Garnache, Cloé; Mérel, Pierre

**Working Paper**

# Environmental Policy in General Equilibrium: When the Choice of Numeraire Matters

CESifo Working Paper, No. 8354

**Provided in Cooperation with:**
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*Suggested Citation:* Garnache, Cloé; Mérel, Pierre (2020) : Environmental Policy in General Equilibrium: When the Choice of Numeraire Matters, CESifo Working Paper, No. 8354, Center for Economic Studies and Ifo Institute (CESifo), Munich

This Version is available at:
https://hdl.handle.net/10419/223426

CESifo WORKING PAPERS

8354
2020

June 2020

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*Cloé Garnache, Pierre Mérel*

CESifo

## Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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# Environmental Policy in General Equilibrium: When the Choice of Numeraire Matters

## Abstract

This paper derives the incidence of a pollution tax in a stylized general equilibrium framework, building on previous work by Fullerton and Heutel (2007a). Using the CPI as numeraire, we show that tax incidence is a simpler problem than previously thought, and that general insights can be derived without the need to restrict the parameter space. In addition, the counter-intuitive possibility that an increase in the tax could lead to worse pollution outcomes vanishes. The choice of the CPI as numeraire is further justified by the fact that environmental taxes, for instance carbon taxes, are typically indexed on inflation.

JEL-Codes: Q520, H230, H220.

Keywords: tax incidence, general equilibrium, relative prices, numeraire.

*Cloé Garnache*
*Department of Economics*
*University of Oslo / Norway*
*cloe.garnache@econ.uio.no*

*Pierre Mérel*
*Department of Agricultural and Resource Economics, University of California*
*Davis / USA*
*merel@primal.ucdavis.edu*

We thank Øyvind Aas, Geir Asheim, Christoph Böhringer, Andrés Carvajal, Nils Christian Framstad, Don Fullerton, Bård Harstad, Garth Heutel, Bas Jacobs, Halvor Mehlum, Erich Muehlegger, Kevin Novan, Paolo Piacquadio, and Christian Traeger for helpful comments.

# 1 Introduction

It has been known since the seminal work of Muth (1964), and later confirmed through the more general results of Heiner (1982) and Braulke (1984), that under fairly general conditions, the law of input demand holds at the industry level even when the prices of outputs or other inputs are allowed to adjust in related markets in response to the input price increase. An interpretation of this law is that an exogenous tax on an industry input reduces the overall use of that input, even if certain firms end up using more of it. That is, demand and "derived demand" both slope down in the aggregate. If the taxed input causes pollution, pollution will unambiguously be reduced by an increase in the tax.[1] This simple economic logic lies at the core of environmental taxation (Baumol and Oates, 1988).

While the literature cited above explicitly allows for the presence of markets besides the one subject to taxation, they rely on partial equilibrium approaches. In seminal contributions to the analysis of the incidence of environmental policy, Fullerton and Heutel (2007a) and Fullerton and Heutel (2010b) show that the desirable effects of environmental taxation on pollution may no longer apply in a general equilibrium context, even in a closed economy and under the assumption of homogenous spending propensities on the demand side.[2] That is, an increase in the tax on a polluting input (e.g., carbon) may in some cases lead to more, not less, pollution. Similarly, in a cap-and-trade system, a decrease in the cap on a polluting input could lead to a decrease in the permit price.

Fullerton and Heutel (2007a) suggest that the ambiguity regarding the applicability of the law of input demand to polluting inputs is due to general equilibrium feedback effects.[3] The present paper shows that whenever the law of input demand is violated for pollution, the equilibrium in their model is unstable in the Marshallian sense.[4] The occurrence of such instability, which may seem tangential to the issue of policy incidence, is in fact closely related to it as all comparative statics, not simply that on the pollution outcome, are reversed in this case. This precludes general conclusions to be drawn regarding policy

[1]The result holds as long as the supply of the dirty input is less than infinitely inelastic. If it is infinitely inelastic, input use remains constant, but it does not increase.

[2]In earlier work, Mieszkowski (1967) relaxes the assumption of identical spending propensities among owners of capital and owners of labor and shows that a series of counterintuitive comparative statics may ensue.

[3]See, for instance, the discussion in footnote 14 of Fullerton and Heutel (2007a). For an example of model parameterization leading to the counterintuitive outcome, see Appendix A.6.

[4]Put simply, Marshallian instability means that if the demand price of a good exceeds the supply price of that good, quantity will move further away from its equilibrium value.

incidence itself. We go on to show that the existence of unstable equilibria is an artefact of the choice of numeraire good. In short, due to the non-market nature of pollution, the tax on the polluting input is a nominal, rather than *ad valorem*, tax. As a result, the effects of this nominal tax on goods provision, resource allocation, and relative prices implicitly depend on which good is chosen as numeraire, as the tax increases the price of the polluting input *relative to that of the numeraire*. Said differently, there are as many *varieties* of environmental taxes as choices of numeraire good.[5]

The question naturally arises as to which good should then be chosen as numeraire, that is, to the price of which good the environmental tax should be anchored. Instead of being anchored to the price of a single good, the tax could conceptually be anchored to a price index reflecting the average level of prices across a set of goods.[6] In that sense, the choice of which *variety* of environmental taxation to model in general equilibrium may seem overwhelming.

In this paper, we argue that any reasonable anchoring price index should be one for which the law of input demand is satisfied, that is, an increase in the price of the dirty input relative to the equilibrium value of the index unambiguously leads to a decrease in input use—and attendant pollution. Such requirement amounts to ruling out equilibria that are unstable in the Marshallian sense and thus ensures that meaningful comparative statics can be derived (Samuelson, 1941). With this constraint in mind, we consider two classes of price indices: those defined over the prices of consumption goods (uses side) and those defined over factor prices (sources side). Using the model of Fullerton and Heutel (2007a), we show that within each class, there exists a unique index that satisfies the stability constraint. We express each index using underlying parameters of the economy. On the uses side, the price index is identical to an inflation index, namely the product of the prices of consumption goods raised to their respective expenditure shares. On the sources side, the price index is equal to the product of factor prices raised

[5]The issue here is not about the undeterminacy surrounding the overall magnitude of prices, but, since the model is used to *compare* equilibria, that an *increase* in the nominal tax rate increases the price of pollution relative to the price of the numeraire while potentially decreasing it relative to other prices. In earlier studies by Bovenberg and Goulder (1997) and De Mooij and Bovenberg (1998) that also consider a pollution tax, capital, the polluting input, and a final consumption good are supplied to/demanded from a small economy in an infinitely elastic fashion, with fixed rates of exchange on the world market. As a result, a tax on the dirty input increases the price of that input relative to that of capital and that of the consumption good. The only price determined endogenously in the model is the wage rate. A government budget constraint imposes that a rise in the pollution tax be matched by a decrease in either the tax on labor or that on capital, giving rise to the possibility of an increase in pollution through an expansion in output. The mechanism behind the increase in pollution in these earlier models is thus distinct from that in Fullerton and Heutel (2007a) and Fullerton and Heutel (2010b).

[6]A similar point is made in Baylis et al. (2014, pg. 63).

to their respective income shares. Importantly, none of these indices depend on behavioral parameters such as substitution elasticities, either in production or consumption.[7] Therefore, a government could anchor the pollution tax to either index without unrealistic informational requirements. For instance, the pollution tax could be tied to the Consumer Price Index (CPI). With a pollution tax so designed, not only is the effect of a rise in the tax in accordance with the law of input demand and the intent of environmental taxation, but the tax incidence results discussed by Fullerton and Heutel (2007a) in the context of specific parameter values also hold generally.

Although fairly recent, the contribution of Fullerton and Heutel (2007a) has had an undeniable influence in the environmental economics literature. Their model has been used in Fullerton and Heutel (2007b), Fullerton and Heutel (2010b), Fullerton and Heutel (2010a), and Fullerton et al. (2011). It has been modified or extended in further contributions by Fullerton and Monti (2013), Baylis et al. (2013), Rivers (2013), Baylis et al. (2014), Rausch and Schwarz (2016), and Goulder et al. (2016). Their analysis has also provided needed clarity on important drivers of the incidence of environmental policy in studies that involve more complex and realistic models of the economy, e.g., Rausch et al. (2011), Rausch and Mowers (2014), or Marten et al. (2019), and in policy discussions, e.g., Morris and Munnings (2013).

Our paper makes four contributions to the literature. First, we show that well-designed pollution taxes need not have ambiguous effects on equilibrium pollution levels in general equilibrium, and that counterintuitive pollution outcomes identified in previous literature imply unstable equilibria. Second, we ask whether there exist anchoring prices that avoid equilibrium instability altogether. We show that the answer is yes, and that, reassuringly perhaps, the set of candidate indices is limited. On the uses side, the only index satisfying the stability constraint for all parameter values is equivalent to the CPI. On the sources side, it is equivalent to a producer price index for primary production factors. Third, using these indices, we demonstrate that tax incidence results previously derived only in special cases hold in fact quite generally. Fourth, we empirically demonstrate that the choice of numeraire in general equilibrium models influences comparative statics with respect to nominal taxes on non-market goods like pollution. Notably, using a model calibrated to the US economy, we show that predicted pollution impacts may differ by up to 40% depending on the choice of anchoring price, and that this choice acts as an essential driver of predicted incidence on the uses side. The fact that our model is extremely stylized suggests that even larger discrepancies could occur in richer models where the number of

[7] Our model assumes homothetic, but not necessarily Cobb-Douglas, consumer preferences.

candidate numeraire goods is larger.

In terms of policy implications, we note that many recent policy proposals for a U.S. carbon tax already choose to index the tax to inflation, albeit for reasons likely unrelated to the results discussed here (Marron et al., 2015; Aldy, 2016; Vail and Burtraw, 2016; Metcalf, 2018; Climate Leadership Council, 2019; H.R. 763, 2019; H.R. 3966, 2019; H.R. 4058, 2019; S. 1128, 2019; S. 2368, 2018; S. 2284, 2019). Furthermore, carbon taxes indexed to inflation have already been implemented in a number of countries, including Denmark and Sweden (Withana et al., 2013), Iceland (PMR, 2017), Norway (Haites et al., 2018), Chile, Colombia, and Mexico (UN, 2019), the Netherlands (Andersen et al., 2007), South Africa (Act No. 15, 2019), and the United Kingdom (National Audit Office, 2007).[8] The arguments laid out in the present paper serve to strengthen the case for such indexation. Our simulation results further suggest that accurately modeling the effects of such policies in general equilibrium requires special attention to the choice of anchoring price.

Although our general equilibrium analysis is of direct relevance to environmental policy, our findings on the comparative statics of tax incidence do not rely on any assumption regarding the actual external effects of the "dirty" input on technology or consumer utility; all that is required for our results is that this input be a non-market good available to the economy in sufficiently large quantity. As such, the framework conceptually applies to other settings, for instance per-period restrictions on the extraction of a natural resource for the purposes of conservation or sustainability.

# 2 Model and notation

We borrow the assumptions of Fullerton and Heutel (2007a), which build upon the seminal model of Harberger (1962). The model is parsimonious and aims to capture the essential drivers and overall magnitude of environmental tax incidence, rather than provide accurate predictions for a given economy, as would a more detailed computable general equilibrium approach. A notable advantage is that it can be solved analytically, providing a "model of the model" (Fullerton and Heutel, 2010a). Relatedly, Fullerton and Ta (2019) demonstrate that a stylized and analytically solvable model of the US economy delivers quantitative predictions on the effects of a carbon tax that are not far from those obtained from the detailed CGE model developed by Goulder and Hafstead (2018), and can help in understanding the drivers of these effects.

There are two economic sectors, $X$ and $Y$, that use labor ($L$) and capital ($K$) as inputs.

[8] California and Québec operate a cap-and-trade program rather than a carbon tax. Yet, the auction price floor and price ceiling are both indexed to inflation (ICAP, 2019b,a).

The dirty sector ($Y$) also uses a dirty input ($Z$, pollution). In each sector, production displays constant returns to scale.[9] The economy is endowed with fixed quantities of labor and capital allocated across the two sectors, and with an unspecified quantity of dirty input. The use of the dirty input by sector $Y$ causes pollution, which a tax on input $Z$ aims to reduce.

The model allows for pre-existing *ad valorem* taxes on all goods, but since the focus of our discussion is on environmental taxes (the tax on input $Z$), we follow Fullerton and Heutel (2007a) and ignore them in the discussion. That is, the only exogenous change is a rise in the tax on the dirty input, and all other taxes are assumed to remain constant. As in Harberger (1962), government is assumed to use additional tax revenue to purchase the two goods in the same proportion as would households under the initial prices. A consequence of this assumption is that the change in the relative aggregate demands for consumption goods only depends on the change in their relative prices. An alternative set of assumptions would be that tax revenue is redistributed to households, who have identical homothetic preferences.

We denote by $L_X$ (resp. $L_Y$) the quantity of labor employed in sector $X$ (resp. sector $Y$), $K_X$ (resp. $K_Y$) the quantity of capital employed in sector $X$ (resp.$Y$), $p_X$ (resp. $p_Y$) the price of good $X$ (resp. good $Y$), $w$ (resp. $r$) the price of labor (resp. capital), and $p_Z$ the price of $Z$ (that is, the tax per unit of $Z$). Small relative changes in equilibrium variables are denoted with a "hat." For instance, $\hat{p}_Y \equiv \frac{dp_Y}{p_Y}$.

The model is solved by differentiating equilibrium conditions pertaining to production, consumption, and resource availability, yielding the following set of linear equations:

$$\hat{L}_X + \gamma_L \hat{L}_Y = 0 \tag{1}$$

$$\hat{K}_X + \gamma_K \hat{K}_Y = 0 \tag{2}$$

$$\hat{X} - \theta_{XL}\hat{L}_X - \theta_{XK}\hat{K}_X = 0 \tag{3}$$

$$\hat{Y} - \theta_{YL}\hat{L}_Y - \theta_{YK}\hat{K}_Y - \theta_{YZ}\hat{Z} = 0 \tag{4}$$

$$\hat{p}_X - \theta_{XL}\hat{w} - \theta_{XK}\hat{r} = 0 \tag{5}$$

$$\hat{p}_Y - \theta_{YL}\hat{w} - \theta_{YK}\hat{r} = \theta_{YZ}\hat{p}_Z \tag{6}$$

$$\hat{L}_X - \hat{K}_X + \sigma_X \hat{w} - \sigma_X \hat{r} = 0 \tag{7}$$

$$\hat{L}_Y - \hat{Z} - \theta_{YL}(e_{LL} - e_{LZ})\hat{w} - \theta_{YK}(e_{LK} - e_{KZ})\hat{r} = \theta_{YZ}(e_{LZ} - e_{ZZ})\hat{p}_Z \tag{8}$$

$$\hat{K}_Y - \hat{Z} - \theta_{YL}(e_{LK} - e_{LZ})\hat{w} - \theta_{YK}(e_{KK} - e_{KZ})\hat{r} = \theta_{YZ}(e_{KZ} - e_{ZZ})\hat{p}_Z \tag{9}$$

$$\hat{X} - \hat{Y} + \sigma_u \hat{p}_X - \sigma_u \hat{p}_Y = 0 \tag{10}$$

[9] In sector $Y$, the constant returns to scale are with respect to all three inputs $L$, $K$, and $Z$.

where $\gamma_L \equiv \frac{L_Y}{L_X} > 0$ and $\gamma_K \equiv \frac{K_Y}{K_X} > 0$ denote the resources allocated to sector $Y$ relative to sector $X$, $\theta_{XL} \equiv \frac{wL_X}{p_X X}$ denotes the cost share of labor in sector $X$ (and similarly for the parameters $\theta_{XK}$, $\theta_{YL}$, $\theta_{YK}$, and $\theta_{YZ}$), $\sigma_u \geq 0$ is the elasticity of substitution in consumption between $X$ and $Y$, $\sigma_X \geq 0$ is the elasticity of substitution between labor and capital in sector $X$, and the parameters $e_{ij}$, for $i, j \in \{L, K, Z\}$, represent Allen elasticities of substitution defined as $e_{ij} = \theta_{Yj} a_{ij}$, where $a_{ij}$ is the conditional input demand elasticity for input $i$ with respect to the price of input $j$ in sector $Y$. Microeconomic theory places restrictions on the acceptable values of the $\theta_{Xj}$, $\theta_{Yj}$, and $e_{ij}$ parameters that are described in Appendix A.1. Given an exogenous change $\hat{p}_Z$, this system has 10 equations for 11 unknowns. Choosing a numeraire good adds the missing relationship, but changing the numeraire also changes the nature of the tax increase and therefore the variety of environmental policy considered. In the next section, we express the tax on pollution, $\hat{p}_Z$, relative to an explicit price index, thereby eliminating the dependency of comparative static results on numeraire choice.

# 3 Anchoring the environmental tax to a price index

Consider the following Cobb-Douglas price index:

$$\mathcal{P} = p_X^{\alpha_X} p_Y^{\alpha_Y} w^{\alpha_L} r^{\alpha_K}$$

with positive exponents and $\alpha_X + \alpha_Y + \alpha_L + \alpha_K = 1$ due to homogeneity of degree one. (Since prices are determined up to a multiplicative constant, the price index so defined is also determined up to the same multiplicative constant.) The relative change in $\mathcal{P}$ can then be expressed as

$$\hat{\mathcal{P}} = \alpha_X \hat{p}_X + \alpha_Y \hat{p}_Y + \alpha_L \hat{w} + \alpha_K \hat{r}.$$

If the price index $\mathcal{P}$ is used to anchor the pollution tax, then $p_Z = \mathcal{P}\tau_Z$, where $\tau_Z$ is now interpretable as an *ad valorem* tax relative to $\mathcal{P}$, and therefore

$$\hat{p}_Z = \hat{\mathcal{P}} + \hat{\tau}_Z.$$

Note that due to the equilibrium relationship $\hat{p}_X = \theta_{XL}\hat{w} + \theta_{XK}\hat{r}$, including $p_X$ in the price index $\mathcal{P}$ is redundant. That is, any change in the weight on $p_X$ can be exactly offset by changes in the weights on $w$ and $r$, leaving $\hat{\mathcal{P}}$ unchanged. This is not the case for $p_Y$ because $\hat{p}_Y = \theta_{YL}\hat{w} + \theta_{YK}\hat{r} + \theta_{YZ}\hat{\tau}_Z$, that is, unlike $\hat{p}_X$, $\hat{p}_Y$ implicitly includes $\hat{\tau}_Z$ independently of $\hat{w}$ and $\hat{r}$. Without loss of generality, we can therefore focus on price

indices of the form

$$\mathcal{P} = p_Y^{\beta} \left(w^{\alpha} r^{1-\alpha}\right)^{1-\beta} \tag{11}$$

where $0 \leq \alpha, \beta \leq 1$. If $\beta = 0$ then the index reflects an average of factor prices, that is, the price of a combined labor-capital input. If $\alpha = \theta_{XL}$, then due to Equation (5) the index reflects an average of prices of consumption goods. Note that our specification of the price index includes as a special case the normalization made by Fullerton and Heutel (2007a), that is, $\mathcal{P} = p_X$ ($\beta = 0$, $\alpha = \theta_{XL}$).[10] It also includes as special cases the choices of $p_Y$ ($\beta = 1$), $w$ ($\beta = 0$, $\alpha = 1$), or $r$ ($\beta = 0$, $\alpha = 0$) as alternative numeraire goods.

In what follows, we focus on price indices that reflect prices either on the uses side ($\alpha = \theta_{XL}$) or the sources side ($\beta = 0$). Although we could analyze each case separately, the price index in Equation (11) allows us to handle both cases within a single framework. We also focus on price indices that can be constructed from the observation of an initial equilibrium allocation and do not require knowledge of substitution elasticities, either in production or consumption. The idea is that since the anchoring price index corresponds to a particular policy choice, it is desirable for implementability to restrict the search to indices that can be designed with readily available economic information.

Note that the restriction to price indices of the Cobb-Douglas form is made here without loss of generality. Consider for instance a CES price index on the uses side

$$\mathcal{P} = \left[\beta p_X^{\frac{\sigma-1}{\sigma}} + (1-\beta) p_Y^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

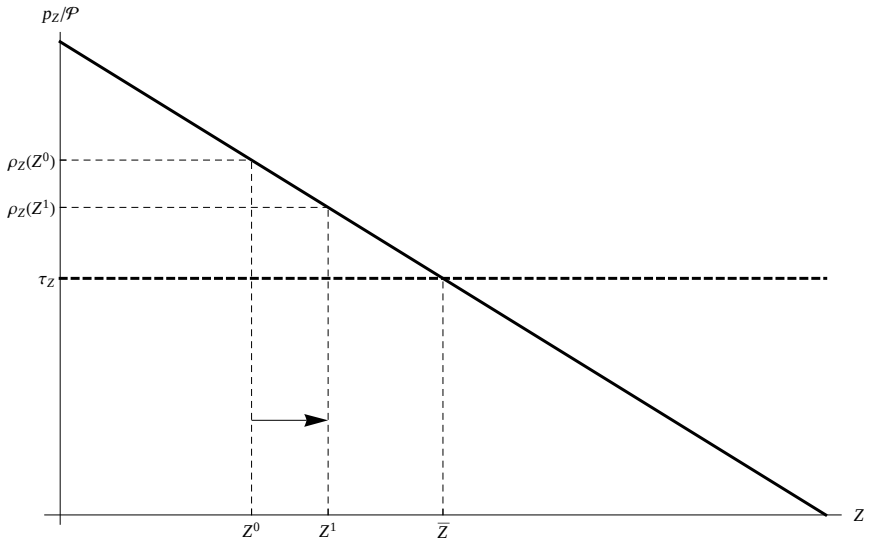
where $0 \leq \beta \leq 1$ and $\sigma \geq 0$. It is easy to show that

$$\hat{\mathcal{P}} = B\hat{p}_X + (1-B)\hat{p}_Y$$

where $B = \frac{\beta p_X^{\frac{\sigma-1}{\sigma}}}{\beta p_X^{\frac{\sigma-1}{\sigma}} + (1-\beta) p_Y^{\frac{\sigma-1}{\sigma}}}$. Therefore, a CES index would not meaningfully expand the set of acceptable indices, as the relative change in the value of the CES index is still reducible to a convex combination of relative price changes. As such, any restriction on the CES parameters $\beta$ and $\sigma$ to ensure a downward-sloping demand for the polluting input would necessarily be channelled through the Cobb-Douglas share $B$.

[10] Fullerton and Heutel (2007a) choose the normalization $\hat{p}_X = 0$ while setting $p_Z = \tau_Z$ (that is, $\tau_Z$ is a nominal tax), which given the equilibrium condition (5) is equivalent to setting $\beta = 0$ and $\alpha = \theta_{XL}$.

Figure 1: The Marshallian adjustment process in the market for the dirty input




## 3.1 Stability of the competitive equilibrium with pollution tax

Here we use the concept of Marshallian stability, as explained for instance in Samuelson (1941) or Plott and George (1992). Marshallian stability mandates that quantity adjusts to the difference between the demand and the supply prices of a good.[11] As explained in Samuelson (1941), "the problem of stability of equilibrium is intimately tied up with the problem of deriving fruitful theorems in comparative statics."

We consider the following Marshallian adjustment process: if at any point in time $t$, the quantity of polluting input $Z_t$ is such that the demand price for that quantity (expressed relative to the price index $\mathcal{P}$) exceeds the supply price (that is, the pollution tax $\tau_Z$), then the quantity must increase. The opposite holds in the case when the demand price is less than the supply price. Figure 1 shows a graphical representation of such adjustment process in the normal case of a downward-sloping derived demand, assuming that in the initial condition the quantity is less than the equilibrium value. Because the derived demand for the input slopes down, the equilibrium is stable: the adjustment process brings quantity closer to its equilibrium value. Had the derived demand sloped up, the equilibrium would have been Marshallian unstable.

A simple mathematical representation of the Marshallian process posits the following

[11] In contrast, Walrasian stability mandates that price adjusts to excess demand, that is, the difference between quantity demanded and quantity supplied. In our model with horizontal supply of polluting input, the quantity of polluting input supplied is not a well-defined function of price, which precludes the use of Walrasian stability.

dynamic relationship:

$$\frac{dZ}{dt} = \rho_Z(Z) - \tau_Z$$

where $\rho_Z(Z)$ represents the inverse demand function for the polluting input, expressed relative to the price index $\mathcal{P}$. Following Samuelson (1941), we can use a first-order expansion of the function $\rho_Z(Z)$ around the equilibrium point $\bar{Z}$ to obtain

$$\frac{dZ}{dt} = \rho'_Z(\bar{Z})\left(Z - \bar{Z}\right)$$

where we have made use of $\rho_Z(\bar{Z}) = \tau_Z$. The solution to this differential equation, together with the initial condition $Z = Z^0$, is simply $Z(t) = \bar{Z} + (Z^0 - \bar{Z})e^{\rho'_Z(\bar{Z})t}$, implying that $\lim_{t\to\infty} Z(t) = \bar{Z}$ if and only if $\rho'_Z(\bar{Z}) \leq 0$, that is, the demand for pollution slopes down. This result establishes the fact that whenever the derived demand for pollution slopes up, the equilibrium cannot be stable in the Marshallian sense. In what follows, we characterize price indices $\mathcal{P}$ that ensure that the derived demand for pollution slopes down, that is, the equilibrium is stable in the Marshallian sense.

## 3.2 Desirable properties of a price index

We are looking for weights $\alpha, \beta$ on the price index in Equation (11) such that the following three conditions are satisfied:

**Condition 1** *The pollution demand elasticity $\frac{\hat{Z}}{\hat{\tau}_z}$ is defined for all parameter values (that is, the equilibrium system denominator is never zero and thus does not change sign).*

**Condition 2** *The weights $\alpha$ and $\beta$ only depend on the observable parameters $\gamma_K$, $\gamma_L$, $\theta_{XK}$, $\theta_{YK}$, and $\theta_{YL}$, or combinations thereof.*

**Condition 3** *The pollution tax has the intended effect for all parameter values, that is, $\frac{\hat{Z}}{\hat{\tau}_z} \leq 0$.*

## 3.3 Derivation of the price indices

In Appendix A.3, we show that Condition 1 and Condition 2 imply the following restriction:

$$\left[\beta\theta_{YK} + (1-\beta)(1-\alpha)\right]\theta_{XL}(1+\gamma_L) = \left[\beta\theta_{YL} + (1-\beta)\alpha\right]\theta_{XK}(1+\gamma_K), \tag{12}$$

while Condition 3 is automatically satisfied as long as Condition 1 is. Equation (12) is violated in Fullerton and Heutel (2007a)'s model where $\beta = 0$ and $\alpha = \theta_{XL}$, except in the special case of equal factor intensities in the two sectors ($\gamma_L = \gamma_K$).

We can now specialize the restriction in Equation (12) to the case of indices on the uses or sources side. On the uses side ($\alpha = \theta_{XL}$), Equation (12) implies that either $\gamma_L = \gamma_K$ or $\beta = \frac{p_Y Y}{p_X X + p_Y Y}$, that is, the desired price index is

$$\mathcal{P} = p_X^{\theta_X} p_Y^{\theta_Y} \tag{13}$$

where $\theta_X$ represents the initial expenditure share on good $X$ and $\theta_Y = 1 - \theta_X$. No other choice of weights will guarantee that the law of input demand holds for all parameter values. In particular, neither $p_X$ nor $p_Y$ as choices of numeraire (and thus anchoring indices) would allow one to rule out upward-sloping demand for the dirty input.[12]

On the sources side ($\beta = 0$), Equation (12) implies that $\alpha = \frac{\theta_{XL}(1+\gamma_L)}{\theta_{XL}(1+\gamma_L)+\theta_{XK}(1+\gamma_K)}$, that is, the price index is

$$\mathcal{R} = w^{\theta_L} r^{\theta_K} \tag{14}$$

where $\theta_L \equiv \frac{w\bar{L}}{w\bar{L}+r\bar{K}} = \frac{\theta_{XL}(1+\gamma_L)}{\theta_{XL}(1+\gamma_L)+\theta_{XK}(1+\gamma_K)}$ represents the initial contribution of labor to national income and $\theta_K = 1 - \theta_L$. Again, these weights are uniquely defined.[13]

Note that the derivation of Equation (12) provides a formal proof of our claim that the choice of numeraire matters when modeling a nominal tax on pollution. Here we model the tax as an *ad valorem* tax anchored to the price index $\mathcal{P}$, which is equivalent to modeling a nominal tax with $\mathcal{P}$ as the numeraire. Fullerton and Heutel (2007a) model a nominal tax with $p_X$ as the numeraire, which is equivalent to modeling an *ad valorem* tax anchored to the price of the clean good. The comparative statics with respect to $\hat{\tau}_Z$ are different across the two normalizations; in particular, the choice of $p_X$ as the anchoring price does not guarantee $\frac{\hat{Z}}{\hat{\tau}_Z} < 0$, whereas that of $\hat{\mathcal{P}}$, with our proposed weights, does. One may wonder whether the counterintuitive comparative static result would be possible in a model with only one clean input, say labor. We show in Appendix B that in that case, the derived demand for the polluting input slopes down for all anchoring price indices, and all parameter values.

[12]It is clear from the analysis in Appendix A.3 that whenever the law of derived input demand is violated, the determinant of the equilibrium system has the "wrong" sign. As a result, all comparative static results, not just the effect on pollution, are reversed relative to the normal case.

[13]More generally, if one is willing to consider price indices that involve prices on both the uses and sources side (that is, $p_X$, $p_Y$, $w$, and $r$), it can be shown that any price index of the form $\mathcal{P}^{\phi}\mathcal{R}^{1-\phi}$, with $\phi \in [0, 1]$, will also satisfy Equation (12).

### 3.4 Equivalence between our price indices and inflation indices

Inflation is typically measured using a price index that reflects the overall cost of a reference basket of goods under varying prices. Consider that the reference basket is the consumption basket before the change in the pollution tax. Our preferred pollution tax is anchored to the price index $\mathcal{P} = p_X^{\theta_X} p_Y^{\theta_Y}$ where $\theta_X$ and $\theta_Y$ are the reference budget shares. When prices change in the economy, the change in our price index is:

$$\begin{aligned}
\hat{\mathcal{P}} &= \theta_X \hat{p}_X + \theta_Y \hat{p}_Y \\
&= \frac{p_X X}{p_X X + p_Y Y} \frac{dp_X}{p_X} + \frac{p_Y Y}{p_X X + p_Y Y} \frac{dp_Y}{p_Y} \\
&= \frac{X dp_X + Y dp_Y}{p_X X + p_Y Y} \\
&= \frac{d\text{CPI}}{\text{CPI}} \\
&= \widehat{\text{CPI}}
\end{aligned}$$

where $\text{CPI} \equiv p_X X + p_Y Y$ represents the value of the reference basket. Therefore, expressing the pollution tax as an *ad valorem* tax anchored to $\mathcal{P}$ is equivalent to adjusting a nominal tax on pollution for inflation as measured by the CPI. In fact, many recent proposals for a carbon tax in the U.S. propose secular adjustments to the per-unit tax to account for inflation (H.R. 763, 2019; H.R. 3966, 2019; H.R. 4058, 2019; S. 1128, 2019; S. 2284, 2019).[14]

A similar argument can be used to demonstrate that our price index on the sources side represents a producer price index, where the weights on factor prices correspond to the reference shares of each primary factor in national income.

## 4 Simple tax incidence

### 4.1 Sources side

We show in Appendix A.4 that, whenever Equation (12) holds,

$$\text{sign}\left\{\frac{\hat{w} - \hat{r}}{\hat{\tau}_Z}\right\} = \text{sign}\left\{\sigma_u(\gamma_K - \gamma_L) + \gamma_L(1 + \gamma_K)e_{LZ} - \gamma_K(1 + \gamma_L)e_{KZ}\right\}.$$

[14]Such adjustments would be on top of any ramping up of the tax meant to increase pollution reduction incentives over time.

Which sector bears proportionately more of the tax burden therefore depends on the consumption elasticity $\sigma_u$, the resource allocation parameters $\gamma_L$ and $\gamma_K$, and the partial substitution elasticities $e_{LZ}$ and $e_{KZ}$, but not on the cost shares in either sector or the substitution elasticity in the untaxed sector.[15]

These few parameters affect the direction of the change in the price of labor relative to capital in an intuitive yet subtle way. If labor and capital are equally substitutable for pollution ($e_{LZ} = e_{KZ} > 0$), then the pollution tax places a disproportionate burden on capital (i.e., $\hat{w} - \hat{r} > 0$) when the polluting sector is capital intensive ($\gamma_K > \gamma_L$) as long as goods are sufficiently substitutable in consumption (i.e., $\sigma_u > e_{LZ} = e_{KZ}$). If not ($\sigma_u < e_{LZ}$), then the input substitution effect dominates and the demand for capital in the capital-intensive polluting sector rises so that the return to capital increases relative to the wage rate. If both sectors are equally capital-intensive (i.e., $\gamma_L = \gamma_K$), then capital bears disproportionately more of the tax burden if and only if it is less substitutable for the polluting input than is labor.

These economic insights were already discussed, albeit for specific classes of model parameterizations, in the original contribution of Fullerton and Heutel (2007a). Indeed, the authors were prevented from drawing general conclusions by the fact that they could not sign the main equilibrium system denominator except in special cases, which as we have shown is a direct consequence of modeling the pollution tax increase relative to the price of good $X$.

## 4.2 Uses side

On the uses side, we show in Appendix A.5 that for our choices of anchoring price indices,

$$\begin{aligned}\text{sign}\left\{\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z}\right\} \;=\; &\text{sign}\left\{\sigma_X(1+\gamma_L\theta_{XL}+\gamma_K\theta_{XK}) + \left(\theta_{YL}\gamma_K(1+\gamma_L)+\theta_{YK}\gamma_L(1+\gamma_K)\right)e_{LK}\right.\\ &\left.+\gamma_L(1+\gamma_K)(\theta_{XK}-\theta_{YK})e_{LZ}+\gamma_K(1+\gamma_L)(\theta_{XL}-\theta_{YL})e_{KZ}\right\}.\end{aligned}$$

This expression generally has an ambiguous sign, although we show that in the case of equal factor intensities ($\gamma_L = \gamma_K$), $\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z} > 0$, that is, users of $Y$ share proportionately more tax burden than users of $X$ irrespective of the values of the substitution elasticities.

[15]The size of $\frac{\hat{w}-\hat{r}}{\hat{\tau}_Z}$ itself depends on the full set of model parameters and the choice of anchoring index, see Section 5 and Appendix C.

Table 1: Model parameters

| Parameter | Value |
|---|---|
| $e_{LK}$ | 1 |
| $\sigma_u$ | 1 |
| $\sigma_X$ | 1 |
| $\gamma_L$ | 0.25 |
| $\gamma_K$ | 0.25 |
| $\theta_{XL}$ | 0.60 |
| $\theta_{YL}$ | 0.45 |
| $\theta_{YK}$ | 0.30 |

Note: Once we have set $\gamma_L$, $\gamma_K$, $\theta_{YL}$, and $\theta_{YK}$, $\theta_{XL}$ is determined by $\theta_{XL} = \frac{\gamma_K \theta_{YL}}{\gamma_K \theta_{YL} + \gamma_L \theta_{YK}}$.

# 5 Does the choice of numeraire matter in practice?

In the previous sections, we have formally shown that comparative static results for a nominal pollution tax depend, from an analytical standpoint, on the choice of numeraire. Appendix A.6 also provides an example whereby using $p_X$ as numeraire leads to the prediction that an increase in the pollution tax increases pollution, whereas the use of our anchoring indices would lead to a decrease in pollution. Importantly, the signs of the predicted tax incidence on the sources and uses sides would also be inconsistent for that parameterization between the two choices of numeraire (i.e., $p_X$ versus our proposed price indices). Intuitively, cases whereby a rise in the pollution tax relative to $p_X$ (or any other price) lead to reversed comparative statics precisely correspond to cases whereby the value of the pollution tax actually decreases relative to our proposed price indices.

Here, we investigate whether the choice of anchoring price index (or, for a nominal tax, the choice of numeraire) matters in practice, that is, for reasonable model parameterizations reflecting existing economies. We use the model parameterizations of Fullerton and Heutel (2007a) for the US economy, which is close to that used in Fullerton and Heutel (2010b). In these papers, the polluting sector is defined by selecting polluting industries based on the EPA's Toxic Release Inventory for 2002. We do not limit our analysis to a comparison of our price indices $\mathcal{P}$ and $\mathcal{R}$ to theirs ($p_X$). Instead, we broaden the scope of the analysis by also considering the following indices: $p_Y$, $w$, $r$. Other indices could be defined, however we believe that the set of chosen indices affords sufficient insights into the empirical question. Baseline parameter values are given in Table 1. All parameter values are fixed, except for the Allen cross-price elasticities $e_{KZ}$ and $e_{LZ}$ which are allowed to take on the values $\{-0.5, 0.0, 0.5, 1.0\}$. We exclude pairs of elasticities that lead to violations of

Table 2: Pollution effect of a 10% increase in the nominal pollution tax (%)

| $e_{KZ}$ | $e_{LZ}$ | $\hat{Z}$ | | | | |
|---|---|---|---|---|---|---|
| | | $p_X$ ($\mathcal{R}$) | $p_Y$ | $\mathcal{P}$ | $w$ | $r$ |
| 0.0 | 0.0 | -2.00 | -2.67 | -2.13 | -2.00 | -2.00 |
| 0.5 | 0.0 | -3.58 | -4.77 | -3.81 | -3.54 | -3.63 |
| 1.0 | 0.0 | -5.10 | -6.80 | -5.44 | -5.00 | -5.26 |
| -0.5 | 0.5 | -2.70 | -3.60 | -2.88 | -2.76 | -2.62 |
| 0.0 | 0.5 | -4.38 | -5.83 | -4.67 | -4.42 | -4.31 |
| 0.5 | 0.5 | -6.00 | -8.00 | -6.40 | -6.00 | -6.00 |
| 1.0 | 0.5 | -7.58 | -10.10 | -8.08 | -7.50 | -7.69 |
| -0.5 | 1.0 | -4.97 | -6.63 | -5.31 | -5.14 | -4.75 |
| 0.0 | 1.0 | -6.70 | -8.93 | -7.15 | -6.84 | -6.50 |
| 0.5 | 1.0 | -8.38 | -11.17 | -8.93 | -8.46 | -8.25 |
| 1.0 | 1.0 | -10.00 | -13.33 | -10.67 | -10.00 | -10.00 |

Note: Since $\gamma_L = \gamma_K$, $\theta_{XL} = \theta_L$ and therefore the index $p_X$ leads to the same results as the index $\mathcal{R}$.

the negative semidefiniteness of the Slutsky matrix in sector $Y$. Because it is assumed that $\gamma_L = \gamma_K$, $\theta_{XL} = \theta_L$ and therefore the index $p_X$ leads to the exact same results as the index $\mathcal{R}$. This assumption is relaxed in additional simulations reported in Appendix C.1.

Results in Table 2 show that the choice of numeraire matters for the calculation of the predicted effect of the nominal tax on pollution, even if there is no reversal in sign. The largest discrepancies are found when comparing the use of the wage rate or the rental on capital to that of the dirty good (good $Y$) as numeraires. In such comparisons, predicted pollution effects ($\hat{Z}$) differ by up to about 40%, and the choice of numeraire leads to variations in predictions that are often comparable to those arising from alternative sets of cross-price elasticities. Predictions using the price index $\mathcal{P}$ are closer to those obtained using $p_X$.

Effects on incidence mirror those on pollution effects, although the size of the effects themselves is smaller. Still, for some model parameterizations, incidence is shown to differ qualitatively between a model that uses $w$ or $r$ as the numeraire and one that uses $p_Y$ as the numeraire. For instance, for $e_{KZ} = -0.5$ and $e_{LZ} = 1.0$, using $r$ as the numeraire yields a predicted increase in $\frac{w}{r}$ (resp. $\frac{p_Y}{p_X}$) of 0.75% (resp. 2.39%), versus an increase of 1.05% (resp. 3.33%) when using $p_Y$. On the uses side, the largest source of variation in predicted effects is the choice of numeraire, not the choice of Allen cross-price substitution elasticities.

In Appendix C.1, we report additional results that hold constant all Allen substitution elasticities but allow factor intensities, as captured by $\gamma_K - \gamma_L$, to vary. Again we follow

Table 3: Incidence effects of a 10% increase in the nominal pollution tax (%)

| $e_{KZ}$ | $e_{LZ}$ | Sources side: $\hat{w} - \hat{r}$ | | | | | Uses side: $\hat{p}_Y - \hat{p}_X$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $p_X$ ($\mathcal{R}$) | $p_Y$ | $\mathcal{P}$ | $w$ | $r$ | $p_X$ ($\mathcal{R}$) | $p_Y$ | $\mathcal{P}$ | $w$ | $r$ |
| 0.0 | 0.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.50 | 3.33 | 2.67 | 2.50 | 2.50 |
| 0.5 | 0.0 | -0.26 | -0.35 | -0.28 | -0.26 | -0.26 | 2.50 | 3.33 | 2.67 | 2.47 | 2.54 |
| 1.0 | 0.0 | -0.51 | -0.68 | -0.54 | -0.50 | -0.53 | 2.50 | 3.33 | 2.67 | 2.45 | 2.58 |
| -0.5 | 0.5 | 0.53 | 0.71 | 0.56 | 0.54 | 0.51 | 2.50 | 3.33 | 2.67 | 2.55 | 2.42 |
| 0.0 | 0.5 | 0.26 | 0.35 | 0.28 | 0.26 | 0.26 | 2.50 | 3.33 | 2.67 | 2.53 | 2.46 |
| 0.5 | 0.5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.50 | 3.33 | 2.67 | 2.50 | 2.50 |
| 1.0 | 0.5 | -0.25 | -0.34 | -0.27 | -0.25 | -0.26 | 2.50 | 3.33 | 2.67 | 2.48 | 2.54 |
| -0.5 | 1.0 | 0.79 | 1.05 | 0.84 | 0.81 | 0.75 | 2.50 | 3.33 | 2.67 | 2.58 | 2.39 |
| 0.0 | 1.0 | 0.52 | 0.69 | 0.55 | 0.53 | 0.50 | 2.50 | 3.33 | 2.67 | 2.55 | 2.43 |
| 0.5 | 1.0 | 0.25 | 0.34 | 0.27 | 0.26 | 0.25 | 2.50 | 3.33 | 2.67 | 2.53 | 2.46 |
| 1.0 | 1.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.50 | 3.33 | 2.67 | 2.50 | 2.50 |

Note: Since $\gamma_L = \gamma_K$, $\theta_{XL} = \theta_L$ and therefore the index $p_X$ leads to the same results as the index $\mathcal{R}$.

the model parameterizations investigated in Fullerton and Heutel (2007a). Our results confirm the importance of the choice of numeraire for predictions on pollution outcomes and tax incidence. Notably, the variation in predictions induced by numeraire choice is not dwarfed by that arising from the choice of factor intensities, and even exceeds it in the case of incidence on the uses side.

Finally, in Appendix C.2 we report results for a calibration of the model relevant for a US carbon tax and adapted from Fullerton and Ta (2019). This calibration leads to discrepancies across anchoring indices even larger than those reported above. Specifically, because the dirty sector is relatively small in terms of its use of labor and capital, and its expenditure share on the polluting input is substantial, using $p_Y$ as the anchoring index gives results very different from those obtained from the alternative indices.

# 6 Conclusion

This paper argues that numeraire choice can matter for assessing the general equilibrium effects of environmental policy on pollution levels and relative prices, both analytically and numerically. The reason is that pollution is a nonmarket good, making a pollution tax a nominal tax with no direct *ad valorem* equivalent. Anchoring the pollution tax to the equilibrium prices of other goods, as we have done here, renders the choice of numeraire innocuous, but comparative statics then depend on the choice of anchoring price, a choice that reflects different varieties of environmental policy. We have shown that if one anchors

the pollution tax to inflation, then pollution always decreases with an increase in the tax rate, the resulting equilibrium is always Marshallian stable, and tax incidence results that have previously been derived for specific model parameterizations in fact hold quite generally. Our analysis further implies that analytical and numerical models aimed at capturing the general equilibrium effects of environmental taxes that are actually indexed on inflation should use the relevant inflation index as numeraire, or explicitly anchor these taxes to inflation, so that comparative static results have the correct sign, and the correct magnitude. The fact that even in the pared-down model we use, the pollution and incidence outcomes of nominal taxes depend critically on the numeraire suggests that the same would be true in larger, more detailed models of the economy with a large set of candidate numeraire goods.

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# Appendices

## A Proofs and derivations

### A.1 Useful equalities and inequalities implied by theory

Constant returns to scale in sectors $X$ and $Y$ imply that $\theta_{XL} + \theta_{XK} = 1$ and $\theta_{YL} + \theta_{YK} + \theta_{YZ} = 1$. Homogeneity of degree one of the conditional input demands implies that $\theta_{YL}e_{mL} + \theta_{YK}e_{mK} + \theta_{YZ}e_{mZ} = 0$, for $m \in \{L, K, Z\}$.

Concavity of the indirect cost function and Shephard's lemma imply that the Jacobian matrix of the conditional input demands is symmetric and negative semidefinite. These properties translate to the matrix of Allen substitution elasticities, defined as

$$\mathbf{E} \equiv \begin{pmatrix} e_{LL} & e_{LK} & e_{LZ} \\ e_{KL} & e_{KK} & e_{KZ} \\ e_{ZL} & e_{ZK} & e_{ZZ} \end{pmatrix}.$$

In particular, $e_{LL}, e_{KK}, e_{ZZ} \leq 0$, $e_{LL}e_{KK} - e_{LK}^2 \geq 0$, and similarly for other input pairs. In addition, $\mathbf{v'Ev} \leq 0$ for any $3 \times 1$ vector $\mathbf{v}$. This property also implies that $\mathbf{v'E}_{LK}\mathbf{v} \leq 0$ for any $2 \times 1$ vector $\mathbf{v}$, where

$$\mathbf{E}_{LK} \equiv \begin{pmatrix} e_{LL} & e_{LK} \\ e_{KL} & e_{KK} \end{pmatrix}.$$

Note that the Allen substitution matrix is singular, and that any triplet of values of substitution elasticities determine the remaining ones through homogeneity. Relatedly, the fact that $\mathbf{E}_{LK}$ be negative semidefinite suffices to ensure that the entire matrix $\mathbf{E}$ be negative semidefinite.

Simple algebra implies that $\frac{\theta_{XL}\gamma_L}{\theta_{YL}} = \frac{\theta_{XK}\gamma_K}{\theta_{YK}} = \frac{p_Y Y}{p_X X}$ and we will denote this ratio as $\delta$. Further note that $\delta(\theta_{YL}\gamma_K + \theta_{YK}\gamma_L) = \gamma_L\gamma_K$.

### A.2 Equilibrium displacement

Once a price index satisfying one-homogeneity, to which the pollution tax can be anchored through the relation $\hat{p}_Z = \hat{\mathcal{P}} + \hat{\tau}_Z$, has been defined, the choice of which price to use as a reference indeed becomes innocuous. That is, identical results on goods provision, resource allocation, and *relative* prices will be obtained whether it is assumed that $\hat{p}_X = 0$, $\hat{w} = 0$, etc. Here we choose to set $\hat{\mathcal{P}} = 0$, but our results would be identical if we chose

any other price normalization.

Using the normalization $\hat{\mathcal{P}} = 0$, the system of equations describing the change in equilibrium values can be written as:

$$\begin{aligned}
\hat{L}_X + \gamma_L \hat{L}_Y &= 0 && \text{(A-1)}\\
\hat{K}_X + \gamma_K \hat{K}_Y &= 0 && \text{(A-2)}\\
\hat{X} - \theta_{XL}\hat{L}_X - \theta_{XK}\hat{K}_X &= 0 && \text{(A-3)}\\
\hat{Y} - \theta_{YL}\hat{L}_Y - \theta_{YK}\hat{K}_Y - \theta_{YZ}\hat{Z} &= 0 && \text{(A-4)}\\
\hat{p}_X - \theta_{XL}\hat{w} - \theta_{XK}\hat{r} &= 0 && \text{(A-5)}\\
\hat{p}_Y - \theta_{YL}\hat{w} - \theta_{YK}\hat{r} &= \theta_{YZ}\hat{\tau}_Z && \text{(A-6)}\\
\hat{L}_X - \hat{K}_X + \sigma_X\hat{w} - \sigma_X\hat{r} &= 0 && \text{(A-7)}\\
\hat{L}_Y - \hat{Z} - \theta_{YL}(e_{LL} - e_{LZ})\hat{w} - \theta_{YK}(e_{LK} - e_{KZ})\hat{r} &= \theta_{YZ}(e_{LZ} - e_{ZZ})\hat{\tau}_Z && \text{(A-8)}\\
\hat{K}_Y - \hat{Z} - \theta_{YL}(e_{LK} - e_{LZ})\hat{w} - \theta_{YK}(e_{KK} - e_{KZ})\hat{r} &= \theta_{YZ}(e_{KZ} - e_{ZZ})\hat{\tau}_Z && \text{(A-9)}\\
\hat{X} - \hat{Y} + \sigma_u\hat{p}_X - \sigma_u\hat{p}_Y &= 0 && \text{(A-10)}\\
\beta\hat{p}_Y + (1-\beta)\alpha\hat{w} + (1-\beta)(1-\alpha)\hat{r} &= 0 && \text{(A-11)}
\end{aligned}$$

which is a linear system that includes 11 equations in 11 unknowns. We are particularly interested in deriving the effect $\frac{\hat{Z}}{\hat{\tau}_z}$ (in order to ensure that Condition 1 holds) and the effects $\frac{\hat{w}}{\hat{\tau}_z}$ and $\frac{\hat{r}}{\hat{\tau}_z}$ (to derive the incidence of the tax increase).

## A.3 Derivation of the pollution effect $\frac{\hat{Z}}{\hat{\tau}_Z}$

Condition 1 mandates that the comparative static $\frac{\hat{Z}}{\hat{\tau}_z}$ be defined for all parameter values. Given that the system describing equilibrium displacement is linear, this implies that the determinant of the system matrix be nonzero, which, by continuity, implies that it must be of a determinate sign. We will show that this requirement is met by imposing a simple functional restriction on the weights of the price index.

Although we could compute the determinant of the system's matrix directly, following Fullerton and Heutel (2007a) it is easier to proceed by substitution to reduce the dimensionality of the problem. Equations (A-1) and (A-2) imply that $\hat{L}_X = -\gamma_L\hat{L}_Y$ and $\hat{K}_X = -\gamma_K\hat{K}_Y$. Using (A-3), we then have $\hat{X} = -\theta_{XL}\gamma_L\hat{L}_Y - \theta_{XK}\gamma_K\hat{K}_Y$ which, together with (A-4), implies that $\hat{X} - \hat{Y} = -(\theta_{XL}\gamma_L + \theta_{YL})\hat{L}_Y - (\gamma_K\theta_{XK} + \theta_{YK})\hat{K}_Y - \theta_{YZ}\hat{Z}$. Using (A-10) and defining $\delta \equiv \frac{\theta_{XL}\gamma_L}{\theta_{YL}} = \frac{\theta_{XK}\gamma_K}{\theta_{YK}}$, this in turn implies that

$$\theta_{YL}(1+\delta)\hat{L}_Y + \theta_{YK}(1+\delta)\hat{K}_Y + \theta_{YZ}\hat{Z} = \sigma_u(\hat{p}_X - \hat{p}_Y). \qquad \text{(A-12)}$$

Using (A-5) and (A-6), we have

$$\hat{p}_X - \hat{p}_Y = (\theta_{XL} - \theta_{YL})\hat{w} + (\theta_{XK} - \theta_{YK})\hat{r} - \theta_{YZ}\hat{\tau}_Z$$

which together with (A-12) implies

$$\theta_{YL}(1+\delta)\hat{L}_Y + \theta_{YK}(1+\delta)\hat{K}_Y + \theta_{YZ}\hat{Z} + \sigma_u(\theta_{YL} - \theta_{XL})\hat{w} + \sigma_u(\theta_{YK} - \theta_{XK})\hat{r} = -\sigma_u\theta_{YZ}\hat{\tau}_Z. \quad \text{(A-13)}$$

One can then use (A-8) and (A-9) to eliminate $\hat{L}_Y$ and $\hat{K}_Y$ in Equation (A-13). To alleviate notation, define

$$\begin{aligned}
\bar{\theta}^1_{YL} &\equiv \theta_{YL}(e_{LL} - e_{LZ}) \\
\bar{\theta}^2_{YL} &\equiv \theta_{YL}(e_{LK} - e_{LZ}) \\
\bar{\theta}^1_{YK} &\equiv \theta_{YK}(e_{LK} - e_{KZ}) \\
\bar{\theta}^2_{YK} &\equiv \theta_{YK}(e_{KK} - e_{KZ}) \\
\bar{\theta}^1_{YZ} &\equiv \theta_{YZ}(e_{LZ} - e_{ZZ}) \\
\bar{\theta}^2_{YZ} &\equiv \theta_{YZ}(e_{KZ} - e_{ZZ})
\end{aligned}$$

After rearrangement, we obtain:

$$\begin{aligned}
&\left[\sigma_u(\theta_{YL} - \theta_{XL}) + (1+\delta)(\theta_{YL}\bar{\theta}^1_{YL} + \theta_{YK}\bar{\theta}^2_{YL})\right]\hat{w} + \left[\sigma_u(\theta_{YK} - \theta_{XK}) + (1+\delta)(\theta_{YL}\bar{\theta}^1_{YK} + \theta_{YK}\bar{\theta}^2_{YK})\right]\hat{r} \\
&\quad + \left[\theta_{YZ} + (1+\delta)(\theta_{YL} + \theta_{YK})\right]\hat{Z} = -\left[\sigma_u\theta_{YZ} + (1+\delta)(\theta_{YL}\bar{\theta}^1_{YZ} + \theta_{YK}\bar{\theta}^2_{YZ})\right]\hat{\tau}_Z \qquad \text{(A-14)}
\end{aligned}$$

Equation (A-7) implies that $-\gamma_L\hat{L}_Y + \gamma_K\hat{K}_Y + \sigma_X\hat{w} - \sigma_X\hat{r} = 0$, which after eliminating $\hat{L}_Y$ and $\hat{K}_Y$ becomes

$$\left[\sigma_X - \gamma_L\bar{\theta}^1_{YL} + \gamma_K\bar{\theta}^2_{YL}\right]\hat{w} + \left[-\sigma_X - \gamma_L\bar{\theta}^1_{YK} + \gamma_K\bar{\theta}^2_{YK}\right]\hat{r} + \left[\gamma_K - \gamma_L\right]\hat{Z} = \left[\gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ}\right]\hat{\tau}_Z. \quad \text{(A-15)}$$

Finally, (A-6) and (A-11) together imply that

$$\left[\beta\theta_{YL} + (1-\beta)\alpha\right]\hat{w} + \left[\beta\theta_{YK} + (1-\beta)(1-\alpha)\right]\hat{r} = -\beta\theta_{YZ}\hat{\tau}_Z. \quad \text{(A-16)}$$

Equations (A-14)-(A-16) constitute a linear system in the three unknowns $\hat{w}$, $\hat{r}$, and $\hat{Z}$. For the comparative static $\frac{\hat{Z}}{\hat{\tau}_Z}$ to be defined for all parameter values, the determinant of

this system should have a constant sign. We can write this determinant as

$$\Delta \equiv \begin{vmatrix} \sigma_u(\theta_{YL}-\theta_{XL})+(1+\delta)(\theta_{YL}\bar{\theta}^1_{YL}+\theta_{YK}\bar{\theta}^2_{YL}) & \sigma_u(\theta_{YK}-\theta_{XK})+(1+\delta)(\theta_{YL}\bar{\theta}^1_{YK}+\theta_{YK}\bar{\theta}^2_{YK}) & \theta_{YZ}+(1+\delta)(\theta_{YL}+\theta_{YK}) \\ \sigma_X-\gamma_L\bar{\theta}^1_{YL}+\gamma_K\bar{\theta}^2_{YL} & -\sigma_X-\gamma_L\bar{\theta}^1_{YK}+\gamma_K\bar{\theta}^2_{YK} & \gamma_K-\gamma_L \\ \beta\theta_{YL}+(1-\beta)\alpha & \beta\theta_{YK}+(1-\beta)(1-\alpha) & 0 \end{vmatrix}$$

and, developing along the third column, we get

$$\begin{aligned} \Delta &= [\theta_{YZ}+(1+\delta)(\theta_{YL}+\theta_{YK})]\left[\left(\sigma_X-\gamma_L\bar{\theta}^1_{YL}+\gamma_K\bar{\theta}^2_{YL}\right)B_K+\left(\sigma_X+\gamma_L\bar{\theta}^1_{YK}-\gamma_K\bar{\theta}^2_{YK}\right)B_L\right] \\ &\quad +(\gamma_L-\gamma_K)\left[\left(\sigma_u(\theta_{YL}-\theta_{XL})+(1+\delta)(\theta_{YL}\bar{\theta}^1_{YL}+\theta_{YK}\bar{\theta}^2_{YL})\right)B_K\right. \\ &\quad \left.-\left(\sigma_u(\theta_{YK}-\theta_{XK})+(1+\delta)(\theta_{YL}\bar{\theta}^1_{YK}+\theta_{YK}\bar{\theta}^2_{YK})\right)B_L\right] \end{aligned}$$

where we have defined $B_L \equiv \beta\theta_{YL}+(1-\beta)\alpha$ and $B_K \equiv \beta\theta_{YK}+(1-\beta)(1-\alpha)$.

Separating the terms in $\sigma_X$ and $\sigma_u$, we can write $\Delta = C_1\sigma_X + C_2\sigma_u + C_3$, where

$$\begin{aligned} C_1 &\equiv (B_L+B_K)\left(1+\delta(\theta_{YL}+\theta_{YK})\right) \\ C_2 &\equiv (\gamma_L-\gamma_K)\left[B_K(\theta_{YL}-\theta_{XL})-B_L(\theta_{YK}-\theta_{XK})\right] \end{aligned}$$

and

$$\begin{aligned} C_3 \equiv\ & B_L\left[(1+(1+\delta)(\theta_{YL}+\theta_{YK}))\left(\gamma_L\bar{\theta}^1_{YK}-\gamma_K\bar{\theta}^2_{YK}\right)-(1+\delta)(\gamma_L-\gamma_K)\left(\theta_{YL}\bar{\theta}^1_{YK}+\theta_{YK}\bar{\theta}^2_{YK}\right)\right] \\ & +B_K\left[(1+(1+\delta)(\theta_{YL}+\theta_{YK}))\left(-\gamma_L\bar{\theta}^1_{YL}+\gamma_K\bar{\theta}^2_{YL}\right)+(1+\delta)(\gamma_L-\gamma_K)\left(\theta_{YL}\bar{\theta}^1_{YL}+\theta_{YK}\bar{\theta}^2_{YL}\right)\right]. \end{aligned}$$

Using the definitions of the terms $\bar{\theta}^1_{YL}$, $\bar{\theta}^2_{YL}$, $\bar{\theta}^1_{YK}$, $\bar{\theta}^2_{YK}$, and using the properties $\theta_{YZ}e_{KZ} = -\theta_{YL}e_{LK}-\theta_{YK}e_{KK}$ and $\theta_{YZ}e_{LZ} = -\theta_{YL}e_{LL}-\theta_{YK}e_{LK}$, we obtain

$$C_3 = -e_{LL}B_K\theta_{YL}\gamma_L(1+\gamma_K)-e_{KK}B_L\theta_{YK}\gamma_K(1+\gamma_L)+e_{LK}\left[B_K\theta_{YL}\gamma_K(1+\gamma_L)+B_L\theta_{YK}\gamma_L(1+\gamma_K)\right].$$

It is clear that $C_1 > 0$, therefore for the sign of $\Delta$ to be invariant to the choice of parameter values, which is required for it to always be nonzero, we need both $C_2 \geq 0$ and $C_3 \geq 0$. Let us start by discussing the sign of $C_3$. The first two terms in $C_3$ are clearly non-negative given that $e_{LL} \leq 0$ and $e_{KK} \leq 0$. The symmetry and negative semidefiniteness of the submatrix $\mathbf{E}_{LK}$ implies that $-e_{LL}v_1^2 - e_{KK}v_2^2 + 2e_{LK}v_1v_2 \geq 0$ for all vectors $\mathbf{v} = (v_1, v_2)$. If we can write $C_3$ in this form for some well-chosen vector $\mathbf{v}$, we can then conclude that $C_3 \geq 0$ for all parameter values. A sufficient condition is that $B_K\theta_{YL}\gamma_K(1+\gamma_L) = B_L\theta_{YK}\gamma_L(1+\gamma_K)$, that is, using the fact that $\gamma_L\theta_{XL}\theta_{YK} = \gamma_K\theta_{XK}\theta_{YL}$,

$$B_K\theta_{XL}(1+\gamma_L) = B_L\theta_{XK}(1+\gamma_K) \tag{A-17}$$

which implies that

$$C_3 = B_K\theta_{YL}\gamma_K(1+\gamma_L)\left(-e_{LL}-e_{KK}+2e_{LK}\right) = -\mathbf{v}'\mathbf{E}_{LK}\mathbf{v}$$

for $\mathbf{v} \equiv \sqrt{B_K\theta_{YL}\gamma_K(1+\gamma_L)}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

The restriction in (A-17) also turns out to be necessary to guarantee that $C_3 \geq 0$ for all parameter values. To see why, first note that $B_L$ and $B_K$ depend on the cost shares $\theta_{YK}$ and $\theta_{YL}$ and the index weights $\alpha$ and $\beta$. Condition 2 further implies that $B_L$ and $B_K$ may not depend on the substitution elasticities $e_{LL}$, $e_{KK}$, or $e_{LK}$ since $\alpha$ and $\beta$ are themselves restricted to be independent of substitution elasticities. This means that values of $\alpha$ and $\beta$ that satisfy Conditions 1 and 2 must ensure that $C_3 \geq 0$ for all possible values of ($e_{LL}$, $e_{KK}$, $e_{LK}$) satisfying the restrictions from production theory, namely: for all $e_{LL}$, $e_{KK}$, and $e_{LK}$ such that $e_{LL} \leq 0$, $e_{KK} \leq 0$, and $e_{LL}e_{KK} - e_{LK}^2 \geq 0$:

$$-e_{LL}B_K\theta_{YL}\gamma_L(1+\gamma_K)-e_{KK}B_L\theta_{YK}\gamma_K(1+\gamma_L)+e_{LK}\left[B_K\theta_{YL}\gamma_K(1+\gamma_L)+B_L\theta_{YK}\gamma_L(1+\gamma_K)\right] \geq 0$$

that is,

$$B_K\theta_{YL}\left[\gamma_K(1+\gamma_L)e_{LK}-\gamma_L(1+\gamma_K)e_{LL}\right] \geq B_L\theta_{YK}\left[\gamma_K(1+\gamma_L)e_{KK}-\gamma_L(1+\gamma_K)e_{LK}\right].$$

Note that if $e_{LK} > 0$, this condition places no restriction on $(B_L, B_K)$ and thus no restriction on $(\alpha, \beta)$. If $e_{LK} < 0$ however, it constrains the set of acceptable values of $(\alpha, \beta)$. Consider the subset of substitution elasticities

$$(e_{LL}, e_{KK}, e_{LK}) = \left(\frac{F\gamma_K(1+\gamma_L)}{\gamma_L(1+\gamma_K)}e_{LK}, \frac{\gamma_L(1+\gamma_K)}{F\gamma_K(1+\gamma_L)}e_{LK}, e_{LK}\right)$$

where $e_{LK} < 0$ and $F > 0$. These elasticities satisfy the theory restrictions. In addition, $\gamma_K(1+\gamma_L)e_{LK}-\gamma_L(1+\gamma_K)e_{LL} > 0 \Leftrightarrow F > 1$. Thus, we have that acceptable values of $\alpha$ and $\beta$ must satisfy

$$B_K \geq B_L\frac{\theta_{YK}\left[\gamma_K(1+\gamma_L)e_{KK}-\gamma_L(1+\gamma_K)e_{LK}\right]}{\theta_{YL}\left[\gamma_K(1+\gamma_L)e_{LK}-\gamma_L(1+\gamma_K)e_{LL}\right]} = B_L\frac{\theta_{YK}\gamma_L(1+\gamma_K)}{\theta_{YL}\gamma_K(1+\gamma_L)}\left(\frac{\frac{1}{F}-1}{1-F}\right) = B_L\frac{\theta_{YK}\gamma_L(1+\gamma_K)}{\theta_{YL}\gamma_K(1+\gamma_L)}\left(\frac{1}{F}\right)$$

for all $F > 1$ while also satisfying

$$B_K \leq B_L\frac{\theta_{YK}\left[\gamma_K(1+\gamma_L)e_{KK}-\gamma_L(1+\gamma_K)e_{LK}\right]}{\theta_{YL}\left[\gamma_K(1+\gamma_L)e_{LK}-\gamma_L(1+\gamma_K)e_{LL}\right]} = B_L\frac{\theta_{YK}\gamma_L(1+\gamma_K)}{\theta_{YL}\gamma_K(1+\gamma_L)}\left(\frac{1}{F}\right)$$

for all $F < 1$. Taking limits as $F \to 1$, $F > 1$, and as $F \to 1$, $F < 1$ yields

$$B_L \frac{\theta_{YK}\gamma_L(1+\gamma_K)}{\theta_{YL}\gamma_K(1+\gamma_L)} \leq B_K \leq B_L \frac{\theta_{YK}\gamma_L(1+\gamma_K)}{\theta_{YL}\gamma_K(1+\gamma_L)}$$

which implies (A-17).

Finally, it is easy to check that (A-17) implies that

$$B_K(\theta_{YL} - \theta_{XL}) - B_L(\theta_{YK} - \theta_{XK}) = \frac{B_L(\gamma_L - \gamma_K)\theta_{YK}(1+\delta)}{\gamma_K(1+\gamma_L)} \tag{A-18}$$

so that $C_2 \geq 0$ as well.

In order to ensure that Condition 3 is satisfied, we use Cramer's rule to derive the effect $\frac{\hat{Z}}{\hat{\tau}_Z}$. Having shown that the system determinant is positive, we only need to ensure that the following determinant

$$\Delta_Z \equiv \begin{vmatrix} \sigma_u(\theta_{YL} - \theta_{XL}) + (1+\delta)(\theta_{YL}\bar{\theta}^1_{YL} + \theta_{YK}\bar{\theta}^2_{YL}) & \sigma_u(\theta_{YK} - \theta_{XK}) + (1+\delta)(\theta_{YL}\bar{\theta}^1_{YK} + \theta_{YK}\bar{\theta}^2_{YK}) & -\sigma_u\theta_{YZ} - (1+\delta)(\theta_{YL}\bar{\theta}^1_{YZ} + \theta_{YK}\bar{\theta}^2_{YZ}) \\ \sigma_X - \gamma_L\bar{\theta}^1_{YL} + \gamma_K\bar{\theta}^2_{YL} & -\sigma_X - \gamma_L\bar{\theta}^1_{YK} + \gamma_K\bar{\theta}^2_{YK} & \gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ} \\ B_L & B_K & -B_Z \end{vmatrix}$$

is non-positive, where we have defined $B_Z \equiv \beta\theta_{YZ}$. (Note that $B_L + B_K + B_Z = 1$.) This determinant can be written as $\Delta_Z = D_1\sigma_X + D_2\sigma_u + D_3\sigma_X\sigma_u + D_4$, with

$$\begin{aligned}
D_1 &\equiv (1+\delta)\left[B_Z(\theta_{YL}\bar{\theta}^1_{YK} + \theta_{YK}\bar{\theta}^2_{YK} + \theta_{YL}\bar{\theta}^1_{YL} + \theta_{YK}\bar{\theta}^2_{YL}) - (B_L + B_K)(\theta_{YL}\bar{\theta}^1_{YZ} + \theta_{YK}\bar{\theta}^2_{YZ})\right] \\
D_2 &\equiv (\theta_{YL} - \theta_{XL})\left[B_Z(\gamma_L\bar{\theta}^1_{YK} - \gamma_K\bar{\theta}^2_{YK}) - B_K(\gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ})\right] \\
&\quad -(\theta_{YK} - \theta_{XK})\left[B_Z(\gamma_L\bar{\theta}^1_{YL} - \gamma_K\bar{\theta}^2_{YL}) - B_L(\gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ})\right] \\
&\quad +\theta_{YZ}\left[B_K(\gamma_L\bar{\theta}^1_{YL} - \gamma_K\bar{\theta}^2_{YL}) - B_L(\gamma_L\bar{\theta}^1_{YK} - \gamma_K\bar{\theta}^2_{YK})\right] \\
D_3 &\equiv (\theta_{YL} - \theta_{XL} + \theta_{YK} - \theta_{XK})B_Z - \theta_{YZ}(B_L + B_K) \\
D_4 &\equiv (1+\delta)\left[B_L\left((\theta_{YL}\bar{\theta}^1_{YK} + \theta_{YK}\bar{\theta}^2_{YK})(\gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ}) - (\gamma_L\bar{\theta}^1_{YK} - \gamma_K\bar{\theta}^2_{YK})(\theta_{YL}\bar{\theta}^1_{YZ} + \theta_{YK}\bar{\theta}^2_{YZ})\right)\right. \\
&\quad -B_K\left((\theta_{YL}\bar{\theta}^1_{YL} + \theta_{YK}\bar{\theta}^2_{YL})(\gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ}) - (\gamma_L\bar{\theta}^1_{YL} - \gamma_K\bar{\theta}^2_{YL})(\theta_{YL}\bar{\theta}^1_{YZ} + \theta_{YK}\bar{\theta}^2_{YZ})\right) \\
&\quad \left.+B_Z\left((\theta_{YL}\bar{\theta}^1_{YL} + \theta_{YK}\bar{\theta}^2_{YL})(\gamma_L\bar{\theta}^1_{YK} - \gamma_K\bar{\theta}^2_{YK}) - (\gamma_L\bar{\theta}^1_{YL} - \gamma_K\bar{\theta}^2_{YL})(\theta_{YL}\bar{\theta}^1_{YK} + \theta_{YK}\bar{\theta}^2_{YK})\right)\right].
\end{aligned}$$

It turns out that all the $D_i$ parameters are non-positive. To start with, note that

$D_3 = -\theta_{YZ}(B_Z + B_L + B_K) = -\theta_{YZ} < 0$. Let us then show that $D_1 \leq 0$. First, note that

$$\begin{aligned}
\theta_{YL}\bar{\theta}^1_{YZ} + \theta_{YK}\bar{\theta}^2_{YZ} &= \theta_{YZ}\left[\theta_{YL}(e_{LZ} - e_{ZZ}) + \theta_{YK}(e_{KZ} - e_{ZZ})\right] \\
&= \theta_{YZ}\left[\theta_{YL}e_{ZL} + \theta_{YK}e_{ZK} - (\theta_{YL} + \theta_{YK})e_{ZZ}\right] \\
&= \theta_{YZ}\left[-\theta_{YZ}e_{ZZ} - (\theta_{YL} + \theta_{YK})e_{ZZ}\right] \\
&= -\theta_{YZ}e_{ZZ} \geq 0
\end{aligned}$$

Second, note that

$$\begin{aligned}
\theta_{YL}\bar{\theta}^1_{YK} + \theta_{YK}\bar{\theta}^2_{YK} + \theta_{YL}\bar{\theta}^1_{YL} + \theta_{YK}\bar{\theta}^2_{YL} &= \theta_{YL}\theta_{YK}(2e_{LK} - e_{KZ} - e_{LZ}) + (\theta_{YL})^2(e_{LL} - e_{LZ}) + (\theta_{YK})^2(e_{KK} - e_{KZ}) \\
&= e_{LL}(\theta_{YL})^2 + e_{KK}(\theta_{YK})^2 + 2e_{LK}\theta_{YL}\theta_{YK} - (\theta_{YL} + \theta_{YK})(\theta_{YL}e_{LZ} + \theta_{YK}e_{KZ}) \\
&= -(\theta_{YL} + \theta_{YK} + \theta_{YZ})(\theta_{YL}e_{LZ} + \theta_{YK}e_{KZ}) \\
&= \theta_{YZ}e_{ZZ} \leq 0
\end{aligned}$$

Therefore, $D_1 = (1+\delta)\theta_{YZ}e_{ZZ} \leq 0$. Let us now show that $D_4 \leq 0$. We can rewrite

$$D_4 = (1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\left[B_L(\bar{\theta}^2_{YK}\bar{\theta}^1_{YZ} - \bar{\theta}^1_{YK}\bar{\theta}^2_{YZ}) + B_K(\bar{\theta}^1_{YL}\bar{\theta}^2_{YZ} - \bar{\theta}^2_{YL}\bar{\theta}^1_{YZ}) + B_Z(\bar{\theta}^2_{YL}\bar{\theta}^1_{YK} - \bar{\theta}^1_{YL}\bar{\theta}^2_{YK})\right].$$

We will show that the coefficients on $B_L$, $B_K$, and $B_Z$ in the square bracket are all non-positive. Consider for instance the coefficient $\bar{\theta}^1_{YL}\bar{\theta}^2_{YZ} - \bar{\theta}^2_{YL}\bar{\theta}^1_{YZ}$ on $B_K$. We have:

$$\begin{aligned}
\frac{\bar{\theta}^1_{YL}\bar{\theta}^2_{YZ} - \bar{\theta}^2_{YL}\bar{\theta}^1_{YZ}}{\theta_{YL}\theta_{YZ}} &= (e_{LL} - e_{LZ})(e_{KZ} - e_{ZZ}) - (e_{LK} - e_{LZ})(e_{LZ} - e_{ZZ}) \\
&= -(e_{LL}e_{ZZ} - e^2_{LZ}) + e_{LL}e_{KZ} - e_{LZ}e_{KZ} - e_{LK}e_{LZ} + e_{LK}e_{ZZ} \\
&= -(e_{LL}e_{ZZ} - e^2_{LZ}) + e_{LL}e_{KZ} + e_{LK}e_{ZZ} - \frac{e_{LZ}}{\theta_{YK}}(\theta_{YK}e_{ZK} + \theta_{YK}e_{LK}) \\
&= -(e_{LL}e_{ZZ} - e^2_{LZ}) + e_{LL}e_{KZ} + e_{LK}e_{ZZ} + \frac{e_{LZ}}{\theta_{YK}}(\theta_{YL}e_{ZL} + \theta_{YZ}e_{ZZ} + \theta_{YL}e_{LL} + \theta_{YZ}e_{LZ}) \\
&= -(e_{LL}e_{ZZ} - e^2_{LZ}) + \frac{1}{\theta_{YK}}\left[e_{LL}(\theta_{YK}e_{KZ} + \theta_{YL}e_{LZ}) + e_{ZZ}(\theta_{YK}e_{LK} + \theta_{YZ}e_{LZ}) + e^2_{LZ}(\theta_{YL} + \theta_{YZ})\right] \\
&= -\frac{(e_{LL}e_{ZZ} - e^2_{LZ})}{\theta_{YK}} \leq 0
\end{aligned}$$

where we have used the fact that $e_{LL}e_{ZZ} - e^2_{LZ} \geq 0$. Similarly, the coefficient on $B_Z$ has the same sign as

$$\begin{aligned}
\frac{\bar{\theta}^2_{YL}\bar{\theta}^1_{YK} - \bar{\theta}^1_{YL}\bar{\theta}^2_{YK}}{\theta_{YL}\theta_{YK}} &= (e_{LK} - e_{LZ})(e_{LK} - e_{KZ}) - (e_{LL} - e_{LZ})(e_{KK} - e_{KZ}) \\
&= -(e_{LL}e_{KK} - e^2_{LK}) + e_{LL}e_{KZ} - e_{LK}e_{KZ} - e_{LK}e_{LZ} + e_{KK}e_{LZ}.
\end{aligned}$$

A reasoning similar to that used to sign the coefficient on $B_K$ can be used by swapping the indices $K$ and $Z$ to determine the sign of this expression, and similarly for the coefficient

on $B_L$. Therefore, $D_4 \leq 0$. It can further be shown that

$$D_4 = -(\gamma_L \theta_{YK} + \gamma_K \theta_{YL} + \gamma_L \gamma_K)\,(\theta_{YL} e_{LK} e_{LZ} + \theta_{YK} e_{LK} e_{KZ} + \theta_{YZ} e_{LZ} e_{KZ})$$

so that $D_4$ is independent of $\alpha$ and $\beta$. It remains to be shown that $D_2 \leq 0$. Rewrite

$$\begin{aligned} D_2 \;=\; & B_L \left[\theta_{YZ}(\gamma_K \bar{\theta}^2_{YK} - \gamma_L \bar{\theta}^1_{YK}) + (\theta_{YK} - \theta_{XK})(\gamma_L \bar{\theta}^1_{YZ} - \gamma_K \bar{\theta}^2_{YZ})\right] \\ & + B_K \left[\theta_{YZ}(\gamma_L \bar{\theta}^1_{YL} - \gamma_K \bar{\theta}^2_{YL}) + (\theta_{YL} - \theta_{XL})(\gamma_K \bar{\theta}^2_{YZ} - \gamma_L \bar{\theta}^1_{YZ})\right] \\ & + B_Z \left[(\theta_{YL} - \theta_{XL})(\gamma_L \bar{\theta}^1_{YK} - \gamma_K \bar{\theta}^2_{YK}) + (\theta_{YK} - \theta_{XK})(\gamma_K \bar{\theta}^2_{YL} - \gamma_L \bar{\theta}^1_{YL})\right]. \end{aligned}$$

Using the definitions of the $\bar{\theta}^i_{Y_j}$ parameters and the relationships $\theta_{YL} e_{mL} + \theta_{YK} e_{mK} + \theta_{YZ} e_{mZ} = 0$, one can show that the three terms in square brackets are equal to each other, so that the value of $D_2$ is independent of $(\alpha, \beta)$ due to $B_L + B_K + B_Z = 1$. Using the first of

these bracketed terms, we thus have

$$
\begin{aligned}
D_2 &= \theta_{YZ}\theta_{YK}\left(\gamma_K(e_{KK}-e_{KZ})-\gamma_L(e_{LK}-e_{KZ})\right)+(\theta_{YK}-\theta_{XK})\theta_{YZ}\left(\gamma_L(e_{LZ}-e_{ZZ})-\gamma_K(e_{KZ}-e_{ZZ})\right)\\
&= e_{KK}\theta_{YZ}\theta_{YK}\gamma_K+e_{ZZ}(\theta_{YK}-\theta_{XK})\theta_{YZ}(\gamma_K-\gamma_L)+e_{KZ}\left(\theta_{YZ}\theta_{YK}(\gamma_L-\gamma_K)-(\theta_{YK}-\theta_{XK})\theta_{YZ}\gamma_K\right)\\
&\quad -e_{LK}\theta_{YZ}\theta_{YK}\gamma_L+e_{LZ}(\theta_{YK}-\theta_{XK})\theta_{YZ}\gamma_L\\
&= e_{KK}\theta_{YZ}\theta_{YK}\left(\gamma_K+\frac{\theta_{YK}\gamma_L}{\theta_{YL}}\right)+e_{ZZ}\theta_{YZ}(\theta_{YK}-\theta_{XK})\left(\gamma_K-\gamma_L-\frac{\theta_{YZ}\gamma_L}{\theta_{YL}}\right)\\
&\quad +e_{KZ}\left(\theta_{YZ}\theta_{YK}(\gamma_L-\gamma_K)-(\theta_{YK}-\theta_{XK})\theta_{YZ}\gamma_K+\theta_{YZ}\theta_{YK}\frac{\theta_{YZ}\gamma_L}{\theta_{YL}}-(\theta_{YK}-\theta_{XK})\theta_{YZ}\frac{\theta_{YK}\gamma_L}{\theta_{YL}}\right)\\
&= e_{KK}\frac{\theta_{YZ}\theta_{YK}\gamma_L}{\delta\theta_{YL}}\left(\frac{\delta\gamma_K\theta_{YL}}{\gamma_L}+\theta_{YK}\delta\right)+e_{ZZ}\frac{\theta_{YZ}\theta_{YK}}{\gamma_K\theta_{YL}}(\gamma_K-\delta)\left((\gamma_K-\gamma_L)\theta_{YL}-\gamma_L\theta_{YZ}\right)\\
&\quad +e_{KZ}\theta_{YZ}\theta_{YK}\left(\gamma_L-\gamma_K-\gamma_K\left(1-\frac{\delta}{\gamma_K}\right)+\frac{\theta_{YZ}\gamma_L}{\theta_{YL}}-\left(1-\frac{\delta}{\gamma_K}\right)\frac{\theta_{YK}\gamma_L}{\theta_{YL}}\right)\\
&= e_{KK}\frac{\theta_{YZ}\theta_{YK}\gamma_L}{\delta\theta_{YL}}(\gamma_K\theta_{XL}+\gamma_K\theta_{XK})+e_{ZZ}\frac{\theta_{YZ}\theta_{YK}}{\gamma_K\theta_{YL}}(\gamma_K-\delta)(\gamma_K\theta_{YL}+\gamma_L\theta_{YK}-\gamma_L)\\
&\quad +e_{KZ}\theta_{YZ}\theta_{YK}\left(\gamma_L-2\gamma_K+\delta+\frac{\theta_{YZ}}{\theta_{YL}}\gamma_L-(\gamma_K-\delta)\frac{\theta_{YK}\gamma_L}{\theta_{YL}\gamma_K}\right)\\
&= e_{KK}\frac{\theta_{YZ}\theta_{YK}\gamma_L\gamma_K}{\delta\theta_{YL}}+e_{ZZ}\frac{\theta_{YZ}\theta_{YK}\gamma_L}{\gamma_K\theta_{YL}\delta}(\gamma_K-\delta)\left(\frac{\delta\gamma_K\theta_{YL}}{\gamma_L}+\delta\theta_{YK}-\delta\right)\\
&\quad +e_{KZ}\theta_{YZ}\theta_{YK}\left(\gamma_L-2\gamma_K+\delta+\frac{\gamma_L}{\theta_{YL}}(1-\theta_{YK}-\theta_{YL})-\frac{\theta_{YK}\gamma_L}{\theta_{YL}}+\delta\frac{\theta_{YK}\gamma_L}{\theta_{YL}\gamma_K}\right)\\
&= e_{KK}\frac{\theta_{YZ}\theta_{YK}\gamma_K}{\theta_{XL}}+e_{ZZ}\frac{\theta_{YZ}\theta_{YK}}{\gamma_K\theta_{XL}}\left(\gamma_K-\delta\right)^2+e_{KZ}\theta_{YZ}\theta_{YK}\left(-2\gamma_K+\delta+\frac{\delta}{\theta_{XL}}-\frac{2\gamma_K\theta_{XK}}{\theta_{XL}}+\frac{\delta\theta_{XK}}{\theta_{XL}}\right)\\
&= e_{KK}\frac{\theta_{YZ}\theta_{YK}\gamma_K}{\theta_{XL}}+e_{ZZ}\frac{\theta_{YZ}\theta_{YK}}{\gamma_K\theta_{XL}}\left(\gamma_K-\delta\right)^2-2e_{KZ}\frac{\theta_{YZ}\theta_{YK}}{\theta_{XL}}\left(\gamma_K-\delta\right)\\
&= \frac{\theta_{YZ}\theta_{YK}}{\theta_{XL}}\left(e_{KK}\gamma_K+e_{ZZ}\frac{\left(\gamma_K-\delta\right)^2}{\gamma_K}-2e_{KZ}(\gamma_K-\delta)\right)
\end{aligned}
$$

which is non-positive from the negative semidefiniteness of the submatrix $\mathbf{E}_{KZ}$.

## A.4 Tax incidence: sources side

We apply Cramer's rule to derive the effect $\frac{\hat{w}}{\hat{\tau}_Z}$. The effect $\frac{\hat{r}}{\hat{\tau}_Z}$ can be obtained using symmetry. We have that $\frac{\hat{w}}{\hat{\tau}_Z}=\frac{\Delta_w}{\Delta}$, where

$$
\Delta_w \equiv \begin{vmatrix}
-\sigma_u\theta_{YZ}-(1+\delta)(\theta_{YL}\bar{\theta}^1_{YZ}+\theta_{YK}\bar{\theta}^2_{YZ}) & \sigma_u(\theta_{YK}-\theta_{XK})+(1+\delta)(\theta_{YL}\bar{\theta}^1_{YK}+\theta_{YK}\bar{\theta}^2_{YK}) & \theta_{YZ}+(1+\delta)(\theta_{YL}+\theta_{YK})\\
\gamma_L\bar{\theta}^1_{YZ}-\gamma_K\bar{\theta}^2_{YZ} & -\sigma_X-\gamma_L\bar{\theta}^1_{YK}+\gamma_K\bar{\theta}^2_{YK} & \gamma_K-\gamma_L\\
-B_Z & B_K & 0
\end{vmatrix}.
$$

This determinant can be written as $\Delta_w = F_1\sigma_X + F_2\sigma_u + F_3$, with

$$\begin{aligned}
F_1 &= -\left[\theta_{YZ} + (1+\delta)(\theta_{YL}+\theta_{YK})\right] B_Z \\
F_2 &= (\gamma_K - \gamma_L)\left[\theta_{YZ}B_K - (\theta_{YK}-\theta_{XK})B_Z\right] \\
F_3 &= (1+\delta)(\gamma_K-\gamma_L)\left[(\theta_{YL}\bar{\theta}^1_{YZ} + \theta_{YK}\bar{\theta}^2_{YZ})B_K - (\theta_{YL}\bar{\theta}^1_{YK} + \theta_{YK}\bar{\theta}^2_{YK})B_Z\right] \\
&\quad + \left[\theta_{YZ} + (1+\delta)(\theta_{YL}+\theta_{YK})\right]\left[(\gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ})B_K - (\gamma_L\bar{\theta}^1_{YK} - \gamma_K\bar{\theta}^2_{YK})B_Z\right].
\end{aligned}$$

Simplifying, we obtain:

$$\begin{aligned}
F_3 &= \theta_{YZ}\left[(\gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ})B_K - (\gamma_L\bar{\theta}^1_{YK} - \gamma_K\bar{\theta}^2_{YK})B_Z\right] \\
&\quad +(1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\left[(\bar{\theta}^1_{YZ} - \bar{\theta}^2_{YZ})B_K - (\bar{\theta}^1_{YK} - \bar{\theta}^2_{YK})B_Z\right] \\
&= \theta_{YZ}\left[(\gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ})B_K - (\gamma_L\bar{\theta}^1_{YK} - \gamma_K\bar{\theta}^2_{YK})B_Z\right] \\
&\quad +(1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\left[\theta_{YZ}(e_{LZ}-e_{KZ})B_K - \theta_{YK}(e_{LK}-e_{KK})B_Z\right].
\end{aligned}$$

Using symmetry, we then have that $\frac{\hat{r}}{\hat{\tau}_Z} = \frac{G_1\sigma_X + G_2\sigma_u + G_3}{\Delta}$ with

$$\begin{aligned}
G_1 &= F_1 \\
G_2 &= (\gamma_K - \gamma_L)\left[-\theta_{YZ}B_L + (\theta_{YL}-\theta_{XL})B_Z\right]
\end{aligned}$$

and

$$\begin{aligned}
G_3 &= \theta_{YZ}\left[-(\gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ})B_L - (\gamma_K\bar{\theta}^2_{YL} - \gamma_L\bar{\theta}^1_{YL})B_Z\right] \\
&\quad +(1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\left[-\theta_{YZ}(e_{LZ}-e_{KZ})B_L - \theta_{YL}(e_{LK}-e_{LL})B_Z\right].
\end{aligned}$$

We have $\frac{\hat{w}-\hat{r}}{\hat{\tau}_Z} = \frac{\sigma_X(F_1-G_1)+\sigma_u(F_2-G_2)+F_3-G_3}{\Delta}$. It is easy to see that $F_2 - G_2 = (\gamma_K-\gamma_L)\theta_{YZ}$. In addition, we have

$$\begin{aligned}
F_3 - G_3 &= \theta_{YZ}\left[(B_L+B_K)(\gamma_L\bar{\theta}^1_{YZ} - \gamma_K\bar{\theta}^2_{YZ}) + B_Z\left(\gamma_K\bar{\theta}^2_{YL} - \gamma_L\bar{\theta}^1_{YL} - \gamma_L\bar{\theta}^1_{YK} + \gamma_K\bar{\theta}^2_{YK}\right)\right] \\
&\quad +(1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\left[\theta_{YZ}(e_{LZ}-e_{KZ})(B_L+B_K) + \left(\theta_{YL}(e_{LK}-e_{LL}) - \theta_{YK}(e_{LK}-e_{KK})\right)B_Z\right] \\
&= e_{ZZ}(\theta_{YZ})^2(B_L+B_K)(\gamma_K-\gamma_L) \\
&\quad +e_{LZ}\left[(\theta_{YZ})^2(B_L+B_K)\gamma_L + \theta_{YZ}\theta_{YL}B_Z(\gamma_L-\gamma_K) + (1+\delta)(\gamma_L\theta_{YK}+\gamma_K\theta_{YL})\theta_{YZ}(B_L+B_K)\right] \\
&\quad +e_{KZ}\left[-(\theta_{YZ})^2(B_L+B_K)\gamma_K + \theta_{YZ}\theta_{YK}B_Z(\gamma_L-\gamma_K) - (1+\delta)(\gamma_L\theta_{YK}+\gamma_K\theta_{YL})\theta_{YZ}(B_L+B_K)\right] \\
&\quad +e_{LL}\left[-\theta_{YZ}\theta_{YL}B_Z\gamma_L - (1+\delta)(\gamma_L\theta_{YK}+\gamma_K\theta_{YL})\theta_{YL}B_Z\right] \\
&\quad +e_{LK}\left[\theta_{YZ}B_Z\left(\gamma_K\theta_{YL} - \gamma_L\theta_{YK}\right) + (1+\delta)(\gamma_L\theta_{YK}+\gamma_K\theta_{YL})(\theta_{YL}-\theta_{YK})B_Z\right] \\
&\quad +e_{KK}\left[\theta_{YZ}\theta_{YK}B_Z\gamma_K + (1+\delta)(\gamma_L\theta_{YK}+\gamma_K\theta_{YL})\theta_{YK}B_Z\right].
\end{aligned}$$

Using $\theta_{YL}e_{LL} = -\theta_{YK}e_{LK} - \theta_{YZ}e_{LZ}$ and $\theta_{YK}e_{KK} = -\theta_{YL}e_{LK} - \theta_{YZ}e_{KZ}$ and $B_L + B_K = 1 - B_Z$, we get

$$\begin{aligned}
F_3 - G_3 &= e_{ZZ}(\theta_{YZ})^2(1-B_Z)(\gamma_K - \gamma_L) \\
&\quad + e_{LZ}\left[(\theta_{YZ})^2\gamma_L + \theta_{YZ}\theta_{YL}B_Z(\gamma_L - \gamma_K) + (1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\theta_{YZ}(1-B_Z)\right] \\
&\quad + e_{KZ}\left[-(\theta_{YZ})^2\gamma_K + \theta_{YZ}\theta_{YK}B_Z(\gamma_L - \gamma_K) - (1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\theta_{YZ}(1-B_Z)\right] \\
&\quad - e_{LL}(1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\theta_{YL}B_Z \\
&\quad + e_{LK}(1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})(\theta_{YL} - \theta_{YK})B_Z \\
&\quad + e_{KK}(1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\theta_{YK}B_Z.
\end{aligned}$$

Now using $\theta_{YZ}e_{ZZ} + \theta_{YL}e_{LZ} + \theta_{YK}e_{KZ} = 0$, we get

$$\begin{aligned}
F_3 - G_3 &= e_{ZZ}(\theta_{YZ})^2(\gamma_K - \gamma_L) \\
&\quad + e_{LZ}\left[(\theta_{YZ})^2\gamma_L + (1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\theta_{YZ}(1-B_Z)\right] \\
&\quad + e_{KZ}\left[-(\theta_{YZ})^2\gamma_K - (1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\theta_{YZ}(1-B_Z)\right] \\
&\quad - e_{LL}(1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\theta_{YL}B_Z \\
&\quad + e_{LK}(1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})(\theta_{YL} - \theta_{YK})B_Z \\
&\quad + e_{KK}(1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\theta_{YK}B_Z \\
&= e_{ZZ}(\theta_{YZ})^2(\gamma_K - \gamma_L) \\
&\quad + e_{LZ}\left[(\theta_{YZ})^2\gamma_L + (1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\theta_{YZ}\right] \\
&\quad + e_{KZ}\left[-(\theta_{YZ})^2\gamma_K - (1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\theta_{YZ}\right] \\
&= e_{LZ}\theta_{YZ}\left[-\theta_{YL}(\gamma_K - \gamma_L) + \theta_{YZ}\gamma_L + (1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\right] \\
&\quad + e_{KZ}\theta_{YZ}\left[-\theta_{YK}(\gamma_K - \gamma_L) - \theta_{YZ}\gamma_K - (1+\delta)(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\right] \\
&= e_{LZ}\theta_{YZ}\left[\gamma_L + \delta(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\right] - e_{KZ}\theta_{YZ}\left[\gamma_K + \delta(\gamma_L\theta_{YK} + \gamma_K\theta_{YL})\right] \\
&= e_{LZ}\theta_{YZ}\gamma_L(1+\gamma_K) - e_{KZ}\theta_{YZ}\gamma_K(1+\gamma_L).
\end{aligned}$$

Therefore, we have

$$\frac{\hat{w} - \hat{r}}{\hat{\tau}_Z} = \frac{\theta_{YZ}\left[\sigma_u(\gamma_K - \gamma_L) + \gamma_L(1+\gamma_K)e_{LZ} - \gamma_K(1+\gamma_L)e_{KZ}\right]}{\Delta}.$$

Note that variations in the choices for $\alpha$ and $\beta$ affect incidence on the sources side only through the denominator $\Delta$.

## A.5 Tax incidence: uses side

Using the equalities $\hat{p}_Y = \theta_{YL}\hat{w} + \theta_{YK}\hat{r} + \theta_{YZ}\hat{\tau}_Z$ and $\hat{p}_X = \theta_{XL}\hat{w} + \theta_{XK}\hat{r}$ together with expressions for $\frac{\hat{w}}{\hat{\tau}_Z}$ and $\frac{\hat{r}}{\hat{\tau}_Z}$, one can derive the following expression for the incidence of the pollution tax on the uses side:

$$\begin{aligned}\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z} &= \frac{\theta_{YZ}}{\Delta}\left[\sigma_X\left(1 + \gamma_L\theta_{XL} + \gamma_K\theta_{XK}\right) + \left(\theta_{YL}\gamma_K(1+\gamma_L) + \theta_{YK}\gamma_L(1+\gamma_K)\right)e_{LK}\right. \\ &\quad \left. +\gamma_L(1+\gamma_K)(\theta_{XK} - \theta_{YK})e_{LZ} + \gamma_K(1+\gamma_L)(\theta_{XL} - \theta_{YL})e_{KZ}\right].\end{aligned}$$

Note that variations in the choices for $\alpha$ and $\beta$ affect incidence on the uses side only through the denominator $\Delta$. In addition, the price of $Y$ could increase or decrease relative to $X$.[16] But note that, as in Fullerton and Heutel (2007a), the ambiguity is resolved when $\gamma_L = \gamma_K \equiv \gamma$. Indeed, we can then write $\theta_{XL} - \theta_{YL} = \theta_{XL} - \theta_{YL}(\theta_{XL} + \theta_{XK}) = \theta_{XL}(1 - \theta_{YL} - \theta_{YK}) = \theta_{XL}\theta_{YZ}$, and similarly $\theta_{XK} - \theta_{YK} = \theta_{XK}\theta_{YZ}$, which yields

$$\begin{aligned}\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z} &= \frac{\theta_{YZ}(1+\gamma)}{\Delta}\left[\sigma_X + \gamma\left((\theta_{YL} + \theta_{YK})e_{LK} + \theta_{XK}\theta_{YZ}e_{LZ} + \theta_{XL}\theta_{YZ}e_{KZ}\right)\right] \\ &= \frac{\theta_{YZ}(1+\gamma)}{\Delta}\left[\sigma_X + \gamma\left((\theta_{XK}\theta_{YL} + \theta_{XL}\theta_{YK})e_{LK} - \theta_{XK}\theta_{YL}e_{LL} - \theta_{XL}\theta_{YK}e_{KK}\right)\right]\end{aligned}$$

which is positive given the negative semidefiniteness of $\mathbf{E}_{LK}$ and the fact that $\theta_{XL}\theta_{YK} = \theta_{XK}\theta_{YL}$.

## A.6 Example of pollution-enhancing tax in Fullerton and Heutel (2007a)

In the modeling approach of Fullerton and Heutel (2007a) ($\beta = 0$, $\alpha = \theta_{XL}$), an increase in the tax on the polluting input may lead to increased pollution. For instance, the following set of parameter values leads to the counterintuitive outcome: $\sigma_u = \sigma_X = 0$, $\theta_{YL} = \frac{1}{8}$, $\theta_{YK} = \frac{1}{2}$, $\gamma_L = 1$, $\gamma_K = \frac{12}{13}$, $e_{LL} = -1$, $e_{KK} = -1.1005$, $e_{ZZ} = -3$. The remaining parameters can be deduced using the relationships $\frac{\gamma_L\theta_{XL}}{\theta_{YL}} = \frac{\gamma_K\theta_{XK}}{\theta_{YK}}$, which determines $\theta_{XL}$, as well as the three equalities $\theta_{YL}e_{mL} + \theta_{YK}e_{mK} + \theta_{YZ}e_{mZ} = 0$, for $m \in \{L, K, Z\}$, which jointly determine the three cross-price Allen elasticities. One can check that the resulting substitution matrix satisfies negative semidefiniteness. With these parameters, we obtain $\hat{Z} = 0.96\hat{\tau}_Z$, that is, a positive elasticity of pollution with respect to the pollution tax. Not surprisingly, in that case the value of the pollution tax actually decreases relative to the value of our preferred price indices. Indeed, denoting $\theta_X = \frac{p_X X}{p_X X + p_Y Y}$ and $\theta_Y = 1 - \theta_X$, we

[16] For an example where $\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z} < 0$, consider the following parameter values: $\theta_{YL} = 0.05$, $\theta_{YK} = 0.005$, $\gamma_L = 1$, $\gamma_K = 0.0025$, $e_{LK} = -1$, $e_{LZ} = 0.01$, $e_{KZ} = 50$, and $\sigma_X = \sigma_u = 0$.

obtain $\hat{p}_Z - \left(\theta_X \hat{p}_X + \theta_Y \hat{p}_Y\right) = -0.8\hat{\tau}_Z$. Denoting $\theta_L = \frac{w\bar{L}}{w\bar{L}+r\bar{K}}$ and $\theta_K = 1 - \theta_L$, we obtain $\hat{p}_Z - (\theta_L \hat{w} + \theta_K \hat{r}) = -1.0\hat{\tau}_Z$.

Note that the numerical example given in Fullerton and Heutel (2007a), footnote 14, to illustrate the counterintuitive outcome violates the negative semidefiniteness property of the substitution matrix in sector $Y$. This is easily seen by computing the determinant of the submatrix $\begin{pmatrix} e_{LL} & e_{LZ} \\ e_{LZ} & e_{ZZ} \end{pmatrix}$, which turns out to be negative rather than non-negative for their choice of substitution elasticities.

# B Model with one clean input

Consider an economy with one clean good, labor. We consider price indices of the form $\mathcal{P} = p_Y^{\beta} p_X^{1-\beta}$. ($\hat{p}_X = \hat{w}$ due to constant returns to scale in sector $X$.) The displaced equilibrium is given by the following set of equations:

$$\begin{aligned}
\hat{L}_X + \gamma_L \hat{L}_Y &= 0 \\
\hat{X} - \hat{L}_X &= 0 \\
\hat{Y} - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z} &= 0 \\
\hat{p}_Y - \theta_{YL}\hat{p}_X &= \theta_{YZ}\hat{\tau}_Z \\
\hat{L}_Y - \hat{Z} + \sigma_Y \hat{p}_X &= \sigma_Y \hat{\tau}_Z \\
\hat{X} - \hat{Y} + \sigma_u \hat{p}_X - \sigma_u \hat{p}_Y &= 0 \\
\beta \hat{p}_Y + (1-\beta)\hat{p}_X &= 0.
\end{aligned}$$

where the notation is as before and $\theta_{YL} + \theta_{YZ} = 1$. It is easy to show that the determinant of the system has a constant sign, and that

$$\frac{\hat{Z}}{\hat{\tau}_Z} = -\frac{\sigma_u \theta_{YZ} + \sigma_Y(\gamma_L + \theta_{YL})}{(1+\gamma_L)(1-\beta\theta_{YZ})} < 0.$$

# C Additional simulations

## C.1 Parameter values considered in Fullerton and Heutel (2007a)

Simulations reported in Tables C.1 and C.2 are performed using the parameter values: $\sigma_u = \sigma_X = e_{LK} = 1$, $e_{LZ} = 1$, $e_{KZ} = -0.5$, and $\theta_{YZ} = 0.25$. As in Fullerton and Heutel (2007a), the values of the remaining model parameters are determined by the values

of $\gamma_K - \gamma_L$ given in the tables and the conditions $K_X + K_Y = 0.4$, $L_X + L_Y = 0.6$, and $L_Y + K_Y = 0.2$.

Table C.1: Pollution effect of a 10% increase in the nominal pollution tax (%)

| $\gamma_K - \gamma_L$ | $\theta_{YK}$ | $\theta_{XK}$ | $\hat{Z}$ | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $p_X$ | $p_Y$ | $\mathcal{P}$ | $w$ | $r$ | $\mathcal{R}$ |
| -0.25 | 0.1515 | 0.4495 | -7.50 | -10.10 | -8.02 | -7.64 | -7.35 | -7.52 |
| -0.20 | 0.1818 | 0.4394 | -7.00 | -9.42 | -7.48 | -7.14 | -6.82 | -7.01 |
| -0.15 | 0.2118 | 0.4294 | -6.49 | -8.72 | -6.93 | -6.65 | -6.29 | -6.50 |
| -0.10 | 0.2416 | 0.4195 | -5.98 | -8.03 | -6.39 | -6.14 | -5.77 | -5.99 |
| -0.05 | 0.2710 | 0.4097 | -5.48 | -7.33 | -5.85 | -5.64 | -5.26 | -5.48 |
| 0.00 | 0.3000 | 0.4000 | -4.97 | -6.63 | -5.31 | -5.14 | -4.75 | -4.97 |
| 0.05 | 0.3286 | 0.3905 | -4.47 | -5.94 | -4.77 | -4.63 | -4.25 | -4.47 |
| 0.10 | 0.3566 | 0.3811 | -3.98 | -5.26 | -4.24 | -4.13 | -3.76 | -3.97 |
| 0.15 | 0.3841 | 0.3720 | -3.49 | -4.59 | -3.71 | -3.63 | -3.28 | -3.48 |
| 0.20 | 0.4110 | 0.3630 | -3.00 | -3.93 | -3.19 | -3.13 | -2.81 | -2.99 |

Table C.2: Incidence effects of a 10% increase in the nominal pollution tax (%)

| $\gamma_K - \gamma_L$ | Sources side: $\hat{w} - \hat{r}$ | | | | | | Uses side: $\hat{p}_Y - \hat{p}_X$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $p_X$ | $p_Y$ | $\mathcal{P}$ | $w$ | $r$ | $\mathcal{R}$ | $p_X$ | $p_Y$ | $\mathcal{P}$ | $w$ | $r$ | $\mathcal{R}$ |
| -0.25 | 0.39 | 0.52 | 0.41 | 0.39 | 0.38 | 0.39 | 2.57 | 3.46 | 2.75 | 2.62 | 2.52 | 2.58 |
| -0.20 | 0.47 | 0.63 | 0.50 | 0.48 | 0.45 | 0.47 | 2.57 | 3.46 | 2.75 | 2.62 | 2.50 | 2.57 |
| -0.15 | 0.55 | 0.73 | 0.58 | 0.56 | 0.53 | 0.55 | 2.56 | 3.44 | 2.74 | 2.62 | 2.48 | 2.56 |
| -0.10 | 0.63 | 0.84 | 0.67 | 0.64 | 0.60 | 0.63 | 2.55 | 3.42 | 2.72 | 2.61 | 2.46 | 2.55 |
| -0.05 | 0.71 | 0.94 | 0.75 | 0.73 | 0.68 | 0.71 | 2.53 | 3.38 | 2.70 | 2.60 | 2.42 | 2.53 |
| 0.00 | 0.79 | 1.05 | 0.84 | 0.81 | 0.75 | 0.79 | 2.50 | 3.33 | 2.67 | 2.58 | 2.39 | 2.50 |
| 0.05 | 0.86 | 1.15 | 0.92 | 0.89 | 0.82 | 0.86 | 2.47 | 3.28 | 2.63 | 2.56 | 2.35 | 2.47 |
| 0.10 | 0.94 | 1.25 | 1.00 | 0.98 | 0.89 | 0.94 | 2.43 | 3.22 | 2.59 | 2.52 | 2.30 | 2.43 |
| 0.15 | 1.02 | 1.34 | 1.09 | 1.06 | 0.96 | 1.02 | 2.39 | 3.15 | 2.54 | 2.49 | 2.25 | 2.39 |
| 0.20 | 1.10 | 1.44 | 1.17 | 1.15 | 1.03 | 1.09 | 2.35 | 3.07 | 2.49 | 2.45 | 2.19 | 2.34 |

## C.2 Parameter values adapted from Fullerton and Ta (2019)

Fullerton and Ta (2019) calibrate a Cobb-Douglas model of a closed economy to U.S. data to analyze the effects of a carbon tax on prices, inputs, outputs, and welfare. Their model specification does not squarely fit the model discussed here because the dirty input (fossil fuels) is used in the production of both electricity (a good consumed by households) and a composite consumption good. The consumption good ($X$) is produced using electricity and fossil fuels. Because production of the consumption good uses mostly labor and

Table C.3: Model parameters adapted from Fullerton and Ta (2019)

| Parameter | Notation in Fullerton and Ta (2019), Table 1 | Value |
|---|---|---|
| $e_{LK}$ | N/A (Cobb-Douglas) | 1 |
| $\sigma_u$ | N/A (Cobb-Douglas) | 1 |
| $\sigma_X$ | N/A (Cobb-Douglas) | 1 |
| $\gamma_L$ | $\frac{L_E^0}{L_X^0}$ | 0.012 |
| $\gamma_K$ | $\frac{K_E^0}{K_X^0}$ | 0.031 |
| $\theta_{XL}$ | $\frac{\beta}{\alpha+\beta}$ | 0.455 |
| $\theta_{YL}$ | $\epsilon$ | 0.138 |
| $\theta_{YK}$ | $\delta$ | 0.415 |

Note: To ensure internal consistency, once we have set $\gamma_L$, $\gamma_K$, $\theta_{YL}$, and $\theta_{YK}$ we recompute $\theta_{XL} = \frac{\gamma_K \theta_{YL}}{\gamma_K \theta_{YL} + \gamma_L \theta_{YK}}$ to ensure that the equality $\frac{\gamma_L \theta_{XL}}{\theta_{YL}} = \frac{\gamma_K \theta_{XK}}{\theta_{YK}}$ holds exactly.

capital (the cost shares on electricity and fossil fuels are very small), to fit their calibration data within our framework we assume that production of $X$ rests solely on capital and labor. Thus sector $Y$ (the dirty sector) is represented by the electricity sector, which uses a large share of fossil fuels in terms of input costs. Table C.3 summarizes the calibration data.

Tables C.4 and C.5 summarize results for a set of selected substitution elasticities. We set $e_{LK} = 1$ but allow the substitution elasticities $e_{KZ}$ and $e_{LZ}$ to vary. We exclude sets of substitution elasticities that violate the negative semidefiniteness of the Slutsky matrix. Note that the choices of $p_X$ and $\mathcal{R}$ as anchoring indices lead to indistinguishable results with the level of precision used here, although they are not identical.

The results indicate that choosing $p_Y$ as the anchoring index leads to predictions very different from those obtained with other indices. These discrepancies are driven by the large expenditure share on the polluting input in sector $Y$ and the small shares of labor and capital used in that sector. To see why, note that differences in predictions are solely due to differences in $\Delta$, the determinant of the equilibrium system. But $\Delta = C_1\sigma_X + C_2\sigma_u + C_3$, and the expressions for $C_1$, $C_2$, and $C_3$ in Section A.3 make it clear that whenever $\gamma_L$ and $\gamma_K$ are small relative to one, which is the case here, the term $C_1$ dominates and thus determines the magnitude of $\Delta$. (Recall that $\sigma_X = \sigma_u = 1$ in the parameterization.) Since the magnitude of $C_1$ directly depends on $B_L + B_K = 1 - B_Z$, choices of anchoring indices leading to very different magnitudes of $B_Z \equiv \beta\theta_{YZ}$ will lead to very different predictions. When $p_Y$ is chosen, $\beta = 1$, while for all other choices considered in Tables C.4 and C.5 $B_Z$ is either close to or equal to zero. It is equal to zero whenever $\beta = 0$ (choice of $p_X$, $w$, $r$, or

Table C.4: Pollution effect of a 10% increase in the nominal pollution tax (%)

| $e_{KZ}$ | $e_{LZ}$ | $\hat{Z}$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | $p_X$ | $p_Y$ | $\mathcal{P}$ | $w$ | $r$ | $\mathcal{R}$ |
| 1.0 | -0.5 | -7.88 | -14.28 | -8.02 | -7.85 | -7.91 | -7.88 |
| 0.0 | 0.0 | -4.36 | -7.88 | -4.44 | -4.38 | -4.35 | -4.36 |
| 0.5 | 0.0 | -6.48 | -11.72 | -6.60 | -6.49 | -6.48 | -6.48 |
| 1.0 | 0.0 | -8.59 | -15.55 | -8.74 | -8.57 | -8.61 | -8.59 |
| 0.0 | 0.5 | -5.06 | -9.13 | -5.15 | -5.09 | -5.03 | -5.06 |
| 0.5 | 0.5 | -7.18 | -12.98 | -7.31 | -7.20 | -7.17 | -7.18 |
| 1.0 | 0.5 | -9.30 | -16.82 | -9.46 | -9.28 | -9.31 | -9.30 |
| 0.0 | 1.0 | -5.75 | -10.37 | -5.85 | -5.79 | -5.72 | -5.75 |
| 0.5 | 1.0 | -7.88 | -14.23 | -8.02 | -7.91 | -7.86 | -7.88 |
| 1.0 | 1.0 | -10.00 | -18.08 | -10.18 | -10.00 | -10.00 | -10.00 |

Table C.5: Incidence effect of a 10% increase in the nominal pollution tax (%)

| $e_{KZ}$ | $e_{LZ}$ | Sources side: $\hat{w} - \hat{r}$ | | | | | | Uses side: $\hat{p}_Y - \hat{p}_X$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $p_X$ | $p_Y$ | $\mathcal{P}$ | $w$ | $r$ | $\mathcal{R}$ | $p_X$ | $p_Y$ | $\mathcal{P}$ | $w$ | $r$ | $\mathcal{R}$ |
| 1.0 | -0.5 | -0.08 | -0.15 | -0.08 | -0.08 | -0.08 | -0.08 | 4.48 | 8.11 | 4.56 | 4.46 | 4.50 | 4.48 |
| 0.0 | 0.0 | 0.08 | 0.14 | 0.08 | 0.08 | 0.08 | 0.08 | 4.46 | 8.05 | 4.54 | 4.48 | 4.44 | 4.46 |
| 0.5 | 0.0 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 4.47 | 8.08 | 4.55 | 4.47 | 4.47 | 4.47 |
| 1.0 | 0.0 | -0.05 | -0.10 | -0.06 | -0.05 | -0.05 | -0.05 | 4.48 | 8.10 | 4.56 | 4.46 | 4.49 | 4.48 |
| 0.0 | 0.5 | 0.11 | 0.19 | 0.11 | 0.11 | 0.11 | 0.11 | 4.46 | 8.04 | 4.54 | 4.48 | 4.44 | 4.46 |
| 0.5 | 0.5 | 0.04 | 0.07 | 0.04 | 0.04 | 0.04 | 0.04 | 4.47 | 8.07 | 4.54 | 4.48 | 4.46 | 4.47 |
| 1.0 | 0.5 | -0.03 | -0.05 | -0.03 | -0.03 | -0.03 | -0.03 | 4.47 | 8.09 | 4.55 | 4.47 | 4.48 | 4.47 |
| 0.0 | 1.0 | 0.13 | 0.24 | 0.14 | 0.14 | 0.13 | 0.13 | 4.45 | 8.03 | 4.53 | 4.49 | 4.43 | 4.45 |
| 0.5 | 1.0 | 0.07 | 0.12 | 0.07 | 0.07 | 0.07 | 0.07 | 4.46 | 8.06 | 4.54 | 4.48 | 4.45 | 4.46 |
| 1.0 | 1.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 4.47 | 8.08 | 4.55 | 4.47 | 4.47 | 4.47 |

$\mathcal{R}$) and it is close to zero when $\mathcal{P}$ is chosen because $\theta_Y$ is small (0.039 for this calibration).