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The Economics of Skyscrapers: A Synthesis

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The Economics of Skyscrapers: A Synthesis

Abstract

This paper provides a synthesis of the state of knowledge on the economics of skyscrapers. First, we document how vertical urban growth has gained pace over the course of the 20th century. Second, we lay out a simple theoretical model of optimal building heights in a competitive market to rationalize this trend. Third, we provide estimates of a range of parameters that shape the urban height profile along with a summary of the related theoretical and empirical literature. Fourth, we discuss factors outside the competitive market framework that explain the rich variation in building height over short distances, such as durability of the structures, height competition, and building regulations. Fifth, we suggest priority areas for future research into the vertical dimension of cities.

JEL-Codes: R300, N900.

Keywords: density, economics, history, skyscraper, urban.

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The usual disclaimer applies.

1 Introduction

Land markets have been subject to economics research since the birth of economics as a modern discipline (Smith, 1776; Malthus, 1798; Ricardo, 1817). Since von Thuenen (1826), the study of the *horizontal* structure of urban space has been one of the main research areas in urban economics. While iconic skylines have long become distinctive features of cities over the course of the 20th century, it is not until recently that urban economics research into the *vertical* structure of cities has gained momentum. In this paper, we document the vertical growth of cities and summarize the theoretical and empirical advances in understanding the urban height profile. We argue that a deeper understanding of the causes and effects of building heights remains a priority research area in urban economics.

From a positive economics perspective, it is noteworthy that at high densities, the convex cost of height is the main force that produces an economic equilibrium in the presence of strong agglomeration economies. Hence, the cost of height critically determines the economically feasible density and, ultimately, a battery of external benefits and costs associated with density (Ahlfeldt and Pietrostefani, 2019). Vertical spillovers and sorting can make tall buildings a further source of agglomeration economies (Liu et al., 2018, 2020). From a normative economics perspective, it is critical to understand the positive productivity spillovers generated within and around tall buildings and the potential negative effects that may arise due to congestion, shadows, and other disamenities.

Importantly, the horizontal structure of cities is not independent of the vertical dimension. Use-specific costs and returns to height give residential or commercial users an edge in the competition for urban land. The price of this inelastically-supplied factor depends as much on the net returns to height as it does on location-specific demand. Understanding the causes and effects of tall buildings is critical to rationalize long-run changes in density and height gradients that are at the very core of urban economics (Alonso, 1964; Mills, 1967; Muth, 1969).

To frame our synthesis of the economics of skyscrapers, in Section 2, we begin by reviewing the history of technological innovations that have dramatically decreased the construction cost of tall buildings since the last decades of the 19th century. The main insight is that progress has been steady and incremental, rather than punctuated; as a result, the notion of their being a first skyscraper is incorrect.

In Section 3, we present a series of stylized facts on the spatio-temporal spread of skyscrapers and the pace at which cities have expanded into the third dimension. Over the past 120 years, the height of the tallest completed buildings in the world

each year has increased by 1.3% per year, on average. The number of completions per year has increased at a remarkable rate of 5% per year. There has been a strong urban bias in this vertical growth. We estimate a city-size elasticity of the number of skyscraper per capita of 0.27 for a global sample of cities exceeding a population of one million.

While skyscrapers first appeared in U.S. cities, the centre of gravity has shifted to Asia since the 1990s. Consistent with this shift, economic growth has become a more significant determinant of vertical growth in absolute and relative terms. We estimate that the cross-country elasticity of skyscraper completions with respect to 10-year GDP growth has increased from about 0.5 in the 1970s to about five in the 2010s, conditional on the effects of population and GDP per capita, which have remained about constant. Although skyscrapers are among the most durable forms of capital, the growth effect is also visible along the time dimension within a country. Since 1910, vertical growth in the U.S. has been highly cyclical, with completions responding to recessions and periods of growth with a lag of about three to five years.

These stylized facts motivate much of our analyses and the discussion of the literature in the subsequent sections. Throughout Sections 4 to 6, we explore the determinants of the urban height profile within a canonical competitive equilibrium framework. In Section 4, we connect horizontal land use patterns to the vertical dimensions of cities. To this end, we lay out a simple model in which developers facing costs and returns to height make profit-maximizing decisions regarding building height. Nesting this model of optimal building height into a conventional bid-rent framework delivers theoretical predictions consistent with the stereotype skyline of "global" cities. Intuitively, construction costs that are convex in height imply that tall buildings are only profitable in central locations where the floor space rent is higher and in large cities, which tend to be denser and more expensive, owing to inelastically supplied land.

Comparative statics deliver stylized predictions that are informative with respect to the expected long-run changes in the urban height gradient. Reductions in the cost of height due to innovations in construction technology should lead to taller buildings and a steeper height gradient, ceteris paribus. The same applies to the rise of interactive knowledge-based urban economies, where spatial productivity spillovers are likely becoming stronger, and more localized (Moretti, 2012). However, improved transportation has led to the decentralization of population and employment since the mid-20th century and to flatter rent gradients, which, ceteris paribus, lead flatter height gradients.

Empirically, we estimate an elasticity of building height with respect to distance from the CBD of slightly below 0.3 for large cities in 1900 and 2015. It appears that demand-side forces pushing for a flatter height gradient and supply-side forces pushing for a steeper height gradient have largely offset each other in their effect on the urban height gradient over the 20th century.

In Section 5, we turn to the cost of height. We estimate values of the height elasticity of per-unit construction cost that range from 0.1 for smaller structures to 0.25 for tall commercial and 0.56 for tall residential structures. For super-tall buildings, the literature has found that the elasticity can increase to beyond unity. In part, the cost of height emerges from a reduction in the share of usable floor area, which, for a given gross floor area, we estimate to decrease in building height at an elasticity of 0.05. Since historical construction cost records are difficult to access, we rely on an indirect approach to quantify the effects of innovations in construction technology on the cost of height. We estimate the land price elasticity of height for New York City and Chicago for various points in time from 1870 to 2010. Our estimates increase from less than 0.25 before 1900 to close to 0.5 after 2000. We use the theoretical framework from Ahlfeldt and McMillen (2018) to translate these estimates into estimates of the cost of height, which we find to have declined by about 2% per year in the long-run, on average.

In Section 6, we focus on returns to height. Exploiting within-building variation, we estimate an elasticity of residential apartment prices with respect to floor height of about 0.07 for New York City as well as for Chicago. For New York City, we estimate a value of 0.033 for commercial rents. For buildings up to 40 stories, these estimates translate into estimates of the building height elasticity of average rent of 0.056 (residential) and 0.026 (commercial). A sizable literature supports the notion of a height premium, although previous studies may confound height and location effects as they mostly do not condition on building fixed effects.

Liu et al. (2018) stand out in that they estimate the commercial within-building vertical rent gradient across a large set of U.S. cities, finding values of the floor elasticity of rent of 0.086 and 0.189 in two different data set. In terms of variation over time, our estimates suggest that the floor elasticity of residential apartment prices in New York City decreased by about 50% since the 1990s following a rapid expansion in the supply of tall residential structures. One interpretation is that the marginal buyer today occupies a lower rank in the distribution of height preferences.

Sections 5 and 6 jointly confirm that the marginal cost of increasing the height of a building exceeds the marginal returns at the margin, which is a requirement for a positive and finite solution for the optimal building height. Another takeaway is

that both costs of and returns to height vary by use, with implications for horizontal land use pattern that are ignored in standard models with an emphasis on two-dimensional space.

While spatial trends in heights tend to conform to the stylized predictions derived from the canonical framework laid out in Section 4, there is a remarkable degree of micro-geographic heterogeneity in building heights that deserves attention. Within one kilometer of the CBD in New York City or Chicago, respectively, the standard deviation in heights is as large as the average height, suggesting additional forces at work that shape the height gradient.

In Section 7, we discuss why the very nature of skyscrapers implies "stickiness" in height adjustments, leading to skylines that are less smooth than predicted by stylized urban models. Extensions of the standard land-use model rationalize fractal structural density through the durability of housing stock (Brueckner, 2000), which is relevant given that skyscrapers are among the most durable forms of capital. The size of skyscrapers implies that the assembly of suitable parcels takes time and can be subject to strategic holdout by sellers Strange (1995). Option values that build up over time can lead to periodical overbuilding by developers seeking to preempt each other (Grenadier, 1995, 1996)) .

In Section 8, we turn to geology as a determinant of skyscraper development. Because tall buildings are heavy and exert strong downward forces, foundations must be constructed to stabilize the structures, so they don't lean, deferentially settle, or fall over. Intuitively, bedrock depths are an (exogenous) determinant of skyscraper heights and locations, since many (but probably not most) skyscrapers are anchored directly to bedrock. However, foundation costs tend to have high fixed costs and low marginal costs with respect to height, so the effect on the cost of height is moderate from an engineering perspective. Research on New York City (Barr et al., 2011; Barr, 2016) suggests bedrock has an effect on the micro-geographic location of skyscrapers within economic clusters such as Downtown but has a limited impact on the location of the clusters.

In Section 9, we engage with the common perception that developers add extra height to project their out-sized egos onto their respective cities. (Clark and Kingston (1930), Al-Kodmany and Ali (2013)). Helsley and Strange (2008) make the theoretical case that in a sequential game where developers compete for the prize of being the tallest (because they gain personal utility from winning a height competition), developers will strategically build beyond a fundamentally-justified height to preempt rivals. Empirical research in this area, however, has yet to achieve a consensus. While some buildings have been estimated to be economically "too tall," the

reasons for the added height is not fully understood. Second-order economic benefits (e.g., tourism, foreign direct investment, advertising, or productivity signaling) may be at work as much as developers' desire to create monuments to themselves or win ego-based competitions. As well, local leaders can use skyscrapers to enhance their political careers, line their pockets, or provide income to their supporters.

Cities, over time, have aimed to curb building heights to varying degrees in the name of limiting externalities, such as shadows, or traffic congestion, or because they feel tall buildings are aesthetically displeasing or reduce the quality of life. Yet, our review of the research on skyscrapers and regulation in Section 10 reveals limited evidence on the causes and effects of skyscraper regulations. To date, most of the related research has focused on the negative collateral effects of height regulations, such as sprawl or affordability problems (Bertaud and Brueckner, 2005; Jedwab et al., 2020), but there is virtually no work in economics on the size and scope of negative externalities or on the impacts on the well-being of residents (Barr and Johnson, 2020).

Overall, it seems reasonable to conclude that research into the vertical dimension of cities is at an early stage. We discuss the potential for future research throughout in Sections 4 to 10 and collect our thoughts on priority areas in Section 11.

2 A brief history of the skyscraper

In this section, we briefly review the technological history of skyscrapers and the innovations that have paved the way for cities to increasingly fill the third dimension.

2.1 The First skyscrapers

Among architectural historians, there has been a vigorous debate about what constitutes the first true skyscraper. This is because a skyscraper is inherently a multifaceted object, and there is no single definition of what makes a skyscraper a skyscraper. Some point to the first tall commercial structures built after the U.S. Civil War. Others point to the first buildings to use all-steel framing, while others point to the first structures to use their height to convey information about the builders.

The common usage of the word "skyscraper" in the press pre-dated steel-framing by several years when it was used to describe the class of relatively tall (8-10 floors) commercial buildings going up in New York and Chicago in the early 1880s (Larson and Geraniotis, 1987). Reviewing the suite of technological elements needed to build tall office buildings, engineering historian, Carl Condit, concludes, "If we are

tracking down the origins the skyscraper we have certainly reached the seminal stage in New York and Chicago around the year 1870" (Condit, 1988, p. 22).

Nonetheless, the popular belief is that the Home Insurance Building in Chicago, designed by William Le Baron Jenney, and completed in 1885, was the first skyscraper in the world (Shultz and Simmons, 1959; Douglas, 2004), Douglas (2004). This belief is now widely considered among architectural and engineering historians to be false. The reason is that there does not exist any structure, let alone the Home Insurance Building, that by itself was fundamentally different from the buildings that preceded it and which generated a radical transformation after it.

The widespread thinking is that Jenney invented the steel frame. But this is not true. Rather, his innovation was to use an iron cage to hold up the floors. But the iron beams were attached to the masonry walls, which bore the load of the building (Larson and Geraniotis, 1987). In this sense, Jenny's building represented one of several that were transitional from the traditional wall-load-bearing building to a steel-framed one.

When Jenney's building was completed in 1885, there was no mention in the public or academic press about his structure being a new building type. To the engineering community, it seemed one of many that added some innovation to make buildings lighter, allow for more windows, or improve fire safety. His structure was innovative, to be sure, but, at the same time, there is nothing so special about it as to make it qualify as the first clear and true skyscraper.

The idea that Jenny's building was the first began to emerge in 1896, based on a flurry of letters written by Jenney's colleagues to the *Engineering Record*. They essentially "voted" him the winner by popularity contest and not by any rigorous engineering, economic, or aesthetic standards. When Jenney died in 1909, virtually all his obituaries declared him the skyscraper's inventor. In short, Jenney's "victory" was due more to the structure of his social networks rather than his building's structure.

But one thing is for certain, by the early 1890s, the key innovations—the steel-framed skeletal structure and the electric elevator—were in place to remove the technological barriers to height. So that from that time forward, skyscraper height decisions were based on balancing the costs with the revenues and were not so much determined by engineering barriers per se.

2.2 The 20th Century

For most of the 20th century, technological innovations were incremental. While engineers learned much more about the physics of stabilizing their buildings from

wind above and from the geology below, even as late as the 1950s, skyscrapers were still steel-framed boxes. However, improvements in glass technology, fluorescent lighting, and air conditioning allowed for a higher fraction of facades to be covered with glass.

In the 1960s, engineers devised new structural techniques that allowed buildings to go taller without requiring as much steel per cubic meter. The theories behind these ideas were well-known in the engineering profession, but it wasn't until mainframe computing came along and allowed for simulating and testing these ideas to validate them for practical use (Baker, 2001).

As buildings rise higher, the wind forces—the so-called lateral loads—rise exponentially with height. After about 15 stories, stabilizing the lateral loads becomes arguably the dominant element driving increasing marginal costs from adding floors.

The most notable innovation of the 1960s was the framed-tube structure. That is, the outer part of the structure is comprised of many closely-spaced columns, which are then attached with horizontal beams. In this way, the building is like a square tube and is sufficiently rigid to prevent significant sway from wind forces.[1] The Twin Towers in New York City used this design. The Sears (Willis) Tower employed a version of this design by utilizing a tube-within-tube structure.

2.3 Innovations in the 21st Century

Though other structural designs were first used in the 1970s, they have been much more widely employed in the 21st century, especially in Asia (Ali and Moon, 2007). For example, the Burj Khalifa uses a buttressed core, which was first implemented in the 1970s. The building is constructed like a type of pyramid. It has a main central core and three additional wings or cores which buttress the main one. Together, these building cores help to create a much stabler building that can rise 0.83 kilometers with reduced impacts from the wind.

Another innovation is to subject various models to wind-tunnel testing, and the one that most efficiently counters the wind forces is used. For example, Gensler, the architectural firm that designed the Shanghai Tower, subjected different designs to wind-tunnel tests. Based on this, they concluded, "Results yielded a structure and shape that reduced the lateral loads to the tower by 24 percent - with each five percent reduction saving about US$12 million in construction costs"(Xia et al., 2010, p. 13).

[1]Note that engineers and developers do not aim to make their buildings perfectly rigid, as this would significantly add to the cost of construction. Instead, the goal is to make tall buildings sufficiently rigid so that the rate of sway is undetectable by the human nervous system under nearly all wind conditions.

Further, many supertall buildings today incorporate mass-tuned dampers. These are large weights hung like a pendulum toward the top of the tower. When the wind forces press against the structure, the damper begins to sway in the opposite diction of the wind, thus dampening the wind's impact (Lago et al., 2018).

Technological innovations in elevators have included ways to use each shaft more efficiently, such as running doubledeckers (one cab on top of another), and eliminating separate machine rooms for raising and lowering the cab cables. Machine-learning algorithms allow cars to more quickly move passengers between floors and reduce waiting times. These innovations not only lowers the marginal costs of going taller, but also improve the occupants' quality of life and lower the building's operating expenses (Al-Kodmany, 2015).

2.4 The Future

Given the rapid economic growth and urbanization around the world, the demand for supertalls continues to be brisk. Competition in the skyscraper construction industry pushes firms to innovate in order to produce taller buildings at lower costs. One of the key remaining problems is that as buildings become taller, the elevator rope needs to be proportionally longer and heavier. However, at some point, the rope becomes so heavy it can no longer carry itself (Al-Kodmany, 2015). ThyssenKrupp, for example, is developing a rope-less elevator that will move via magnetic levitation. Presumably, once the elevator rope is made obsolete, it will remove a bottleneck to constructing the first mile-high skyscraper.

3 Stylized facts

In this section, we present stylized evidence on the spatio-temporal diffusion of skyscrapers.

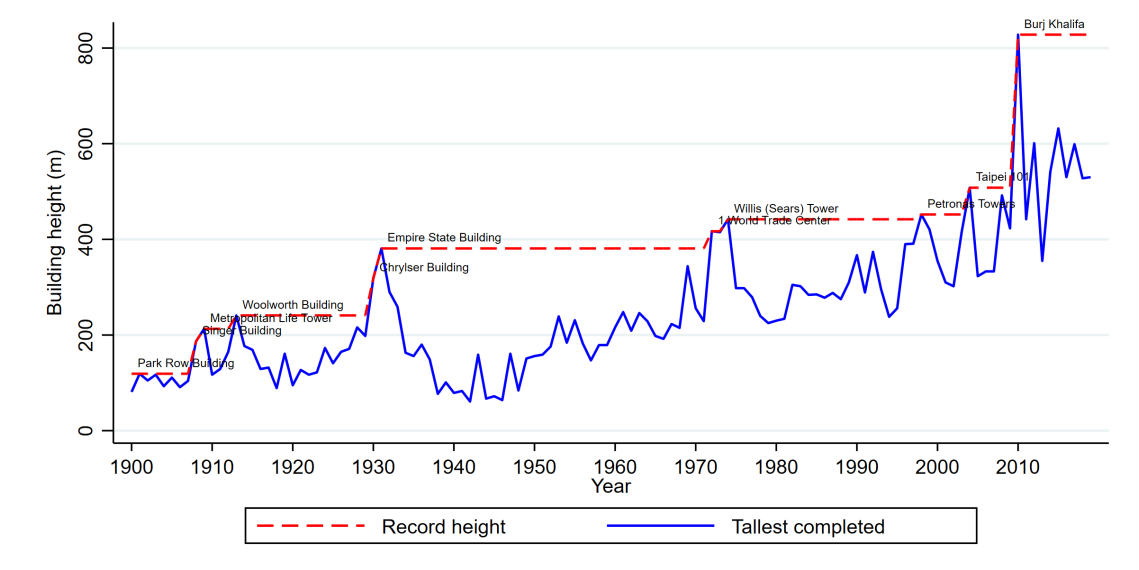
3.1 Evidence

We begin by illustrating the tallest skyscraper completions by year over the 20th and 21st centuries in Figure 1. The pattern is cyclical, and there are several peaks throughout history. From 1908 to 1913, three buildings held the title of the tallest building in the world, all exceeding 200 meters and all located in New York City. World War I induced a period of less ambitious construction before the next wave of skyscraper development culminated in the famous skyscraper race in which the Chrysler Building was defeated by the iconic 380-meter-tall Empire State Building.

Following the Great Depression and World War II, it took until the 1950s before the tallest buildings started to reach the levels of the late 1920s. A new level was reached in the 1970s when the World Trade Centre took the crown from the Empire State Building after nearly four decades. It was rapidly overtaken by the 440-meter-tall Willis (Sears) Tower in Chicago, the first record-breaking skyscraper outside of New York City.

Following the oil crisis, there was a dip in tallest constructions until the 1990s when skyscraper development gained new momentum, this time in a much more international context. The Petronas Towers (1998) and the Taipei 101 (2004) pushed the limits to 452 and 508 meters, respectively, before the Burj Khalifa set the record of 828 meters in 2009. Since then, it has become the norm that the tallest completions within a year exceed 500 meters (about 100 floors).

Figure 1: Tallest completions

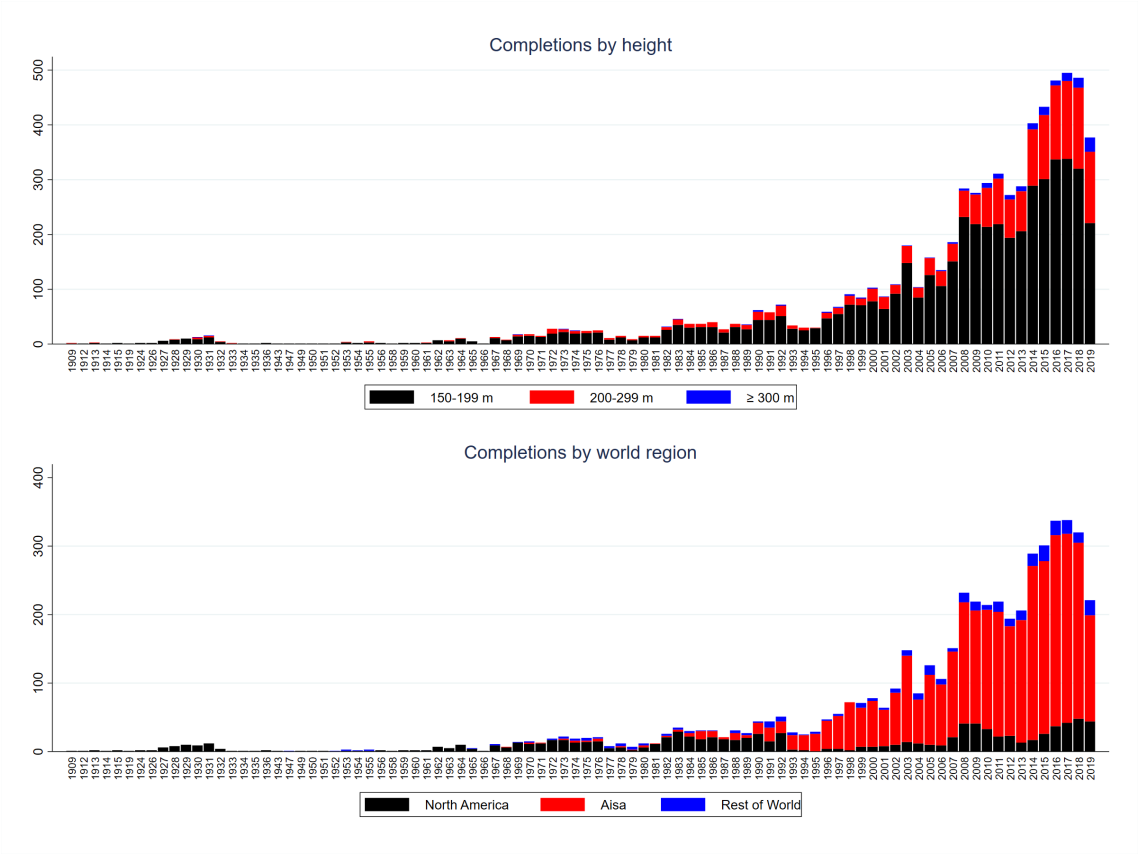


Sources: Data from https://www.emporis.com/ and https://www.skyscrapercenter.com/

The same cyclically is visible in the volume of skyscraper completions depicted in Figure 2. If anything, the pace of vertical growth since the 1990s is even more impressive in this chart. Since the 1990s, construction activity has shifted to Asia. In 2002, Asia took over from North America as the region with the largest cumulative number of skyscrapers. Today, 43% of the world's 150-meter or taller towers are in China alone (including Hong Kong).

To quantify the positive long-run trends in heights and volumes, we regressed the log of heights and completions against a yearly time trend in Table 1. Over 120 years, the heights of tallest completions have increased at an average rate of 1.3%.

Figure 2: Skyscraper completions by year



Note: A skyscraper is a building that is at least 150 m tall. Source: https://www.skyscrapercenter.com/

The volume of completions exceeding 150 meters has increased at an even larger percentage of 4.9%. Given that simple log-linear trends explain 63% and 82% of the variation in tallest heights and skyscraper volumes over time, it seems fair to conclude that pronounced vertical growth, historically, has been the norm rather than the exception.

In Figure 3, we take a closer look at the spatial diffusion of skyscrapers. The first skyscraper outside the U.S. was completed in Brazil (the Altino Arantes Building) in the 1940s. Skyscrapers reached many of the larger and economically more developed countries during the second half of the 20th century. However, many countries, including developed countries, have remained resistant to adopting the technology. It took until the 2010s for skyscrapers to reach Italy and Switzerland. Ireland and Portugal do not have any skyscrapers (150 m or taller) to date.

Only 59 out of 193 nations (30%) have at least one skyscraper. And 80% of all skyscrapers are in eight countries. In per-capita terms, skyscraper penetration today is generally largest in North America, South-East Asia, and Australia. Small

Table 1: Long-run trends in skyscraperization

	(1) Ln(height tallest construction)	(2) Ln(height record-holder)	(3) Ln(# completions)	(4) Ln(# cumulative completions)
Year	0.013***	0.011***	0.049***	0.064***
	(0.00)	(0.00)	(0.00)	(0.00)
Observations	120	120	120	120
R^2	.627	.787	.819	.906

Notes: Unit of observation is years. * p < 0.1, ** p < 0.05, *** p < 0.01

states in which the largest city dominates the city system, such as Panama or the United Arab Emirates reach the highest penetration, in line with skyscrapers being a distinctively urban phenomenon.

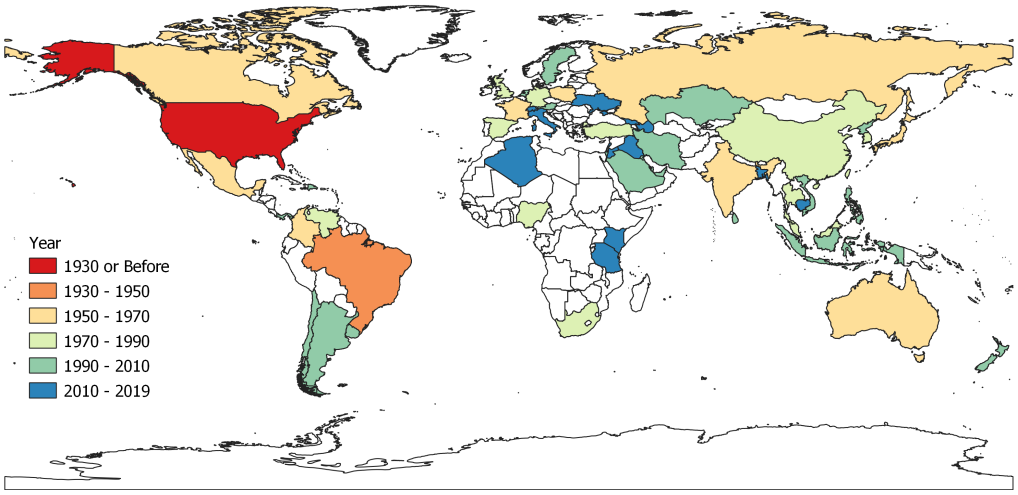
The degree of urban bias in the distribution of skyscrapers is striking as 50% of the world's skyscrapers are located in just 17 cities, though a total of 315 cities worldwide have at least one. Hong Kong leads the list with 1453 buildings of at least 100 meters, with New York City coming in second at 823. Of the top twenty cities around the world, eight are in China (including Hong Kong). 15 out of 20 are in east Asia. Only three cities in North America (New York, Chicago, and Toronto) make the list (https://www.emporis.com/, Accessed May 1, 2020).

Figure 4 sheds additional light on the urban bias in the distribution of skyscrapers. There appears to be a size-threshold beyond which the skyscraper technology becomes increasingly viable. For cities that exceed a population of one million, the number of skyscrapers increases more than proportionately in city size. Our estimates from a global sample of cities imply a city size elasticity of the per capita number of skyscrapers of $1.27 - 1 = 0.27$. We find somewhat larger estimates when restricting the sample to cities in the U.S. (0.34) or China (0.45). In contrast, if we consider cities with a population of less than one million, the relationship between city size and the number of skyscrapers is much weaker.

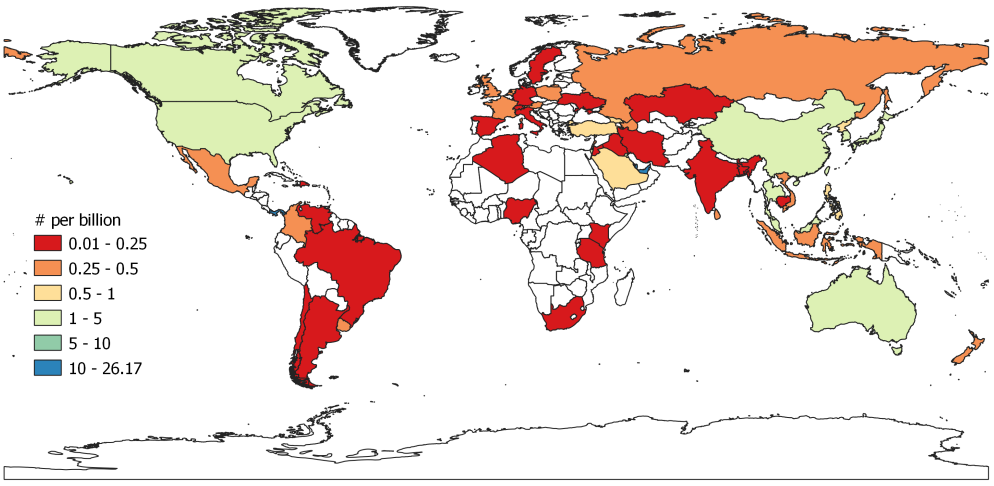
Figure 3 is broadly consistent with the spread of skyscrapers from economically developed countries to countries that are developing. This impression is substantiated by Figure 5. Since the 1970s, economic growth has become a much more powerful predictor of a country's skyscraper completions, whereas the effect of GDP per capita (and population) has remained about the same.

The relationship between economic growth and skyscraper construction is also visible in the time-series of U.S. skyscraper completions depicted in Figure 6. Volumes of skyscraper completions along with tallest heights tend to increase during boom periods. Once the economy contracts, completions and heights plummet. There is a lag of about three to five years in the economic growth effect, which

Figure 3: Skyscraperisation by country



(a) Year of first skyscraper



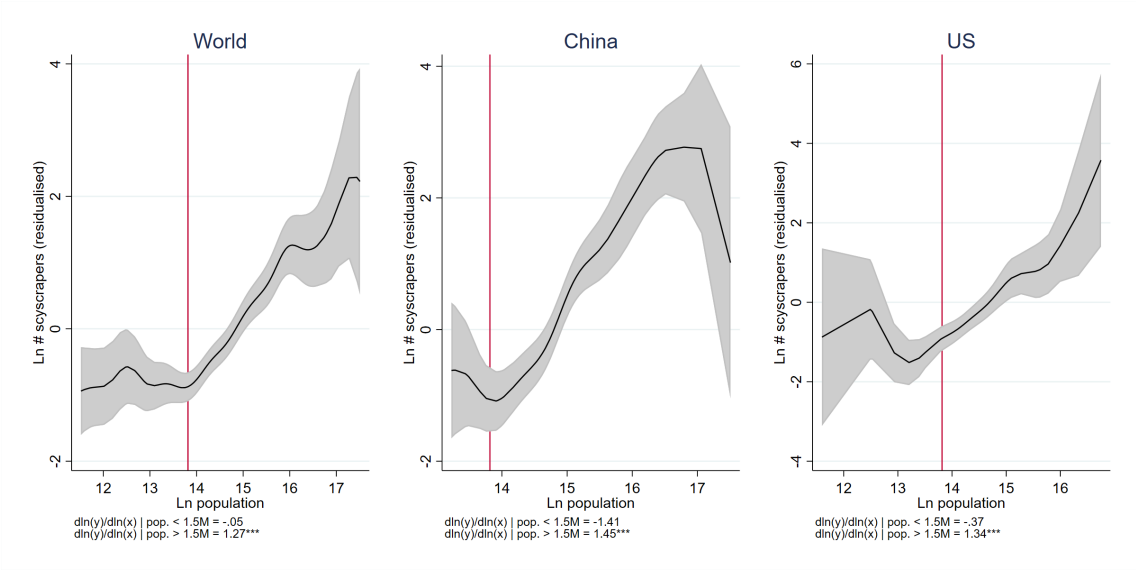
(b) Skyscrapers per capita

Note: A skyscraper is a building that is at least 150 m tall. Source: https://www.skyscrapercenter.com/, as of 2018. Population data from https://data.worldbank.org/.

is intuitive since planning and building a skyscraper takes time (panel b). In this context, it is worth noting that there is no evidence for the common belief that skyscraper heights can be used to forecast economic downturns (Barr et al., 2015).

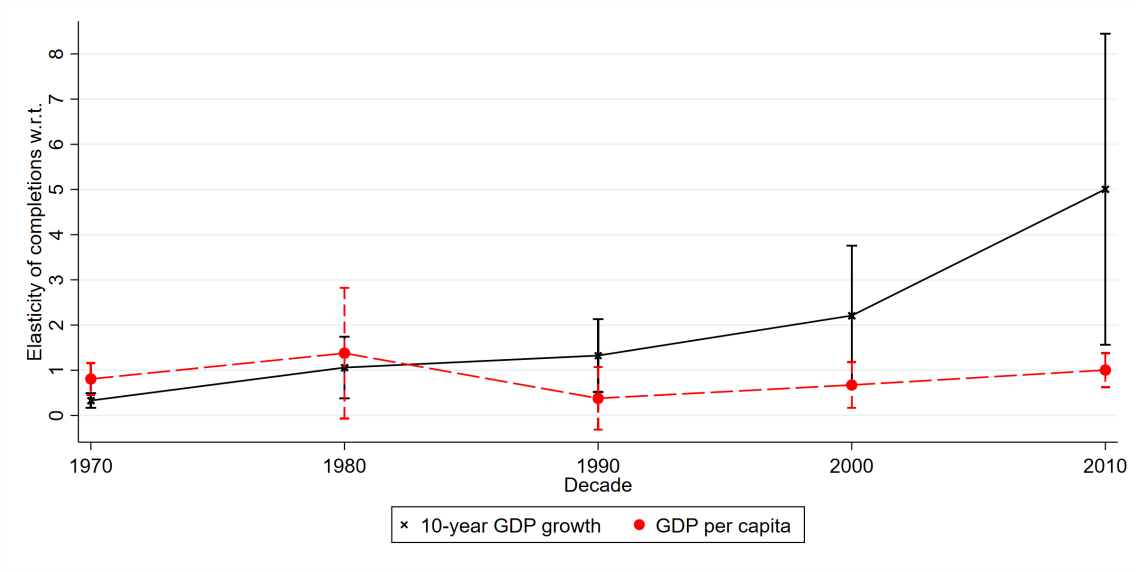
In any case, the sensitivity of vertical growth to the short-run economic cycle is quite striking, given that skyscrapers are among the most durable forms of capital. Emporis, who maintains one of the most comprehensive databases on tall buildings, record only a hand full of teardowns out of nearly 4,000 skyscrapers. The only recorded demoliton or destruction in the class of tall buildings exceeding 250 meters are are New York's Twin Towers in the World Trade Centre.

Figure 4: Skyscrapers and city size



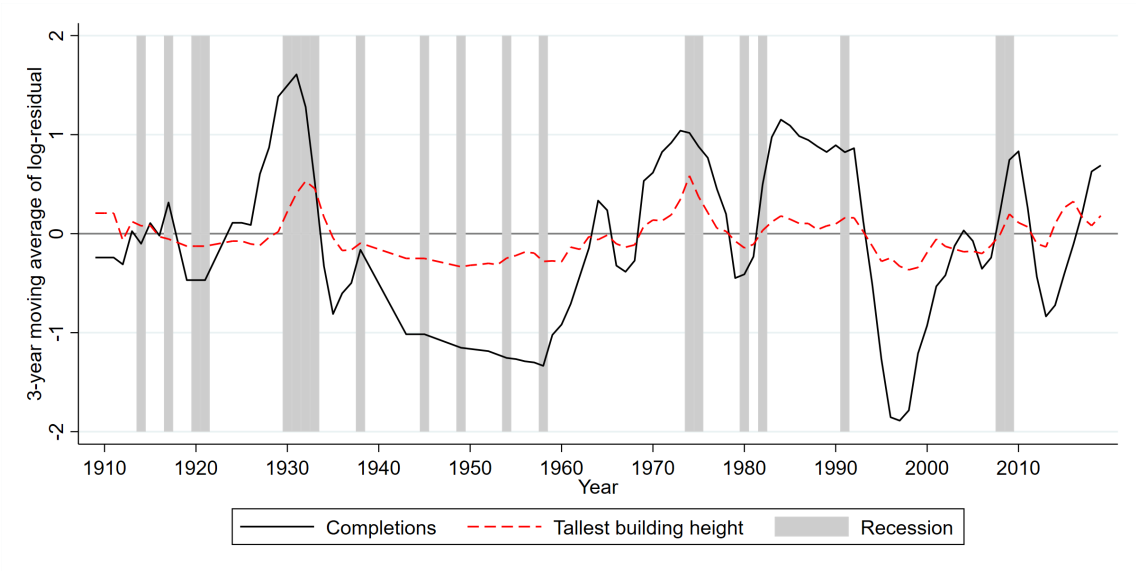
Note: Skyscrapers and population for cities around with world, with at least one 150m+ skyscraper. ln(# skyscrapers) is residualised in regressions against country fixed effects. Black solid line are from locally weighted regressions using a Gaussian kernel and a bandwidth of 0.25. Confidence bands are at the 95% level. Vertical line marks the log of 1 M. Skyscraper data is from https://www.skyscrapercenter.com/; accessed Feb. 2020. Population data are from several sources (available upon request) and are the most currently available counts for the metropolitan regions, which includes the central city and surrounding population agglomerations.

Figure 5: Determinants of skyscraper completions

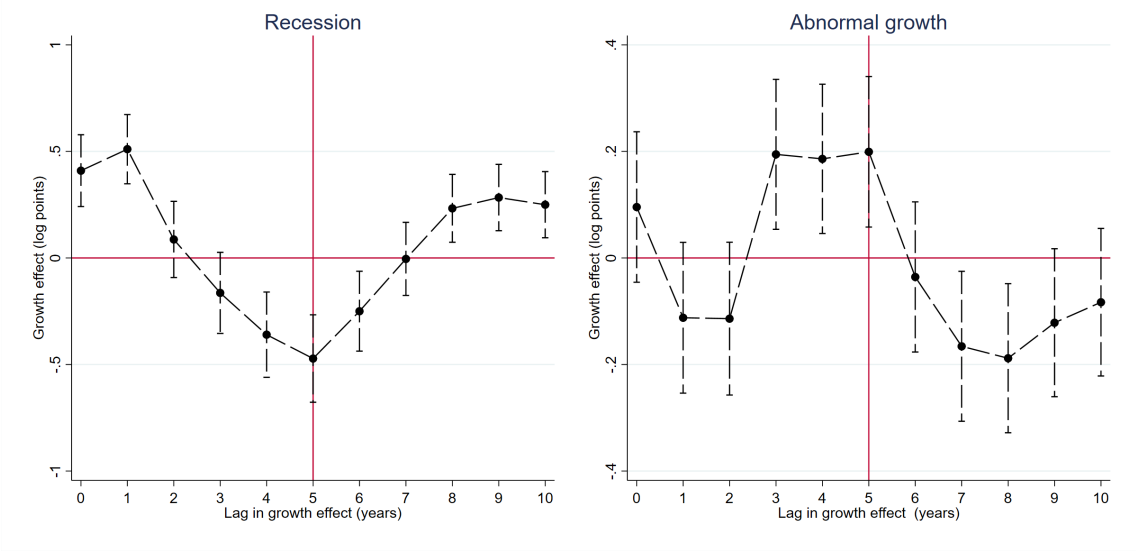


Note: Markers illustrate estimates from country-level regressions of log # completions against the log of 10-year GDP growth, log of GDP per capita and log of population (not reported for clarity of the graph; the estimated elasticity is close to 0.5 throughout) by decade. Confidence bands are at the 95% level. Sources: Skyscraper data is from https://www.skyscrapercenter.com/. Country level population and GDP is from https://data.worldbank.org/. GDP is constant 2010 US$.

Figure 6: Skyscraper completions in the U.S.



(a) Time trend



(b) Lag in growth effect on completions

Note: In panel (a), completions and tallest building heights are residualised in regressions against semi-log-linear trends. The residuals are then smoothed using exponential three-year moving averages. Recessions are defined as years with negative real GDP per capita growth. In panel (b), point estimates and 95% confidence intervals are from Poisson regressions of the number of completions against indicators for recessions (left) or abnormal growth (right) lagged by the indicated number of years, controlling for a time trend. Recessions are defined as years with negative real GDP per capita growth. Abnormal growth periods are periods with the real GDP per capita growth exceeds the long-run median (about 2%). Skyscrapers: https://www.skyscrapercenter.com/. Real GDP per capita: https://www.measuringworth.com/

3.2 Summary

Countries adopt the skyscraper technology if GDP per capita is sufficiently high, and there is at least one large city. Besides wealth and urbanization, economic

growth as become an increasingly important determinant. However, these demand-side conditions are not sufficient, as many wealthy countries, especially in Europe, tend to avoid building skyscrapers.

Skyscrapers have shown a secular increase in their heights over the long run, suggesting steady, if not predictable, improvements in building technology. Skyscraper construction also demonstrates clear building cycles, which are tied, in part, to the business cycle (Glaeser, 2013).

4 Vertical and horizontal city structure

In this section, we explore how the urban height profile is shaped by demand and supply in competitive land markets. To this end, we introduce a simple partial equilibrium framework in which building heights are set by profit-maximizing developers who face height-related costs and returns. We use this framework to explore how the internal structure of cities endogenously responds to exogenous changes such as innovations in construction technology. We provide evidence that empirically substantiates the stylized predictions and review the related theoretical and empirical literature.

4.1 An illustrative partial equilibrium framework

We start from a simplistic demand side of urban real estate markets, which we assume to be fully described by indifferent users that trade a ground-floor bid rent p^U against the value of an amenity A^U that depends on distance D from an arbitrary central point.

$$p^U = p^U(A^U(D)) \tag{1}$$

We assume that $\frac{\partial p^U}{\partial A^U} > 0$ and $\frac{\partial A^U}{\partial D} < 0$, consistent with the standard monocentric city model (Brueckner, 1987). $U \in C, R$ indexes commercial (C) and residential (R) use. Turning to the supply side, we follow Ahlfeldt and McMillen (2018) and assume that homogeneous developers face the following profit function in per unit of building footprint L terms:

$$\pi(S^U) = \tilde{p}^U(S^U)S^U - \tilde{c}^U(S^U)S^U - \frac{1}{d^U}r^U, \tag{2}$$

where $S^U = \frac{F^U}{L^U}$ is a measure of building height that multiplies building footprint to homogeneous floor space F^U. r^U is the land bid-rent offered by the developer, with $0 < d^U \leq 1$ giving the fraction of a parcel that is developable. Per-unit construction

costs $\tilde{c}^U(c^U, S^U)$ depend on the baseline construction cost c^U for a one-storey building and are a convex function of height since taller buildings require more sophisticated structural engineering, facilities such as elevators, and more building materials, so $\frac{\partial \tilde{c}^U}{\partial S^U} > 0$ and $\frac{\partial^2 \tilde{c}^U}{\partial S^U} > 0$. The average per-unit rent $\tilde{p}^U(p^U(D), S^U)$ depends on the ground floor rent $p^U(D)$ and on height S^U since being at a higher floor is an amenity (Liu et al., 2018). Profit-maximisation delivers the optimal building height:

$$S^{U*} = S^{U*}(p^U(D)), \tag{3}$$

with a unique positive solution as long as $p^U > c^U$ and $\frac{\partial^2 \tilde{c}^U}{\partial S^U} > \frac{\partial^2 \tilde{p}^U}{\partial S^U}$. Furthermore, assuming perfect competition, free entry and exit, and zero profits, the equilibrium land price r^{U*} is pinned down as the residual in the profit function. Since S^{U*} depends on p^U, which depends on D, r^{U*} is also a function of D.

$$r^{U*} = r^{U*}(p^U(D)) \tag{4}$$

Intuitively, a more central location leads to a higher ground-floor rent, leading to a greater optimal building height and higher profits per land unit, which capitalise into the land rent.

For a graphical illustration of the urban gradients and some simple comparative statics we impose convenient parametrizations. We assume homogenous firms and households, Cobb-Douglas production (with firms using floor space and freely traded capital as inputs), Cobb-Douglas consumption (with households consuming a basket of tradable goods and non-tradable floor space under an income constraint), profit maximisation and utility maximisation, and perfect spatial competition leading to spatially invariant profits and utility. We can then derive the ground floor bid rent as $p^U = a^U(A^U)^{\frac{1}{1-\alpha^U}}$, where $0 < 1-\alpha^C < 1$ is the share of floor space at firm inputs, $0 < 1 - \alpha^R < 0$ is the consumption expenditure share on housing, a^C is constant determined by α^C, and a^R is a constant that depends on α^R and household income. We further assume $A^U = b^u e^{-\tau^U D}$ where $b^u > 0$ is a constant that determines the amenity level at the central point, $\tilde{p} = p^U S^{U^{\omega^U}}$, and $\tilde{c} = c^U S^{U^{\theta^U}}$, where $\omega^U > 0$ and $\theta^U > 0$ are the height elasticities of average per-floor space rent and construction cost. For more detail on the chosen functional forms, we refer to Section A1.1 in the appendix.

We list the parameter values we use in 2. These values are not directly taken from the literature and do not necessarily conform to the estimates we provide later in the paper. However, they are roughly consistent with values in the literature and suitable for the stylized representation of an abstract city. Based on these values,

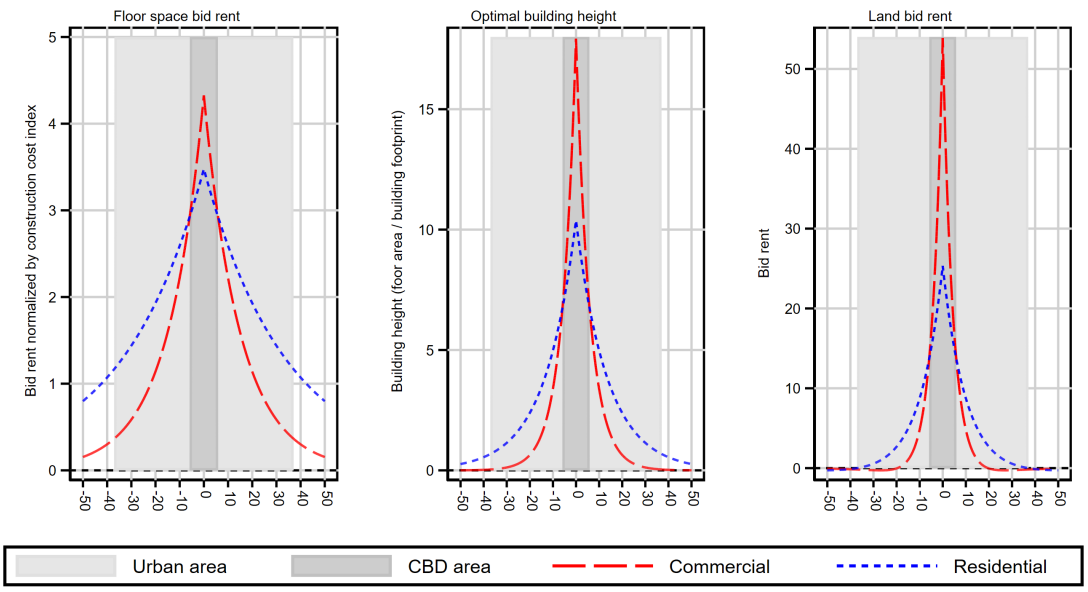
we derive the parametric equivalents to (1), (3) and (4) in Figure 9.

Table 2: Parameters

	Parameter	Value	Further reading
α^C	Share of floor space at inputs	0.15	(Lucas and Rossi-Hansberg, 2002)
α^R	Share of floor space at consumption	0.33	(Combes et al., 2018)
θ^C	Commercial height elasticity of construction cost	0.5	(Ahlfeldt and McMillen, 2018)
θ^R	Residential height elasticity of construction cost	0.5	(Ahlfeldt and McMillen, 2018)
ω^C	Commercial height elasticity of rent	0.1	(Liu et al., 2018)
ω^R	Residential height elasticity of rent	0.1	(Liu et al., 2018)
τ^C	Productivity decay	0.01	(Ahlfeldt et al., 2020)[a]
τ^R	Utility decay	0.01	(Ahlfeldt et al., 2015)

Notes: These parameter values are chosen in an ad-hoc fashion for a stylized representation. They are not taken from individual papers and do not confirm to the estimates we provide later in the paper. The last columns provides a references for the interested reader for further reading, but not necessarily the source of a point estimate. [a] The parameter value is consistent with the commercial rent gradient estimated by Ahlfeldt et al. (2020) for a large set of global cities, assuming $\alpha^C = 0.15$. We set the amenity scale parameter arbitrarily to $b^C = b^{R^3}$ and $b^R = 10000$ to generate the conventional land use pattern. We set the share of developable land to $d^C = d^R = 1$ for simplicity.

Figure 7: Urban gradients



Note: Parameter choices: $\alpha^C = 0.15, \alpha^R = 0.66, b^C = b^{R^3}, b^R = 10000, \tau^C = \tau^R = 0.01, d^C = d^R = 1, c^C = c^R = 1, \omega^C = \omega^R = 0.1, \theta^C = \theta^R = 0.5$. These parameter values are chosen in an ad-hoc fashion for a stylized representation. While they are broadly consistent with values in the literature (see Table 2), they do not necessarily conform to the empirical estimates we contribute to the literature.

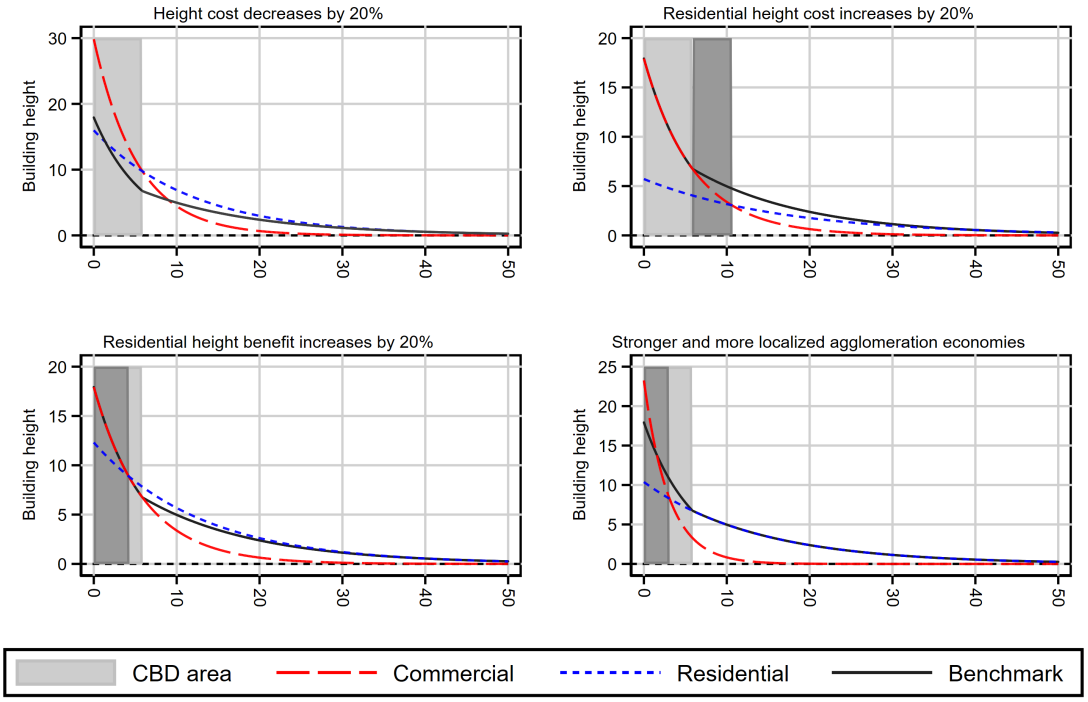
The ground floor bid rent curves for commercial and residential users naturally decrease in distance from the central point to compensate users for the associated

amenity loss. Note that we assume the same spatial decay τ^U in the amenity spillover for firms and residents. In our parametrization, the commercial bid rent curve is steeper because $\alpha^C > \alpha^R$, which implies that firms can adjust their use of space more easily to changes in prices. In central areas, firms, therefore, have a relative advantage over residents when competing for space. The height gradient mirrors the ground floor rent gradient in that it decreases faster for commercial than for residential buildings. In keeping with the stereotype of global cities, tall commercial buildings in a central business district (CBD) are surrounded by smaller residential buildings. Because the height elasticity of net costs $\theta^U - \omega^U = 0.4$ is smaller than unity, changes in ground floor bid rents trigger more than proportionate changes in building height, resulting in a height gradient that is steeper than the ground floor bid rent gradient. The developer's ability to multiply land to floor space via height results in land bid rents that increase more than proportionately in floor space rents. Hence, Figure 9 illustrates how tall buildings rationalize extreme differentials in land prices across small areas such as documented by Ahlfeldt and McMillen (2014a).

From the perspective of an individual developer, a high land price may be exogenously given, seemingly creating an incentive to build taller to use an expensive input factor more intensely. Within the canonical framework developed here, however, it is a higher floor space rent (left panel in Figure 9) that makes a taller building more profitable (middle panel), resulting in greater profits, which eventually capitalize into the price of the inelastically supplied factor land (right panel). Profits can become negative if the ground floor rent falls below the minimum construction cost, leading to a negative land bid rent. In Figure 9 the city ends where both commercial and residential land bid rents turn negative. Competition from a non-urban user with a positive bid rent will naturally result in a smaller urban area.

While the purpose of of the theoretical framework is to provide a stylized representation of an abstract city, we note that the chosen parametrization generates a pattern that is roughly consistent with real-world settings in which D corresponds to distance from the CBD in kilometers. It is reassuring that the chosen parametrization implies the following semi-elasticities that are within the range of extant estimates: $\frac{\partial \ln p^C}{\partial D} = -\frac{\tau^C}{1-\alpha^C} = -0.067$ and $\frac{\partial \ln p^R}{\partial D} = -\frac{\tau^R}{1-\alpha^R} = -0.03$, in line with in line with Ahlfeldt et al. (2020); $\frac{\partial \ln S^C}{\partial D} = \frac{\tau^C}{(1-\alpha^C)(\omega^C-\theta^C)} = -0.17$ and $\frac{\partial \ln S^R}{\partial D} = \frac{\tau^R}{(1-\alpha^C)(\omega^R-\theta^R)} = -0.13$, in line with Ahlfeldt and McMillen (2018). The analytical solution for the land rent semi-elasticity is complex and not constant in D, so we derive the following average semi-elasticity estimates in ancillary regressions: $\frac{\partial \ln r^C}{\partial D} = -0.32$; $\frac{\partial \ln r^C}{\partial D} = -0.15$, both well within the range of extant estimates summarized by Ahlfeldt and Wendland (2011).

Figure 8: Comparative statics



Note: Baseline choices for baseline: $\alpha^C = 0.15, \alpha^R = 0.66, b^C = b^{R^3}, b^R = 10000, \tau^C = \tau^R = 0.01, d^C = d^R = 1, c^C = c^R = 1, \omega^C = \omega^R = 0.1, \theta^C = \theta^R = 0.5$. These parameter values are chosen in an ad-hoc fashion for a stylized representation. While they are broadly consistent with values in the literature (see Table 2), they do not necessarily conform to the empirical estimates we contribute to the literature.

In Figure 8, we use the same framework to conduct some simple comparative statics analyses. In the upper-left panel, we decrease θ^U to simulate the effects of seminal improvements in construction technology discussed in Section 3. A lower cost of height results in taller buildings, which is intuitive. Perhaps more interestingly, a 10-percent reduction in the cost elasticity of height from $\theta^U = 0.5$ to $\theta^U = 0.45$ leads to an increase in height at the central point by more than 50%. This strong sensitivity of efficient building heights to changes in construction technology may play a critical role in explaining the rapid vertical growth global cities have been experiencing.

In the upper-right panel, we increase the cost of height for residential buildings by increasing $\theta^C = 0.6$. Tall residential buildings typically have a smaller floor plate size (due to the need for more exterior walls), use different materials (e.g., all concrete due to acoustic reasons), and have more complex facades (with balconies and sun rooms), none of which are advantageous for the construction of very tall buildings (Smith et al., 2014). In keeping with intuition, the optimal building height of residential buildings decreases. The area over which commercial developments

are relatively more profitable increases (indicated by the dark-shaded area). This counterfactual illustrates how vertical costs can shape horizontal land use pattern. In our framework, an increase in height is isomorphic to a reduction in the returns to height. Hence, an increase in the residential height elasticity of floor space rent has the opposite effects on the residential height gradient and the size of the central business district as illustrated in the bottom-left panel.

In the last counterfactual in the bottom-right panel, we consider the effect of an increase in the degree of interactiveness in urban economic activity that has been documented by Michaels et al. (2018). More interactive occupations are typically in more knowledge intense industries which face highly localized external returns to scale (Rosenthal and Strange, 2001). Hence, we increase the value of the commercial amenity a^C and the decay of commercial spillovers τ^C at the same time. The intuitive result is that commercial building heights increase near the central point, owing to increasing bid rents, but decrease more steeply in distance.

There is a strong intuition that the cost of height must have decreased since the late 19th century due to seminal innovations such as the elevator and the steel frame and incremental improvements in structural engineering and material science ever since. The urban bias in knowledge-based tradable services that have fueled economic growth since the mid 20th century has led to increasing rents in growing cities, further promoting vertical growth. Given that the willingness-to-pay for amenities generally seems to be increasing over time, it is also likely that returns height increased over time. The rather unambiguous prediction is that building heights should have increased over time, which is in line with the stylized facts provided in Section 3.

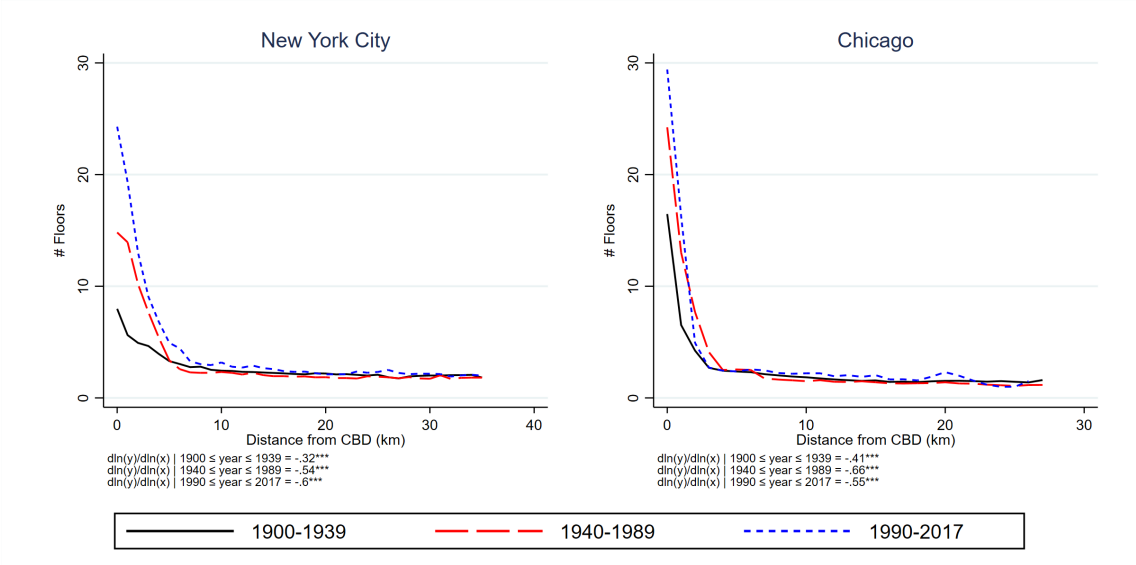
For the height gradient, predictions are more ambiguous. Ceteris paribus, innovations in construction technology imply a steeper height gradient. Increasingly important localized spillovers likely work in the same direction. However, decentralization of employment and population has been a major urban trend (Baum-Snow, 2007; Baum-Snow et al., 2017), going hand in glove with flattening price gradients (McMillen, 1996; Ahlfeldt and Wendland, 2011), which in turn lead to flatter height gradients in theory. Height gradients per se, are therefore not directly informative about the effects that technological innovations on the housing supply side have on urban economies.

4.2 Estimating the height gradient

To contrast the comparative statics with data, we begin by illustrating the spatial distribution of average building heights at different points in time in the two most

vertical U.S. cities in Figure 9. Over the course of the 19th century, building heights increased in absolute and relative terms in the centre of New York city. The elasticity of building heights with respect to distance from the CBD almost doubled. In contrast, the height gradient is more stable in Chicago, suggesting that relative increase in building heights in New York may be caused by city-specific factors.

Figure 9: Height gradients in New York City and Chicago

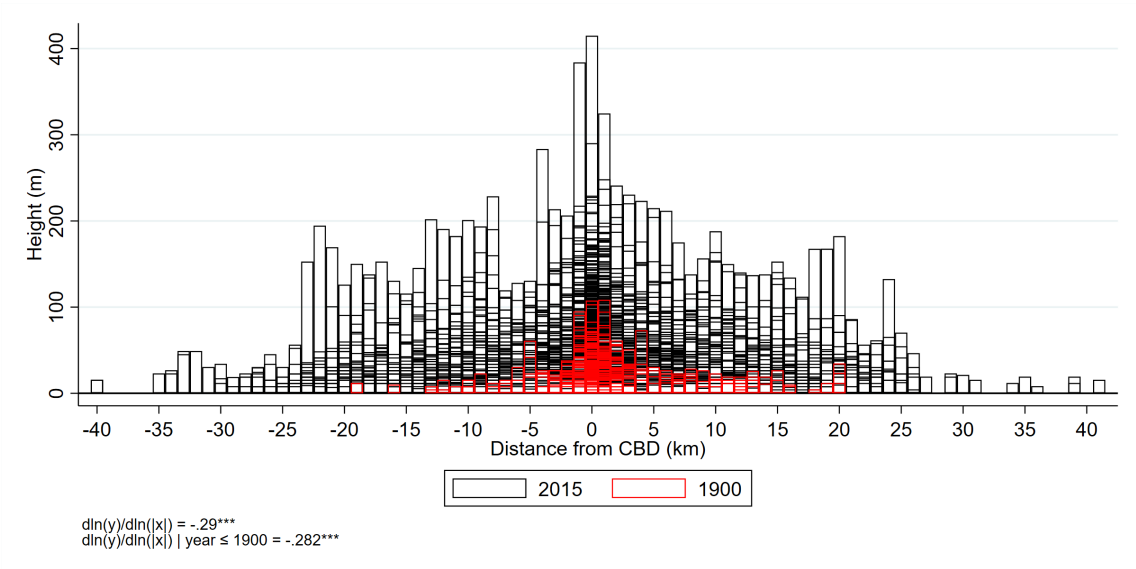


Note: CBD in New York City is the Empire State Building. CBD in Chicago is .... Data are from...

Indeed, pooling 55 North American cities sampled by Ahlfeldt et al. (2020), shown in Figure 10, reveals that the urban height profile, in relative terms, remained rather stable over more than one hundred years. While, of course, absolute heights increased significantly, our estimates of the elasticity of height with respect to distance from the CBD, at 0.28, are almost identical for buildings that existed in 2015 and in 1900.

In Table 3, we further zoom out to cover all 125 global cities for which Ahlfeldt et al. (2020) identify "global" prime locations (our CBD proxy). We focus on the tallest building within a one-kilometer distance ring from the CBD since we presumably measure the height of the tallest building with less error than the height of the average building. We estimate a semi-elasticity of height with respect to distance that is smaller in the contemporary cross-section than in the historic sample (columns 1 and 2). The implied contemporary elasticity estimate is somewhat larger since the sample mean distance is greater. Regarding the counterfactual in the upper-left panel of Figure 8, we may tentatively conclude that over the 20th century, reductions in the cost of height (θ) and the cost of CBD access (τ) have about offset each other

Figure 10: Tallest buildings by North American city, and distance from the CBD



Note: Each bar illustrates the height of the tallest building within a one-km bin to the west or the east of the CBD in one of 55 North American cities. Height data from Emporis. CBDs are the "global prime locations" identified by Ahlfeldt et al. (2020). Negative (positive) distance values indicate a location in the west (east) where the x-coordinate in the World Mercator projection is smaller (larger) than the x-coordinate of the CBD. Average height elasticities estimated conditional on city fixed effects. Data from https://www.emporis.com/ (see Ahlfeldt and McMillen (2018) for details).

in their impact on the (relative) height gradient.

In columns (3) and (4) of Table 3, we find that the commercial height gradient is steeper than the residential height gradient. A convenient way of rationalizing such differences in a bid-rent framework is to assume that the cost of distance from the CBD is larger for commercial users sine agglomeration spillovers decay faster in distance than does residential utility owing to commuting costs. However, as we have illustrated in Figure 9, relatively steep commercial rent and height gradients can also be rationalized at a uniform cost of distance ($\tau^C = \tau^R$) if the cost share of floor space in production is smaller than the expenditure share of floor space in consumption ($\alpha^C < \alpha^R$). As illustrated by the counterfactual in the upper-right panel of Figure 8, a relatively flatter residential height gradient may also originate from the supply side if the cost of height is relatively larger for tall residential buildings ($\theta^R > \theta^C$). We will return to this scenario in Section 5. The remaining columns of Table 3 illustrate how the height gradient is steeper in North American cities than in European or Asian cities. This suggests a role for land use regulation, which we discuss in Section 10.

Table 3: Height gradient estimates

	(1) Ln(height)	(2) Ln(height)	(3) Ln(height)	(4) Ln(height)	(5) Ln(height)	(6) Ln(height)	(7) Ln(height)
Distance from	-0.069***	-0.042***	-0.052***	-0.036***	-0.024***	-0.023***	-0.048***
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
City fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$d\ln(y)/d\ln(x)$	-.23	-.27	-.33	-.26	-.12	-.12	-.35
Building Sample	$t^a \leq 1900$	All	Comm.^{b}	Residential	Asia	Europe	North A.^{b}
Observations	344	3469	1294	1185	2081	662	419
R^2	.494	.559	.679	.647	.427	.331	.376

Notes: ^{a}: Completion year. ^{b} Commercial. ^{c}: North America. Unit of observation is city-distance bin (1 km). Height is the heigh of the talles building within a one-km distance bin. Building data from Emporis. CBD definitions for 125 global cities from Ahlfeldt et al. (2020). Elasticities computed at the sample means. Data from Empiris (see Ahlfeldt et al. (2020) for details). * p < 0.1, ** p < 0.05, *** p < 0.01

4.3 Related theoretical research

Height is typically not modeled explicitly in urban economics models. Yet, urban economics models of the housing supply side typically feature some notion of structural density, broadly defined as housing services per land unit (Epple et al., 2010). Housing developers optimally adjust the use of capital and land to produce housing, leading to higher structural densities. While structural density is technically closer to the floor-area-ratio (FAR) than height, the two measures are mechanically correlated since there are natural bounds for the site occupancy index.

A range of models has linked the supply side to the demand side to rationalize the internal structure of cities. To this end, the monocentric city model has been a workhorse tool in urban economics for at least half a century. As in the above toy model, spatial competition leads to bid rents that decline with distance from the CBD to offset for transport cost. The canonical Brueckner (1987) version of the model, which draws from Alonso (1964), Mills (1967), and Muth (1969), features a supply side in which profit-maximizing developers respond to changes in bid rents by providing structural densities that decline with tne distance from the CBD. Unlike in the quantitative framework outlined in Section 4.1, there is aggregate demand and supply. The market-clearing condition can be used to determine either the utility of residents in a city or the population size of a city, depending on whether the open or closed-city model is employed. However, land use segregation is not a feature of this class of models that focus on the housing sector.

More recent models of internal city structure such as Fujita and Ogawa (1982) and Lucas and Rossi-Hansberg (2002) account for the spatial distribution of land uses but exclude from their model the housing and office supply side. Grimaud (1989) shows how to incorporate a housing supply side into a framework akin to Fu-

jita and Ogawa (1982). In the quantitative spatial model of internal city structure developed by Ahlfeldt et al. (2015), land use is endogenously determined with developers producing structural density. Still, since the cost of height is not use-specific, the horizontal land use pattern is independent of the supply side and the vertical dimension of cities. Most extant models of internal city structure also do not differentiate between within- and between- and building transport cost (Sullivan, 1991) and do not take into within-building spillovers that may arise if skyscrapers promote interactions (Helsley and Strange, 2007).

A notable exception in the theoretical urban economics literature is the model by Henderson et al. (2016), which not only incorporates height but also distinguishes between a building technology for the formal (tall and durable) and the informal (flat and malleable) sector. Their model predicts that in developing cities, land will be developed informally first, and then formally, with periodical adjustments to changing economic circumstances. Curci (2017) is an example of how to model the housing supply side through convex cost of height in a monocentric city model that is nested in a Rosen-Roback type spatial equilibrium framework.

4.4 Related empirical research

The literature on the evolution of height gradients is still nascent and largely confined to particular cities. For New York City, a few studies have looked at the evolution of building heights and densities across space and time, and how they may be influenced by geology, agglomeration benefits, and other factors. (Barr and Tassier, 2016; Barr et al., 2011; Barr, 2012). Barr and Cohen (2014) study the evolution of the floor area ratio (FAR) gradient for commercial buildings in New York City across both time and space. They find that the FAR gradient for the city as a whole flattened over the first half of the 20th century, then remained relatively steady between the late-1940s and mid-1980s, and then flattened to a new "plateau" over the last quarter-century.

For Chicago, Ahlfeldt and McMillen (2018) evaluate the evolution of height and land price gradients over time, providing estimates similar to the ones reported here. Henderson et al. (2016) provide a unique analysis of the urban height profile in the context of a developing city. They document that in Nairobi, the built volume in the core city increased by more than 50% over 12 years. They also show that the height gradient is flatter in the informal than in the formal housing sector in Nairobi. This finding is consistent with the lack of capital access among those who build in the informal sectors (Bertaud, 2018). More loosely related, there is an older literature that has estimated the rate at which population density decreases in distance from the CBD summarized in McDonald (1989). Thus, there is an intimate,

which is consistent with the standard monocentric model (Brueckner, 1987). But as discussed in Bertaud (2018), the negative density gradients can reverse in non-market economies, such as China or the former Soviet Republics, where planning often places high-rise social housing projects far from the city center.

Since skyscrapers accommodate density at the extreme, they are a relevant phenomenon for the study of agglomeration economies. Moretti (2012) argues that the increase in the returns to agglomeration has increased the demand for central city locations and hence increased the steepness of the height gradient in these locations, a reasoning that we echo in the bottom-right panel of Figure 8. Solid bedrock has been proposed as an instrumental variable for agglomeration since it presumably reduces the cost of tall buildings (Combes et al., 2010). Building on this idea, Curci (2020) shows that skyscrapers add to the productivity of locations above and beyond a generic density effect. This finding is consistent with Liu et al. (2020) who, focusing on vertical density gradients, provide evidence that is suggestive of productivity spillovers from nearby employment within a building.

4.5 Potential for future research

One avenue for future theoretical research is to incorporate height-related agglomeration and dispersion forces into models of internal city structure. One step towards accounting for implications of the vertical dimension of cities for horizontal land use pattern would be to allow for use-specific costs of height.

While natural amenities, endogenous agglomeration, and transport networks have been explored as sources of persistence in the internal structure of cities (Lee and Lin, 2017; Brooks and Lutz, 2019; Ahlfeldt et al., 2020), the durability of building stock has been overlooked. Skyscrapers typically occupy the most productive urban areas and potentially represent an additional source of path-dependency. At the same time, vintage effects may encourage shifts in the spatial structure of cities as building capital depreciates (Brueckner and Rosenthal, 2009). Hence, aging tall building stock could theoretically promote the emergence of edge cities (Henderson and Mitra, 1996). Since skyscrapers are extremely durable, they may play a significant role in moderating how urban economies shift between multiple steady states. In this context, it may be worth revisiting the current workhorse models with a view to incorporating durable building stock.

On the empirical front, there is further need for additional studies that analyse the evolution of height gradients overtime beyond a case study context. The analysis of within-skyscraper productivity spillovers and transport costs also is a priority area for empirical research into the vertical dimension of cities. In the longer run, evidence

may motivate theoretical research to incorporate skyscraper-related agglomeration and congestion forces into models of the internal structure of cities.

5 The costs of height

To avoid hyper-concentration of economic activity into a singular point, urban economics models require a dispersion force. Inelastically provided land represents a natural source of such a dispersion force. In models that incorporate a housing supply side, the amount of usable housing services is not per-se limited. The dispersion force then emerges from marginal costs of housing services that increase in structural density.

Building height is an important factor in this context since beyond a certain density level, increases in density can only be achieved through increases in height. In the theoretical framework laid out in Section 4.1, the cost of height is monitored by θ^U, the height elasticity of per-unit construction cost. In this section, we provide estimates of θ^U and explore how innovations in construction technology may have affected the cost of height. In doing so, we also review the related literature.

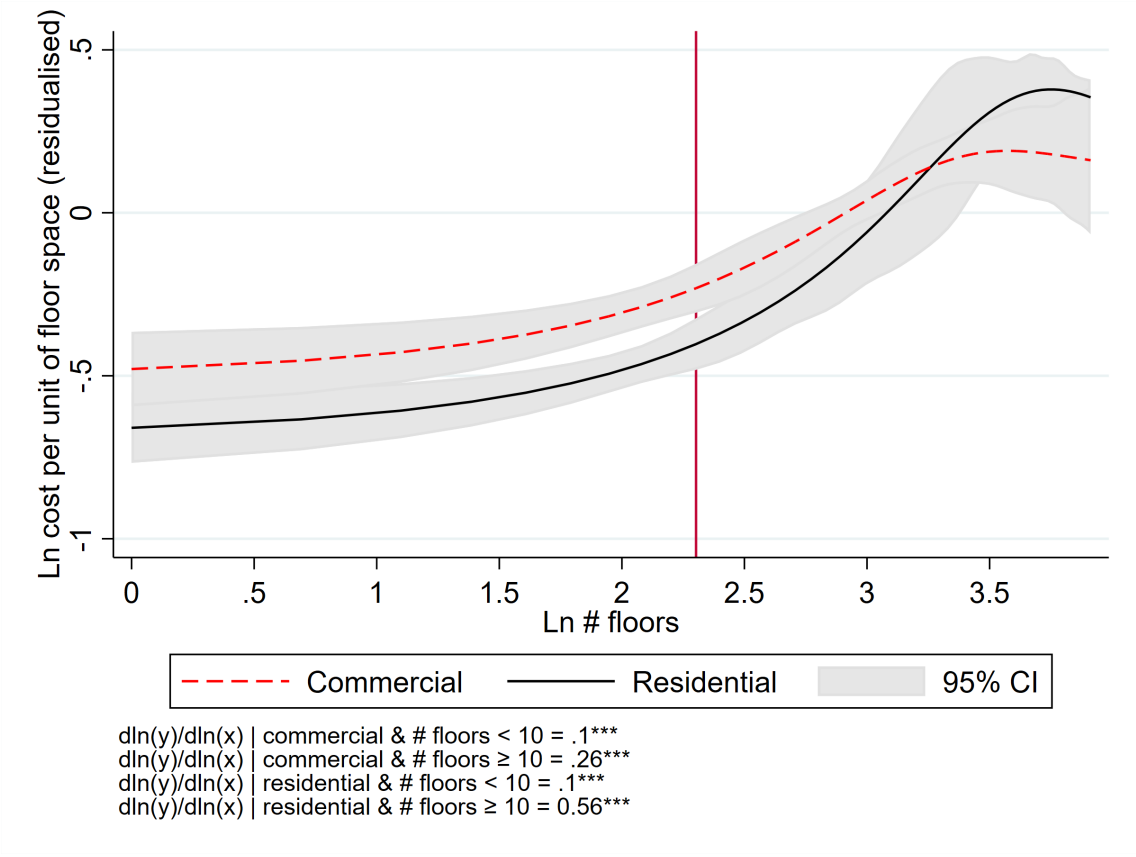
5.1 Estimating the cost of height

Figure 11 illustrates how the per-unit construction cost of commercial and residential buildings change in height within a global cross-section of buildings. Evidently, there is an increasing marginal cost from height as the height elasticity of per-unit construction cost is positive. Moreover, the elasticity increases in height. For buildings up to a height of nine floors, we estimate a height elasticity of 0.1. Beyond that point, the cost of height increases significantly.

From an engineering perspective, several factors rationalize the cost of height. First, there is the cost of wind bracing because the wind loads increase as buildings get taller. Second, taller buildings need longer and thicker elevator cables. Third, high-rises also have greater downward pressure and therefore need stronger foundations. Fourth, the size of plant and equipment increases with height (Picken and Ilozor, 2015; Barr, 2016).

There is also an important implicit cost that rises with height. As buildings get taller, on average, there is a reduction in the ratio of usable or rental area to gross building area. Taller buildings require more space for elevator shafts, and plant and equipment. Therefore, one of the key trade-offs that developers face when going taller is whether the revenue or benefits from adding floors offsets the lost income from the lower floors when an additional elevator shaft is installed (Barr,

Figure 11: Cost of height



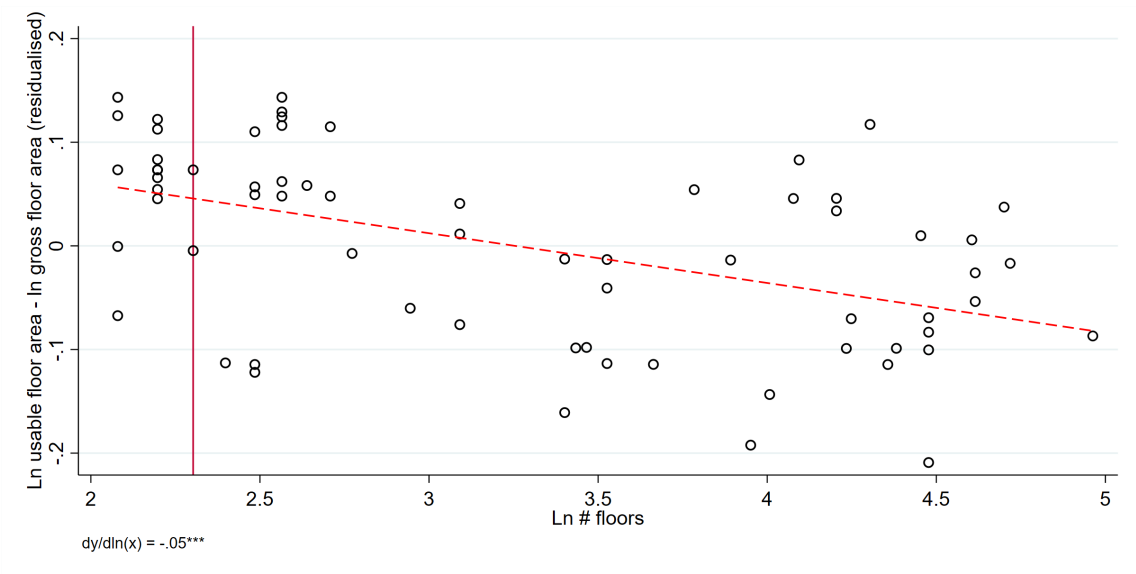
Note: We first regress the log of the ratio of building construction cost over building floor space against decade fixed effects and country fixed effects and number-of-floor fixed effects. The displayed non-linear functions are the outcome of locally weighted regressions of the estimated number-of-floor fixed effects against the number of floors, using a Gaussian kernel and a bandwidth of 10. Confidence bands are at the 95% level. The vertical line marks the natural log of 10. Data are from from https://www.emporis.com/ (see Ahlfeldt and McMillen (2018) for details).

2016). Figure 12 illustrates this mechanism based on a hand-collected data set. We estimate that for a given gross floor area, the usable floor area decreases in the height of the building at an elasticity of 0.05.

Figure 11 not only suggests that the cost of height increases in height, but also suggests there is significant heterogeneity across uses. For taller buildings, we estimate a height elasticity of 0.26 for commercial buildings and about twice as large elasticity for residential buildings of 0.56. There are several reasons that may account for this remarkable difference. Residential units require more rooms with windows and, therefore, typically have smaller floor plates. Moreover, they need more sophisticated plumbing since residential units are equipped with bathrooms. Often, they have more complex facades with balconies and sunrooms (Smith et al., 2014).

The upper-right panel of Figure 8 exemplifies how use-specific differences in the

Figure 12: Usable floor area vs. height



Note: Each marker is an individual building. The dependent variable is residualied in regressions against country fixed effects and a time trend. The vertical line marks the natural log of 10. Data was created from several sources; data and sources are available upon request.

cost of height should affect the height gradient, theoretically. Indeed, we estimate a height gradient that is about 1.5 times as steep for commercial than for residential buildings in Table 3, columns (3 and (4). The use-specific cost of height that comes out of Figure 11 may explain some of the differences in the height gradient to the extent that there is a sizable fraction of tall buildings. Since the sampled cities used in Table 3 are global cities that are generally large and have many tall buildings, this seems plausible.

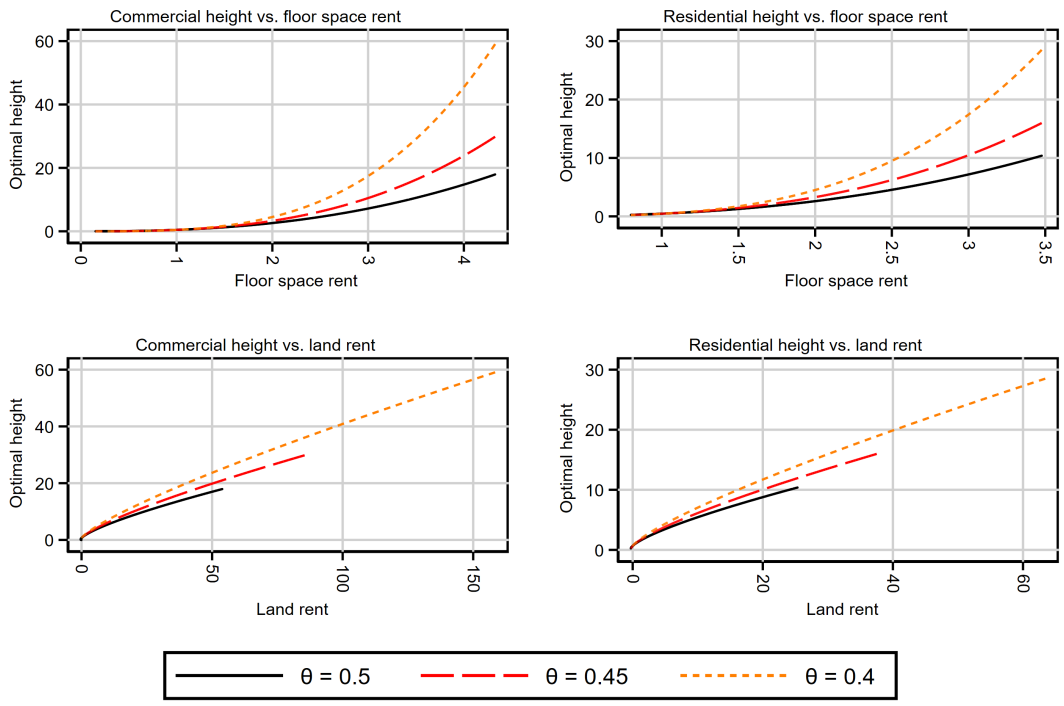
5.2 Quantifying the effects of innovation on the cost of height

As summarised in Section 2, the history of skyscrapers is marked by technological innovations. Quantifying the effect of technological innovations on the cost of height is empirically challenging since historical records of construction costs are not easily accessible. The height gradient is not directly informative of the cost of height since it also reflects demand-side factors that shape the spatial structure of a city (see Section 4 for a detailed discussion). In the absence of feasible alternatives, we exploit that within a canonical urban model, the relationship between heights and rents (which collect all demand-side factors) is shaped by the supply side.

In Figure 13, we use our theoretical framework introduced in Section 4.1 to illustrates how changes in the functional form of the relationship between bid rents and building heights are informative of reductions in the cost of height. The implication

is that we expect reductions in the cost of height over time to lead to an increase in the elasticity of height with respect to land rent, a prediction that we can test since (unlike with historical construction cost records) we have access to historic height and land value data.

Figure 13: The role of the cost of height for the height-rent relationship

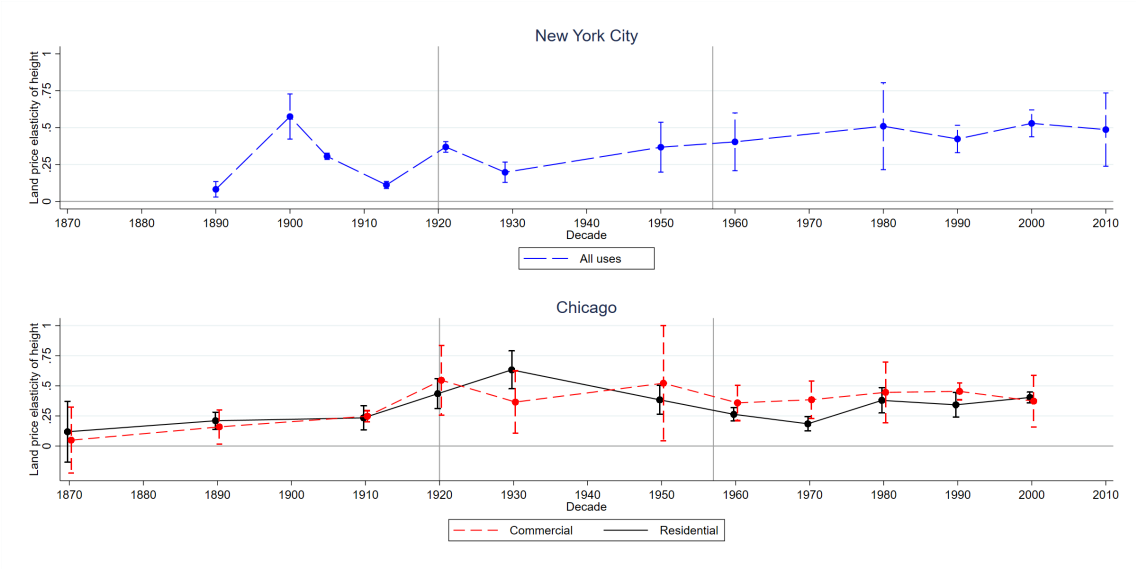


Note: Baseline choices for baseline: $\alpha^C = 0.15, \alpha^R = 0.66, b^C = b^{R3}, b^R = 10000, \tau^C = \tau^R = 0.01, d^C = d^R = 1, c^C = c^R = 1, \omega^C = \omega^R = 0.1, \theta^C = \theta^R = 0.5$. These parameter values are chosen in an ad-hoc fashion for a stylized representation. While they are broadly consistent with values in the literature (see Table 2), they do not necessarily confirm to the empirical estimates we contribute to the literature.

Figure 14 summarises our estimates of the elasticity of heights with respect to land prices for New York City and Chicago at different points in time from 1870 to 2010. In line with our theoretical expectation, we find that the elasticity estimates increase over time. It is noteworthy that there were major reforms in the zoning regime in Chicago in 1920, 1923 and 1957, which likely affected the land price elasticity of height. Unlike for Chicago, the trend for New York is quite smooth across those dates.

Under the parametrisation chosen in Section 4.1, we can use equation (2) along with the first-order condition of profit-maximisation to obtain the following straightforward mapping from the land price elasticity of height to the height elasticity of cost: $\frac{\partial \ln S^U}{\partial \ln r^U} = \frac{1}{1+\theta^U}$. To simplify the toy model, we have abstracted from any role of regulation and imposed a Cobb-Douglas housing production function. Allowing for

Figure 14: Elasticity of height with respect to land price in New York and Chicago



Note: Each point estimate is from a separate regression of the log of height of buildings constructed over a decade against the log of land value at the beginning of the decade, using the following instrumental variables. Log distance from Empire State Building and log distance from Wall Street for New York; Log distance from CBD and log distance from Lake Michigan for Chicago. 1940 missing in both data sets due to lack of completions. 1880 and 1900 missing for Chicago due to missing land value data. 1970 effect for New York is an imprecisely estimated outlier which is dropped to improve readability. Chicago data are from Ahlfeldt and McMillen (2018) and New York city land values data from Barr (2016) and Spengler (1930). Building height data from the 2018 NYC PLUTO file from the NYC Dept. of City Planning

a CES production function and a height-dependent share of the lot that is developable, we obtain the more general formulation of the land price elasticity of height derived by Ahlfeldt and McMillen (2018):

$$\frac{\partial \ln S^U}{\partial \ln r^U} = \frac{\sigma^U}{1 + \theta^U - \lambda^U}, \tag{5}$$

where σ^U is the elasticity of subsstitution between land and capital and λ^U is the height elasticity of extra space which determines how the developable fraction of a parcel depends on height, so that $d^U = lS^{-\lambda^U}$, with l being a constant. Since we prefer this more general formulation for a quantification of the changes in the cost of height, we relax the constraints $\sigma^U = 0$, $\lambda^U = 0$. Borrowing the parameter values $\sigma^C = 0.66, \sigma^R = 0.61, \lambda^C = 0.15, \lambda^R = 0.1$ values from Ahlfeldt and McMillen (2018), we compute decade-specific θ^U values using equation (5) and the elasticity estimate reported in Figure 14. We then regress the log of θ^U against a trend variable, weighting observations by the inverse of the standard errors of the parameter estimates reported in Figure 13 and controlling for period effects in the case of Chicago. Our tentative interpretation of the results reported in Table 4 is that the cost of height decreased by about 2% per year over the course of the 20th century,

although we stress that this interpretation hinges on assuming constant values for the elasticity of substitution between land and capital and the height elasticity of extra space.

Table 4: Cost of height over time

	(1) Log of height elasticity of cost: $\ln(\theta^{(C,R)})$	(2) Log of height elasticity of cost: $\ln(\theta^{C})$	(3) Log of height elasticity of cost: $\ln(\theta^{R})$
Decade	-0.017***	-0.023*	-0.018*
	(0.01)	(0.01)	(0.01)
Period effect	-	Yes	Yes
city	New York City	Chicago	Chicago
Use	All	Commercial	Residential
Observations	12	11	11
R^2	0.355	.826	.595

Notes: Unit of observation is decade. Height elasticity of cost inferred from the land price elasticity of height estimates reported in Figure 13 using equation (5) and parameter values from Ahlfeldt and McMillen (2018). Observations are weighted by the inverse of the standard errors in 13. Period effects control for level shifts in 1920 and 1957 owing to changes in zoning regime in Chicago. + p < 0.15, * p < 0.1, ** p < 0.05, *** p < 0.01

5.3 Related research

There is a strand in economics research concerned with the supply of housing services, which in per-land-unit terms corresponds to structural density. It is conventional in this literature to assume a Cobb-Douglas housing production function in which developers produce housing services H rented out at price p using capital and land L as inputs at the factor shares δ and $1-\delta$. The first-order conditions of profit-maximization along with perfect competition and zero profits deliver an intensive-margin services supply elasticity of $\frac{d\ln H/L}{d\ln p} = \frac{\delta}{1-\delta}$ (Epple et al., 2010; Ahlfeldt and McMillen, 2014b; Combes et al., 2016; Baum-Snow and Han, 2019). Since under the assumptions, there is a one-to-one mapping from marginal costs c to rents, the implied cost elasticity of structural density is simply the inverse of the housing supply elasticity, i.e. $\frac{d\ln c}{d\ln H/L} = \frac{1-\delta}{\delta}$. While structural density is not the same as height, they are correlated, especially for taller buildings where the site occupancy index varies less. For typical land shares in the range of 10% to one third, the implied cost elasticity of structural density is in line with our estimates of the height elasticity of construction cost reported in Figure 11. There is a debate, however, whether the Cobb-Douglas formulation which implies an elasticity of substitution between land and capital of $\sigma = 1$ is an appropriate approximation for the housing production function. While the literature has not achieved consensus on this question it

appears that σ is closer to one for smaller structures (Epple et al., 2010; Ahlfeldt and McMillen, 2014b; Combes et al., 2016; Baum-Snow and Han, 2019) than for tall buildings (Ahlfeldt and McMillen, 2018).

There is less economics research that explicitly focuses on the cost of height. Arguably, the first detailed research on the economics of skyscraper height was that from Clark and Kingston (1930). In their work, the authors play the role of a hypothetical developer in Manhattan to investigate the skyscraper height that produces the highest return on investment. They cost out buildings of different heights on the same lot to see how the costs change with height. With this approach, they find that for a large lot in midtown Manhattan, using land values and prices from 1929, the profit-maximising building height was 63 stories. Average total construction costs (total cost divided by gross building area) are minimized at 22 stories, after which they increase at an increasing rate. Based on their costings, we estimate an elasticity of average cost for gross floor space with respect to height of 0.17.

More recent work on the shape of skyscraper cost functions has mostly focused on data from Hong Kong and other Asian cities. This work is summarized in Picken and Ilozor (2015). They discuss the various studies which aim to find the height where average costs are minimized. Interestingly, different studies find different turning points, which range from 30 meters to 100 meters. However, all studies show that after a minimum point, costs rise with height.

Most closely related to our estimates of the cost of height are Ahlfeldt and McMillen (2018) who also exploit the Emporis data set. Employing a more restrictive parametric estimation approach and extending the sample to very tall buildings they estimate somewhat larger height elasticities of construction cost.

Outside economics, there is a literature that provides engineering cost estimates. The rule of thumb is that construction costs tend to increase by 2% per floor (Department of the Environment, 1971), which is in line with more recent estimates (Tan, 1999; Lee et al., 2011).

There have been few works in economics exploring the rate of technological improvements in skyscrapers over time. This may be in part due to the difficulty of getting detailed data to estimate total factor productivity (TFP), for example. Skyscraper developers tend to keep their cost data private. For that matter, they do not tend to itemize costs in a way that readily lend themselves to estimating production functions, which require estimates of the quantities used of labor, natural resources, and capital.

Gort et al. (1999) measure the rate of technological change in building structures by using a vintage capital model, where technological progress is embodied in the

form of new capital goods, namely, equipment and structures. Using estimates from a panel data set of 200 office buildings in the United States, they find technological progress increased the TFP of the U.S. construction sector at a rate of 1% per year, and a contribution of 15% of GDP growth over the 1988-1996 period. Chau and Walker (1988) infer TFP growth in the Hong Kong construction industry using various construction cost and price indexes. Their TFP index fell in the 1970s but rose to be about 30% higher by 1984. Other studies confirm that there has been significant TFP growth in building construction (Zhi et al., 2003; Abdel-Wahab and Vogl, 2011; Chau et al., 2005; Wang et al., 2013).

5.4 Potential for future research

The economics literature on the cost of height is still at an early stage. There are few empirical estimates of how construction costs depend on building heights. In the Emporis data used here and in Ahlfeldt and McMillen (2018) is arguably the most comprehensive database on tall buildings used in economics research so far. However, construction costs are missing for about 85% of the buildings, so there is a natural concern about sample selection. More estimates of the cost of height are needed, ideally from more comprehensive samples and for different classes of buildings. It is likely that the cost of height is larger for smaller lot sizes, and there are likely threshold effects in marginal costs at certain heights, but these are difficult to evaluate with the relatively sparse data that have been used so far.

A better understanding of how costs change with respect to building height may inform a broader literature in urban economics concerned with the supply side of housing. This literature is far from reaching a consensus on how substitutable land and capital are in the production of housing. One line of potential research could investigate more systematically how the elasticity of substitution between land and capital changes as places get denser and buildings get taller.

There is a severe lack of evidence on how the cost of height has changed over time. Technological progress in the construction of tall buildings is obvious, and our indirect estimates suggest a sizable reduction in the cost of height over time. To the extent that historical records on construction costs may become accessible, estimating the change in the cost of height over time is a priority area for research into the vertical dimension of cities. We require reliable estimates of the change in construction cost to distinguish between demand-side and supply-side forces that have shaped many skylines over the 20th century.

6 The returns to height

Traditional urban economics models focus on how accessibility leads to variation in firm profits and household utility that capitalizes in horizontal rent gradients. However, productivity and amenity may also vary within buildings, leading to vertical rent and density gradients. In the theoretical framework laid out in Section 4.1, returns to height are monitored by ω^U, the rent elasticity of per-unit construction cost. In this section, we provide estimates of ω^U and explore whether there is a time trend. In doing so, we also review the related literature.

6.1 Estimating returns to height

Figure 15 illustrates how the per-unit rent changes across floors within tall buildings in New York City and Chicago. Evidently, there is a positive height gradient. For commercial units, the positive height gradient can originate from increased revenues due to a signaling of quality to customers, or a lower wage bill if height is a workplace amenity for which workers are willing to accept a compensating wage differential (Liu et al., 2018). For residential units, the rent increase likely reflects an amenity effect, e.g., due to better views. Note that we use an implied residential rent measure based on observed purchase prices and a constant discount rate to mitigate effects of rent regulation.

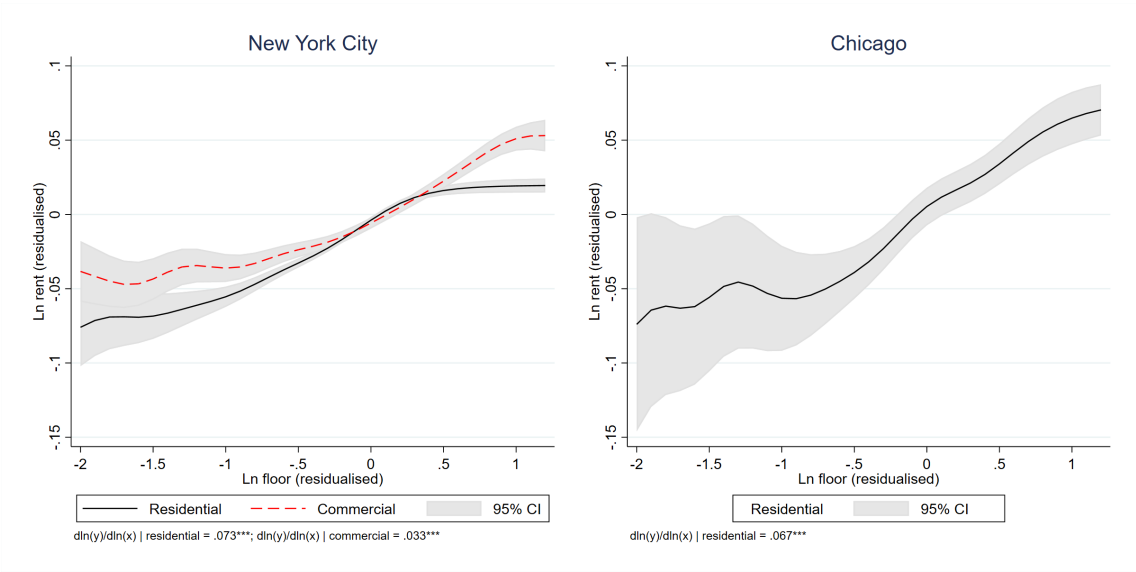
Reassuringly, we estimate an almost identical floor elasticity of residential rent of about 0.07 for both cities. For New York City, where we also have access to commercial rent data, we estimate a slightly lower height elasticity of commercial rent of 0.03. Note that this elasticity is not quite the same as ω^U introduced in Section 4.1, which gives the elasticity of average rent with respect to the height (the number of floors) of a building.

To relate the two elasticities, let's denote the floor at which we observe a rent by x, so that the elasticity we have estimated is $\frac{\partial \ln p^U(x)}{\partial \ln(x)} = \beta^U$. The aggregate rental revenue per unit of land a building generates is given by $R^U = \int_0^{S^U} p^U(x)dx = \frac{1}{1+\beta^U} S^{U(1+\beta^U)}$. The average rent is $\frac{R^U}{S^U} = \frac{1}{1+\beta^U} S^{U\beta^U}$. So, for marginal changes, the floor elasticity of rent is identical to the height elasticity of average rent $\frac{\ln(R^U/S^U)}{\ln S^U} = \beta^U = \omega^U$. In practice, changes in height are typically non-marginal due to the integer floor constraint so that the two elasticities are only asymptotically identical for infinitely tall buildings. Therefore, to convert the estimated floor elasticity of rent β^U in Figure 15 into a height elasticity of average rent ω^U, we generate a series of artificial buildings with $S^U \in 1, 2, ..., S^{MAX}$ and compute the aggregate rental revenue per land unit as $R^U_{S^U} = \sum_{s=1}^{S^U} p^U s^{\beta^U}$. To obtain an estimate of ω^U, we

regress the log of R^U_{SU} against the log of S^U. With this approach, we obtain an estimate of the height elasticity of average rent of $\omega^R = 0.056$ given a residential floor elasticity of rent $\beta^R = 0.07$ for buildings up to a height of $S^{MAX} = 40$ stories. For buildings up to $S^{MAX} = 10$, ω^R drops to 0.047. For commercial properties, we find $\omega^C = 0.026$ for $S^{MAX} = 40$ and $\omega^C = 0.022$ for $S^{MAX} = 10$.

The somewhat lower floor elasticity for commercial than for residential buildings is consistent with luxurious condominiums occupying the upper floors in mixed-used skyscrapers such as the Woolworth Tower in New York or the Shard in London. However, we wish to highlight that the commercial data we use are office rents, while the residential data are condominium purchasing prices. The conclusion that returns to height are greater within residential than commercial buildings rests on the assumption that the capitalization rate of rents is independent of the floor.

Figure 15: Returns to height



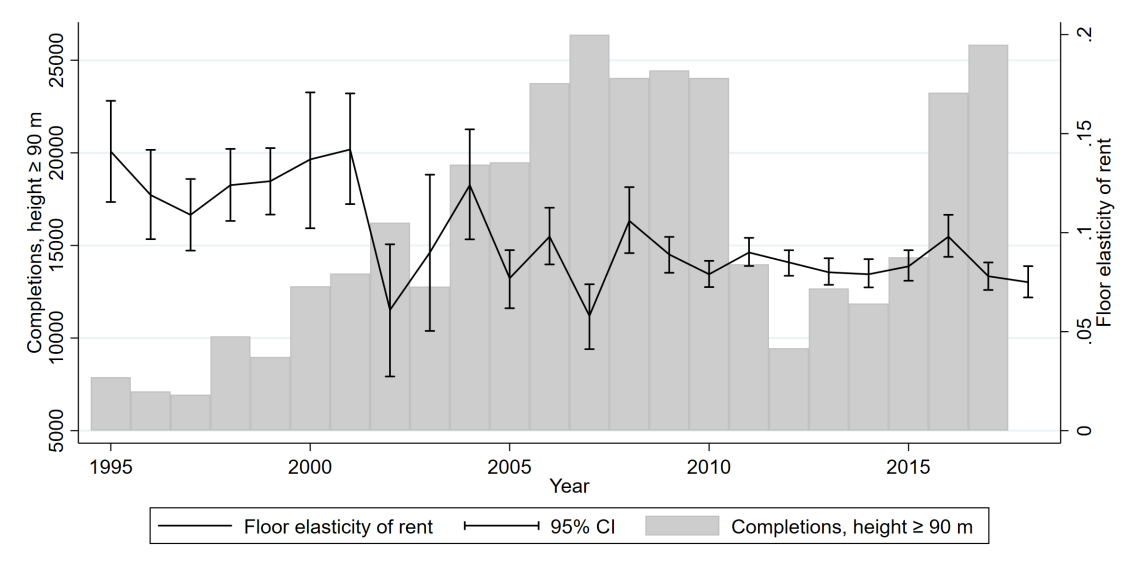
Note: Log rent and log floor are residualised in regressions against unit characteristics and building fixed effects and time effects. The displayed non-linear functions are the outcome of locally weighted regressions using a Gaussian kernel and a bandwidth of 0.2. Confidence bands are at the 95% level. We trim the data set to exclude outliers that fall into the bottom or top percentile in terms of residualised ln rent or residualised ln floor before estimating the non-parametric gradient to improve the presentation. The parametric elasticity estimates are from the full sample. Residential rents are implied rents based on apartment prices data from StreetEasy, assuming a constant discount rate. Commercial asking rents are from Cushman Wakefield

6.2 Returns to height over time

Figure 16 summarizes year-specific estimates of the residential floor elasticity of rent over 22 years in New York City. Since the early 2000s, there has been a remarkable decline in the returns to height that is seemingly at odds with the notion that skilled workers increasingly demand amenities (Glaeser et al., 2001). However, it is quite

notable that the decrease in the height premium coincides with an expansion of the supply of tall residential buildings. The number of completed residential buildings that exceed 90 meters more than doubled over the course of the 2000s, before the financial crisis led to a temporary dip. If there is heterogeneity in preferences, (implicit) prices are generally not independent of supply. One interpretation of the patterns observed in Figure 16 is that there is significant dispersion in tastes for height. As the availability of the height amenity has increased, the valuation of the marginal buyer has decreased.

Figure 16: Returns to height over time in the New York City residential market



Note: The floor elasticity of rent is estimated in regressions of log rent against log floor by year, controlling for unit characteristics and building fixed effects. Confidence bands are at the 95% level. Residential rents are implied rents based on apartment prices data from StreetEasy, assuming a constant discount rate. Building completions count from Emporis.

6.3 Related research

Extant empirical work substantiates our empirical finding that there is a rent premium at higher floors. Unlike in the above estimates, however, many existing studies do not control for building fixed effects. This is a limitation since estimates of vertical rent gradients may be confounded by attributes of horizontal space that may correlate with average building heights, such as distance from the CBD.

Koster et al. (2013) study rents in Dutch office buildings. They find that firms are willing to pay a 4% premium to be in a building that is 10 meters taller. Their findings suggest that the premium comes from a mix of agglomeration benefits, views, and the status associated with working in a structure that stands out in the

skyline (i.e., a "landmark" effect). Shilton and Zaccaria (1994) and Colwell et al. (1998) similarly find higher office rents in taller buildings. An exception in this literature is Eichholtz et al. (2010), who find mixed results. Compared to previous work, Liu et al. (2018) improve on the identification in that they control for building fixed effects when estimating commercial vertical rent gradients across multiple U.S. cities. They estimate significantly larger values of the floor elasticity of rent β^C of 0.086 within the CompStat data set and even 0.189 within the Offering memo data set. Using the same procedure as above, we can translate these estimates into estimates of the height elasticity of average rent ω^C of 0.07 and 0.15 for buildings up to $S^{MAX} = 40$ floors.

As for residential height premiums, Wong et al. (2011) find a vertical price gradient in Hong Kong apartments while Chau et al. (2007) find a price premium in Hong Kong for those units that have a sea view. Danton and Himbert (2018) estimate that within buildings in Switzerland, residential rents increase by 1.5% per floor. Their sample mostly consists of smaller structures. For the average floor of two within their sample, the implied floor elasticity of rent β^R is 0.03, somewhat less than what we find of taller residential structures in New York City and Chicago.

Some of the more recent studies on vertical rent gradients have attempted to identify the underlying mechanisms. In perhaps the first paper to include a measure that controls for views, Nase et al. (2019) find that for the Amsterdam office market, 27% of the height premium is related to the view, while 70% is due to firm-level signaling and other firm-specific factors. Liu et al. (2018) find evidence that firms in the U.S. pay premiums not only for better amenities (views and sunlight) but also to signal their productivity. They also document vertical sorting, with businesses that generate higher revenues per worker, such as law firms, being located on higher floors.

In contrast, businesses that value accessibility, such as retail, located on lower floors, even paying a significant ground floor premium. Related to the "signaling effect", Dorfman et al. (2017) perform a novel behavioral study in which they explore the perceptions of status and building height. In particular, they investigate how people view the concept of power in regards to floor height. From their survey analysis, they find that people perceive those residing on higher floors as being more powerful. Ben-Shahar et al. (2007) apply a cooperative game theory model to allocate the land and construction costs among the stories of the building. They show how the desire for status can generate a height premium in the cost allocation game.

Liu et al. (2020) show that the vertical employment density gradient follows the

vertical rent gradient, just like the horizontal density gradient follows the horizontal rent gradient. Hence, there is a u-shaped relationship between worker density and height, consistent with firms optimally adjusting factor inputs to factor prices.

6.4 Potential for future research

Despite recent progress, there are still relatively few studies that explore vertical rent gradients controlling for location via building effects. In particular, the evidence is thin for commercial buildings outside the U.S. and tall residential buildings more generally.

There is no substantial evidence on how returns to height have changed in the long run. In determining optimal building heights, returns to height are isomorphic to the costs of height (see Section 4.1). Hence, in terms of rationalizing the evolution of the urban height gradient over time, understanding changes in returns to height is as important as understanding changes in the cost of height.

The origins of the residential vertical rent gradient have remained understudied. Disentangling the effect of a view amenity from other height effects such as prestige is a fairly obvious research question. Similarly, little is known about heterogeneity in the valuation of the height amenity. Estimates of the income elasticity of the height amenity would be informative with respect to vertical and horizontal sorting. If richer households were willing to pay a greater height premium, tall buildings could contribute to spatial income segregation through a preference channel, in addition to an affordability channel.

Despite the notion that certain agglomeration effects such as knowledge spillovers are highly localized (Rosenthal and Strange, 2001), productivity spillovers within buildings have remained under-researched. Decades of research into horizontal agglomeration effects naturally suggest that priority areas for research into vertical spillovers should include causal estimates of the vertical within-building spillovers after controlling for selection; the attenuation of spillovers over vertical distances; the relative importance of information and input sharing, matching, and learning within buildings; and the co-location benefits across industries (see Combes and Gobillon (2015) for a recent review of the empirics of agglomeration).

7 Sticky Adjustment

So far, our analysis has been guided by a competitive equilibrium framework in which, for given costs and returns to height, there is a direct mapping from location-specific rents to optimal location-specific building heights. We have assumed that

the distribution of fundamental productivity and amenity is continuous, and the city is perfectly malleable; hence, all gradients are "smooth". In this section, we discuss why, in reality, short buildings survive in central cities.

7.1 Anecdotal evidence

Recent research into the fractal dimensions of cities reveals that the shape of skylines is more irregular than the already pretty irregular shape of city boundaries (Anas et al., 1998; Qin et al., 2015). Economically, an irregular building height profile can be rationalized with differences in the amenity and productivity of locations that translate into variations in rents. Since short and tall buildings often stand side-by-side, this explanation is not fully satisfactory. A walk down a typical block in Lower Manhattan will reveal 19th-century low-rise buildings wedged next to 50-story skyscrapers. At this scale, it is difficult to think of differences in fundamentals that give rise to correspondingly large variation in rents.

To document micro-geographic variation in height more systematically, we illustrate the average height within one-kilometer distance-from-the-CBD bin along side the standard deviation in heights within these bins for New York City and Chicago in Figure 17. Even within just one kilometer from the the CBDs in New York City and Chicago, the standard deviation in building heights is about as large as the average height. Even with highly localized agglomeration spillovers in knowledge-based tradable services (Arzaghi and Henderson, 2008), it is difficult to rationalize this degree of variation in heights within such a small area. The degree of variation in heights within bins near the CBD is all the more striking as the coefficient of variation drops sharply after about five kilometers. Although more remote bins are much larger, the relative variation in heights is much lower.

There are several institutional explanations for this remarkable degree of micro-geographic variation in in downtown building heights. Land use regulations and historic preservation are obvious candidates. New York City zoning rules, for example, limit the amount of floor area on each block. Owners of smaller buildings with "extra" floor area can sell it to owners of neighboring properties. They then forego any future densification of their properties, and the result is a persistent difference in the height of neighboring buildings.

Property rights also matter. Public ownership may deter teardowns since evicting residents can be politically costly. As an example, municipal governments in China typically sign 70-year land leases with residential developers. When these leases run out in the coming decades, governments may face pressure to renew them to avoid redevelopment and mass displacement. Large cities in the U.S. that own public

housing projects face similar pressures.

The protection of sitting tenants, such as under the U.S. rent control and stabilization laws, implies that landlords cannot redevelop their properties if some tenants exercise their right of lease renewals. If ownership is private but fragmented, such as in a coop, condominium, or homeowner association, selected owners can veto the redevelopment of a property, even if there is a clear economic case.

In addition to these institutional origins, there are some more sources of stickiness that have received more attention in economics research and which we review below.

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Figure 17: Variation in height

Note: Point zero n New York City is the Empire State Building. CBD in Chicago is W. Jackson Blvd. and S. Dearborn Sts. Data from the NYC PLUTO File and the Chicago Building Footprints Shapefile

7.2 Related literature

The economics literature has brought forth at least three not mutually exclusive explanations for why the sheer scale of skyscrapers may lead to "stickiness" in adjustments to changing economic circumstances.

Holdouts Skyscrapers generally require large lots to make the economics work. In dense older cities like New York, lots on a block can be owned by several owners. A developer of a tall building then needs to assemble lots. During this process, owners of one or two strategically-located lots may refuse to sell in an effort to extra monopoly rents (Cunningham, 2013; Strange, 1995). This is the holdout problem.

To avoid this problem, developers typically buy lots in secret. Still, if a block has many small lots with different owners, it may take years or decades even to assemble a sufficiently large parcel for the development of a skyscraper.

Durability Tall buildings by their very nature are frequently expected to last decades if not centuries. Beyond marginal changes, it is not economically feasible to adjust the height of existing tall structures as this would require additional elevator shafts and reinforcements of structural components and foundations. Brueckner (2000) provides a summary of the standard land-use models that incorporate the durable nature of structures. Some models assume that structures are infinitely-lived, and new ones get added on the urban core as the city expands. Other models assume redevelopment based on depreciation and changing prices. Henderson et al. (2016) nicely illustrate how adding a dynamic element to the land use model can produce "jagged" or fractal spatial structure in regard to its structural density.

Options Value An owner of a central-city vacant or under-utilized lot must decide on the timing of construction (Williams, 1991). Because of the inherent uncertainty and the long lag between ground breaking and opening, developers cannot always correctly predict the revenue they will obtain upon opening. Consistent with options theory, Barr (2010) shows empirically that price uncertainty delays skyscraper construction in Manhattan. Due to strategic interaction, option values that build up over time are capitalized periodically, leading to waves of local development under different fundamentals with correspondingly differing economic heights (Grenadier, 1995, 1996; Schwartz and Torous, 2007).

7.3 Potential for future research

Except for Barr (2010), there is no work that empirically explores the timing of skyscraper construction, or how tall building durability might affect future construction. In most larger, older cities, like New York and London, the vast majority of structures were built before World War II. This suggests a tremendous lock-in force at work once the decision to build is made. Skyscrapers rarely get torn down, and the implications for spatial structure, agglomeration effects, and the quality of urban life are worth exploring. Closely related, there is also scope for research linking the increasing option value of vacant land (Barr et al., 2018) to the timing of development, the malleability of urban spatial structure, and the cost of housing and office space.

8 Geology

Intuitively, skyscrapers are so tall and heavy that they need to be stabilized in some way so that they do not lean or deferentially settle (or fall over). This intuition underpins the use of geological conditions as relevant instruments for density in the identification of agglomeration spillovers (Rosenthal and Strange, 2008; Combes et al., 2011). In this section, we review the research into geology as a determinant of the cost of construction of tall buildings.

8.1 Related research

Deeply embedded in New York's historiography is that skyscrapers are "missing" between Downtown and Midtown because the bedrock in that area (which includes Greenwich Village and the Lower East Side) is far below the surface. From a detailed inspection of building heights and bedrock depths, however, Barr et al. (2011) conclude that subsoil supply-related conditions play a much less important role for skyscraper formation than demand for space in the two economic centers. Where engineers encountered difficulties regarding the geological conditions, they were able to devise innovative methods to make skyscrapers cost effective. That said, there is some evidence that, conditional on the decision to develop a skyscraper within Downtown, developers may have "moved" their skyscrapers to locations with more favorable geological conditions. So, bedrock appears to be a relevant determinant of building heights in New York at a micro-geographic level primarily.

Closely related, Barr and Tassier (2016) show that the rise of Midtown as a separate skyscraper district was due to the agglomerative forces related to shopping and commercial activity north of 14th Street. This commercial district emerged after the U.S. Civil War, following the northward expansion of Manhattan's residential districts. Having become the natural focal point on a long and narrow island, Midtown was the second location after Downtown where demand drove rents above the threshold that made skyscrapers economically viable. Taken together, Barr et al. (2011) and Barr and Tassier (2016) demonstrate that Manhattan's skyline shape was driven by the demand side and not the supply side.

The evidence does not substantiate that bedrock depths are a major determinant of building heights in New York. To rationalize this result, it is worth considering that Chicago, another early adopter of the skyscraper technology, was built on "swampy soil" (Bentley and Masengarb, 2015). Thus, bedrock is not a binding requirement for the construction of skyscrapers. Although skyscrapers are anchored directly to the bedrock where it is easily accessible, there are relatively cost-effective

alternatives. Typically, if there is no bedrock near the surface, foundations are created by boring long piles into the subsoil and then placing a concrete mat on top of them, with the structure built on top of the mat. It is not clear that anchoring a skyscraper to bedrock is necessarily always the cheaper alternative. As an example, accessing bedrock in downtown Manhattan is quite difficult because on top of the rock floor is viscous, wet quicksand with boulders scattered throughout. Engineers had to devise expensive caisson technology in order to build foundations in lower Manhattan (Barr, 2016). And even if establishing the foundations for a tall building on bedrock is cheaper, the cost of stabilizing a building may not vary greatly in height. In fact, in 1929 Manhattan foundation costs were found to be essentially independent of height (Clark and Kingston, 1930).

8.2 Potential for future research

The role of environmental factors of the location and heights of skyscrapers remains understudied. Recent work has looked at the role of geology on housing prices (Saiz, 2010; Burchfield et al., 2006), but there are no studies have analysed how geology effects foundation costs since the late 19th century. Further evidence on the impact of below-ground conditions on the cost of tall buildings from a larger sample of cities would be useful to substantiate the case for using geology as an instrument for agglomeration.

9 "Too Tall" Height

A profit-maximizing developer will build a structure such that at the last floor, the marginal cost of adding that floor equals the marginal revenues. In this section, we review the literature that has conjectured that skyscrapers, measured against this benchmark, may be "too tall".

9.1 Anecdotal evidence

As discussed in Sections 5 and 6, average construction costs are positively convex in height and exceeding returns to height at the margin. Thus, at a given time and location, there exists an optimal economic height of the building that maximizes the profits. However, if tall buildings serve non-economic objectives, some buildings may be too tall, in the sense that the chosen height had a marginal cost exceeds the marginal revenue, at the time of completion.[2] The debate about whether skyscrap-

[2] Bercea et al. (2005) argue that in the 16th century, the Roman Catholic church used cathedrals to signal their power when faced with competition from Protestantism.

ers, as a class of structures, or particular buildings, are too tall has a long and contentious history, which began in New York City at the end of the 19th-century.[3]

In 1930, Clark and Kingston (1930) wrote their book, *The Skyscraper: Study in the Economic Height of Modern Office Buildings*, to argue that those who called skyscrapers "freak buildings," misunderstood the economics of building tall. They argued that tall buildings were inherently a function of high land values, and they aimed to silence critics who felt that tall buildings were an inefficient use of land.

Nonetheless, they published their book during a famous "height race" in New York City, at the end of the Roaring Twenties (Tauranac, 1997). In 1930, the Bank of Manhattan (283 meters) topped out its world-record-breaking building, only to soon be beaten by the Chrysler Building (318 meters), which within a year was beaten by the Empire State Building (381 meters) in 1931. This race to dominate the skyline has only fueled the perception that ego-driven developers are imposing their will on the skyline, and the economics be damned (Barr, 2016).

There is also the perception that developers of the world's tallest building add extra-height to preempt would-be entrants, thus allowing the record holders to keep their records for extra time. Supporters of this theory can point to the Empire State Building, which held its record for 40 years. Based on the data in Figure 1, 40% of record holders held their title for more than ten years. However, there are also cases of rapid-fire succession. 40 Wall Street held its record for a matter of months. The Twin Towers lost their record within a year. And the Petronas Towers (1999) (only a record due to its spires, not its floor count) lost its record to the Taipei 101 within five years.

More broadly, the ability to use case studies to generalize about the motivations of record-breaking developers is limited. The reasons for building "too tall" are multifaceted. Cities in Asia have more complex land markets and greater government involvement in supertall projects. Skyscrapers may have additional second-order economic benefits that can be confused for or correlated with "ego-based" height. This can include various types of advertising or signaling, and which may come from the builder, the occupants, the city, or the nation (Watts et al. (2007), Garza (2017), Garza (2017)). Developers may also use their structures as "loss leaders" to increase land values of surrounding properties and to draw income from tourism. Observation decks in the clouds are profitable ventures.

[3]Note that in 1893, Chicago began a half-century of capping it building heights, thus disqualifying itself from the debate about whether its buildings were too tall or not.

9.2 The Literature on Height Competition

Helsley and Strange (2008) offer a game-theoretic model of height competition. They observe that across the world, a city's tallest building is often much higher than the second tallest building. They argue that if two developers are vying to claim the prize of the "tallest building" for bragging rights or personal satisfaction (i.e., ego), then a pure-strategy equilibrium will have one builder constructing a building so tall it will have zero economic profit. In a mixed-strategy equilibrium, each builder will assign a positive probability to building too tall. In a sequential game, developers deliberately build extra tall to deter competitors. In short, they demonstrate that "too tall" construction can be rationalized theoretically.

Prompted by this theoretical contribution, there has been some work on empirically measuring the effects of height competition. However, it has remained a challenge to separate competition effects from the effects of unobserved fundamentals, so the evidence is best interpreted as suggestive.

Barr (2010) studies the economics of skyscraper construction in Manhattan using time series data from 1895 to 2004. He estimates the economic heights and number of completions with a regression that includes economic fundamentals. While he cannot rule out height competition from time to time and for particular buildings, skyscraper heights, on average, are consistent with economic fundamentals.

Barr (2012) analyses the effects of height competition by regressing the heights of a newly-completed buildings on a weighted average of the heightsof surrounding structures (completed before or contemporaneously). The estimate for the spatial autoregressive parameter turns out to be positive and statistically significant, but is economically small. Extra height is mostly added during boom times when the forgone profits are relatively more affordable.

Barr (2013) investigates height competition between two rivaling skyscraper cities, namely New York and Chicago. He uses time-series measures of each other city's skyscraper construction (average heights and completions) to estimate skyscraper reaction functions. He finds that each city adds heights in response to the (lag of) building activity in the other city, but, again, the interaction coefficients are economically small.

Barr and Luo (2020) analyze skyscraper construction across Chinese cities. They find that local GDP and population are strong predictors of skyscraper height and completions, which suggests that China's rapid urbanization is driving the rise of its skylines. They also find that cities with younger officials build taller skyscrapers, suggesting the desire to use icons for career promotion in a system where municipal officials control land use through land leases (Brueckner et al., 2017). From spatial

autoregressions, they infer that same-tier cities are competing against each other in the height market, in order to advertise their cities and its officials.

Using a different approach that draws from the spatial point-pattern literature (Duranton and Overman, 2005), Ahlfeldt and McMillen (2015) document that the completion of a very tall building is followed by a period of less ambitious constructions in the immediate neighbourhood. While this finding is consistent with the sequential-game predictions of the Helsley and Strange (2008) model, an existing tall building may also lead to a lower economic height for subsequent buildings due to a lower view amenity.

Taken together, the evidence suggests that there may be a strategic element in height decisions, but that the effects of fundamentals dominate.

9.3 Other related research

White Elephants Gjerløw and Knutsen (2019) test the hypothesis that autocratic leaders build "white elephants" as symbols of their reigns, or to demonstrate their power to muster significant resources, or to provide work or provide incomes to their supporters. They show that going from maximum democracy to maximum autocracy is associated with about one extra skyscraper, ceteris paribus. They also find that autocracies are more likely to add more "vanity height" to their structures, where vanity height is the difference between the roof-line and the height of the topmost part of the building. They interpret this extra height as the desire to signal the power of autocratic leaders.

Urban Planning and Growth Anecdotally, cities world-over appear to be using supertall buildings as catalysts of urban development and renewal. For example, the so-called Three Brothers of Shanghai (the Jin Mao Tower, the Shanghai World Financial Center, and the Shanghai Tower) were part of the master plan for Lujiazui financial district. The Burj Kalifa and the Jeddah Tower (under-construction) are central features of new neighborhoods being developed. The original Twin Towers in New York were part of a strategy to modernize and re-invigorate lower Manhattan, which had fallen on hard times after World War II. Unlike for sports stadia (Coates, 2007; Ahlfeldt and Kavetsos, 2014), there is no work in economics that explores how skyscrapers induce land values, house prices, or foreign direct investment.

The skyscraper curse The so-called Skyscraper Curse alleges that supertall building completions are a herald of economic doom (Lawrence, 1999). Supporters point to the Empire State Building, completed in 1931, and the Burj Khalifa,

completed in 2010, to show that these towers were finished during severe downturns. Since there are about twice as many record-breaking builds than economic crises over the past 100 years, it is relatively easy to "manually" pair record-breaking buildings with business cycle peaks. Barr et al. (2015) are the first to rigorously test the skyscraper curse hypothesis using Granger-causality analysis. The result is that height cannot be used to predict the business cycles, but economic growth can be used to forecast building heights.

9.4 Potential or future research

Each generation redefines its own "normal" building height. In the late 19th century, 15 stories was considered excessive. By the 1920s, 40 stories was not uncommon. Today 100-story structures are regularly built. As the barriers to building height continue to fall away, the debate about whether skyscrapers are too tall keeps renewing itself.

Empirical research into whether the tallest structures in the world are, in fact, economically too tall remains challenging due to data limitations. Thus far, comprehensive data that are readily accessible are confined to completed heights of buildings and the general economic climate in which they are built. Building-level data on revenues and the costs of land acquisition and construction cost will be key to evaluating the economic case for building super-tall.

Since skyscrapers are durable and there are significant construction lags, expectations matter for building height decisions. Seemingly excessive structures may be result of perfectly foreseen economic growth or a myopically extrapolated real estate boom. Understanding the extent to which cyclical vertical growth can be rationalized under perfect foresight may be informative with respect to the role of expectations on real estate markets more generally.

10 Regulation and externalities

In this section, we review the literature on causes and effects of regulation that seeks to address external effects skyscraper may have on their cities and neighbourhoods.

10.1 Anecdotal evidence

When tall buildings started to rise in U.S. cities at the end of the 19th century, planners and officials were concerned about shadows, increased traffic congestion, increased risk of conflagrations, and correspondingly reduced property values of surrounding buildings (Heights of Buildings Commission, 1913; Hoxie, 1915).

While some cities, like Chicago and Boston, placed direct height caps on their buildings, New York never did. In 1916, it implemented the first comprehensive zoning regulations in the nation. Rather than capping heights, it mandated setback rules that required tall buildings to set back from the street line as they rose taller. This gave rise to the so-called "wedding-cake style" architecture. The purpose of this regulation was to increase sunlight on the streets and reduce total building area and congestion. New York's codes were widely copied throughout the country in the 1920s (Weiss, 1992). These policies were not based on rigorous social cost-benefit analyses. Instead, planners followed their own normative judgements under the constraint that the imposed regulation should not be overturned by courts (Weiss, 1992; Bertaud, 2018).

Today, cities around the world generally attempt to control building height and density by capping the maximum allowable floor area ratios (FARs). The FAR is the total building area divided by the lot area. For example, the maximum allowable FAR for Manhattan office buildings is 15 (or 18 if open space is provided). Similarly, for Chicago, the maximum FAR is 12. To put these numbers in perspective, the maximum FAR for Paris, a "flat" city, is 3 (Brueckner and Sridhar, 2012).

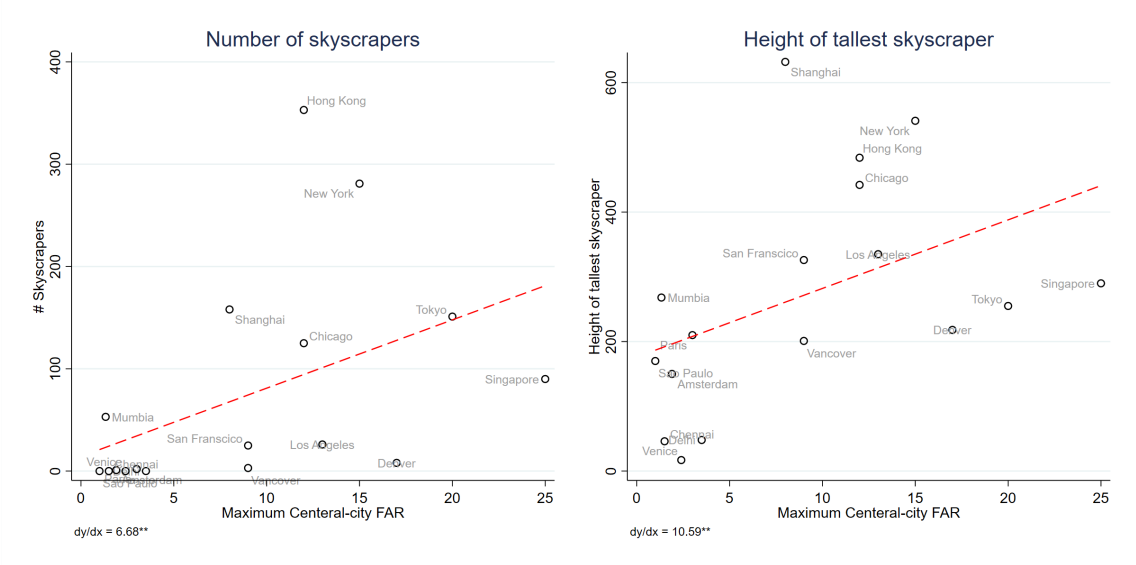
In keeping with intuition, Figure 18 shows a positive correlation between maximum allowable central-city FARs on the one hand, and the number and height of skyscrapers on the other, for cities around the world. The relationship, however, is quite noisy, suggesting that the causes and effects of regulation of vertical growth are highly context-dependent. As an example, the most vertical cities of the world, Hong Kong and New York have many and tall skyscrapers despite the FAR regulation being more restrictive than in Tokyo or Singapore.

10.2 Related research

Gyourko and Molloy (2015) provide a review of a sizable literature on the causes and effects of effects of building regulation. The literature specifically concerned with height regulation is somewhat thinner but growing.

In general, it appears that FAR regulation tends to be binding. In several analyses, Barr (2010, 2012, 2013) concludes that zoning regulations in New York and Chicago reduce building heights. Ahlfeldt and McMillen (2018) find that in Chicago, the building footprints of taller buildings cover a smaller fraction of the land parcel. This relationship reflects that for developers to go tall, they must reduce the floor plate sizes in order to maintain the maximum allowable FAR. Sometimes, extra height is allowed in return for publicly accessible open space, further reducing the floor plate. Theoretically, the partial equilibrium effect of a forced reduction in

Figure 18: Skyscrapers and FAR regulation US



Sources: Floor area ratio data from Brueckner and Sridhar (2012). Skyscraper data: https://www.skyscrapercenter.com/

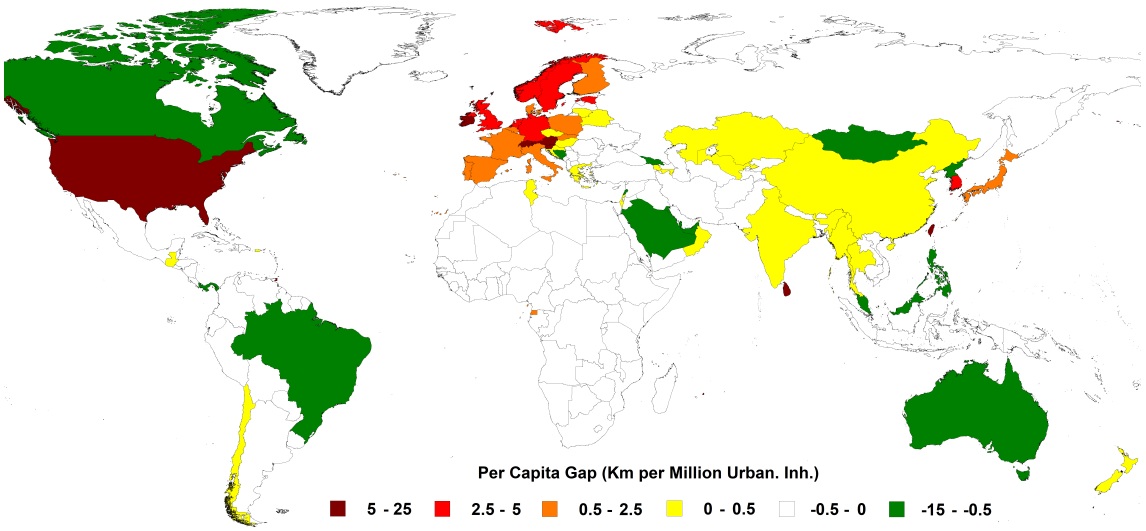
the intensity of development of a land parcel must be to restrict its value. Indeed, Brueckner et al. (2017) show that the greater the FAR stringency imposed by local officials in China, the greater is the reduction in land values. Brueckner and Singh (2020) find a similar effect on vacant land sales prices in five cities in the U.S. Moon (2019) substantiates this finding in a detailed analysis of the New York City land market.

Naturally, height restrictions have welfare consequences, some of which are unintended. Bertaud and Brueckner (2005) and Brueckner and Sridhar (2012) demonstrate that the very low allowable FARs in Indian cities increase sprawl and traffic congestion. Hence, the attempt to reduce externalities from building density has produced a new suite of negative externalities. Since binding height restrictions, ceteris paribus, reduce housing supply, the general equilibrium effect is to raise house prices above the marginal cost of construction, adding to affordability problems (Glaeser et al., 2005).

Using a database of nearly every 80-meter or taller building on the planet, Jedwab et al. (2020) compute the building-height gap by country. The measure, depicted in Figure 19, summarizes how much building height per capita falls short of the predicted value for given fundamentals under laissez-faire regulation. Considering their economic potential, European countries and the U.S. under-perform in terms of vertical growth, most likely due to planning regulations. The gap measure positively correlates with measures of sprawl, housing prices, and air pollution, suggesting costs

of regulation.

Figure 19: Per-capita height gaps by country



Note: The height gap measures by how much total height of tall buildings in per-capita terms falls short of the predicted value under laissez-fair regulation (for given fundamentals). The measure is constructed for 149 countries in 2020. Positive values (red shades) indicate more stringent regulation. Source: Jedwab et al. (2020).

In some cities, planners seek to mitigate negative externalities by ensuring that tall buildings are aesthetically appealing. Cheshire and Dericks (2020) study the impact of tight building height regulations in London, where skyscrapers have to go through an approval process since there is no "as-of-right" development as in New York or Chicago. The authors document that developers employing so-called Trophy Architects—those that won one of three prestigious architectural awards—are allowed to build 14-floors taller than otherwise, a sizable magnitude in a reasonably flat city. They interpret the one-and-a-half-fold increase in site value due to the extra permitted height as the compensation of the cost of rent-seeking and, thus, an indirect measure of dead-weight loss.

Ironically, some of the unintended negative effects of height restrictions may have socially desirable second-order effects. Borck (2016) shows theoretically how building height regulation leads to lower housing consumption due to a supply-driven increase in house prices, which leads to a reduction in greenhouse gas emissions. In his model with a global emissions externality, the welfare effect is non-monotonic and, depending on the value of the externality, either the absence of height regulation or a very stringent regulation can be socially desirable.

10.3 Potential for future research

Much of the literature on height regulation is concerned with the negative collateral effects. One naturally wonders how these costs compare to potential benefits that motivate height regulations in the first instance. There is some evidence pointing to amenity values of sunshine (Fleming et al., 2018) and distinctive design (Ahlfeldt and Holman, 2018), suggesting that a regulation that reducse shadowing and improves the design of tall buildings may have positive effects. Also, while very tall buildings likely reduce sprawl and encourage mass transit, they tend to produce more CO_2 on per square meter, so that their environmental impact is theoretically ambiguous.

There is an evident polarization between opponents and proponents of tall buildings. The former argue that skyscrapers are, aesthetically speaking, "too big" for the "human scale" (Gehl, 2013) and an anathema to the vibrant city that Jason Jacobs (1961) argued for. The latter focus on the cost of height restrictions that materialize in sprawl and affordability problems. Yet, only a quantitative evaluation of the positive and negative external effects of tall buildings enables setting the appropriate Pigovian tax for a transition into a first-best optimum of building heights.

Likely, the degree of stringency of height regulation varies significantly across cities around the world because the external costs and benefits vary, too. Developing an understanding of external costs and benefits of skyscrapers that account of heterogeneity across institutional context is an ambitious research agenda that will likely remain topical for quite some time.

11 Conclusion

The high-level conclusion of our synthesis is that the literature engaging with the economics of tall buildings is still at an early stage. We have discussed the potential for future research throughout Sections 4 to 10. Below, we offer an admittedly subjective selection of three priority areas for research into the vertical dimension of cities.

First, there is a long way to go in understanding how the positive and negative externalities associated with tall buildings play out on balance. Skyscrapers may not just accommodate productive workers in productive locations but also facilitate gains in productivity, ceteris paribus, if the vertical cost of interaction is sufficiently low. If well-designed skyscrapers put neighbourhoods or cities on the map, they may attract businesses and tourism. At this same time, skyscrapers may generate external costs in the form of shadowing, congestion, or the loss of a coherent historic

fabric.

Second, there is scope for exploring the costs of and returns to height more fully. Costs of and returns to height appear to be non-linear, and there are likely threshold effects that could be explored with richer data sets. Heterogeneity in the cost of height across uses and the value of the height amenities across users is not only interesting in its own right but also has implications for the horizontal pattern of land use and sorting. The net cost of height is one of the main congestive forces and a better understanding of how it evolves over time is helpful with respect to rationalizing changes in the spatial structure in the past as well as anticipating the evolution of cities in the future.

Third, there are forces outside the canonical competitive equilibrium framework that shape the urban height profile and deserve more attention. As an example, disentangling the effects of height competition from fundamentals that would justify extreme building heights remains an empirical challenge. Progress on this front will be essential for economically rationalizing skyscrapers such as the Empire State Building or the Burj Khalifa, which dominate height rankings for decades and exemplify the frontier of technological innovation and human ambition.

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A1 Online appendix

This online appendix contain supplementary material not intended for publication.

A1.1 Parametrization

In this section, we provide additional detail on the parametrization of the theoretical model introduced in 4.1.

Residential use Consider homogeneous residents that maximize the Cobb-Douglas utility

$$U = A^R X^{\alpha^R} F^{1-\alpha^R}$$

subject to the budget constraint

$$y = p^R F + C,$$

where A^R is a location-specific residential utility shifter, C is a tradable good with a price normalized to unity, F is floor space, p^R is the residential floor space price, and α^R is the expenditure share on floor space. Using the Marshallian demand functions $C = \alpha^R y, F = \frac{(1-\alpha^R)y}{p^R}$ and imposing that the spatial equilibrium anchors indirect utility to an exogenous reservation level $\bar{U} = 1$, the residential bid rent is defined as:

$$p^R = a^R (A^R)^{\frac{1}{1-\alpha^R}},$$

where $a^R = (1-\alpha^R)(\alpha^R)^{\frac{\alpha^R}{1-\alpha^R}} y^{\frac{1}{1-\alpha^R}}$ is a scale parameter that collects some constants.

Commercial use Consider homogeneous firms that produce according to the following production function

$$X = A^C K^{\alpha^C} F^{1-\alpha^C}$$

and maximize profits

$$\pi = X - p^C F - K,$$

where A^C is a location-specific commercial productivity shifter, X is a tradable

good with a price normalized to unity, K is tradable capital with a price normalized to unity, and p^C is the commercial floor space price. Using the marginal rate of substitution $\frac{K}{F} = \frac{\alpha^C}{1-\alpha^C} p^C$ and assuming zero economic profits in a perfectly competitive market, the commercial bid rent is defined as

$$p^C = a^C (A^C)^{\frac{1}{1-\alpha^C}},$$

where $a^C = (1-\alpha^C)(\alpha^C)^{\frac{\alpha^C}{1-\alpha^C}}$ is a scale parameter that collects some constants.

Bid rent In the below we index use by $U \in R, C$ for convenience and assume that productivity and amenity decrease in distance from the CBD D:

$$A^U = b^u e^{-\tau^U D}$$

so that

$$p^U = a^U (b^u)^{\frac{1}{1-\alpha^U}} e^{-\frac{\tau^U}{1-\alpha^U} D},$$

where $b^u > 0$ is a constant that determines the amenity level at the central point.

Developers We define height $S^U = \frac{F^U}{L}$ as the ratio of floor space F^U over the land L occupied by a building at any location. Homogeneous developers choose S^U to maximize profits

$$\pi^U(S^U) = \tilde{p}^U S^U - \tilde{c}^U S^U - \frac{1}{d^U} r^U,$$

where the average rent $\tilde{p}^U = p^U {S^U}^{\omega^U}$ and the average cost $\tilde{c}^U = c^U {S^U}^{\theta^U}$ increase in height at the elasticities $\omega^U > 0$ and $\theta^U > 0$. $d^U = 1$ is the fraction of a parcel that is developable, which along with the land rent r^U determines the land cost. Profit maximization delivers the economic height

$$S^{U*} = f^U (c^U)^{\frac{1}{\omega^U - \theta^U}} e^{\frac{\tau^U}{(1-\alpha^U)(\omega^U - \theta^U)}},$$

where $f^U = (a^U)^{-\frac{1}{\omega^U - \theta^U}} (b^U)^{-\frac{1}{(1-\alpha^U)(\omega^U - \theta^U)}} \left(\frac{1+\theta^U}{1+\omega^U}\right)^{\frac{1}{\omega^U - \theta^U}}$ collects various constants.

Land rent Using S^{U*} in the developer profit function and assuming zero profits due to perfect competition delivers the residential land rent.

$$
\begin{aligned}
r^U = d^U \big[& a^U (b^U)^{\frac{1}{1-\alpha}} (f^U)^{1+\omega^U} (c^U)^{\frac{1+\omega^U}{\omega^U-\theta^U}} d^{\frac{(1+\omega^U)-(\omega^U-\theta^U)}{(1-\alpha^U)(\omega^U-\theta^U)}\tau^U D} \\
& - c^U (f^U)^{1+\theta^U} (c^U)^{\frac{1+\theta^U}{\omega^U-\theta^U}} e^{\frac{(1+\theta^U)}{(1-\alpha^U)(\omega^U-\theta^U)}\tau^U D} \big]
\end{aligned}
$$