Chen, Yu-Fu; Snower, Dennis J.; Gylfi, Zoega

Working Paper

Firing costs: eurosclerosis or eurosuccesses?

Dundee discussion papers in economics, No. 94

Provided in Cooperation with:
Kiel Institute for the World Economy – Leibniz Center for Research on Global Economic Challenges

Suggested Citation: Chen, Yu-Fu; Snower, Dennis J.; Gylfi, Zoega (1999) : Firing costs: eurosclerosis or eurosuccesses?, Dundee discussion papers in economics, No. 94, Department of Economic Studies, University of Dundee, Dundee

This Version is available at:
https://hdl.handle.net/10419/2227

Dundee Discussion Papers
in Economics

Firing Costs:
Eurosclerosis Or Eurosuccesses

Yu-Fu Chen,

Dennis Snower

and

Gylfi Zoega

Department of
Economic Studies,
University of Dundee,
Dundee.
DD1 4HN

Working Paper
No. 94
September 1999
ISSN:1473-236X

Firing Costs: Eurosclerosis or Eurosuccesses?

by

Yu-Fu Chen*

Department of Economic Studies
University of Dundee
Dundee DD1 4HN
United Kingdom

Dennis Snower
and
Gylfi Zoega

Department of Economics
Birkbeck College
7-15 Gresse Street, London W1P 2LL
United Kingdom

September 1999

ABSTRACT

In this paper we analyse the employment implications of firing restrictions. We find that when a recession is expected and the trend rate of productivity growth is small, a rise in firing costs affects mainly the hiring decision. Thus there is a negative effect on average employment. When, on the other hand, a boom is expected and the rate of productivity growth is large, firing costs affect mainly the firing decision. Then, as a result, average employment is increased. Our analysis suggests that while firing restrictions might have stimulated employment and reduced unemployment in Europe in the first two decades following World War II—when large supply shocks were absent and the average rate of growth was high—these same restrictions may have had the opposite effects in the 1970s and 1980s, when significant negative supply shocks occurred.

Keywords: Firing restrictions, stochastic demand, hiring and firing, real options

JEL: E32, J23, J24, J54

* Corresponding author. E-mail: y.f.chen@dundee.ac.uk. Fax: 44-1382-344691. We thank Juan Dolado and Margaret Stevens for comments.

Politicians and journalists often view labor market rigidities as an important source of the European unemployment problem. This syndrome is commonly called *Eurosclerosis*. One of the clearest manifestations of such rigidities are state-mandated restrictions on firing. Economic theory, however, has not identified an unambigously negative effect of firing costs on employment. For example, Bentolila and Bertola (1990), show that, assuming fixed wages and demand following a geometric Brownian motion, firing costs raise average employment over the business cycle because they discourage firing more than they discourage hiring (on account of quits and time discounting). The robustness of this result, however, depends on assumptions about the nature of the stochastic process governing demand. Bentolila and Saint-Paul (1994), considering an alternative demand process which consists of white-noise shocks in a discrete-time context, find a large negative effect of firing costs on average employment for low values of firing costs, but a positive effect for larger values. Other adverse effects of firing restrictions on employment include the indirect effect working through wage setting (Díaz and Snower, 1996) and the adverse effect on capital investment (Bertola, 1991).

In this paper, we show that firing costs tend to lower average employment when a recession is expected, the recession being characterized by large negative demand shocks—larger than any positive shocks. The opposite holds when a boom is expected, viz, relatively large positive demand shocks. When firm managers expect the negative shocks to be larger than the positive ones, the ability to fire in the future takes centre stage in any hiring decision. Firms are not willing to hire new workers unless they can be reasonably certain that they can fire them cheaply at a later stage, if they so choose. As firing restrictions are imposed under these conditions, the ability to fire is reduced, along with the incentives to hire. But, more interestingly, this may not prevent workers from being fired.

The reason is straightforward. The cost of firing consists of the sum of the direct firing costs (severence pay) and an indirect cost which takes the form of a sacrificed option to fire the worker later. As firing costs go up, the former is raised while the latter is reduced. The two effects cancel if firms expect the negative shocks to be very large and the only effect of raising firing restrictions is found in reduced hiring.

We argue that in the two decades following World War II, firing costs may have helped keep European employment rates high, while in the post-1970 environment—when

growth rates have been lower and adverse shocks frequent—firing costs are likely to have had a detrimental effect on employment. The model explains the empirical observation by Saint-Paul (1996) that most labour market restrictions in Europe were introduced during periods of economic expansions but partly relaxed during recessions.[1]

1. The Stochastic Environment and Firm Behaviour

We consider the behaviour of a representative firm which finds itself facing stochastic demand for its output and linear costs of hiring and firing workers. We assume that the firm has to pay wages to all employed workers and hence that they are engaged in production at all times. In this context, the firm's hiring and firing are intertemporal investment decisions. We will describe these decisions by deriving the profit maximizing threshholds at which the firm engages in hiring and firing.[2]

Let the representative firm have a linear production technology (1) and face a linear output demand function (2):

$$Q = gN, \tag{1}$$

$$P = Z - bQ, \tag{2}$$

where Q denotes production and sales, N is the size of the representative firm's workforce, g is the level of labour productivity, P is the product price, and Z is an index for the position of the direct demand curve. The only factor input in the firm is labour. The average quit rate per unit time is constant over time and equal to δ which gives:

$$dN = -d\ N\ dt \tag{3}$$

in the absence of any hiring and firing. Labour productivity grows at a deterministic exponential rate h_g:

$$dg = h_g g\ dt, \tag{4}$$

and Z follows a combined geometric Brownian motion and jump process:

$$dZ = h_Z dt + s_Z Z d\nu - Z dq_1 + Z dq_2 \tag{5}$$

[1] This includes the introduction of fixed-term contracts in France and in Spain in the early 1980s. See for example Bentolila and Saint-Paul (1992) and Saint-Paul (1996).

[2] We do not consider the effect of inventories in the analysis that follows. We also exclude the possibility of temporary layoffs.

where v is a Wiener process; $dz = e\sqrt{dt}$ (since e is a normally distributed random variable with mean zero and a standard deviation of unity), h_Z is the drift parameter and s_Z the variance parameter, dq_1 and dq_1 are the increments of Poisson processes (with mean arrival rates l_1 and l_2), and dq_1, dq_1 and dv are independent to each other [so that E(dwdq_1)=0, E(dwdq_2)=0, and E(dq_1dq_2)=0].

It is assumed that if a "recession" (or "boom") occurs, q_1 (or q_2) falls (or increases) by some fixed percentage f_1 (or f_2) with probability 1. Thus equation (5) implies that demand will behave as a geometric Brownian motion, but over each time interval dt there is a small probability $l_1 dt$ (or $l_2 dt$) that it will drop (or rise) to $1-f_1$ (or $1+f_2$) times its original value, and it will then continue fluctuating until another event occurs.

We model expectations about the future through the parameters σ, l_1, l_2, f_1, f_2 and η_Z. When σ is large and l_1 and l_2 close to zero, there is much uncertainty about the future but neither large negative nor positive shocks expected. However when σ is close to zero and l_1 (or l_2) is positive, we expect large discrete negative (positive) shocks. We are interested in testing the implications of different parameter configurations for the effect of firing costs on average employment.

Combining (1) and (2) gives

$$P \cdot f(Z, g, N) = gZN - bg^2 N^2. \tag{6}$$

The firm's revenue function is concave in labour productivity and employment. The representative firm bears labour adjustment costs: a hiring cost T per new employee and a firing cost F per dismissed worker, and pays an wage w (which grows at the same rate as expected demand) to its workers. Thus, the growth rate of wage is equal to $(h_Z - l_1 f_1 + l_2 f_2)$. If the worker leaves voluntarily—which they do at rate δ—the firm bears no firing cost.

Using Itô's Lemma, we derive the following Bellman equation for the value $V(Z, g, N)$ of the firm's stock of workers, at time zero, in the continuation region—defined by the hiring- and the firing thresholds—where the value of future hires or fires is not taken into account,

$$rV = \left(gZN - bg^2N^2\right) - wN - dNV_N + h_g gV_g$$
$$+ h_Z ZV_Z + \frac{1}{2}s_Z^2 Z^2 V_{ZZ} - l_1\{V - V[(1-f_1)Z]\} + l_2\{V[(1+f_2)Z - V]\}, \qquad (7)$$

and r is the real rate of interest. The first term on the right-hand side is the current profit (the difference between output and the total wage bill). The second term is the loss due to quits. The third term is the gain due to productivity growth. The last term is the change in the value of the firm caused by changes in demand.

From (7) we can derive the value of the marginal employed worker. (The derivation is given in the appendix.) The particular integral of equation (7) gives the expected present value of the marginal employed worker, $v = V_N$, which can be written as,

$$v^P(Z, g, N) = K_1 gZ - 2K_2 bg^2 N - K_3 w, \qquad (8)$$

where the three terms $K_1 = \left(r + d + l_1 f_1 - l_2 f_2 - h_g - h_z\right)^{-1}$, $K_2 = \left(\rho + 2\delta - 2\eta_g\right)^{-1}$, and $K_3 = \left(r + d + l_1 f_1 - l_2 f_2 - h_Z\right)^{-1}$ are the discount factors.

The general solutions for the hiring and firing options have the following forms respectively,

$$v_H^G(Z, g, N) = A_1(gZ)^{b_1}, \qquad (9)$$

$$v_F^G(Z, g, N) = A_2(gZ)^{\beta_2}. \qquad (10)$$

To satisfy the boundary conditions that $v_H^G(0, g, N) = 0$ and $v_F^G(\infty, g, N) = 0$, we use the positive solution for v_H^G and the negative solution for v_F^G.

The value of the marginal, employed worker is equal to the sum of v^P and v_F^G in the continuation region. In order to derive the two thresholds for hiring and firing, we then compare the value of the worker to the direct and indirect costs of hiring (firing) the workers. The definitions of the hiring and firing barriers, Z_H and Z_F, are given by the standard value-matching and smooth-pasting conditions below. The firm would find it optimal to exercise its option to hire or fire the marginal worker once Z hits one of the two barriers. The value-matching conditions follow;

$$K_1 gZ_H - 2K_2 bg^2 N - K_3 w + A_2(gZ_H)^{b_2} = T + A_1(gZ_H)^{b_1}, \qquad (11)$$

$$-\left[K_1 gZ_F - 2K_2 bg^2 N - K_3 w\right] + A_1(gZ_F)^{b_1} = F + A_2(gZ_F)^{b_2}, \qquad (12)$$

where T and F denote hiring and firing costs respectively.

The left-hand sides of (11) and (12) show the marginal benefit from hiring/firing a worker and the right-hand sides the marginal costs. The marginal benefit of hiring a worker is equal to the sum of the present discounted value of his productivity net of wages, on the one hand, and the value of the option to fire him, on the other hand. Thus a disposable worker is more valuable than one who cannot be dismissed; the ability to fire raises the benefit from employing a worker. On the other hand, the marginal cost of hiring is the sum of the direct hiring costs and the sacrificed option to hire him in the future. By hiring a worker today, the opportunity to do so in the future—when conditions may be more favourable—is sacrificed.

The interpretation of the firing decision is similar. By firing a worker, the opportunity to do so in the future—when demand conditions may be even more adverse—is sacrificed, and the opportunity to hire him again is gained.

The value of the two options depends on expectations about changes in demand. The option to hire is valuable if firms expect demand to increase in the future, while the option to fire is the more important if they expect it to fall. As this affects the marginal benefit and the marginal cost of hiring and firing, the level of the thresholds is affected and also the way they depend on the model's parameters; if the firing option is relatively important, parameters affecting its value become relatively important and the same for the hiring option. This will become important in our numerical solutions in Section 2 below.

The smooth-pasting conditions follow.[3]

$$K_1 g + A_2 b_2 Z_H^{b_2 - 1} g^{b_2} = A_1 b_1 Z_H^{b_1 - 1} g^{b_1}, \tag{13}$$

$$-K_1 g + A_1 b_1 Z_H^{b_1 - 1} g^{b_1} = A_2 b_2 Z_H^{b_2 - 1} g^{b_2}. \tag{14}$$

Equations (11), (12), (13) and (14) form a non-linear system of equations with four unknown parameters, Z_H, Z_F, A_1 and A_2, and can be solved for numerically once the solutions for β_1 and β_2 are found from the characeristic equaiton (A13). The demand thresholds for hiring and firing a marginal worker can be found once numerical values for Z_H and Z_F are known.

[3] These ensure that hiring (firing) is not optimal after the hiring- (firing) theshold is reached. If the smooth-pasting conditions were not satisfied, waiting longer and observing changes in demand would see different changes in the marginal benefit and the marginal cost of hiring (firing). By waiting and choosing the better of two options we can then always do better than acting at the threshold.

We can now solve the firm's optimisation problem for different values of the firing costs and show how they affect the two thresholds.[4]

2. The Influence of Firing Costs on Average Employment

Let us use the Bentolila-Bertola setup as a benchmark and set the probability of a Poisson jump to zero and let the remaining parameters be assigned the values given by them.[5] Figure 1 shows results very similar to theirs: the firing threshold (Z_F) is affected by more than the hiring threshold (Z_H) so that the slope of the firing threshold is greater than that of the hiring threshold. Thus firing costs stimulate employment.

2.1 The Case of Large and Frequent Negative Demand Shocks

Whereas the above result presupposes that firms are just confused about future demand (as represented by a high value of σ), we now assume that firms expect a "recession." In particular, they expect a discrete negative demand shock that is either *larger* than the expected positive shock or *more likely* to happen. In addition to the Geometric Brownian motion in the benchmark case (now with a lower value of σ), there is now a constant probability per unit time (0.20) of a discrete drop in the level of demand (30%). The probability of an equally sized demand increase is only 0.05. This makes the expected change in demand equal to –4.5% per unit of time. Figure 1 shows the effect of the firing costs on the hiring- and firing thresholds under these circumstances (Z'_H and Z'_F). Compared to the earlier results—both sets of thresholds have been normalised to start at the same value—the hiring theshold is affected by more and the firing threshold by a lot less. The firing threshold is now relatively flat—implying that raising firing costs does not affect the firing decision by much— while the hiring threshold is steeper.

[4]The method described here is different from that used by Bentolila and Bertola. In their paper, the representative firm's profit-maximisation problem is represented by a regulated stochastic process. Here the firm's profit-maximisation problem is transformed into a partial differentiation equation with two boundary conditions. Both methods give the same results mathematically although the form of the equations differs.

[5]They are ρ=0.05, δ=0.05, η_g=0.0, η_Z=0.0, b=0.5, w=1, and T = 0.083. Note that our demand function is different from that used in Bentolia and Bertola (1990).

In Figure 2, however, the probability of the two kinds of shocks is equal but the negative one is expected to be much larger (40%) instead of being only 10% in the case of the positive shock. Observe that the effect of the firing costs on the hiring- and the firing thresholds is similar to that in Figure 1.

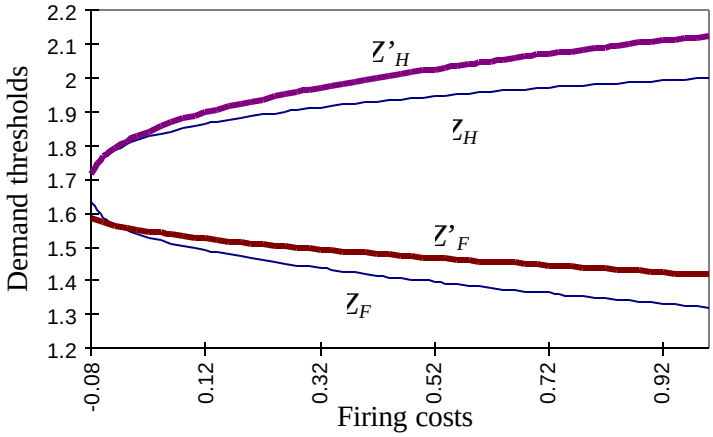

Figure 1. The effect of firing costs on the hiring- and firing thresholds of N with a jump process (s_Z =0.01, l_1=0.20, l_2=0.05, f_1=0.3, and f_2 =0.3), and without a jump (s_Z =0.12, l_1=l_2=0). The former thresholds are distinguished by a prime. Other parameters: r=0.05, d=0.05, h_g =0.0, h_Z=0.0, b=0.5, w=1, g_0 = 1, Z_0 = 2, N_0 = 1, and T = 0.083.

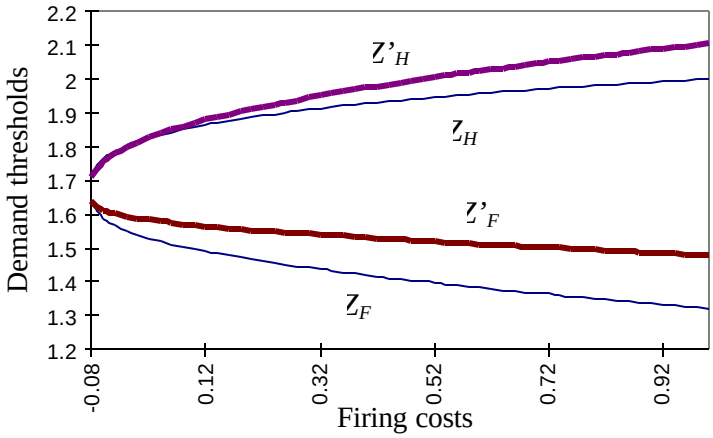

Figure 2. The effect of firing costs on the hiring- and firing thresholds of N with a jump process (σ_Z =0.01, l_1=l_2=0.15, f_1=0.4, and f_2=0.1), and without a jump (s_Z =0.12, l_1=l_2=0). The former thresholds are distinguished by a prime. Other parameters: r=0.05, d=0.05, h_g=0.0, h_Z =0.0, b=0.5, w=1, g_0 = 1, Z_0 = 2, N_0 = 1, and T = 0.083.

The intuition behind these results is straightforward. When the probability of a "recession" is an important constituent of a firm's uncertainty about the future (viz, a negative demand shock is more likely or greater in magnitude than a positive demand shock) then the option to fire is important. In particular, the ability to change the timing of hiring is worth much less

than the ability to change the timing of firing; by waiting, the firm is much more likely to gain valuable information about the optimal timing of firing than about the optimal timing of hiring. For this reason, the firing option is much more valuable than the hiring option. As firing costs increase, the option value of firing falls as it becomes more expensive to dismiss workers. Thus the total cost of firing—the direct firing cost plus the cost of sacrificing the firing option —rises less than the direct firing cost. As a result, the firing threshold becomes relatively flat.

However, the slope of the hiring threshold is affected. When a fall in demand is expected, firms are hesitant to hire a new worker unless they think they will be able to fire him later. Rising firing costs make it difficult to fire workers and this reduces the value of the firing option and the benefit from hiring. As a result, the hiring threshold becomes steeper.

We conclude that firing costs are not very effective at reducing layoffs when the risk of future demand shocks is asymmetric such that firms expect bad shocks in the future. But the effect on hiring can be substantial. We are lead to an apparently paradoxical conclusion; when a recession is expected and common sense tells us to impose restrictions on firing, the model tells us that such actions will only affect hiring adversely with little gain in the form of reduced firing.

2.2 The Case of Large or Frequent Positive Demand Shocks

We now turn to the "boom" outlook. Here firms expect relatively large or frequent discrete jumps in demand in the future. In Figure 2, we take into account a fixed probability (equal to 0.20) of a jump in demand to a higher level (30% higher) while there is a smaller probability of a negative shock (0.05). The firing threshold $\left(Z'_F \right)$ is now very steep while the hiring threshold $\left(Z'_H \right)$ is virtually horizontal. We again also show the benchmark case of no jumps, as in Figure 1. Again for comparison, both sets of thresholds have been normalised to start at the same value. The same occurs when we expect the positive shocks to be larger (40%) than the negative ones (10%) in Figure 4.

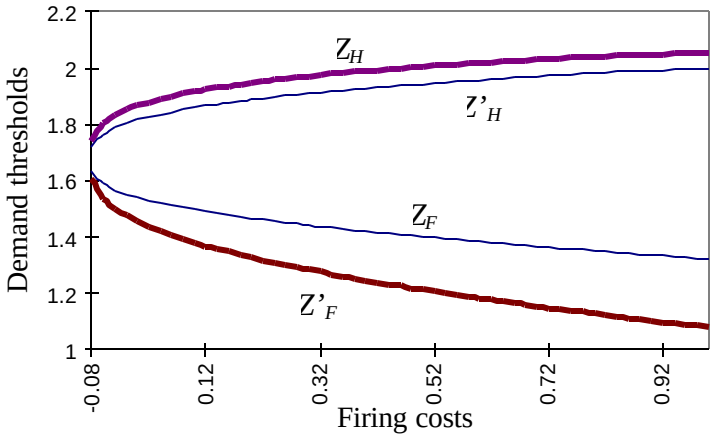

Figure 3. The effect of firing costs on the hiring- and firing thresholds of N with a jump process (s_Z =0.01, l_1=0.05, l_2=0.20, f_1=0.3, and f_2 =0.3), and without a jump (s_Z =0.12, l_1=l_2=0). The former thresholds are distinguished by a prime. Other parameters: r=0.05, d=0.05, h_g =0.0, h_Z=0.0, b=0.5, w=1, g_0 = 1, Z_0 = 2, N_0 = 1, and T = 0.083.

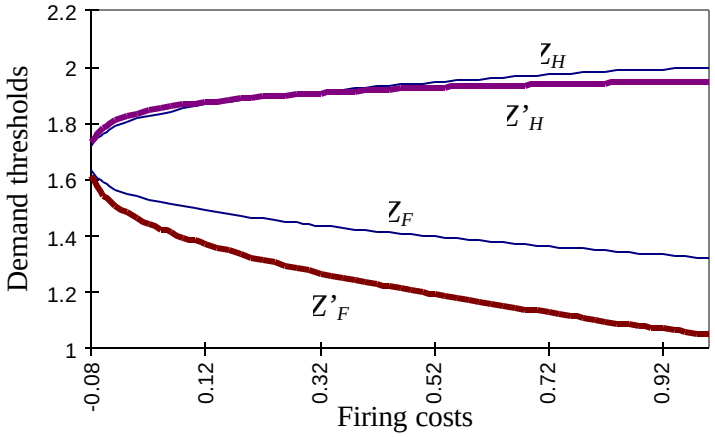

Figure 4. The effect of firing costs on the hiring- and firing thresholds of N with a jump process (σ_Z =0.01, l_1=l_2=0.15, f_1=0.1, and f_2=0.4), and without a jump (s_Z =0.12, l_1=λ_2=0). The former thresholds are distinguished by a prime. Other parameters: r=0.05, d=0.05, h_g=0.0, h_Z=0.0, b=0.5, w=1, g_0 = 1, Z_0 = 2, N_0 = 1, and T = 0.083.

In this case it is not likely that any given worker will be fired at any point in the future. The value of the firing option is low and it is hence not much affected by the level of firing costs. However, firms know that demand is very likely to go up in the future and the value of the hiring option is for that reason high. An increase in the cost of firing, by reducing slightly the value of the hiring option, reduces somewhat the marginal benefit from firing workers. So when the marginal cost of firing goes up with rising firing costs, the marginal benefit of firing

is reduced a little and the effect on the firing threshold is hence magnified. But the hiring threshold is not affected by much since the effect on the hiring option is relatively small.[6]

3.3 Productivity Slowdown and Adverse Shocks

We finally derive the combined effect of a slowdown in productivity growth and an increased probability of negative shocks, similar to what occurred in the 1970s and 1980s in most OECD countries. In particular, we derive the two thresholds; for the case of 2.5% rate of growth of productivity and zero probability of adverse shocks $\left(Z_H \text{ and } Z_F\right)$—a scenario resembling that in the 1950s and 1960s—and then for the case of a 1% productivity growth rate and a 20% probability of a large downturn and a 5% probability of a positive jump in demand $\left(Z'_H \text{ and } Z'_F\right)$ where the size of the jumps is equal— a scenario more resembling the economic situation around the large supply shocks of the 1970s and 1980s. The results are in Figure 3 where both sets of thresholds have been normalised to start at the same value.

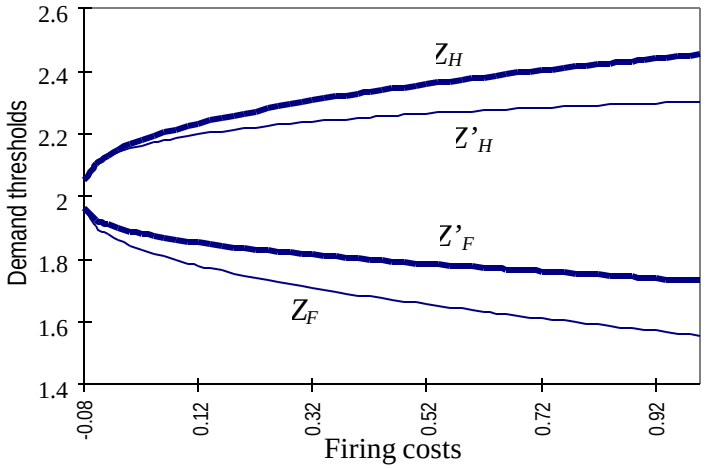

Figure 5. The effect of firing costs on the hiring- and firing thresholds with parameters corresponding to a high growth, no supply-shock, period (h_g =0.025, $l_1 = l_2 = 0$, s_Z =0.12) and a low growth, supply-shock period (h_g =0.01, l_1 = 0.20, l_2 = 0.05, $f_1 = f_2$ = 0.3, s_Z =0.01). The latter thresholds are distinguished by a prime. Other parameters the same.

We see that in the former case the firing restrictions affect mainly the firing threshold, while in the latter case it is the hiring threshold which is most affected. For this reason, restrictions on firing are more likely to have reduced average employment in the more recent period.

[6] The effect on the hiring option is indirect and operates only through the firing option.

Moreover, we find complementarities such that the effect of the growth slowdown on the hiring threshold is larger when adverse shocks are expected and vice versa.

3. Some Implications

At the aggregate level, firing restrictions may help—that is if, and this is a big if, they do not raise wages too much—when productivity is growing and the possibility of large adverse demand shocks remote. This was probably the case in Europe in the first two decades following the World War II. But lower growth in the past two decades (Maddison, 1987) and the higher probability of adverse shocks may have turned firing restrictions from a potentially helpful policy instrument to a likely cause of high unemployment.

We conclude that the imposition of firing costs can have surprisingly adverse effects on the rate of employment. Job security legislation may have unfavorable side-effects that make job security legislation self-defeating, especially at times when recession is anticipated. This is precisely the time when politicians often rely on this policy to preserve jobs. We have shown that when employment is threatened, firing costs are not likely to prevent layoffs. However, they may prevent firms experiencing positive idiosyncratic shocks from hiring.

In addition, labour market rigidities are subject to great inertia, difficult to reverse when they are no longer in the public interest, such as at times of adverse macroeconomic shocks and low productivity growth. For this reason, too, firing costs are a very dangerous instrument to stimulate employment.

References

Bentolila, Samuel and Giuseppe Bertola (1990), "Firing Costs and Labor Demand: How Bad is Eurosclerosis?", *Review of Economic Studies*, 57, 381-402.

Bentolila, Samuel and Gilles Saint-Paul (1992), "The Macroeconomic Impact of Flexible Labor Contracts, with an Application to Spain", *European Economic Review*, 36, 1013-53.

Bentolila, Samuel and Gilles Saint-Paul (1994), "A Model of Labor Demand with Linear Adjustment Costs", *Labour Economics*, 1, 303-26.

Bertola, Guiseppe (1991), "Flexibility, Investment, and Growth", mimeo, Princeton University.

Cohen, Daniel, Arnaud Lefrance and Gilles Saint-Paul (1997), "French Unemployment: A Transatlantic perspective," *Economic Policy*, 265-292.

Díaz, M. Pilar and Dennis J. Snower (1996), "Employment, Macroeconomic Fluctuations, and Job Security", CEPR Working Paper # 1430.

Dixit, Avinash K. (1993), *The Art of Smooth Pasting*, Vol. 55 in *Fundamentals of Pure and Applied Economics*, eds. Jacques Lesourne and Hugo Sonnenschein, Chur, Switzerland, Harwood Academic Publishers.

Dixit, Avinash K. and Robert S. Pindyck (1994), *Investment under Uncertainty*, Princeton University Press, Princeton, New Jersey.

Lazear, Edward P. (1990), "Job Security Provisions and Employment", *Quarterly Journal of Economics*, 105, 699-726.

Maddison, Angus (1987), "Growth and Slowdown in Advanced Capitalist Economies: Techniques of Quantitative Assessment," *Journal of Economic Literature*, XXV, 649-699.

McDonald, Robert, and Daniel R. Siegel, (1987), "Time to Build, Option Value, and Investment Decisions," *Quarterly Journal of Economics*, 101, 707-728.

Malliaris, A. G. and William A. Brock (1982), *Stochastic Methods in Economics and Finance*, New York: North-Holland.

Saint-Paul, Gilles (1996), "Exploring the Political Economy of Labour Market Regulations," *Economic Policy*, 23, 263-316.

Saint-Paul, Gilles (1996*), Dual Labor Markets*, The MIT Press, Cambridge, MA.

Appendix
Derivation of Equations (8)-(14)

Taking the derivative of (7) with respect to N gives;

$$(r+d)v = gZ - 2bg^2N - w - dNv_N + h_g g v_g + h_Z Z v_Z + \frac{1}{2}s_Z^2 Z^2 v_{ZZ} - l_1\{v - v[(1-f_1)Z]\} + l_2\{v[(1+f_2)Z - v]\}, \quad \text{(A1)}$$

where $v = V_N$. The problem now is to solve for $v(Z, g, N)$, which is the value of employing the marginal worker. The solution for $v(Z, g, N)$ consists of the particular integral and the complementary function. The particular integral of equation (A1), which is the expected present value of the marginal employed worker, can be written as,

$$v(Z, g, N) = E\int_0^\infty \left[g_t Z_t - 2bg_t^2 N_t - w_t\right] e^{-(\rho+\delta)t}\, dt. \quad \text{(A2)}$$

which simplifies to

$$v^P(Z, g, N) = K_1 gZ - 2K_2 bg^2 N - K_3 w, \quad \text{(A3)}$$

where the three terms $K_1 = (r + d + l_1 f_1 - l_2 f_2 - h_g - h_z)^{-1}$, $K_2 = (\rho + 2\delta - 2\eta_g)^{-1}$, and $K_3 = (r + d + l_1 f_1 - l_2 f_2 - h_Z)^{-1}$ are the discount factors.

The firm takes into account the option value of hiring in the future. There is also the option to fire the worker once he is employed. The two option values are measured by the complementary function. Focusing on the homogenous part of equation (A1), and letting v^G be the value of the marginal option,

$$(r+d)v = -dNv_N + h_g g v_g + h_Z Z v_Z + \frac{1}{2}s_Z^2 Z^2 v_{ZZ} - l_1\{v - v[(1-f_1)Z]\} + l_2\{v[(1+f_2)Z - v]\}, \quad \text{(A4)}$$

The general solution to equation (A4) has the same component as the complementary ones. That is, the general solution has the following functional form

$$v = A(gZ)^b. \quad \text{(A5)}$$

This gives the following relationships

$$h_g g v_g = h_g b v, \quad \text{(A6)}$$

$$dNv_N = 0. \quad \text{(A7)}$$

$$h_Z Z v_Z = h_Z b v, \quad \text{(A8)}$$

$$\frac{1}{2}s_Z^2 Z^2 v_{ZZ} = \frac{1}{2}s_Z^2 b(b-1)v, \quad \text{(A9)}$$

$$v[(1-f_1)Z] = (1-f_1)^b v. \quad \text{(A10)}$$

$$v[(1+f_2)Z] = (1+f_2)^b v. \quad \text{(A11)}$$

Substituting (A6), (A7), (A8), (A9) (A10) and (A11) into (8) in the text gives

$$v\left[\frac{1}{2}s_Z^2 b(b-1) + h_Z b + h_g b - l_1\left[1-(1-f_1)^b\right] + l_2\left[(1+f_2)^b - 1\right] - (r+d)\right] = 0. \quad \text{(A12)}$$

Equation (*A*12) must hold for any value of v, so that bracketed terms must equal zero:

$$\frac{1}{2}s_Z^2 b(b-1) + h_Z b + h_g b - l_1\left[1-(1-f_1)^b\right] + l_2\left[(1+f_2)^b - 1\right] - (r+d). \quad \text{(A13)}$$

Thus, (*A*5) becomes

$$v = A_1(gZ)^{b_1} + A_2(gZ)^{b_2}. \quad \text{(A14)}$$

where b_1 and b_2 are the positive and negative roots of (A12).

The general solutions are equal to the value of the options to fire or hire the marginal worker. When *Z* goes to infinity, the value of the option to fire has to go to zero. Hence A_1 is equal to zero for the value of option to fire.[7] Similarly, when Z approaches zero, the value of the option to hire has to go to zero. Hence we set $A_2 = 0$ for the value of option to fire. The general solutions for the hiring and firing options have the following forms respectively,

$$v_H^G(N, Z, g) = A_1(gZ)^{b_1}, \quad \text{(A15)}$$

$$v_F^G(N, Z, g) = A_2(gZ)^{b_2}. \quad \text{(A16)}$$

[7] Note that b_1 is positive and b_2 is negative.