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Decision making under uncertainty: An experimental study in market settings

Discussion Paper, No. 197

Provided in Cooperation with:

University of Munich (LMU) and Humboldt University Berlin, Collaborative Research Center Transregio 190: Rationality and Competition

Suggested Citation: Echenique, Federico; Imai, Taisuke; Saito, Kota (2019): Decision making under uncertainty: An experimental study in market settings, Discussion Paper, No. 197, Ludwig-Maximilians-Universität München und Humboldt-Universität zu Berlin, Collaborative Research Center Transregio 190 - Rationality and Competition, München und Berlin

This Version is available at: https://hdl.handle.net/10419/222094

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Decision Making under Uncertainty: An Experimental Study in Market Settings

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Discussion Paper No. 197

November 7, 2019

Decision Making under Uncertainty: An Experimental Study in Market Settings

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Abstract

We design and implement a novel experimental test of subjective expected utility theory and its generalizations. Our experiments are implemented in the laboratory with a student population, and pushed out through a large-scale panel to a general sample of the US population. We find that a majority of subjects' choices are consistent with maximization of *some* utility function, but not with subjective utility theory. The theory is tested by gauging how subjects respond to price changes. A majority of subjects respond to price changes in the direction predicted by the theory, but not to a degree that makes them fully consistent with subjective expected utility. Surprisingly, maxmin expected utility adds no explanatory power to subjective expected utility.

Our findings remain the same regardless of whether we look at laboratory data or the panel survey, even though the two subject populations are very different. The degree of violations of subjective expected utility theory is not affected by age nor cognitive ability, but it is correlated with financial literacy.

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1 Introduction

We present an empirical investigation of the most widely used theories of decision under uncertainty, including subjective expected utility and maxmin expected utility. We consider economic environments, where an agent has to choose a portfolio of state-dependent payoffs, given state prices and a budget. Such environments are ubiquitous in economic theory, where agents choose a portfolio of Arrow-Debreu securities in complete markets. In our study, we record subjects' choices in a laboratory setting, and in a large-scale field panel. In consequence, we obtain results for very different populations, ranging from undergraduate students to older retirees, acting in economic environments that resemble real-world financial decisions. Our data allows us to see if subjects' demographic characteristics, such as age, income, and education, as well as cognitive ability and financial literacy, are related to how well they comply with the theories. We can also relate the results of our experiment to traditional measures of ambiguity aversion. Finally, experiments speak to the external validity of laboratory studies, since we can compare behaviors in the lab to the behaviors of a sample of the general US population.

Subjective expected utility theory (SEU; Savage, 1954) is the standard model of decision making under uncertainty (that is, where states of the world are uncertain, and no objective probabilities are known). The theory postulates an agent that has a subjective probabilistic belief over states of the world, and who maximizes the expected utility with respect to this belief. The starting point of our analysis is a methodological innovation: a nonparametric test for SEU using data on market choices (Echenique and Saito, 2015). Our experiments were conducted with the purpose of recreating the economic settings that are commonly assumed in economic theory: the choice of a financial state-contingent portfolio under uncertainty, and with an eye to using the new nonparametric tests to gauge the empirical performance of SEU.

While SEU is the dominant theory of choice under uncertainty, it is well known to face empirical challenges. In an influential paper, Ellsberg (1961) suggested that many agents would not conform to SEU. The phenomenon he uncovered, known as the "Ellsberg paradox," suggests that agents may seek to avoid betting on uncertain events in ways that cannot be represented with a subjective probability. Such avoidance of uncertain bets is termed ambiguity aversion. The Ellsberg paradox is based on a thought experiment, however, using bets on drawing a particular colored ball from an urn. The Ellsberg paradox therefore assumes an abstract, artificial, choice environment. One of our contributions is to empirically assess SEU in an economic environment that resembles the real-world financial markets where economists routinely assume that SEU guides agents' choices.

To account for the Ellsberg paradox, researchers have developed generalizations of SEU. Gilboa and Schmeidler (1989) suggest that an agent in Ellsberg's example may have too little information to form a unique subjective belief, and hence entertains multiple subjective probabilities. Being ambiguity averse, the agent maximizes the minimal expected utility over all subjective probabilities she entertains. The resulting theory is called maxmin expected utility (MEU). On the other hand, Machina and Schmeidler (1992) postulate that agents may have a unique subjective probability, but not necessarily decide according to the expected utility with respect to the probability. Such agents are called probabilistically sophisticated.

Our understanding of ambiguity aversion is incomplete. It has been identified in different contexts, and in different subject populations (Trautmann and van de Kuilen, 2015); but the literature has relied almost exclusively on the paradigm introduced by Ellsberg (1961), where agents are offered bets on the color of balls drawn from urns whose composition is not fully specified. The simple binary choice structure of Ellsberg makes it easy to identify violations of SEU through violations of the so-called "sure-thing principle" (postulates P2 and P4 of Savage, 1954). But the artificial nature of the experiment may question the external validity of its findings. Despite its difficulty, designing choice environment that are more "natural," while providing clean identification, is an important task in the empirical literature on ambiguity aversion (Baillon et al., 2018b). In our paper, we investigate deviations from SEU and MEU in economic environments, combining a novel experimental paradigm and measurement techniques that are inspired by recent work on revealed preference theory. We are also able to partially test for probabilistic sophistication.

Echenique and Saito (2015) provide a necessary and sufficient condition for an agent's behavior in the market to be consistent with (risk-averse) SEU. Chambers et al. (2016) provide a similar condition for MEU when there are two states of the world. Echenique et al. (2018) characterize "approximate" SEU by relaxing conditions in the model. These revealed preference characterizations provide nonparametric tests for SEU and MEU as well as a measure quantifying "how much" a dataset deviates from these theories. While the cited studies focus mostly on establishing theoretical revealed preference conditions, the main motivation of the current paper is to bring the theoretical machinery to actual choices people make in the face of uncertainty. Our empirical approach is *nonparametric* in the sense that we do not impose any specific functional form, such as CRRA or CARA. We do assume that agents are risk averse or risk neutral (they have a concave von-Neumann-Morgenstern utility).

The theoretical revealed preference results assume data on an agent's behavior in the market: meaning a collection of purchases of Arrow-Debreu securities at different budget constraints. This setting naturally translates into our proposed experimental design, which follows the spirit

of portfolio choice tasks introduced by Loomes (1991) and Choi et al. (2007), and later used in many other studies (e.g., Ahn et al., 2014; Carvalho and Silverman, 2019; Choi et al., 2014; Hey and Pace, 2014). Subjects in our experiment are asked to allocate "tokens" into two accounts. Each account has an associated exchange rate which converts tokens into actual monetary rewards. These exchange rates define a budget set. Two accounts correspond to two mutually exclusive events, and subjects are told that they receive payment based on the chosen allocation, and on the realized event. Importantly, subjects are provided no information regarding the probabilities of these events. We generate uncertainty from two different sources. The first source is the classical Ellsberg-style "urns and balls." The second source comes from simulated stock prices.

As said, we ran our experiments in the laboratory where we used undergraduate students as subjects, and on a large-scale panel where we recruited representative of the US population. See Section 2.2 for details.

1.1 Overview of Results

Our main findings are that: 1) subjects are consistent with utility maximization and probabilistic sophistication, but not SEU¹; 2) MEU has no added explanatory power to SEU; 3) demand responds to price in the direction predicted by SEU, but not enough to make the data consistent with SEU; 4) subjects in the lab and in the panel display the same patterns; and 5) correlations with demographics exist but are limited.

The main purpose of our study was to nonparametrically test theories of decision under uncertainty. We find that most subjects are utility maximizers (they satisfy the Generalized Axiom of Revealed Preference), and satisfy Epstein's (2000) necessary condition for probabilistic sophistication. But the news is not good for the more restrictive theories. In our experiments, across lab and panel, the vast majority of subjects do not conform to SEU. This finding would be in line with the message of the Ellsberg paradox, except that pass rates for MEU are just as low as for SEU. In fact, in all of our sample, only one subject's choice is consistent with MEU but not SEU.

One might conjecture that the theories could be reconciled with the data if one allows for small mistakes, but our measures of the distance of the data to being rationalizable do not suggest so. A more forgiving test is to check if price changes are negatively correlated with quantity changes: we refer to this property as "downward-sloping demand," and it bears a close connection to SEU (see Echenique et al. (2018) for details). The vast majority of subjects exhibit the downward-sloping demand property, at least to some degree (meaning that the correlation between price and

¹Since we check a necessary condition for probabilistic sophistication, we can only say that subjects are not inconsistent with probabilistic sophistication. See Section 3.

quantity changes is negative), but not to the extent needed to make them fully consistent with SEU. The downward-sloping demand property is strongly correlated with our measure of the distance between the data and SEU, so there is a precise sense in which the degree of compliance with downward-sloping demand can be tied to the violations of SEU.²

Our panel experiment allows us to compare the distance to SEU between subjects with different sociodemographic characteristics. We find that distance to SEU is correlated with financial literacy, with more financially literate subjects being closer to SEU than less literate subjects; and gender differences, with males being closer to SEU than females. A notable finding is the absence of a significant correlation with factors that have been shown to matter for related theories of choice (Choi et al., 2014; Echenique et al., 2018). In particular, older subjects, subjects with lower educational backgrounds, and subjects with lower cognitive ability, do not necessarily exhibit lower degrees of compliance with SEU.

One final implication of our results is worth discussing. Our experiments included a version of the standard Ellsberg question. The distance to SEU, or the degree of compliance with downward-sloping demand, are not related to the answers to the Ellsberg question, but the variability of uncertainty in our market experiment is. The experiments included a treatment on the variability of the uncertain environment, specifically the variability in the sample paths of the stock price whose outcomes subjects were betting on. Subjects who were exposed to more variable uncertainty seem less ambiguity averse (in the sense of Ellsberg) than subjects who were exposed to less variable uncertainty.

1.2 Related Literature

Starting with an influential thought experiment by Ellsberg (1961), many studies have tested SEU and related models of decision making under uncertainty using data from laboratory experiments. Trautmann and van de Kuilen (2015) provide an overview of this large but still growing empirical literature. Typical experiments involve "urns and colored-balls" following Ellsberg's (1961) original thought experiment, and individual's attitude towards ambiguity is inferred by looking at valuations or beliefs elicited through a series of binary choices (e.g., Abdellaoui et al., 2011; Baillon and Bleichordt, 2015; Chew et al., 2017; Epstein and Halevy, 2019; Halevy, 2007). Other studies try to parametrically estimate the models under consideration (e.g., Ahn et al., 2014; Dimmock et al., 2015; Hey et al., 2010; Hey and Pace, 2014). Unlike these studies, our approach is nonparametric, imposing no assumptions on functional form other than risk-aversion. While

²Echenique et al. (2018) contain a detailed discussion.

the use of artificially generated ambiguity as in Ellsberg-style urns and balls has attractive features that make the interpretation of choice behavior, and experimental implementation, simple, it has been argued that researchers should not rely too much on a paradigm that uses an artificial source of ambiguity. Instead, one should study more "natural" sources of ambiguity.³ In response to these concerns, several studies use non-artificial sources of ambiguity such as stock market indices and temperature (Abdellaoui et al., 2011; Baillon and Bleichordt, 2015; Baillon et al., 2018a). Baillon et al. (2018b) introduce a method that elicits ambiguity attitudes for natural events while controlling for unobservable subjective likelihoods.

It is also important to note that there are several studies that try to understand the relationship between sociodemographic characteristics, ambiguity attitudes, and real-world behavior (especially financial).⁴ This is a subset of a growing empirical literature that seeks to understand the common foundation of a wide class of (behavioral) preferences and to relate cross-/within-country heterogeneity and cultural or sociodemographic characteristics (e.g., Bianchi and Tallon, 2019; Bonsang and Dohmen, 2015; Dimmock et al., 2015, 2016a,b; Dohmen et al., 2018; Falk et al., 2018; Huffman et al., forthcoming; Sunde and Dohmen, 2016; Tymula et al., 2013).

Finally, the analysis of our data uses theoretical tools developed and discussed in Chambers et al. (2016), Echenique and Saito (2015), and Echenique et al. (2018). They require coupling SEU and MEU with risk-aversion. The methods in Polisson et al. (2017) avoid the assumption of risk-aversion, but are computationally hard to implement in the case of SEU (their paper contains an application to objective EU, for which their method is efficient). Polisson et. al. also develop a test for first-order stochastic dominance in models with known (objective) probabilities. Their test could be seen as a first step towards an understanding of probabilistic sophistication.

2 Experimental Design

We conducted experiments at the Experimental Social Science Laboratory (ESSL) at the University of California, Irvine (hereafter *the lab*), and on the Understanding America Study (UAS) panel,

³For example, Camerer and Weber (1992) note that: "Experimental studies that do not directly test a specific theory should contribute to a broader understanding of betting on natural events in a wider variety of conditions where information is missing. There are diminishing returns to studying urns!" (p. 361). Similarly, Gilboa (2009) writes: "David Schmeidler often says, 'Real life is not about balls and urns.' Indeed,important decisions involve war and peace, recessions and booms, diseases and cures" (p. 136).

⁴Trautmann and van de Kuilen (2015) note the importance of this direction: "Interestingly, the empirical literature has so far provided little evidence linking individual attitudes toward ambiguity to behavior outside the lab. Are those agents who show the strongest degree of ambiguity aversion in some decision task also the ones who are most likely to avoid ambiguous investments?" (p. 89).

Table 1: Order of the tasks.

Platform	Treatment	Task 1	Task 2	Task 3	Task 4
Laboratory	Large volatility Small volatility		Market-Ellsberg Market-Ellsberg	Standard Ellsberg Standard Ellsberg	•
Panel			Standard Ellsberg Standard Ellsberg	- -	_ _

a longitudinal survey platform (hereafter *the panel*).⁵ The general structure of tasks in the lab and in the panel were the same. We shall first in Section 2.1 describe the basic tasks, which were common to the lab and the panel experiments. Then in Section 2.2 we turn to the features that were unique to each. Further details and instructions are presented in the online appendices D and E.

2.1 Tasks

We first describe the two basic tasks used in our experiments: the market task (also referred to as the allocation task), and the Ellsberg two-urn choice task. The market task has two versions, depending on the source of uncertainty. The exact set of tasks differed somewhat depending on the platform: the lab or the panel. Table 1 summarizes the lab and the panel experiments.

Market task. The market task is meant to represent the most basic economic problem of choice under uncertainty. An agent chooses among Arrow-Debreu commodities, given state prices and a budget. Experimental implementations of such portfolio choice problems were introduced by Loomes (1991) and Choi et al. (2007), and later used in Ahn et al. (2014), Choi et al. (2014), and Hey and Pace (2014), among others.

Uncertainty is represented through a *state space* $\Omega = \{\omega_1, \omega_2, \omega_3\}$. For each choice problem there are two relevant *events*, denoted by E_s , s = 1, 2. Events are sets of states, which are lumped together in ways that will be clear below. The events E_1 and E_2 are mutually exclusive. Subjects are endowed with 100 (divisible) tokens in each round. An event-contingent payoff may be purchased at a price, which experimentally is captured through an "exchange value." Exchange values, denoted z_s , s = 1, 2, relate tokens allocated to an event, and monetary outcomes. Given exchange values (z_1, z_2), subjects are asked to decide on the allocation of tokens, (a_1, a_2), between

⁵Our experiment was approved by the Institutional Review Board of California Institute of Technology (#15-0478). It was then reviewed and approved by the director of ESSL and the board of UAS. The module number of our UAS survey is 116.

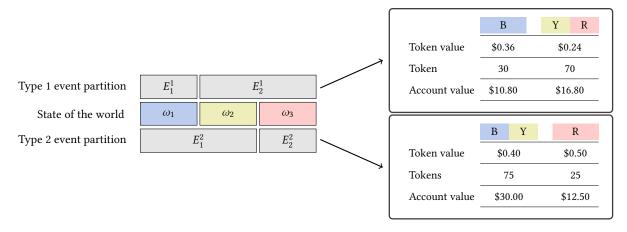


FIGURE 1: (Left) Event structure in two types of questions. (Right) Illustration of the allocation table for a type 1 question (top) and a type 2 question (bottom).

the two events. A subjects who decides on an allocation (a_1, a_2) earns $x_s = a_s \times z_s$ if event E_s occurs. The sets of exchange values (z_1, z_2) used in the experiments are presented in Table D.1 in online appendix.

An allocation (a_1, a_2) of tokens is equivalent to buying a x_s units of an Arrow-Debreu security that pays \$1 per unit if event E_s holds, from a budget set satisfying $p_1x_1 + p_2x_2 = I$, where prices and income (p_1, p_2, I) are determined by the token exchange values (z_1, z_2) in the round.⁶

Our design deviates from the other studies mentioned above by introducing a novel event structure. There are three underlying states of the world: ω_i , i=1,2,3, and we introduce two types of questions. In Type 1 questions, event 1 is $E_1^1=\{\omega_1\}$ and event 2 is $E_2^1=\{\omega_2,\omega_3\}$. In Type 2 questions, event 1 is $E_1^2=\{\omega_1,\omega_2\}$ and event 2 is $E_2^2=\{\omega_3\}$. See Figure 1 for an illustration. This event structure requires SEU decision makers to behave consistently not only within each type of questions but also across two types of questions.

The design allows us to examine one aspect of SEU rationality, monotonicity of choice.⁷ The monotonicity follows from the fact that SEU rational agent should consider event E_1^2 is more likely than event E_1^1 and, hence, the agent should allocate more tokens on event E_1^2 than on event E_1^1 if the prices are the same. We present a more detailed discussion later in the paper.

Subjects in our experiments make decisions through a computer interface. The *allocation table* on the computer screen contains all the information subjects need to make their decisions in each

⁶We set $p_1 = 1$ (normalization) and $p_2 = z_1/z_2$. Then, the income is given by $I = 100 \times z_1$.

⁷Hey and Pace's (2014) design is the closest to ours. In their experiment, uncertainty was generated by the colors of balls in the Bingo Blower and subjects were asked to make 76 allocation decisions in two different types. In the first type of problems, subjects were asked to allocate between two of the colors. In the second type, they were asked to allocate between one of the colors and the other two. Note that the motivation of Hey and Pace (2014) is a parametric estimation of leading models of ambiguity aversion.

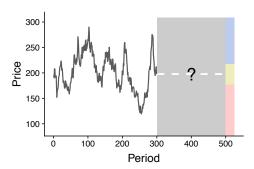
question; see right panels in Figure 1. The table displays exchange values (z_1, z_2) for the current question, their current allocation of tokens (a_1, a_2) , and implied monetary value of each account, referred to as the "account value," $(a_1 \times z_1, a_2 \times z_2)$. Subjects can allocate tokens between two events using a slider at the bottom of the screen; every change in allocation is instantaneously reflected in the allocation table.⁸

An important feature of our design is that we implement the task under two different sources of uncertainty. Subjects face two versions of the market task, as we change the source of uncertainty. In the first version, called "market-Ellsberg", uncertainty is generated with an Ellsberg urn. In the second version, termed "market-stock", uncertainty is generated through a stochastic process that resembles the uncertain price of a financial asset, or a market index. The market-Ellsberg version follows Ellsberg (1961), and the empirical literature on ambiguity aversion (Trautmann and van de Kuilen, 2015). Subjects are presented with a bag containing 30 red, yellow, and blue chips, but they are not told anything about the composition of the bag. The three states of the world are then defined by the color of a chip drawn from the bag: state 1 (ω_1) corresponds to drawing a blue chip, state 2 (ω_2) corresponds to drawing a yellow chip, and state 3 (ω_3) corresponds to drawing a red chip.

In the market-stock task, uncertainty is generated through the realization of simulated stock prices. Subjects are presented with a history of stock prices, as in Figure 2.9 The chart shows the evolution of a stock price for 300 periods; the next 200 periods are unknown, and left blank. Subject are told that prices are determined through a model used in financial economics to approximate real world stock prices. They are told that the chart represents the realized stock price up to period 300, and that the remaining periods will be determined according to the same model from financial economics. Let the price at period 300 be the "starting value" and the price at period 500 be the "target value." We define three states, given some threshold $R \in (0, 1)$: $\omega_1 = (R, +\infty)$, in which the target value rises by more than 100R% compared to the starting value (see the blue region in the figure), $\omega_2 = [-R, R]$, in which the price varies by at most 100R% between the starting value and the target value (the yellow region in the figure), and $\omega_3 = [-1, -R)$, in which the target value falls by more than 100R% compared to the starting value (the red region in Figure 2).

⁸Tokens are divisible (the slider moves in the increment of 0.01). This ensures that the point on the budget line which equalizes the payouts in the two events (i.e., on the 45-degree line) is technically feasible.

⁹We used a Geometric Brownian Motion to simulate 100 stock price paths that share the common starting price and the time horizon. After visually inspecting the pattern of each price path, we handpicked 28 paths and then asked workers on Amazon Mechanical Turk what they believed the future price of each path would be. The elicited belief distributions were then averaged across subjects. Some price paths, especially those with clear upward or downward trends, tend to be associated with skewed distributions. Others have more symmetric distributions. We thus selected two relatively "neutral" ones from the latter set for the main experiment. See online appendix D.2.



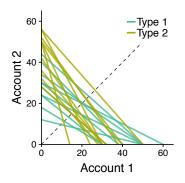


FIGURE 2: Source of uncertainty in the market-stock task.

FIGURE 3: Set of 20 budgets.

We chose token exchange values (z_1, z_2) for each question to increase the power of our tests. After running several choice simulations to calculate the power of our tests, we select 20 budgets (10 for type 1, 10 for type 2) shown in Figure 3 (and Table D.1 in online appendix). Note that event 1 is "more likely" in type 2 decision problems since $E_1^1 \subseteq E_1^2$. In constructing budget sets, we made assets in account 1 more "relatively expensive" than assets in account 2 in type 2 questions. This is reflected in the steeper slope of the budget lines presented in Figure 3.

Several remarks about our experimental design are in order. First, we use the movement of stock prices as a source of uncertainty, not balls and urns. We are not the first to use financial information as the source of uncertainty (see Abdellaoui et al., 2011; Baillon and Bleichordt, 2015; Baillon et al., 2018a), but it is rare in the experimental literature. Second, subjects were allowed to make fractional allocations of tokens between accounts. Our fractional allocation design sought to mimic choices from a continuous budget line, as in the theoretical models we try to test. Third, we asked two types of allocation decisions. This makes our task demanding for subjects, but it creates a powerful environment for our revealed preference analysis, and allows for natural within-subject comparisons.

Ellsberg two-urn choice task. In addition to the market task described above, we presented our subjects with a standard two-urn version of Ellsberg's (1961) choice question. The purpose of including this standard task is to compare the behavior of subjects in the different designs. By this comparison, we can investigate how traditional evaluations of ambiguity aversion relate to market choices, and see if the market setting affects subjects' attitude toward uncertainty.

Subjects confront two bags: bag A and bag B, each of which contains 20 chips. They receive the following information: Bag A contains 10 orange chips and 10 green chips. Bag B contains 20 chips, each of which is either orange or green. The number of chips of each color in bag B is unknown to them, so there can be anywhere from 0 to 20 orange chips, and anywhere from 0

to 20 green chips, as long as the total number of orange and green chips sums to 20.

Subjects were offered choices between bets on the color of the chip that would be drawn at the end of the experiment. Before choosing between bets, subjects were first asked to choose a fixed color (orange or green; called "Your Color") for which they would be paid if they chose certain bets. They were then asked three questions.¹⁰

The first question asks to choose between a bet that pays X+b if the color of the ball drawn from bag A is "Your Color" (and nothing otherwise), and a bet that pays X if the color of a ball drawn from bag B is "Your Color" (and nothing otherwise). Similarly, the second question asks to choose between a bet that pays X if the color of the ball drawn from bag A is "Your Color," and a bet that pays X if the color of a ball drawn from bag B is "Your Color". Finally, the third question asks to choose between a bet that pays X if the color of the ball drawn from bag A is "Your Color" and a bet that pays X+b if the color of a ball drawn from bag B is "Your Color". The payoff X and the bonus X+b if the color of a ball drawn from bag B is "Your Color". The payoff X+b in the panel. In our lab experiments, the content of bag B had already been determined at the beginning of the experiment by an assistant. The timing is important to ensure that there is no incentive to hedge (Baillon et al., 2015; Saito, 2015; Epstein and Halevy, 2019). The subjects were allowed to inspect the content of each bag after completing the experiment.

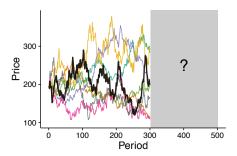
Post-experiment survey. In the lab experiment, subjects were asked to fill out a short survey asking for their age, gender, major in college, the three-item cognitive reflection test (CRT; Frederick, 2005), and strategies they employed in the allocation tasks if any (see online appendix D.3). In the panel study, subjects answered a standard questionnaire that the Understanding America Study (UAS) asks of all its panelist households.

2.2 Implementation

Interface. We prepared an experimental interface that runs on a web browser. In the panel study, our interface was embedded in the survey page of the UAS. Therefore, subjects in both the lab and panel experiments interacted with the exact same interface.

Recruiting and sampling. Subjects for our lab study were recruited from a database of undergraduate students enrolled in the University of California at Irvine. The recruiting methodology

¹⁰We adopted the three-question setting akin to Epstein and Halevy (2019), as a way of identifying strict ambiguity preferences. The typical Ellsberg-style experiment would ask only one question, namely the second one.



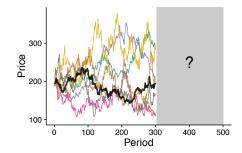


FIGURE 4: Context of market information: large volatility (left) and small volatility (right).

for the UAS survey is described in detail in the survey website.¹¹ Within the UAS sample, we drew a stratified random sub-sample with the aim of obtaining a representative sample of subjects in different age cohorts. In particular, we recruited subjects in three age groups: from 20 to 39, from 40 to 59, and 60 and above, randomly from the pool of survey participants. The purpose of stratifying the sample was to be able to assess the relation between age and pass rates for our revealed preference tests.

Treatments. In the market-stock task, we prepared two simulated paths of stock prices with different degree of volatility, so that one path seems relatively more volatile than the other, while keeping the general trend in prices as similar as possible between the two paths. Since the perception of volatility is only relative, we embed each path in the common market "context" as shown in Figure 4. Here, the bold black lines indicate the stock under consideration, and the other lines in the background are the same in the two treatments.

Our treatment variation is the perceived volatility of simulated stock prices (we call the two treatments Large and Small). The subjects were randomly assigned to either a large volatility condition (left panel in Figure 4), or a small volatility condition (right panel).¹² The instructions for the market-stock task included one of the two charts of Figure 4, depending on the treatment (see online appendix E).

Order of the tasks. Subjects in the lab study performed three tasks in the following order: market-stock, market-Ellsberg, and standard Ellsberg. Subjects in the Panel study performed two tasks, market-stock and standard Ellsberg, but due to time constraints we did not implement market-Ellsberg in the panel. Table 1, which has a summary of the structure of the experiments

¹¹https://uasdata.usc.edu/index.php.

¹²In the lab study, random assignment to one of the two treatments was done at the session level, meaning that all subjects in the same session were shown the same price path.

and treatments, lists the order in which the tasks were completed.

Incentives. In the lab study, we used the standard incentive structure of paying-one-choice-at-random. Subjects received a sealed envelope when they entered the laboratory room. The envelope contained a piece of paper, on which two numbers were written. The first number indicated the task number, and the second number indicated the question number in that task. Both numbers were randomly selected beforehand. At the end of the experiment, subjects brought the envelope to the experimenter's computer station. If the selected task was the market task with stock price information, the simulated "future" price path was presented on the screen. If, on the other hand, the selected task involved the Ellsberg urn, the subject was asked to pick one chip from the relevant bag. All subjects received a \$7 showup fee.

In the panel study, four subjects were randomly selected to receive the bonus payment based on their choices in the experiments. Unlike the lab study, the bonus payment for these subjects was determined by a randomization implemented by the computer program, but payments were of a much larger scale. All subjects received a participation fee of \$10 by completing the entire survey.

3 Results

This section presents results from the lab and the panel. For each dataset, we first discuss the basic patterns of subjects' choices, and then proceed to present our revealed preference tests.

3.1 Results from the Lab

We conducted seven experimental sessions at the Experimental Social Science Laboratory (ESSL) of the University of California, Irvine. A total of 127 subjects (62 in treatment Small and 65 in treatment Large; age mean = 20.16, SD = 1.58; 35% male) participated in the study.¹³ Each session lasted about an hour, and subjects earned on average \$21.3 (including a \$7 showup fee, SD = 9.21).

¹³Three additional subjects participated in the study, but we excluded their data from the analysis. One subject accidentally participated in two sessions (thus, the data from the second appearance was excluded). Two subjects spent significantly longer time for each decision than anyone else. They had to be eliminated from our data, because they were delaying the experiment for the rest of the subjects in their session. We distributed the instructions for each task of the experiment just before they were to perform that task, so each subject would had to wait until all the other subjects in the session completed the task. We had to "nudge" the two subjects that were extremely slow, and hence eliminated their choices from our data.

Choices in the market tasks. In the two market tasks, subjects face the same set of 20 budgets in random order, with the exception of two budgets for which the order was fixed (see below). The choices made by about three-quarters of the subjects are positively correlated between the two tasks (Figure C.1 in online appendix), and 36% of those subjects exhibited significantly positive correlation (one-sided, p < 0.05).

Subjects faced budgets in random order, with one exception. The exception is that we prepared two consecutive questions: questions #5 and #6, that had the same budget, but with different event structures. These were the only questions that were not presented in random order. We included them to check that subjects had a basic understanding of the task. The 5th question was presented as a type 1 question while the 6th question was presented as a type 2 question. Since the event upon which the first account pays off is a larger set in question #6 than in question #5 ($\{\omega_1\} = E_1^1 \subseteq E_1^2 = \{\omega_1, \omega_2\}$ by construction), while prices and budget remain the same, subjects should allocate more to the first account in question #6 than in question #5: we term this property monotonicity of allocation with respect to event structure.

More than 70% of the subjects satisfied monotonicity with respect to event structure, and this number increased to 90% if we allow for a small margin of error of five tokens. Moreover, choices are clustered around the allocation which equalizes payout from the two accounts, which can be interpreted as the subjects' ambiguity aversion. See Figure C.2 in online appendix.

As Echenique et al. (2018) discuss in depth, the empirical content of expected utility is captured in part by a negative relation between state prices and allocations: a property that can be thought of as "downward-sloping demand." We thus look at how subjects' choices responded to price variability between budgets; in particular, we focus on the relation between price ratios, $\log (p_2/p_1)$, and allocation ratios, $\log (x_2/x_1)$, pooling choices from all subjects. Figure 5 shows a negative relation between these two quantities, confirming the downward-sloping demand property at the aggregate level. It holds for both types of questions and in both tasks.

We also quantify the degree of compliance with the downward-sloping demand property at the individual level by calculating correlation $\rho^{\rm dsd}$ between $\log{(p_2/p_1)}$ and $\log{(x_2/x_1)}$.¹⁴ A significant majority of the subjects made choices that responded to prices negatively ($\rho^{\rm dsd} < 0$; Figure C.3 in online appendix).

Revealed preference tests. Did the subjects in our experiment make choices that are consistent with basic economic models of utility maximization, including the standard subjective

¹⁴Let $ρ_t$ be the (Spearman's) correlation coefficient in type t questions. We obtain the "average" correlation coefficient, $ρ^{\rm dsd}$, by Fisher's z-transformation $ρ^{\rm dsd} = \tanh\left(\sum_{t=1}^2\tanh^{-1}(\rho_t)/2\right)$.

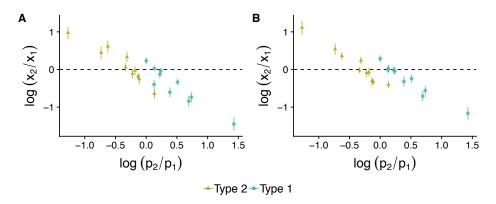


FIGURE 5: Downward-sloping demand in the market-stock task (A) and the market-Ellsberg task (B). Each dot represents mean $\log(x_2/x_1)$ at each $\log(p_2/p_1)$ and bars indicate standard error of means.

Table 2: Pass rates.

	GARP	PS	SEU	MEU
Market-stock	0.5827	0.4803	0.0000	0.0000
Market-Ellsberg	0.6693	0.6220	0.0157	0.0157

Note: Since Epstein's (2000) condition is only necessary for probabilistic sophistication, the numbers reported here capture the fraction of the subjects who are *n*ot inconsistent with probabilistic sophistication.

expected utility (SEU) theory? In order to answer this question, we implement nonparametric, revealed-preference tests on each individual subject's choice data. These tests include: GARP, probabilistic sophistication (hereafter PS; Machina and Schmeidler, 1992), SEU (based on and extended from Echenique and Saito, 2015), and MEU (based on Chambers et al., 2016).

Recall that depending on how we partition the state space, we have two types of decision problem. For GARP and PS, we first test each type of problem separately and then combine the results. We say that a subject's data satisfies GARP if it passes the GARP test for both types. Similarly, we say that a subject's data is not inconsistent with PS if it is not inconsistent with PS in the sense of Epstein's (2000) condition for both types, and satisfies monotonicity of allocation with respect to event structure. For SEU and MEU, we implement the test directly on the data combining two types of problems. It is not obvious that this can be done: see the online appendices A and B for details.

Table 2 presents the *pass rate* of each test. That is, the fraction of subjects (out of 127) who passed each test. We find that a majority of subjects satisfy GARP, meaning that their choices are consistent with maximization of *some* utility function. On the contrary, subjects clearly did not make choices that are consistent with SEU. The SEU pass rates are below 0.1, and not a single

Table 3: Distance measures.

	CCEI			e _* (SEU)			e _* (MEU)		
Task	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
Market-stock	0.9805	1.0000	0.0450	0.8508	0.8238	0.3749	0.8448	0.8148	0.3784
Market-Ellsberg	0.9892	1.0000	0.0317	0.7631	0.6931	0.3834	0.7605	0.6931	0.3839

agent passed the SEU test in the market-stock task.¹⁵

Perhaps surprisingly, allowing for multiple priors via MEU does not change the result. Pass rates for MEU are the same as for SEU, implying that *MEU does not capture violations of SEU in our experiment*. These findings are consistent with data from the experiment in Hey and Pace (2014): see Chambers et al. (2016), which performs the same kind of analysis as we do in the present paper for Hey and Pace's (2014) data.

Finally, we look at PS to investigate whether observed behavior is (in)consistent with preferences being based on probabilities, using the necessary condition proposed by Epstein (2000) and checking monotonicity of allocation in questions #5 and #6. We find that 48% to 62% of subjects are not inconsistent with PS.

Distance measures. The Critical Cost Efficiency Index (CCEI; Afriat, 1972) is a measure of the degree of compliance with GARP. It is heavily used in the recent experimental literature to gauge how close subjects are to being rational economic agents (e.g., Choi et al., 2014). In our lab data, the average CCEI is above 0.98, which implies that on average budget lines needed to be shifted down by about two percent to eliminate a subject's GARP violations (Table 3). The CCEI scores reported in Table 3 are substantially higher than those reported in Choi et al. (2014), but close to the CCEI scores in Choi et al. (2007). This would seem to indicate a higher level of compliance with utility maximizing behavior than in the 2014 experiment, and about the same as the 2007 experiment. Note, however, that there are several substantial differences in the settings and the designs between the two aforementioned studies and ours. We had two types of events (other studies typically have one fixed event structure), each type involved 10 budgets (i.e., total 20 budgets) while the aforementioned studies had 25 and 50 budgets respectively. Importantly, objective probabilities were not provided in our study.

The pass rates for SEU are very small, but it is possible that small mistakes could account

¹⁵Along similar lines, Echenique et al. (2018) find that only five out of more than 3,000 participants in three online surveys (Carvalho et al., 2016; Carvalho and Silverman, 2019; Choi et al., 2014) make choices that are consistent with *objective* expected utility theory.

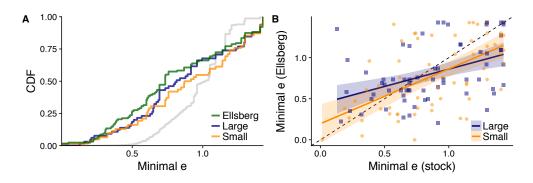


Figure 6: Comparing e_* across tasks.

for a subjects' violation of SEU. We turn to a measure of the severity of violatios of SEU. Table 3 reports e_* (minimal e), a measure of the degree of deviation from SEU theory proposed by Echenique et al. (2018). The number e_* comes from a perturbation to the model that allows SEU to accommodate the data: It can be interpreted as the size of a utility perturbation that can rationalized the observed choices. Thus, the number e_* is zero if a choice data is consistent with SEU, meaning that no perturbation is needed to rationalize the data by means of SEU, but takes a positive value if it violates SEU. The larger is e_* , the larger is the size of the perturbation needed to rationalize the data by means of a perturbed version of SEU. See Echenique et al. (2018) for details.

One basic finding from our experiments is that the joint e_* (i.e., calculated from data combining both types of questions) in the market-stock task is significantly higher than in the market-Ellsberg task (paired-sample t-test; t(126) = 2.635, p = 0.009). See also Figure 6A. This finding suggests that subjects made choices that were closer to SEU when the source of information was an Ellsberg urn than when the source was a stock price, but the result has to be qualified because the order of the two market tasks was not counterbalanced.

As we mentioned above, subjects' choices in the two market tasks are correlated (Figure C.1). This correlation is reflected in the degree of violation of SEU—Figure 6B shows that e_* from two tasks are highly correlated (Spearman's correlation coefficient: r=0.406 for treatment Large, r=0.583 for treatment Small).

We also find that e_* and the downward-sloping demand property (specifically, the aggregate correlation coefficient between price and quantity, as described above) are closely related; see Figure C.5. The subjects' e_* tend to be large when their choices do not respond to price changes, indicating larger deviation from SEU. This is particularly true when the subjects are choosing allocations that are close to the 45-degree line in order to hedge against uncertainty. On the contrary, CCEI can be (close to) one even when choices are not responding to price changes

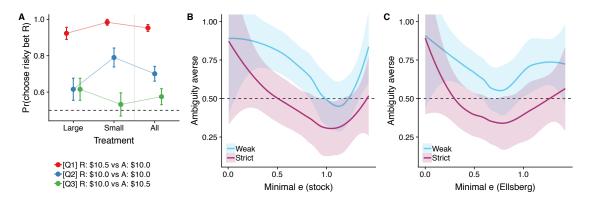


FIGURE 7: (A) Probability of choosing a risky bet in each question in the standard-Ellsberg task in the lab data. Bars indicate standard errors of means. (BC) LOESS curves relating e_* and ambiguity attitude.

(Figure C.6). The connection between e_* and downward sloping demand (and the disconnection with CCEI) is natural. Compliance with SEU requires a certain kind of downward sloping demand property (Echenique and Saito, 2015). Our subjects largely display responses to the price that go in the direction predicted by SEU, but not to the degree that the theory demands.

Table 3 also shows that the data is not much closer to MEU than to SEU. The MEU model has little added explanatory power beyond SEU. In other words, the way in which subjects' choices deviate from SEU is not captured by the MEU model. In MEU, agents' beliefs can depend on choices, as in the perturbation of the SEU model behind our e_* calculation. However, in MEU, the dependency is specific: beliefs are chosen so as to minimize expected utility. Our finding suggests that subjects' belief may depend on choices, but not determined pessimistically. Therefore, the MEU model cannot explain the subjects' choices better than SEU; the needed size of perturbation for MEU is not much lower than that of SEU.

We do not observe gender differences on e_* . We do, however, observe an effect of cognitive ability as measured with the three-item Cognitive Reflection Test (CRT; Frederick, 2005). Subjects who answered all three questions correctly exhibit lower e_* than those who answered none of them correctly. This effect is statistically significant only in the market-stock task (Figure C.7, Table C.2).

Ambiguity attitude. Finally, we look at the relation between behavior in the market tasks and subjects' attitudes toward ambiguity, measured with a standard Ellsberg-paradox design. As explained in Section 2.1, we asked three questions regarding choices between an ambiguous bet and a risky bet to identify subjects' attitude toward ambiguity. Figure 7 shows the frequency with which subjects preferred to bet on the risky urn, for each question.

In the first question, the risky bet pays an additional \$0.5 in case of winning. This bonus made almost all (95.3%) subjects choose the risky bet. The third question has instead a bonus for choosing the ambiguous bet, which then pays an additional \$0.5 in case of winning. A little more than half of the subjects (61.5% in the Large treatment, 53.2% in the Small treatment) preferred the risky bet, but the difference from 50% (i.e., indifference at the aggregate level) is not significantly large (z-test for proportion; p = 0.063 in the Large treatment and p = 0.612 in the Small treatment). In the second question, which pays the equal winning prize in the two bets (as in many other Ellsberg-style studies), subjects in the Small treatment chose the risky bet more frequently than those in the Large treatment (61.5% in the Large treatment and 73.0% in the Small treatment; two-sample z-test for proportion, p = 0.031).

We classify subjects as weakly ambiguity averse if they chose the risky bet, both in the first and in the second question (68.5% of the subjects). Similarly, we classify subjects as strictly ambiguity averse if they chose the risky bet in all three questions (44.1% of the subjects). In order to connect the deviation from SEU captured by e_* and a measure of ambiguity attitude standard in the literature, we nonparametrically estimate how the probability of being classified as ambiguity averse depends on e_* . Figure 7BC suggest a weak but quadratic relationship between these two. Ambiguity aversion is the leading explanation for violations of SEU, so our finding may seem counterintuitive. One might instead expect a monotonic relation between e_* and ambiguity aversion. It is, however, important to emphasize that e_* captures any deviation from SEU. Not only those that could be traced to ambiguity aversion.

3.2 Results from the Panel

A total of 764 subjects (age mean = 50.26, SD = 15.45; 50.39% male) completed the study. The median survey length was 29.1 minutes. In addition to \$10 baseline payment for completing the survey, four randomly selected subjects received additional payment from one of the choices they made during the survey (average \$137.56).

We tried to get subjects to do our experiment on a desktop or laptop computer, but a significant proportion of them took it with their mobile devices—such as smartphones or tablets. These devices usually have screens that are smaller than desktop/laptop computers, which makes it quite difficult to understand our experiments, and perform the tasks we request them to complete. We thus analyze the data following three inclusion criteria, (i) desktop/laptop computer only (66%), (ii) desktop/laptop computer and tablet (76%), and (iii) all devices combined. We treat the first as the "core" sample. Table 4 provides summary statistics of individual sociodemographic characteristics across the three inclusion criteria. We present the entire sample as well as the core

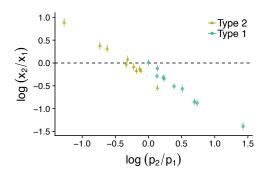


FIGURE 8: Downward-sloping demand. Each dot represents mean $\log(x_2/x_1)$ at each $\log(p_2/p_1)$ and bars indicate standard error of means.

sample (those who used desktop/laptop computers), and the excluded sample (those who did not use desktop/laptop computers). It is evident that the type of device used is correlated with some of the demographic variables (age: $\chi^2(2) = 17.79$, p < 0.001; education level: $\chi^2(3) = 53.70$, p < 0.001; income level: $\chi^2(4) = 43.97$, p < 0.001). The sub-samples of subjects exhibited markedly different patterns of behavior as well. Throughout the rest of the paper, we analyze data from the core sample. ¹⁶

The set of 20 budgets used in the market task is the 10-times scaled-up version of the one used in the lab (Figure 3). This keeps the relative prices the same between our two studies, making the distance measure e_* comparable between data from the lab and the panel.¹⁷

We start by checking the monotonicity of allocations with respect to event structure, along the lines of our discussion for the lab experiment. Our subjects' choices on questions 5 and 6 are informative about how attentive they are when they perform the tasks in our experiment. We find that about 60% of subjects satisfy monotonicity, and that this number jumps to 78% if we allow for a margin of error of five tokens (see Figure C.8). There are no treatment differences. Our subjects also made choices that are, to some extent, responding to underlying price changes (Figure 8).

Revealed preference tests, distance measures, and ambiguity attitude. The pass rates for GARP, SEU and MEU presented in Table 5 are very similar to those of our lab data. We find high GARP pass rates, but very low rates for SEU and MEU. Importantly, MEU does not have more explanatory power than SEU: there is no room for additional rationalizations by allowing for multiple priors (only one non-SEU subject is rationalized by MEU). High compliance with GARP

¹⁶Results from the same analyses on the entire subjects, or comparison across sub-samples, are available upon request.

¹⁷The distance is measured in units of relative price.

Table 4: Sociodemographic information.

		Device		
Variable	All	Desktop/laptop	Other	
Gender				
Male	0.504	0.529	0.456	
Age group				
20-39	0.319	0.279	0.395	
40-59	0.353	0.345	0.369	
60-	0.327	0.375	0.236	
Education level				
Less than high school	0.258	0.190	0.388	
Some college	0.219	0.200	0.255	
Assoc./professional degree	0.187	0.200	0.163	
College or post-graduate	0.336	0.410	0.194	
Household annual income				
-\$25k	0.211	0.148	0.331	
\$25k-\$50k	0.258	0.246	0.281	
\$50k-\$75k	0.202	0.230	0.148	
\$75k-\$150k	0.262	0.297	0.194	
\$150k-	0.068	0.080	0.046	
Occupation type				
Full-time	0.497	0.509	0.475	
Part-time	0.102	0.100	0.106	
Not working	0.401	0.319	0.418	
Marital status				
Married/live with partner	0.690	0.713	0.646	
# of obs. in the sample	764	501	263	

pushes the average CCEI score above 0.97 (Table 6). The average e_* of 0.907 is not statistically different from the average 0.878 in the lab study (two-sample t-test, t(626) = 0.772, p = 0.441). As in our lab study, we find that e_* and how well choices respond to prices are positively associated (Figure C.10). Subjects who violated monotonicity with respect to event structure (monotonicity in questions #5 and #6) for more than five-token margin have significantly higher e_* on aver-

TABLE 5: Pass rates.

Treatment	N	GARP	PS	SEU	MEU
Large variance	245	0.4492	0.3945	0.0122	0.0122
Small variance	256	0.4367	0.3959	0.0156	0.0195
Combined	501	0.4431	0.3952	0.0140	0.0160

Note: Since Epstein's (2000) condition is only necessary for probabilistic sophistication, the numbers reported here capture the fraction of the subjects who are not inconsistent with probabilistic sophistication.

TABLE 6: Distance measures.

		CCEI			e _* (SEU)			e_* (MEU)		
Treatment	N	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
Large variance	245	0.9720	0.9950	0.0509	0.8974	0.9309	0.3740	0.8927	0.9163	0.3765
Small variance	256	0.9688	0.9958	0.0552	0.8868	0.9062	0.3820	0.8852	0.9062	0.3829
Combined	501	0.9704	0.9954	0.0531	0.8920	0.9163	0.3778	0.8888	0.9163	0.3794

age (mean 0.999 vs. 0.881, two-sample t-test, t(499) = 2.925, p < 0.01), but the difference is not significant when we do not allow for this margin (mean 0.928 vs. 0.894, two-sample t-test, t(499) = 0.988, p = 0.324). Among the subjects who satisfied (exact) monotonicity with respect to event structure, the larger the difference between tokens allocated in two questions becomes, the higher e_* becomes (Spearman's correlation coefficient r = 0.127, p = 0.024). So there is some evidence that the degree of violation of monotonicity in questions #5 and #6 is related to the magnitude of deviation from SEU.

The pattern of choices in the standard-Ellsberg task is also similar to what we observed in the lab data, but the overall frequency with which the risky bet is chosen is smaller. In particular, only 70% of the subjects (regardless of treatment) chose the risky bet in the first question, in which the risky bet pays a \$5 more than the ambiguous bet in case of winning (note that almost everybody chose the risky bet in the lab, albeit with a reward magnitude that is 1/10th of what we used in the panel). There are thus 44% (26%) of subjects who are weakly (strictly) ambiguity averse (Figure 9). These numbers are lower than in the lab data. Now, using this classification, we look at the relationship between ambiguity aversion and e_* . Unlike Figure 7BC, using lab data, Figure 9A exhibits a decreasing relation between the two (there is a slight indication of reflection around $e_* \approx 0.8$, but it is not as strong as Figure 7BC). Combining these two observations, we

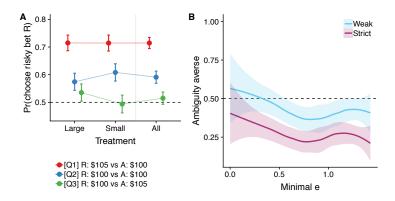


FIGURE 9: (A) Probability of choosing a risky bet in each question in the standard-Ellsberg task in the panel data. (B) LOESS curves relating e_* and ambiguity attitude.

can see that subjects with small e_* (close to SEU) are not necessarily less ambiguity averse in the standard Ellsberg task.

Sociodemographic correlation. One of the great advantages of using the UAS survey is that registered researchers can access datasets from past surveys, and use subject responses in related surveys and experiments. In particular, we use basic demographic information collected through the survey, as well as measures of cognitive ability, financial literacy, and other behavioral data from relevant experiments.¹⁸

We estimated a linear model

$$y_i = X_i \beta + \varepsilon_i,$$

where the dependent variable y_i is subject i's value of e_* or downward-sloping demand measured by correlation $\rho^{\rm dsd}$ between $\log{(p_2/p_1)}$ and $\log{(x_2/x_1)}$, and X_i is a vector of sociodemographic characteristics. These explanatory variables include: age group (omitted category is "20-39 years old"), above-median financial literacy (measured in UAS modules #1 and #6; omitted category is "below-median score"), cognitive ability measured with CRT (omitted category is "score is 0"), education level (omitted category is "high school or less"), annual income group (omitted category is "less than \$25,000"), gender, and employment status. The model is estimated by OLS with robust standard errors. We also estimate logistic regressions where the dependent variable y_i is monotonicity of allocation with respect to event structure (= 1 if monotonicity is violated with a margin of five tokens) and ambiguity attitude in the sense of standard Ellsberg (= 1 if choices

¹⁸The cognitive ability measure is taken from survey module #1. Two financial literacy measures are taken from modules #1 and #6, which asked both the basic and sophisticated financial literacy questions in Lusardi and Mitchell (2017). One caveat to this approach is the time lag between previous the surveys and ours. For example, the first survey module #1 was administered in May 2014.

indicate weak ambiguity aversion).

Regression results are presented in the first two columns of Table 7. First, there is no effect of age on e_* . Cognitive ability measured with CRT is negatively associated but the effect is not strong. The financial literacy variable measured in UAS module #6 is negatively correlated with e_* (i.e., subjects with higher financial literacy are closer to SEU). Subjects in higher income brackets have larger e_* (i.e., further away from SEU), compared to those in the lowest bracket in our sample. Educational background has an effect in the expected direction, but only in the category "associate or professional degree," not in "college or post-graduate degree." Demographic characteristics do not capture variation in the compliance with the downward-sloping demand property (column 2), but the similar effect of income is observed. Other two measures, violation of monotonicity with respect to event structure and ambiguity attitude in the sense of Ellsberg, also exhibit non-significant association with demographic characteristics (except that high CRT score subjects tend to be ambiguity averse compared to low CRT score counterpart).

3.3 Comparing the Lab and the Panel

Finally, we compare the distribution of e_* in the lab and panel data. We can make this comparison because the same set of prices was used in the two experiments. Budgets were very different, but e_* is about relative prices and not about budgets (in contrast with CCEI; see Echenique et al. (2018) for details). It is evident from Figure 10 that there is no difference in distributions. As a basic check to compare that subjects' decisions are at least different than what random choices would offer, we compared the observed distributions to what purely random choices would give rise to: the two distributions are significantly different from the distribution of e_* when simulated subjects make uniformly random choices.

4 Conclusion

Motivated by recent theoretical advances that provide revealed-preference characterizations of expected utility theory, we design and implement a novel experimental test of the theory. We find that subjects are mostly consistent with utility maximization, and respond to price changes in the expected direction: they satisfy the downward-sloping demand property, at least to some degree, but not enough to make their choices consistent with SEU. Our findings are the same,

 $^{^{19}}$ In contrast to these observations, Echenique et al. (2018) find that older subjects have larger e_* for OEU (i.e., further away from OEU, not SEU) than younger subjects; a robust finding in the sense that it holds across data from three different panel surveys (Choi et al., 2014; Carvalho et al., 2016; Carvalho and Silverman, 2019).

TABLE 7: Relation between demographic characteristics and measures for several aspects of behavior in the experiment.

	OLS	3	logistic		
-	(1)	(2)	(3)	(4)	
Dependent variable	e_*	$ ho^{ m dsd}$	Violate mon.	Weak AA	
Treatment: Large	0.016	0.055	0.003	0.182	
	(0.034)	(0.036)	(0.233)	(0.198)	
Age: 40-59	-0.012	-0.008	-0.109	-0.134	
	(0.045)	(0.049)	(0.316)	(0.263)	
Age: 60+	0.026	-0.041	0.365	-0.249	
	(0.048)	(0.052)	(0.319)	(0.288)	
Financial literacy (UAS #1): High	0.034	0.033	-0.291	0.307	
	(0.043)	(0.043)	(0.263)	(0.250)	
Financial literacy (UAS #6): High	-0.106**	-0.054	0.067	0.204	
	(0.041)	(0.041)	(0.268)	(0.247)	
CRT score (UAS #1): 1	-0.013	-0.031	-0.362	0.436	
	(0.040)	(0.041)	(0.264)	(0.232)	
CRT score (UAS #1): 2+	-0.059	-0.029	-0.609	0.711^{*}	
	(0.051)	(0.053)	(0.362)	(0.286)	
Education: Some college	0.046	-0.000	0.142	-0.070	
	(0.053)	(0.059)	(0.342)	(0.331)	
Education: Assoc. or pro. degree	-0.107^{*}	-0.095	-0.167	-0.026	
	(0.054)	(0.059)	(0.374)	(0.324)	
Education: College or postgrad	-0.015	-0.050	-0.478	0.574	
	(0.050)	(0.055)	(0.346)	(0.299)	
Income: 25,000-49,999	0.109	0.096	-0.055	0.470	
	(0.059)	(0.059)	(0.368)	(0.335)	
Income: 50,000-74,999	0.184^{**}	0.122^{*}	0.635	0.071	
	(0.058)	(0.059)	(0.374)	(0.353)	
Income: 75,000-149,999	0.155**	0.142^{*}	-0.112	0.226	
	(0.060)	(0.061)	(0.414)	(0.349)	
Income: 150,000+	0.124	0.187	-0.087	0.675	
	(0.085)	(0.101)	(0.614)	(0.484)	
Male	-0.062	-0.019	-0.130	0.277	
	(0.036)	(0.036)	(0.247)	(0.210)	
Working	0.024	-0.005	-0.311	-0.309	
-	(0.040)	(0.043)	(0.299)	(0.250)	
Constant	0.838***	-0.302***	-0.779	-1.237**	
	(0.070)	(0.075)	(0.431)	(0.430)	
Observations	490	490	490	490	
R^2	0.070	0.033			
Adjusted R^2	0.039	0.001			
Log likelihood			-239.470	-309.069	

 $\overline{\textit{Note}}$: Robust standard errors are presented in parentheses. *p < 0.05; **p < 0.01; ***p < 0.001.

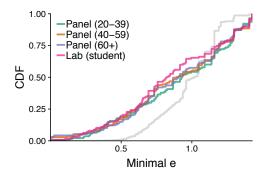


Figure 10: Comparing distributions of e_* from the panel study and the lab study.

regardless of whether we look at lab or panel data. In fact, there is a striking similarity in how SEU is violated across the two studies. The subject populations are very different, but look very similar in terms of the distribution of the degree of violation of SEU.

Motivated by the literature on ambiguity aversion, we study the possibility that violations of SEU are due to ambiguity aversion, and look at whether maxmin expected utility (MEU) can explain the data. MEU adds no explanatory power to SEU: with a single exception, *all* subjects who fail to satisfy SEU also fail MEU. It is possible that other models of ambiguity aversion could do a better job of accounting for our experimental data. We are restricted to MEU because it is the only model for which there exists nonparametric tests of the kind that we use in our paper; it is also arguably the best known, and most widely applied, model in the ambiguity literature. The testable implications of other models of ambiguity-averse choice is an interesting direction for future research.

Finally, the results in our experiments are markedly unaffected by some of the demographic characteristics that other studies (on risky choice, not uncertain) have found significant. Older subjects do not seem to violate SEU to a larger degree than younger subjects. Neither do we see higher degrees of SEU violations in our broad sample of the US population, compared to our laboratory experiment conducted on undergraduate students. There are modest effects of income and education. Financial literacy is correlated with subjects' distance to SEU.

There is no doubt that further studies are necessary to fully understand the behavior in environments that are more "natural" than traditional artificial Ellsberg-style settings. Our non-parametric revealed preference tests and the empirical approach driven by these theories should hopefully be a useful tool to collect more evidence in this direction.

Acknowledgement. The authors wish to thank Aurélien Baillon, Yoram Halevy, Yves Le Yaouanq, Olivia Mitchell, Pietro Ortoleva, Maricano Siniscalchi, and Charles Sprenger for helpful discussions. The authors also

thank Noriko Imai for developing the software for the experiment, Yimeng Li for research assistance, John Duffy, Michael McBride, and Jason Ralston for supporting our experiments at ESSL, and Tania Gutsche and Bart Orriens at CESR (University of Southern California) for setting up and implementing the survey on the Understanding America Study. This research is supported by Grant SES1558757 from the National Science Foundation and the TIAA Institute and Wharton School's Pension Research Council/Boettner Center. In addition, Echenique thanks the NSF for its support through the grant CNS-1518941, and Imai acknowledges financial support by the Deutsche Forschungsgemeinschaft through CRC TRR 190. The project described received funding from the TIAA Institute and Wharton School's Pension Research Council/Boettner Center. The content is solely the responsibility of the authors and does not necessarily represent official views of the TIAA Institute or Wharton School's Pension Research Council/Boettner Center. The project described in this paper relies on data from surveys administered by the Understanding America Study, which is maintained by the Center for Economic and Social Research (CESR) at the University of Southern California. The content of this paper is solely the responsibility of the authors and does not necessarily represent the official views of USC or UAS. A summary of our research was published in the TIAA Institute's "Trends and issues," and a first draft of this paper on their "Research dialogue."

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Supplementary Material

Decision Making under Uncertainty: An Experimental Study in Market Settings

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A Theoretical Background

Let *S* be a finite set of *states*. We occasionally use *S* to denote the number |S| of states. Let $\Delta_{++} = \{\mu \in \mathbb{R}^S_{++} : \sum_{s=1}^S \mu_s = 1\}$ denote the set of strictly positive probability measures on *S*. In the models we consider below, the objects of choice are state-contingent monetary payoffs, or *monetary acts*, which is a vector in \mathbb{R}^S_+ .

A *dataset* is a finite collection $(p^k, x^k)_{k=1}^K$, where each $p^k \in \mathbf{R}_{++}^S$ is a vector of strictly positive (Arrow-Debreu) prices, and each $x^k \in \mathbf{R}_+^S$ is a monetary act. The interpretation of a dataset is that each pair (p^k, x^k) consists of a monetary act x^k chosen from the budget $B(p^k, p^k \cdot x^k) = \{x \in \mathbf{R}_+^S : p^k \cdot x \leq p^k \cdot x^k\}$ of affordable acts. We now introduce several concepts of rationalization of the dataset.

Following Echenique and Saito (2015), we say that a dataset $(p^k, x^k)_{k=1}^K$ is *subjective expected utility (SEU) rational* if there is $\mu \in \Delta_{++}$ and a concave and strictly increasing function $u : \mathbf{R}_+ \to \mathbf{R}$ such that, for all k,

$$y \in B(p^k, p^k \cdot x^k) \Longrightarrow \sum_{s \in S} \mu_s u(y_s) \le \sum_{s \in S} \mu_s u(x_s^k).$$

Similarly, following Chambers et al. (2016), we say that a dataset $(p^k, x^k)_{k=1}^K$ is *maxmin expected utility (MEU) rational* if there is a convex set $\Pi \subseteq \Delta_{++}$ and a concave and strictly increasing function $u : \mathbb{R}_+ \to \mathbb{R}$ such that, for all k,

$$y \in B(p^k, p^k \cdot x^k) \Longrightarrow \inf_{\pi \in \Pi} \sum_{s \in S} \pi_s u(y_s) \le \inf_{\pi \in \Pi} \sum_{s \in S} \pi_s u(x_s^k).$$

When imposed on a dataset, expected utility maximization may be too demanding. In order to capture situations where the model holds *approximately*, Echenique et al. (2018) relax the previous definition of SEU rationality by "perturbing" some elements of the model.

Let $e \in \mathbb{R}_+$. A dataset $(x^k, p^k)_{k=1}^K$ is *e-belief-perturbed SEU rational* if there exist $\mu^k \in \Delta_{++}$ for each $k \in K$ and a concave and strictly increasing function $u : \mathbb{R}_+ \to \mathbb{R}$ such that, for all k,

$$y \in B(p^k, p^k \cdot x^k) \Longrightarrow \sum_{s \in S} \mu_s^k u(y_s) \le \sum_{s \in S} \mu_s^k u(x_s^k),$$

and for each $k, l \in K$ and $s, t \in S$,

$$\frac{\mu_s^k/\mu_t^k}{\mu_s^l/\mu_t^l} \le 1 + e.$$

Echenique et al. (2018) introduce perturbation of prices and beliefs and show that three sources of perturbations are equivalent. More precisely, for any $e \in \mathbb{R}_+$, a dataset is e-perturbed rationalizable according to one of the sources if and only if it is also rationalizable according to any of the other sources with the same e.

B Implementation

This section presents a method to implement the revealed preference tests.

B.1 Exact Revealed Preference Tests

We are able to check whether a given dataset is consistent with SEU or MEU by solving the linear programming problem implied by the corresponding axiom. The construction of linear programming problems closely follows the argument in the proofs of Theorems appeared in Echenique and Saito (2015) and Chambers et al. (2016). For example, Echenique and Saito (2015) prove in Lemma 7 that a dataset $(x^k, p^k)_{k=1}^K$ is SEU rational if and only if there are strictly positive numbers v_s^k , λ^k , and μ_s for $s=1,\ldots,S$ and $k=1,\ldots,K$ such that

$$\mu_s v_s^k = \lambda^k p_s^k, \quad x_s^k > x_{s'}^{k'} \Longrightarrow v_s^k \le v_{s'}^{k'},$$

or equivalently, in a log-linearlized form,

$$\log v_s^k + \log \mu_s - \log \lambda^k - \log p_s^k = 0, \quad x_s^k > x_{s'}^{k'} \Rightarrow \log v_s^k \le \log v_{s'}^{k'}.$$

In this way, testing SEU rationality boils down to checking for existence of a solution to the above system, which is expressed as a system of linear equalities and inequalities:

$$\begin{cases} A \cdot z = 0 \\ B \cdot z \ge 0 \\ E \cdot z > 0 \end{cases}$$

A system of linear inequalities. We now construct three key ingredients of the system, matrices *A*, *B*, and *E* for testing SEU.

The first matrix A has $K \times S$ rows and $K \times S + S + K + 1$ columns, defined as follows: We have one row for every pair (k, s); one column for every pair (k, s); one column for every s, one column for each k; and one last column. In the row corresponding to (k, s) the matrix has zeroes everywhere with the following exceptions: it has a 1 in the column for (k, s); it has a 1 in the column for s; it has a -1 in the column for k; and $-\log p_s^k$ in the very last column. This finalizes

the construction of *A*. The resulting matrix looks as follows:

Next, we construct matrix B that has $K \times S + S + K + 1$ columns and there is one row for every pair (k, s) and (k', s') for which $x_s^k > x_{s'}^{k'}$. In the row corresponding to $x_s^k > x_{s'}^{k'}$ we have zeroes everywhere with the exception of a -1 in the column for (k, s) and a 1 in the column for (k', s').

Finally, we prepare a matrix that captures the requirement that the last component of a solution be strictly positive. The matrix E has a single row and $K \times S + S + K + 1$ columns. It has zeroes everywhere except for 1 in the last column.

In order to test MEU, we need to modify matrices A, B, and E appropriately, following characterization provided by Chambers et al. (2016), for the case of two states of the world. Let $K^0 = \{k : x_1^k = x_2^k\}$, $K^1 = \{k : x_1^k < x_2^k\}$ and $K^2 = \{k : x_1^k > x_2^k\}$. The first-order conditions are:

$$\mu_s^k v_s^k = \lambda^k p_s^k,$$

for s = 1, 2 and $k \in \{1, ..., K\}$, where $\mu_1^k = \bar{\mu}_1$ if $k \in K^1$, $\mu_1^k = \underline{\mu}_1$ if $k \in K^2$, $\mu_1^k \in [\underline{\mu}_1, \bar{\mu}_1]$ if $k \in K^0$. We now use $\pi = \mu_1/\mu_2$ instead of μ_1 . Then we can rewrite the first-order conditions:

$$\pi^k v_1^k = \lambda^k p_1^k$$
 and $v_2^k = \lambda^k p_2^k$,

for $k \in \{1, ..., K\}$, where $\pi^k = \bar{\pi}$ if $k \in K^1$, $\pi^k = \underline{\pi}$ if $k \in K^2$, $\pi^k \in [\underline{\pi}, \bar{\pi}]$ if $k \in K^0$.

Let A be a matrix with $2K + 2 + |K^0| + K + 1$ columns. The first 2K columns are labeled with a different pair (k, s). The next two columns are labeled $\bar{\pi}$ and $\underline{\pi}$. The next $|K^0|$ columns are for choices on the 45-degree line. The next K columns are labeled with K. Finally the last column is labeled K.

For each (k, 2), A has a row with all zero entries with the following exception: It has a 1 in the column labeled (k, 2); It has a -1 in the column labeled k; It has $-\log p_s^k$ in the column labeled p. For each (k, 1) with $k \in K^1$, A has a row with all zero entries with the following exception: It has a 1 in the column labeled (k, 1); It has a -1 in the column labeled k; it has $-\log p_s^k$ in the column labeled p; It has a 1 in the column labeled $\bar{\pi}$. For each (k, 1) with $k \in K^2 \cup K^0$, A has a row defined as above. The only difference is that it has a 1 in the column labeled $\bar{\pi}$ if $k \in K^2$ and in the column labeled $\bar{\pi}$ if $k \in K^0$, instead of having a 1 in the column labeled $\bar{\pi}$.

The resulting matrix *A* looks as follows:

Let B be a matrix with the same number of columns as A. The columns of B are labeled like those of A. B has a row for each pair $(x_s^k, x_{s'}^{k'})$ with $x_s^k > x_{s'}^{k'}$. The row for $x_s^k > x_{s'}^{k'}$ has all zeroes except for a 1 in column (k', s') and a -1 in column (k, s). Finally, B has a row which has a 1 in the column for $\bar{\pi}$ and a -1 in the column for $\bar{\pi}$ and $2|K^0|$ additional rows to capture $\pi^k \in [\underline{\pi}, \bar{\pi}]$ for $k \in K^0$. The resulting matrix B looks as follows:

Solve the system. Our task is to check if there is a vector z that solves the following system of linear inequalities corresponding to model $M \in \{SEU, MEU\}$. If there is a solution z to this system, we say that the dataset is "M rational."

Extension. There are three underlying states of the world, $S = \{\omega_1, \omega_2, \omega_3\}$ in the experiments. There are two types of questions: in type 1 questions, two events are $s_1 = \{\omega_1\}$ and $s_{23} = \{\omega_2, \omega_3\}$; in type 2 questions, two events are $s_{12} = \{\omega_1, \omega_2\}$ and $s_3 = \{\omega_3\}$. Let $S_1 = \{s_1, s_{23}\}$ denote the set of events in type 1 questions and $S_2 = \{s_{12}, s_3\}$ denote the set of events in type 2 questions. Suppose we have K observations in the data $(x^k, p^k)_{k=1}^K$. Let $k_1 \in K_1$ and $k_2 \in K_2$ denote indices for type 1 and type 2 questions, respectively (thus $K = K_1 \cup K_2$). Note that there is no type 1 question with state s_1 . Therefore, indices for observations s_1 0 and states s_2 1 and s_3 2 are the consistent, i.e., s_1 3 and s_2 4 for each s_3 5 for each s_3 5 for each s_3 6 for each s_3 7 for each s_3 7 for each s_3 8 for each s_3 9 for eac

In order to test SEU in this environment, we use the following proposition.

Proposition B.1. There exist strictly positive numbers μ_1 , μ_{23} , μ_{12} , μ_3 , $v_1^{k_1}$, $v_{23}^{k_2}$, $v_{12}^{k_1}$, $v_3^{k_1}$, λ^{k_1} , λ^{k_2} such that

$$\mu_1 v_1^{k_1} = \lambda^{k_2} p_1^{k_1}$$
 for each k_1 , (FOC1)

$$\mu_{23}v_{23}^{k_1} = \lambda^{k_2}p_{23}^{k_1}$$
 for each k_1 , (FOC2)

$$\mu_{12}v_{12}^{k_2} = \lambda^{k_1}p_{12}^{k_2} \text{ for each } k_2,$$
 (FOC3)

$$\mu_3 v_3^{k_2} = \lambda^{k_1} p_3^{k_2}$$
 for each k_2 , (FOC4)

$$\mu_{12} + \mu_3 = 1,$$
 (NO1)

$$\mu_1 + \mu_{23} = 1,$$
 (NO2)

$$\mu_{12} \ge \mu_1,\tag{MO1}$$

$$\mu_{23} \ge \mu_3,\tag{MO2}$$

$$\mu_{12} - \mu_1 = \mu_{23} - \mu_3. \tag{EQ}$$

if and only if there exist strictly positive numbers $\tilde{\mu}_{23}$, $\tilde{\mu}_{12}$, $\upsilon_1^{k_1}$, $\upsilon_{23}^{k_2}$, $\upsilon_{12}^{k_2}$, $\upsilon_3^{k_2}$, $\tilde{\lambda}^{k_1}$, $\tilde{\lambda}^{k_2}$ such that

$$v_1^{k_1} = \tilde{\lambda}^{k_1} p_1^{k_1} \quad \text{for each } k_1, \tag{FOC1'}$$

$$\tilde{\mu}_{23}v_{23}^{k_1} = \tilde{\lambda}^{k_1}p_{23}^{k_1} \text{ for each } k_1,$$
 (FOC2')

$$\tilde{\mu}_{12}v_{12}^{k_2} = \tilde{\lambda}^{k_2}p_{12}^{k_2} \text{ for each } k_2,$$
 (FOC3')

$$v_3^{k_2} = \tilde{\lambda}^{k_2} p_3^{k_2} \quad \text{for each} \quad k_2, \tag{FOC4'}$$

$$\tilde{\mu}_{23}\tilde{\mu}_{12} \ge 1. \tag{MO1'}$$

Proof. Define $\mu_1 = 1/(1 + \tilde{\mu}_{23})$, $\mu_{23} = \tilde{\mu}_{23}/(1 + \tilde{\mu}_{23})$, $\mu_{12} = \tilde{\mu}_{12}/(1 + \tilde{\mu}_{12})$, $\mu_3 = 1/(1 + \tilde{\mu}_{12})$, $\lambda^{k_1} = \tilde{\lambda}^{k_1}/(1 + \tilde{\mu}_{23})$, and $\lambda^{k_2} = \tilde{\lambda}^{k_2}/(1 + \tilde{\mu}_{12})$. Then, conditions FOC1-FOC1 are equivalent to FOC1'-FOC4' since, for example,

$$\mu_{12}v_{12}^{k_2} = \lambda^{k_2}p_{12}^{k_2} \quad \Longleftrightarrow \quad \frac{\tilde{\mu}_{12}}{1 + \tilde{\mu}_{12}}v_{12}^{k_2} = \frac{\tilde{\lambda}^{k_2}}{1 + \tilde{\mu}_{12}}p_{12}^{k_2} \quad \Longleftrightarrow \quad \tilde{\mu}_{12}v_{12}^{k_2} = \tilde{\lambda}^{k_2}p_{12}^{k_2}.$$

Condition MO1 is equivalent to MO1' since

$$\mu_{12} \geq \mu_1 \quad \Longleftrightarrow \quad \frac{\tilde{\mu}_{12}}{1 + \tilde{\mu}_{12}} \geq \frac{1}{1 + \tilde{\mu}_{23}} \quad \Longleftrightarrow \quad \tilde{\mu}_{12} + \tilde{\mu}_{12}\tilde{\mu}_{23} \geq 1 + \tilde{\mu}_{12} \quad \Longleftrightarrow \quad \tilde{\mu}_{12}\tilde{\mu}_{23} \geq 1,$$

and similarly MO2 is equivalent to MO1'. Condition EQ is satisfied since

$$\mu_{12} - \mu_1 = \frac{\tilde{\mu}_{12}}{1 + \tilde{\mu}_{12}} - \frac{1}{1 + \tilde{\mu}_{23}} = \frac{\tilde{\mu}_{12}\tilde{\mu}_{23} - 1}{(1 + \tilde{\mu}_{12})(1 + \tilde{\mu}_{23})} = \frac{\tilde{\mu}_{23}}{1 + \tilde{\mu}_{23}} - \frac{1}{1 + \tilde{\mu}_{12}} = \mu_{23} - \mu_3.$$

In order to implement the test, we first assemble matrices A (capturing v, μ , and λ ; equality constraints in the linear programming problem) and B (capturing concavity of u; weak inequality constraints). The above proposition has two implications: (i) We need to find only two strictly positive numbers capturing subjective beliefs, $\tilde{\mu}_{23}$ and $\tilde{\mu}_{12}$, instead of four numbers μ_1 , μ_{23} , μ_{12} , and μ_3 . (ii) We need to add one row in B to take care of additional weak inequality constraint $\tilde{\mu}_{23}\tilde{\mu}_{12} \geq 1$ (or equivalently, $\log \tilde{\mu}_{23} + \log \tilde{\mu}_{12} \geq 0$).

Let us now consider MEU. Suppose that in the first m_1 observations we have a partition $\{\{\omega_1\}, \{\omega_2, \omega_3\}\}$ (i.e., type 1 questions), and in the second m_2 observations we have a partition $\{\{\omega_1, \omega_2\}, \{\omega_3\}\}$ (i.e., type 2 questions).

• Partition 1: $\{\{\omega_1\}, \{\omega_2, \omega_3\}\}$. Let $O^0 = \{k: x_1^k = x_{23}^k\}$, $O^1 = \{k: x_1^k < x_{23}^k\}$ and $O^2 = \{k: x_1^k > x_{23}^k\}$. Afriat inequalities are now:

$$\theta^k v_1^k = \lambda^k p_1^k$$
 and $v_{23}^k = \lambda^k p_{23}^k$,

for $k \in \{1, ..., m_1\}$, where $\theta^k = \bar{\theta}$ if $k \in O^1$, $\theta^k = \underline{\theta}$ if $k \in O^2$, and $\theta^k \in [\underline{\theta}, \bar{\theta}]$ if $k \in O^0$.

• Partition 2: $\{\{\omega_1, \omega_2\}, \{\omega_3\}\}$. Let $T^0 = \{k : x_{12}^k = x_3^k\}$, $T^1 = \{k : x_{12}^k < x_3^k\}$ and $T^2 = \{k : x_{12}^k > x_3^k\}$. Afriat inequalities are now:

$$\pi^k v_{12}^k = \lambda^k p_{12}^k \text{ and } v_3^k = \lambda^k p_3^k,$$

for $k \in \{m_1 + 1, \dots, m_1 + m_2\}$, where $\pi^k = \bar{\pi}$ if $k \in T^1$, $\pi^k = \underline{\pi}$ if $k \in T^2$, and $\pi^k \in [\underline{\pi}, \bar{\pi}]$ if $k \in T^0$.

The unknowns are

$$\theta, \bar{\theta}, \theta^k, v_s^k, \lambda^k$$

for all $k = 1, ..., m_1$, and $s \in \{\{\omega_1\}, \{\omega_2, \omega_3\}\}$, and

$$\pi, \bar{\pi}, \pi^k, v_s^k, \lambda^k$$

for all $k=m_1+1,\ldots,m_1+m_2,s\in\{\{\omega_1,\omega_2\},\{\omega_3\}\}$. The system of inequalities are:

$$\begin{cases} \theta^k v_1^k = \lambda^k p_1^k & \text{if } k \in O^0 \\ \bar{\theta} v_1^k = \lambda^k p_1^k & \text{if } k \in O^1 \\ \underline{\theta} v_1^k = \lambda^k p_1^k & \text{if } k \in O^1 \\ v_{23}^k = \lambda^k p_{23}^k & \text{if } k \in O^0 \cup O^1 \cup O^2 \\ \underline{\theta} \leq \theta^k \leq \bar{\theta} & \text{if } k \in O^0 \cup O^1 \cup O^2 \end{cases}, \begin{cases} \pi^k v_{12}^k = \lambda^k p_{12}^k & \text{if } k \in T^0 \\ \bar{\pi} v_{12}^k = \lambda^k p_{12}^k & \text{if } k \in T^1 \\ \underline{\pi} v_{12}^k = \lambda^k p_{12}^k & \text{if } k \in T^2 \\ v_3^k = \lambda^k p_3^k & \text{if } k \in T^0 \cup T^1 \cup T^2 \\ \underline{\pi} \leq \pi^k \leq \bar{\pi} & \text{if } k \in T^0 \cup T^1 \cup T^2 \end{cases}$$

and, in addition, the constraints $\underline{\theta} \leq \bar{\theta}$, $\underline{\pi} \leq \bar{\pi}$, $\underline{\theta} \leq \underline{\pi}$, and $\bar{\theta} \leq \bar{\pi}$.

Note that this system of inequalities is linear after we take the log of each variable. In particular the constraint that $\pi^k \in [\pi, \bar{\pi}]$ is written as $\log(\pi) \le \log(\pi^k) \le \log(\bar{\pi})$.

Proposition B.2. A solution to the previous Afriat inequalities gives a solution to the FOCs.

Proof. Define

$$\mu_1^k = \theta^k / (1 + \theta^k), \quad \mu_{23}^k = 1/(1 + \theta^k)$$

if $k = 1, ..., m_1$, and

$$\mu_{12}^{k'} = \pi^{k'}/(1+\pi^{k'}), \quad \mu_{3}^{k'} = 1/(1+\pi^{k'})$$

if $k' = m_1 + 1, ..., m_1 + m_2$. Then

$$1 = \mu_1^k + \mu_{23}^k = \mu_{12}^{k'} + \mu_3^{k'}.$$

Observe that

(a)
$$\underline{\theta} \leq \bar{\theta} \Longrightarrow \underline{\mu}_1 \leq \bar{\mu}_1$$
.

(b)
$$\underline{\pi} \leq \bar{\pi} \Longrightarrow \underline{\mu}_{12} \leq \bar{\mu}_{12}$$
.

(c)
$$\underline{\theta} \leq \underline{\pi} \Longrightarrow \underline{\mu}_1 \leq \underline{\mu}_{12}$$
.

(d)
$$\bar{\theta} \leq \bar{\pi} \Longrightarrow \bar{\mu}_1 \leq \bar{\mu}_{12}$$
.

Define $\underline{\mu}_{2} = \underline{\mu}_{12} - \underline{\mu}_{1}$. Note that $\underline{\mu}_{2} = \bar{\mu}_{23} - \bar{\mu}_{3}$ because

$$1 = \underline{\mu}_{12} + \bar{\mu}_3 = \underline{\mu}_1 + \bar{\mu}_{23} \Longrightarrow \underline{\mu}_{12} - \underline{\mu}_1 = \bar{\mu}_{23} - \bar{\mu}_3.$$

Note also that $\underline{\mu}_2 \ge 0$, as (c) implies that $\underline{\mu}_{12} \ge \underline{\mu}_1$. Similarly, if we define $\bar{\mu}_2 = \bar{\mu}_{12} - \bar{\mu}_1$. Then using (d) we obtain that

$$0 \le \bar{\mu}_2 = \underline{\mu}_{23} - \underline{\mu}_3.$$

B.2 Approximate Revealed Preference Tests

Proposition B.3. Given $e \in \mathbb{R}_+$, a dataset $(x^k, p^k)_{k=1}^K$ is e-price-perturbed SEU rational if and only if there exist strictly positive numbers v_s^k , λ^k , μ_s , and ε_s^k such that:

1. for all $(k, s), (k', s') \in \bigcup_{i=1}^{2} (K_i \times S_i),$

$$\mu_{s}v_{s}^{k} = \lambda^{k}\varepsilon_{s}^{k}p_{s}^{k}, \quad x_{s}^{k} > x_{s'}^{k'} \Longrightarrow v_{s}^{k} \leq v_{s'}^{k'},$$

2. for all i = 1, 2,

$$\sum_{s\in S_i}\mu_s=1,$$

3. $\mu_{12} \ge \mu_1$ and $\mu_{23} \ge \mu_3$,

4.
$$\mu_{12} - \mu_1 = \mu_{23} - \mu_3$$
, and

5. for all $k, l \in K$ and $s, t \in S$,

$$\frac{\varepsilon_s^k/\varepsilon_t^k}{\varepsilon_s^l/\varepsilon_t^l} \le 1 + e.$$

Proposition B.4. Given $e \in \mathbb{R}_+$, a dataset $(x^k, p^k)_{k=1}^K$ is e-price-perturbed SEU rational if and only if there exist strictly positive numbers v_s^k , $\tilde{\lambda}^k$, $\tilde{\mu}_s$, and ε_s^k such that:

1. for all $k_1 \in K_1$,

$$v_1^{k_1} = \tilde{\lambda}^{k_1} \varepsilon_1^{k_1} p_1^{k_1}, \quad and \quad \tilde{\mu}_{23} v_{23}^{k_1} = \tilde{\lambda}^{k_1} \varepsilon_{23}^{k_1} p_{23}^{k_1}, \quad (e\text{-FOC1})$$

2. for all $k_2 \in K_2$,

$$\tilde{\mu}_{12}v_{12}^{k_2} = \tilde{\lambda}^{k_2}\varepsilon_{12}^{k_2}p_{12}^{k_2}$$
 and $v_3^{k_2} = \tilde{\lambda}^{k_2}\varepsilon_3^{k_2}p_3^{k_2}$, (e-FOC2)

3. for all $(k, s), (k', s') \in \bigcup_{i=1}^{2} (K_i \times S_i),$

$$x_s^k > x_{s'}^{k'} \Longrightarrow v_s^k \le v_{s'}^{k'},$$
 (e-CON)

- 4. $\tilde{\mu}_{23}\tilde{\mu}_{12} \geq 1$ (e-MON), and
- 5. for all $k, l \in K$ and $s, t \in S$,

$$\frac{\varepsilon_s^k/\varepsilon_t^k}{\varepsilon_s^l/\varepsilon_t^l} \le 1 + e.$$

Definition B.1. Given a dataset $(x^k, p^k)_{k=1}^K$, minimal e for SEU, e_*^{SEU} , is the solution to the following problem:

$$\min_{(\tilde{\mu}_{s}, v_{s}^{k}, \tilde{\lambda}^{k}, \varepsilon_{s}^{k})_{k,s}} \max_{k,l \in K, s, t \in S} \frac{\varepsilon_{s}^{k}/\varepsilon_{t}^{k}}{\varepsilon_{s}^{l}/\varepsilon_{t}^{l}}$$
 (\(\psi\))

subject to (e-FOC1), (e-FOC2), (e-CON), and (e-MON).

Remark B.1. Notice that in the objective function of the problem (\bigstar) ,

$$\frac{\varepsilon_s^k/\varepsilon_t^k}{\varepsilon_s^l/\varepsilon_t^l},$$

two states s, t are fixed and observations k, l are different. In our experimental setup, it means that either $k, l \in K_1$ or $k, l \in K_2$.

Remark B.2. By log-linearizing and substituting equality constraints in the objective function in the problem (\bigstar) , we obtain

$$\begin{split} \log\left(\frac{\varepsilon_s^k/\varepsilon_t^k}{\varepsilon_s^l/\varepsilon_t^l}\right) &= \log \varepsilon_s^k - \log \varepsilon_t^k - \log \varepsilon_s^l + \log \varepsilon_t^l \\ &= (\log \tilde{\mu}_s + \log v_s^k - \log \lambda^k - \log p_s^k) - (\log \tilde{\mu}_t + \log v_t^k - \log \lambda^k - \log p_t^k) \\ &- (\log \tilde{\mu}_s + \log v_s^l - \log \lambda^l - \log p_s^l) + (\log \tilde{\mu}_t + \log v_t^l - \log \lambda^l - \log p_t^l) \\ &= (\log \tilde{\mu}_s + \log v_s^k - \log p_s^k) - (\log \tilde{\mu}_t + \log v_t^k - \log p_t^k) \\ &- (\log \tilde{\mu}_s + \log v_s^l - \log p_s^l) + (\log \tilde{\mu}_t + \log v_t^l - \log p_t^l) \\ &= (\log v_s^k - \log p_s^k) - (\log v_t^k - \log p_t^k) - (\log v_s^l - \log p_s^l) + (\log v_t^l - \log p_t^l) \end{split}$$

B.3 Probabilistic Sophistication

Machina and Schmeidler (1992) propose the most basic Bayesian model of decision under uncertainty. An agent is *probabilistically sophisticated* if $x \in \mathbb{R}_+^S$ is evaluated by the distribution it induces given some prior $\mu \in \Delta_{++}$. Epstein (2000) shows the following.

Theorem B.1 (Epstein (2000)). If a dataset $(x^k, p^k)_{k=1}^K$ is probabilitsically sophisticated, then there cannot exist $k, k' \in K$ and $s, t \in S$ such that

1. $p_t^k \ge p_s^k$ and $p_s^{k'} \ge p_t^{k'}$, with at least one inequality being strict, and

2.
$$x_t^k > x_s^k$$
 and $x_s^{k'} > x_t^{k'}$.

C Additional Results

C.1 Lab Data

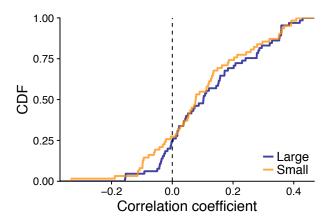


FIGURE C.1: Empirical CDF of within-subject (Spearman's) correlation between allocations in the market-stock task and the market-Ellsberg task. The two distributions are not significantly different (two-sample Kolmogorov-Smirnov test, p = 0.57).

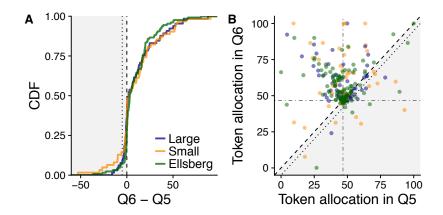


FIGURE C.2: Monotonicity of allocations with respect to event structures. (A) Empirical CDFs of token allocation difference. The dotted line represents a 5-token margin. No two pairs of distributions is significantly different (two-sample Kolmogorov-Smirnov test). (B) Token allocations in two questions. The dot-dashed lines at 46.67 indicate the number of tokens which equalizes payouts in two events.

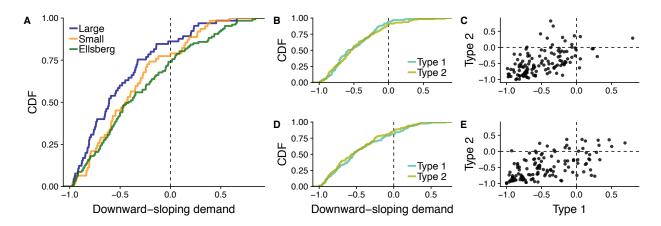


Figure C.3: Downward-sloping demand at the individual level (measured by $\rho^{\rm dsd}$). (A) Comparison across tasks. (BC) market-stock. (DE) Market-Ellsberg.

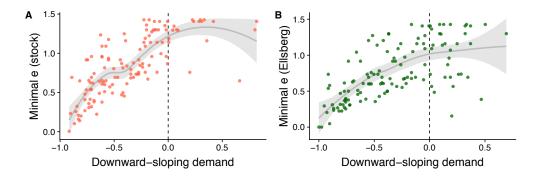


Figure C.4: Relation between the degree of conformity to downward-sloping demand and e_* . Note: We first calculate Spearman's correlation coefficient for each type of problem and then take the maximum as a conservative measure of compliance with the downward-sloping demand property. Gray lines represent LOESS curves together with confidence bands.

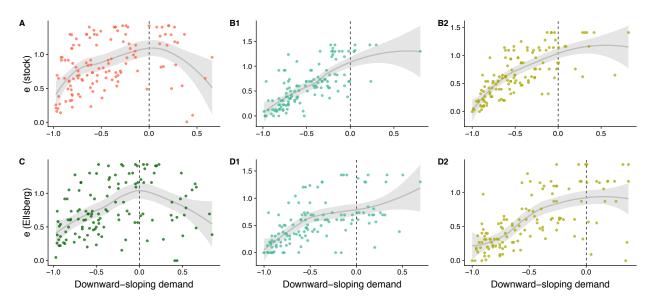


FIGURE C.5: Relation between the degree of conformity to downward-sloping demand (measured by $\rho^{\rm dsd}$) and e_* . (AB) the market-stock task, (CD) the market-Ellsberg task, (B1, D1) type 1 questions, (B2, D2) type 2 questions. Note: Correlation coefficient $\rho^{\rm dsd}$ in panels A and C are first calculated for each type of problem and then aggregated by Fisher's z-transformation ($\bar{r} = \tanh(\sum_{i=1}^n \tanh^{-1}(r_i)/n)$). Gray lines represent LOESS curves together with confidence bands.

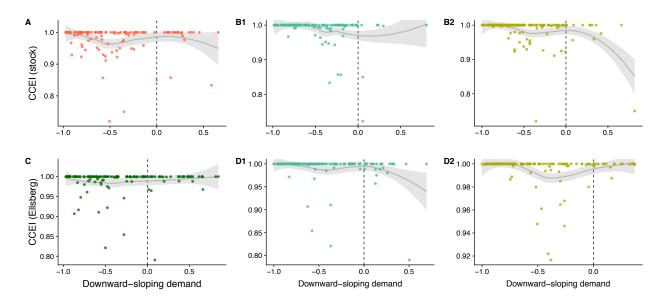


FIGURE C.6: Relation between the degree of conformity to downward-sloping demand (measured by $\rho^{\rm dsd}$) and CCEI: (AB) the market-stock task, (CD) the market-Ellsberg task, (B1, D1) type 1 questions, (B2, D2) type 2 questions.

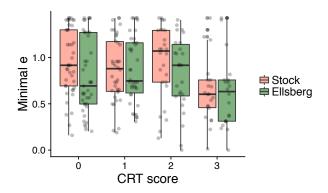


Figure C.7: Cognitive Reflection Test score and e_* .

Table C.1: Pass rates.

	GARP			SEU			MEU			PS		
Task	Type 1	Type 2	Joint									
Market-stock	0.7638	0.6850	0.5827	0.0472	0.0158	0.0000	0.0472	0.0158	0.0000	0.7323	0.8110	0.4803
Market-Ellsberg	0.8268	0.5650	0.6693	0.0787	0.0315	0.0157	0.0787	0.0315	0.0157	0.8110	0.8346	0.6220

Note: A subject satisfies GARP "jointly" if the subject passes GARP for both types. A subject is not inconsistent with PS "jointly" if the subject is not inconsistent with PS in the sense of Epstein for both types, and satisfies monotonicity of allocation with respect to event structure. Since Epstein's (2000) condition is only necessary for probabilistic sophistication, the numbers reported here capture the fraction of the subjects who are not inconsistent with probabilistic sophistication.

Table C.2: Effect of cognitive ability on e_* .

	(1)	(2)
market-Ellsberg	-0.175**	-0.154**
	(0.057)	(0.067)
Score 1	-0.084	-0.083
	(0.067)	(0.063)
Score 2	-0.099	-0.093
	(0.066)	(0.064)
Score 3	-0.324^{**}	-0.326**
	(0.096)	(0.099)
market-Ellsberg \times Score 1	0.156^{*}	0.156^{*}
	(0.076)	(0.075)
$market\text{-}Ellsberg \times Score~2$	0.081	0.080
	(0.073)	(0.074)
market-Ellsberg \times Score 3	0.140	0.146
	(0.114)	(0.118)
Treatment: Small		0.045
		(0.077)
Gender: Male		-0.028
		(0.098)
Constant	0.952**	0.939**
	(0.048)	(0.067)
R^2	0.0798	0.0826
# observations	254	254
# clusters	7	7

Note: Standard errors in parentheses are clustered at the session level. Significance levels: $^*p < 0.05$; $^{**}p < 0.01$.

C.2 Panel Data

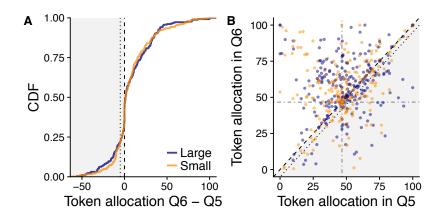


FIGURE C.8: Monotonicity of allocations with respect to event structure. (A) Empirical CDFs of token allocation difference. The dotted line represents a 5-token margin. No two pairs of distributions are significantly different (two-sample Kolmogorov-Smirnov test). (B) Token allocations in two questions. The dot-dashed lines at 46.67 indicate the number of tokens which equalizes payouts in two events.

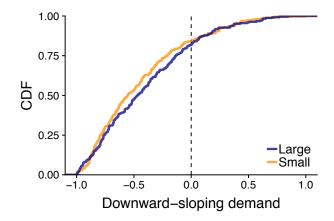


FIGURE C.9: Downward-sloping demand at the individual level (measured by $\rho^{\rm dsd}$).

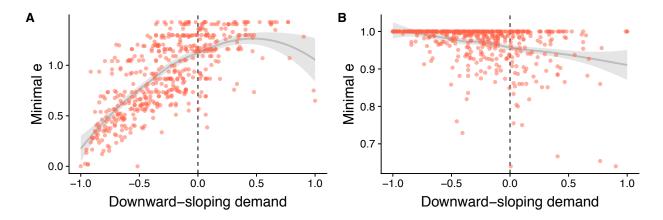


FIGURE C.10: Relation between the degree of conformity to downward-sloping demand and e_* (A) and CCEI (B). Note: We first calculate Spearman's correlation coefficient for each type of problem and then take the maximum as a conservative measure of compliance with the downward-sloping demand property. Gray lines represent LOESS curves together with confidence bands.

Table C.3: Pass rates.

		GARP		SEU			MEU			PS			
Treatment	N	Type 1	Type 2	Joint	Type 1	Type 2	Joint	Type 1	Type 2	Joint	Type 1	Type 2	Joint
Large variance Small variance		0.6653 0.6523			0.0653 0.0430				0.0490 0.0430		0.6776 0.6680	0.8041 0.7813	
Combined	501	0.6587	0.5968	0.4431	0.0539	0.0459	0.0140	0.0539	0.0459	0.0160	0.6727	0.7924	0.3952

Note: A subject satisfies GARP "jointly" if the subject passes GARP for both types. A subject is not inconsistent with PS "jointly" if the subject is not inconsistent with PS in the sense of Epstein for both types, and satisfies monotonicity of allocation with respect to event structure. Since Epstein's (2000) condition is only necessary for probabilistic sophistication, the numbers reported here capture the fraction of the subjects who are not inconsistent with probabilistic sophistication.

C.3 Sample Comparison in the Panel Study

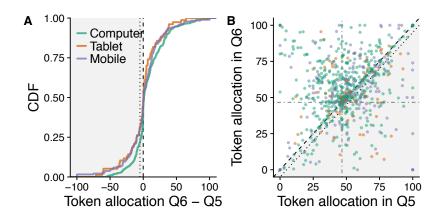


FIGURE C.11: (Panel) Monotonicity of allocation with respect to event structure. (A) Empirical CDFs of token allocation difference. The dotted line represents 5-token margin. Two-sample Kolmogorov-Smirnov test p-values: Computer vs. Tablet, p=0.116; Computer vs. Mobile, p=0.044; Tablet vs. Mobile, p=0.575. (B) Token allocations in two questions. The dot-dashed lines at 46.67 indicate the number of tokens which equalizes payouts in two events.

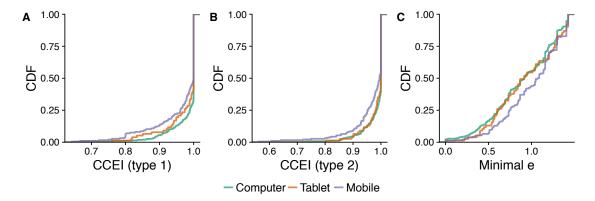


FIGURE C.12: (Panel) Distribution of measures, by subjects using computer, tablet, or mobile.

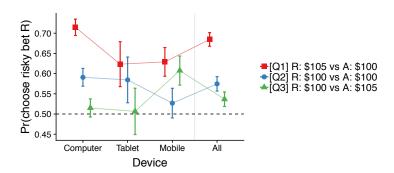


FIGURE C.13: (Panel) Probability of choosing a risky bet in each question in the standard-Ellsberg task, by subjects using computer, tablet, or mobile.

C.4 Power Calculation

It is well known that tests in revealed preference theory can have low power when used on certain configurations of budget sets. As a result, it is common to assess the power of a test by comparing the pass rates (the fraction of choices that pass the relevant revealed preference axiom) of the observed choice data from some benchmark behavior such as purely random choices. ¹

We assess the power of the tests using two kinds of data-generating process. In the first benchmark, we use the simple bootstrap procedure to look at the power from an ex-post perspective (Andreoni and Miller, 2002). More precisely, for each budget set, we randomly pick one choice from the set of choices observed in the experiment. We repeat this to generate 10,000 synthetic choice data. In the second benchmark, we generate 10,000 datasets in which choices are made at random and uniformly distributed on the frontier of the budget set (Method 1 of Bronars, 1987). Table C.4 report pass rates. The simulated choices almost always violate SEU and MEU. Pass rates for GARP test range from 0.23 to 0.68, depending on underlying data-generating process. These numbers are higher than those reported in other studies (e.g., Choi et al., 2007, 2014), but given that each type of problems has only 10 budgets, the configuration of budgets in our design has reasonable power to detect GARP violations.

Table C.4: Pass rates under different data generating process. Simulation sample size N = 10,000.

			GARP			PS			SEU			MEU	
Data-generating process	Device	Type 1	Type 2	Joint									
The Lab													
Bootstrap from Market-stock (Large)		0.4591	0.5577	0.2562	0.5762	0.8742	0.3913	0.0007	0.0001	0.0000	0.0011	0.0001	0.0000
Bootstrap from Market-stock (Small)		0.4769	0.4005	0.1934	0.6689	0.7269	0.4003	0.0002	0.0000	0.0000	0.0002	0.0000	0.0000
Bootstrap from Market-Ellsberg	Bootstrap from Market-Ellsberg		0.5365	0.2560	0.7429	0.8373	0.5196	0.0002	0.0002	0.0000	0.0002	0.0004	0.0000
The Panel													
Bootstrap from Market-stock (Large)	All	0.3642	0.3162	0.1172	0.4894	0.7299	0.2545	0.0001	0.0000	0.0000	0.0001	0.0000	0.0000
Bootstrap from Market-stock (Small)	All	0.3184	0.2842	0.0881	0.4857	0.7136	0.2521	0.0001	0.0000	0.0000	0.0001	0.0000	0.0000
Bootstrap from Market-stock (Large)	Desktop/laptop	0.3940	0.3533	0.1389	0.4878	0.7319	0.2661	0.0002	0.0000	0.0000	0.0002	0.0000	0.0000
Bootstrap from Market-stock (Small)	Desktop/laptop	0.3314	0.2908	0.0967	0.5106	0.7245	0.2789	0.0001	0.0001	0.0000	0.0001	0.0001	0.0000
Uniform random		0.2270	0.1440	0.0332	0.5002	0.5350	0.1925	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

¹The idea of using random choices as a benchmark is first applied to revealed preference theory by Bronars (1987). This approach is the most popular in empirical application: see, among others, Andreoni and Miller (2002), Fisman et al. (2007), Choi et al. (2007), Crawford (2010), Beatty and Crawford (2011), Adams et al. (2014), and Dean and Martin (2016). For overview of power calculation, see discussion in Andreoni et al. (2013) and Crawford and De Rock (2014).

TABLE C.5: CCEI and e^* calculated with randomly generated choice data. Simulation sample size N=10,000.

	CCEI				e*(SEU)		$e^*(MEU)$			
	Type 1	Type 2	Joint	Type 1	Type 2	Joint	Type 1	Type 2	Joint	
Mean	0.9123	0.9194	0.8716	0.7761	0.8257	0.9777	0.7631	0.8257	0.9725	
Median	0.9256	0.9436	0.8841	0.7340	0.9365	1.0166	0.7326	0.9365	1.0145	
SD	0.0829	0.0817	0.0855	0.2583	0.2890	0.2035	0.2633	0.2890	0.2094	

D Design Detail

D.1 The Set of Budgets

Table D.1: The set of 20 budgets. The numbers indicate "exchange value" for each account (z_1, z_2) .

			La	ab	Pa	nel
	Type	Order	Account 1	Account 2	Account 1	Account 2
1	1	random	0.30	0.18	3.0	1.8
2	1	random	0.30	0.24	3.0	2.4
3	1	random	0.38	0.30	3.8	3.0
4	1	random	0.40	0.40	4.0	4.0
5	1	random	0.50	0.12	5.0	1.2
6	1	random	0.50	0.24	5.0	2.4
7	1	random	0.50	0.34	5.0	3.4
8	1	random	0.50	0.44	5.0	4.4
9	1	random	0.60	0.30	6.0	3.0
10	1	fixed (5 th)	0.32	0.28	3.2	2.8
11	2	random	0.14	0.50	1.4	5.0
12	2	random	0.24	0.50	2.4	5.0
13	2	random	0.28	0.32	2.8	3.2
14	2	random	0.30	0.36	3.0	3.6
15	2	random	0.30	0.42	3.0	4.2
16	2	random	0.30	0.56	3.0	5.6
17	2	random	0.38	0.52	3.8	5.2
18	2	random	0.40	0.50	4.0	5.0
19	2	random	0.50	0.56	5.0	5.6
20	2	fixed (6 th)	0.32	0.28	3.2	2.8

D.2 Simulated Price Paths for the Market-Stock Task

In order to simulate price paths, we use a Geometric Brownian Motion (GBM):

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where W_t is a Wiener process, μ is a drift parameter, and σ is a volatility parameter. To simulate trajectories of GBM, we calculate increments of S:

$$S_{t+h} = S_t \times \exp\left((\mu - \sigma^2/2)h + \sigma\sqrt{h}Z\right),$$

with $Z \sim N(0, 1)$.

As a first step, we generated N paths of GBM, where each path $P^n = (P_0^n, P_1^n, \dots, P_T^n)$ has the common starting price P_0 and T periods of prices. We then group these N paths into several categories, based on several observable features: (i) absolute movement within T periods; (ii) final price is higher than the initial price; (iii) final price is lower than the initial price; (iv) trends such as up-down, down-up, straight-gain, straight-loss, and cycle.

After visually inspecting the pattern of each price path, we handpicked 28 paths and then asked workers on Amazon Mechanical Turk what they believed the future price of each path would be. We used a "bins-and-balls" belief elicitation task (also known as a histogram elicitation method) introduced by Delavande and Rohwedder (2008) to elicit subjective belief distribution. The method, later refined by (Delavande et al., 2011) and Rothchild (2012), is simple and easy to understand. It has been shown to work well in experiments conducted at developing countries (Delavande et al., 2011) and online survey (Huck et al., 2016).

The idea of the task is as follows. First, the (continuous) state of the world (ranges of future prices) is partitioned into 20 disjoint and exhaustive bins. Second, subjects are asked to place 20 "balls," each representing 5% probability mass, into these bins. The subjects were then asked to express how likely they believed that the price to be in each or the 20 ranges. Figure D.1 illustrates the task.

The elicited belief distributions were then averaged across subjects. Some price paths, especially those with clear upward or downward trend, tend to be associated with skewed distributions. Others have more symmetric distributions. We thus selected two relatively "neutral" ones from the latter set for the main experiment.

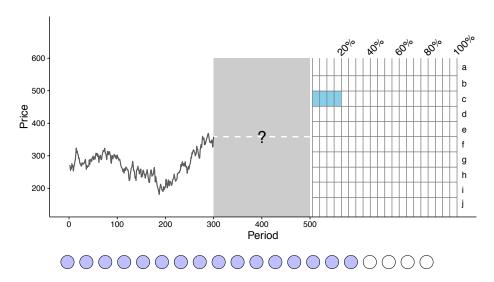


FIGURE D.1: Illustration of the bins-and-balls belief elicitation task.

D.3 Post-Experiment Survey in the Laboratory Study

Demographic information.

- 1. What is your age?
- 2. What is your gender?
- 3. What is your ethnicity?
- 4. What is your major?

Three-item cognitive reflection test.

- 1. If it takes 5 people 5 months to save a total of \$5,000, how many months would it take 100 people to save a total of \$100,000?
- 2. A TV and a radio cost \$110 in total. The TV costs \$100 more than the radio. How much does the radio cost?
- 3. In a lake, there is a patch of lily pads. Each day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?

E Instructions for the Experiments

Welcome!

Thank you for participating in today's experiment.

Please turn off all electronic devices, especially phones and tablets. During the experiment you are **not** allowed to open or use any other applications on these laboratory computers, except for the interface of the experiment.

This experiment is designed to study decision making. You will be paid for your participation in cash privately at the end of the session. Please follow the instructions carefully and do not hesitate to ask the experimenter any questions by raising your hand. The experimenter will then come to your desk.

Structure of the experiment

The experiment consists of 3 tasks and a survey. We will hand out specific instructions for each of the tasks just before you are to perform that task.

At the front of this laboratory you will see several opaque bags labeled **A**, **B**, and so on, which we will use in some of the tasks during the experiment. Each of these bags contains colored chips. The exact composition of chips in each bag (for example, how many of them are red) may or may not be announced to you. If you wish, you can inspect these bags **after** completing all sections of the experiment.

Payment

In order to determine the payment, **one task** and **one question from that task** will be randomly selected. Your payoff in the experiment will consist of the amounts you earned in the selected question plus a \$10 show-up fee if you complete the experiment as announced. The specific rules applied to determine payoffs for each section will be described in detail in the instructions for that part.

To select **one task** and **one question** that will determine your payment, the assistant rolled two fair dice for each participant. The assistant wrote down two numbers, one indicating the task and another indicating the question in that particular task that counts for payment. The note was placed into a sealed envelope. Please write your participant ID on the envelope once you receive it. Please do not open the envelope until you are instructed by the experimenter.

Remember that the question determining your payment is selected before you make any decisions in the experiment. This protocol of determining payments suggests that you should make a decision in each question as if it is the question that determines your payment.

Important rules

In the experiment we use a web browser. It is important that you ...

- 1. do not close or refresh the browser,
- 2. do not open other windows/tabs on the browser,
- 3. do not exit the full screen mode, and
- 4. do not open other applications and programs.

If you exit the full screen mode, please click the button at the top right corner to enter the full screen mode again.

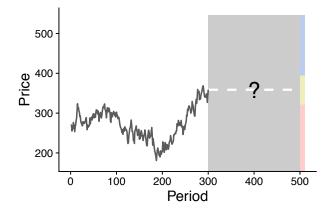
Please raise your hand if you have any questions regarding the structure of the experiment.

Task 1

Overview

In this part of the experiment, you will be asked a total of 20 independent questions that share a common structure. Your goal is to invest tokens in two different accounts. The accounts pay off according to the value of a stock.

There is a **hypothetical** company which we refer to as **Company X**. We simulated a history of stock prices of this company using a model frequently used in financial economics. You will be presented a chart plotting the history of Company X's stock prices. The figure below is an illustration of such chart. Note that the price history presented in the image is meant to be an example, and is not the same one as you will see in the task.



The chart shows stock prices from period 1 to 300 (imagine that period 300 is "today"). You do not know the movement beyond period 300, the area shaded in gray. Your payoff in this task depends on the "future" value of Company X's stock price at period 500, i.e., at the end of the chart. More precisely, it depends on whether the final price lies in the Blue area (increase by more than the threshold), Yellow area (change up to the threshold), or Red area (decrease by more than the threshold). In this example, the threshold is set to 10%.

How it works

Now we will explain the task in detail.

You will be asked a total of 20 independent questions. In each decision problem, you will be endowed with 100 tokens and asked to choose the portion of this amount (between 0 and 100 tokens, inclusive and divisible) that you wish to allocate between two accounts. Tokens allocated to each account may have different monetary values. Your payoff in this task will be determined by the following three components:

- (i) the monetary value of tokens in each account, which is given in the question,
- (ii) your allocation of tokens in each of the two accounts, and
- (iii) in which colored area Company X's stock price lies at period 500.

Two types of questions

There are two types of questions.

In Type 1 questions, two accounts are

Account **Blue-or-Yellow** : Stock price increases by a positive percentage

or decreases by at most 10%.

Account **Red** : Stock price decreases by more than 10%.

In Type 2 questions, two accounts are

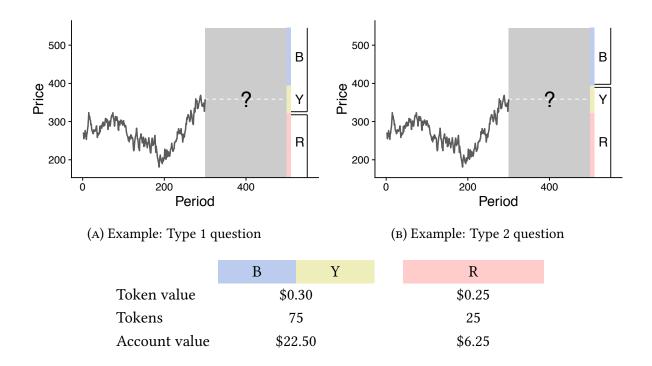
Account **Blue** : Stock price increases by more than 10%.

Account **Yellow-or-Red** : Stock price decreases by any percentage

or increases at most 10%.

These two types of questions appear in random order. To understand the decision problem for the given trial correctly, it would be of your best interest to check the type of the question (1 or 2) on the right of the stock chart and also at the top of the "allocation table" which will be explained below.

The "allocation table" at the bottom block of the screen shows information on the monetary values of tokens. In the example of Type 1 question below, each token you allocate to the Blueor-Yellow account is worth \$0.30 (30 cents), while each token you allocate to the **Red** account is worth \$0.25 (25 cents). Notice that monetary values of tokens may change across questions.



How to make a decision

You can allocate 100 tokens between two accounts using the slider. The table will be updated instantly once you move the slider, showing current allocations of tokens and their implied payment amounts if the stock ends up in the corresponding color region. No cursor appears at the start of the experiment—you need to click anywhere on the slider line to activate it.

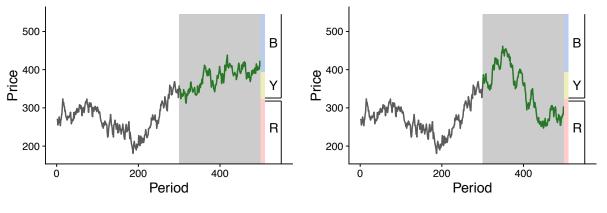
Alternatively, you can allocate tokens by directly putting numbers in one of the boxes, or clicking **up/down** arrow (which appears when you mouse-over the box) to make small adjustments.

How your payoff for this task is determined

Your payoff is determined by the number of tokens in your two accounts, and the "future" value of stock X at the end of the chart (period 500), which will be simulated after you complete all questions.

Suppose you allocated 75 tokens to account **Blue-or-Yellow** and 25 tokens to account **Red** as in the above example. If this question has been chosen to determine your payoff, your payoff will be determined by the price of company X's stock in period 500. If stock X hits blue or yellow area at period 500 (as in panel (a) below), then you will earn $75 \times \$0.30 = \22.50 (22 dollars 50 cents). On the other hand, if stock X hits red area (as in panel (b) below), then you will earn

 $25 \times \$0.25 = \6.25 (6 dollars 25 cents). Amount below one cent will be rounded up.



(A) Example: Stock price hits Blue area

(B) Example: Stock price hits Red area

Important

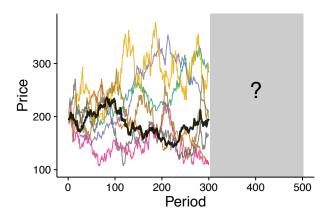
- The history of stock prices of Company X up to period 300 ("today") is the same throughout this task.
- You will not know "future" prices (between period 301 and period 500) until you complete all tasks in the experiment.
- Token values and question types can vary between questions.
- Once you hit the Proceed button, you cannot change your decision. You **cannot** go back to previous pages, either. Note also that you **cannot** change the question by refreshing the browser once it is displayed.
- Remember that the question that will determine your payment has already been selected at the start of the experiment. It is your best interest to treat each question as if it is the question that determines you payment.

Hypothetical Stock Market

As we mentioned before, we simulated a history of stock prices of this company using a model frequently used in financial economics. The following chart illustrates eight such simulated stocks in our "hypothetical stock market".

Let's imagine that we are at period 300 ("today") and we do not know the "future" stock prices (periods 301 to 500).

The black solid line represents the price history of our Company X. You will see ONLY this price history during this task.



Task 2

Overview

In this part of the experiment, you will be asked a total of 20 independent questions that share a common structure. Your goal is to invest tokens in two different accounts. The accounts pay off according to the color of a chip drawn from a bag at the end of the experiment.

There is an opaque **bag Z** which contains 30 colored chips. Each chip is either Blue, Yellow, or Red. The number of chips of each color is unknown to you: There can be anywhere from 0 to 30 Blue chips, anywhere from 0 to 30 Yellow chips, and anywhere from 0 to 30 Red chips, as long as the total number of Blue, Yellow, and Red chips sums to 30. Your payoff in this task depends on the color of a chip you will draw at the end of the experiment.

Bag Z: Total 30 chips



The contents of bag Z has already been determined by an assistant at the beginning of the experiment. If you wish, you can inspect the contents of the bag **after** completing the experiment.

How it works

Now we will explain the task in detail.

You will be asked a total of 20 independent questions. In each decision problem, you will be endowed with 100 tokens and asked to choose the portion of this amount (between 0 and 100 tokens, inclusive and divisible) that you wish to allocate between two accounts. Tokens allocated to each account may have different monetary values. Your payoff in this task will be determined by the following three components:

- (i) the monetary value of tokens in each account,
- (ii) your allocation of tokens in each of the two accounts, and
- (iii) the color of the chip you will draw from the bag at the end of the experiment.

Two types of questions

There are two types of questions.

In Type 1 questions, two accounts are

Account **Blue-or-Yellow**: The color of chip drawn from the bag is either Blue or Yellow.

Account **Red** : The color of chip drawn from the bag is Red.

In Type 2 questions, two accounts are

Account **Blue** : The color of chip drawn from the bag is Blue.

Account **Yellow-or-Red**: The color of chip drawn from the bag is either Yellow or Red.

These two types of questions appear in random order. To understand the decision problem for the given trial correctly, it would be of your best interest to check the account structure at the top of the "allocation table" which will be explained below.

The "allocation table" at the bottom block of the screen shows information on the monetary values of tokens. In the example of Type 2 question below, each token you allocate to the **Blue** account is worth \$0.36 (36 cents), while each token you allocate to the **Yellow-or-Red** account is worth \$0.24 (24 cents). Notice that monetary values of tokens may change across questions.

	В	Y	R	
Token value	\$0.36	\$0.24		
Token	30	7	0	
Account value	\$10.80	\$16	5.80	

How to make a decision

You can allocate 100 tokens between two accounts using the slider. The table will be updated instantly once you move the slider, showing current allocations of tokens and their implied payment amounts. No cursor appears at the start of the experiment—you need to click anywhere on the slider line to activate it.

Alternatively, you can allocate tokens by directly putting numbers in one of the boxes, or clicking **up/down** arrow (which appears when you mouse-over the box) to make small adjustments.

How your payoff for this task is determined

Your payoff is determined by the number of tokens in your two accounts, and the color of a chip you will draw from the bag at the end of the experiment.

Suppose you allocated 30 tokens to account **Blue** and 70 tokens to account **Yellow-or-Red** as in the above example. If this question has been chosen to determine your payoff, your payoff will be determined by the color of a chip you will draw at the end of the experiment. If it is Blue, then you will earn $30 \times \$0.36 = \10.80 (10 dollars 80 cents). On the other hand, if it is Yellow or Red, then you will earn $70 \times \$0.24 = \16.80 (16 dollars 80 cents). Amount below one cent will be rounded up.

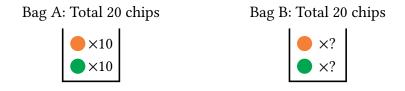
Important

- The composition of the bag (how many chips are blue, yellow, or red) is the same throughout this task.
- You will not know the actual composition until you complete all tasks in the experiment.
- Token values and question types can vary between questions.
- Once you hit the Proceed button, you cannot change your decision. You cannot go back
 to previous pages, either. Note also that you cannot change the question by refreshing the
 browser once it is displayed.
- Remember that the question that will determine your payment has already been selected at the start of the experiment. It is your best interest to treat each question as if it is the only question that determines you payment.

Task 3

There are two bags, bag A and bag B, each of which contains 30 chips. Each chip is either orange or green. The contents of each bag is as follows:

- Bag A contains 10 orange chips and 10 green chips.
- Bag B contains 20 chips. Each chip is either orange or green. The number of chips of each color is unknown to you: There can be anywhere from 0 to 20 orange chips, and anywhere from 0 to 20 green chips, as long as the total number of orange and green chips sums to 20.



The contents of bag B has already been determined at the beginning of the experiment. If you wish, you can inspect the contents of each bag **after** completing the experiment.

You will now answer several questions, each of which offers you a choice between bets on the color of a chip that you will draw from one of two bags at the end of the experiment (if this section is chosen for payment).

You will first be asked to choose one of the two colors. We will call this **Your Color**. You will be paid only if a chip of this color is drawn from the bag at the end of the experiment.

You will then be asked to answer the following three questions.

- Question: Please select a bet
 - 1. \$10.50 if a chip drawn from bag A is of **Your Color** and \$0 otherwise.
 - 2. \$10.00 if a chip drawn from bag B is of **Your Color** and \$0 otherwise.
- Question: Please select a bet
 - 1. \$10.00 if a chip drawn from bag A is of **Your Color** and \$0 otherwise.
 - 2. \$10.00 if a chip drawn from bag B is of **Your Color** and \$0 otherwise.
- Question: Please select a bet
 - 1. \$10.00 if a chip drawn from bag A is of **Your Color** and \$0 otherwise.
 - 2. \$10.50 if a chip drawn from bag B is of **Your Color** and \$0 otherwise.

How your payoff for this section is determined

Suppose one of the three questions in this section is selected for determining your payment. If you chose bet 1 in that particular question, you will draw a chip from bag A. On the other hand, if you chose bet 2 in that particular question, you will draw a chip from bag B. In either case, you will get payment if the color of the drawn chip matches with Your Color.

How to make a decision

You will see four questions on the screen. The first one asks which color you want to use as **Your Color** and the following three questions ask which bet you would like to play.

For each question, you can make your selection by clicking on the check box for the option you would like to choose. You can change your selection as many times as you want, and there is no time limit. Once you make your selections for all three questions, you can submit them by clicking Proceed. You will not be able to change your decision after that.

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