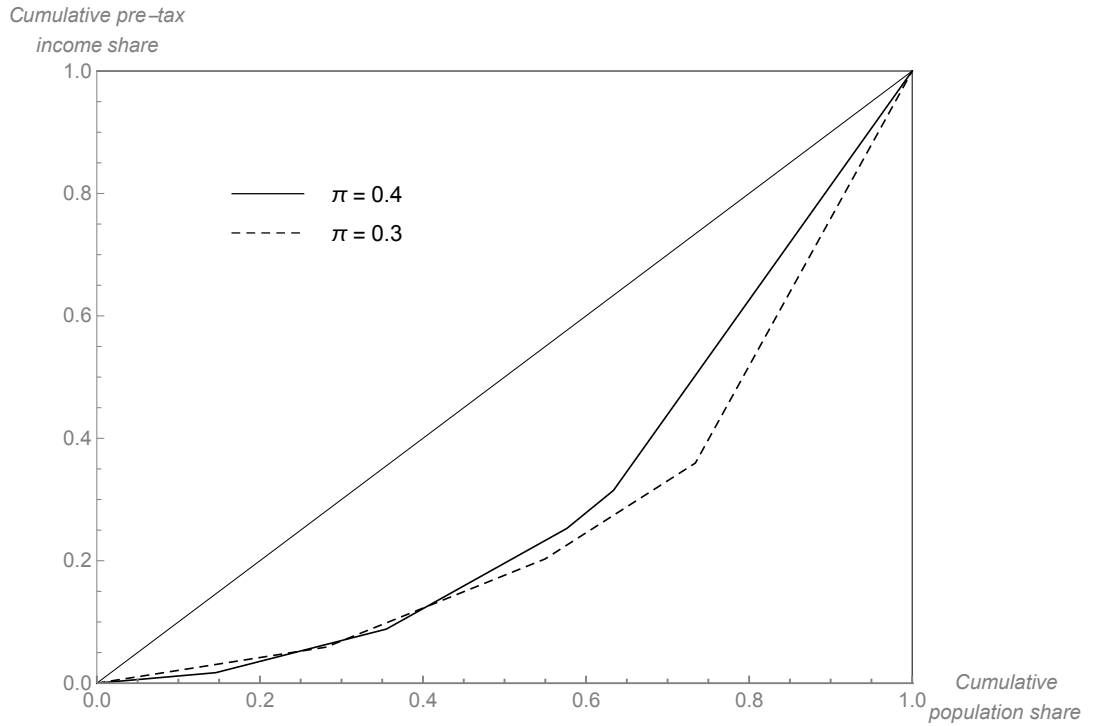


Figure 5: Output volatility and income concentration

the 85th percentile of the distribution when  $\pi = 0.4$  is lower, at approximately 36%. Approximate values of the Gini coefficient for  $\pi = 0.4$  and  $\pi = 0.3$  are respectively 0.39 and 0.45.

In our discussion, we have characterized the effect of economic globalization in product markets as exclusively consisting of an increase in the variance of outcomes for individual projects. However, the same conclusions continue to apply if, additionally, globalization also results in an increase in productivity.<sup>17</sup> To see this, take the distribution of productivity types in our model, which has a support  $[0, 1]$ , to represent the ‘baseline’ distribution. Now suppose that globalization results in an economy-wide, type-neutral productivity shift that increases the productivity of each worker by a factor  $\lambda > 1$ . Further, let the cutoff productivity of the individual that is indifferent between contract types before globalization (scenario 0), for a tax rate  $t_0$  and tax revenue  $g_0$ , be  $\hat{a}_0$ . After globalization, the productivity of this same type is  $\lambda\hat{a}_0$ . If this individual is also the cut-off productivity type following globalization (scenario 1), so that  $\hat{a}_1 = \lambda\hat{a}_0$ , then the model remains structurally unchanged. For an unchanged tax rate,  $t_1 = t_0$ , total tax revenues will then equal  $g_1 = \lambda g_0$  (as the income of all workers is multiplied by  $\lambda$ , irrespective of contract type); and from (13) we see that the condition  $\hat{a}_1 = \lambda\hat{a}_0 = \lambda g_0 / ((1 - t)K)$  will indeed be met if the tax rate remains the same.<sup>18</sup> We argue below that there is indeed no reason for the tax rate to change in scenario 1. It follows that our problem is fully invariant to uniform changes in the productivity of all types. Hence, all the results derived in our analysis continue to hold if, in addition to increasing the volatility of output for each worker, the globalization of product markets also leads to productivity gains.

## 5 Optimal redistributive taxation and economic globalization

We now turn to our main question—how the choice of redistributive taxes is affected by an increase in output risk brought about by market integration.

Since agents are risk-averse in our model and firms are risk-neutral, optimal con-

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<sup>17</sup>The model we present in Appendix A, which provides microfoundations for our interpretation of higher output risk as being equivalent to product market integration, does predict a positive productivity shift.

<sup>18</sup>With logarithmic utility, the trade-off between the fixed wage contract and the incentive contract remains unaffected when incomes under both contract types increase proportionally and effort costs in the incentive contract stay nominally unchanged.

tracts under full information (i.e. with effort being observable) would be directly conditioned on effort and would fully eliminate income risk for workers. Under asymmetric information, performance-based contracts that impose risks on risk-averse agents arise endogenously, by deliberate design, as a second-best market remedy. As a result, the linear income tax cannot play any role in mitigating this risk.<sup>19</sup>

By redistributing income between different productivity types, linear income taxes can still insure individuals against their individual productivity draw.<sup>20</sup> Income based taxation, however, restricts redistribution to be conditioned on income realizations rather than on productivity types. In a setting where workers in an incentive contract face income volatility, the ranking of realized earnings does not fully reflect the ranking of productivity types. By redistributing from ex-post high earners to ex-post low earners the indirectly progressive income tax achieves redistribution “on average” from high-productivity to low-productivity types, thus providing ex-ante insurance against adverse productivity draws. But it does so at the cost of interfering with contract design: a linear income tax that reduces the after-tax wage gap in the two states of the world will require a correspondingly higher before-tax wage differential for the contract to remain incentive-compatible. This, in turn, will put a break on redistribution.<sup>21</sup>

A change in output risk caused by economic globalization will affect the extent to which contract choices respond to taxes and will thus be reflected in the level of redistributive taxation that is optimally selected by the tax planner.

## 5.1 Revenue maximization

A natural starting point for studying how changes in the economic environment translate into changes in tax policy choices is the case where a government aims at maximizing tax revenues. Revenue maximization is consistent with the government pursuing

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<sup>19</sup>This corresponds to the finding that social insurance leads to a crowding-out of private insurance (e.g. Chetty and Saez, 2010).

<sup>20</sup>The optimal income tax literature often distinguishes between an *insurance role* of the income tax, which reduces the variance of exogenous earnings risk for a given productivity type, and a *redistributive role*, which reduces the after-tax variation in the incomes of different productivity types. See Boadway and Sato (2015) for a theoretical survey and Hoynes and Luttmer (2011) for an empirical analysis distinguishing the insurance and redistributive effects of state tax-and-transfer programmes in the United States. In this paper we interpret the redistributive role of the income tax equivalently as an insurance against individual productivity draws.

<sup>21</sup>In the following analysis, we restrict attention to interior optima. This requires that the productivity gap between fixed-wage and performance-wage contracts is sufficiently large (i.e.,  $\eta$  is sufficiently below unity) that it is never optimal for the government to opt for a confiscatory tax rate ( $t = 1$ ).

redistribution under a Rawlsian social welfare objective where only the utility level of the least well-off type enters as an argument. In an economy where the lowest productivity type receives no wage income under either contract ( $\underline{\alpha} = 0$ ), this is the same as maximizing tax revenues and hence the per-capita transfer that can be redistributed to all individuals, including the least well-off.<sup>22</sup>

From (18), the first-order condition for a revenue-maximizing tax rate,  $t^*$ , is

$$\frac{dg}{dt^*} = Q + t^* \frac{dQ}{dt^*} = 0, \quad (19)$$

where

$$\frac{dQ}{dt} = -(1 - \eta) f(\hat{\alpha}) \hat{\alpha} \frac{d\hat{\alpha}}{dt}; \quad \frac{d\hat{\alpha}}{dt} = \frac{\partial \hat{\alpha}}{\partial t} + \frac{\partial \hat{\alpha}}{\partial g} \frac{dg}{dt} > 0, \quad (20)$$

where the expression for  $\partial \hat{\alpha} / \partial t > 0$  is given in (17), and  $dg/dt$  in (20) vanishes at an optimum.

In order to examine how market integration, modelled as an exogenous reduction in  $\pi$ , affects the optimal tax rate  $t^*$ , it is sufficient to focus on expression  $K$  in (13)—since  $\pi$  enters (19) and (20) only through  $K$ . From (B.1) in the appendix,  $K$  is positively related to  $\pi$ . Relying on the implicit function theorem and assuming the second-order condition for an optimum to hold,<sup>23</sup> we get, at  $t = t^*$ :

$$\frac{dt}{dK} = \frac{d^2g/(dt dK)}{d^2g/dt^2} \Rightarrow \text{sign}\left(\frac{dt}{dK}\right) = \text{sign}\left(\frac{d^2g}{dt dK}\right).$$

Differentiating (19) with respect to  $K$  yields (see Appendix B.4):

$$\frac{d^2g}{dt dK} = \Phi \left( f(\hat{\alpha}) + \frac{t}{1-t} \left( f'(\hat{\alpha}) \hat{\alpha} + 2f(\hat{\alpha}) \right) \right), \quad (21)$$

where  $\Phi \equiv (1 - \eta) \hat{\alpha} g / (K ((1 - t) K + t (1 - \eta) f(\hat{\alpha}) \hat{\alpha})) > 0$ . The expression on the right-hand side of (21) is positive if the sum of terms in parentheses is positive, i.e. if

$$\frac{\partial f(\hat{\alpha})}{\partial \alpha} \frac{\hat{\alpha}}{f(\hat{\alpha})} \equiv \xi_{f,\alpha} \geq -\frac{1+t}{t}. \quad (22)$$

<sup>22</sup>From an ex-ante perspective, the tax planner's objective in this case does *not* coincide with the ex-ante expected utility of an individual as implied by an expectation over  $u(x_{\alpha_s}) - e_\alpha c$ , with  $s \in \{0, 1\}$  denoting the output-relevant state. Rather, it corresponds to a case where individuals exhibit ex-ante infinite risk aversion, which corresponds to an ex-ante expected utility equal to  $\min_{\alpha,s} \{u(x_{\alpha_s}) - e_\alpha c\}$ .

<sup>23</sup>The second-order condition is met if revenue is concave in  $t$ , i.e. if  $2 dQ/dt + t d^2Q/dt^2 < 0$ ; or  $-t(d^2Q/dt^2)/(dQ/dt) < 2$ . Using the fact that  $dg/dt = 0$  at an optimum, this can be shown to require  $\xi_{f,\alpha} < 1 + 2/t$ . This condition is always met by a distribution of ability types such that  $f'(\alpha) \leq 0$  (sufficiency).

A sufficient condition for this to be satisfied for all values of  $t \in (0, 1)$  is  $\xi_{f,\alpha} > -2$ . If this condition is met, then globalization (a reduction in  $\pi$ ) will *lower* the equilibrium tax rate  $t^*$ . This will be the case when  $f(\alpha)$  is either non-decreasing, or only moderately decreasing in the productivity type  $\alpha$ . Condition (22) will always be fulfilled, for example, under a uniform distribution of productivity types, where  $f'(\hat{\alpha}) = 0$ .<sup>24</sup>

A mean-preserving output spread thus causes the efficiency costs of taxation to rise, as incentive contracts respond more sensitive to taxation. Intuitively, a fall in  $\pi$  increases income volatility in a performance-based contract and increases the cutoff productivity level,  $\hat{\alpha}$ , above which workers select into this contract. At the higher level of  $\hat{\alpha}$ , the level of the tax base is reduced and the remaining tax base responds more strongly to taxation. Both of these effects imply that the elasticity  $\varepsilon = (t/Q)(dQ/dt)$  with which the tax base responds to a change in the tax rate tends to increase in absolute value. This increase in the elasticity of the tax base is similar to models where the high-income workers are internationally mobile (Lehmann et al., 2014). The mechanism, however, is entirely different—there is no inter-jurisdictional factor mobility in our model.

Under revenue maximization, the optimal tax rate will therefore fall unless, at the higher level of  $\hat{\alpha}$ , there are substantially fewer workers whose contract choice is distorted by the tax. Since the government is only concerned with the ex-post welfare of the poorest individual, any changes in the insurance role that the indirectly progressive income tax takes in our model, are not relevant under this objective function.

We summarize our results in this section in the following proposition:

**Proposition 4.** *A mean-preserving spread in output realizations (a fall in  $\pi$ ) reduces the revenue-maximizing tax rate: (i) if the distribution of productivity types is uniform; (ii) for any distribution of types that fulfills condition (22).*

PROOF: See Appendix B.4.

Note that if we additionally associate globalization with a positive, proportional productivity shift, as discussed at the end of Section 3, this has no effect on the optimal tax under a revenue-maximizing objective: in (19), both  $Q$  and  $dQ/dt^*$  are multi-

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<sup>24</sup>Condition (22) refers to the distribution of productivity types and does *not* imply a condition on the distribution of realized wages. The latter results in our setting from a mixture of distributions of type-specific distributions of wage realizations (cf. Section 3.2). In a generalized version of our model where the type-specific distributions of wage realizations are continuous (e.g. Pareto), the shape and tail behaviour of the overall distribution of realized wages (e.g. a Pareto tail) would thus be inherited from the shape of the type-specific distributions, not from the shape of the distribution of productivity types.



plied by a common productivity shift factor in this case, leaving the optimal tax rate unchanged.

Once more, it is worth emphasizing that the uniform distribution of types singled out in Proposition 4 does *not* imply a uniform distribution of incomes. Rather, it generates a right tail as only some of the highest-productivity workers will be successful, and these receive high bonus payments in equilibrium. Moreover, this right tail becomes fatter as market integration increases, as fewer high-productivity workers will achieve comparatively greater success.

## 5.2 Utilitarian objective

A utilitarian social welfare function can be written as

$$W = \int_0^{\hat{\alpha}} EU^F(\alpha) dF(\alpha) + \int_{\hat{\alpha}}^1 EU^E(\alpha) dF(\alpha). \quad (23)$$

The first term on the right-hand side sums over the expected utilities of workers with a fixed wage contract, whereas the second term represents workers with an incentive contract. This is a non-trivial change from the Rawlsian case (revenue maximization), because now all individuals, with different types of contracts, enter the government's objective.

Since  $EU^E(\hat{\alpha}) = EU^F(\hat{\alpha})$  (by the definition of  $\hat{\alpha}$ ), the first-order condition for a welfare-maximizing tax rate can be written as

$$\frac{dW}{dt} = \int_0^{\hat{\alpha}} \frac{dEU^F(\alpha)}{dt} dF(\alpha) + \int_{\hat{\alpha}}^1 \frac{dEU^E(\alpha)}{dt} dF(\alpha) = 0. \quad (24)$$

Expanding the terms,  $dEU^Z(\alpha)/dt$ ,  $Z \in \{E, F\}$ , we can express them as (see Appendix B.5)

$$\frac{dEU(\alpha)}{dt} = \frac{dg/dt - b(\alpha)\alpha}{g + (1-t)b(\alpha)\alpha}, \quad (25)$$

with  $b(\alpha) = \eta$  for  $\alpha < \hat{\alpha}$  and  $b(\alpha) = 1$  for  $\alpha \geq \hat{\alpha}$ . Just by inspecting (25) and (24), we can conclude that, since  $b(\alpha) > 0$  for all  $\alpha$ , we must have  $dg/dt > 0$  at an optimum. Thus, under a utilitarian objective the optimal tax falls short of the level that maximizes tax revenues.<sup>25</sup> As noted earlier, a welfare-maximizing tax could never exceed the level at which revenues are maximized, and will only reach that level when the redistribution motive is at its strongest—under a Rawlsian objective.

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<sup>25</sup>The Rawlsian case corresponds to  $b(\alpha) = 1$  for  $\alpha = 0$  and  $b(\alpha) = 0$  for all  $\alpha > 0$ ; (24) then becomes  $(dg/dt)/g = 0$ .

To understand how a mean-preserving spread in output realizations (a reduction in  $\pi$ ) is reflected in the optimal choice of tax, note first that all effects only flow through the induced increase in the cutoff productivity  $\hat{\alpha}$ . In the absence of a change in  $\hat{\alpha}$ , there would be no change in  $dg/dt$  (nor in  $g$ ), nor would there be any change in the  $b(\alpha)$  terms that enter expression (25).<sup>26</sup> With a change in  $\hat{\alpha}$ , on the other hand, there are two separate effects on (25)—and hence on the optimality condition for the tax rate chosen by a utilitarian planner.

The first effect, how  $dg/dt$  changes following a reduction in  $\pi$ , is the same as for the revenue-maximizing case in (21): a fall in  $\pi$  makes the tax base respond more elastically to a tax increase, thus lowering  $dg/dt$ . The new element under a utilitarian objective comes from a change in the redistributive effect of the tax. This has two components. First, a fall in tax revenues ( $g$ ) lowers the denominator in (25). While this is true for all individuals, the relative importance of this effect is higher for lower-ability types—more specifically, those for which the numerator in (25) is positive at an optimum. There is a further effect, operating through the second term in the numerator of (25): the switch in contract form for the marginal type  $\hat{\alpha}$  causes the tax base for that type—a comparatively lower ability type—to fall from  $\hat{\alpha}$  to  $\eta\hat{\alpha}$ . Both effects work to raise the attractiveness of tax-funded redistributive transfers.

The intuition behind these effects is that there is a ‘leisure’ component of welfare, having a consumption value equal to  $(1 - \eta)\alpha$  and producing a utility differential equal to the effort cost  $c$ , that at the discontinuity point goes from being untaxed (below  $\hat{\alpha}$ ) to being taxed (above  $\hat{\alpha}$ ). Since higher-income individuals bear a comparatively greater tax burden, this adds to the redistributive effectiveness of the tax for a given  $\hat{\alpha}$ . If then  $\hat{\alpha}$  rises following a decrease in  $\pi$ , the redistributive effectiveness of the tax further improves, with comparatively lower productivity types previously above the threshold now moving below the threshold and benefiting comparatively more from a higher tax. Other things being equal, this makes a higher tax more attractive to a utilitarian planner.

Appendix B.6 derives a general condition—analogous to condition (22)—that must be met for a fall in  $\pi$  to bring about a fall in the optimal tax rate. This involves a comparison of a term (capturing the change in the marginal revenue effectiveness of the tax) that only depends on the level and the slope of  $f(\alpha)$  at  $\hat{\alpha}$  with a term (capturing the change in the re-distributional effectiveness of the tax) that depends on the shape of  $f(\alpha)$  over its full support  $[0, 1]$ —a term that is absent in the revenue-maximization

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<sup>26</sup>If there is also a productivity shift, all terms  $dEU(\alpha)/dt$  in (25) remain unchanged, as both the numerator and the denominator on the right-hand side are multiplied by a common factor. Therefore, the optimal tax rate implied by the first-order condition (24) must again remain unchanged.

case. This makes the condition more stringent than condition (22), and the more so the larger is the mass of individuals below  $\hat{a}$  who are positively affected by redistribution becoming more targeted following an increase in  $\hat{a}$ .

We have the standard optimal income tax argument: the efficiency loss is proportional to the number of workers whose decision is distorted at the margin, whereas the redistributive value of the tax depends on the total number of individuals who are positively or negatively affected by the redistribution induced by the tax (Sheshinski, 1972). In the presence of performance contracts, however, we need to apply this logic to the distribution of *expected gross earnings* (the  $\alpha$ s) rather than to the distribution of the income realizations, the latter resulting from the scrambling of the former with an uninsurable stochastic component. For a given observed distribution of income realizations, the presence of contracts implies a comparatively less bottom-heavy distribution of  $\alpha$ . Hence, other things equal, the effect on the efficiency cost of the tax is comparatively more likely to dominate the effect on its redistributive value.

It is therefore not difficult to identify scenarios where the first effect remains the dominating effect and so a decrease in  $\pi$  unambiguously lowers  $t^*$ : this is the case when the distribution of productivity types is uniform (which, as already noted, can produce a right-skewed distribution of earnings realizations). The uniform case is a natural benchmark case because it implies that the trade-off between the efficiency costs of the tax and its redistributive value does not change along the ability distribution, and so changes in  $K$  affect the efficiency costs and the redistributive value of the tax in the same proportion.

To characterize the optimal tax response to a change in  $\pi$  for the uniform case, we proceed in the same way as in the previous section: assuming that the second-order condition for an optimum is satisfied and relying on the positive relationship between  $K$  and  $\pi$  in (B.1), the effect of a change in  $\pi$  on the optimal tax rate has the same sign as  $d^2W/(dt dK)$ . With a uniform distribution of ability types, this is given by (see Appendix B.7):

$$\frac{d^2W}{dt dK} = \frac{(1-t)(1-\eta)K}{((1-t)K + \mu)\mu} \left( \frac{t^2}{K} \left( \frac{1}{(K + \mu)} + \frac{2}{(1-t)K} \right) - \frac{1}{(K + 1)(K + \eta)} \right). \quad (26)$$

where  $\mu \equiv \sqrt{(1-t)^2K^2 + (1-\eta)t^2} > 0$ . The first bracketed term in (26) gives the higher efficiency costs of the tax. This term is always positive and is rising in the tax rate  $t$ . The negative second term in the bracket gives the higher redistributive value of the tax. The appendix then proceeds to prove that the first effect always dominates the second; which leads to the following result:

**Proposition 5.** *Under a utilitarian objective, and for a uniform distribution of productivity types, a mean-preserving spread in output realizations (a fall in  $\pi$ ) reduces the welfare-maximizing tax rate.*

PROOF: See Appendix B.8.

Proposition 5 corresponds to the result in Proposition 4 for the Rawlsian case. With a uniform distribution of productivity types, our model therefore unambiguously predicts a *negative* effect of market integration (characterized here as a mean-preserving spread in output realizations) on the optimal redistributive income tax rate.

The intuition for this result is as follows. At the increased cutoff value  $\hat{a}$  induced by globalization (Proposition 3), a tax increase has a larger marginal effect on incentive contracts (see (17)). When the distribution of ability types is uniform, this translates into a larger absolute number of incentive contracts being destroyed by a marginal tax increase, and hence to a larger efficiency cost of the tax. As we have seen from our discussion of (25) above, the redistributive value of the tax simultaneously rises; however, the higher redistributive value arises only from the fall in wage income that is caused by the destruction of incentive contracts. In other words, the increase in the redistributive value of the tax is an indirect effect that cannot compensate the direct effect caused by the higher efficiency costs of the tax.

Figures 6 and 7 conclude the example that we have described in the previous sections, using the same parameterization but now making the level of taxation equal to its optimum under a utilitarian objective. This optimum rate is approximately 58% under the baseline value of  $\pi$  of 0.4. If  $\pi$  is reduced to 0.3, the optimal tax rate falls to approximately 51%. Figure 6 shows that the negative effects that the change in  $\pi$  has on the number of incentive contracts is largely, though not fully, offset by the reduction in the optimal tax rate (cf. Proposition 2). The gap in gross wages earned in the high and the low state, which is increased by globalization (cf. Figure 4), remains largely unaffected by the optimal adjustment of the income tax rate.

Figure 7 shows that globalization (a fall in  $\pi$ ) and the optimal adjustment of the income tax rate unambiguously increase the concentration of after-tax earnings. The isolated reduction in  $\pi$  raises, in particular, the concentration in the upper tail of the distribution of earnings realizations (see also Figure 5). The optimal, downward adjustment of the income tax rate increases inequality further by reducing net transfers to the lower tail of the earnings distribution. The Gini coefficient for after-tax income is 0.19 in the baseline scenario ( $\pi = 0.4, t = 58\%$ ), rising to 0.21 for  $\pi = 0.3$  if  $t$  remains at 58%, and rising further to 0.24 if  $t$  adjusts to the optimal rate of 51%.<sup>27</sup>

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<sup>27</sup>Pre-tax earnings also become more concentrated—approximate values of the pre-tax income Gini

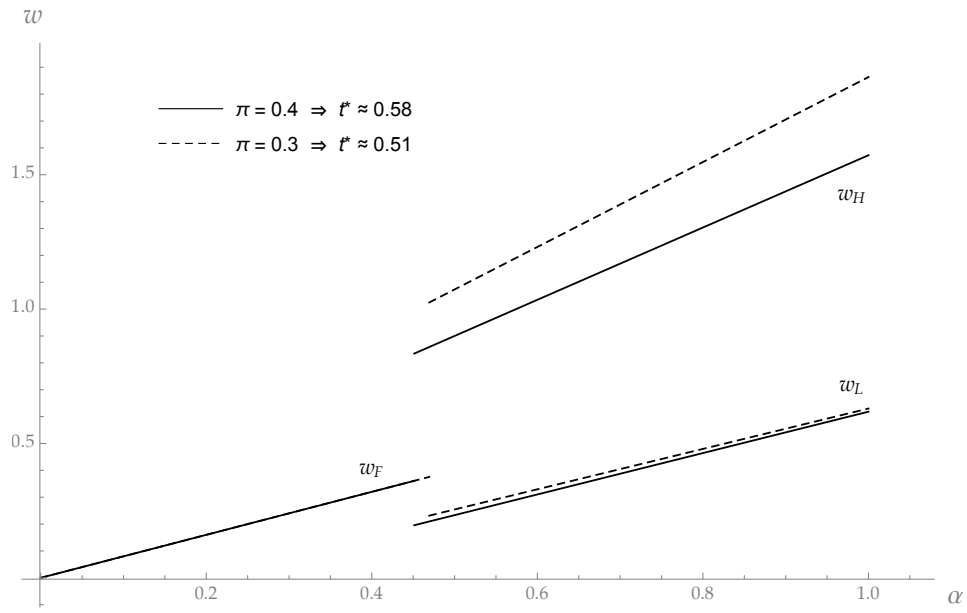


Figure 6: Output volatility, optimal taxes, and equilibrium contracts – utilitarian objective

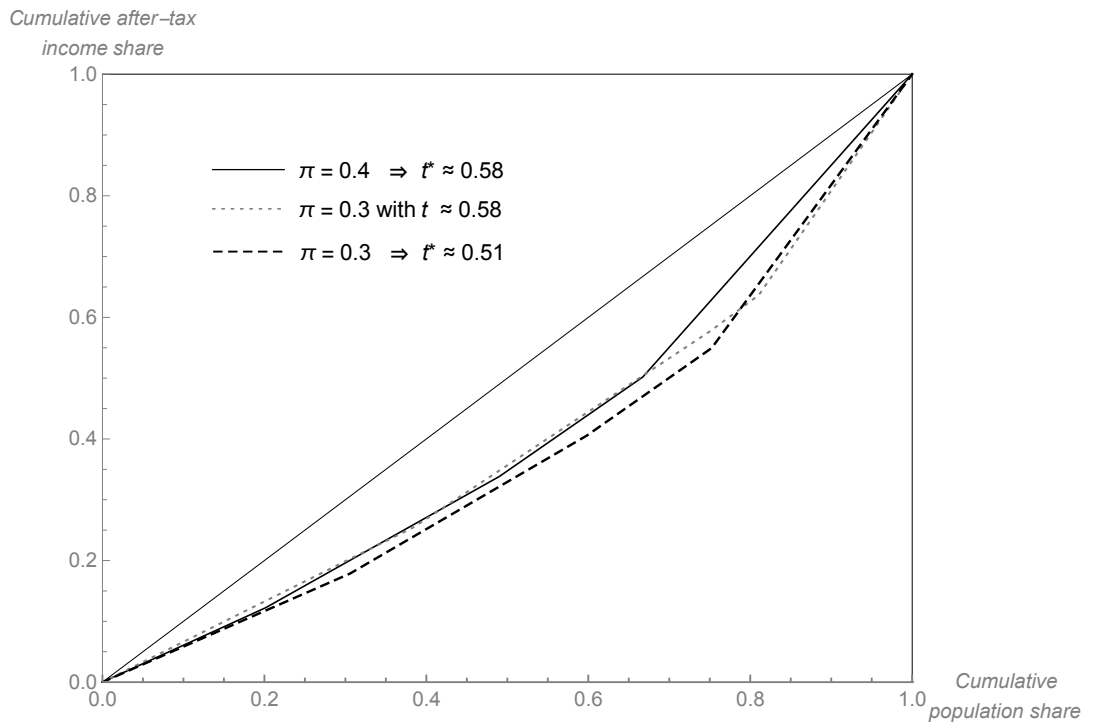


Figure 7: Output volatility, optimal taxes, and after-tax income concentration – utilitarian objective

## 6 Globalization, inequality and redistribution

The predictions we have derived in the previous sections have direct implications for the relationship between changes in tax policy choices and changes in the distribution of income, when this relationship is mediated through a change in globalization-induced income risk, as measured by  $\pi$ . In Section 4, we have shown that economic globalization, characterized as a decrease in  $\pi$ , raises wage inequality and the concentration of earnings at the top of the income distribution (Proposition 3). And our results in Section 5 state that this can be accompanied by a fall in the optimal rate of redistributive taxation (Propositions 4 and 5).

Our results are thus fully consistent with the simultaneous increase in pre-tax inequality and the reduction in redistributive income tax rates that has been documented in the empirical literature (Immervoll and Richardson, 2011; Egger et al., 2019). In our model, the decrease in taxation and the increase in earnings inequality are both consequences of structural changes associated with economic globalization.

The increase in income inequality is generated in our model by the wider dispersion of pre-tax wages under incentive contracts that can be attributed to increased competition under market integration. This characterization is consistent with the increase in *between-firm* wage inequality that has been documented in the empirical labour economics literature (Song et al., 2019). If performance incentives are dynamically interpreted, it is also consistent with the observation of rising *within-firm inequality* that has been documented in the empirical literature on life-cycle wage dynamics (Heathcote et al., 2010; Bayer and Kuhn, 2019).

Market integration raises income inequality in our model even when it does not cause incentive contracts to become widespread—a fall in  $\pi$  causes the range of productivity types selecting into incentive contracts to narrow (i.e.  $\hat{a}$  to fall). But the model can be extended to account for a rising share of incentive contracts if, in addition to lowering  $\pi$ , market integration also results in a positive productivity shift (as discussed in Section 3.1). If this productivity shift affects only high-ability workers in incentive contracts but not lower ability types in fixed-wage contracts, then the combination of a reduction in  $\pi$  and an increase in the productivity of incentive contracts can cause the optimal tax rate to fall *and* the share of incentive contracts to rise (i.e.  $\hat{a}$  to fall).<sup>28</sup>

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coefficient for  $\pi = 0.4$  and  $\pi = 0.3$  are respectively 0.42 and 0.45.

<sup>28</sup>An alternative explanation for a rising share of incentive contracts is to introduce monitoring costs for incentive contracts falling over time as a result of new technological developments (see Lemieux et al., 2009).

The fact that globalization induces changes in income inequality differentiates our analysis from the literature focusing on the increased mobility of high income earners under labor market integration (Simula and Trannoy, 2010; Bierbrauer et al., 2013; Lehmann et al., 2014). In these models, changes in optimal non-linear income tax rates are exclusively driven by efficiency considerations, which are in turn determined by the level and the slope of the semi-elasticity of migration.<sup>29</sup> Our analysis also considers optimal tax responses that simultaneously weigh changes in the elasticity of the tax base against changes in the redistributive value of the income tax.

The trade-off between higher efficiency costs and a higher redistributive value of the income tax is also present in the ‘superstar’ model of Scheuer and Werning (2017), who show that introducing superstar effects for a given distribution of *abilities* leaves the optimal redistributive tax rate constant. This result obtains because the higher redistributive gain from the tax is exactly offset by the larger elasticity of the tax base. Formally, this is directly comparable to the opposing effects that we highlight in our analysis of utilitarian optima, but our results are different. In our model, the fall in  $\pi$  induced by market integration generates an effect analogous to a ‘superstar effect’, but (under weak conditions) the increase in the elasticity of the tax base dominates the redistributive gains. One core difference between our analysis and that of Scheuer and Werning (2017) is that the *expected wage* of high-ability types does not rise in our model, even though the observed wage inequality rises. This once again shows that the importance of redistributive motives is systematically reduced by the presence of performance pay, and more so the larger is the contribution of incentive contracts to observed income inequality.

Our analysis captures only a subset of the relevant effects that are associated with economic globalization. While our results are invariant to an equiproportional change in productivity levels (as discussed in Section 4.2), a key assumption in our analysis is that *relative* productivity levels remain unchanged. To the extent that globalization raises the relative productivity of high-ability vis-à-vis low-ability types, the redistributive argument for a progressive income tax is strengthened. Even in this extended setting, however, the increase in observed income inequality is partly driven by the higher income spread in incentive contracts, and this part will not give any reason for increased redistributive taxation.

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<sup>29</sup>In line with their focus on efficiency effects, Simula and Trannoy (2010) and Lehmann et al. (2014) only consider revenue maximizing governments. Bierbrauer et al. (2013) assume utilitarian governments, but relocation of all workers is costless in their model. As they show, this makes it impossible to levy positive taxes on individuals with above-average incomes.

## 7 Conclusion

Empirical studies have shown that economic globalization is associated with both an increase in income inequality and a reduction in the progressivity of income taxes (Egger et al., 2019). In this paper we have offered an explanation for these simultaneous developments, which seemingly contradict the predictions of optimal tax theory.

In our model, increased competition in integrated product markets translates into higher idiosyncratic income risk that performance-based contracts must impose on high productivity workers to overcome moral hazard. The higher volatility of individual earnings makes incentive contracts more sensitive to the disincentive effects caused by a progressive income tax, raising the efficiency cost of redistributive taxation. At the same time, redistributive taxation is unable to counter the higher income inequality that arises endogenously from steeper incentive contracts. As a result, optimal redistributive income tax rates will fall.

Our analysis could be extended in several further directions. One example is to allow for continuous wage contracts in the context of a continuous distribution of possible output realizations. Additionally allowing for heterogeneous effects on both output volatility and productivity across different ability types and occupations could yield a framework that lends itself to structural estimation in future empirical work.

## References

- Abraham, A., Alvarez-Parra, F., Forstner, S. (2017). The effects of moral hazard on wage inequality in a frictional labor market. Mimeo, European University Institute, Florence.
- Agrawal, D., Foremny, D. (2019). Relocation of the rich: Migration in response to top tax rate changes from Spanish reforms. *Review of Economics and Statistics* 101, 214-232.
- Autor, D., Dorn, D., Katz, L.F., Patterson, C., Van Reenen, J. (2017). The fall of the labor share and the rise of superstar firms. NBER Working Paper No. 23396. Cambridge, Mass.
- Alvaredo, F., Chancel, L., Piketty, T., Saez, E., Zucman, G. (2017). Global inequality dynamics: New Findings from WID.world. *American Economic Review, Papers and Proceedings* 107, 404-409.
- Atkinson, A., Piketty, T., Saez, E. (2011). Top incomes in the long run of history. *Journal of Economic Literature* 49, 3-71.



- Bargain, O., Dolls, M., Immervoll, H., Neumann, D., Peichl, A., Pestel, N., Siegloch, S. (2015). Tax policy and income inequality in the United States, 1979-2007. *Economic Inquiry* 53, 1061-1085.
- Battisti, M., Felbermayr, G., Peri, G., Poutvaara, P. (2018). Immigration, search and redistribution: A quantitative assessment of native welfare. *Journal of the European Economic Association* 16, 1137-1188.
- Bayer, C., Kuhn, M. (2019). Which ladder to climb? Decomposing life cycle wage dynamics. CEPR Working Paper No. 13158. London.
- Bell, B., Van Reenen, J. (2014). Bankers and their bonuses. *The Economic Journal* 124, F1-F21.
- Bierbrauer, F., Brett, C., Weymark, J. (2013). Strategic nonlinear income tax competition with perfect labor mobility. *Games and Economic Behavior* 82, 292-311.
- Boadway, R., Marchand, M., Pestieau, P. (1991). Optimal linear income taxation in models with occupational choice. *Journal of Public Economics* 46, 133-162.
- Boadway, R., Sato, M. (2015). Optimal income taxation with risky earnings: A synthesis. *Journal of Public Economic Theory* 17, 773-801.
- Bryson, A., Freeman, R., Lucifora, C., Pelizzari, M., Perotin, V. (2012). Paying for performance: incentive pay schemes and employees' financial participation. Centre for Economic Performance Discussion Paper No. 1112. London School of Economics.
- Chetty, R., Saez, E. (2010). Optimal taxation and social insurance with endogenous private insurance. *American Economic Journal* 2(2), 85-114.
- Cuñat, V., Guadalupe, M. (2009). Globalization and the provision of incentives inside the firm: The Effect of foreign competition. *Journal of Labor Economics* 27, 179-212.
- Diamond, P.A., Saez, E. (2011). The case for a progressive tax: From basic research to policy recommendations. *Journal of Economic Perspectives* 25, 165-190.
- Doligalski, P., Ndiaye, A., Werquin, N. (2020). Redistribution with performance pay. CESifo Working Paper No. 8267. Munich.
- Dorn, F., Fuest, C., Potrafke, N. (2018). Globalization and income inequality revisited. CESifo Working Paper No. 6859. Munich.
- Dreher, A., Gaston, N. (2008). Has globalization increased inequality? *Review of International Economics* 16, 516-536.
- Eaton, J., Kortum, S. (2002). Technology, geography, and trade. *Econometrica* 70, 1741-1779.
- Edmans, A., Gabaix, X. (2016). Executive compensation: A modern primer. *Journal of Economic Literature* 54, 1232-1287.
- Egger, P., Nigal, S., Strecker, N. (2019). The taxing deed of globalization. *American Economic Review* 109, 353-390.

- Eurofound (2017). Exploring self-employment in the European Union. Publications Office of the European Union, Luxembourg.
- Fama, E. (1980). Agency problems and the theory of the firm. *Journal of Political Economy* 88, 288-307.
- Frederiksen, A. (2013). Incentives and earnings growth. *Journal of Economic Behavior and Organization* 85, 97-107.
- Goldberg, P., Pavcnik, N. (2007). Distributional effects of globalization in developing countries. *Journal of Economic Literature* 45, 39-82.
- Golosov, M., Tsyvinski, A. (2007). Optimal taxation with endogenous insurance markets. *Quarterly Journal of Economics* 122, 487-534.
- Hamilton, B. (2000). Does entrepreneurship pay? An empirical analysis of the returns to self-employment. *Journal of Political Economy* 108, 604-631.
- Heathcote, J., Storesletten, K., Violante, G. (2010). The macroeconomic implications of rising wage inequality in the United States. *Journal of Political Economy* 2010, 681-722.
- Holmstrom, B. (1999). Managerial incentive problems: Dynamic perspective. *Review of Economic Studies* 66, 169-182.
- Hoynes, H., Luttmer, E. (2011). The insurance value of state tax-and-transfer programmes. *Journal of Public Economics* 95, 1466-1484.
- Immervoll, H., Richardson, L. (2011). Redistribution policy and inequality reduction in OECD countries. OECD Social, Employment and Migration Working Paper No. 122. Paris.
- Kleven, H., Landais, C., Saez, E., Schultz, E. (2014). Migration and wage effects of taxing top earners. Evidence from the foreigners' tax scheme in Denmark. *Quarterly Journal of Economics* 129, 333-378.
- Kleven, H., Landais, C., Muñoz, M., Stantcheva, S. (2020). Taxation and migration: Evidence and policy implications. *Journal of Economic Perspectives* 34, 119-142.
- Krueger, D., Perri, F. (2011). Public versus private risk sharing. *Journal of Economic Theory* 146, 920-956.
- Lehmann, E., Simula, L., Trannoy, A. (2014). Tax me if you can! Optimal non-linear income tax between competing governments. *Quarterly Journal of Economics* 129, 1995-2030.
- Lemieux, T., MacLeod, W.B., Parent, D. (2009). Performance pay and wage inequality. *The Quarterly Journal of Economics* 124, 1-49.
- Office for National Statistics (2017). *Average weekly earnings, bonus payments in Great Britain: 2017*.
- Lucas, R.E. (1978). On the size distribution of business firms. *Bell Journal of Economics* 9, 508-523.

- Piketty, T., Saez, E., Stantcheva, S. (2014). Optimal taxation of top labor incomes: A tale of three elasticities. *American Economic Journal: Economic Policy* 6(1), 230–271.
- Piketty, T., Saez, E., Zucman, G. (2018). Distributional national accounts: Methods and estimates for the United States. *Quarterly Journal of Economics* 133, 553–609.
- Raith, M. (2003). Competition, risk, and managerial incentives. *American Economic Review* 93, 1425–36.
- Rodrik, D. (1997). Has globalization gone too far? Institute for International Economics, Washington, D.C.
- Rodrik, D. (1998). Why do more open economies have bigger governments? *Journal of Political Economy* 106, 997–1032.
- Saez, E. (2001). Using elasticities to derive optimal income tax rates. *Review of Economic Studies* 68, 205–229.
- Scheuer, F., Slemrod, J. (2019). Taxation and the superrich. NBER Working Paper No. 26207. Cambridge, Mass.
- Scheuer, F., Werning, I. (2017). The taxation of superstars. *Quarterly Journal of Economics* 132, 211–270.
- Schmidt, K. (1997). Managerial incentives and product market competition. *Review of Economic Studies* 64, 191–213.
- Sheshinski, E. (1972). The optimal linear income-tax, *Review of Economic Studies* 39, 297–302.
- Simula, L., Trannoy, A. (2010). Optimal income tax under the threat of migration by top-income earners. *Journal of Public Economics* 94, 163–173.
- Slemrod, J., Bakija, J. (2001). Growing inequality and decreased tax progressivity. In: K.A. Hassett and R.G. Hubbard (eds.), *Inequality and Tax Policy*, Chapter 6, 192–226. American Enterprise Institute, Washington, D.C.
- Sliwka, D., Werner, P. (2017). Wage increases and the dynamics of reciprocity. *Journal of Labor Economics* 35, 299–344.
- Song, J., Price, D.J., Guvenen, F., Bloom, N., von Wachter, T. (2019). Firming up inequality. *Quarterly Journal of Economics* 134, 1–50.
- Tóbiás, A. (2016). Income redistribution in open economies. *Journal of Public Economics* 134, 19–34.
- Varian, H. (1980). Redistributive taxation as social insurance. *Journal of Public Economics* 14, 49–68.

# Appendix

## A Market integration and revenue volatility

In this appendix, we describe a model of product market integration whose predictions can be mapped to the predictions of the model we use to derive results in the main text.

There are  $N$  symmetric economies. In each of these economies, agents are endowed with a given amount of a non-produced good that can be either consumed or used as an input into the production of consumption goods. There is a continuum of consumption good types,  $i \in [0, 1] \equiv I$ , that can (potentially) be produced in each economy. In each economy, there is a unit mass of identical consumers with preferences

$$u(y, x) = y + \int_0^1 \min \{ \bar{x}(i), x(i) \} di,$$

where  $y$  is consumption of the endowment good and  $x(i)$  is consumption of product  $i$ . Consumers view products of the same type,  $i$ , that are produced in different economies as being homogeneous in consumption. Without loss of generality, we assume that products are ordered so that  $i'' > i' \Rightarrow x(i'') > x(i')$ . We also assume that  $\bar{x}(i)$  is a continuous function of  $i$ . Given these preferences, a consumer is willing to buy up to  $\bar{x}(i)$  units of product  $i$  at any price  $p(i) \leq 1$ .

In each economy, and for each product  $i$ , there is a finite number,  $M \geq 2$ , of workers/managers of type  $i$  that can be employed to oversee the production of good  $i$ . Each worker/manager of type  $i$  is hired by a single firm—and so, for each good  $i$ , there are at most  $M$  firms (potentially) producing product  $i$  in each economy. There are otherwise no barriers to entry. A worker of type  $i$  can be either employed to produce good  $i$  or, alternatively, employed to produce one unit of the endowment good. Doing the latter always requires positive effort on the part of the worker. Each worker/manager is also a consumer. With a unit mass of workers/consumers in the economy, this implies that the mass of product types equals  $1/M$ .

Each of the firms that has hired a worker of type  $i$ —i.e. each firm that operates in sector  $i$ —can produce any quantity of product  $i$  using the endowment good as an input at a constant marginal input cost equal to  $1/\phi$ , where  $\phi$  is a firm-specific productivity draw. The probability of a firm experiencing a certain productivity draw depends on whether or not the worker/manager the firm has hired chooses to exert positive effort—at a positive cost to the worker/manager. If the manager exerts positive effort, the draw is  $\bar{\phi} > 1$  with probability  $\gamma$  and  $\phi \in [1, \bar{\phi})$  otherwise. If the worker/manager exerts no effort, the draw is  $\phi$  with certainty. Once productivity draws are realized, firms compete with each other in prices in all consumer markets to which they have access.

Consider next an autarkic economy. With  $M$  firms in sector  $i$ , and assuming that each worker/manager in each firm has exerted positive effort, with probability

$$M \gamma (1 - \gamma)^{M-1} \equiv M \pi,$$

a single firm will experience a draw  $\bar{\phi}$  and all other firms will experience a productivity draw  $\underline{\phi}$ . In this case, the firm that has experienced the favourable draw will be able, by pricing just marginally below  $\bar{p} = 1/\underline{\phi}$ , to keep other firms from producing, thus securing profits equal to

$$(1/\underline{\phi} - 1/\bar{\phi}) \bar{x}(i) = \mu \bar{x}(i) \equiv R(i).$$

In all other realizations, there will either be at least two firms experiencing  $\bar{\phi}$ , or all firms will experience  $\underline{\phi}$ ; in all these cases, price competition will drive prices down to a point where  $p(i) = 1/\phi$ . The market will then be shared by more than one firm and all firms, whether they are active or inactive, will experience zero profits. Thus, under autarky, any given firm in sector  $i$  experiences a profit  $R(i)$  with probability  $\pi$  and a profit equal to zero with probability  $1 - \pi$ . Expected interim profits with effort for any firm operating in sector  $i$  are therefore  $\pi R(i) = \gamma(1 - \gamma)^M \mu \bar{x}(i) \equiv \alpha(i)$ .

We assume  $\bar{x}(i) > 1/(\pi \mu)$ ,  $\forall i$ . This implies  $\alpha(i) > 1$ ,  $\forall i$ ; i.e. the expected return of a worker of type  $i$  in the production of good  $i$  is greater than the alternative of employing the worker to produce one unit of the endowment good. Only interim profits,  $R(i)$ , are verifiable, not the productivity draw; and so payments to worker/managers can only be conditioned on  $R(i)$ : the worker/manager's pay is  $w_L(i)$  when the firm experiences a zero interim profit realization and  $w_H(i)$  when it experiences a positive interim profit realization. The compensation level  $w_H(i)$  is formally structured as follows:  $w_H(i) = w_L(i) + \sigma(i)R(i)$ ; i.e.  $w_H(i)$  equals the base pay,  $w_L(i)$ , plus a bonus component consisting of a share,  $\sigma(i)$ , of the interim profits,  $R(i)$ . Structuring pay in this way ensures that the interim incentives of the managers when making pricing decisions (once the effort choice has been made) are aligned with those of the firm.

The ex-ante expected profits of the firm, under positive effort, will then equal  $E[\Pi(i)] = \pi(R(i) - w_H(i)) - (1 - \pi)w_L(i)$ . Absent barriers to entry, these will have to be zero in equilibrium. The equilibrium contract for each worker type will therefore be a contract  $(w_L(i), w_H(i) = w_L(i) + \sigma(i)R(i))$  that satisfies the incentive compatibility constraint with equality (as we detail discuss in the main text) and results in zero ex-ante expected profits for the firm.

Now suppose that the  $N$  economies become integrated. In this case, we have

$$\pi(N) = \gamma(1 - \gamma)^{NM-1}, \quad R(i, N) = N \mu \bar{x}(i),$$

with  $d\pi(N)/dN < 0$  and  $dR(i, N)/dN > 0$ . Being the only firm in sector  $i$  to experience  $\bar{\phi}$  is now less likely; but if that happens, the firm that experiences  $\bar{\phi}$  earns positive profits in each of the  $N$  consumer markets.<sup>30</sup> Expected interim profits, gross of the expected payments to worker/managers, equal

$$\pi(N) R(i, N) = \gamma(1 - \gamma)^{NM-1} N \mu \bar{x}(i).$$

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<sup>30</sup>This is similar to the mechanism at work in trade models where Ricardian comparative advantage results from idiosyncratic technology shocks, as in Eaton and Kortum (2002). When trade opens up, the country with the lowest production cost for any given product variety will secure the demand from all countries for that variety—a 'winner-takes-all' effect.

These are not invariant to a change in  $N$ , i.e. an increase in  $N$  combines a mean-preserving spread with a positive expected productivity shift that affects all productivity types uniformly. As discussed in Section 4, however, such a uniform productivity shift is of no consequence for our results.

## B Derivations of expressions and proofs of results

### B.1 Lemma 1

*For all productivity types,  $\alpha > \hat{\alpha}$ , that select into performance-based contracts, the low wage level,  $w_L$ , is strictly positive.*

PROOF: From (8a), the productivity level below which  $w_L \leq 0$  in a performance-based contract is given by  $\pi(\Gamma - 1)g/(1 - t) \equiv \alpha^0$ . The expected payoff to an individual in a performance-based contract must equal  $EU^N$  in equilibrium; this is the level of expected utility associated with receiving  $w_H$  with probability  $\eta\pi$  and  $w_L$  with probability  $1 - \eta\pi$ , i.e. a lottery with expected value

$$\eta\pi w_H + (1 - \eta\pi)w_L = \frac{\alpha(1 + \eta\pi(\Gamma - 1)) - \pi g(\Gamma - 1)(1 - \eta)/(1 - t)}{1 + \pi(\Gamma - 1)} \equiv EW^N.$$

The productivity level for which  $EW^N = EW^F = \eta\alpha$  is  $(\Gamma - 1)g/(1 - t) \equiv \tilde{\alpha} > \alpha^0$ . By risk aversion, an individual will always prefer a certain prospect to an uncertain one of equal expected value; and so, for  $\alpha = \tilde{\alpha}$ , it must be the case that  $EU^F > EU^N$ . By (12), this implies  $\tilde{\alpha} < \hat{\alpha}$  and  $\alpha^0 < \hat{\alpha}$  (since  $\alpha^0 < \tilde{\alpha}$ ). Thus, productivity types that select into performance-based contracts will receive a positive wage even when unsuccessful. Since  $w_F > 0$  and  $w_H > w_L$ , wages will be positive for all productivity types in all realizations, implying that a limited liability constraint  $w \geq 0$ , if present, can never be binding.  $\square$

### B.2 Proof of Proposition 1

If  $\eta \rightarrow 1$ , then there can be no positive solution  $\hat{\alpha} < 1$ : the insurance motive dominates, and the fixed-wage contract becomes preferred to the performance-based contract by all productivity types. For  $\eta < 1$ , the expression  $K$  is negative for  $c$  approaching  $\bar{c}$  and approaches infinity for  $c$  approaching zero; and so we can always find a threshold,  $\tilde{c} < \bar{c}$ , such that for  $c < \tilde{c}$  the condition  $K > g/(1 - t)$  is met and therefore  $\hat{\alpha} \in (0, \bar{\alpha})$ .

The rest of the results follow from (12).  $\square$

### B.3 Proof of Proposition 3

Differentiating the expression for  $K$  in (13) with respect to  $\pi$  and letting

$\Psi \equiv -(1 - \eta) \Gamma^{\eta\pi-1} / \left(1 + \pi(\Gamma - 1) - \Gamma^{\eta\pi}\right)^2 < 0$ , we obtain

$$\frac{dK}{d\pi} = \Psi \left( \Gamma \left( 1 - \frac{c}{(1 - \eta) \pi} \right) - 1 \right) > 0, \quad (\text{B.1})$$

where the sign follows from (6) and from the expression in parentheses being negative.

The expression for  $\hat{\alpha}$  is  $g / ((1 - t)K)$ , which is decreasing in  $K$  and increasing in  $g$ . In turn, for a given  $t$ ,  $g$  is decreasing in  $\hat{\alpha}$ . Consider then an increase in  $K$  accompanied by a reduction in  $\hat{\alpha}$ . If the increase in  $g$  caused by the fall in  $\hat{\alpha}$  were large enough to offset the negative effect on  $\hat{\alpha}$  of a higher  $K$ , then  $\hat{\alpha}$  would rise, and so  $g$  would have to fall rather than rise: a contradiction. So, for a given  $t$ ,  $\hat{\alpha}$  must be decreasing in  $K$  (and thus in  $\pi$ ), as stated in part (i) of the proposition.

For part (ii),  $w_H$  is unambiguously decreasing in  $\pi$  from (8b). For given  $t$ , and with  $\hat{\alpha}$  rising from (13) and (B.1), a counteracting effect comes from the simultaneous reduction in the tax base, and hence in  $g$ . This effect, however, can never dominate the direct effect of a change in  $\pi$ . If it did, then it would not only lower  $w_H$ , but also raise  $w_L$  from (8a). This would align the pre-tax wage levels for an unchanged expected wage, and would thus make the performance-based contract unambiguously more attractive, relative to the fixed-wage contract. But then  $\hat{\alpha}$  would need to fall, a contradiction to part (i) proven above.  $\square$

### B.4 Proof of Proposition 4

Differentiating (19) with respect to  $K$  gives

$$\frac{\partial^2 g}{\partial t \partial K} = \frac{dQ}{dK} + t \frac{d^2 Q}{dt dK}, \quad (\text{B.2})$$

where

$$t \frac{d^2 Q}{dt dK} = -t(1 - \eta) \left( \frac{df(\hat{\alpha})}{d\alpha} \hat{\alpha} \frac{d\hat{\alpha}}{dt} \frac{d\hat{\alpha}}{dK} + f(\hat{\alpha}) \frac{d\hat{\alpha}}{dt} \frac{d\hat{\alpha}}{dK} + f(\hat{\alpha}) \hat{\alpha} \frac{d^2 \hat{\alpha}}{dt dK} \right). \quad (\text{B.3})$$

From the definitions of  $\hat{\alpha}$  and  $Q$  in (13) and (18), we get the following derivatives

$$\frac{d\hat{\alpha}}{dt} = \frac{\partial \hat{\alpha}}{\partial t} = \frac{g}{(1 - t)^2 K} = \frac{\hat{\alpha}}{1 - t}, \quad \frac{dQ}{dK} = -(1 - \eta) f(\hat{\alpha}) \hat{\alpha} \frac{d\hat{\alpha}}{dK}. \quad (\text{B.4})$$

From (13) and (B.4), and letting  $Z \equiv ((1 - t)K + t(1 - \eta)f(\hat{\alpha})\hat{\alpha})K$ :

$$\frac{d\hat{\alpha}}{dK} = -\frac{g}{(1 - t)K^2} + \frac{t}{(1 - t)K} \frac{dQ}{dK} = -\frac{g}{Z}, \quad \frac{d^2 \hat{\alpha}}{dt dK} = \frac{d\hat{\alpha}}{dK} \frac{1}{1 - t} = -\frac{g}{(1 - t)Z}. \quad (\text{B.5})$$

Substituting (B.4)–(B.5) into (B.3) gives

$$t \frac{d^2 Q}{dt dK} = \frac{t(1 - \eta)\hat{\alpha}g}{(1 - t)Z} \left( \frac{df(\hat{\alpha})}{d\alpha} \hat{\alpha} + 2f(\hat{\alpha}) \right). \quad (\text{B.6})$$

Using the first term in (B.5) to rewrite  $dQ/dK$  and substituting this along with (B.6) into (B.2) yields (21) in the main text.  $\square$

## B.5 Derivation of (25)

From the utilitarian welfare function (23), the first-order condition for a welfare-maximizing  $t$  is

$$\frac{dW}{dt} = \int_{\underline{\alpha}}^{\hat{\alpha}} \frac{dEU^F(\alpha)}{dt} d\alpha + \int_{\hat{\alpha}}^{\bar{\alpha}} \frac{dEU^E(\alpha)}{dt} d\alpha - \left( EU^E(\hat{\alpha}) - EU^F(\hat{\alpha}) \right) \frac{d\hat{\alpha}}{dt} = 0. \quad (\text{B.7})$$

The last term on the LHS of (B.7) is zero because  $EU^E(\hat{\alpha}) = EU^F(\hat{\alpha})$  by the definition of  $\hat{\alpha}$ . Expanding the remaining terms  $dEU^Z(\alpha)/dt$ ,  $Z \in \{E, F\}$ , we get

$$\frac{dEU^Z(\alpha)}{dt} = \frac{\partial EU^Z(\alpha)}{\partial g} \frac{dg}{dt} + \frac{\partial EU^Z(\alpha)}{\partial t}, \quad Z \in \{E, F\}; \quad (\text{B.8})$$

where

$$\frac{\partial EU^E(\alpha)}{\partial g} = \frac{1}{g + (1-t)\alpha} > 0; \quad \frac{\partial EU^F(\alpha)}{\partial g} = \frac{1}{g + (1-t)\eta\alpha} > 0; \quad (\text{B.9})$$

$$\frac{\partial EU^E(\alpha)}{\partial t} = \frac{-\alpha}{g + (1-t)\alpha} < 0; \quad \frac{\partial EU^F(\alpha)}{\partial t} = \frac{-\eta\alpha}{g + (1-t)\eta\alpha} < 0. \quad (\text{B.10})$$

The terms in (B.10) are summarized in (25) in the main text. Since  $\partial EU^Z(\alpha)/\partial g$  is positive and  $\partial EU^Z(\alpha)/\partial t$  is negative, we must have  $dg/dt > 0$  at an optimum.  $\square$

## B.6 Condition for $dt/d\pi > 0$ —utilitarian objective, general distribution

The FONC for a welfare-maximizing  $t$  can be written as

$$\frac{dW}{dt} = \int_0^1 \frac{dEU(\alpha)}{dt} dF(\alpha) = 0. \quad (\text{B.11})$$

Letting  $z(\alpha) = (1 - (1 - \eta) \mathbb{1}_{(\alpha < \hat{\alpha})}) \alpha$ , where  $\mathbb{1}(\cdot)$  is an indicator function, and using the expressions

$$\frac{dEU(\alpha)}{dt} = \frac{dg/dt - z(\alpha)}{g + (1-t)z(\alpha)}, \quad (\text{B.12})$$

the FONC can be written as

$$\frac{dg}{dt} \Xi - \tilde{\Xi} \equiv \Omega = 0, \quad (\text{B.13})$$

where  $\Xi \equiv \int_0^1 \xi(\alpha) dF(\alpha) > 0$ ,  $\tilde{\Xi} \equiv \int_0^1 z(\alpha) \xi(\alpha) dF(\alpha) > 0$ , and  $\xi(\alpha) \equiv 1/(g + (1-t)z(\alpha))$ . Condition (B.13) can also be expressed as  $dg/dt = \Lambda \equiv \tilde{\Xi}/\Xi$ . The expression  $\Lambda$  measures the marginal welfare cost of the tax relative to the marginal welfare brought about by the associated marginal increase in revenue.<sup>31</sup>

<sup>31</sup>In the extreme case of a Rawlsian objective, we have  $z(0) = 1$ ,  $z(\alpha) = 0$ ,  $\alpha > 0$ ; and so  $\tilde{Q} = 0$ , implying that an optimal tax structure simply maximizes tax revenues. In the other extreme case of linear utility (risk neutrality), we have  $\xi(\alpha) = 1$  (implying  $\Xi = 1$ ) and  $\tilde{\Xi}$  equals the tax base,  $Q$ ; and so, since  $dg/dt = Q + t dQ/dt$ , the left-hand side of (B.13) simplifies to  $t dQ/dt$ , which is negative and implies a corner solution at  $t = 0$ .



The sign of  $dt/d\pi$  coincides with the sign of  $d\Omega/dK \equiv \Omega_K$ . Denoting  $dg/dt$  as  $g_t$ , and using further subscripts to denote total derivatives, the condition  $\Omega_K > 0$  can be expressed as

$$\Omega_K = g_{tK} \Xi + g_t \Xi_K - \tilde{\Xi}_K > 0. \quad (\text{B.14})$$

Substituting  $\Xi$  with  $\tilde{\Xi}/g_t$  from B.13, condition (B.14) can be expressed as

$$\begin{aligned} & \frac{\Xi}{K} \left( K \frac{g_{tK}}{g_t} + K \frac{\Xi_K}{\Xi} - K \frac{\tilde{\Xi}_K}{\tilde{\Xi}} \right) \\ &= \frac{\Xi}{K} \left( \epsilon_K^{g_t} + \epsilon_K^{\Xi} - \epsilon_K^{\tilde{\Xi}} \right) > 0, \end{aligned} \quad (\text{B.15})$$

where the  $\epsilon$ 's denote elasticities. Since  $\Xi/K > 0$ , (B.15) will be satisfied iff

$$\epsilon_K^{g_t} > \epsilon_K^{\Lambda}, \quad (\text{B.16})$$

i.e. if the elasticity of marginal revenue with respect to an increase in  $K$  exceeds the corresponding elasticity of  $\Lambda$ .

The elasticity condition (B.16) involves a comparison of a change in the marginal revenue effectiveness of the tax (on the LHS) with a change in the distributional effectiveness of the tax (on the RHS). While the former only depends on the level and the slope of  $f(\alpha)$  at  $\hat{\alpha}$ , the latter incorporates an integral ( $\tilde{\Xi}_K$ ) that depends on the shape of  $f(\alpha)$  over its full support  $[0, 1]$ . In Appendix B.8 we show that condition (B.16) is always fulfilled when the distribution of ability types is uniform. If instead there is a comparatively larger mass of individuals below  $\hat{\alpha}$ , a rise in  $\hat{\alpha}$  triggered by a fall in  $\pi$ , and hence  $K$ , tends to reduce the efficiency costs of the tax proportionately more than it reduces the redistributive value of the tax.

## B.7 Derivation of (26)

For the uniform ability distribution  $F(\alpha) = \alpha$ , expression (18) simplifies to

$$g = tQ = \frac{t}{2} (1 - (1 - \eta) \hat{\alpha}^2), \quad (\text{B.17})$$

and the response of the tax base  $Q$  with respect to the tax rate  $t$  becomes

$$\frac{dQ}{dt} = -(1 - \eta) \hat{\alpha} \frac{d\hat{\alpha}}{dt}. \quad (\text{B.18})$$

Using (B.9)-(B.10) in (B.7) gives

$$\frac{dW}{dt} = \frac{\Theta}{\eta} \left( \frac{g}{1-t} + \frac{dg}{dt} \right) - 1 = 0, \quad \Theta \equiv \eta \ln \frac{g + (1-t)}{g + (1-t)\hat{\alpha}} + \ln \frac{g + (1-t)\eta\hat{\alpha}}{g} > 0. \quad (\text{B.19})$$

The term  $\Theta$  in (B.19) is positive since, with  $\hat{\alpha} \in (0, 1)$  the fractions for which the logarithms are taken are both greater than unity.

With a uniform ability distribution, it is possible to obtain a closed-form solution for  $g$ . Solving the balanced-budget condition (B.17) and using  $\hat{\alpha}$  in (13) gives

$$g = \frac{t(1-t)K}{(1-t)K + \mu}, \quad \mu \equiv \sqrt{(1-t)^2 K^2 + (1-\eta)t^2} > 0. \quad (\text{B.20})$$

Using (B.20) in (B.19) gives the final expression for the first-order condition:

$$\frac{dW}{dt} = \frac{\Theta}{\eta} \frac{(1-t)K^2}{\mu((1-t)K + \mu)} - 1 = 0. \quad (\text{B.21})$$

Differentiating the first-order condition (B.21) with respect to  $K$  gives, in a first step

$$\frac{d^2W}{dt dK} = \frac{(1-t)K}{((1-t)K + \mu)\mu^2} \left( A \frac{\Theta}{\eta} + \frac{B}{(K + \mu)(K + 1)} \right), \quad (\text{B.22})$$

where

$$A \equiv \frac{(1-t)K + 2\mu}{\mu} (\mu - (1-t)K) > 0,$$

$$B \equiv K(K + 1)((1-t)^2K + \mu) - \mu(K + \mu)(K + (K + 1)/(K + \eta)).$$

Using the first-order condition (B.21) to eliminate  $\Theta/\eta$  gives, after some manipulations,

$$\begin{aligned} \frac{d^2W}{dt dK} = \frac{(1-t)K}{((1-t)K + \mu)\mu^2} & \left( \frac{((1-t)K + 2\mu)(\mu^2 - (1-t)^2K^2)}{(1-t)K^2} \right. \\ & \left. + \frac{K\mu + (1-t)^2(K + 1)K^2 - \mu^2K}{(K + \mu)(K + 1)} - \frac{\mu}{K + \mu} \right). \end{aligned} \quad (\text{B.23})$$

Using  $\mu^2 - (1-t)^2K^2 = t^2(1 - \eta)$ , this can be further simplified to yield (26) in the main text.  $\square$

## B.8 Proof of Proposition 5

We need to show that (26) is positive for all levels of  $K$  and  $t$ . Ignoring the first positive term in parentheses, a sufficient condition for this to be true is that

$$\frac{2t^2(K + 1)(K + \eta) - (1-t)K^2}{(1-t)K^2(K + 1)(K + \eta)} > 0 \Rightarrow (2t^2 + t - 1)K^2 + 2t^2(K + \eta + \eta K) \equiv H(K) > 0, \quad (\text{B.24})$$

for all levels of  $K$ . We first evaluate  $H(K)$  in (B.24) for  $K \rightarrow 0$ . This gives  $H(K)|_{K \rightarrow 0} \rightarrow +\infty$ , since  $t$  approaches zero less quickly than  $K$  does. This follows from the fact that  $\hat{a}$  in (13) must approach a maximum when  $K \rightarrow 0$  and hence effort costs  $c$  are at a maximum:

$$\frac{\partial K}{\partial c} = \Psi \pi ((1 - \eta) + (\Gamma - 1)(1 - \eta\pi)) < 0, \quad (\text{B.25})$$

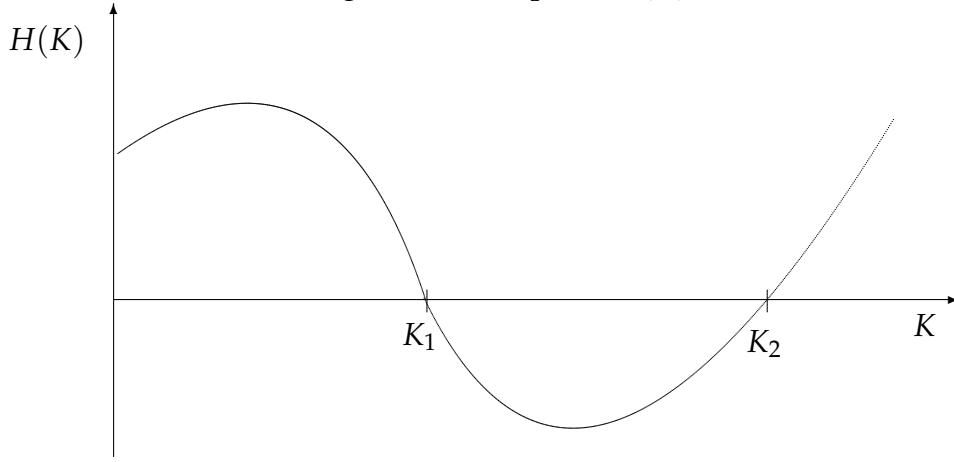
where  $\Psi$  has been defined in the Proof of Proposition 3.

Next, we evaluate  $H(K)$  for  $K \rightarrow \infty$ . In this case  $\hat{a} \rightarrow 0$  from (13) and the tax becomes a non-distortive instrument. Hence  $t \rightarrow 1$ , which implies  $H(K)|_{K \rightarrow \infty} > 0$  from (B.24).

It remains to show that  $H(K) > 0$  holds also for all intermediate values of  $K$ . Differentiating  $H(K)$  in (B.24) with respect to  $K$  gives

$$\frac{dH}{dK} = 2(2t^2 + t - 1)K + 2t^2(1 + \eta) + (4t(K + \eta + \eta K + K^2) + K^2) \frac{dt}{dK}. \quad (\text{B.26})$$

Figure A.1: Graph of  $H(K)$



We first show that  $dH/dK$  is positive for both  $K \rightarrow 0$  and  $K \rightarrow \infty$ . Evaluating (B.26) at  $K \rightarrow 0$  leaves the second and third terms. These must both be positive, since  $H(K)|_{K \rightarrow 0} > 0$  is a sufficient condition for  $(dt/dK)|_{K \rightarrow 0} > 0$ . Evaluating (B.26) at  $K \rightarrow \infty$ , the first term must be positive, as  $t \rightarrow 1$  from the argument above. Moreover, the third term in (B.26) is positive given that  $H|_{K \rightarrow \infty} > 0$  is a sufficient condition for  $(dt/dK)|_{K \rightarrow \infty} > 0$ . It follows that  $H(K) < 0$  can only occur for some intermediate range of  $K$  when the graph of  $H(K)$  has two local extrema, as shown in Figure 8.

We proceed by contradiction and assume that this is indeed the case. Then the  $H(K)$  curve must cross the  $K$ -axis twice, at levels  $K_1$  and  $K_2$ . We rewrite  $H(K)$  in (B.24) as

$$H(K) = K \left( (2t^2 + t - 1)K + 2t^2(1 + \eta) + \frac{2t^2\eta}{K} \right). \quad (\text{B.27})$$

Assume that (B.27) is zero at  $K = K_1$ , implying that the sum of terms in the large parentheses is zero. Moreover, by assumption,  $H(K)$  must be negative between  $K_1$  and  $K_2$ . Hence  $t|_{K_2}$  must be smaller than  $t|_{K_1}$ .<sup>32</sup> But then a comparison of the three terms in the large parentheses of (B.27) shows that these are all smaller at  $K_2$  than at  $K_1$ . The first term must be negative if  $H(K_1) = 0$  is to hold, and it is smaller (i.e., more negative) at  $K_2$ , since  $K_2 > K_1$  and  $t|_{K_2} < t|_{K_1}$ . Moreover, under these conditions the positive second and third terms are both unambiguously smaller at  $K_2$  than they are at  $K_1$ . Hence when  $H(K_1) = 0$ , there cannot be a value  $K_2 > K_1$  for which  $H(K_2) = 0$  holds. But then the graph of  $H(K)$  cannot be as depicted in Figure 8. By contradiction, this implies that  $H(K) > 0$  must hold for any level of  $K$ . Since  $H(K) > 0$  is sufficient for  $dt/dK > 0$ , this proves the result.  $\square$

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<sup>32</sup>To be precise,  $H(K) < 0$  is only a necessary, but not a sufficient, condition for  $t$  to fall between  $K_1$  and  $K_2$ . But if  $t$  does not fall between  $K_1$  and  $K_2$ , then it cannot fall for any level of  $K$ , because  $H(K) > 0$  holds for  $K < K_1$  and for  $K > K_2$ . The result would then follow immediately.