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Divergence in Labour Force Growth:

Should Wages and Prices Grow Faster in Germany?

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Abstract

We develop a model which shows that wages, prices and real income should grow faster in countries with low increase in their labour force. If not, other countries experience growing unemployment and/or trade deficit. This result is applied to the case of Germany, which has displayed a significantly lower increase in its labour force than its trade partners, except in the moment of the reunification. By assuming that goods are differentiated according to their country of origin (Armington’s hypothesis), a low growth of the working population constrains the production of German goods, which entails an increase in their prices and in German wages. This mechanism is magnified by the low price elasticity of the demand for German goods. Hence, the German policy of wage moderation could severely constrain other countries’ policy options. The simulations of an extended model which encompasses offshoring to emerging countries and labour market imperfections suggest that (i) the impact of differences in labour force growth upon unemployment in Eurozone countries has been significant and (ii) the German demographic shock following unification could explain a large part of the 1995-2005 German economic turmoil.

J.E.L. E24; F16; J11; O57.

Keywords. Population growth; Labour force; Inflation; Wages; Germany

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1. Introduction

We develop a model which shows that wages, prices, and real income should grow faster in countries with low increase in their labour force. If not, other countries suffer from growing unemployment and/or trade deficit. The model is used to explain the developments of the German economy relative to other advanced countries.

In the last fifty years, the German economy has exhibited a number of specificities. First, German inflation has been significantly lower than that of other advanced economies. As regards Eurozone countries, the resulting gain in price competitiveness has only been partially offset by the appreciation of the German currency (the DM until 1999) and the German effective real exchange rate has depreciated in the long run. Second, German unemployment has permanently been lower than that of most other advanced economies, except in the 15 years following the reunification in 1990. Third, the German trade surplus has regularly increased, except once again in the years following the reunification. Finally, Germany has experienced a significantly lower increase in its labour force than other advanced economies, with however a one-shot shock in the unification year. Several explanations have been given to the first three German specificities: a stable and strict monetary policy, the social consensus which drives the employer-employee relationship, a quality and productivity gap with other countries, and a large propensity to save of German households.

The model developed here provides an additional explanation of the German economic performance in comparison to that of other advanced economies. This explanation is based on the divergence in labour force growth across countries.

From a simple theoretical model, we firstly determine a relationship between the growth in labour force and the rates of variation in nominal wages, prices and real income. We show that (i) when assuming perfect competition in labour markets (full employment) and balanced trade in all countries, a country with a relatively low increase in its labour force should exhibit a higher growth in its wages, prices and real income than its trade partners, and (ii) that if the latter outcomes are prevented, then the low increase in the country’s labour force generates growing unemployment and/or trade deficits of its trade partners.

Those results are subsequently used to explain the economic differences and relationships between Germany and other advanced countries. When goods are differentiated according to their country of origin (Armington’s hypothesis), the low increase in the German labour force limits the production of German goods, which in turn increases their price in relation to that of
other goods at the full employment equilibrium. This mechanism is magnified if the price elasticity of the demand for German goods is low because a low elasticity implies that prices must rise significantly to reduce demand. Finally, higher prices of German goods imply higher wages of German workers and a higher real income per capita in Germany compared to other advanced countries since consumer prices partly depend on the prices of imported goods. This is what should have happened in case of perfect competition in all labour markets and balanced trade.

In contrast, if prices and wages of other countries do not decrease in relation to those of Germany, this lessens the demand for their goods and generates growing unemployment. In addition, if a share of the German total income moves to other countries, this can reduce unemployment in those countries, but it generates a German trade surplus. This is what has been observed before the German reunification and after 2005, and the effects have been strengthened by the low price elasticity of German goods.

The major analytical results of the paper are in a first step derived from a simple stylised model with two advanced economies, each producing a country-specific good, and one of them being characterised by a low growth of its labour force.

This simple framework is convenient to show the results ceteris paribus, but it cannot account for certain developments which can modify the analysed outcomes, such as the large offshoring of low-skilled production stages to emerging countries which has characterised German industries since the mid-90s and the differences between Eurozone and non-Eurozone countries in their relations with Germany.

So as to analyse the mechanisms within a more realistic framework, we construct in a second step an extended model in which (i) there are three advanced areas (Germany, Eurozone and non-Eurozone advanced economies) and one emerging area, and (ii) we introduce growing offshoring of low-skilled production stages to emerging countries. This model is subsequently utilised to simulate several scenarios: (1) a situation characterised by differences in labour force growth, competitive labour markets and balanced trade, (2) the combination of differences in labour force growth with offshoring, wage rigidity in the unskilled labour markets and income transfers, and (3) a counterfactual scenario which assumes no difference in labour force growth across countries within the second scenario.

The parameters and exogenous variables for the simulations have been derived in line with the structure of the model and the groups of countries (Germany, Eurozone and non-Eurozone advanced economies) considered. Data from the World Input-Output Database (WIOD) has been used to calculate time series for the offshoring intensities for the three country areas.
taking into account the input-output linkages between countries along global value chains. The same database has been utilised for the estimation of export equations to derive the price elasticity of exports and the elasticity of substitution between tradable goods produced in the three areas.

Section 2 puts forward the major stylised facts and briefly reviews the literature on the subject. Section 3 develops a general equilibrium model to show our key results. Section 4 presents the extended model utilised for the simulations of the German case and Section 5 describes the implementation and results of those simulations. The major findings are summarised and discussed in Section 6.

2. Stylised facts and literature

2.1. Stylised facts

We make a distinction between three areas, namely, Germany, other advanced euro area economies called Eurozone, and advanced non-Eurozone countries called North. This distinction is motivated both by different developments in Eurozone and North and because Germany and Eurozone share the same currency since 1999. The term Eurozone will be utilised even when referring to the time before the introduction of the euro.

2.1.1. Low growth of the German labour force

Structurally, Germany exhibits a lower increase in its labour force than other advanced economies (Fig. 1a). This can be observed before (1970-1989) and after (since 1990) the unification. However, the unification acted as a one-shot jump in the German labour force which increased from 30.7 million in 1990 to 39.6 million in 1991.

A comparison of labour force developments should also take into account the decrease in the average annual hours worked which has been more pronounced in Germany than in other advanced countries (Fig. 1b). This shows that the labour force growth gap between Germany and other advanced economies has even been more pronounced in terms of hours worked than in terms of number of persons (Fig. 1c).
Fig. 1. Growth in the labour force, 1970-2015 (initial year = 100)


2.1.2. Low inflation and the real exchange rate

Since 1970, inflation has been continuously lower in Germany than in other advanced countries, except (i) in the three years following the reunification, and (ii) since 2008 because of the decrease in inflation in Southern Europe. From 1970 up to 2015, prices (GDP deflator) have quintupled in Eurozone compared to Germany and doubled in North (Fig. 2). The appreciation of the German currency (D-Mark until 1999 and euro afterwards, Fig. 3) has counteracted the inflation gap, but the total impact differs across areas (Fig. 4). In the long term, German prices have decreased compared to Eurozone prices. This decrease has been particularly large between 1995 and 2005, with German prices falling by almost 25% in relation to Eurozone prices. In contrast, in the long term, the real exchange rate of Germany in relation to non-Eurozone advanced economies has slightly increased, albeit with very large fluctuations due to the volatility of the US dollar (Fig. 4).
2.1.3. Low unemployment except in the post-reunification period

As shown in Fig. 5, Germany experienced lower unemployment throughout the last fifty years compared to Eurozone countries, except in the years 2001-2008. Compared to other advanced economies, the evidence is mixed. German unemployment is lower until the early 80s, similar from the early 80s to the early 90s, higher from the early 90s to 2008 and lower since then. A general observation for years after the reunification (1990) is that German unemployment increased relative to both areas up to 2006 and decreased afterwards.

Notes: The unemployment gap is the difference in unemployment rates between the respective area and Germany. The unemployment rate is the ratio of unemployment to the labour force in each area. Data source: OECD Stat. (Population and Labour Force).
2.1.4. Permanent trade surplus and large offshoring

In the last fifty years, Germany has had a permanent and large trade and current account surplus with both non-Eurozone and Eurozone advanced economies, except in the eight years after the reunification regarding the latter area (Fig. 6). Since the 2008 financial crisis, the surplus with non-Eurozone countries has continued to grow, whereas the surplus with Eurozone has decreased because of the severe recession in Southern Europe. Moreover, since the mid-90s, Germany has experienced an increase in offshoring to emerging countries which is much larger than that of other advanced countries (Fig. 7).

**Fig. 6.** German current account surplus (in % of German GDP)

**Fig. 7.** Imports of intermediate goods from emerging countries (in % of area GDP)

Data sources: Deutsche Bundesbank (Fig. 6) and the CHELEM database (Fig. 7).

2.2. Related literature: Demographic changes and German economic performance

The impact of demographic changes on relative prices across countries has been essentially analysed through the influence of ageing on the real effective exchange rate (REER), see Hassan et al. (2011) for a survey. The key mechanism can be summarised as follows. According to the life cycle hypothesis (LCH), individuals save during their working life and dissave once retired. Applying the LCH to open economies with mobility of capital flows and differences in age structures, countries with a large proportion of retired households tend to save less and hence to benefit from net capital inflows whereas countries with a large proportion of working households experience net capital outflows. As capital inflows lead to exchange rate appreciation, the REER tends to appreciate in ageing countries.\(^1\) Georges et al. (2013) combine these life-cycle features with an Armington trade structure in a multi-country OLG model and show that the rapid ageing of North countries results in an appreciation of their REER relative to South countries. Another channel of ageing results from the fact that elderly people demand more non-tradable services, which raises the country’s price index.

\(^1\) This impact can be counteracted by the fact that countries with ageing populations tend to invest less. Then, a decrease in domestic investment may lead to capital outflows and REER depreciation (Fougère & Mérette, 1998).
through the Balassa-Samuelson effect, and thereby its REER (Van Ewijk & Volkerink, 2012; Groneck & Kaufmann, 2017). If a country is a net creditor, a real appreciation may also be induced by a declining birth rate and the resulting increase in wealth per capita which acts on consumption (Aloy & Gente, 2009). The impact of ageing on the REER seems to be confirmed by the empirical work of Anderson & Osterholm (2006) and Salim & Hassan (2012), among others. In addition, Ito & Tabata (2010) analysed the impact of ageing in one country on welfare in another country. At the efficient steady state equilibrium of an overlapping-generation model with between-country capital mobility, they show that the rise in old age survival probability in one country is welfare-enhancing for the other country when this probability is low, but welfare-deteriorating when this probability is high.$^2$

To the best of our knowledge, the demographic channel put forward in our paper, i.e. the impact of divergence in labour force growth on countries’ economic outcomes, has not been explored by the literature which has focused on ageing.

In addition to the theoretical results related to labour force divergence, our work utilises these results in the German context and contributes to the literature explaining key aspects of the German economic performance. In the last fifty years, the German economic performance in comparison to that of other advanced economies can be broadly divided in three phases (Giersch et al., 1992, chap. 5, for the 70s and 80s; Carlin & Soskice, 1997, for the 80s and 90s). From the early 70s up to the unification, Germany was characterised by low inflation, lasting revaluation of the D-Mark and quasi-permanent trade surpluses. In addition, Germany appeared less hurt by the two oil shocks than most advanced countries. During the decade following the unification, Germany was diagnosed as the ‘sick man’ in Europe, with its growing unemployment and low growth. Since 2005, Germany has experienced a new age of ‘economic miracle’ with permanently decreasing unemployment, economic growth rates higher than those of other European countries and large trade surpluses.

The determinants of the German economic performance highlighted in the literature have varied across time. The explanations of the German characteristics of the 70s and 80s (low inflation, DM appreciation, trade surplus and relatively low unemployment) have been based on three factors: (i) a monetary policy which targeted long-term stability, (ii) bargaining practices based on social consensus, and (iii) permanent increases in productivity and quality. The post-unification breakdown of the German economy has been linked to the loss of competitiveness brought about by high wages and low labour market flexibility in a period of

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$^2$ The impact of ageing on per capita output has also been analysed from endogenous and semi-endogenous growth models by Prettner (2013).
high unification-related public expenditure. The good performance of the German economy since 2005 has received great attention in the literature. Several explanations have been put forward. The first one considers the labour market reforms (Hartz reforms implemented from 2003 to 2005) as the starting point of an improvement in German competitiveness (e.g., Kirkegaard, 2014, and Rinne & Zimmermann, 2013). Dustmann et al. (2014) however noted that the improvement in German competitiveness and trade began in the 90s and could result from the significant changes in the labour market structure prior to the Hartz reforms (see also Carlin & Soskice, 2009). Burda & Hunt (2011) show that the German firms’ decision to lessen the working time rather than the workforce was a key factor explaining the German low unemployment after the 2008 financial crisis. Beissinger et al. (2016) point to the fact that the improvement in German competitiveness and trade could essentially result from the offshoring behaviour of firms, the labour market reforms permitting to erase the offshoring-related unemployment.

Changes in international price competitiveness are typically reflected in changes in the real effective exchange rate (REER). Those changes may explain the imbalances in the euro area and the special position of Germany because price divergence can no longer be offset by exchange rate adjustments, as argued by Belke & Dreger (2013). Related to this is the ongoing policy debate on whether Germany should foster higher growth in wages and prices that could lessen other countries’ relative prices, making it easier for unbalanced countries to adjust (Economist, 2017; Sinn, 2014). Another explanation for the changes in the REERs in the euro area can be found in the catching-up of Southern Europe combined with the Balassa-Samuelson effect (Diaz del Hoyo et al., 2017).

In those debates, the fact that Germany has had a lastingly low increase in its labour force compared to other countries has to our knowledge not been considered.

### 3. A stylised model

We construct a stylised model which assumes two countries and permits to determine our major findings *ceteris paribus*. It must be stressed that the theoretical results can be applied to any two countries. In line with the simulations we perform later on, the two countries are named country $G$ (for Germany) and $E$ (for euro area countries other than Germany).³

³ In the extended model considered in Section 4 a third advanced country, called North, and one emerging country, called South, are included in the analysis.
There are two tradable goods which are differentiated according to their country of origin (Armington’s hypothesis), and one non-tradable service. Tradable goods are produced with both skilled labour $H$ and unskilled labour $L$, whereas the production of the non-tradable service utilises unskilled labour only. The markets for goods and services are competitive.

3.1. Cross-country differences in labour force growth

The country endowments with unskilled and skilled labour at time $t$ are denoted $\bar{L}_i(t)$ and $\bar{H}_i(t)$, $i = G, E$. To focus on the impact of divergence in labour force growth, we assume that the relative endowments $\bar{\lambda}_i$ are unchanged over time in each country:

$$\frac{\bar{H}_i(t)}{\bar{L}_i(t)} = \bar{\lambda}_i, \quad i = G, E$$

Consequently, $\bar{L}_i(t)$ and $\bar{H}_i(t)$ grow at the same exogenous rate $n_i$, $i = G, E$:

$$\bar{L}_i(t) = \bar{L}_i(0) \times e^{n_i t}$$
$$\bar{H}_i(t) = \bar{H}_i(0) \times e^{n_i t}$$

The growth rate of the labour force is lower in country $G$ compared to country $E$:

$$n_G < n_E$$

3.2. Demand for goods and services

To keep the model as simple as possible, savings decisions are ignored, reducing the consumers’ decision to a static optimization problem. The representative household in each country maximises the following utility function subject to the usual income constraint:

$$u = y_{NT}^{1-\beta} y_{T}^{\beta}, \quad \text{with: } y_T = \left( a_G y_G^{(\sigma-1)/\sigma} + a_E y_E^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

where $y_{NT}$ denotes the representative household’s consumption of the non-tradable service $NT$ and $y_T$ the quantity index comprising the household’s consumptions of country–specific tradable goods $y_G$ and $y_E$. The parameter $a_i$ depicts the attractiveness of tradable good $i$, and $\sigma$ is the elasticity of substitution between goods $G$ and $E$.

The utility function defines the consumer price $P_i$ in each country $i = G, E$:

$$P_i = \left( p_{NT} \right)^{1-\beta} \left( P_T \right)^{\beta}, \quad \text{with: } P_T = \left( a_G^{\sigma} p_G^{1-\sigma} + a_E^{\sigma} p_E^{1-\sigma} \right)^{1/(1-\sigma)}$$

where $p_{NT}$ and $P_T$ respectively depict the price of the non-tradable service in country $i$ and the price index of tradable goods, and $p_i$ is the price of tradable good $i = G, E$.

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4 To simplify the notation the time index is omitted wherever possible.
Because of function (4), consumers’ expenditures for the NT service account for the proportion \((1 - \beta)\) of total income in each country.

### 3.3. Production

In both countries, the non-tradable service NT is produced with unskilled labour. One unit of unskilled labour produces one unit of NT. Consequently:

\[ Y_{NT}^i = L_{NT}^i, \quad i = G,E \]

where \(Y_{NT}^i\) is the quantity of non-tradable service NT produced in country \(i\) and \(L_{NT}^i\) the unskilled labour utilised in country \(i\) for the production of NT.

The sector of tradables (T) comprises two country-specific goods \(i = G,E\), produced with both skilled and unskilled labour.

Good \((T,i)\) is produced in country \(i\) with the Cobb-Douglas technology:

\[ Y_{Ti} = A_i \left( L_{Ti} \right)^{\alpha} \left( H_{Ti} \right)^{1-\alpha} \]  \quad (6)

where \(L_{Ti}\) (resp. \(H_{Ti}\)) is the unskilled (resp. skilled) labour utilised in country \(i\) for the production of good \((T,i)\).

In the sector of tradables, the country-specific technologies can differ in their total factor productivity \(A_i\), but not in their factor intensity \(\alpha\).

This production structure defines the total factor demands in each country (superscript \(d\) denotes demand): \(L_{di}^i = L_{NT}^i + L_{Ti}\) and \(H_{di}^i = H_{Ti}, \quad i = G,E\).

### 3.4. Equilibrium in perfect competition

Assume that labour markets are perfectly competitive. Consequently, the unit wages of skilled and unskilled labour, \(w_H^i\) and \(w_L^i\), ensure full employment of both types of labour in both countries. We thus have \(H_i = \bar{H}_i\) and \(L_{Ti} + L_{NT}^i = \bar{L}_i, \quad i = G,E\).

The full employment equilibrium of the model is characterised by balanced trade between the two countries and by the following relations (Appendix A):

\[
\frac{P_G(t)}{P_E(t)} = \frac{a_G \left( A_E \right)^{\frac{1}{\sigma}} \left( \frac{\lambda_E}{\lambda_G} \right)^{\frac{1-\alpha}{\sigma}} \left( \frac{L_E(0)}{L_G(0)} \right)^{\frac{1}{\sigma}}} {a_E \left( A_G \right)^{\frac{1}{\sigma}} \left( \frac{\lambda_G}{\lambda_E} \right)^{\frac{1-\alpha}{\sigma}} \left( \frac{L_G(0)}{L_E(0)} \right)^{\frac{1}{\sigma}}} \times e^{-\left( \frac{n_E - n_G}{\sigma} \right)} \]  \quad (7)
Proportion 1. Assume a given relative attractiveness $a_G / a_E$, a given relative TFP $A_G / A_E$, and given relative factor endowments $\lambda_G$ and $\lambda_E$. If the growth rate of country G’s labour force $n_G$ is lower than that of country E, $n_E$, then at the full employment equilibrium:

1) The relative price $p_G / p_E$ and the relative wages $w_L^G / w_L^E$ and $w_H^G / w_H^E$ increase with time at rate $(1-\sigma(1-\sigma)) = \sigma > 0$, the relative consumer price $P_G / P_E$ at rate $0 < (1-\beta(n_E - n_G)) / \sigma < \sigma$, and the relative real income per head $i_G / i_E$ as well as relative real wages $v_L^G / v_L^E$ and $v_H^G / v_H^E$ at rate $0 < \beta(n_E - n_G) / \sigma < \sigma$.

2) Those rates increase with the difference in labour force growth $n_E - n_G$.

3) Those rates are all the larger the smaller the elasticity of substitution $\sigma$.

Proof. Relations (7) – (9). Appendix A for consumer prices, relative real wages and income.

Proposition 2. For given relative factor endowments $(\lambda_G, \lambda_E)$, the relative price $p_G / p_E$ is:

1) An increasing function of country G’s relative attractiveness $a_G / a_E$.

2) A decreasing function of country G’s relative TFP $A_G / A_E$.

Proof. Relation (7).

Proposition 3. For given relative factor endowments $(\lambda_G, \lambda_E)$, the relative wages $w_L^G / w_L^E$ and $w_H^G / w_H^E$ are:

1) An increasing function of country G’s relative attractiveness $a_G / a_E$.

2) An increasing function of country G’s relative TFP, $A_G / A_E$, if the elasticity of substitution $\sigma$ is higher than one, and a decreasing function if $\sigma$ is less than one.

Proof. Relations (8) and (9).

Proposition 1 reveals that, at full employment in both countries, the country with the lower growth of its labour force experiences a permanent increase in its prices, in its real and
nominal wages and in its real income per capita compared to the other country. These results can be easily extended to the case of several countries, the hierarchy of price and wage growth being then the reverse of the hierarchy in labour force growth.

It must be noted that relations (7) – (9) are only valid at the full employment equilibrium. If, for any reason, the price and wage adjustments are impeded, the adjustment should operate through employment \((L_i \text{ and/or } \lambda_i)\) which then differ from \(\bar{L}_i \text{ and } \bar{\lambda}_i, i = G, E\). The section below analyses the case in which imperfections in the unskilled labour market prevent wage and price adjustment.

3.5. Equilibria with imperfect unskilled labour markets

Two types of unskilled labour market imperfections could be considered.

First, we could assume that country \(i\)’s skill premium \(w_i = w_i^H / w_i^L, \ i = G \text{ and/or } E\), is upward rigid, i.e., it cannot go above a certain level which is lower than the full employment skill premium. This portrays a situation in which the nominal wage of unskilled workers is bound to that of skilled workers, to the average wage or to the country’s production prices. Straightforwardly, such imperfection leads to unskilled unemployment in country \(i\).

We will focus on a second market imperfection which consists in binding the unskilled wages of both countries. More specifically, we assume (i) that the relative unskilled wage \(\sigma = w^{G}_L / w^{E}_L\) is growing more slowly than its full employment rate \(\sigma = (n_E - n_G) / \sigma\) (see Proposition 1), which makes \(\sigma\) increasingly deviate from its full employment value, and (ii) that perfect competition prevails in the skilled labour market, leading to full employment of skilled workers in both countries. In assumption (i) the constrained country is country \(E\).

This market imperfection is a more suitable constraint because it is in line with the model’s framework which focuses on the variations in the between-country prices and wages as the main adjustment mechanism, and because it can portray a larger range of market misadjustments. When all labour markets are perfectly competitive, the full employment adjustment prevails. However, when wages are to a large extent institutionally determined through employer-employees bargaining, public policies and labour legislations, then assumption (i) implies that both countries’ institutional decisions interact with each other and jointly influence the wage and price dynamics in each country, particularly when the exchange rate adjustment does not offset the changes in production costs expressed in national currencies.
Proposition 4. Assume imperfections in country E’s unskilled labour market causing the relative unskilled wage \( \sigma = \frac{w^G_L}{w^E_L} \) (i) to be lower than its full employment value and (ii) to grow at rate \( \gamma^* \) lower than \( \overline{\gamma} = \frac{n_E - n_G}{\sigma} \). Then:

1) Country E experiences growing unskilled unemployment.
2) Country E’s skill premium decreases, which implies an increase in country G’s relative skill premium \( \frac{w_G}{w_E} \).

Proof. Appendix B.

3.6. Income transfers

Within our general equilibrium model, income transfers across countries are a necessary condition for trade imbalances. Income transfers can cover a number of mechanisms, the most usual being savings transfers through financial flows. Since our purpose is not to analyse the reasons for income transfers but their impacts on trade balance, we do not explicitly introduce savings decisions into our model. Instead, we assume (net) income transfers from one country to the other without specifying the source of this transfer. Note that two mechanisms may create such transfers within our model, i.e., (i) public transfers and (ii) remittances linked to migration.

Proposition 5. A net income transfer \( \varphi \) from country G to country E entails:

1) A trade surplus of country G (equal to \( \varphi \)), and an equivalent trade deficit of country E.
2) A reduction in country E’s unemployment and an increase in both countries’ skill premia if country E experiences unemployment of unskilled workers due to the binding of its unskilled wage to country G’s unskilled wage.

Proof. Appendix C. Note that, if country G experiences unemployment of unskilled workers, the transfer from G to E raises G’s unemployment.

4. The extended model

The creation of the euro area has typically erased one adjustment mechanism for the involved countries. Without exchange rate adjustment, the differences in price variations between Germany and other euro area countries which ensure full employment can only result from adjustments in wages. This can be difficult with imperfectly competitive labour markets. In addition, the US dollar and the currencies influenced by the dollar have been highly volatile
relative to the German D-Mark and the euro, and those wide variations are to a large extent independent from inflation differentials. To account for those elements, we have extended the model by assuming three advanced areas, namely Germany \((G)\), Eurozone countries except Germany \((E)\) and other advanced countries (labelled \(N\) for North). By assuming three advanced economies, we can also differentiate Germany from other countries in terms of elasticity of substitution and reveal thereby the impact of this difference on the price and wage variations.

In the globalised economy, many low-skill intensive production segments are relocated to emerging countries. Hence, the price of each country-specific good does not only depend on the production costs in the country itself, but also on the costs in emerging countries where those segments are offshored. This can modify the above findings because the offshoring intensity has been very divergent across advanced economies (Fig. 7). To model those impacts, we extend our approach by adding two assumptions:

1) A new area called South \((S)\) is introduced which is characterised by a large amount of unskilled labour with a low cost (wage). Hence, the world comprises four areas: three advanced areas \((G, E\) and \(N)\) and the South which is endowed with unskilled labour only with a wage significantly lower than low-skilled wages in all advanced countries.

2) The production of tradable goods is decomposed in different segments, and the segments utilising unskilled labour can be offshored to the South, involving however an extra cost which differs across segments and countries.

For simplicity, it is assumed that the South only produces non-traded services and the offshored segments of goods \(G, E\) and \(N\).

4.1. The demand for goods and services

We extend the utility function (4) of the representative consumer by assuming three country-specific tradable goods denoted \(G, E\) and \(N\):

\[
u = y_N^{1-\beta} y_T^\beta \quad \text{with} \quad y_T = \left[ a_G y_G^{\theta_1} + \left( a_E y_E^{\theta_2} + a_N y_N^{\theta_2} \right)^{\theta_1/\theta_2} \right]^{1/\theta_1}
\]  

where \(\sigma_1 = (1-\theta_1)^{-1}\) is the elasticity of substitution between country \(G\)’s tradable good and other tradable goods and \(\sigma_2 = (1-\theta_2)^{-1}\) denotes the elasticity of substitution between tradable goods \(E\) and \(N\). The total demand functions for goods and services are shown in Appendix D1.
4.2. Production

The production of the tradable good \( Y_{ti} \), \( i = G, E, N \), uses a Cobb-Douglas technology which combines (i) an intermediate good produced one-to-one with high-skilled labour and (ii) \( F \) unskilled segments, each one utilising one unit of low-skilled labour to produce one unit of the respective segment. The production function is:

\[
Y_{ti} = A_i F_i \alpha H_i^{1-\alpha} \prod_{f=1}^{F} L_{if}^{\alpha/F}, \quad i = G, E, N; \tag{11}
\]

where \( L_{if} \) denotes unskilled labour that is employed in the production of segment \( f \) in country \( i \). Advanced countries can offshore some or all low-skill intensive segments to countries where their cost of production is lower.

4.3. Offshoring

The cost of relocating production abroad combines two components. First, wherever the production is offshored (advanced country or South), the unskilled unit labour cost is augmented by a fixed amount which accounts for the costs of transporting the intermediate goods across countries and organising the global production at the world level. We assume for simplicity that this cost is high enough to prevent offshoring across advanced countries considering the limited difference in unskilled wages between them. Consequently, offshoring only concerns the South.

There is a second cost which is specific to offshoring to the South and which can differ across the three advanced economies. This reflects the facts (i) that infrastructures, workers’ personal productivity, organisation, enforcement of property rights etc. can be partially deficient in emerging countries and (ii) that the cost of relocating production abroad depends, for each advanced country, on the geographical, cultural and historical links it has with certain emerging countries. As the impact of these factors on production costs can substantially diverge across production segments, we assume that for each country \( i \) the cost of relocating segment \( f \) to the South at time \( t \), \( \omega_{ft}^j \), differs across segments \( 1,...,F \). Ordering the segments by increasing offshoring cost, this cost is defined by:

\[
\omega_{ft}^j = w_{L}^S \times (\kappa_{ft})^{f/F}, \quad \kappa_{ft} > 1, \; f = 1,...,F, \; i = G, E, N; \tag{12}
\]

\(^5\) Inserting \( F^{\alpha} \) in the production function makes the cost of production and the prices of goods independent from the number of unskilled segments. This will permit to assume a continuum of stages.
where $w^S_L$ is the unit wage of low-skilled labour in South including the fixed amount representing the common offshoring cost, and $(K^F_{it})^{f/F} > 1$ is the multiplicative factor determining the extra cost of producing the segment $f$ of good $i$ in the South.

Let $K_{it}$ be the segment such that its production cost is the same in advanced country $i$ and in the South at time $t$. Hence, $K_{it}$ is the number of offshored segments in the production of good $i = G,E,N$ and $k_{it} = K_{it} / F$ its proportion (the proportion of segments remaining in country $i$ is thus $1 - k_{it}$). We can write (time index $t$ is omitted for simplicity):$^6$

$$\kappa_i = \left( \frac{w^L_i}{w^S_L} \right)^{1/k_i}$$

(13)

Consider country $i = G,E,N$, which offshores the proportion $k_i$ of unskilled segments in the production of good $i$. Assuming a continuum of low-skilled production segments, the price of the tradable good $i$ is (proof in Appendix D.2):

$$p_i = A_i^{-1} \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \left( \frac{w^f_{it}}{w^L_i} \right)^{-\alpha} \left( \frac{w^S_L}{w^L_i} \right)^{\alpha k_i/2} \left( \frac{w^L_i}{w^S_L} \right)^{\alpha (1-k_i/2)}$$

(14)

4.4. Factor demands and general equilibrium

Table 1 depicts the system of equations defining the general equilibrium of the extended model which is used for simulations. This model is built in Appendix D. We summarise here the successive steps generating this system.

The maximisation of utility and profit firstly permits to define the demands for goods and factors in each country in relation to the world total income $W = IG + IE + IN + IS$. From the South’s balanced trade, we subsequently determine the South income $IS$ and thereby the world income $W$ in relation to the income of advanced countries ($IG + IE + IN$).

By equalising supply and demand, we obtain the equilibrium equations in labour markets which determine relative wages as functions of relative labour supply for each advanced country. The labour markets equilibrium relations are combined with the price and offshoring equations to generate 13 equations with 13 unknown variables defining the full employment general equilibrium.

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$^6$ By definition of $K_i$: $\omega_{K_i} = w^S_L \times \kappa^k = w^L_i \Rightarrow \kappa = \left( \frac{w^L_i}{w^S_L} \right)^{1/k_i} \Leftrightarrow k_i = \log \left( \frac{w^L_i}{w^S_L} \right) / \log \kappa$
Based on the equilibrium, we can calculate for each country $i = G, E, N$, (i) the consumer price $p_i$, (ii) the real wages and real income per capita ($w_i^j / P_i$, $w_i^j / P_i$ and $I_i / P_i(L_i + \bar{H}_i)$), the relative production prices $p_G / p_j$ and the relative consumer prices $p_G / P_j$, $j = E, N$.

### Table 1. The general equilibrium equations

| Exogenous parameters: $\alpha$, $\beta$, $A_G, A_E, A_N$, $a_G, a_E, a_N$. |
| Exogenous variables: $k_G, k_E, k_N, w_i^L, L_i, T_i, T_N, \Pi_G, \Pi_E, \Pi_N, w_H^G = 1$. |
| 13 equations ($i = G, E, N$): |
| $p_i = A_i^{-1} \left( w_i^j / (1 - \alpha) \right)^{1-\alpha} \alpha^{-\alpha} \left( w_L^G \right)^{\alpha k/2} \left( w_L^I \right)^{\alpha(1-k/2)}$ (3 equations) |
| $P_{EN} = \left( a_G \sigma_G P_G^{-1-\sigma_G} + p_N^{-1-\sigma_N} \right)^{1-\sigma_N}$ (1 equation) |
| $I_i = w_i^j L_i + \bar{w}_i^j \bar{H}_i$ (3 equations) |
| $w_H^j = \beta(1-\alpha) \left( 1 - (1-\alpha)/(1-k_1) \right) \bar{H}_i w_L^j$ (3 equations) |
| $H_G = a_G \sigma_G \left( P_{EN} / P_G \right)^{\alpha - 1} \left( 1 - \alpha \right) \beta (I_G + I_E + I_N) \left( P_T / P_{EN} \right)^{1-\sigma_N} - \alpha \left( k_G a_G \sigma_G / P_G \right)^{\alpha - 1 - 1} + \sum_{i=E,N} \left( k_i a_i \sigma_i / P_i \right)^{\alpha - 1 - 1}$ (1 equation) |
| $H_E = a_E \sigma_E \left( P_{EN} / P_E \right)^{\alpha E - 1} \left( 1 - \alpha \right) \beta (I_G + I_E + I_N) \left( P_T / P_{EN} \right)^{1-\sigma_N} - \alpha \left( k_G a_G \sigma_G / P_G \right)^{\alpha - 1 - 1} + \sum_{i=E,N} \left( k_i a_i \sigma_i / P_i \right)^{\alpha - 1 - 1}$ (1 equation) |

Additional related variables: $P_i = \left( w_i^j \right)^{1-\beta} \left( P_T \right)^{\beta}$, $p_G / p_j$, $P_G / P_j$, $w_i = w_L^j / w_H^j$, $w_L^G / w_L^I$, $w_G / w_j$, $w_L^G / P_i$, $w_H^G / P_i$, $i = (I_i / P_i) / (L_i + \bar{H}_i)$, $i = G, E, N$, $j = E, N$. 

In the case of imperfections in unskilled labour markets, the system comprises as many additional equations and endogenous variables as the number of imperfect markets. The additional equations define the unskilled wage indexations and the additional unknown

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7 The equilibrium equation on the market for skilled labour in North $H_N$ is deleted because of Walras’ Law and because the wage of skilled workers in Germany is selected as numeraire: $w_H^G = 1$. We also assume $a_N = 1$. 
variables are the constrained $L_s$. After $L_s$ has been determined, the unemployment rate of unskilled workers $u^i_L = (L^i - L) / L$ can be calculated. As regards income transfers, they are inserted in the demand functions for goods and factors.

5. Simulations

We simulate the extended CGE model built in Section 4. The simulations are based on values of the parameters and of the exogenous variables in line with the structure of the model and calculated from the economic developments observed in each area (Germany, Eurozone and North). Though North is included in the simulations, the focus is still on a comparison between Germany and Eurozone. In Section 5.1 we define the three specifications selected for the simulations. Section 5.2 explains the empirical determination of the values of the parameters and exogenous variables used in the simulations. Section 5.3 presents and interprets the simulations results.

5.1. Setup of the three simulations

We perform three simulations, each covering the period 1975–2015.

The first simulation refers to a variant of the extended model with no offshoring, competitive labour markets and balanced trade. This simulation intends to assess the cross-area dynamics of wages and prices caused by divergent labour force growth rates that would be in line with full employment and balanced trade in all countries. Comparing these results with observed facts, we highlight the model predictions in terms of unemployment and trade balance when the adjustment through wages and prices is impeded, i.e., when wages and prices do not vary in accordance with the differences in labour force growth rates.

The second simulation integrates all the extensions made in Sections 3 and 4: pegged unskilled wages between Eurozone and Germany (reflecting imperfect unskilled labour markets), income transfers between the areas and offshoring to the South which can diverge across areas. Note that even though Simulation 2 represents a more realistic setup than Simulation 1, it does not aim at portraying the real economic developments since a large range of events which had differing impacts on each area are disregarded (oil shocks, technological changes, geopolitical changes, financial crises, euro area debt crisis etc.). The simulation rather intends to diagnose the impacts of the factors analysed in our propositions (divergence in labour force growth rates with imperfect unskilled labour markets and income
transfers) as well as the impact of offshoring by utilising values of the parameters and of the exogenous variables calculated from available databases.

In Simulations 1 and 2, the impact of the unification-related jump in the German labour force has been smoothed over the period 1990-2005 (see the explanations in Section 5.2.1).

We finally simulate a counterfactual scenario (Simulation 3) which replicates all the characteristics of Simulation 2 except the differences in labour force growth which are assumed to be zero in the three areas. This counterfactual exercise will be compared with the second simulation so as to measure the specific contribution of the divergence in labour force growth rates to the simulated outcomes.

Table 2 summarises the setup of the three simulations.

<table>
<thead>
<tr>
<th>Differences in labour force size</th>
<th>Simulation 1 Basic Model</th>
<th>Simulation 2 Extended model</th>
<th>Simulation 3 Counterfactual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differences in factor endowments</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Differences in labour force growth</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Offshoring to the South</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Unskilled labour market imperfection</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Income transfers</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

A large number of additional simulations have been implemented which are available from the authors upon request.

5.2. Variables and parameters

We give a brief overview of the way we have selected the data and constructed the series utilised in the simulations concerning (i) the labour force in each area (Germany, Eurozone and North), (ii) the introduction of labour market imperfections, (iii) the income transfers across countries, (iv) the offshoring dynamics, (v) the elasticities of substitution and (vi) the other parameters and exogenous variables. A detailed presentation and justification of those choices and measurements are given in Appendix E.

5.2.1. Labour force

The labour forces and their variations are measured annually in terms of working hours as depicted in Fig. 1c. This is important because countries and firms have, to different degrees, utilised the tools of working time reductions and working time accounts to lessen or avoid unemployment.
The impact of the unification-related jump in the German labour force has been smoothed over ten years so as to account for the policies taken by the German governments to prevent the increase in unemployment in the transition period following the reunification. In addition, we have not simulated the 1990-1995 period for which the model is evidently inadequate because it assumes a ‘unified’ German structure which was not the case in this period.

We account for between–area differences in skill endowments. However, we do not consider the increase in the skill level of the three areas from 1975 to 2015.

A discussion of this last assumption and a detailed presentation of the data and methods utilised for labour force calculations can be found in Appendix E.1.

5.2.2. Labour market imperfections

In line with the theoretical approach, imperfections in the unskilled labour markets are introduced in Simulations 2 and 3 by pegging the wages between Germany and Eurozone. This can generate unemployment in these areas.

Pegged wages are introduced by inserting in the simulations the observed ratio of labour costs per hour employed in Germany to those employed in Eurozone in the same currency. As the structure of the model implies identical variations in labour productivity in the two areas, we have erased the effect of differences in labour productivity growth by considering the following productivity-adjusted ratio of unit labour costs:

\[
\lambda_{G/E} = \frac{\text{labour share of total income (in %) in Germany} \times \frac{\text{GDP in current US dollar in Germany}}{\text{employment in hours in Germany}}}{\text{labour share of total income (in %) in Eurozone} \times \frac{\text{GDP in current US dollar in Eurozone}}{\text{employment in hours in Eurozone}}}
\]

To take into account the lag between changes in unit labour costs and changes in prices, demand and production, we have applied the mean of the pegged wages of years \((t-1)\) and \(t\) to the simulation in year \(t\). The data and methods as well as a discussion of the constraints and possible biases related to the selected data can be found in Appendix E.2.

We have not assumed pegged wages between North and the other areas because, within a general equilibrium model with price adjustments, the large volatility of the US dollar would have generated large, rapid and unrealistic volatility in unemployment. Since pegging the unit labour costs between Germany and North is not possible, we have introduced in Simulations 2 and 3 the North employment (labour force minus unemployment) as an

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8 See also the discussion in Appendix E.2.
exogenous variable, the unskilled wage of the North becoming endogenous. Hence, the model no longer determines unemployment in North and the simulations focus only on the differences between Germany and Eurozone.

5.2.3. Income transfers

Inter–area income transfers have been introduced by calculating the current account balances between Germany and the other two areas. From Deutsche Bundesbank data, we have calculated for each year the ratio of the current account balance between Germany and area $j$ ($j = E, N$) relative to German GDP. In the simulations, we have applied those ratios to the German total income (sum of the factor incomes) and transferred this amount to the related area’s total income. Those ratios are depicted in Fig. 6.

5.2.4. Offshoring

In the model, $k_{it}$ denotes the proportion of low-skilled segments offshored to the South in the production of tradable good $i = G, E, N$ in period $t$. We quantify $k_{it}$ by calculating:

$$ k_{it} = \frac{\psi_{Si,t}}{\psi_{Si,t} + \psi_{ii,t}}, \quad i = G, E, N. $$

where $\psi_{Si,t}$ and $\psi_{ii,t}$ respectively denote the value-added contribution of Southern unskilled workers and of country $i$’s unskilled workers to the final production of tradable good $i$ in period $t$.

Note that $\psi_{Si,t}$ does not only include the low-skilled value added which is embedded in direct flows of intermediate goods from the South to area $i$, but also the indirect flows of Southern intermediate goods embedded in the imports of area $i$ from other advanced areas. In other words, the calculation of $\psi_{Si,t}$ correctly reflects the global value chains between the South and region $i$. As is outlined in Appendix E.3, we calculate $k_{it}$ using the 2013 release of the World Input-Output Database (WIOD), see Timmer et al. (2015).

Appendix E.4 (i) explains in more detail how the $k_{it}$s are calculated, (ii) depicts these values and (iii) explains how the $k_{it}$ have been extrapolated to obtain values for the missing years.

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5.2.5. Calculation of the elasticities of substitution $\sigma_1$ and $\sigma_2$

From our theoretical model, we firstly determine the elasticities of substitution $\sigma_1$ and $\sigma_2$ as functions of the price elasticities of exports $\varepsilon_i$ for the three areas (see Appendix E.5.1 for details):

$$
\sigma_1 = \frac{\varepsilon_G}{1-S_G}; \quad \sigma_2 = \frac{1}{1-(S_j/(1-S_G))}\left(\varepsilon_j-\sigma_1 S_G \sum_i - S_j\right), \quad j = E, N
$$

where $S_i, i = G, E, N$, denotes share of the world demand for tradable goods met by country $i$: $S_i = p_i Y^d / \beta I_w$.

We then estimate the export price elasticities $\varepsilon_i, i = G, E, N$, using the 2013 release of the WIOD. We perform fixed-effects panel estimations at the sectoral level to obtain export demand elasticities with respect to the effective real exchange rate for Germany, Eurozone and North for the period 1995-2007. Using relationships (16), we finally determine the following estimated values for the elasticities of substitution: $\hat{\sigma}_1 = 0.64$ and $\hat{\sigma}_2 = 0.87$. It should be noted that the estimated elasticity of substitution between the German good and other goods is lower than the estimated elasticity of substitution between Eurozone and North goods ($\hat{\sigma}_1 < \hat{\sigma}_2$). This magnifies the impact of the differences in labour force growth rates (Proposition 1). Appendices E.5.2 – E.5.4 provide a detailed exposition on how the estimated values of $\sigma_1$ and $\sigma_2$ have been obtained as well as a comparison of our estimates of the export price elasticities with those from the literature.

5.2.6. Other parameters and exogenous variables

Table 3 shows the parameters and exogenous variables utilised in the three simulations.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$A_G, A_E, A_N$</th>
<th>$a_G$</th>
<th>$a_E$</th>
<th>$a_N$</th>
<th>$w^S_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.75</td>
<td>1</td>
<td>0.1232</td>
<td>0.318</td>
<td>1.0</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The value of $\alpha$ means that about 30% of the total labour income goes to less skilled workers in the tradable sector, which is what was observed on average in advanced economies in the late 70s and in the 80s. The value of $\beta$ makes the non-tradable unskilled-intensive service to represent 25% of the total expense, in line with what is observed in the considered
economies. The attractiveness coefficients, \( a_i, i = G, E, N \), have been selected to generate the observed unemployment gap between Germany and Eurozone in 1975.

The wage in the South \( W^S_L \) (which includes the fixed offshoring cost but not the variable costs related to each production segment) represents between 1/3 and 1/4 of the unskilled wage in the three advanced areas.

The total factor productivities have been chosen to be identical and equal to 1 in the production of tradable goods in the three advanced areas. Since the results of the simulated model are formulated in terms of relative changes, this is equivalent to assuming the same rate of TFP growth in all areas.\(^{10}\)

### 5.3. Results

We focus on the results and on the comparison of simulated and observed data for Germany and Eurozone only, as explained in Section 5.2.2 and Appendices E.1 and E.2.

In a first step, we compare the variations in relative wages and prices as determined by Simulation 1 (characterised by differences in labour force growth, competitive labour markets, no income transfer and no offshoring) with the observed facts. We highlight the differences in wage and price variations between Simulation 1 and the observed developments, and their implications in terms of market adjustment, unemployment and trade balance as predicted by the model.

In a second step, we compare the results of Simulations 2 and 3 to evaluate *ceteris paribus* the contribution of the differences in labour force growth to observed developments. As the Eurozone/Germany relative unskilled wage, the current account balances and the employed labour force of the North are exogenously introduced in Simulation 2 (see Section 5.2 and Appendix E.2), the comparison will focus on the unemployment gap between Eurozone and Germany as the key endogenous variable determined by the model.

Finally, we compare the results of Simulation 2 with observed facts. As already noted, Simulation 2 cannot perfectly mimic observed developments since many shocks (oil shocks, financial crises etc.) are not taken into account and the model behind Simulation 2 does not portray the full complexity of the mechanisms determining the economic situation of the three areas. We therefore highlight the omitted variables and missing mechanisms which can explain the divergence between Simulation 2 and observed facts.

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\(^{10}\) Even though Eurozone comprises several Southern European countries characterized by a significantly higher TFP growth than Germany since 1975, this difference is accounted for in the labour cost pegging.
5.3.1. Simulation 1

Fig. 8a contrasts the relative wage and price (Germany/Eurozone) obtained from Simulation 1 (solid line) with the observed counterpart (dashed line). Both series are depicted as index values with 1975 as the base year.11 A rise in the simulated values signifies that German wages and prices should have increased relatively to those in Eurozone to maintain full employment and balanced trade. The shaded surface represents the non-simulated years 1990–1995 (see Appendix E.1 for the explanation why this period has been excluded from the simulation).

**Fig. 8. Relative wages and production prices (Germany/Eurozone, 1975=100)**

Note: The shaded surface covers the non-simulated years 1990-1995.

Fig. 8b depicts the difference between the simulated and observed values of the German wages and prices in relation to other countries. An increase in this differential means that German wages and prices have grown too slowly compared to other countries given the variations in labour forces, which leads to the prediction of an increase in Eurozone’s unemployment in relation to Germany (Proposition 2) and/or an income transfer from Germany to Eurozone, generating a growing German current account surplus (Proposition 3). A reduction of this differential leads to the opposite prediction (decrease in Eurozone relative unemployment or/and decrease in the German current account surplus).

According to Fig. 8, three periods can be distinguished:

1. From the late 70s to the unification, the actual German/Eurozone relative wages and prices decreased whereas the simulated ones increased by 15%. In this case, the model predicts that unemployment should have increased in Eurozone compared to Germany and/or the German current account surplus should have risen.

---

11 In Simulation 1, relative prices and wages (Germany/Eurozone) have identical variations. This is because, without offshoring and with perfect competition, balanced trade and given skill endowments, both skilled and unskilled labour wages have identical growth rates as well as prices which are combinations of both wages. We do not display the comparison of the relative real income per capita (available from the authors) because this would lead to the same diagnosis.
2. From 1995 to 2002, the difference between the simulated and observed wages and prices has continuously decreased, and the simulated and observed values broadly coincided from 2002 to 2005. Our model then predicts that the unemployment gap (Eurozone – Germany) should have decreased and/or the German current surplus should have declined.

3. From 2005 to 2010, the difference between the simulated and observed values of the German/Eurozone relative wages and prices increased again and this difference has remained roughly constant from 2010 to 2015. Here, the model predicts that the unemployment gap (Eurozone – Germany) and/or the German current account surplus should have increased, with a stabilisation in the early 2010s.

The observed developments of the unemployment gap (Eurozone – Germany) and of the German current account depicted in Section 2 (Fig. 5 and 6) confirm the model predictions.

5.3.2. Simulations 2 and 3

Simulation 2 combines differences in labour force growth rates with offshoring, labour market imperfections and income transfers. The counterfactual Simulation 3 replicates the same developments except the differences in labour force growth. By comparing Simulations 2 and 3, we can calculate the *ceteris paribus* impact of the divergence in labour force growth rates on the model results.

Fig. 9 shows the unemployment gap (Eurozone – Germany) calculated by Simulation 2 (solid line) as well as the difference in the unemployment gap between Simulation 2 and Simulation 3 (dashed line). The latter is interpreted as the contribution of the differences in labour force growth in the three areas to the unemployment gap (Eurozone–Germany) predicted by the model. Fig. 10 depicts the contribution of the difference in labour force growth across areas to the annual change in the unemployment gap (Eurozone–Germany). This contribution is calculated as the difference between the annual change in the unemployment gap from Simulation 2 and the annual change in the unemployment gap from Simulation 3.

Fig. 9 and 10 show that, according to our extended model (including offshoring, labour market imperfections and income transfers), the difference in labour force growth across areas was a key driver of the increase in the unemployment differential (Eurozone – Germany) from 1980 to 1990, of its decrease from 1995 to 2006 and of its increase from 2006 to 2010. Over the periods 1975-1989 and 1996-2015, the differences in labour force growth explain on average more than 50% of the unemployment gap between Eurozone and Germany.
Fig. 9. Contribution of the difference in labour force growth across all three areas to the unemployment gap (Eurozone–Germany)

Note: The shaded surface covers the non-simulated years 1990-1995

Fig. 10. Annual contribution of the difference in labour force growth to the variation of the unemployment gap (Eurozone–Germany)

Note: The shaded surface covers the non-simulated years 1990-1995

5.3.3. Simulation 2 vs. observed facts

Fig. 11 compares the unemployment gap (Eurozone–Germany) calculated by Simulation 2 with its observed level. It appears that, even though both series are not congruent, the general trends and turning points of the simulated data are clearly consistent with the observed ones. Moreover, two periods can be distinguished based on how both series evolve relatively to each other.

Fig. 11. Unemployment gap (Eurozone-Germany): simulated vs. observed data

Note: The shaded surface covers the non-simulated years 1990-1995
In the first period, from the late 70s to 1990, the increase in the simulated unemployment gap is significantly higher than the observed increase. Three major facts and developments could explain this difference:

1. The existence of a public sector is disregarded in our model. This increases the simulated unemployment compared to the observed unemployment because the public sector can absorb workers made redundant in other sectors. The public sector accounts for between 20 and 30% of total employment in Germany and Eurozone countries and this share was higher in Eurozone than in Germany in the period 1975-1990. In particular, the share of public employment in total employment increased significantly in France (representing 30% of the Eurozone labour force), moving from 23% in the mid-70s to 30% in the early 90s. In contrast, this share was only 21% in Germany in 1991.

2. As mentioned in Section 5.2.1, we have assumed no change in the areas’ skill endowments. This assumption could have induced higher simulated unemployment in Eurozone in the late 70s and in the 80s compared to the observed unemployment because those periods were characterized by a large increase in the skill endowments in Southern Europe (Greece, Portugal and Spain).

3. Finally, our model assumes no change in the attractiveness of the country-specific tradable goods, $a_i, i = G, E, N$. However, the 70s and 80s were characterised by a catching-up process in several Southern European countries (Greece, Portugal, Spain). In our context, this can be interpreted as an improvement in the attractiveness of the goods they produce, which depicts the quality and adaptation to demand, therefore implying an increase in the value of $a_E$ relative to $a_G$. Taking this change into account would lower the simulated unemployment in Eurozone and move the simulation results closer to the observed variations in Fig. 11.

In the second period, from 1995 to the early 2010s, the positive difference between the simulated and observed unemployment gaps (Eurozone–Germany) substantially decreases. Several factors can explain this reduction. A first explanation can be the very restrictive policies pursued by Southern European countries (and Ireland) after the 2008 financial crisis and during the euro debt crisis. Those policies have had a huge impact on these countries in terms of GDP and unemployment, thus increasing the unemployment rate of Eurozone. Another possible explanation is a decrease in Southern European goods attractiveness, lowering coefficient $a_E$. This could result from the relocation of German (but also French and Dutch) production units from Southern Europe to Eastern Europe in the 90s and 2000s.
6. Discussion and conclusion

The model developed in this paper puts forward the impact of between-country divergence in labour force growth upon relative wages and prices. It shows that a country which is characterised by a structurally low increase in its labour force experiences higher growth in wages and prices than its trade partners if full employment prevails in all countries. Otherwise, these partners suffer from unemployment and/or trade deficits. This can generate severe constraints on the trade partners’ policy options, particularly when the country pursues a policy of wage moderation.

The simulations implemented with parameters calculated from the World Input-Output Database and with exogenous variables observed in the three groups of countries (Germany, Eurozone and North) show that this ‘demographic channel’ is consistent with the German experience and relationship with other advanced economies since the mid-70s. In addition, our estimates show that the elasticity of substitution between German goods and goods from the other two areas is lower than the elasticity of substitution between goods produced in those areas. This magnifies the impact of the demographic channel on wages and prices.

In the periods when Germany had a labour force growth lower than its trade partners without a higher increase in German prices and wages (1975-1990 and 2005-2013), those partners have typically experienced higher unemployment and/or a large trade deficit with Germany. In contrast, in the 1995-2005 period which followed the upward jump in the German working population due to the unification, the maintenance of relatively high wages in Germany resulted in a huge rise in unemployment in this country as well as a significant decrease in its trade surplus.

Consequently, our model provides a broader explanation for the German economic turmoil experienced in the 1995-2005 period. The German ‘sickness’ was not only a consequence of low flexibility in the labour market as often argued in the literature. After all, the German economy had done quite well with relatively inflexible labour market institutions before 1990. Following our model, the German disorder resulted from the conflict between the changes imposed by the unification and the labour market structure. Unification led to a huge increase in the labour force without similar increase in German goods attractiveness. Consequently, Germany had to decrease its wages and prices to maintain full employment, which was at odds with the existing labour market structures. The changes in labour market institutions implemented in the late 1990s (Dustman et al, 2014) and the Hartz reforms of 2003–2005 permitted to adapt the labour market institutions to the new conditions imposed by the
unification. It must however be emphasised that, on top of the unification, the large offshoring implemented by German firms from the early 1990s constituted a supplementary shock to the German labour market. This has to a large extent been treated by expanding a non-tradable service sector with low labour costs and part time jobs, which has resulted in labour market polarization between stable and well-paid positions (in manufacturing) and short term contracts with low pay and part-time work (Beissinger et al., 2016).

From 2005 onward, the lasting low growth in the German labour force has re-established the requirement of higher wage and price growth in this country.

Can the labour force channel highlighted here be a key element of the relationship between Germany and other euro area members in the forthcoming years? The answer to this question depends on the between-country differences in labour force growth. In this respect, Southern European countries have experienced a lasting and substantial decrease in their natural demographic growth over the last two decades and their labour force now grows slower than that of Germany. But other Eurozone countries continue to show a labour force growth which is higher than the German one, such as France, Ireland, and Belgium. Our model shows that, if those countries want to avoid unemployment and trade deficits, they must either have an increase in wages and prices permanently lower than Germany or have a permanent increase in their goods attractiveness compared to German goods (Propositions 2 and 3). In addition, a permanent rise in the country’s productivity in relation to that of Germany increases the price gap (relative German prices must grow faster) but lowers the wage gap (relative German wages grow slower) at the full employment equilibrium. These findings show that, if Germany pursues a policy which leads to wage moderation, its euro area partners with a larger labour force growth may have to implement an even stricter wage policy. This constraint would be relaxed if Germany can increase its labour force growth. As an active pro-birth policy can only be efficient for the next working generation, the two means to raise labour force growth are an increase in the average number of hours worked and immigration. Of course, the constraint could also be relaxed by a higher increase in German wages.
Appendix A. Proof of Proposition 1

1. Households and demand
Due to the weak separability of the utility function (4), the household’s maximisation described in Section 3.2 can be easily determined by two-stage budgeting, leading to

\[ p^i_{NT} Y^i_{NT} = (1 - \beta) I_i \]  (A1)

\[ Y^i_G = a^\sigma_G \left( \frac{p_G}{P_T} \right)^{-\sigma} \beta I_i P_T \]  and  \[ Y^i_E = a^\sigma_E \left( \frac{p_E}{P_T} \right)^{-\sigma} \beta I_i P_T, \]  (A2)

where \( Y^i_j \) is the total demand for good \( j \) in country \( i \), \( I_i \) country i’s total income and \( P_T \) is defined by eq. (5) in the text.

Total demand (denoted by superscript \( d \)) for each tradable good is therefore

\[ Y^d_G = a^\sigma_G \left( \frac{p_G}{P_T} \right)^{-\sigma} \beta (I_G + I_E) P_T \]  (A3)

\[ Y^d_E = a^\sigma_E \left( \frac{p_E}{P_T} \right)^{-\sigma} \beta (I_G + I_E) P_T \]  (A4)

This determines the ratio of tradable prices in terms of relative demand:

\[ \frac{p_G}{p_E} = a_G \left( \frac{Y^d_G}{Y^d_E} \right)^{1/\sigma} \]  (A5)

2. Production and equilibrium

2.1. Non-tradable service
The market for the non-tradable service being perfectly competitive, the zero profit condition entails \( p^i_{NT} = w^i_L \). Inserting the production function \( Y^i_{NT} = L^i_{NT} \) and \( p^i_{NT} = w^i_L \) into eq. (A1) yields \( w^i_L L^i_{NT} = (1 - \beta) I_i \). As \( I_i = w^i_L L_i + w^i_H H_i \), one obtains the following demand for unskilled labour in the non-tradable sector of country \( i = G, E \):

\[ L^i_{NT} = (1 - \beta)(L_i + w^i_H H_i), \]  (A6)

with \( w_i \equiv w^i_H / w^i_L \) being the skill premium in country \( i \).
2.2. **Tradables**

The tradable good \((T, i)\) is produced in country \(i, i = G, E\), with the Cobb-Douglas technology:

\[
Y_{Ti} = A_i \left( L_{Ti} \right)^\alpha \left( H_{Ti} \right)^{1-\alpha} \tag{A7}
\]

The skill premium \(w_i = \frac{w^i_H}{w^i_L}\) in country \(i\) equals the marginal rate of technical substitution between \(L_{Ti}\) and \(H_{Ti}\). Since in equilibrium \(H_{Ti} = \bar{H}_i\), it holds that \(w_i = \frac{1-\alpha}{\alpha} \frac{L_{Ti}}{\bar{H}_i}\), and since \(\bar{L}_i = L^i_{NT} + L_{Ti}\), one obtains \(w_i = \frac{1-\alpha}{\alpha} \frac{L_{Ti} - L^i_{NT}}{\bar{H}_i}\). Combining this equation with eq. (A6) yields:

\[
w_i = \frac{(1-\alpha)\beta}{1-(1-\alpha)\beta} \bar{L}_i^{-1} \tag{A8}
\]

Inserting eq. (A8) in (A6) gives:

\[
L^i_{NT} = \frac{1-\beta}{1-(1-\alpha)\beta} \bar{L}_i \tag{A9}
\]

and since \(L_{Ti} = \bar{L}_i - L^i_{NT}\):

\[
L_{Ti} = \frac{\alpha\beta}{1-(1-\alpha)\beta} \bar{L}_i \tag{A10}
\]

Hence:

\[
Y^i_{NT} = \frac{1-\beta}{1-(1-\alpha)\beta} \bar{L}_i \tag{A11}
\]

\[
Y_i = A_i \left( \frac{\alpha\beta}{1-(1-\alpha)\beta} \bar{L}_i \right)^\alpha \left( \bar{H}_i \right)^{1-\alpha} \tag{A12}
\]

Since prices equal marginal costs:

\[
p_i = \frac{w_{Li}^{1-\alpha}}{A_i \alpha^{\alpha} (1-\alpha)^{1-\alpha}} \Leftrightarrow w_L = \frac{A_i \alpha^{\alpha} (1-\alpha)^{1-\alpha}}{w_{Li}^{1-\alpha}} p_i \tag{A13}
\]

Inserting eq. (A8) in (A13) determines the relationship between the tradable goods price and the unskilled wage in country \(i\):

\[
w_{Li} = \frac{1-(1-\alpha)\beta}{\beta} \bar{L}_i^{1-\alpha} p_i \tag{A14}
\]
3. Equilibrium and the relative prices, wages and income per head

Combining eqs. (A5) and (A12) yields:

\[
P_G(t) = \frac{a_G}{a_E} \left( \frac{A_E}{A_G} \right)^{\frac{1-\alpha}{\sigma}} \left( \frac{\lambda_E}{\lambda_G} \right)^{\frac{1-\alpha}{\sigma}} \left( \frac{T_E(t)}{T_G(t)} \right)^{\frac{1}{\sigma}}
\]  
(A15)

Combining this equation with eq. (A14) gives:

\[
w_L^G(t) = \frac{a_G}{a_E} \left( \frac{A_E}{A_G} \right)^{\frac{1-\alpha}{\sigma}} \left( \frac{\lambda_E}{\lambda_G} \right)^{\frac{1-\alpha}{\sigma}} \left( \frac{T_E(t)}{T_G(t)} \right)^{\frac{1}{\sigma}}
\]  
(A16)

Combining eqs. (A16) and (A8) yields:

\[
w_H^G(t) = \frac{a_G}{a_E} \left( \frac{A_E}{A_G} \right)^{\frac{1-\alpha}{\sigma}} \left( \frac{\lambda_E}{\lambda_G} \right)^{\frac{1+\alpha(\sigma-1)}{\sigma}} \left( \frac{T_E(t)}{T_G(t)} \right)^{\frac{1}{\sigma}}
\]  
(A17)

Inserting \( \bar{L}_i(t) = \bar{L}_i(0)e^{n_i t} , i = G, E \) into (A15)–(A17) determines relations (7)–(9) in the text. 

Eq. (5) in the text determines the consumer price in country \( i \): \( P_i = \left( \frac{p_{NT}^i}{P_T} \right)^{1-\beta} \). As \( p_{NT}^i = w_L^i \), we have: \( P_i = \left( \frac{w_L^i}{w_L^E} \right)^{1-\beta} \) and: \( \frac{P_G}{P_E} = \left( \frac{w_L^G}{w_L^E} \right)^{1-\beta} \).

As \( \frac{w_L^G}{w_L^E} \) grows at rate \( (n_E - n_G) / \sigma \), \( P_G / P_E \) grows at rate \( (1-\beta)(n_E - n_G) / \sigma \).

The relative real wages are \( \frac{v_L^G}{v_L^E} = \frac{w_L^G}{P_G} / \frac{w_L^E}{P_E} = \left( \frac{w_L^G}{w_L^E} \right)^{\beta} \) for unskilled labour and \( \frac{v_H^G}{v_H^E} = \frac{w_H^G}{P_G} / \frac{w_H^E}{P_E} = \frac{w_H^G}{w_H^E} \left( \frac{w_L^G}{w_L^E} \right)^{\beta} \) for skilled labour. As \( \frac{w_L^G}{w_L^E} \) grows at rate \( (n_E - n_G) / \sigma \), \( \frac{v_L^G}{v_L^E} \) grows at rate \( \beta(n_E - n_G) / \sigma \), as well as \( \frac{v_H^G}{v_H^E} \) since \( \frac{w_G}{w_E} \) is constant.

Finally, the relative real income per head is \( \frac{\bar{I}_G}{\bar{I}_E} = \frac{v_L^G\bar{T}_G + v_H^G\bar{H}_G}{v_L^E\bar{T}_E + v_H^E\bar{H}_E} \). As \( \frac{\bar{L}_i + \bar{H}_i}{\bar{L}_i + \bar{H}_i} \) are constant for \( i = G, E \), and since \( \frac{v_L^G}{v_L^E} \) and \( \frac{v_H^G}{v_H^E} \) grow at the same constant rate \( \beta(n_E - n_G) / \sigma \), then \( \frac{\bar{I}_G}{\bar{I}_E} \) grows at the same rate \( \beta(n_E - n_G) / \sigma \).
Appendix B. Proof of Proposition 4

We denote \( \sigma(t) \equiv w^G_L(t) / w^E_L(t) \) the exogenous relation which binds country \( E \)'s unskilled wage to that of country \( G \), \( \sigma(t) \) being lower than the full employment value determined by eq. (A16), and \( \tilde{\gamma} < \overline{\gamma} = (n_E - n_G) / \sigma \) denoting its exogenous growth rate.

The above-defined relations (A1)–(A17) are still valid except that \( \sigma(t) \equiv w^G_L(t) / w^E_L(t) \) is now exogenous and \( \lambda_E \) has to be replaced by \( \lambda_E(t) \equiv \overline{H}_E(t) / L_E(t) \), where \( L_E(t) = \left( L^E_{NT}(t) + L^E_E(t) \right) < \overline{L}_E(t) \) denotes total unskilled employment in country at time \( t \).

**Proof of feature 1**

The modified eq. (A16) is

\[
\sigma(t) = \frac{a^G_A}{a^E_A} \left( \frac{A^G}{A^E} \right)^{\sigma - 1} \left( \frac{\lambda_E}{\overline{H}_E(t)} \right)^{(1-\alpha)(\sigma - 1) / \sigma} \left( \frac{L_E(t)}{\overline{L}_G(t)} \right)^{(1-\alpha)(\sigma + 1) / \sigma} \frac{1}{\sigma} \quad (B1)
\]

Hence:

\[
L_E(t) = \overline{C} \left( (\sigma(t))^{\sigma} \times (\overline{H}_E(t))^{(1-\alpha)(\sigma - 1)} \times \overline{L}_G(t) \right)^{(1-\alpha)(\sigma - 1) + 1} \quad (B2)
\]

where \( \overline{C} = \left( \frac{a^E_A}{a^G_A} \right)^{\sigma} \left( \frac{A^E_A}{A^G_A} \right)^{\sigma - 1} \left( \frac{\lambda_E}{\overline{H}_E(t)} \right)^{(1-\alpha)(\sigma - 1) + 1} \) is constant.

Since \( \sigma(t) = \sigma(0) \times e^{\tilde{\gamma}t} \), \( \overline{H}_E(t) = \overline{H}_E(0) \times e^{\tilde{\gamma}t} \) and \( \overline{L}_G(t) = \overline{L}_G(0) e^{n_E t} \), then:

\[
L_E(t) = \overline{C} \left( (\sigma(0))^{\sigma} \times (\overline{H}_E(0))^{(1-\alpha)(\sigma - 1)} \times \overline{L}_G(0) \right)^{(1-\alpha)(\sigma - 1) + 1} \times e^{\sigma(1-\alpha)(\sigma - 1) n_E + n_G / (1-\alpha)(\sigma - 1) + 1} \quad (B3)
\]

Denote the growth rate of \( L_E \) as:

\[
\tilde{n}_L^E = \frac{\sigma \overline{\gamma} + (1-\alpha)(\sigma - 1) n_E + n_G}{(1-\alpha)(\sigma - 1) + 1} \quad (B4)
\]

By assumption, \( \tilde{\gamma} < \overline{\gamma} = (n_E - n_G) / \sigma \). Inserting \( \overline{\gamma} \) as an upper bound in eq. (B4) leads to:

\[
\frac{\sigma \overline{\gamma} + (1-\alpha)(\sigma - 1) n_E + n_G}{(1-\alpha)(\sigma - 1) + 1} = n_E \quad \text{Hence,} \quad \tilde{n}_L^E < n_E.
\]
Proof of feature 2

Because of eq. (A8), \( w_G = \frac{(1-\alpha)\beta}{1-(1-\alpha)\beta} \lambda_G^{-1} \) and \( w_E(t) = \frac{(1-\alpha)\beta}{1-(1-\alpha)\beta} (\lambda_E(t))^{-1} \), with \( \lambda_E(t) = \Pi_E(t) / L_E(t) \).

An increase in \( \sigma \) which is lower than its full employment value entails an increase in \( \lambda_E \) and thereby a decrease in the skill premium \( w_E \) and an increase in the relative skill premia \( w_G / w_E \) since full employment in country \( G \) keeps \( w_G \) constant over time.

Appendix C. Proof of Proposition 5

Proof of feature 1

The transfer increases country \( E \)'s income and decreases country \( G \)'s income by the same amount \( \varphi \). Let \( Y^d_i \) denote the total demand for tradable good \( i \), \( Y^j_i \) the demand for good \( i \) by country \( j \), \( Y^s_i \) the supply of good \( i \) and \( Y^{NT}_i \) the supply of service \( NT \) in country \( i \). The equality ‘income = expenditure’ implies for country \( E \): \( p_Y^E Y^E + p_{NT} Y^{NT}_E + \varphi = p_Y^E + p_{G} Y^E + p_{NT} Y^{NT} \) and as the market for \( NT \) is balanced (\( Y^E = Y^{NT} \)): \( p_Y^E + \varphi = p_Y^E + p_{G} Y^E \). As the market for good \( E \) is balanced: \( p_Y^E = p_Y^E + p_{E} Y^E \). Combining both equalities yields: \( p_{E} Y^E - p_{E} Y^E = \varphi \), where \( p_{E} Y^E \) are the imports of country \( E \) and \( p_{E} Y^E \) its exports. This establishes feature 1.

Proof of feature 2.

A proportion \( \tau \), \( 0 \leq \tau < 1 \), of country \( G \)'s total income is transferred to country \( E \).

Country \( E \)'s unskilled wage is bound to country \( G \)'s unskilled wage by relation \( w_L^E = w_L^G / \sigma \) such that the ratio \( w_L^E / w_L^G \) is above its full employment level, leading to unemployment of unskilled workers in country \( E \). Hence, the employment of unskilled labour in country \( E \), \( L_E \), becomes an endogenous variable which can be lower than the supply \( L_E \).

Demand in the markets for goods and services

The income available to consumers in country \( G \) in equilibrium is

\[
I_G = (1-\tau) \left( w_L^G L_G + w_H^G H_G \right),
\]

while the income available to consumers in country \( E \) is
As shown in eq. (A1), consumers in each country spend a fraction \((1 - \beta)\) of their income on non-tradable services. Hence, because of eqs. (C1) and (C2) and the zero-profit condition \(p_{NT}^i = w_i^i\), the demands for non-tradable services are

\[
Y_{NT}^G = (1 - \beta)(1 - \tau)(L_G + w_G\bar{H}_G)
\]

\[
Y_{NT}^E = (1 - \beta)[(L_E + w_E\bar{H}_E) + \tau \omega(L_G + w_G\bar{H}_G)],
\]

where \(w_i = w_i^G / w_i^L\) denotes the skill premium in country \(i = G, E\). For total demand for each tradable good total income \((I_G + I_E)\) matters. Hence, based on eq. (A4) and the definition of the price index for tradable goods in eq. (A3), goods demand can be written as

\[
Y_{G}^d = \beta a_G \sigma \frac{w_L^G (L_G + w_G\bar{H}_G) + w_L^E (L_E + w_E\bar{H}_E)}{(a_E^\sigma p_E^{1-\sigma} + a_G^\sigma p_G^{1-\sigma}) p_G^\sigma}
\]

\[
Y_{E}^d = \beta a_E \sigma \frac{w_L^G (L_G + w_G\bar{H}_G) + w_L^E (L_E + w_E\bar{H}_E)}{(a_E^\sigma p_E^{1-\sigma} + a_G^\sigma p_G^{1-\sigma}) p_E^\sigma}
\]

### Production of goods and services

According to eq. (A7), the production functions for the tradable goods are

\[Y_{i} = A_i L_i^\alpha (H_i)^{1-\alpha}\]

Moreover, the production function for non-tradable services in country \(i\) is \(Y_{NT}^i = L_{NT}^i\), \(i = G, E\). This determines the factor demands in each sector in country \(i = G, E\):

\[L_i = \alpha \frac{p_i Y_i}{w_i}
\]

\[L_{NT}^i = Y_{NT}^i
\]

\[H_i = (1 - \alpha) \frac{p_i Y_i}{w_i}
\]

### Equilibria in the markets for goods and services

#### Equilibrium in the market for non-tradable services

Since \(Y_{NT}^i = L_{NT}^i\), eqs. (C.3) and (C.4) can be written as

\[
L_{NT}^G = (1 - \beta)(1 - \tau)(L_G + w_G\bar{H}_G)
\]

\[
L_{NT}^E = (1 - \beta)[(L_E + w_E\bar{H}_E) + \tau \omega(L_G + w_G\bar{H}_G)]
\]
Equilibrium in good G-market

Using eq. (C5), the nominal demand for good G is:

\[ p_G Y^d = \beta \frac{w_G}{a_E} \left( \frac{L_G + w_G \bar{H}_G}{a_G} \right) + \frac{w_E}{a_G} \left( L_E + w_E \bar{H}_E \right) \left( \frac{p_G}{p_E} \right)^{\sigma - 1} + 1 \]  

(C12)

Due to eq. (A13) \( p_i = \frac{w_i^{\alpha}}{A_i \alpha (1-\alpha)^{1-\alpha}} w_i^j \), \( i = G, E \). Hence, \( p_G = \frac{A_E}{A_G} \left( \frac{w_G}{w_E} \right)^{1-\alpha} \frac{w_G}{w_E} \). Inserting this in eq. (C12) gives:

\[ p_G Y^d = \beta \frac{w_G}{a_E} \left( \frac{L_G + w_G \bar{H}_G}{a_G} \right) + \frac{w_E}{a_G} \left( L_E + w_E \bar{H}_E \right) \left( \frac{A_E}{A_G} \left( \frac{w_G}{w_E} \right)^{1-\alpha} \frac{w_G}{w_E} \right)^{\sigma - 1} + 1 \]  

(C13)

Since \( H_{TG} = \bar{H}_G \), eq. (C9) for country G can be written as

\[ p_G Y^d = \frac{w_G \bar{H}_G}{1-\alpha} \]  

(C14)

Equalising supply and demand \( p_G Y^d = p_G Y^d \) using eqs. (C14) and (C13) leads to:

\[ \beta (1-\alpha) \left( w_G \left( L_G + w_G \bar{H}_G \right) + w_E \left( L_E + w_E \bar{H}_E \right) \right) = w_G \bar{H}_G \left( \frac{a_E}{a_G} \left( \frac{A_E}{A_G} \left( \frac{w_G}{w_E} \right)^{1-\alpha} \frac{w_G}{w_E} \right)^{\sigma - 1} + 1 \right) \]  

(C15)

Equilibrium in good E-market

The same reasoning as for good G’s market yields:

\[ \beta (1-\alpha) \left( w_G \left( L_G + w_G \bar{H}_G \right) + w_E \left( L_E + w_E \bar{H}_E \right) \right) = w_E \bar{H}_E \left( \frac{a_G}{a_E} \left( \frac{A_E}{A_G} \left( \frac{w_G}{w_E} \right)^{1-\alpha} \frac{w_E}{w_G} \right)^{\sigma - 1} + 1 \right) \]  

(C16)

Combining eqs. (C15) and (C16):

\[ \frac{w_G}{w_E} \frac{\bar{H}_G}{\bar{H}_E} \left( \frac{a_E}{a_G} \left( \frac{A_E}{A_G} \left( \frac{w_G}{w_E} \right)^{1-\alpha} \frac{w_G}{w_E} \right)^{\sigma - 1} + 1 \right) = \frac{w_E}{w_G} \frac{\bar{H}_E}{\bar{H}_G} \left( \frac{a_G}{a_E} \left( \frac{A_E}{A_G} \left( \frac{w_G}{w_E} \right)^{1-\alpha} \frac{w_E}{w_G} \right)^{\sigma - 1} + 1 \right) \]  

Multiplying both sides by \( \frac{a_G}{a_E} \left( \frac{A_E}{A_G} \left( \frac{w_G}{w_E} \right)^{1-\alpha} \frac{w_E}{w_G} \right)^{1-\sigma} \) gives

\[ \frac{w_G}{w_E} = \left( \frac{\bar{H}_E}{\bar{H}_G} \right)^{1-\sigma} \left( \frac{A_E}{A_G} \right)^{1-\alpha} \left( \frac{w_E}{w_G} \right)^{\sigma - 1} \]  

(C17)
Equilibria on the L-markets

The \(L_G\)-market

The total demand for unskilled labour in country \(G\) is \(L_G^d = L_{TG} + L_{NT}^G\).

\[ L_{TG} = \frac{\alpha}{1-\alpha} w_G \bar{H}_G \]  
because of the Cobb-Douglas technology and full employment in the \(H_G\) - 
market, and \(L_{NT}^G\) is given by eq. (C10). Consequently:

\[ L_G^d = \frac{\alpha}{1-\alpha} w_G \bar{H}_G + (1-\beta)(1-\tau)(\bar{L}_G + w_G \bar{H}_G) \]

Full employment in the \(L_G\) market (\(L_G^d = L_G^s = \bar{L}_G\)) entails

\[ w_G = \left( \frac{1-(1-\beta)(1-\tau)}{\frac{\alpha}{1-\alpha} + (1-\beta)(1-\tau)} \right) \bar{H}_G^{-1}, \quad \frac{\partial w_G}{\partial \tau} > 0 \]  
(C18)

Inserting eq. (C18) into eq. (C17):

\[ w_E = \frac{1-(1-\beta)(1-\tau)}{\bar{H}_G \left( \frac{\alpha}{1-\alpha} + (1-\beta)(1-\tau) \right)} \left( \bar{H}_G \left( \frac{a_G}{A_G} \right)^\sigma \left( \frac{A_G}{A_E} \right)^{(1-\sigma)} \bar{H}_E \right)^\frac{1}{\alpha+(1-\alpha)\sigma}, \quad \frac{\partial w_E}{\partial \tau} > 0 \]  
(C19)

The \(L_E\)-market

The demand for \(L_E\) is \(L_E^d = L_{TE} + L_{NT}^E\). Inserting \(L_{TE} = \frac{\alpha}{1-\alpha} w_E \bar{H}_E\) and eq. (C11) in this equality yields:

\[ L_E^d = \frac{\alpha}{1-\alpha} w_E \bar{H}_E + (1-\beta) \left( L_E + w_E \bar{H}_E \right) + (1-\beta)\tau \sigma \left( \bar{L}_G + w_G \bar{H}_G \right) \]  
(C20)

Since \(L_E = L_E^d\):

\[ L_E = \left( \frac{\alpha}{1-\alpha} \beta^{-1} + \frac{1-\beta}{\beta} \right) w_E \bar{H}_E + \frac{1-\beta}{\beta} \tau \sigma \left( \bar{L}_G + w_G \bar{H}_G \right) \]  
(C21)

Inserting (C18) and (C19) in (C21) yields

\[ L_E = \left( \frac{1-\beta(1-\alpha)(1-(1-\beta)(1-\tau))}{\beta \bar{H}_E \left( \frac{\alpha}{1-\alpha} + (1-\alpha)(1-\beta)(1-\tau) \right)} \right) \left( \bar{H}_G \left( \frac{a_E}{a_G} \right)^\sigma \left( \frac{A_G}{A_E} \right)^{(1-\sigma)} \bar{H}_E \right)^\frac{1}{\alpha+(1-\alpha)\sigma} \frac{1}{\beta \sigma \tau} \bar{L}_G \frac{1}{L_E} \left( \frac{\alpha}{1-\alpha} \beta^{-1} + \frac{1-\beta}{\beta} \right), \quad E_1(\tau) \]

We can thus write: \(L_E = E_1(\tau) + E_2(\tau)\), with \(\frac{\partial E_1}{\partial \tau} > 0\) and \(\frac{\partial E_2}{\partial \tau} > 0\). Hence: \(\frac{\partial L_E}{\partial \tau} > 0\).
Finally, $\frac{\partial E}{\partial \tau} > 0$, $\frac{\partial W_G}{\partial \tau} > 0$ and $\frac{\partial W_E}{\partial \tau} > 0$ establish feature 2 of Proposition 5.

Appendix D. The Extended Model

D.1. Demand for goods and services

The utility function of the representative household is:

$$u = y_{NT}^{1-\beta} y_T^{\beta}, \quad \text{with} \quad y_T = \left[ a_G y_G^{\theta_1} + (a_E y_E^{\theta_2} + a_N y_N^{\theta_2})^{\theta_1/\theta_2} \right]^{1/\theta_1}$$

where $y_{NT}$ and $y_i$ denote the household's consumption of the non-tradable service and good $i = G, E, N$, respectively, and $y_T$ the consumption index of the combination of tradable goods $G, E$ and $N$.

$\sigma_1 = (1-\theta_1)^{-1}$ is the elasticity of substitution between country $G$'s tradable good and other tradable goods and $\sigma_2 = (1-\theta_2)^{-1}$ denotes the elasticity of substitution between tradable goods $E$ and $N$.

The maximisation of utility determines the total demands for goods and services:

$$Y^d_G = a_G^{\sigma_1} \frac{\beta I_w / P_T}{(P_G / P_T)^{\sigma_1}}, \quad p_G Y^d_G = a_G^{\sigma_1} \frac{\beta I_w}{(P_G / P_T)^{\sigma_1 - 1}} \quad (D1)$$

$$Y^d_E = a_E^{\sigma_2} \frac{\beta I_w / P_T}{(P_E / P_{EN})^{\sigma_2} (P_{EN} / P_T)^{\sigma_1}}, \quad p_E Y^d_E = a_E^{\sigma_2} \frac{\beta I_w}{(P_E / P_T)^{\sigma_2 - 1} (P_T / P_{EN})^{\sigma_2 - \sigma_1}} \quad (D2)$$

$$Y^d_N = a_N^{\sigma_2} \frac{\beta I_w / P_T}{(P_N / P_{EN})^{\sigma_2} (P_{EN} / P_T)^{\sigma_1}}, \quad p_N Y^d_N = a_N^{\sigma_2} \frac{\beta I_w}{(P_N / P_T)^{\sigma_2 - 1} (P_T / P_{EN})^{\sigma_2 - \sigma_1}} \quad (D3)$$

$$Y^d_{NT,i} = (1-\beta)I_i / w^i_L, \quad i = G, E, N \quad (D4)$$

$$I_W = I_G + I_E + I_N + I_S \quad (D5)$$

$$P_{EN} = \left( a_E^{\sigma_2} P_E^{1-\sigma_2} + a_N^{\sigma_2} P_N^{1-\sigma_2} \right)^{1/\sigma_2} \quad (D6)$$

$$P_T = \left( a_G^{\sigma_1} P_G^{1-\sigma_1} + P_{EN}^{1-\sigma_1} \right)^{1/\sigma_1} \quad (D7)$$

$$p_i^d = P_T^{\beta} \left( p_{NT}^{i} \right)^{1-\beta} \quad (D8)$$

$Y^d_i$ is the total demand (at the world level) of tradable good $i = G, E, N$ and $Y^d_{NT,i}$ the demand for the non-tradable service in country $i = G, E, N$. 
$I_w$ is the world’s total income, $I_j$ country $j$’s total income, $j = G,E,N,S$.

$p_i$ is the price of tradable good $i = G,E,N$, $p^T_{NT}$ the price of service $NT$ in country $i$, $P_{E,N}$ is the price index for tradable goods from countries $E$ and $N$, $P_i$ is the price index for the tradable goods from all three countries, and $P_i$ the consumer price index in country $i = G,E,N$.

**D.2. Production and factor demand in relation to world’s total income**

**a) Production prices**

Segments are ordered by increasing offshoring cost. In line with the notation introduced in Section 4.2 of the main text, $F$ denotes the total number of segments and $K_i \leq F$ the number of segments offshored to the South in the production of good $i = G,E,N$.

Because of the Cobb-Douglas technologies, the prices $p_i$, $i = G,E,N$, at the firms’ optimum are (with $w_j^f$ the cost of unskilled labour in the $f$-segment of country $i$’s tradable good production):

$$p_i = \frac{1}{A'} \left( \frac{w^j_H}{1-\alpha} \right)^{1-\alpha} \prod_{j=1}^{F} \left( \frac{F}{\alpha} \right)^{\alpha/F} = \frac{1}{A'} \left( \frac{w^j_H}{1-\alpha} \right)^{1-\alpha} \prod_{j=1}^{F} \left( \frac{F}{\alpha} \right)^{\alpha/F}$$

$$p_j = \frac{1}{A'} \left( \frac{w^j_H}{1-\alpha} \right)^{1-\alpha} \prod_{j=1}^{F} \left( \frac{F}{\alpha} \right)^{\alpha/F}$$

where $k_i \equiv \frac{K_i}{F}$, $\omega_j^f = w^S_L \times k_i^{f/F}$.

Noting that $\prod_{f=1}^{K_i} (\omega_j^F)^{\alpha/F} = (\omega_j^F)^{\alpha/F} \times (\omega_j^F)^{\alpha/F} \times \cdots (\omega_j^F)^{\alpha/F}$ and $\omega_j^f = w^S_L \times k_i^{f/F}$, it follows that

$$\prod_{j=1}^{K_i} (\omega_j^F)^{\alpha/F} = \left( w^S_L \right)^{\alpha/K_i} \times \left( \frac{w^S_L}{w^S_L} \right)^{\alpha/F} \times \frac{K_i}{F} \times \frac{1+2+\cdots+K_i}{F^2} = \left( w^S_L \right)^{\alpha/K_i} \times \frac{K_i}{F} \times \frac{K_i+1}{2F^2}$$

For a continuum of segments ($F \to \infty$), $k_i K_i + 1 \to \frac{k_i F (K_i + 1)}{2F^2} = \frac{k_i F (K_i + 1)}{2F^2}$, $F \to \infty \to k_i^2 / 2$ and

$$\prod_{j=1}^{K_i} (\omega_j^F)^{\alpha/F} = \left( w^S_L \right)^{\alpha/K_i} \times \frac{K_i}{F} \times \frac{K_i+1}{2F^2} \to \left( w^S_L \right)^{\alpha/K_i} \times \frac{K_i}{F} \times \frac{K_i+1}{2F^2} \to k_i^2 / 2$$

Hence:

$$p_j = \frac{1}{A'} \left( \frac{w^j_H}{1-\alpha} \right)^{1-\alpha} \left( \frac{w^S_L}{w^S_L} \right)^{\alpha/K_i} \times \frac{K_i}{F} \times \frac{K_i+1}{2F^2}$$

By inserting eq. (13) of the main text ($\kappa_i = \left( w^S_L / w^S_L \right)^{1/K_i}$) in this equation:

$$p_j = \frac{1}{A'} \left( \frac{w^j_H}{1-\alpha} \right)^{1-\alpha} \left( \frac{w^S_L^{k_i^2/2}}{w^S_L^{1-k_i^2/2}} \right)^{\alpha}$$

(D9)
Note that eq. (D9) is only valid for $\kappa_i > 1 \Leftrightarrow w^i_L > w_L^S$ and $k_i > 0$.

**b) Demand for Skilled labour**

The demand for skilled labour in each advanced country is:

$$H_i = (1 - \alpha) \frac{P_Y}{w_H}, \quad i = G, E, N$$  \hspace{1cm} (D10)

Inserting (D1) – (D3) into (D10):

$$H_G = (1 - \alpha) \frac{a_G \sigma_1}{w_H^G} \frac{\beta I_w}{(p_G / P_{EN})^{\alpha_1 - 1} (p_T / P_{EN})^{1 - \sigma_1}}$$ \hspace{1cm} (D11)

$$H_E = (1 - \alpha) \frac{a_E \sigma_2}{w_H^E} \frac{\beta I_w}{(p_E / P_{EN})^{\alpha_2 - 1} (p_T / P_{EN})^{1 - \sigma_1}}$$ \hspace{1cm} (D12)

$$H_N = (1 - \alpha) \frac{a_N \sigma_2}{w_H^N} \frac{\beta I_w}{(p_N / P_{EN})^{\alpha_2 - 1} (p_T / P_{EN})^{1 - \sigma_1}}$$ \hspace{1cm} (D13)

**c) Demand for Unskilled labour**

The demand for unskilled labour by the sector of non-tradable services in country $i$ is:

$$L_{NT}^i = Y_{NT}^i, \quad i = G, E, N.$$  

Because of the Cobb-Douglas technology, the demand $L_f^i$ for low skilled labour for the production of intermediate good $f$ utilised in the production of tradable good $i$ is:

$$L_f^i = \frac{\alpha p_Y}{F w_f^i}$$  

where $w_f^i$ is the unit cost of the unskilled labour utilised in the production of intermediate good $f$ in country $i$. This cost can be either the unskilled labour cost in country $i$, or the unskilled labour cost (including the offshoring cost) in the South when intermediate good $f$ is offshored.

Let $[1, ..., K_f]$ be the intermediate goods produced in the South for the tradable good $i$, $i = G, E, N$. Then:

$$L_{Ti}^i = (1 - k_i) \frac{\alpha p_Y}{w_L^i}, \quad k_i = K_f / F, \quad i = G, E, N.$$  

where $L_{Ti}^i$ is the unskilled labour demanded in country $i$ for the production of tradable good $(T,i)$ (the subscript indicates the good and the superscript the country).

Finally, the demand for low skilled labour in advanced country $i = G, E, N$ is:
\[ L_i = L_i^{NT} + L_i^I = Y^{i}_{NT} + (1 - k_i) \alpha \frac{p^{Y}_i}{w^{i}_L} \]  

By inserting eqs. (D1) – (D4) into eq. (D14), we obtain:

\[ L_G = (1 - \beta) \frac{I_G}{w^G_L} + (1 - k_G) \frac{a^G_G (1 - \sigma)}{w^G_L} (p_G / P_T)^{-\sigma} I_W \]  

\[ L_E = (1 - \beta) \frac{I_E}{w^E_L} + (1 - k_E) \frac{a^E_E (1 - \sigma)}{w^E_L} (p_E / P_{EN})^{1-\sigma} I_W \]  

\[ L_N = (1 - \beta) \frac{I_N}{w^N_L} + (1 - k_N) \frac{a^N_N (1 - \sigma)}{w^N_L} (p_N / P_{EN})^{1-\sigma} I_W \]

**D.3. Balanced trade of the South and determination of \( I_W \)**

The demands for factors depend on \( I_W = I_G + I_E + I_N + I_S \), with \( I_i = w^i_L \bar{L}_i + w^i_h \bar{P}_i \), \( i = G, E, N \). To determine \( I_S \) in terms of \( I_G + I_E + I_N \), we assume balanced trade of the South (imports = exports in nominal terms).

**Imports of the South:** All the tradable goods consumed in the South are imported. Hence:

\[ M_S = \beta I_S \]

**Exports of the South:** The South exports all the offshored segments in the production of goods \( G, E \) and \( N \): \( X_S = \sum_{f=1}^{\bar{K}_G} \alpha_f G L_{Gf} + \sum_{f=1}^{\bar{K}_E} \alpha_f E L_{Ef} + \sum_{f=1}^{\bar{K}_N} \alpha_f N L_{Nf} = \frac{K_G}{F} \alpha p_G Y_G + \frac{K_E}{F} \alpha p_E Y_E + \frac{K_N}{F} \alpha p_N Y_N \). This can be written:

\[ X_S = \alpha (k_G p_G Y_G + k_E p_E Y_E + k_N p_N Y_N) \]

Inserting eqs. (D1)-(D3) into this expression:

\[ X_S = \alpha \beta \left( \frac{k_G a_G^\sigma (1 - \sigma)}{(p_G / P_T)^{\sigma-1}} + \frac{k_E a_E^\sigma (1 - \sigma)}{(p_E / P_T)^{\sigma-1}(P_T / P_{EN})^{\sigma-\sigma}} + \frac{k_N a_N^\sigma (1 - \sigma)}{(p_N / P_T)^{\sigma-1}(P_T / P_{EN})^{\sigma-\sigma}} \right) I_W \]

**Balanced trade:** \( M_S = X_S \)

\[ \Rightarrow \beta I_S = \alpha \beta \left( \frac{k_G a_G^\sigma (1 - \sigma)}{(p_G / P_T)^{\sigma-1}} + \frac{k_E a_E^\sigma (1 - \sigma)}{(p_E / P_T)^{\sigma-1}(P_T / P_{EN})^{\sigma-\sigma}} + \frac{k_N a_N^\sigma (1 - \sigma)}{(p_N / P_T)^{\sigma-1}(P_T / P_{EN})^{\sigma-\sigma}} \right) I_W \]

\[ I_S = I_W - (I_G + I_E + I_N) = \alpha \left( \frac{k_G a_G^\sigma (1 - \sigma)}{(p_G / P_T)^{\sigma-1}} + \frac{k_E a_E^\sigma (1 - \sigma)}{(p_E / P_T)^{\sigma-1}(P_T / P_{EN})^{\sigma-\sigma}} + \frac{k_N a_N^\sigma (1 - \sigma)}{(p_N / P_T)^{\sigma-1}(P_T / P_{EN})^{\sigma-\sigma}} \right) I_W \]
Finally:

\[ I_w = \frac{I_G + I_E + I_N}{1 - \alpha \left( \frac{k_G a_G}{(p_G / p_T)^{\sigma_1}} + \frac{k_E a_E}{(p_E / p_T)^{\sigma_2}} + \frac{k_N a_N}{(p_N / p_T)^{\sigma_2-\sigma_1}} \right)} \quad (2.18) \]

\[ \text{D.4. Equilibria in the markets for factors} \]

Inserting eq. (D18) in eqs. (D11)–(D13) and in eqs. (D15)–(D17) to determine the factor demands and equalising demand and supply for each factor \((I_i^* = \bar{I}_i\) and \(H_i^* = \bar{H}_i, \ i = G, E, N\), yields:

\[ \bar{H}_G = a_G^{\sigma_1} \left( \frac{P_{EN}}{w_H p_G} \right)^{\sigma_1} \left( \frac{P_T}{p_G} \right)^{-\omega_1} \left( \frac{1}{\omega_1} \right) \beta (I_G + I_E + I_N) \quad (D19) \]

\[ \bar{H}_E = a_E^{\sigma_2} \left( \frac{P_{EN}}{w_H p_E} \right)^{\sigma_2} \left( \frac{P_T}{p_E} \right)^{-\omega_1} \left( \frac{1}{\omega_1} \right) \beta (I_G + I_E + I_N) \quad (D20) \]

\[ \bar{H}_N = a_N^{\sigma_2} \left( \frac{P_{EN}}{w_H p_N} \right)^{\sigma_2} \left( \frac{P_T}{p_N} \right)^{-\omega_1} \left( \frac{1}{\omega_1} \right) \beta (I_G + I_E + I_N) \quad (D21) \]

\[ \bar{I}_G = (1 - \beta) I_G + (1 - k_G) a_G^{\sigma_1} \left( \frac{P_{EN}}{w_G p_G} \right)^{\omega_1} \left( \frac{P_T}{p_G} \right)^{-\omega_1} \beta (I_G + I_E + I_N) \quad (D22) \]

\[ \bar{I}_E = (1 - \beta) I_E + (1 - k_E) a_E^{\sigma_2} \left( \frac{P_{EN}}{w_E p_E} \right)^{\omega_2} \left( \frac{P_T}{p_E} \right)^{-\omega_1} \beta (I_G + I_E + I_N) \quad (D23) \]

\[ \bar{I}_N = (1 - \beta) I_N + (1 - k_N) a_N^{\sigma_2} \left( \frac{P_{EN}}{w_N p_N} \right)^{\omega_2} \left( \frac{P_T}{p_N} \right)^{-\omega_1} \beta (I_G + I_E + I_N) \quad (D24) \]

with:

\[ I_i = w_i^j \bar{I}_i + w_H^j \bar{H}_i, \quad i = G, E, N \quad (D25) \]
By combining the equations defining $\bar{H}_i$ and $\bar{L}_i$ for each country $i = G, E, N$, we can simplify the system and replace eqs. (2.22) – (2.24) by the following:

$$w_H^i = \frac{\beta(1-\alpha)}{(1-\beta)(1-\alpha)+(1-k_j)\alpha} \frac{\bar{L}_i}{\bar{H}_i} w_L^i, \quad i = G, E, N$$  \hspace{1cm} (D26)

Because of Walras’ Law, we omit eq. (D21). We select $w_H^G = 1$ as numeraire. This makes a system of 13 equations: (D7), (D8), (D9) (3 equations), (D19), (D20), (D25) (3 equations) and (D26) (3 equations).

The exogenous parameters are: $\alpha, \beta, \beta_A, \beta_E, \beta_N, a_G, a_E, a_N$.

The exogenous variables are: $\bar{L}_i, \bar{H}_i, k_i, i = G, E, N$, and $w_L^G$. Additionally, $w_H^G = 1$.

The 13 endogenous variables are:

$$w_L^G, w_L^E, w_L^N, w_H^G, w_H^E, p_G, p_E, p_N, P_{EN}, P_T / P_{EN}, I_G, I_E, I_N.$$  \hspace{1cm} (D25)

D.5. Calculations in the case of unskilled labour market imperfection

The Eurozone unskilled wage $w_L^E$ and the German unskilled wage $w_L^G$ are pegged by the relation $\frac{w_L^E}{\sigma} = \frac{w_L^G}{\sigma}$. Then, we add $\frac{w_L^E}{\sigma} = \frac{w_L^G}{\sigma}$ to the system of equations and one of the unskilled labour utilisation $L_j, j = G, E$, becomes an endogenous variable defined by:

1) If $\sigma = \frac{w_L^G}{w_L^E} < \frac{\hat{w}_L^G}{\hat{w}_L^E}$ where $\frac{\hat{w}_L^G}{\hat{w}_L^E}$ corresponds to full employment in the three countries:

$$L_E = (1-\beta) \frac{I_E}{w_L^E} + (1-k_G) \frac{a_G}{w_L^E} \left( \frac{P_{EN}}{P_E} \right)^{\alpha - 1} \alpha \left( I_E + I_N \right) \left( \frac{P_L}{P_{EN}} \right)^{-\sigma} - \alpha \left( k_{G} a_{G} \left( \frac{P_{EN}}{P_E} \right)^{\sigma - 1} + \sum_{j=E,N} k_{j} a_{j} \right)$$

2) If $\sigma = \frac{w_L^G}{w_L^E} > \frac{\hat{w}_L^G}{\hat{w}_L^E}$ where $\frac{\hat{w}_L^G}{\hat{w}_L^E}$ corresponds to full employment in the three countries:

$$L_G = (1-\beta) \frac{I_G}{w_L^G} + (1-k_G) \frac{a_G}{w_L^G} \left( \frac{P_{EN}}{P_G} \right)^{\alpha - 1} \alpha \left( I_G + I_N \right) \left( \frac{P_L}{P_{EN}} \right)^{-\sigma_G} - \alpha \left( k_{G} a_{G} \left( \frac{P_{EN}}{P_G} \right)^{\sigma_G - 1} + \sum_{j=E,N} k_{j} a_{j} \right)$$

Case 1) corresponds to unskilled unemployment in Eurozone and case 2) to unskilled unemployment in Germany.

From the above equation(s) we can calculate the total number of unskilled workers and the total unemployment rates:

$$u_L^i = \frac{\bar{L}_i - L_i}{\bar{L}_i}, \quad u_i = \frac{\bar{L}_i - L_i}{\bar{L}_i + \bar{H}_i}, i = G, E$$
Appendix E. Empirical calculations

E.1. Labour force

1. The labour force is measured annually in terms of working hours, i.e.:

\[
\text{Labour force} = \text{number of persons in the labour force} \times \text{average number of hours worked}
\]

This is important because countries and firms have, to different degrees, utilised the tools of working time reductions and working time accounts to lessen or avoid unemployment.\(^\text{12}\)

2. The impact of the unification-related jump in the German labour force has been smoothed. In our model, Germany is an integrated area in terms of production and wage structure. In this model, the unification generates a 30% one-shot jump in the labour force of the German integrated economy. Inserting directly this jump in the simulations would be misleading because the unification put together two very different areas in terms of production and wages and the convergence of the two areas to form a unified economy was a staggered process. Lindlar & Scheremet (1998) note (i) that the decline in East German employment from 1990 to 1992 did not raise unemployment because of early retirement and migration from the East to the West, and (ii) that employment has been artificially maintained by the German policy from 1992 to 1995 (maintenance of jobs in the East German firms by the Treuhandanstalt, large increase in jobs in the sector of construction and in the public sector). Consequently:

a) We have firstly ignored the 1990-1995 period for which the model structure is evidently inadequate.

b) We have secondly smoothed the variation in the labour force by adding each year 1/10 of the 1990-2005 variation from 1995 to 2005. This corresponds to a smoothed integration of East Germany to the (West) German economy which comes to an end in 2005.

3. We do not consider the increase in the skill level of the three areas from 1975 to 2015. Considering this would lead to a significant decrease in the skill premia whereas, in fact, they increased over this time period, in particular because of skill-biased technological change. The latter is not introduced in our model because we focus on the sole impacts of labour force changes and offshoring. This means that our results correspond to a situation in which the changes in skill endowments are totally offset by skill-biased technological change. As this is obviously not the case in reality, the results must be interpreted ceteris paribus. As skilled biased technological change has certainly had a much greater impact in the US than in other

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\(^{12}\) For example, working time reductions have been implemented in France in the late 1990s and early 2000s, and in Germany in the 1980s and early 1990s. In Germany, the firms’ use of working time accounts and short-time work has been a key factor for explaining the relatively moderate decline in employment in the 2008 financial crisis in contrast to what occurred in other countries (Burda & Hunt, 2011).
countries, and considering that the US accounts for more than 50% of the labour force of the North area, this introduces a bias when comparing North with the other two areas. This is one of the reasons why the comparison is centred on Germany and Eurozone.

4. Even though we do not consider changes in skill levels, we account for between-area differences in skill endowments by calculating the average share of skilled workers as a percentage of the total labour force in the 1990s and 2000s (from the OECD database; skilled workers are defined as workers with tertiary and post-secondary non tertiary education).

E.2. Wage pegging (imperfections in the unskilled labour markets)

The unskilled wages between Germany and Eurozone are pegged to introduce imperfections in the unskilled labour markets (Simulations 2 and 3; see section 5.1 for information on the setup of the simulations). This can generate unemployment in these two areas.

1. Pegged wages are introduced by inserting in the simulations the observed ratio of labour costs per hour employed in Germany to those employed in Eurozone in the same currency (current US dollar). As the model assumes identical variations in labour productivity in the two areas, we need to erase the effect of differences in productivity growth by considering the following productivity-adjusted ratio of unit labour costs:

$$\lambda_{G/E} = \frac{\text{labour share of total income (in %) in Germany} \times \text{GDP in current US dollar in Germany}}{\text{labour share of total income (in %) in Eurozone} \times \text{GDP in current US dollar in Eurozone}} \times \frac{\text{employment in hours in Germany}}{\text{employment in hours in Eurozone}}$$

We calculate this ratio from the following data provided by the OECD (OECD.Stat): the labour share of total income (in %) for each year and each country, the GDP in current US dollar, the total employment (number of persons) and the average annual worked hours per employed worker for all the selected countries.

In a first round of simulations we entered the pegged wage of Eurozone unskilled workers of the year we simulated (i.e., the pegged wage of year t for the simulation in year t; these simulations are available from the authors) and we noted a lag in the observed values compared to the simulated values. This is not surprising considering the lag between changes in unit labour costs and their impacts on prices, demand and production. We have therefore applied the mean of the pegged wages of years (t-1) and t to the simulation in year t.

2. We have not assumed pegged wages between North and the other two areas because of the large volatility of the US dollar. The US accounts for more than half of the North labour force and production. As the dollar has been characterised by high volatility in relation to other currencies, the North unit labour costs exhibit wide and rapid fluctuations in relation to

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13 This is also motivated by the fact that North is a highly heterogeneous area which covers countries greatly differing in their social and labour market institutions, public interventions etc. (the US, Scandinavia and Japan).
the unit labour costs of other areas. In our general equilibrium model where demand adjusts immediately to price changes, this generates large and rapid movements in the demand for goods and in unemployment. Such movements are at odds with observed developments because, in the real economy, firms are insured against exchange rate volatility. In consequence, in contrast to Eurozone, non-adjustment in relative wages between Germany and North cannot be introduced through pegged unit labour costs. Since pegging the unit labour costs between Germany and North is not possible, we have introduced in Simulations 2 and 3 the North employment (labour force minus unemployment) as an exogenous variable, the unskilled wage of the North becoming then endogenous. Since the simulated model no longer determines unemployment in the North, the simulations focus on the differences between Germany and Eurozone.

3. Pegging the unskilled labour wages between Germany and Eurozone from the ratio of productivity-adjusted unit labour costs has two implications:

a) The pegging is based on costs of all workers because no comparable data are available for the unskilled workers’ wage in the different countries covering the analysed time period. This means that the imperfections in the labour market for unskilled workers could be underestimated because the market for skilled labour is typically more flexible than that for unskilled labour.

b) When $\hat{w}_L^G / w_L^E < \hat{w}_L^G / \hat{w}_L^E$ (here, $\hat{\cdot}$ indicates the full employment value), this generates unskilled workers’ unemployment in Eurozone and full employment in Germany, and $\hat{w}_L^G / w_L^E > \hat{w}_L^G / \hat{w}_L^E$ generates the opposite situation. Hence, one of the two areas is always at full employment and the unemployment calculated for the constrained area depicts in fact the unemployment gap between the two areas.

E.3. Data for the calculations of offshoring shares and elasticities of substitution

1) **Original data:** World Input-Output Database (Release 2013), see Timmer et al. (2015):

   a) World Input-Output Tables (WIOT)
      - 40 countries (27 EU members and 13 other major economies) and the “rest of the world” region (RoW)
      - 35 sectors (according to ISIC Rev. 3 classification)
      - Tables in *current prices*, in millions of dollars: available for the period 1995 – 2011
      - Tables in *previous years’ prices*, in millions of dollars: available for the period 1996 – 2009
b) Socio-Economic Accounts (SEA) – Updated in June 2014:
- Compatible with WIOT: the same countries and sectors
- Nominal variables, e.g. labour compensation, are denoted in millions of national currencies
- Dataset available for the period 1995 – 2011 but some variables are available only until 2009.

2) Transformed data

- Nominal variables in the SEA are converted from values in national currencies to dollar values using a table with exchange rates provided in WIOD.
- Elimination of certain sectors, sector and country aggregation (for a detailed explanation of the transformation of the original data, see the Appendix of Marczak and Beissinger (2018). The same transformation is applied to WIOT in current prices, WIOT in previous years’ prices and SEA
- Since SEA does not cover information on RoW, this region is for consistency excluded also from WIOT in current prices and WIOT in previous years’ prices.
- The transformed dataset (without RoW) covers 36 countries and 29 sectors.
- Even though data in current prices is available until 2011, data in previous years’ prices is required for the computation of real values, e.g. real exports. Tables in previous years’ prices are available only until 2009. Moreover, variables from the SEA dataset that are needed for the calculation of the share of production stages offshored to the South \( k_{it} \) are available also only until 2009. For export estimations, the years 2008 and 2009 are excluded from the final sample. In this case, extreme observations at the end of the sample might otherwise induce biased results. Therefore, for export estimations the sample is restricted to the period 1995-2007.

3) Assignment of individual countries to the regions considered in the paper: Eurozone, North and South:

a) Eurozone: Austria, Belgium, France, Finland, Greece, Ireland, Italy, Netherlands, Portugal, Spain
b) North: Australia, Denmark, Canada, Japan, Sweden, UK, US
c) South: Bulgaria, Brazil, China, Czech Republic, Baltic countries (Estonia, Lithuania, Latvia), Hungary, Indonesia, India, Mexico, Poland, Romania, Russia, Slovakia, Slovenia, Turkey
A few remarks must be made as regards the assignment of countries to the above regions. First, note that South includes some euro area countries. Second, there are some countries which are not assigned to any of these regions. Third, according to the definition of the regions in the paper, North also encompasses New Zealand and Norway. However, WIOD Release 2013 does not include these countries.

4) Assignment of individual sectors to the aggregate sector of tradable goods:

- The first considered option is to treat manufacturing as the tradable sector. In this case, the composition of the tradable sector is the same in all countries due to the same sectoral classification.
- The second option considered in this paper is to use the following definition of the tradable sector: for each individual country, a sector is classified as a tradable sector if its nominal exports lie above the 25th percentile of exports distribution of this country in 1995. This implies that sectors belonging to the tradable sector may differ across regions.

E.4. Formal derivation of $k_{it}$

In the model, $k_{it}$ denotes the share of low-skilled production segments offshored to the South in the total of low-skilled segments utilised to produce the tradable good $i = G,E,N$ in period $t$. We quantify $k_{it}$ by calculating

$$k_{it} = \frac{\psi_{Si,t}}{\psi_{Si,t} + \psi_{iit}}, \quad i = G, E, N,$$

where $\psi_{Si,t}$ denotes the value-added contribution of low-skilled workers in the South to final goods production in the tradable goods sector of country (group) $i$ in period $t$, and $\psi_{iit}$ the value added generated by unskilled labour in country (group) $i$ to produce country $i$’s tradable goods.

The calculation of $k_{it}$ is based on the World Input-Output Database (WIOD) that comprises the World Input-Output Tables (WIOT) and Socio-Economic Accounts. The WIOT provides detailed information on global production linkages at the country-sector level for the years 1995-2011. The SEA contain, among others, information on labour incomes for different skill levels for the years 1995-2009. Since we need the information from both databases, we can only calculate $k_{it}$ for the years 1995-2009. Appendix E.3 lists the countries belonging to Eurozone, North and South in the calculations and explains the data selection and construction in more detail. For Germany and for each country belonging to Eurozone and North, an industry is considered as non-tradable (resp. tradable) if its gross
exports lie below (resp. above) the 25th percentile of the gross exports’ distribution in this country in 1995. Consequently, the tradable sector in a country may include manufacturing as well as service sectors. As an alternative for the tradable sector, we also considered the whole manufacturing sector for the computation of $k_{ii}$. In the simulations, however, we use the values for the tradable sector. The results for $k_{ii}$ based on the manufacturing sector are available from the authors upon request.

The concepts presented below are compatible with any global inter-country input-output (ICIO) table. Global ICIO tables provide detailed information at a disaggregated country-sector level which allows us to precisely derive $\psi_{Si,t}$ and $\psi_{ti,t}$ for an aggregated tradable goods sector in the aggregated regions $i = G, E, N$. The information on international intersectoral linkages offers the advantage that all direct and indirect connections between South and a particular developed country (region) $i$ are correctly recorded.

Global ICIO tables are available at an annual frequency so that the quantities derived below can be computed for each year. In the following calculations we will omit the time index for convenience. A simplified ICIO table for $C$ countries and $M$ sectors consists of four parts: (i) the $(CM \times CM)$ matrix $Z$ of intermediate sales, (ii) the $(CM \times C)$ matrix $F$ of final demand, (iii) the $(CM \times 1)$ vector $y$ of value added, and (iv) the $(CM \times 1)$ vector $q$ of gross output, where

$$Z = \begin{pmatrix} Z_{11} & \cdots & Z_{1C} \\ \vdots & \ddots & \vdots \\ Z_{C1} & \cdots & Z_{CC} \end{pmatrix}, \quad Z_{cd} = \begin{pmatrix} z_{cd}^{11} & \cdots & z_{cd}^{1M} \\ \vdots & \ddots & \vdots \\ z_{cd}^{M1} & \cdots & z_{cd}^{MM} \end{pmatrix}, \quad F = \begin{pmatrix} F_1 \\ \vdots \\ F_C \end{pmatrix}, \quad F_c = \begin{pmatrix} f_{c1}^1 & \cdots & f_{cc}^1 \\ \vdots & \ddots & \vdots \\ f_{c1}^C & \cdots & f_{cc}^C \end{pmatrix}.$$  

$q = (q_1', ..., q_C')', \quad q_c = (q_{c1}', ..., q_{cM}')', \quad y = (y_1', ..., y_C')', \quad y_c = (y_{c1}', ..., y_{cM}')'.

Gross output satisfies the following accounting relationship: $q = Z \cdot e_{CM} + F \cdot e_C = A \cdot q + F \cdot e_C$, where $A \equiv Z \cdot [diag(q)]^{-1}$ is the matrix of technical coefficients, and $e_{CM}$ and $e_C$ denote vectors of ones with dimension $(CM \times 1)$ and $(C \times 1)$, respectively. The solution for $q$ is $q = B \cdot F \cdot e_C$, with $B \equiv (I_{CM} - A)^{-1}$ and $I_{CM}$ denoting the $(CM \times CM)$ identity matrix. The matrix $B$ is the Leontief inverse with elements $b_{cd}^{mn}$ indicating how much output of industry $m$ in country $c$ is needed to produce one extra unit of final good $n$ in country $d$. $B$ takes global value chains into account, i.e. all direct and indirect intermediate good linkages between sectors. To obtain unskilled value added contained in $b_{cd}^{mn}$, i.e. the unskilled value-added contribution of industry $m$ in country $c$ to the final good produced in industry $n$ in country $d$, matrix $V^u$ is first defined:

$$V^u \equiv \Phi \cdot \Gamma \cdot V,$$

where $V = diag(y) \cdot [diag(q)]^{-1}$ is the matrix of direct sectoral value-added shares. The matrix $\Gamma$ is given by $\Gamma = diag(y)$, where the $(CM \times 1)$ vector $y$ contains labour shares in sectoral value added. The matrix $\Phi$ is also a diagonal matrix, $\Phi = diag(\phi)$, with the
elements of the \((CM \times 1)\) vector \(\Phi\) describing unskilled shares in sectoral labour income. The matrix \(V^u\) is a diagonal matrix where each element on the diagonal represents the sectoral share of unskilled value added in the sector’s own gross output. Then, the matrix of unskilled value-added contributions is defined as \(\Psi = V^u \cdot B\).

Note that \(\Psi\) is given at the level of individual countries and sectors. It is possible to obtain \(\psi_{si}\) and \(\psi_{ii}\), the quantities needed for the computation of \(k_{it}\), from \(\Psi\) after an appropriate aggregation. For that purpose, let \(I_S\) denote the set containing all indices of countries belonging to the South. Then, the \((CM \times 1)\) vector \(r_S = (r_{S,1}^1, ..., r_{S,1}^M, ..., r_{S,C}^1, ..., r_{S,C}^M)^\top\) is defined such that \(r_{S,c}^m = 1\), if \(c \in I_S\), and \(r_{S,c}^m = 0\) otherwise. Similarly, let \(I_i, i = G, E, N, \) be the set with indices of countries from a developed region (Germany, Eurozone, North) and the corresponding vector \((CM \times 1)\) vector \(r_i = (r_{i,1}^1, ..., r_{i,1}^M, ..., r_{i,C}^1, ..., r_{i,C}^M)^\top\) has elements \(r_{i,c}^m = 1\), if \(c \in I_i\), and \(r_{i,c}^m = 0\) otherwise. Further, let \(I_T^c\) denote the country-specific set containing all indices of sectors belonging to the tradable goods sector. It is to be noted that this allows for a general definition of an aggregate tradable goods sector that may lead to a different sectoral composition across individual countries. If the same sectoral composition of the tradable sector in all countries is considered, then \(I_T^c = I_T\) for all \(c = 1, ..., C\). An important example for such a case is when total manufacturing sector (with the same sectoral classification in all countries) is considered as the tradable sector. Based on \(I_i\) and \(I_T^c\) the matrix \((CM \times CM)\) matrix \(R_i^T\) is given by \(R_i^T = \text{diag}(r_i^T)\), where \(r_i^T = (r_{i,1}^{T,1}, ..., r_{i,1}^{T,M}, ..., r_{i,C}^{T,1}, ..., r_{i,C}^{T,M})^\top\). The elements \(r_{i,c}^{T,m}\) are defined as follows: \(r_{i,c}^{T,m} = 1\), if \(c \in I_i\) and \(m \in I_T^c\), and \(r_{i,c}^{T,m} = 0\) otherwise.

Finally, \(\theta_{si}\) and \(\theta_{ii}\) can be obtained using \(r_s, r_i\) and \(R_i^T\):

\[
\psi_{si} = r_s^\top \cdot \Psi \cdot R_i^T \cdot f
\]
\[
\psi_{ii} = r_i^\top \cdot \Psi \cdot R_i^T \cdot f,
\]

where \(f = F \cdot e_c\).

**Calculation of \(k_{it}\) using WIOD for 1995-2009**

The calculation of \(k_{it}\) is based on the transformed WIOD dataset described in Appendix E.1. Since the data needed for the computation is available for the period 1995 - 2009, the resulting \(k_{it}\) will be obtained for this period.

In the following explanations, the time index will be omitted for convenience. One of the components common to both \(\psi_{si}\) and \(\psi_{ii}\) is matrix \(\Psi\), as defined above. This matrix requires

\[\text{Whereas } V \text{ can be directly computed using an ICIO table, calculation of } \Gamma \text{ and } \Phi \text{ requires additional information that is not available in an ICIO table. In this paper, we resort to the WIOD for the calculation of } k_{it}. \]

\[\text{In this case, this required additional information is provided in Socio-Economic Accounts (SEA) which is available in the WIOD compatible with the World-Input Output Tables (WIOT).}\]
the calculation of $V$, $F$, $\Phi$ and $B$. Matrix $V$ is calculated using value added and gross output – both provided in WIOT. Matrix $B$ is also obtained from WIOT using the matrix of intermediate sales and gross output. The source for $F$ and $\Phi$ is the SEA dataset. As regards $F$, its diagonal elements are computed as shares of labour compensation (label LAB) in value added. As for $\Phi$, its diagonal elements – shares of unskilled labour – are computed as the sum of the shares of low-skilled labour compensation (label LABLS) and medium-skilled labour compensation (LABMS). Another component common to $\psi_{Si}$ and $\psi_{it}$ is vector $f$ obtained from the final demand matrix provided in WIOT. Vectors $r_s$ and $r_t$, and matrix $R^T_t$ are obtained using the definition of regions (see point 3 in Appendix E.3) and two alternative definitions of the tradable sector. The first definition is based on the individual countries’ export distribution and is therefore country-specific (see point 4 in Appendix E.3). The second definition treats manufacturing as the aggregate tradable sector.

Results for $k_{it}$ and extrapolation

Table 1 summarizes values for $k_{it}$ where the aggregate sector in country (region) $i = G, E, N$ that offshores its production segments is the tradable goods sector.\(^{15}\)

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</tr>
</thead>
<tbody>
<tr>
<td>$k_{Gi}$</td>
<td>0.34</td>
<td>0.32</td>
<td>0.34</td>
<td>0.36</td>
<td>0.38</td>
<td>0.39</td>
<td>0.37</td>
<td>0.37</td>
<td>0.41</td>
<td>0.43</td>
<td>0.44</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.41</td>
</tr>
<tr>
<td>$k_{Et}$</td>
<td>0.27</td>
<td>0.25</td>
<td>0.27</td>
<td>0.28</td>
<td>0.30</td>
<td>0.30</td>
<td>0.31</td>
<td>0.31</td>
<td>0.33</td>
<td>0.32</td>
<td>0.32</td>
<td>0.31</td>
<td>0.30</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>$k_{Nt}$</td>
<td>0.22</td>
<td>0.20</td>
<td>0.23</td>
<td>0.23</td>
<td>0.24</td>
<td>0.25</td>
<td>0.24</td>
<td>0.25</td>
<td>0.26</td>
<td>0.31</td>
<td>0.31</td>
<td>0.32</td>
<td>0.31</td>
<td>0.30</td>
<td>0.27</td>
</tr>
</tbody>
</table>

In the next step, we extrapolate $k_{it}$ obtained with WIOD since the simulations require values of $k_{it}$ for the whole simulation period 1975 – 2015. The extrapolation is based on a linear regression of $k_{it}$ on $m_{it}$ denoting intermediate goods imports of region $i = G, E, N$, from the South relative to GDP of region $i$. The variable $m_{it}$ represents a simple offshoring measure which, in contrast to $k_{it}$, does not precisely capture the contribution of low-skilled workers from the South to the production in a tradable sector in a developed region as the intermediate goods imported from the South also contain value-added contribution of high-skilled workers. Nevertheless, $k_{it}$ should be related to $m_{it}$ since higher intermediate goods imports from the South are expected to embed more low-skilled labour. The data on country-level imports required for the calculation of $m_{it}$ are easily available for a longer time period which makes $m_{it}$ a suitable variable for the extrapolation purpose.

We calculate $m_{it}$ using the CHELEM database and the same definition of regions (Germany, Eurozone, North and South) as in the case of $k_{it}$ and obtain values for the period 1970 – 2015.

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\(^{15}\) The results using the manufacturing sector are available on request.
The regression period is reduced to 1995 – 2006 to exclude years of financial and economic crisis that might lead to unreliable regression results. In fact, scatterplots of $k_{it}$ versus $m_{it}$ for all three developed areas, especially for the Eurozone, reveal that years 2007 and 2009 can be treated as outliers. Based on the estimated coefficients for $m_{it}$ we calculate predictions $\hat{k}_{it}$ for the out-of-sample period 1970 – 1994 and 2007 – 2015. Fig. 1 depicts the extrapolated values $\hat{k}_{it}$ along with the values computed with WIOD for all three developed areas and the tradable goods sector. This time series is then finally used in the simulations.

**E.5. Calculation of the elasticities of substitution $\sigma_1$ and $\sigma_2$**

**E.5.1. Theoretical considerations**

*Export elasticity of Germany and identification of $\sigma_1$*

In the extended model the German exports (exports of good $G$) are:

$$X_G^d = a_G^\sigma \left( \frac{P_G}{P_T} \right)^{-\sigma_1} \beta \frac{I_W - I_G}{P_T}$$  \hspace{1cm} (E1)

where

$$P_T = \left( a_G^\sigma P_G^{1-\sigma_1} + P_{EN}^{1-\sigma_1} \right)^{1/(1-\sigma_1)}$$  \hspace{1cm} (E2)
At this level of aggregation, it must be taken into account that $P_T$ is affected by changes in national price levels. If $p_G$ has a non-negligible influence on $P_T$, the price elasticity of German exports is not equal to $\sigma_1$.

We denote the expression $\frac{I_w - I_G}{P_T}$ as

$$ E_G^w = \frac{I_w - I_G}{P_T} \quad \text{(E3)} $$

Eq. (E1) can be written:

$$ X_G^d = a_G^{\sigma} \left( \frac{p_G}{P_T} \right)^{-\sigma} \beta E_G^w \quad \text{(E4)} $$

The elasticity of German exports with respect to the German export price keeping $E_G^w$ constant is

$$ \frac{\partial X_G^d}{\partial p_G} \bigg|_{de_e=0} = -\sigma \left( 1 - \frac{\partial P_T}{\partial p_G} \frac{p_G}{P_T} \right) \quad \text{(E5)} $$

Hence, the export elasticity is equal to $\sigma_1$ only in the case where $P_T$ does not react to changes in $p_G$. In the general case the elasticity of $P_T$ with respect to $p_G$ must be taken into account.

It holds that

$$ \frac{\partial P_T}{\partial p_G} \frac{p_G}{P_T} = a_G^{\sigma} \left( \frac{p_G}{P_T} \right)^{1-\sigma} = \frac{p_G X_G^d}{\beta (I_w - I_G)} \quad \text{(E6)} $$

The last expression derives from eq. (E1).

Alternatively, using the relationship $Y_G^d = a_G^{\sigma} \left( \frac{p_G}{P_T} \right)^{-\sigma} \beta I_w$ one gets:

$$ \frac{\partial P_T}{\partial p_G} \frac{p_G}{P_T} = \frac{p_G Y_G^d}{\beta I_w} \quad \text{(E7)} $$

We define:

$$ S_G = \frac{p_G X_G^d}{\beta (I_w - I_G)} = \frac{p_G Y_G^d}{\beta I_w} \quad \text{(E8)} $$

Hence, the elasticity of German exports with respect to its own export price (in absolute value), for a given level of $E_G^w$, denoted $\varepsilon_G$, can be written as

$$ \frac{\partial X_G^d}{\partial p_G} \bigg|_{de_e=0} = \sigma (1 - S_G) = \varepsilon_G \quad \text{(E9)} $$
From the estimated value of \( \varepsilon_G, \hat{\varepsilon}_G \), we can identify \( \hat{\sigma}_1 \) by
\[
\hat{\sigma}_1 = \frac{\hat{\varepsilon}_G}{1 - \bar{S}_G}
\]  
(E10)
where \( \bar{S}_G \) denotes the average value of \( S_G \) over time.

**Export elasticities of Eurozone and North and identification of \( \sigma_2 \)**

In the extended model it holds that
\[
X^d_E = a_E \sigma_2 \left( \frac{P_E}{P^*_E} \right)^{-\sigma_2} \left( \frac{P^*_E}{P_E} \right)^{-\sigma_1} \beta(I^*_W - I_E) = a_E \sigma_2 \left( \frac{P_E}{P^*_E} \right)^{-\sigma_2} \left( \frac{P^*_E}{P_E} \right)^{-\sigma_1} \left( \frac{P^*_E}{P_E} \right)^{-\sigma_1} \left( \frac{P_E}{P^*_E} \right)^{-\sigma_1} E^W_E
\]  
(E11)
where \( P_T \) is defined in eq. (E2) and \( P_{EN} \) is defined as
\[
P_{EN} = \left( a_E \sigma_2^{-1} a_N \sigma_2^{-1} + p_N \sigma_2^{-1} \right)^{(1 - \sigma_2)}
\]  
(E12)
The elasticity of Eurozone exports with respect to the Eurozone export price keeping \( E^W_E \) constant is
\[
\frac{\partial X^d_E}{\partial P_E} \bigg|_{E^w_E = 0} = -\sigma_2 \left( 1 - \frac{\partial P^*_E}{\partial P_E} \frac{P_E}{P^*_E} \right) - \sigma_1 \left( 1 - \frac{\partial P^*_E}{\partial P_E} \frac{P_E}{P^*_E} \right) \left( 1 - \frac{\partial P^*_E}{\partial P_E} \frac{P_E}{P^*_E} \right)
\]  
(E13)
The last term describes the percentage change in \( P_T \) due to one percent increase in \( p_E \) that happens via the change in \( P_{EN} \) that is caused by the change in \( p_E \). For the export elasticity to be equal to \( \sigma_2 \) it would be sufficient that \( p_E \) has no influence on \( P_{EN} \). If that is not true, the resulting expression for the export elasticity is more complicated.

Computing the derivative of \( P_{EN} \) with respect to \( p_E \) and taking account of eq. (E11), it holds that
\[
\frac{\partial P_{EN}}{\partial P_E} P_E = a_E \sigma_2 \left( \frac{P_E}{P^*_E} \right)^{-\sigma_2} \left( \frac{P^*_E}{P_E} \right)^{-\sigma_1} \left( \frac{P^*_E}{P_E} \right)^{-\sigma_1} \left( \frac{P_E}{P^*_E} \right)^{-\sigma_1} \frac{P_E X^d_E}{P_E X^d_E} \frac{\beta(I^*_W - I_E)}{\beta(I^*_W - I_E)}
\]  
(E14)
Because of the definition of \( P_T \) in eq. (E2):
\[
a^*_G \left( \frac{P_G}{P_T} \right)^{-\sigma_1} \left( \frac{P_T}{P_T} \right)^{-\sigma_1} = 1
\]  
(E15)
Hence, using eqs. (E6) and (E7):
\[
\left( \frac{P_{EN}}{P_T} \right)^{-\sigma_1} = 1 - a_G \left( \frac{P_G}{P_T} \right)^{-\sigma_1} = 1 - \frac{P_G X^d_G}{\beta(I^*_W - I_G)} = 1 - \frac{P_G Y^d_G}{\beta I^*_W} = 1 - S_G
\]  
(E16)
where \( S_G \) is defined by eq. (E8).
Therefore:

\[ \left( \frac{P_T}{P_{EN}} \right)^{1-\sigma_i} = \frac{1}{1-S_G} \]  

(E17)

Inserting (E17) into (E14) leads to:

\[ \frac{\partial P_{EN}}{\partial P_T} \frac{P_E}{P_{EN}} = \frac{S_E}{1-S_G} \]

where

\[ S_E = \frac{P_E X^d_E}{\beta(I_W - I_E)} = \frac{P_E Y^d_E}{\beta I_W} \]

(E18)

The expression \( \frac{\partial P_T}{\partial P_{EN}} \frac{P_{EN}}{P_T} \) in eq. (E.13) can be written because of eq. (E.16):

\[ \frac{\partial P_T}{\partial P_{EN}} \frac{P_{EN}}{P_T} = \left( \frac{P_{EN}}{P_T} \right)^{1-\sigma_i} = 1 - S_G \]

(E19)

Hence, eq. (E.13) can be written:

\[ \frac{\partial X^d_E}{\partial P_T} \frac{P_E}{X^d_E} \bigg|_{d_{EF}^u = 0} = -\sigma_2 + (\sigma_2 - \sigma_1) \frac{S_E}{1-S_G} + \sigma_1 (1 - S_G) \frac{S_E}{1-S_G} = -\sigma_2 \left( 1 - \frac{S_E}{1-S_G} \right) - \sigma_1 S_G \frac{S_E}{1-S_G} \]

The price elasticity of Eurozone exports (in absolute value), denoted \( \varepsilon_E \), can be written:

\[ \varepsilon_E = \left| \frac{\partial X^d_E}{\partial P_T} \frac{P_E}{X^d_E} \bigg|_{d_{EF}^u = 0} \right| = \sigma_2 \left( 1 - \frac{S_E}{1-S_G} \right) + \sigma_1 S_G \frac{S_E}{1-S_G} \]

(E20)

By indicating the estimated values with a hat, and with \( \hat{\sigma}_1 \) being determined according to eq. (E10), we have:

\[
\hat{\sigma}_2 = \frac{1}{1 - (\overline{S}_E/(1 - \overline{S}_G))} \left( \varepsilon_E - \hat{\sigma}_1 \overline{S}_G \frac{\overline{S}_E}{1 - \overline{S}_G} \right)
\]

(E21)

where \( \overline{S}_G \) and \( \overline{S}_E \) respectively denotes the average values of \( S_G \) and \( S_E \) over time.

Extrapolating the calculation of (E21) to the case of North, and given that \( \sigma_2 \) holds for both Eurozone and North, we must also have:

\[
\hat{\sigma}_2 = \frac{1}{1 - (\overline{S}_N/(1 - \overline{S}_G))} \left( \varepsilon_N - \hat{\sigma}_1 \overline{S}_G \frac{\overline{S}_N}{1 - \overline{S}_G} \right)
\]

(E22)
E.5.2. Data construction for export estimations

All variables are calculated based on the data of the 2013 release of the WIOD, see Appendix E.3 for details.

1) Real exports

- Nominal exports are computed as the sum of final goods exports and intermediate goods exports. Exports at the level of individual sectors or a sectoral aggregate (manufacturing or tradable sector) are considered as sales to:
  - other countries (in the case of data without aggregation of countries to regions); for example, for France exports are sales to all other countries
  - or other regions (in the case of data for country regions); for example, for Eurozone exports are sales to Germany, North and the remaining region.
- This computation is performed for the data in current prices and in previous years’ prices
- Using nominal exports in current prices and previous years’ prices, real exports are obtained with the chain-linking procedure in three steps
  - Step 1: Yearly growth rates of nominal values are computed holding the prices at the level of a previous year:
    \[ g_{t-1,t} = \frac{P_{t-1} Y_t}{P_{t-1} Y_{t-1}} - 1 \]
    \( P \): price; \( Y \): quantity
  - Step 2: (chain-linking) with 1 as the value for the base year \( b \) and the growth rates from Step 1, chain index is computed for all other years:
    \[ I_{d,b} = 1 \]
    \[ I_{d,b+1} = I_{d,b+1} \cdot (1 + g_{b,b+1}) \]
    \[ I_{d,b+2} = I_{d,b+2} \cdot (1 + g_{b+1,b+2}) \]
    ...
- Step 3: Chain-linked volumes are computed by multiplying the index of each year with the nominal value in the base year.

2) Real foreign demand

- Nominal foreign demand corresponding to an individual sector or a sectoral aggregate in a particular country or country region is obtained as the weighted sum of:
  - Imports of all countries or country regions (apart from the considered country or country region) from the considered sector in all other countries or country regions; imports comprise imports of final goods and intermediate goods
- Sales of final goods and intermediate goods from the considered sector or sectoral aggregate to the own country (for all countries or country regions apart from the considered country)

- Weights for countries or country groups in the weighted sum are given by the share of nominal exports from the considered sector (or sectoral aggregate) in the considered country (or country group) to a particular receiving country or region included in the sum in total exports of the considered sending country-sector pair.

- The above computations are performed for data in current and previous years’ prices.

- Real foreign demand is in the last step obtained using the chain-linking procedure, analogously as in the case of real exports.

3) **Real effective exchange rate (REER)**

- The account below refers to the case of individual countries and sectors.

- REER in a considered sector is computed as the geometric mean of the relative price deflators:

\[
\varphi^m_c = \prod_{c=1,c\neq d}^C \left( \frac{\rho^m_c}{\rho^m_d} \right)^{\delta_{cd}^m}
\]

\( \varphi^m_c \): REER in sector \( m \) in country \( c \)

\( \rho^m_c \): deflator in sector \( m \) in country \( c \)

\( \rho^m_d \): deflator in sector \( m \) in a trading partner country \( d \)

\( \delta_{cd}^m \): weight of trading partner country \( d \)

- The number of trading partners \( (C - 1) \) is 35 in the case of individual countries.

- As a deflator \( \rho \) in the above formula, we used the producer price index. To compute this price index, we resorted to the final data described in a previous step (WIOT in current and previous years’ prices). More specifically, based on gross output in current and previous years’ prices, we derived real gross output by applying the chain-linking procedure (for details on chain-linking, see also the computation of real exports). We obtained producer price index by dividing real gross output by gross output in current prices.

- Weights \( \delta \) reflects the importance of trading partner \( d \) for sector \( m \) in country \( c \).

  - They are computed as a weighted average of double export weights and import weights. Computation of double export weights follows here the definition of BIS (see Turner and Van’t dack, 1993).

  - Double export weights take into account not only the direct competition between the exporter and its trading partner (i.e. on the home market of the trading partner) but also third country competition.
Calculation of REER in a particular period involves weight $\gamma$ in the same period (for simplicity, time index is omitted in the above formula). However, since weights are based on exports, this could induce endogeneity bias in the export estimations when REER is used a regressor. To mitigate this problem, weights of the previous year (i.e. based on export of the previous year) are used in the construction of REER for the current period.

- In the case of country regions, number of trading partners is 3. For example, trading partners of Germany are Eurozone, North and the remaining region (that includes South and some other countries).

### E.5.3. Estimation of the price elasticity of export demand

All estimations are fixed-effects estimations at the sectoral level for the period 1995-2007. The aim is to estimate a uniform (‘average’) export elasticity across sectors of the respective area (Germany, Eurozone or North). In the next section, the estimation results are compared to results from the literature.

The baseline model is:

$$\text{exp}_{jt} = c_i + \alpha_i \text{reer}_{jt} + \mu_j + \gamma_i \text{fd}_{jt} + u_{jt},$$  \hspace{1cm} (Model 1a)

where $\text{exp}$ denotes log real exports, $\text{reer}$ denotes the real effective exchange rate (in logs) and $\text{fd}$ denotes log foreign demand. Appendix E.2 explains how these variables have been generated. The index $i \in \{G, E, N\}$ denotes Germany, Eurozone and North. The index $j$ denotes the manufacturing sectors considered in the analysis, and $t$ is the time index. As can be seen from the above equation, the estimations are done separately for these regional aggregates, leading to region-specific export elasticities $\alpha_i$ and $\gamma_i$. The parameter $c_i$ denotes a region-specific constant term. For a single country, such as Germany, 12 manufacturing sectors are considered. Since the Eurozone comprises 10 countries in our sample (see Appendix E.3.3), there are $10 \times 12 = 120$ sectors for the Eurozone. North consists of 7 countries implying $7 \times 12 = 84$ sectors for the North. We have 13 observations in the time dimension since we consider annual data from 1995 to 2007. In the theoretical model $\gamma = 1$, implying that the log export shares ($\text{exp}_{jt} - \text{fd}_{jt}$) could be used as dependent variable. This restriction has been explicitly tested for all estimated model variants and has in almost all cases been clearly rejected. The results for the restricted estimations are therefore not shown here. Though in the long run, as suggested by the model, the elasticity of exports with respect to foreign demand should be unity, deviations from a unit elasticity are possible over a short sample period. For example, for our considered time span from 1995-2007 it is quite likely that China’s or India’s increasing share in world trade reduced the export shares of developed countries which may affect the estimate of $\gamma_i$. Anderton et al. (2004) point out that exports
may not grow in line with foreign demand if FDI outflows result in multinational firms substituting their exports with increased production abroad. To capture the possibility of declining exports at a given level of foreign demand, we also include a linear time trend in the estimation equation, leading to

\[
\text{exp}_{ijt} = \alpha_i + \alpha_i \text{rer}_{ijt} + \gamma_i \text{fd}_{ijt} + \mu_j + \delta_i t + \epsilon_{ijt},
\]

(Model 2a)

Export equations with a linear trend have often been estimated in the literature using aggregate data. Since we are using panel data, we can instead take account of time fixed effects:

\[
\text{exp}_{ijt} = \alpha_i + \alpha_i \text{rer}_{ijt} + \gamma_i \text{fd}_{ijt} + \mu_j + \nu_t + \epsilon_{ijt},
\]

(Model 3a)

In the models (1a) to (3a) each sector has the same weight in the estimation of the “average” export elasticities for the regional aggregate. To take account of the sector size, each model is also estimated as a weighted regression where the weights correspond to the export share of a sector within the respective regional aggregate \(i \in \{G, E, N\}\). This leads to models (1b) to (3b).

For the variance-covariance matrix the Huber/White/sandwich estimator has been used. The estimation results for Germany are presented in Table 2. Based on the adjusted R\(^2\) and the Akaike Information Criterium (AIC) the estimated models with a linear trend or with year dummies (models 2 and 3) are preferred, with a slight preference for the weighted regression (model 2b and model 3b).

| Table E2. Estimation of the export price elasticity for German manufacturing |
|-------------------------|-------------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
|                         | (1a)                         | (1b)                | (2a)                | (2b)                | (3a)                | (3b)                |
| l_exp                  | 1_exp                        | 1_exp               | 1_exp               | 1_exp               | 1_exp               | 1_exp               |
| rer                    | -1.159**                     | -0.844***           | -0.559*             | -0.573*             | -0.492              | -0.546              |
|                        | (-3.91)                      | (-4.45)             | (-2.23)             | (-2.40)             | (-0.91)             | (-1.08)             |
| fd                     | 1.720***                     | 1.572***            | 0.563**             | 0.631***            | 0.540*              | 0.639***            |
|                        | (7.78)                       | (7.16)              | (4.15)              | (8.91)              | (3.10)              | (7.98)              |
| trend                  |                              |                     |                     | 0.0496***           | 0.0459***           |
|                        |                              |                     |                     | (8.11)              | (7.95)              |
|                        | (-3.49)                      | (-2.71)             | (2.69)              | (4.58)              | (2.25)              | (4.02)              |
| Adjusted R\(^2\)      | 0.786                        | 0.840               | 0.939               | 0.949               | 0.938               | 0.949               |
| AIC                    | -212.2                       | -251.0              | -406.6              | -429.1              | -399.4              | -424.1              |
| Observations           | 156                          | 156                 | 156                 | 156                 | 156                 | 156                 |

\(t\) statistics in parentheses; * \(p < 0.05\), ** \(p < 0.01\), *** \(p < 0.001\). Models (3a) and (3b) include year fixed effects.
As explained in Appendix E.5.1 (see, in particular, equations (E8) and (E10)), the calculation of the elasticity of substitution $\sigma_1$ requires, apart from the estimated price elasticity of exports, also the average share of expenditures on German tradable goods in total expenditures on tradable goods, denoted by $\bar{S}_G$. The latter is computed using the same dataset (WIOD) as for the estimations of the export price elasticities to have a consistent calculation of the elasticity of substitution. First, for each year in the considered time period, the world final demand for German manufacturing goods is obtained as a sum of final demand in all German manufacturing sectors across all countries in the dataset, not only those belonging to any of the developed area. Then, the world final demand for German manufacturing goods is divided by the world final demand in the manufacturing sectors of all three developed areas together. Finally, $\bar{S}_G$ is obtained as the time average of the annual shares from the previous step.

With the estimation results from models 2a, 2b, 3a and 3b as well as with $\bar{S}_G$, we arrive at the estimated $\sigma_1$ given in Table E3.

| Table E3. Calculation of $\sigma_1$ based on estimation results |
|-------------------|------------------|
| $\alpha_G$       | $\bar{S}_1$     |
| 0.49              | 0.56             |
| 0.55              | 0.63             |
| 0.56              | 0.64             |
| 0.57              | 0.66             |

The estimation results for the price elasticity of exports for the Eurozone are summarized in Table 4.

| Table E4. Estimation of the export price elasticity for Eurozone manufacturing |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                   | (1)              | (1b)             | (2a)             | (2b)             | (3a)             |
| l_exp             | 1.473***         | 1.258***         | 0.847***         | 0.695***         | 0.825***         |
| l_reer_qpi        | -0.735***        | -0.646***        | -0.793***        | -0.659***        | -0.746***        |
|                   | (-6.27)          | (-4.00)          | (-7.62)          | (-6.06)          | (-6.45)          |
| l_fd              | 1.473***         | 1.258***         | 0.847***         | 0.695***         | 0.825***         |
|                   | (13.74)          | (11.24)          | (7.01)           | (6.19)           | (6.09)           |
| trend             | 0.0257***        | 0.0259***        |                   |                   |                   |
|                   | (6.00)           | (7.25)           |                   |                   |                   |
| Constant          | -8.038***        | -4.698***        | -1.257           | 1.601            | -1.026           |
|                   | (-6.75)          | (-3.64)          | (-0.95)          | (1.25)           | (-0.69)          |
| Adjusted $R^2$    | 0.463            | 0.628            | 0.540            | 0.711            | 0.544            |
| AIC               | -985.2           | -1876.5          | -1220.7          | -2260.5          | -1224.4          |
| Observations      | 1534             | 1534             | 1534             | 1534             | 1534             |

$t$ statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Models (3a) and (3b) include year fixed effects.

Based on the adjusted $R^2$ and the Akaike Information Criterium (AIC) the weighted regressions with a linear trend or with year dummies (model 2b and 3b) are preferred. In comparison to these estimates model (1b) is slightly inferior, but the estimated coefficient is
very similar to that of model (2b) and, for completeness, is also considered in the calculation of \( \sigma_2 \). Based on eq. (E.21) in Appendix E.5.1 this leads to values for the estimated \( \sigma_2 \) summarized in Table 6. Note that the computation of \( \sigma_2 \) requires the estimated export price elasticities for both the Eurozone and Germany which, with four different values of \( \hat{\alpha}_G \) (see Table 5) and three different values of \( \hat{\alpha}_E \), leads to 12 estimates of \( \sigma_2 \). Moreover, in the calculations we use \( \bar{S}_G \) and \( \bar{S}_E \) (the average expenditures on tradable goods produced in Germany and Eurozone, respectively, relative to total expenditures on tradable goods). \( \bar{S}_E \) is obtained analogously as \( \bar{S}_G \) (see the explanations above).

### Table E5. Calculation of \( \sigma_2 \) based on estimation results for the Eurozone

<table>
<thead>
<tr>
<th>( \hat{\alpha}_G )</th>
<th>( \hat{\alpha}_E )</th>
<th>( \hat{\sigma}_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.57</td>
<td>0.59</td>
<td>0.796</td>
</tr>
<tr>
<td>0.56</td>
<td>0.59</td>
<td>0.797</td>
</tr>
<tr>
<td>0.55</td>
<td>0.59</td>
<td>0.798</td>
</tr>
<tr>
<td>0.49</td>
<td>0.59</td>
<td>0.801</td>
</tr>
<tr>
<td>0.57</td>
<td>0.65</td>
<td>0.811</td>
</tr>
<tr>
<td>0.56</td>
<td>0.65</td>
<td>0.882</td>
</tr>
<tr>
<td>0.55</td>
<td>0.65</td>
<td>0.882</td>
</tr>
<tr>
<td>0.49</td>
<td>0.65</td>
<td>0.886</td>
</tr>
<tr>
<td>0.57</td>
<td>0.66</td>
<td>0.895</td>
</tr>
<tr>
<td>0.56</td>
<td>0.66</td>
<td>0.896</td>
</tr>
<tr>
<td>0.55</td>
<td>0.66</td>
<td>0.896</td>
</tr>
<tr>
<td>0.49</td>
<td>0.66</td>
<td>0.900</td>
</tr>
</tbody>
</table>

According to our theoretical model and the explanations given in Appendix E.5.1, the elasticity of substitution \( \sigma_2 \) could be alternatively calculated using the price elasticity of North exports instead of Eurozone exports. The results of the estimations of the export equations for North appear in Table 6.

### Table E6. Estimation of the export price elasticity for North manufacturing

<table>
<thead>
<tr>
<th></th>
<th>(1a)</th>
<th>(1b)</th>
<th>(2a)</th>
<th>(2b)</th>
<th>(3a)</th>
<th>(3b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>l_exp</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>l_reer_qpi</td>
<td>-0.347**</td>
<td>-0.311*</td>
<td>-0.476***</td>
<td>-0.440**</td>
<td>-0.496***</td>
<td>-0.636***</td>
</tr>
<tr>
<td>(-3.99)</td>
<td>(-2.04)</td>
<td>(-4.54)</td>
<td>(-3.19)</td>
<td>(-4.76)</td>
<td>(-4.55)</td>
<td></td>
</tr>
<tr>
<td>l_fd</td>
<td>0.816***</td>
<td>0.805***</td>
<td>0.498***</td>
<td>0.506***</td>
<td>0.468***</td>
<td>0.397***</td>
</tr>
<tr>
<td>(8.12)</td>
<td>(11.58)</td>
<td>(4.35)</td>
<td>(5.11)</td>
<td>(3.97)</td>
<td>(3.64)</td>
<td></td>
</tr>
<tr>
<td>Trend</td>
<td>0.0165***</td>
<td>0.0175***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.21)</td>
<td>(3.66)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.601</td>
<td>0.888</td>
<td>2.947*</td>
<td>4.338***</td>
<td>3.212*</td>
<td>5.545***</td>
</tr>
</tbody>
</table>
As in the estimations for Eurozone and Germany, the weighted regressions with time trend (model 2b) or year dummies (3b) perform best. However, the difference in goodness of fit to the weighted model without time trend or year dummies (model 1b) is less pronounced than in the other estimations. Based on the estimation results for North from models 1b, 2b and 3b as well as four different values for $\alpha_c$ (see Table 4) we arrive at 12 different estimates for $\sigma_2$. (see Table 7). Note that, in the derivation of $\sigma_2$, analogously as in the case of the Eurozone (where we used $\bar{S}_G$ and $\bar{S}_E$), we also need $\bar{S}_G$ and $\bar{S}_N$ (the average expenditures on tradable goods produced in Germany and North, respectively, relative to total expenditures on tradable goods).

<table>
<thead>
<tr>
<th>Model</th>
<th>Adjusted $R^2$</th>
<th>AIC</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3a)</td>
<td>0.417</td>
<td>-1252.0</td>
<td>1092</td>
</tr>
<tr>
<td>(3b)</td>
<td>0.679</td>
<td>-1618.1</td>
<td>1092</td>
</tr>
<tr>
<td>(3c)</td>
<td>0.475</td>
<td>-1366.1</td>
<td>1092</td>
</tr>
<tr>
<td>(3d)</td>
<td>0.713</td>
<td>-1740.9</td>
<td>1092</td>
</tr>
<tr>
<td>(3e)</td>
<td>0.538</td>
<td>-1493.8</td>
<td>1092</td>
</tr>
<tr>
<td>(3f)</td>
<td>0.776</td>
<td>-2001.0</td>
<td>1092</td>
</tr>
</tbody>
</table>

$t$ statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Models (3a) and (3b) include year fixed effects.

Using the estimate $\bar{S}_N = 0.31$ one obtains values for $\hat{\sigma}_2$ that are completely in line with the values obtained with the Eurozone estimates (compare Table 5). However, with the other estimates $\bar{S}_N$ much higher values for $\hat{\sigma}_2$ are obtained. Alternative estimations done at the aggregate level instead of the sectoral level showed that the estimations for North are less reliable than the Eurozone estimates (we even got the wrong sign for the North price elasticity of exports at the aggregate level). We therefore concentrate on the values of the Eurozone estimates for $\hat{\sigma}_2$ (keeping in mind that at least some estimates for the North export price elasticity led to comparable values for $\hat{\sigma}_2$).

To sum up, $\hat{\sigma}_1$ equals 0.63 or 0.65, and $\hat{\sigma}_2$ is around 0.88 (according to Eurozone estimates) or 0.86 (according to North estimates where the results overlap). We therefore decided to take the values $\hat{\sigma}_1 = 0.64$ and $\hat{\sigma}_2 = 0.87$ for the simulations.

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Table E7. Calculation of $\sigma_2$ based on estimation results for North

<table>
<thead>
<tr>
<th>$\alpha_G$</th>
<th>$\alpha_N$</th>
<th>$\bar{S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.57</td>
<td>0.31</td>
<td>0.857</td>
</tr>
<tr>
<td>0.56</td>
<td>0.31</td>
<td>0.861</td>
</tr>
<tr>
<td>0.55</td>
<td>0.31</td>
<td>0.864</td>
</tr>
<tr>
<td>0.49</td>
<td>0.31</td>
<td>0.887</td>
</tr>
<tr>
<td>0.57</td>
<td>0.44</td>
<td>1.305</td>
</tr>
<tr>
<td>0.56</td>
<td>0.44</td>
<td>1.308</td>
</tr>
<tr>
<td>0.55</td>
<td>0.44</td>
<td>1.312</td>
</tr>
<tr>
<td>0.49</td>
<td>0.44</td>
<td>1.334</td>
</tr>
<tr>
<td>0.57</td>
<td>0.64</td>
<td>1.992</td>
</tr>
<tr>
<td>0.56</td>
<td>0.64</td>
<td>2.000</td>
</tr>
<tr>
<td>0.55</td>
<td>0.64</td>
<td>2.000</td>
</tr>
<tr>
<td>0.49</td>
<td>0.64</td>
<td>2.022</td>
</tr>
</tbody>
</table>
E.5.4 Comparison with results from the literature

This section gives a short overview over export elasticities estimated in the literature. The focus is on estimations for the euro area and for Germany, because (i) in contrast to the group of countries belonging to North in our simulations these are well-defined geographical areas which makes a comparison with results from the literature easier and (ii) we considered the results for the euro area in our own estimations to be more reliable than those for North. We will show that our results are quite in line with the findings in the literature. Nevertheless, our own estimations (documented in the above sections) had been necessary to guarantee that the estimated export price elasticities (and the derived elasticities of substitution) are consistent with the group of trading partner countries and with the selected euro area countries used in our simulations.

Some studies estimating price and income elasticities of exports for the euro area are summarized in Table 9. These studies perform the estimations at the aggregate (country) level. If one focuses on studies reporting extra-euro area export price elasticities, the estimated values are -0.5 and -0.58 in the studies of Anderton et al. (2004) and di Mauro et al (2005), respectively. In the study of Ca’Zorzi and Schnatz (2007) six different measures of cost or price competitiveness are used. Their partner countries of the euro area fit quite well to the composition of North in this paper (Hong Kong, Singapore, South Korea and Switzerland are additionally included in their study). As in Anderton et al. (2004), their estimation period comes relatively close to our own estimation period. They find export price elasticities ranging from -0.3 to -0.4 for all competitiveness indicators except for export prices where an elasticity of -0.6 is found. Algieri (2011) analyses the stability of estimated export elasticities of euro area countries performing separate estimations for each decade starting from 1978. She finds that the elasticities range between -0.44 and -0.51 and remained quite stable over the sample period. This makes us confident to apply our estimated elasticities (based on the estimation sample starting in 1995) in the simulations covering a longer time period (starting in 1975). To sum up, our estimates ranging from -0.59 to -0.66 (see Table 8) are quite in line with the results from other studies.

Table 10 gives an overview over estimated export elasticities for Germany found in the literature. For example, the sample period of Heinze (2018) is roughly similar to ours.17 For extra-EMU (with EMU denoting the Economic and Monetary Union) trade Heinze (2018) finds the German export price elasticity ranging from -0.68 to -0.74 which is in line with our estimates. However, for intra-EMU trade the price elasticity is found to be insignificant. For a sample starting already in the 80s, Stahn (2006) reports a price elasticity of -0.92 for extra-EMU exports and -0.63 for intra-EMU exports. However, for a reduced sample starting in the 90s the elasticities become smaller. Other studies find stable export price elasticities over a

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17 Heinze (2018) provides a more extensive survey over papers estimating export elasticities for Germany.
long sample period. For example, the study of Barrell and Pomerantz (2007) covering the period 1978 - 2004 reports a price elasticity of -0.38. An EMU dummy, modelling a structural break related to the introduction of the EMU, is found to be insignificant. The study of Clostermann (1998) using data for the period 1975-1996 documents export price elasticities between -0.65 and -0.97. The values in the lower range are in line with our results obtained with a shorter sample period than in Clostermann (1998). All in all, our estimation results for Germany are in the middle range of export price elasticities found in the literature.

**Table E8. Estimated export elasticities for the euro area in the literature**

<table>
<thead>
<tr>
<th>Publication</th>
<th>Frequency and period</th>
<th>Method</th>
<th>Trading partners</th>
<th>Type of exports</th>
<th>Foreign demand</th>
<th>REER</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anderton et al. (2004)</td>
<td>quarterly 1989.q1-2001.q4</td>
<td>Not described; estimates done by ECB staff</td>
<td>extra-euro area</td>
<td>G</td>
<td>import volumes 1.0</td>
<td>XUV -0.5</td>
<td></td>
</tr>
<tr>
<td>Di Mauro et al. (2005)</td>
<td>quarterly 1992-2003</td>
<td>NA</td>
<td>intra- and extra euro area</td>
<td>G+S</td>
<td>Restricted to 1.0</td>
<td>XUV -0.58</td>
<td>Estimates based on Area Wide Model (AWM) for euro area</td>
</tr>
<tr>
<td>Ca'Zorzi and Schnatz (2007)</td>
<td>quarterly 1992.q1-2006.q1</td>
<td>ECM</td>
<td>12 extra-euro countries</td>
<td>G</td>
<td>Import volumes usually 0.75 – 0.8 for XUV not significantly different from 1</td>
<td>PPI, ULCM, ULCT; GDP; XUV; CPI -0.3 to -0.4 XUV: -0.6</td>
<td></td>
</tr>
<tr>
<td>Bayoumi et al. (2011)</td>
<td>annual 1980-2009</td>
<td>Panel regression at country level (11 euro area countries)</td>
<td>extra-euro area; intra-euro area</td>
<td>G</td>
<td>Extra-euro elasticity 1.58 to 1.86</td>
<td>extra-euro elasticities insignificant for CPI, PPI, ULCM, (-0.1 to -0.27); only for XUV significant: -0.3</td>
<td></td>
</tr>
<tr>
<td>Algieri (2011)</td>
<td>quarterly 1978.q1-2009.q1</td>
<td>ECM with unobserved components</td>
<td>intra- and extra-euro area</td>
<td>G+S</td>
<td>Restricted to 1.0</td>
<td>Total period: -0.48 1978.q1-1987.q4: -0.46 1988.q1-1997.q4: -0.44 1998.q1-2009.q1: -0.51</td>
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<tr>
<td>Bobeica et al. (2016)</td>
<td>quarterly 1995.q1-2013.q3</td>
<td>Panel-ECM</td>
<td>intra- and extra-euro area</td>
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<td>Import volumes Restricted to 1.0</td>
<td>GDP, CPI; ULC -0.7 to -1.0</td>
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**Notes**

**Methods:** ECM: Error-correction model; SUR: Seemingly unrelated regression; NA: not available

**Type of exports:** G: Goods; G+S: Goods and Services

**Foreign demand:** this column reports the income elasticity of exports.

**REER:** Real effective exchange rate; this column reports export price elasticities.

Values of the export elasticities are in bold.

**Price deflators** (used for the REER): PPI: producer price index; GDP: GDP deflator; XUV: Export unit values; ULCM: unit labour costs for manufacturing; ULCT: unit labour costs for the total economy.
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<td>Clostermann (1998)</td>
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<td>Carlin et al. (2001)</td>
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<td>ULC at sectoral level for Germany: -0.12 in case a) -0.24 in case b) a) country regression pooled across industries b) mean by country from individual industry-country-regressions</td>
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<td>Di Mauro et al. (2005)</td>
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<td>Stahn (2006)</td>
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<td>Intra – and extra-EMU</td>
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<td>0.88</td>
<td>-0.92 for intra-EMU -0.63 for extra-EMU For sample starting 1993:3 the price elasticities are smaller (-0.32 and -0.30 respectively).</td>
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<td>Barrell and Pomerantz (2007)</td>
<td>quarterly 1978.q1-2004.q4</td>
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<td></td>
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**Notes**

Values of the export elasticities are in bold.

**Methods:** ECM: Error-correction model; SUR: Seemingly unrelated regression; NA: not available

**Trading partners:** RoW: rest of the world; EMU: the Economic and Monetary Union

**Type of exports:** G: Goods; G+S: Goods and Services

**Foreign demand:** this column reports the income elasticity of exports.

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**Price deflators** (used for the REER): PPI: producer price index; GDP: GDP deflator; XUV: Export unit values; ULCM: unit labour costs for manufacturing; ULCT: unit labour costs for the total economy.
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