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# Financial Market Globalization and Endogenous Inequality of Nations

By Kiminori Matsuyama<sup>1</sup>

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## Abstract

This paper analyzes the effects of financial market globalization on the cross-country pattern of development in the world economy. To this end, it develops a dynamic macroeconomic model of imperfect credit markets, in which the domestic investment becomes borrowing-constrained at the lower stage of development. In the absence of the international financial market, the world economy converges to the symmetric steady state, and the cross-country difference disappears in the long run. It is shown that, under some parameter values, financial market globalization causes the instability of the symmetric steady state and generates stable asymmetric steady states, in which the world economy is polarized into the rich and the poor. The world output is smaller, the rich are richer and the poor are poorer in these asymmetric steady states than in the (unstable) symmetric steady state. The model thus demonstrates the possibility that financial market globalization may cause, or at least magnify, inequality among nations, and that the international financial market is a mechanism through which some countries become rich at the expense of others. Furthermore, the poor countries cannot jointly escape from the poverty trap by merely cutting their links to the rich. Nor would foreign aids from the rich to the poor eliminate inequality; as in a game of musical chairs, some countries must be excluded from being rich.

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*Keywords:* imperfect credit markets, borrowing-constraint, convergence versus divergence, the international financial market, polarization, symmetry-breaking, poverty traps

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## 1. Introduction

The role of the international financial market in economic development is one of the most controversial issues in macroeconomics. The standard, neoclassical view suggests that an integration of financial markets leads to the efficient allocation of the world saving by facilitating the flows of financial capital from rich countries, where the return to investment is low, to poor countries, where the return is high. This accelerates economic development in poor countries. Without borrowing from abroad, poor countries would have to finance their investment entirely by the domestic saving, which would slow down their development. According to this view, financial market globalization is an equalizing force, which will bring about a greater, faster convergence of economic performances across countries.

As many have pointed out, however, even casual observations seem to refute this textbook view. In reality, many poor countries receive little private credit from abroad.<sup>2</sup> They are indeed more concerned that the access to the international financial market might lead to an outflow of domestic funds, and continue to impose restrictions in their efforts to channel more domestic saving into the investment at home. These restrictive policies did not seem to prevent some former developing countries, such as Korea and Taiwan, from achieving rapid growth; some even argue that these policies were essential elements of their successful development strategies. Furthermore, a greater integration of financial markets after WWII seems to have done little to reduce the cross-country difference in economic performances. On the contrary, the evidence, reported by Quah (1993, 1997) and others, suggests that the world economy is increasingly polarized into the rich and the poor.

There is indeed the popular view that the international financial market magnifies the gap between the rich and the poor. According to this view, financial market globalization is an unequalizing force. The believers of this view often advocate that poor countries should impose more controls to stem the outflows of the domestic saving and that official aids from rich countries are needed for the development of poor countries. Some even hold a radical view that the World Bank and the IMF, which promote financial market globalization, are agents of the global corporate capitalism that exploits developing countries. These radical economists often

suggest that the poor countries should jointly cut their links to the rich countries and unite among themselves to escape the poverty.

The standard neoclassical framework, which builds on the dual assumptions of diminishing returns in capital and of the perfect credit market, is simply inadequate to deal with these issues. An alternative theoretical framework is needed, which allows for the divergence of economic performances, and the lack of the private financial capital flows from the rich to the poor, and the possibility that the international financial market may be a cause of the inequality of nations. Only within such a framework could one examine the validity of policy proposals offered by the radical economists.

One natural departure from the neoclassical framework, pursued by many, is to introduce some aggregate increasing returns at the national level.<sup>3</sup> The presence of such increasing returns creates agglomeration economies, which lead to a divergence and reverse flows of financial capital. According to this approach, financial capital flows from the poor to the rich, *because* the return to investment is higher in rich countries. If this is the only mechanism through which globalization generates a divergence, the world economy as a whole may benefit from globalization and inequality. Even the poor countries may be better off than in autarky.<sup>4</sup> Furthermore, this effect is not unique to financial market globalization. Whether globalization takes place in financial markets (that is, borrowing and lending) or in factor markets (that is, foreign direct investment or trade in the capital goods), it would lead to a divergence. Driven by agglomeration economies, both physical and financial capital would flow from the poor to the rich.

This paper explores an alternative departure from the standard framework, by dropping the assumption of the perfect credit markets. To highlight the role of imperfect credit markets, the assumption of diminishing returns in capital at the national level is maintained (without denying the empirical relevance of aggregate increasing returns). Due to the credit market

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<sup>2</sup>This prompted Lucas (1990) to pose now the famous question, “Why doesn’t capital flow from rich to poor countries?”

<sup>3</sup>This includes all the models with endogenous total factor productivity (based on, for example, external economies of human capital or innovation of new goods) so that the rich countries would have better aggregate production technologies in equilibrium. Much of the work that followed Lucas (1990) may be classified in this category.

<sup>4</sup>See, for example, Krugman and Venables (1995) and Matsuyama (1996). This is a familiar feature of models with agglomeration economies.

imperfection, financial capital can flow from the poor to the rich, as a result of financial market globalization, *despite* that the return to investment is higher in poor countries. Unlike those driven by agglomeration economies, the reverse flows of financial capital driven by the credit market imperfection lead to a misallocation of the world saving, and hence, the world output declines, and rich countries become richer at the expense of poor countries. Furthermore, this divergence result is unique to globalization of financial markets. Globalization of factor markets would lead to a convergence, by facilitating flows of physical capital from the rich countries to the poor.

In the model developed below, the credit market imperfection arises due to potential defaults by the borrowers (and imperfect sanction against them). Due to the imperfection, the potential entrepreneurs can borrow only a limited amount, and hence need to own enough wealth to start the investment project. This makes the domestic investment be borrowing-constrained at the lower stage of development. The model is examined in three different environments: i) the autarky; ii) the small open economy that faces the exogenously given world interest rate; and iii) the world economy consisting of a continuum of inherently identical economies.

In autarky, the dynamics of capital formation is determined entirely by the domestic saving, and the economy converges to the unique steady state in the long run. Even though the domestic investment is borrowing-constrained at the lower stage of development, the equilibrium interest rate adjusts to equate the domestic saving and investment. In the small economy case, the domestic investment no longer need to be equal to the domestic saving and the interest rate is fixed in the international financial market. At a low stage of development, the entrepreneurs have little wealth and the borrowing constraint is binding. Since the interest rate cannot decline to offset the effects of the credit market imperfection, the borrowing constraint leads to a low domestic investment, thereby slowing down the development process. At a high stage of development, the entrepreneurs have enough wealth and the borrowing constraint is not binding. Without an offsetting rise in the interest rate, this leads to a high domestic investment. Under some conditions, this effect is strong enough to create multiple steady states in the dynamics of capital formation in the small open economy.

Having examined the autarky and small open economy cases, the paper turns to the analysis of the world economy. In the absence of the international financial market, the world economy is simply a collection of autarky economies, and hence converges to the unique symmetric steady state, in which all the countries have the same level of the capital stock. The cross-country difference will disappear in the long run. The symmetric steady state is always stable, because the domestic interest rates could adjust independently within each country, when different countries are hit by different shocks. In the presence of the international financial market, the world economy is a collection of small open economies (with the interest rate being endogenously determined in the international financial market). Under some conditions, the symmetric steady state loses its stability. This is because unrestricted flows of financial capital forces the interest rates in different countries to move together. In other words, all the entrepreneurs must compete directly for the world saving to finance their investment. This put the entrepreneurs living in the countries hit by relatively bad shocks in disadvantage, and the domestic investment in these countries decline, creating a downward spiral of low-wealth/low-investment. The same force operates in the opposite direction within the countries hit by relatively good shocks, creating an upward spiral of high-wealth/high-investment. This process would continue until the world economy is polarized into the rich and the poor. In these (stable) asymmetric steady states, the rich countries are richer and the poor countries are poorer and the world output is smaller than in the (unstable) symmetric steady state. Therefore, the model shows not only the possibility that financial market globalization and unrestricted flows of financial capital may cause or at least magnify the inequality of nations. It also offers a theoretical justification for the view that the international financial market is a mechanism through which some countries become rich at the expense of others. At the same time, the model suggests that poor countries cannot jointly escape from the poverty trap by merely cutting their links to rich countries and that official aids from the rich would not eliminate inequality. Just as in a game of musical chairs, some countries must be excluded from being rich.

It should also be emphasized that financial market globalization does not always lead to the symmetry-breaking and the polarization. Under different conditions, the model predicts the convergence. One major advantage of the present model is that it is capable of generating the two

alternative scenarios, convergence and divergence, thereby providing theoretical justifications for the two conflicting views of the world. More importantly, which of these two alternative scenarios will materialize depends on a few key parameters in an interesting way. (Roughly speaking, divergence occurs when the productivity of the investment projects is neither too high nor too low.) The present model thus serves as an organizing principle on these controversial and seemingly intractable issues.

Before proceeding, mention should be made of the title. The term, “financial market globalization” is chosen instead of “capital mobility” to emphasize two points. First, the perspective adopted in this paper is global. We are interested not so much in the effects of capital mobility on poor countries, but in the effects of financial market integration on the *cross-country* pattern of development in the *world economy*. And, as will be discussed later, the global perspective offers different policy implications. Second, the paper is concerned with the effect of international mobility of *financial* capital, or the possibility of international *lending* and *borrowing*, which is modeled as *intertemporal* trade in the final good. Throughout the paper, it is assumed that physical capital, i.e., the capital good used in the production of the final good, is *nontradeable*,<sup>5</sup> and that the agents can start their investment projects only in their own countries. The main focus of the paper is globalization of financial markets, not globalization of factor markets. What is demonstrated is the possibility that financial market globalization cause or magnify the inequality among nations, when the factor markets are not fully integrated. The use of the term, “capital mobility,” is avoided, because it could mean, to many, the tradeability of the capital good and/or foreign direct investment.<sup>6</sup>

The rest of the paper is organized as follows. Section 2 develops the building blocks of the model. The three alternative environments, --autarky, a small open economy, and the world economy--, are examined in sections 3, 4, and 5, respectively. Section 6 considers an extension

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<sup>5</sup>Here, the adjective “physical” is used as opposed to “financial,” not as opposed to “human.” What truly matters in the following analysis is that some accumulative forms of factor inputs have some nontradeable components. Human capital could equally play the same role, and hence “physical capital” may be broadly interpreted to include human capital as well.

<sup>6</sup>The distinction between the mobility of financial capital and the mobility of the capital good would not be important if the credit market were perfect. In a world of the perfect credit market, nontradeable capital goods would become effectively tradeable with the access to the international financial market, because the economy could finance the production of the nontradeable capital good by borrowing from abroad. This distinction is critical, however, when the credit market is imperfect.

that allows for heterogeneous agents. Section 7 discusses the related work in the literature. Section 8 concludes.

## 2. The Model

The model comes in three versions: the autarky, the small open economy, and the world economy consisting of a continuum of inherently identical economies. This section explains the common elements.

Time is discrete and extends from zero to infinity ( $t = 0, 1, 2, \dots$ ). There are two goods, a final good and physical capital, and one primary factor of production, labor. Physical capital may be interpreted broadly to include human capital or any other nonprimary (i.e., produced) factors of production. (Throughout the paper, the adjective “physical” is used as opposed to “financial.”) Both physical capital and labor are nontradeable. Only the final good can be traded (intertemporally) between countries. The final good produced in period  $t$  may be consumed in period  $t$  or may be invested in the production of physical capital, which become available in period  $t+1$ . When physical capital is interpreted to include human capital, this technology may be interpreted as education.

The technology of the final goods sector satisfies standard, neoclassical properties. It is given by a linear homogeneous production function,  $Y_t = F(K_t, L_t)$ , where  $K_t$  and  $L_t$  are aggregate domestic supplies of physical capital and labor in period  $t$ . Let  $y_t \equiv Y_t/L_t = F(K_t/L_t, 1) \equiv f(k_t)$  where  $k_t \equiv K_t/L_t$  and  $f(k)$  is  $C^2$  and satisfies  $f'(k) > 0 > f''(k)$ ,  $f(0) = 0$ , and  $f''(0) = \infty$ . The factor markets are competitive, and the factor rewards for physical capital and for labor are equal to  $\rho_t = f'(k_t)$  and  $w_t = f(k_t) - k_t f'(k_t) \equiv W(k_t)$ , which are both paid in the final good. Note that  $f''(k) < 0$  implies that a higher  $k_t$  increases  $w_t$  and reduces  $\rho_t$ . For simplicity, physical capital is assumed to depreciate fully in one period.

There are overlapping generations of two-period lived agents. Each generation consists a continuum of homogenous agents with unit mass. Each agent supplies one unit of labor inelastically to the final goods sector only in the first period, and consumes only in the second.<sup>7</sup> Thus,  $L_t = 1$ , and the wage income,  $w_t$ , is also equal to the level of wealth held by the young



agents at the end of period  $t$ . They allocate their wealth,  $w_t$ , in order to finance their consumption in period  $t+1$ . They have two options. First, they may lend it in the competitive credit market, which earns the gross return equal to  $r_{t+1}$  per unit. If they lend the entire wealth, their second-period consumption is equal to  $r_{t+1}w_t$ . Second, they may become an entrepreneur and start a project. The project comes in discrete, nondivisible units and each young agent can manage only one project.<sup>8</sup> The project transforms one unit of the final good in period  $t$  into  $R > 0$  units of physical capital in period  $t+1$ . It is assumed that the investment project is not too productive so that

$$(A1) \quad W(R) < 1$$

or equivalently  $R \in (0, R^+)$ , where  $R^+$  is defined by  $W(R^+) = 1$ . As seen later, (A1) ensures that  $w_t < 1$ , so that the agent needs to borrow  $1 - w_t > 0$  in the competitive credit market, in order to start the project.<sup>9</sup> It is also assumed that the agent cannot start a project abroad (or it is prohibitively costly to do so). In other words, foreign direct investment is ruled out.<sup>10</sup> This assumption, as well as that of the nontradeability of physical capital, is imposed to focus on the effects of globalization via financial market integration, instead of globalization via factor market integration.

We are now ready to look at the investment decision. The second period consumption, if the agent starts the project, is equal to  $\rho_{t+1}R - r_{t+1}(1 - w_t)$ . This is greater than or equal to  $r_{t+1}w_t$  (the second period consumption if the agent lends the entire wage income) when the net present discounted value of the project,  $\rho_{t+1}R/r_{t+1} - 1$ , is nonnegative. This condition can be expressed as

$$(1) \quad Rf'(k_{t+1}) \geq r_{t+1}.$$

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<sup>7</sup>It is straightforward to allow the agent to work also in the second period. Such an extension may be appealing if we are to interpret physical capital broadly to include human capital.

<sup>8</sup>Note that, even though each agent faces an indivisible investment technology, aggregate technology is convex, because there is a continuum of agents in each country that invest in the same indivisible project.

<sup>9</sup>The purpose of (A1) is merely to avoid a taxonomical exposition.

The young agents are willing to borrow and to start the project, when (1) holds. We shall call (1) *the profitability constraint*.

The credit market is competitive in the sense that both lenders and borrowers take the equilibrium rate,  $r_{t+1}$ , given. It is not competitive, however, in the sense that one cannot borrow any amount at the equilibrium rate. The borrowing limit exists because of the enforcement problem: the payment is made only when it is the borrower's interest to do so. More specifically, after having borrowed  $1-w_t$ , and the project being completed, the entrepreneur would refuse to honor its payment obligation,  $r_{t+1}(1-w_t)$ , if it is greater than the cost of default, which is taken to be a fraction of the project revenue,  $\lambda\rho_{t+1}R$ .<sup>11</sup> Knowing this, the lender would allow the entrepreneur to borrow only up to  $\lambda\rho_{t+1}R/r_{t+1}$ . Thus, the agent can start the project only if  $1-w_t \leq \lambda\rho_{t+1}R/r_{t+1}$ , or

$$(2) \quad \lambda Rf'(k_{t+1}) \geq r_{t+1}(1-W(k_t)).$$

We shall call (2) *the borrowing constraint*.<sup>12</sup> The parameter,  $0 < \lambda \leq 1$ , can be naturally taken to be the degree of the efficiency of the credit market. Note that there is no default in equilibrium. It is the possibility of default that makes the credit market imperfect. It should also be noted that the same enforcement problem rules out the possibility that different agents may pool their wealth to overcome the borrowing constraint.

In order for the young agents in period  $t$  to start the project, both the profitability constraint (1) and the borrowing constraint (2) must be satisfied. In other words, they must be both willing and able to borrow. These constraints can be summarized as

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<sup>10</sup>This restriction is also reasonable if physical capital and the investment project are interpreted as human capital and education.

<sup>11</sup>A natural interpretation of the cost is that the creditor seizes a fraction  $\lambda$  of the project revenue in the event of default. One may also interpret that this fraction of the revenue will be dissipated in the borrower's attempt to default. This makes no difference, because the default does not occur in equilibrium.

<sup>12</sup>One may also call (2) *the collateral constraint*, because it can be rewritten as  $w_t \geq C_{t+1} \equiv 1 - \lambda\rho_{t+1}R/r_{t+1}$ , where  $C_{t+1}$  may be interpreted as the collateral requirement. However, this interpretation assumes a particular form of loan contract. What is essential here is that, because of the borrowing constraint, the agents must self-finance a fraction  $C_{t+1}$  of the investment, whether it is interpreted as the collateral or not.

$$(3) \quad R \geq R_t \equiv \begin{cases} (r_{t+1}/f'(k_{t+1}))(1-W(k_t))/\lambda & \text{if } k_t < K(\lambda), \\ r_{t+1}/f'(k_{t+1}) & \text{if } k_t \geq K(\lambda), \end{cases}$$

where  $R_t$  may be interpreted as the project productivity *required* in order for the project to be undertaken in period  $t$ , and  $K(\lambda)$  is defined implicitly by  $W(K(\lambda)) = 1-\lambda$ . Note that which of the two constraints is binding depends entirely on  $k_t$ . The borrowing constraint (2) is binding if  $k_t < K(\lambda)$ ; the profitability constraint (1) is binding if  $k_t > K(\lambda)$ . Thus, the investment is borrowing constrained only at the lower stage of economic development. The intuition should be clear. With a low level of the capital stock, the agents accumulate less wealth and hence must borrow more to finance the project. Note that the critical value of  $k$ ,  $K(\lambda)$ , is decreasing in  $\lambda$ , with  $K(+0) = R^+$ , and  $K(1) = 0$ . Thus, the more efficient the credit market becomes, the less important the borrowing constraint becomes, and if the credit market is perfect ( $\lambda = 1$ ), the borrowing constraint is never binding.

### 3. The Autarky Case.

In autarky, there is no possibility of intertemporal trade in the final good with the rest of the world, which precludes international lending and borrowing. Thus, the domestic investment (by the young) must be equal to the domestic saving (by the young) in equilibrium.<sup>13</sup> This condition is illustrated in Figure 1. The domestic saving is equal to  $W(k_t)$ , given by the vertical line. The domestic investment is equal to zero if  $R_t > R$ , and to one, if  $R_t < R$ . If  $k_t < R$ , (A1) implies that the equilibrium holds at the horizontal segment of the investment schedule, where  $R_t = R$ . In equilibrium, the aggregate investment is made equal to  $W(k_t)$ . Thus, the fraction of the young agents who become borrowers/entrepreneurs is equal to  $W(k_t)$ , while the rest,  $1-W(k_t)$ , become lenders. If  $k_t \geq K(\lambda)$ , the young agents are indifferent between borrowing and lending.

<sup>13</sup>The GNP accounting of a closed economy, of course, implies that the saving by all the residents is equal to the investment by all the residents, including not only the young but also the old. However, in this model, the old is never engaged in the investment activity and the old consumes all their income, so that their saving is zero. Hence, the equality of the saving and the investment by the young is indeed the equilibrium condition when the economy is in autarky. In what follows, we shall simply call the domestic saving and the domestic investment, without specifically mentioning "by the young."

When  $k_t < K(\lambda)$ , on the other hand, they strictly prefer borrowing to lending. Therefore, the equilibrium allocation necessarily involves credit rationing, where the fraction  $1-W(k_t)$  of the young agents are denied the credit. Those who are denied the credit cannot entice the potential lenders by raising the interest rate, because the lenders would know that the borrowers would default at a higher rate. (In the present model, credit rationing is an inevitable feature of the equilibrium whenever the borrowing constraint is binding. As will be explained in section 6, however, what is essential is the borrowing constraint, not credit rationing.<sup>14</sup>)

Therefore, regardless of  $k_t < K(\lambda)$  or  $k_t \geq K(\lambda)$ , the measure of the young agents who start the project is equal to  $W(k_t)$ . Since every one of them produces  $R$  units of capital in period  $t+1$ ,

$$(4) \quad k_{t+1} = RW(k_t).$$

Eq. (4) completely describes the dynamics of capital formation in autarky. Note that, if  $k_t < R$ ,  $k_{t+1} = RW(k_t) < RW(R) < R$ . Therefore,  $k_0 < R$  implies  $k_t < R$  and  $w_t = W(k_t) < 1$  for all  $t > 0$ , as has been assumed.

Notably, the dynamics of  $k$ , (4), is entirely independent of  $\lambda$ ; the credit market imperfection has no effect on the capital formation in the autarky case. This is because the domestic investment is determined entirely by the domestic saving. Any effect of the credit market imperfection is completely absorbed by the interest rate movements. From (3), (4), and  $R = R$ , the equilibrium interest rate is given by

$$(5) \quad r_{t+1} = \begin{cases} \lambda Rf'(RW(k_t))/(1-W(k_t)) < Rf'(RW(k_t)) & \text{if } k_t < K(\lambda), \\ Rf'(RW(k_t)) & \text{if } k_t \geq K(\lambda). \end{cases}$$

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<sup>14</sup>While some authors use the term, “credit-rationing,” whenever some credit limits exist, here it is used to describe the situation that the aggregate supply of credit falls short of the aggregate demand, so that some borrowers cannot borrow up to their credit limit. In other words, there is no credit rationing if every borrower can borrow up to its limit. In such a situation, their borrowing may be constraint by their wealth, which affects the credit limit, but not because they are credit-rationed. This is consistent with the following definition of credit rationing by Freixas and

Note that a greater imperfection in the credit market (a smaller  $\lambda$ ) manifests itself in the reduction of the interest rate.

Clearly, the result that the dynamics of capital formation in autarky is unaffected by the credit market imperfection is not a robust feature of the model. In particular, it critically depends on the fact that the aggregate supply of the credit is inelastic. Nevertheless, this feature of the model makes the autarky case a useful benchmark for examining the effects of financial market globalization in the presence of the imperfection. What is essential here is that the aggregate supply of the credit is less elastic in autarky than in an open economy.

The dynamics of capital formation in autarky, given by eq. (4), even though it is independent of  $\lambda$  and unaffected by credit markets imperfection, may still have multiple steady states. It is well-known (see, e.g. Azariadis 1993) that the overlapping generations model imposes less restrictions on the equilibrium dynamics than the infinitely-lived representative agent model. This is a nuisance that has nothing to do with the credit market imperfection. To avoid any unnecessary complications that arise from this feature of overlapping generations model, we impose the following assumption:

$$(A2) \quad W'(0) = \infty \text{ and } W''(k) < 0.$$

Many standard production functions imply (A2). For example, if  $y = f(k) = A(k)^\alpha$  with  $0 < \alpha < 1$ ,  $W(k) = (1-\alpha)A(k)^\alpha$ , which satisfies (A2).

Under (A2), for any  $R \in (0, R^+)$ , eq. (4) has the unique steady state,  $k^* = K^*(R) \in (0, R)$ , defined implicitly by  $k^* = RW(k^*)$ , and for  $k_0 \in (0, R)$ ,  $k_t$  converges monotonically to  $k^* = K^*(R)$ , as shown in Figure 2a. The function,  $K^*(R)$ , is increasing and satisfies  $K^*(0) = 0$  and  $K^*(R^+) = R^+$ . It is worth emphasizing that  $K^*(R)$ , the steady state level of  $k$ , is independent of  $\lambda$ , and  $K(\lambda)$ , the critical level of  $k$ , below which the borrowing constraint is binding, is independent of  $R$ . Therefore, the borrowing constraint may or may not be binding in the steady state.

To summarize,

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Rochet (1997, Ch.5), who attributed it to Baltensperger: “some borrower’s demand for credit is turned down, even if this borrower is willing to pay all the price and nonprice elements of the loan contract.”

Proposition 1. In autarky, the dynamics of  $k$  is given by  $k_{t+1} = RW(k_t)$ , which is independent of  $\lambda$ , and converges monotonically to the unique steady state,  $K^*(R)$ , where  $K^*(R)$  is increasing in  $R$  and satisfies  $K^*(0) = 0$  and  $K^*(R^+) = R^+$ . If  $K^*(R) < K(\lambda)$ , the borrowing constraint is binding in the steady state. If  $K^*(R) > K(\lambda)$ , the profitability constraint is binding in the steady state.

Figures 2a and Figure 2b illustrate Proposition 1. The downward-sloping curve in Figure 2b is given by  $K^*(R) = K(\lambda)$ , which connects  $(\lambda, R) = (0, R^+)$  and  $(\lambda, R) = (1, 0)$ . Below and left to this curve, the autarky steady state is borrowing-constrained.

#### 4. The Small Open Economy

The goal of this section is twofold. First, it examines the effect of financial market globalization on the capital formation of the small open economy. Second, it offers a preliminary step for the analysis of the world economy in the presence of the international financial market.

The agents in the small open economy are allowed to trade intertemporally the final good with the rest of the world at exogenously given prices. In other words, international lending and borrowing is allowed. The interest rate, the intertemporal price of the final good, is exogenously given in the international financial market and assumed to be invariant over time:  $r_{t+1} = r$ .

In what follows, we will focus on the case  $Rf'(R) < r$  for the ease of exposition.<sup>15</sup> Then, the equilibrium condition is given by setting  $R_t = R$  in (3), which can be further rewritten as

$$(6) \quad k_{t+1} = \Psi(k_t) \equiv \begin{cases} \Phi(r(1-W(k_t))/\lambda R) & \text{if } k_t < K(\lambda), \\ \Phi(r/R) & \text{if } k_t \geq K(\lambda), \end{cases}$$

where  $\Phi$  is the inverse of  $f'$ , which is a decreasing function and satisfies  $\Phi(\infty) = 0$ .

<sup>15</sup>If  $Rf'(R) \geq r$ , the dynamics is given by  $k_{t+1} = \min\{R, \Psi(k_t)\}$ , where  $\Psi(k_t)$  is defined as in eq. (6). Assuming  $Rf'(R) < r$  ensures  $k_{t+1} = \Psi(k_t) < R$ , and hence the equilibrium is never at the corner. This restriction helps to reduce the notational burden significantly, but the result can be easily extended to the case where  $Rf'(R) \geq r$  as

Eq. (6) governs the dynamics of the small open economy. Unlike in the autarky case, the domestic investment is no longer equal to the domestic saving. Instead, the investment is determined entirely by the profitability and borrowing constraints. If the credit market were perfect ( $\lambda = 1$  and  $K(1) = 0$ ), the economy would immediately jumps to  $\Phi(r/R)$ , from any initial condition. In the presence of the imperfection, this occurs only when the economy is at the higher level of development ( $k_t \geq K(\lambda)$ ), where the profitability of the project is the only binding constraint. At the lower level of development ( $k_t < K(\lambda)$ ), the borrowing constraint is binding, which creates the gap between the return to investment and the interest rate. In this range, the map is increasing in  $k_t$ . This is because, with a higher capital stock, the young agents earn a high wage income, accumulate more wealth, which alleviates the borrowing constraint, so that more investment takes place. This effect is essentially the same with the credit multiplier effect identified by Bernanke and Gertler (1989) and others. In this range, the map is also increasing in  $\lambda R/r$ . In particular, a reduction in  $\lambda$  reduces  $k_{t+1}$ . In a small open economy, the interest rate is fixed in the international financial market. Therefore, a greater imperfection has the effect of reducing the domestic investment (and channeling more of the domestic saving into investment abroad). This differs significantly from the autarky case, where the domestic investment was determined by the domestic saving, and a reduction in  $\lambda$  reduces  $r_{t+1}$ , but has no effect on  $k_{t+1}$ .

The steady states of the small open economy are given by the fixed points of the map (6), satisfying  $k = \Psi(k)$ . The following lemma summarizes some properties of the set of the fixed points. While elementary, they turn out to be quite useful, and will be evoked repeatedly in the subsequent discussion.

Lemma.

- a) Eq. (6) has at least one steady state.
- b) Eq. (6) has at most one steady state above  $K(\lambda)$ . If it exists, it is stable and equal to  $\Phi(r/R)$ .

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well. This restriction can alternatively be justified on the ground that, in the world economy version of the model developed later, the world interest rate prevailing in any steady state satisfies  $Rf'(R) < r$ .

c) Eq. (6) has at most two steady states below  $K(\lambda)$ . If there is only one,  $k_L$ , either it satisfies  $0 < k_L < \lambda R/r$  and is stable, or,  $k_L = \lambda R/r$  at which  $\Psi$  is tangent to the  $45^\circ$  line. If there are two,  $k_L$  and  $k_M$ , they satisfy  $0 < k_L < \lambda R/r < k_M < K(\lambda)$ , and  $k_L$  is stable and  $k_M$  is unstable.

Proof. See Appendix.

One immediate implication of Lemma is that there are only three generic cases of the dynamics generated by (6). They are illustrated in Figures 3a-3c. In Figure 3a, the unique fixed point,  $k_L$ , is located below  $K(\lambda)$ , to which  $k_t$  converges from any  $k_0 \in (0, R)$ . In Figure 3c, the unique fixed point,  $k_H = \Phi(r/R)$ , is located above  $K(\lambda)$ , to which  $k_t$  converges from any  $k_0 \in (0, R)$ . In Figure 3b, there are three fixed points; two stable steady states,  $k_L$  and  $k_H$ , are separated by the third (unstable) steady state,  $k_M$ , which is located between  $k_L$  and  $K(\lambda)$ , and  $k_t$  converges to  $k_L$  if  $k_0 < k_M$  and to  $k_H$  if  $k_0 > k_M$ .<sup>16</sup>

The following proposition provides the exact condition for each of the three cases.

Proposition 2. Let  $\lambda_c \in (0, 1)$  be defined by  $f(K(\lambda_c)) = 1$ . Then,

- a) If  $Rf'(K(\lambda)) < r$ , there exists a unique steady state,  $k_L$ . It is stable and satisfies  $k_L < K(\lambda)$ .
- b) If  $Rf'(K(\lambda)) > r$ ,  $f(\lambda R/r) < 1$ , and  $\lambda < \lambda_c$ , there exist three steady states,  $k_L$ ,  $k_M$ , and  $k_H$ . They satisfy  $k_L < k_M < K(\lambda) < k_H$ , and  $k_L$  and  $k_H$  are stable and  $k_M$  is unstable.
- c) If  $Rf'(K(\lambda)) > r$  and either  $f(\lambda R/r) > 1$  or  $\lambda > \lambda_c$ , there exists a unique steady state,  $k_H$ . It is stable and satisfies  $k_H > K(\lambda)$ .

Proof. See Appendix.

Proposition 2 is illustrated by Figure 4. The conditions for Proposition 2a), 2b) and 2c) are satisfied in Region A, B, and C, respectively. The outer limit of Region A is given by  $Rf'(K(\lambda)) = r$ , and the border between Regions B and C are given by  $f(\lambda R/r) = 1$ . These two downward-sloping curves meet tangentially at  $\lambda = \lambda_c$ .

<sup>16</sup>Figures 3a-3c are drawn so that  $\Psi'' > 0$  for  $k < K(\lambda)$ . This may or may not be true. Note that Lemma c) does not say that the map is convex in this range. It says that it cannot intersect the  $45^\circ$  line more than twice in this range.



Proposition 2 states that the dynamics of capital formation in the small open economy differ drastically from the autarky case. The difference is most significant when the world interest rate is such that the parameters lie in Region B, as illustrated by point P in Figure 4. In this case, an integration of this economy to the international financial market creates multiple steady states, as shown in Figure 3b. Around  $k_M$ , the investment is borrowing constrained, and the dynamics is unstable. If the integration occurs slightly below  $k_M$ , the economy experiences vicious circles of low-wealth/low-investment, and will gravitate toward the lower stable steady state,  $k_L$ , in which the borrowing constraint is binding. On the other hand, if the integration takes place slightly above  $k_M$ , the economy experiences virtuous circles of high-wealth/high-investment, and eventually converges to the higher stable steady state,  $k_H$ , in which the borrowing constraint is no longer binding. This case thus suggests that the timing of the integration has significant permanent effects on the capital formation.

This does not mean, however, that the integration would have negligible effects on the capital formation in other cases. For example, suppose that the world interest rate is such that the parameters lie in Region C. In this case, the economy will eventually converge to the unique steady state, in which the borrowing constraint is not binding. The convergence could take long time, however, because the economy must go through the “narrow corridor” between the map and the  $45^\circ$  line, as illustrated in Figure 3c.

More generally, a comparison between the shapes of the two maps,  $k_{t+1} = RW(k_t)$  for the autarky case and  $k_{t+1} = \Psi(k_t)$  for the small open economy case, suggests that the integration have the effect of slowing down the growth process of middle-income economies.

Let us now consider the effect of a change in the world interest rate on the capital formation of the small open economy. We focus on the case, where the parameters lie in Region B, depicted by P in Figure 4, and the dynamics is hence illustrated by Figure 3b. Suppose that the economy is trapped in  $k_L$ . A decline in the world interest rate, illustrated in Figure 4 as the vertical move from point P in Region B to point P' in Region C eliminates  $k_L$  and the dynamics is now illustrated by Figure 3c. The decline in the interest rate thus helps the economy to escape from the trap and to start a (perhaps long and slow) process of growth toward  $k_H$ . Furthermore, even a temporary decline in the interest rate could have similar long run effects. Once the

economy accumulates enough capital, the economy will not fall back to the trap, when the interest rate returns to the original level. Therefore, even a small, temporary decline in the interest rate could have a significant permanent effect.<sup>17</sup> Similarly, one could show that even a small, temporary rise in the world interest rate could lead to a permanent stagnation of the economy, if it is initially located at  $k_H$  in Figure 3b.

One might be tempted to argue that Region B of Figure 4, which gives rise to the dynamics illustrated in Figure 3b with multiple stable steady states, can be used to explain the divergence of economic performance across the countries. Imagine that there are two small open countries, called N and S, which share the same technology, the same demographic structure, etc. Furthermore, both countries are fully integrated into the international financial market and face the same world interest rate. The only difference is that the capital stock in N is equal to  $k_H$  and the capital stock in S is equal to  $k_L$ . The model does explain why this situation can persist, because both  $k_H$  and  $k_L$  are stable steady states of the dynamics, if the parameters lie in Region B of Figure 4.

While suggestive, this argument explains only *the possibility* that we may not observe the convergence of the two otherwise identical countries, but does not predict the *inevitability* of the divergence. This is because the model also allows for the possibility of convergence. Indeed, the situation in which the capital stocks are both equal to  $k_H$  in N and S and the situation in which they are both equal to  $k_L$  in N and S (as well as the situation in which it is equal to  $k_H$  in S and  $k_L$  in N) are also stable steady states under the same condition. The argument does not offer any reason why one should believe that the divergence is a more plausible outcome than the convergence. In other words, the small open economy version of the model cannot impose any restriction on the degree of inequality that might be observed in the world economy. It therefore fails to *predict* the divergence, or the empirical finding reported in Quah (1993, 1997) that the distribution of the per capita income tends to converge to a bimodal, or “twin-peaked,” distribution in the long run. The small open economy version of the model imposes no restriction

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<sup>17</sup>Of course, how small the decline can be in order to have the permanent effect depends on the distance between point P and the border between Regions B and Region C. Furthermore, the larger the decline, the shorter it can be to have the permanent effect.

on the cross-country difference because it takes into account no interaction between the dynamics of different countries.<sup>18</sup>

To resolve this problem, therefore, one must move beyond the small open economy framework, and analyze the model from a global perspective. In the next section, the world economy version of the model is analyzed. This helps not only to endogenize the world interest rate, but also to address the issue of divergence versus convergence in a more satisfactory manner.

Analyzing the model from a global perspective is also important for the policy analysis. From the prospective of an individual country, escaping from the poverty trap may appear simple. One might be tempted to argue that the poor countries should temporarily cut their financial links or that foreign aids from the rich countries should solve the problem. The global perspective will show, however, why these measures may not be able to eliminate the poverty trap.

## 5. The World Economy

In the world economy version of the model, there is a continuum of inherently identical countries with unit mass. In the absence of the international financial market, this is merely a collection of the autarky economies analyzed in section 3. Hence one can immediately conclude that the world economy would converge to the symmetric steady state, in which each country holds  $K^*(R)$  units of the capital stock.

In what follows, let us assume that all the countries are fully integrated in the international financial market, where each country faces the same interest rate. The world economy can hence be viewed as a collection of inherently identical small open economies of the type analyzed in section 4. Since the world as a whole is a closed economy, the interest rate is now endogenously determined to equate the world saving and the world investment.

The presence of the international financial market does not change the fact that the state in which every country has the capital stock equal to  $K^*(R)$  is a steady state. However, it may change the stability property of the symmetric steady state, in which case the world economy

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<sup>18</sup>This drawback is not limited to the use of small open economy models with multiple steady states. Any attempt to explain the divergence by using closed economy models with multiple steady states, like those in Azariadis and Drazen (1990), Ciccone and Matsuyama (1996) and others, may be criticized on the same ground.

cannot be expected to converge to it in the long run. Furthermore, it may create other steady states. We need to characterize the *entire* set of *stable* steady states of the world economy.

In any stable steady state of the world economy, each country must be at a stable steady state of the small open economy. As stated in Lemma, there are at most two stable steady states in which each small open economy can be located. This means that a stable steady state of the world economy must be one of the following two types. The first type is the case of perfect equality, or the case of convergence. In such a steady state, all the countries have the same level of capital,  $k^*$ . The second type is the case of endogenous inequality, or the case of divergence. In such a steady state, the world economy is polarized into the rich and the poor, in which the poor (rich) countries have the same level of capital stock, given by  $k_L$  ( $k_H$ ), which satisfies  $k_L < K(\lambda) < k_H$ . The next two subsections derive the condition for the existence of these two types of stable steady states. (The reader not interested in the derivation may want to skim through these subsections and move onto Section 5.3., at least on the first reading.)

### 5.1. The Steady State with Equality of Nations: The Case of Convergence.

Suppose that all the countries have the same level of capital stock,  $k^*$ , in a steady state. Then, the world saving is equal to  $W(k^*)$ . Since the world economy as a whole is closed, the measure of the young agents that invest in this steady state must be equal to  $W(k^*)$ . Since every one of them produces  $R$  units of capital, the steady state capital must satisfy  $k^* = RW(k^*)$ , or equivalently,  $k^* = K^*(R)$ . If  $k^* = K^*(R) > K(\lambda)$ , the borrowing constraint is not binding, hence the world interest rate in this steady state is  $r = Rf'(K^*(R)) < Rf'(K(\lambda))$ . This inequality can be rewritten as  $\Phi(r/R) > K(\lambda)$ , which is exactly the condition under which a small open economy has a stable steady state,  $k_H = \Phi(r/R) = K^*(R) = k^*$ . (See also Proposition 2b)-2c.) This proves that  $K^*(R) > K(\lambda)$  is the condition under which there exists a stable steady state in which all the countries have the same level of capital stock,  $k^* = K^*(R) > K(\lambda)$ .

If  $k^* = K^*(R) < K(\lambda)$ , the borrowing constraint is binding, hence the world interest rate in this steady state is  $r = \lambda Rf'(K^*(R))/[1 - W(K^*(R))]$ . From c) of Lemma,  $k^* = K^*(R) < K(\lambda)$  is a stable steady state for each small open economy, if and only if it satisfies  $k^* = K^*(R) < \lambda R/r =$

$[1-W(K^*(R))]/f'(K^*(R))$ . This condition can be rewritten to  $K^*(R)f'(K^*(R)) + W(K^*(R)) = f(K^*(R)) < 1$ . This proves that  $K^*(R) < K(\lambda)$  and  $f(K^*(R)) < 1$  are the condition under which there exists a stable steady state in which all the countries have the same level of capital stock,  $k^* = K^*(R) < K(\lambda)$ .

The above argument also shows that, if  $K^*(R) < K(\lambda)$  and  $f(K^*(R)) > 1$ , a symmetric steady state, in which all the countries have the same level of capital stock, is unstable. To see this, in such a steady state, the capital stock in each country must be equal to  $k^* = K^*(R) < K(\lambda)$ , which means that the borrowing constraint is binding. Therefore, the world interest rate is equal to  $r = \lambda R f'(K^*(R)) / [1 - W(K^*(R))]$ . When  $f(K^*(R)) > 1$ , this implies  $k^* = K^*(R) > \lambda R / r$ , which means that  $k^* = k_M$  from Lemma c). Thus, it is unstable. Figure 5 illustrates this situation. Suppose that there is no international financial market at the beginning. Then, the dynamics of every country follows  $k_{t+1} = RW(k_t)$ , which converges to  $K^*(R)$ . In this steady state, the interest rates are equal across countries, even though there is no international lending and borrowing. If the international financial market is open at this point, the dynamics of each country is now governed by  $k_{t+1} = \Psi(k_t)$ , which cut the 45° line from below at  $K^*(R)$ . This situation is unstable, even though it is still a steady state.

To summarize the above,

**Proposition 3.** Let  $R_c \in (0, R^+)$  be defined by  $f(K^*(R_c)) = 1$ . Then,

- a) If  $K^*(R) < K(\lambda)$  and  $R < R_c$ , the state in which all the countries have  $k^* = K^*(R)$ , is a stable steady state of the world economy.
- b) If  $K^*(R) < K(\lambda)$  and  $R > R_c$ , there exists no stable steady state in which all the countries have the same level of capital stock.
- c) If  $K^*(R) > K(\lambda)$ , the state in which all the countries have  $k^* = K^*(R)$ , is a stable steady state of the world economy.

Note  $R_c$  satisfies  $K^*(R_c) = K(\lambda_c)$ ; it is well-defined in  $(0, R^+)$ , since  $f(K^*(0)) = 0 < 1 = W(R^+) < f(K^*(R^+))$  and  $f(K^*(R))$  is strictly increasing and continuous in  $R$ .

Figure 6 illustrates the conditions in Proposition 3. In Regions A and AB, the condition in Proposition 3a) is satisfied. In Region B, the condition in Proposition 3b) is satisfied. In Regions BC and C, the condition in Proposition 3c) is satisfied. The border between Regions AB and B is given by  $f(K^*(R)) = 1$ , or  $R = R_c$ . The border between Regions B and BC (as well as the border between A and C) is given by  $K^*(R) = K(\lambda)$ . Note that, when the credit market imperfection is significant ( $\lambda < \lambda_c$ ), the stability of the symmetric steady state requires that the productivity of the investment project,  $R$ , is either sufficiently high or sufficiently low. For an intermediate range of  $R$ , the condition in Proposition 3b) holds and the symmetric steady state is unstable.

## 5.2. Steady States with Endogenous Inequality of Nations: The Case of Divergence.

Suppose now that the world economy is a stable steady state, in which a fraction  $X$  of the countries have the capital stock equal to  $k_L < K(\lambda)$ , and a fraction  $1-X$  of the countries have the capital stock equal to  $k_H > K(\lambda)$ . Since all the countries face the same world interest rate,  $k_L$  and  $k_H$  must satisfy  $Rf'(k_H) = r = \lambda Rf'(k_L)/(1-W(k_L))$ , or

$$(7) \quad f'(k_H) = \lambda f'(k_L)/(1-W(k_L)),$$

in addition to

$$(8) \quad k_L < K(\lambda) < k_H.$$

From Lemma b),  $k_t = k_H$  is a stable steady state for each small open economy. From Lemma c), the stability of  $k_t = k_L$  requires  $k_L < \lambda R/r = [1-W(k_L)]/f'(k_L)$ , which can be rewritten to  $k_L f'(k_L) + W(k_L) = f(k_L) < 1$ , or

$$(9) \quad k_L < K^*(R_c) = K(\lambda_c).$$

Since the young agents in the fraction  $X$  of the countries earn  $W(k_L)$  and those in the fraction  $1-X$  earn  $W(k_H)$ , the world saving is given by  $XW(k_L) + (1-X)W(k_H)$ , which is equal to

the world investment, which produces  $R$  units of capital per unit. Hence, the total capital stock must satisfy

$$(10) \quad Xk_L + (1-X)k_H = XRW(k_L) + (1-X)RW(k_H).$$

A stable steady state with endogenous inequality exists if there are  $k_L$  and  $k_H$  that solve (7)-(10).

**Proposition 4.** Let  $R_c \in (0, R^+)$  and  $\lambda_c \in (0, 1)$  be defined by  $f(K^*(R_c)) = f(K(\lambda_c)) = 1$ . The world economy has a continuum of stable steady states, in which a fraction  $X \in (X^-, X^+) \subset (0, 1)$  of the countries have the capital stock,  $k_L < K(\lambda)$ , and a fraction  $1 - X$  of the countries have the capital stock equal to  $k_H > K(\lambda)$ , if and only if  $\lambda < \lambda_c$ ,  $f'(K(\lambda)) > \lambda f'(K^*(R)) / [1 - W(K^*(R))]$  where  $R < R_c$ , and  $\lambda < f'(K^*(R))K(\lambda_c)$ . Furthermore,  $X^- > 0$  if  $R > R_c$  and  $X^+ < 1$  if  $K^*(R) < K(\lambda)$ .

Proof. See Appendix.

The condition of Proposition 4 is satisfied in Regions AB, B, and BC of Figure 6. The border between A and AB is given by  $f'(K(\lambda)) = \lambda f'(K^*(R)) / [1 - W(K^*(R))]$  with  $R < R_c$  and  $\lambda < \lambda_c$ . It is upward-sloping and connecting  $(\lambda, R) = (0, 0)$  and  $(\lambda, R) = (\lambda_c, R_c)$ . The border between BC and C is given by  $f'(K^*(R))K(\lambda_c) = \lambda$ . This curve is downward-sloping, and stays above  $K^*(R) = K(\lambda)$  for  $\lambda < \lambda_c$ , and tangent to it at  $(\lambda, R) = (\lambda_c, R_c)$ .<sup>19</sup> Note that the existence of these asymmetric steady states requires that the credit market imperfection is significant ( $\lambda < \lambda_c$ ), and that the productivity of the investment project,  $R$ , is not too low.<sup>20</sup>

### 5.3. The Effects of Financial Market Globalization: Discussion.

<sup>19</sup>To see this, let  $\Theta(\lambda) \equiv f'(K(\lambda))K(\lambda_c) - \lambda$ . Then,  $\Theta(\lambda_c) = f'(K(\lambda_c))K(\lambda_c) - \lambda_c = f'(K(\lambda_c))K(\lambda_c) - f(K(\lambda_c)) + (1 - \lambda_c) = (1 - \lambda_c) - W(K(\lambda_c)) = 0$ , and  $\Theta'(\lambda) \equiv f'(K(\lambda))K(\lambda_c)K'(\lambda) - 1 = K(\lambda_c)/K(\lambda) - 1 < 0$  for  $\lambda < \lambda_c$ , since  $K'(\lambda) = 1/f'(K(\lambda))K(\lambda)$  by differentiating  $W(K(\lambda)) = 1 - \lambda$ . Therefore,  $\Theta(\lambda) > \Theta(\lambda_c) = 0$  for  $\lambda < \lambda_c$ . Thus,  $\lambda = f'(K^*(R))K(\lambda_c)$  implies  $f'(K(\lambda))K(\lambda_c) > \lambda = f'(K^*(R))K(\lambda_c)$  or  $K^*(R) > K(\lambda)$  for  $\lambda < \lambda_c$ . The tangency follows from  $\Theta'(\lambda_c) = 0$ .

<sup>20</sup>In fact, these asymmetric steady states disappear not only when  $R$  is sufficiently low, but also when it is sufficiently high, if we drop (A1) and allow  $R$  to be greater than  $R^+$ .

Having characterized all the stable steady states, we are now ready to discuss the effects of financial market globalization. In Regions A and C of Figure 6, there is a unique stable steady state, which is symmetric. In both cases, the model predicts the convergence of economic performances across countries. In Region A, the investment is borrowing-constrained in each country. In Region C, the borrowing constraint is not binding in any country. In Region B, there is no stable steady state with perfect equality. Even though there is a continuum of stable steady states, they all show that the long-run distribution of the capital stock, and hence those of the income, the wage, etc, have two mass points. In Region B, the model predicts the divergence; the co-existence of rich and poor nations is an inevitable feature of the world economy. In Region AB, and Region BC, these two types of the steady states co-exist.

The prediction of the model is most stark when the parameters lie in Region B of Figure 6. See also Figure 5. In this case,  $K^*(R) < K(\lambda)$  so that, in the absence of the international financial market, each country is in autarky and will converge to the same steady state, in which the borrowing constraint is binding. Despite that each country is borrowing-constrained, this symmetric steady state is stable. This is because the interest rates can adjust independently across countries, when different countries are hit by different shocks. In the presence of the international financial market, however, the symmetric steady state loses its stability. This is because unrestricted flows of financial capital forces the interest rates in different countries to move together. In other words, all the agents must compete for the world saving in the international financial market. This put the agents living in countries hit by worse shocks in disadvantage, compared to those living in countries hit by better shocks. This leads to financial capital flows from the unlucky countries to the lucky ones. Then, this creates vicious circles of low-investment/low-wealth in the former and virtuous circles of high-investment/high wealth in the latter. This process continues until the world is polarized into the rich and the poor. In any stable steady state, the rich accumulate enough capital that the borrowing constraint is no longer binding, while it is binding for the poor ( $k_L < K(\lambda) < k_H$ ). One can also show that, from (A2) and (10),  $k_L < K^*(R) < k_H$  in these steady states. That is to say, the rich countries become richer and the poor become poorer than in autarky. Furthermore, the world output in these steady states is



strictly lower than in the symmetric steady state.<sup>21</sup> Therefore, this case offers a theoretical justification for the view that the international financial market is a mechanism through which rich countries become richer at the expense of poor countries and at the expense of the world economy as a whole.

When the world economy is polarized, the countries that became poor find themselves in the stable steady state with the binding borrowing constraint,  $k_L$  in Figure 3b. From a perspective of an individual country, the problems of poor countries may seem easy to solve. It may appear that, in order to escape the poverty trap and to join the club of rich countries, all the government has to do is to cut its link to the international financial market temporarily. The global perspective, however, offers a different view. Such temporary isolationist policy cannot work when attempted by all the countries. This is because, once the restriction is removed, a positive measure of countries must find themselves in the lower steady state. (Note that, in Region B, a fraction of the countries that become poor is bounded away from zero.) Similar points can be made for a joint attempt for the poor countries to cut their links to the rich countries and to unite among themselves. It is impossible for all of them to escape from the poverty trap. Nor would a one-time redistribution from the rich countries eliminate inequality. This is because  $K^*(R) < K(\lambda)$  in Region B. That is, one of the reasons why the symmetric steady state is unstable is that there is not enough saving in the world economy to finance the investment required to make all the countries rich. As long as the parameters lie in Region B of Figure 6, some countries must be excluded from being rich, just as in a game of musical chairs.<sup>22</sup>

Let us now consider the following thought experiment, which arguably traces the evolution of the world economy. Suppose  $\lambda < \lambda_c$  and  $R$  is sufficient small so that the parameters lie in Region A. Then, let  $R$  increase gradually. Imagine that this exogenous technological progress is sufficiently slow (or the convergence to a stable steady state is sufficiently fast) that

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<sup>21</sup> To see this, consider the problem of maximizing the steady state world output subject to the steady state resource constraint: Maximize  $\int_0^1 f(k(z))dz$ , s.t.  $\int_0^1 k(z)dz \leq \int_0^1 RW(k(z))dz$ , where  $k(z)$  is the capital stock in country  $z \in [0,1]$ . Since the feasibility set is convex and the objective function is symmetric and strictly quasi-concave, the solution is  $k(z) = k^*$  for all  $z \in [0,1]$ , where  $k^*$  satisfies  $k^* = RW(k^*)$ . That is, the world output is maximized when  $k(z) = K^*(R)$  for all  $z \in [0,1]$ .

one could approximate the state of the world economy by a stable steady state. Initially, the world economy is in A, so that all the countries are equally poor and the borrowing constraint is binding in each country. Even when an increase in  $R$  pushes the world economy in Region AB, this situation does not change, because the symmetric steady state remains stable. This changes when a further increase in  $R$  makes  $R > R_c$  and the world economy enters Region B. Then, divergence begins. Some, but not all, countries start growing rapidly. These countries become sufficiently rich and that the borrowing constraint is no longer binding. The rest of the world is left behind. As  $R$  continues to rise, more and more countries start growing, and once  $R$  becomes big enough to push the world economy in Region C, then the convergence occurs.<sup>23</sup>

## 6. Heterogeneous Agents

In the models presented above, the agents are assumed to be homogeneous. This assumption, while simplifying the analysis significantly, implies that the agents are equally willing and equally credit-worthy as an entrepreneur. This means that the saving and the investment can be equalized in the autarky case only by means of credit-rationing, when the borrowing constraint is binding. This section briefly sketches a model with heterogeneous agents, and demonstrates that, even though equilibrium credit rationing does not occur, much of the results obtained above carry over. This should help to convince the reader that what matters in the analysis is the borrowing constraint, not the presence of equilibrium credit rationing.

Let us assume that the agents are heterogeneous in terms of their productivity as an entrepreneur. More specifically,  $R$  is now an agent-specific, and its cumulative distribution is given by  $G(R)$ , without any mass point, with the density function,  $g(R) = G'(R) > 0$ . In period  $t$ , only the young agents whose productivity satisfies  $R \geq R_t$  are willing to borrow and credit-worthy. If  $k_t \geq K(\lambda)$  so that  $R_t = r_{t+1}/f'(k_{t+1})$ , the agents with  $R < R_t$  are not willing to start the projects, because they are not profitable. If  $k_t < K(\lambda)$  so that  $R_t = r_{t+1}(1-W(k_t))/\lambda f'(k_{t+1})$ , the agents with  $R < R_t$  want to borrow but they are not denied credit, because they are not as credit-

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<sup>22</sup>When the parameters lie in Region BC and the world economy is in the polarized steady state, a one-time redistribution from the rich to the poor can eliminate inequality and move the world economy into the symmetric steady state.

<sup>23</sup>If (A1) is dropped and  $R$  is allowed to be greater than  $R^+$ , then it can be shown that, for any  $\lambda < \lambda_c$ , a sufficiently large  $R$  pushes the world economy in C.

worthy as the agents with  $R \geq R_t$ . Thus, the domestic investment in period  $t$  is equal to  $1-G(R_t)$ , which is a well-defined function, and decreasing in  $R_t$ . The capital stock in period  $t+1$  is now given by

$$(11) \quad k_{t+1} = \int_{R_t}^{\infty} Rg(R)dR \equiv H(R_t),$$

where  $H$  is decreasing in  $R_t$  with  $H'(R_t) = -R_t g(R_t) < 0$ .

In the autarky case, the domestic investment is equal to the domestic saving:

$$(12) \quad W(k_t) = 1 - G(R_t)$$

Since the RHS is a well-defined decreasing function in  $R_t$ , eq. (12) determines  $R_t$  uniquely as a decreasing function of  $k_t$ . Since  $R_t$  adjusts to ensure the saving-investment balances, there is no equilibrium credit rationing. The dynamics is described entirely by (11) and (12), or

$$(13) \quad k_{t+1} = H(G^{-1}(1-W(k_t))) \equiv \Lambda(k_t)$$

which is independent of  $\lambda$ . When  $k_t < K(\lambda)$ , a greater credit market imperfection reduces the interest rate, but not the dynamics of capital formation. Some algebra verifies  $\Lambda'(k_t) = R_t W'(k_t) = G^{-1}(1-W(k_t))W'(k_t) > 0$ , and that (A2) ensures that  $\Lambda'(0) = \infty$  and  $\Lambda''(k_t) = R_t W''(k_t) - (W'(k_t))^2/g(R_t) < 0$ . Therefore, for any distribution  $G$ ,  $k_t$  converges to the unique steady state,  $K^*(G) > 0$ . In the steady state, the borrowing constraint is binding if and only if  $K^*(G) < K(\lambda)$ .

In the small open economy, eq. (3) with  $r_{t+1} = r$  and (11) yield

$$H^{-1}(k_{t+1})f'(k_{t+1}) = \begin{cases} r(1-W(k_t))/\lambda, & \text{if } k_t < K(\lambda), \\ r & \text{if } k_t \geq K(\lambda), \end{cases}$$

where the LHS is strictly decreasing in  $k_{t+1}$ . By denoting the inverse function of  $H^{-1}(k)f'(k)$  by  $\Omega$ , the dynamics of the small open economy is described by

$$(14) \quad k_{t+1} = \begin{cases} \Omega(r(1-W(k_t))/\lambda) & \text{if } k_t < K(\lambda), \\ \Omega(r) & \text{if } k_t \geq K(\lambda), \end{cases}$$

which defines a map from  $k_t$  to  $k_{t+1}$ , which is continuous, increasing in  $k_t < K(\lambda)$ , and constant in  $k_t \geq K(\lambda)$ . A greater credit market imperfection reduces the rate of capital formation, when  $k_t < K(\lambda)$ , but not when  $k_t > K(\lambda)$ . For a sufficiently high  $r$  or a sufficiently low  $\lambda$ ,  $\Omega(r) < K(\lambda)$  holds, and there are, generically speaking,  $m$  stable and  $m - 1$  unstable steady states ( $m = 1, 2, \dots$ ), all of which are located below  $K(\lambda)$ . If  $\Omega(r) > K(\lambda)$ , there is one and only stable steady state above  $K(\lambda)$ , in addition to the same number of stable and unstable steady states below  $K(\lambda)$ .

Qualitatively, (14) differs from (6) only in that there may be additional pairs of stable and unstable steady states below  $K(\lambda)$ . Thus, (14) is capable of generating any qualitative feature of the dynamics generated by (6), which is nothing but a special case of (14).

A characterization of the steady states in the world economy case is hopelessly complicated. This is only because that there may be more than two stable steady states of the small open economy, which dramatically increases the number of the types of the steady states of the world economy. If the existence of  $m$  stable steady states of the small open economy cannot be ruled out,  $2^m - 1$  types of the steady states of the world economy need to be distinguished. It should be obvious, however, that a sufficiently small heterogeneity, which makes  $H^{-1}(k)$  almost constant, would not change the nature of the model. This can be verified, for example, by letting  $G$  the uniform distribution with the support  $[R-\delta, R+\delta]$  and  $\delta \rightarrow 0$ .

## 7. Related Work in the Literature.

Starting with Bernanke and Gertler (1989), many recent studies have examined the implications of imperfect credit markets on the aggregate investment behavior. The critical feature

of the present model, --an increase in the entrepreneur's wealth eases the borrowing constraint--, is common in this literature. Many of these studies assume the presence of an alternative storage technology that helps to pin down the interest rate, which makes their models effectively partial equilibrium ones, as in the small open economy case above. By making the aggregate supply of the credit infinitely elastic, this assumption helps to generate the credit multiplier effect in these models. What is crucial in this paper is that the extent to which the interest rate can adjust endogenously changes with financial market globalization.

Most of these studies introduce imperfect credit markets through moral hazard, adverse selection, and costly state verification models. Among macroeconomic studies that introduce the imperfect credit markets through the threat of potential defaults are Kiyotaki and Moore (1997) and Obstfeld and Rogoff (1996, Ch.6.1 and Ch.6.2). The specification here follows Matsuyama (2000). The main advantage of using potential defaults as a source of imperfection is its simplicity and tractability.

A large number of recent studies examine imperfect credit markets in an open economy context. They mostly focus on the issue of short-run volatility, motivated by recent economic crises in emerging markets. Only a few studies have addressed the role of the international financial market on the cross-country pattern of development in the presence of imperfect credit markets. The seminal work is Gertler and Rogoff (1990). In their static model, the country's wealth is given by an exogenous endowment. They examined how the distribution of the endowment across countries affects the investment and financial capital flows in the presence of imperfect credit markets, but, due to the static nature of the model, there is no feedback effect from the investment to the distribution. Furthermore, in the simple version of their model, presented in Obstfeld and Rogoff (1996, Ch.6.4), the credit market imperfection only reduces the flow of financial capital from the rich to the poor, but does not generate a reverse flow. This can happen in their model only when the poor country initially has a sufficiently high external debt.

Boyd and Smith (1997) succeeded in eliminating these limitations of the Gertler-Rogoff model. They developed an overlapping generations model of a two-country world economy, with the imperfect credit market arising from a costly-state-verification problem. However, their model is so complicated that they had to assume that the borrowing constraint is always binding

for both countries, both in and out of the steady states, and even then, they had to rely on the numerical simulation to prove the stability of asymmetric steady states. They also restricted their parameters in such a way the symmetric steady state is always unstable. The model presented in this paper has advantage of being tractable, which makes it possible to characterize all the stable steady states for the full set of the parameter values, without making any auxiliary assumption. In other words, the present model allows one to derive the analytical conditions for the stability of the symmetric and of the asymmetric steady states and for the borrowing constraint to be binding in these steady states. This in turn makes it possible to examine the effects of changing the parameter values, making the model useful as an intuition-building device on the issue of convergence versus divergence. Furthermore, the analysis have shown that the rich are not borrowing-constrained and the poor remain borrowing-constrained in all asymmetric stable steady states, which exist whenever the symmetric steady state is unstable.

Acemoglu and Zilibotti (1997, Section VI) and Martin and Rey (2001) explored the role of incomplete markets (in the sense of Arrow-Debreu securities) on the cross-country patterns of development. The key mechanism in these models is that rich countries have better financial markets than poor countries, which provide with more opportunities to diversify, and hence encourage more investment. In other words, the divergence occurs because the entrepreneurs from poor countries do not enjoy the equal access to the financial markets with those from rich countries. In the present paper, as well as in the models of Gertler-Rogoff-Boyd-Smith, it is assumed that countries do not differ in their degree of credit market imperfection. The divergence occurs because the entrepreneurs from poor countries, who have less wealth, have to compete directly with those from rich countries for the credit in the international financial market.<sup>24</sup>

The present paper may remind some readers of the recent literature on wealth distribution across households; see Aghion and Bolton (1997), Banerjee and Newman (1993), and Matsuyama (2000, 2001). Despite some resemblance, the present model differs fundamentally from these models in many dimensions. First, in all these models, the assumption that each household faces a

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<sup>24</sup>Another difference is that, unlike in the present model, the aggregate increasing returns exist in the models of Acemoglu-Zilibotti-Martin-Rey (as in the Dixit-Stiglitz-Ethier-Romer model of monopolistic competition). This feature of their models makes it difficult to separate the role of financial factors from that of agglomeration economies in generating the divergence. (Indeed, one may interpret that they explored the role of agglomeration economies that operate through financial markets.)

nonconvex technology plays an essential role in generating the inequality among households. In the present model, the inequality among nations is generated despite that each nation has a convex technology.<sup>25</sup> Second, inequality is transmitted over time through bequest motives in these studies. Here, they are transmitted through nontraded factor markets, or more precisely, due to an incomplete integration of the factor markets. These differences in the specifications lead to the difference in the predictions, as well. For example, in the model of Matsuyama (2000), which uses the same specification of the credit market imperfection with the present model, endogenous inequality across households occurs when the productivity of the investment projects is sufficiently low. In the present model, endogenous inequality across nations occurs when the productivity of the investment projects is neither too low nor too high.<sup>26</sup>

## 8. Concluding Remarks

This paper analyzed the effects of financial market globalization on the cross-country pattern of development in the world economy. To this end, it developed a dynamic macroeconomic model of imperfect credit markets, in which the investment becomes borrowing-constrained at the lower stage of development. In the absence of the international financial market, the world economy converges to the symmetric steady state, and the cross-country difference disappears in the long run. In the presence of the international financial market, the symmetric steady state could lose its stability. When this happens, there are stable asymmetric steady states, in which the cross-country distribution of the capital stocks is concentrated into two mass points. The symmetry-breaking caused by unrestricted flows of financial capital leads to a polarization of the world economy into the rich and the poor. In any of these polarized steady states, the rich are richer, the poor are poorer and the world output is smaller than in the

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<sup>25</sup>Recall that, even though each agent faces an investment threshold, there is no threshold for the aggregate investment, because there is a continuum of agents in each nation. When presenting a working paper version of Matsuyama (2000), I was frequently asked if one could reinterpret “households” as “countries.” Such a reinterpretation of the model would be awkward, because one would have to assume that the investment threshold is so large and cannot be financed even if a whole country pools its entire resources.

<sup>26</sup>Furthermore, in the model of Matsuyama (2000), when endogenous inequality occurs, the steady state in which all the households own the same amount of wealth may not even exist. In the present model, the steady state in which all the countries own the same amount of the capital stock always exist, even though it is unstable. Another way of seeing the difference between the two models is to consider the case of autarky. In the model of Matsuyama

(unstable) symmetric steady state. The model thus demonstrates the possibility that financial market globalization may cause, or at least magnify, inequality of nations, and the international financial market is a mechanism through which some countries become rich at the expense of others. At the same time, the model also suggests that poor countries cannot jointly escape from the poverty trap by merely cutting their links to rich countries. Nor could foreign aids from the rich countries to the poor eliminate inequality.

Needless to say, the model presented is highly stylized, and there is no doubt that it has many counterfactual implications. In particular, the model is not designed to match or calibrate the empirical patterns of international capital flows and economic development. An attempt to fit the model better with some empirical regularities would merely obscure the key insights.<sup>27</sup> The following extensions, however, might be worth pursuing for a better understanding of the mechanism explored here.

First, in the analysis above, the effects of financial market globalization were examined by comparing the two extreme cases, autarky and full financial market integration. It would be more satisfactory to introduce some parameters (say, financial transaction costs, the Tobin tax, etc.) that may be interpreted as a measure of financial market globalization.

Second, the model assumes that the capital good is nontradeable. It is also assumed that an agent can start an investment project only in his/her own country. These assumptions are introduced as a simplest way of modeling the factor markets that are not fully integrated.<sup>28</sup> With an incomplete integration of the factor markets, the result that financial market globalization can cause the inequality among nations should carry over. Nevertheless, it seems highly plausible that, as the factor markets become more integrated, the condition for endogenous inequality might become more stringent. To address this issue, the model needs to be extended to introduce some parameters that can be interpreted as a measure of factor market integration. For example, one could introduce foreign direct investment, by allowing the agents to start a project in a foreign country with some additional costs, which may arise from the difficulty of operating in an

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(2000), no investment is made, if each household is in autarky. In the present study, the stability of the steady state with equal distribution is restored if each country is in autarky.

<sup>27</sup>For example, in reality, some poor countries receive some capital flows, while other poor countries are exporters of financial capital. This can be accommodated only by dropping the assumption of the homogeneity of nations.



unfamiliar environment or from legal and other obstacles for repatriating the project revenue. In such an extension, one might also be able to demonstrate the possibility that globalization via financial markets may magnify inequality, while globalization via foreign direct investment may help the poor to catch up with the rich.

Third, the model assumes that globalization has no effect on the penalty for defaults, and hence the efficiency of the credit markets. This assumption may be justified as a benchmark case, because it is not obvious in which direction globalization might affect the operation of credit markets.<sup>29</sup> Yet, the reader should keep in mind that the results of this paper are conditional on this assumption. At the same time, it would be desirable to combine the present macroeconomic model with a variety of microeconomics of credit markets in such a way that one could examine full impacts of globalization of financial markets.

Fourth, the model does not allow for sustainable growth of the world economy as a whole. It would be interesting to examine the condition under which endogenous inequality of nations occurs in a growing global economy. This would require the model to be extended in such a way that the minimum investment requirement for the project would increase with the growth of the world economy.

Fifth, the model has only one type of the capital good and one final goods industry. In a model with many capital goods or final goods industries, which differ in the minimum investment requirements or in the default penalty, poor countries may find comparative advantages in the sectors with less stringent borrowing constraints. It would be interesting to investigate how such a sorting of countries affects the impacts of financial market globalization on cross-country patterns of development.

Sixth, the present model deliberately excluded increasing returns at the national level, not because it is empirically insignificant, but to highlight the role of imperfect credit markets in generating inequality. The evidence seems to suggest that the rich countries tend to have higher

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<sup>28</sup>Indeed, if the factor markets were fully integrated, the very notion of the “country” would lose its operational meaning.

<sup>29</sup>On one hand, one might argue that, the lower the cost of international financial transactions is, the harder it becomes to catch the borrowers that defaulted. If so, globalization has the effect of reducing the efficiency of credit markets. On the other hand, one might also argue that the globalization and resulting competition for the world saving provide a greater incentive for an individual country to improve legal and other protections for both domestic and foreign creditors. If so, globalization has the effect of enhancing the efficiency of credit markets.

total factor productivity, which implies the presence of aggregate increasing returns. Introducing aggregate increasing returns in the present model would be useful, not merely for the sake of realism. It is also important to see how these two inequality-generating mechanisms interact with each other, because each may amplify the power of the other in generating inequality. In other words, the combined effects of the two mechanisms may be larger than the simple sum of the effects of the two mechanisms operating in isolation.

Finally, the model may be extended to introduce the heterogeneity of the agents in such a way that one could discuss the impacts of financial market globalization on the income distribution within each country. Such an extension would be essential in order to discuss the political economy of globalization.

## Appendix

Proof of Lemma.

- a) This follows from that  $\Psi$  is a continuous map on  $[0, R]$  into itself.
- b) This is trivial, because the map,  $\Psi$ , is constant above  $K(\lambda)$  and equal to  $\Phi(r/R)$ .
- c) Differentiating (6) yields  $\Psi'(k_t) = k_t[f'(k_t)/f'(\Psi(k_t))](r/\lambda R)$  for  $k_t < K(\lambda)$ . By setting  $k_t = \Psi(k_t) = k$ , the slope of the map at a steady state,  $k < K(\lambda)$ , is equal to  $\Psi'(k) = k(r/\lambda R)$ , which is increasing in  $k$ . Also,  $\Psi(0) = \Phi(r/\lambda R) > 0$ . Therefore, at the smallest steady state,  $0 < k_L < K(\lambda)$ , if there is one, either  $\Psi$  is tangent to the  $45^\circ$  line (i.e.,  $\Psi'(k_L) = k_L(r/\lambda R) = 1$  or  $k_L = \lambda R/r$ ), in which case it is the only intersection below  $K(\lambda)$ , or  $\Psi$  cuts the  $45^\circ$  line from above (i.e.,  $\Psi'(k_L) = k_L(r/\lambda R) < 1$  or  $k_L < \lambda R/r$ ), in which case it is stable. At the second smallest steady state,  $k_M$ , if it exists,  $\Psi$  cuts the  $45^\circ$  line from below (i.e.,  $\Psi'(k_M) = k_M(r/\lambda R) > 1$ , or  $k_M > \lambda R/r$ ) and hence it is unstable, which also implies that  $\Psi$  cannot cut the  $45^\circ$  line from above between  $k_M$  and  $K(\lambda)$ , ruling out the existence of a third steady state below  $K(\lambda)$ . This completes the proof.

Proof of Proposition 2.

The proof consists of four steps.

*Step 1.* Since  $f(K(\lambda))$  is strictly decreasing and continuous in  $\lambda$  and  $f(K(1)) = f(0) = 0 < 1 = W(R^+) < f(R^+) = f(K(0))$ ,  $\lambda_c \in (0, 1)$  is well-defined and  $f(K(\lambda)) > (<) 1$  if and only if  $\lambda < (>) \lambda_c$ .

*Step 2.* Consider the nongeneric case of  $Rf'(K(\lambda)) = r$ . Then,  $K(\lambda) = \Phi(r/R)$  and hence  $K(\lambda)$  is a fixed point of the map,  $\Psi$ . Because  $f(K(\lambda)) - 1 = K(\lambda)f'(K(\lambda)) + W(K(\lambda)) - 1 = K(\lambda)r/R - \lambda = \lambda[\lim_{k \uparrow K(\lambda)} \Psi'(k) - 1]$ , the left derivative of the map at  $K(\lambda)$  is greater (less) than one if and only if  $f(K(\lambda)) > (<) 1$  or  $\lambda < (>) \lambda_c$ . These properties are illustrated in Figure A1 for  $\lambda < \lambda_c$  and Figure A2 for  $\lambda \geq \lambda_c$ . Note that, from Lemma,  $\Psi$  has another intersection,  $0 < k_L < K(\lambda)$ , in Figure A1, and has no other intersection in Figure A2.

*Step 3.* Consider the case where  $Rf'(K(\lambda)) < r$ . This case can be studied by reducing  $R$ , starting from the case,  $Rf'(K(\lambda)) = r$ , while fixing  $\lambda$  and  $r$ . This change is captured by a downward shift of the map,  $\Psi$ , in Figures A1 and A2. Clearly, with any downward shift,  $\Psi$  has the unique stable fixed point, which satisfies  $k_L < K(\lambda)$ . This proves Proposition 2a).

*Step 4.* Consider the case where  $Rf'(K(\lambda)) > r$ , which can be studied by increasing  $R$ , starting from the case,  $Rf'(K(\lambda)) = r$ , while fixing  $\lambda$  and  $r$ . This change is captured by an upward shift of the map  $\Psi$  in Figures A1 and A2. In Figure A2, i.e., if  $f(K(\lambda)) \leq 1$ ,  $\Psi$  has the stable unique fixed point,  $k_H = \Phi(r/R) > K(\lambda)$ , after any upward shift. In Figure A1, i.e., if  $f(K(\lambda)) > 1$ , there is a critical value of  $R$ ,  $R'$ , such that, if  $r/f'(K(\lambda)) < R < R'$ , there are three fixed points,  $k_L < k_M < K(\lambda) < k_H$ , and, if  $R > R'$ , there is the unique fixed point,  $k_H = \Phi(r/R) > K(\lambda)$ . In the borderline case,  $R = R'$ ,  $\Psi$  is tangent to the  $45^\circ$  line below  $K(\lambda)$ . From Lemma c), the value of  $k$  at the tangency is equal to  $\lambda R'/r$ , and hence  $\Psi(\lambda R'/r) = \lambda R'/r$ , which can be rewritten as  $(\lambda R'/r)f'(\lambda R'/r) = 1 - W(\lambda R'/r)$ , or  $f(\lambda R'/r) = 1$ . Thus,  $f(\lambda R/r) < 1$  implies the three fixed points and  $f(\lambda R/r) > 1$  implies the unique steady state,  $k_H = \Phi(r/R) > K(\lambda)$ . This proves Proposition 2b) and 2c).

#### Proof of Proposition 4.

First, note that (7) defines  $k_H$  as a function of  $k_L$ . Differentiating (7) shows that this function, denoted by  $k_H = \phi(k_L)$ , is increasing if and only if  $f(k_L) < 1$  or equivalently  $k_L < K^*(R_c) = K(\lambda_c)$ . Furthermore, it satisfies  $\phi(0) = 0$  and  $\phi(K(\lambda)) = K(\lambda)$ . If  $\lambda \geq \lambda_c$ ,  $K(\lambda) \leq K(\lambda_c)$  and hence  $k_L < K(\lambda)$  implies  $k_H = \phi(k_L) < \phi(K(\lambda)) = K(\lambda)$ , which violates (8). If  $\lambda < \lambda_c$ , the set of  $(k_L, k_M)$  that satisfies (7), (8), and (9) is nonempty, and illustrated by the solid curve in Figure A3.

Second, (A2) and (8) imply that (10) has a solution,  $X \in (X^-, X^+) \subset (0,1)$ , if and only if  $k_L < K^*(R) < k_H$ . This condition is illustrated by the shaded area in Figure A3. Therefore, a stable steady state,  $(k_L, k_H, X)$ , exists if and only if the solid curve (the segment of  $k_H = \phi(k_L)$  that satisfies (8) and (9)), overlaps with the shaded area, or equivalently, if and only if  $K(\lambda) < \phi(K^*(R))$  and  $\phi(K(\lambda_c)) = \phi(K^*(R_c)) > K^*(R)$ . The first condition can be rewritten to  $f'(K(\lambda)) > \lambda f'(K^*(R)) / [1 - W(K^*(R))]$  where  $R < R_c$  and the second to  $\lambda < f'(K^*(R))K(\lambda_c) = f'(K^*(R))K^*(R_c)$ .

That  $X^- > 0$  requires that the upper-right end of the solid curve is strictly inside the shaded area, or equivalently  $R > R_c$ . Similarly,  $X^+ < 1$  if and only if the lower-left end of the solid curve is strictly inside the shaded area, or equivalently,  $K(\lambda) > K^*(R)$ .

This completes the proof.

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