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Heifetz, Aviad; Spiegel, Yossi

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ON THE EVOLUTIONARY EMERGENCE OF OPTIMISM

Aviad Heifetz[¤] and Yossi Spiegel[y]

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Abstract

Successful individuals were frequently found to be overly optimistic. These …nding are puzzling, as one could expect that realists would perform best in the long run. We show, however, that in a large class of strategic interactions of either cooperation or competition, the equilibrium payo¤s of optimists may be higher than those of realists. This is because the very fact of being optimistic changes the game, and drives the adversary to change her equilibrium behavior, possibly to the bene…t of the optimist. Suppose, then, that a population consists initially of individuals with various perceptional tendencies – pessimists and optimists to various extents, as well as of realists. Individuals meet in pairs to interact, and more successful tendencies proliferate faster. We show that as time goes by, some moderate degree of optimism will take over, and outnumber all other tendencies.

[¤]The Division of Humanities and Social Sciences, California Instituite of Technology, mail code 228-77, Pasadena, CA 91125, and The Eitan Berglass School of Economics, Tel Aviv University 69978, Israel. email: heifetz@post.tau.ac.il

[y]Department of Economics, Northwestern University, 2003 Sheridan Road, Evanston, IL 60208, and the Eitan Berglass School of Economics, Tel Aviv University, Tel Aviv 69978, Israel. email: spiegel@post.tau.ac.il.

1 Introduction

There is by now a considerable body of evidence, that in many kinds of circumstances, successful individuals are overly optimistic regarding the return to their own investment or e¤ort. Taylor and Brown (1988) found that most mentally healthy people have somewhat unrealistically positive self-views, while the less mentally healthy perceive themselves more accurately. In peer reviews, for example, non-depressed individuals' self-ratings were considerably more favorable than those given to them by others (Lewinsohn, Mischel, Chaplin and Barton 1980). Non-depressed individuals exhibited an illusion of control in a dice-throwing experiment (Fleming and Darley 1986). Most individuals believe that their driving ability is above average (Svenson 1981), and most young Americans know that half of U.S. marriages end in divorce, but they are con…dent that theirs will not (Lehman and Nisbett, 1985).[1] In the Economic arena, excess entry of new businesses that fail within several years is common in the US, and recent experimental work (Camerer and Lovallo, 1999) suggests that this phenomenon may be due to entrepreneurs being overcon…dent regarding their own ability in comparison with other entrepreneurs. Babcock and Loewenstein (1997) review experimental work that suggests that parties to legal disputes are reluctant to settle out of court because they hold overly optimistic beliefs about the merits of their case.

These observations are puzzling because it might be thought that optimistic individuals, who consistently overestimate their eventual payo¤s, will not do as well as realistic individuals who assess the situation more accurately, and hence will not survive evolutionary pressures. If success pays o¤ in wealth, which translates to more supportable descendants and more imitators, one might have guessed that individuals whose estimations are not biased would perform best on average, and thus would outnumber the biased types in the long run.

In this paper we show that this intuition need not be right and in fact, there is a wide range of circumstances where optimistic individuals not only survive, but also prosper and take over the entire population. Our model is based on the consequences of biased perceptions in strategic interactions between individuals in a large population, who are continuously matched at random to interact. The individuals di¤er from one another in the way they perceive their payo¤s from interacting with their rivals: optimists overes-

[1] In a similar vein, most cigarette smokers smoke high-tar brands, but only 17 percent believe their brand to have a more hazardous tar level than most others (Segerstrom et al, 1993), and sexually active undergraduate women, especially those who do not consistently use e¤ective contraception, perceive their vulnerability to unwanted pregnancy as smaller than the other women at their university perceive this risk (Burger and Burns, 1988). In a study of optimism and behavior, Seligman and Schulman (1986) compared the sales made by new life insurance agents and found that those who put an optimistic spin on setbacks by seeing them as ‡ukes or as suggesting a new approach rather than viewing them as signs of incompetence sold more policies during their …rst year and were half as likely to quit.

timate the positive impact of their own actions, pessimists underestimate it, and only realists assess it correctly. Biased individuals do not react optimally to their environment as shaped by the objective circumstances and the actions of others. However, it turns out that in many strategic settings, being recognized as optimistic may give individuals a strategic advantage, by inducing opponents to alter their behavior in a favorable way. When individuals misperceive the impact of their own actions on their payo¤s, the resulting equilibrium behavior and true payo¤s are di¤erent than those that would appear if the interacting parties were both realistic. Given a speci…c way the other party perceives her utility, it may very well "pay" to misperceive one's own utility, so that the resulting equilibrium would be better according to the true payo¤s. For moderate levels of optimism, this bene…cial e¤ect outweighs the losses due to the biases in judgement, in a large class of interactions.

Interestingly, cautious optimism may pay o¤ both in games in which the actions are strategic substitutes in the sense of Bulow, Geanakoplos and Klemperer (1985) as well as games in which the actions are strategic complements. The reason for this is that an optimist who overestimates the return to his actions, behaves more "aggressively" than a realist and chooses a higher level of action.[2] When actions are strategic substitutes and the actions of one individual impose a negative externality on the payo¤s of rivals, this aggressive behavior triggers a favorable soft behavior from the rival. Examples for such interactions include the tragedy of the commons (the joint use of congested common resources) and some tournaments.[3] When the actions are strategic complements and the actions of one individual impose a positive externality on the payo¤ of rivals, the aggressive behavior of the optimist triggers a favorable aggressive behavior from rivals. Examples for this kind of interaction include the Bertrand duopoly model with di¤erentiated products and the Cournot duopoly model with complementary products. Hence, in both cases, cautiously optimistic types fare better on average than realists. The reason why overly optimistic types do not do as well is that the strategic advantage from being "too" optimistic is outweighed by the associated loss from having a biased perception and hence from taking suboptimal actions.[4]

The idea that a biased objective function that does not coincide with the true payo¤

[2] In this paper, we use the term aggressiveness to mean a higher level of action, not a tendency to hurt the other player.

[3] For instance, consider the Tournament model of Lazear and Rosen (1981), where two individuals 1 and 2, compete for a prize, w. Each of the individuals expands e¤ort, θ_i $(i = 1; 2)$, to produce an output $q_i = \theta_i + \varepsilon_i$, where ε_i is a random luck component. The individual with the higher output wins the prize. If the random luck components are independently drawn from the same exponential distribution, $F(\varepsilon_i) = 1 - e^{-\lambda\varepsilon_i}$; then the expected payo¤ of individual i is $\pi_i = \frac{we^{\lambda(\theta_i - \theta_j)}}{2} - C(\theta_i)$; where $C(\theta_i)$ is the disutility of e¤ort which is increasing and convex. It can be veri…ed that this model, the e¤orts are strategic substitutes and the e¤ort of one individual lowers the expected payo¤ of the other individual.

[4] For an alternative exploration of optimism and self-con…dence see Benabou and Tirole (1999a,b) and Brocas and Carrillo (1999).

may confer a strategic advantage is well established and goes back at least to Schelling (1960). It was used extensively in many areas in economics including, Macroeconomics (Rogo¤, 1985), International Trade (e.g. Brander and Spencer 1985, Eaton and Grossman 1986), Industrial Organization, (e.g., Brander and Lewis 1986, Fershtman and Judd 1987), and delegation (e.g. Green 1992, Fershtman, Judd and Kalai 1991, Fershtman and Kalai 1997, Katz 1991). In a similar vein, the literature on the evolution of preferences has shown that a population of "irrational" types who care about fairness (Güth and Yaari 1992, Huck and Oechssler 1998), are socially minded (Fershtman and Weiss 1997, 1998), altruistic (Bester and Güth 1998), spiteful (Possajennikov 2000, Bolle 2000), concerned with relative success (Koçkesen, Ok and Sethi 1998), or overcon…dent in …nancial investments (Kyle and Wang 1997, Benos 1998), may be evolutionary stable, i.e. immune to the appearance of few sel…sh or rational "mutants".[5]

In the current work, we take these ideas one step further. We show under what conditions optimism would evolve in a full-‡edged, dynamic evolutionary context. That is, instead of just showing that optimism is evolutionary stable, we show under what conditions the distribution of individual types will converge over time to a unit mass on some level of optimism. Speci…cally, we consider a large population of individuals who are continuously matched in pairs at random and interact with one another. Individuals di¤er in the way they perceive the impact of their own actions on their payo¤s, with optimists overestimating the impact and pessimists underestimating it. Over time, more successful types of individuals proliferate. This is either because they are more …t in the biological sense and therefore have higher reproduction rates, or because they are imitated by others. For a large class of interactions, we show that some degree of cautious optimism would reign the population in the long run and wipe out all other types.[6]

To prove our result, we posit an arti…cial, preliminary game, in which two players commit simultaneously to their degree of optimism or pessimism, knowing that consequently they will be playing the equilibrium of the game de…ned by the types to which they committed, but get the true payo¤s that result from their behavior. When this arti…cial game is dominance solvable, a regular payo¤-monotonic dynamics will wipe out all serially dominated types, and the population will converge in distribution to a unit mass at some level of biased perception. This result is known for a distribution of …nitely many strategies or types (Samuelson and Zhang 1992), and is proved in the appendix

[5] Related ideas appear already in the works of Frank (1987, 1988). The indirect evolutionary approach, where the preferences rather than the strategies are the subject of evolutionary pressures, is employed also by Dekel and Scotchmer (1999), Dufwenberg and Güth (1999), Rogers (1994), Robson (1996a,b), Waldman (1994) Vega-Redondo (1997) and Bergman and Bergman (2000). See also further references in the sequel.

[6] In a similar dynamic setting, Huck, Kirchsteiger and Oechssler (1997) examine the emergence of an endowment e¤ect – an excess valuation of one's own endowment in bargaining. They show that the proportion of realists with no such e¤ect will shrink to zero with time, as will types with a very high endowment e¤ect. However, unlike in our case, the dynamics is not shown to converge in the long run.

for general distributions when the payo¤ function is continuous and the set of types is a compact interval.

The paper is organized as follows. Section 2 brings a simple example that exhibits most of the properties of the model. Section 3 discusses several interpretational issues. Section 4 explores general conditions on the payo¤ functions which are su¢cient for our results to hold, and elaborates on the dynamic selection process and its properties. Section 5 concludes.

2 A Simple Example

Consider a large population of individuals who are continuously and randomly matched in pairs to interact with one another. In this interaction, the matched individuals $i = 1; 2$ choose actions $x^i \in \mathbb{R}$. These actions are to be interpreted as the degree of e¤ort or the level of investment the individuals put into the interaction.[7] Given a pair of actions $x^1; x^2$; the payo¤s of the individuals are

$$\Pi^i(x^i; x^j) = (\alpha - bx^j - x^i)x^i \quad \text{for } i = 1; 2 \text{ and } j \neq i; \tag{2.1}$$

where $\alpha > 0$ and $-1 < b < 1$: When b is positive, the individuals are competing with each other: The larger the action of the other party, the lower is the return to every unit of one's own action. Moreover, since $\Pi^i_{ij} = b > 0$ (subscripts are used to denote partial derivatives), the best-response functions are decreasing in the $(x^1; x^2)$ space, so the actions are strategic substitutes in the sense of Bulow, Geanakoplos and Klemperer (1985). With a negative b, the individuals cooperate with one another since now there is a positive, linear correlation between the return to one's action and the action of the other individual. And, since $\Pi^i_{ij} = b < 0$, the best-response functions are increasing in the $(x^1; x^2)$ space, so the actions are strategic complements.

Although the payo¤s of all individuals are symmetric, individuals di¤er from one another in the way they perceive the interaction between them: Pessimistic types underestimate the value of α; optimistic types overestimate it, and only realistic types assess it correctly. Speci…cally, player i conceives the value of α to be

$$\alpha^i = \alpha + \varepsilon^i; \tag{2.2}$$

where ε^i is positive (zero, negative) when the individual is optimistic (realistic, pessimistic) regarding the return to her action for any given action of the other individual. Thus, the individuals conceive their utility functions to be

[7] For some interpretations, it may be suitable to consider only non-negative actions. Our arguments continue to hold with such a restriction, though the analysis gets more involved.

$$U^i(x^i; x^j) = (\alpha^i - bx^j - x^i)x^i \quad \text{for } i = 1; 2 \text{ and } j \neq i \tag{2.3}$$

Let us assume that the players play the unique Nash equilibrium of the game with these utility functions:

$$\hat{x}^i = \frac{(2 - b)\alpha + 2\theta^i - b\theta^j}{4 - b^2} \quad \text{for } i = 1; 2 \text{ and } j \neq i: \tag{2.4}$$

An important assumption here is that in every pairwise meeting, the players know each other's type. This is either because the types $\theta^1; \theta^2$ are "written on the players' foreheads" and are thus immediately recognized, or alternatively because each interaction consists of several rounds, in which the players' actions converge relatively quickly to the equilibrium behavior (e.g. because in those rounds both players play their best response to the other's previous action or some average of the other's previous actions).[8]

Substituting $\hat{x}^1$ and $\hat{x}^2$ into (2.1), the true equilibrium payoffs of the players are given by

$$f^i(\theta^i; \theta^j) \equiv \pi^i(\hat{x}^i; \hat{x}^j) = \frac{((2 - b)\alpha + 2\theta^i - b\theta^j)((2 - b)\alpha - (2 - b^2)\theta^i - b\theta^j)}{(4 - b^2)^2}. \tag{2.5}$$

Now imagine that these true payoffs translate into fitness terms, so that the instantaneous growth rates of the types are monotonic in their average true payoffs when interacting with randomly matched individuals from the current population of types. That is, the proportion of types with high current average payoffs tends to increase, at the expense of types with low current average payoffs. The evolution of types therefore follows a regular, payoff monotonic dynamics, as shall be formally defined in section 5. The mechanism by which the frequency of types evolves over time can be seen as either purely biological (types with higher payoffs have a higher ability to reproduce), as a process by which parents transmit their attitudes to life (i.e. their degree of "optimism") to their children via education or parental influence, or as a process by which more successful attitudes to life are imitated more often and increase in popularity.

Given this evolution of attitudes, which levels of optimism will perform best and survive in the long run? To provide an answer, we consider a preliminary, artificial two-player "types game". In this game, each of the players $i = 1; 2$ chooses a type $\theta^i \in [-\alpha;$

[8] If types were not mutually observed, individuals would not be able to gain a strategic advantage from being recognized as optimistic or aggressive. Individuals would then choose their actions to maximize their expected payoffs in a random match with an individual from the population, and realistic types would fare best. Consequently, as shown by Dekel, Ely and Yilankaya (1998) and Ok and Vega-Redondo (1999) (see also an example in Possajenikov 1999), in an evolutionary stable distribution of the population, all the individuals play a (true) Nash equilibrium strategy.

or, equivalently, an assessment $\alpha^i = \alpha + \theta^i$ which is non-negative. The payoffs $f^i(\theta^i;\theta^j)$ as a function of these types are given by (2.5) above.

The best-response functions in this game are

$$BR^i(\theta^j) = \frac{b^2((2-b)\alpha - b\theta^j)}{4(2-b^2)} \quad \text{for } i = 1;2 \text{ and } j \neq i: \tag{2.6}$$

These functions are downward sloping in the $(\theta^1;\theta^2)$ space when $b > 0$; and upward sloping when $b < 0$: In other words, the types are strategic substitutes (complements) in this artificial game whenever the actions are strategic substitutes (complements) in the original game itself. Moreover, since $|b| < 1$; the slope of the best-response functions BR^i is less than $\frac{1}{4}$ in absolute value (each as a function of its variable). Therefore, they have a unique intersection in the $(\theta^1;\theta^2)$ space at $(\hat{\theta}^1;\hat{\theta}^2)$; where

$$\hat{\theta} \equiv \hat{\theta}^1 = \hat{\theta}^2 = \frac{b^2}{4+2b-b^2}\alpha: \tag{2.7}$$

are the unique Nash equilibrium assessments. The following graph depicts $\frac{\hat{\theta}}{\alpha} = \frac{b^2}{4+2b-b^2}$: As can be seen, the players will stick with an optimistic assessment ($\frac{\hat{\theta}}{\alpha} > 0$) for any $b \neq 0$: When $b = 1$ (the case of competition and strategic substitutes), the degree of optimism will be 20%. When $b = -1$ (the case of cooperation and strategic complements), the degree of optimism will reach 100%. Only when $b = 0$; where the individuals do not face a genuine strategic interaction but rather a one-person decision problem, no optimism will appear.

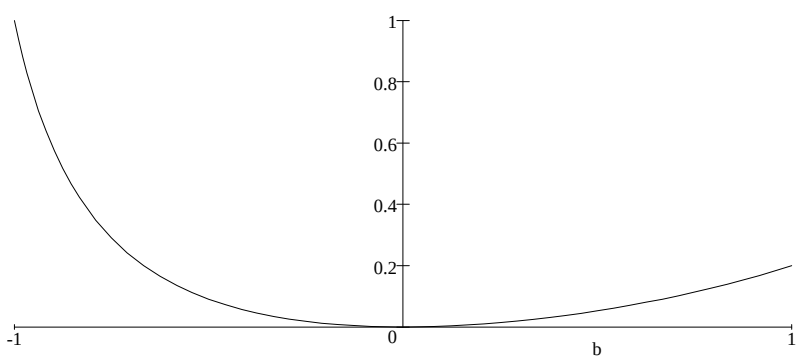

Fig.1: The optimism factor $\frac{\hat{\theta}}{\alpha}$ at equilibrium as a function of b

Since the slopes of the best-response functions BR^i are smaller than 1 in absolute value, this artificial "types game" can be also solved by a process of iterative elimination of strictly dominated strategies, in which every type but $\hat{\theta}$ will eventually be eliminated.[9] By theorem 1 below (which is a generalization of a result of Samuelson and Zhang (1992)),

[9] For instance, suppose that $b = 1$; so that the payoff functions (2.5) in the "types game" are

the serially dominated types (those that do not survive iterative elimination of strictly dominated strategies) are wiped out in a regular, payo¤ monotonic dynamics. Precisely, for any initial distribution of types whose support is an interval that contains b; the distribution of types will converge in distribution to the unit mass at b; so that the density of all other types will converge to zero. Figure 1 above depicts, therefore, the relative optimism of the surviving type as a function of b.

The intuition underlying the evolution of optimism is as follows. Individuals with optimistic types play more aggressively than realists and choose larger actions. When b is positive, the actions are strategic substitutes, so the aggressive behavior of optimists invites a softer behavior from opponents. Since the interactions are competitive when b is positive, this soft behavior from opponents is bene…cial, so optimists end up with a higher payo¤ than realists or pessimists, and hence end up with a higher payo¤. Of course, the return to aggression has its limits, as overly aggressive strategies harm not only the opponent but also the aggressor. Hence, wildly optimistic types do not do as well as more moderately optimistic types. When b is negative, actions are strategic complements, so typically there is a free-rider problem: The players fail to take into account the bene…t of their actions for their opponents, and therefore the equilibrium actions tend to be "too low". Since optimism induces players to be more aggressive than they would be otherwise, it invites a similar reaction from the opponents. And since when b is negative the interactions between individuals are cooperative, this eventually bene…ts both players. Once again, wildly optimistic types end up doing "too much", while realistic or pessimistic types do "too little". Hence, cautious optimists fare better on average and gradually take over the population.

3 Discussion

Before we continue with more general results, we sidestep to discuss several interpretational issues of the model.

$f^i(¿^i;¿^j) = \frac{(®+2¿^i ¡ ¿^j)(® ¡ ¿^i ¡ ¿^j)}{9}$; and the best-response functions (2.6) are $BR^i(¿^j) = \frac{® ¡ ¿^j}{4}$: Since player 1's assessment of ® is non-negative, i.e. $¿^1 ¸ ¡®$; it is better for player 2 to be of type $¿^2 = \frac{®}{2}$ and assess $®^2 = \frac{3}{2}®$ than to have any higher assessment. Understanding this, having type $¿^1 = \frac{®}{8}$ and assessing $®^1 = \frac{9}{8}®$ is better for player 1 than having any lower assessment. But with this in mind, player 2 is better o¤ having the type $¿^2 = \frac{7}{32}®$ and the assessment $®^2 = \frac{39}{32}®$ than any higher value, and so forth. The only type that survives this elimination process is $\frac{®}{5}$; the Nash equilibrium type b.

3.1 Learning about ® over time

The above example immediately raises the following question: How come that optimists do not come to realize that they overestimate ® once they observe their true payo¤s (2.5)? A persistent overestimation of ® may be compatible with what is called in the psychology literature the belief perseverance phenomenon, which is the tendency to cling to one's beliefs in the face of contrary evidence.[10]

Another possible interpretation is that individuals face uncertainty regarding the true current value of ®; which is drawn at random from some distribution. To illustrate, suppose that each period, ® could either assume a high value, $®_h$; with probability p; or a low value, $®_`$; with probability 1 ¡ p; independently across periods. Individuals do not know the value of p, but can observe past realizations of ® and use them to update their beliefs about p: Using p^0 to denote the posterior estimate of p; and noting that the payo¤ functions (2.1) are linear in ®; substituting $®^0 = p^0®_h + (1 ¡ p^0)®_`$ in place of ® in (2.1) gives the expected payo¤ functions.

Initially, individuals have in mind a prior distribution over values of p; that contains the true value p in its support. Realists update their beliefs about p using Bayes rule, so by the strong law of large numbers, their posterior distribution will converge in distribution to a point mass on p almost surely. As a result, their $®^0$ will converge almost surely to the true mean $p®_h + (1 ¡ p)®_`$, and they will get closer and closer to maximizing their true expected payo¤.

In contrast, optimists (pessimists) do not use all the relevant information they have to update their beliefs about p: Rather, they discard some low (high) realizations of ®; attributing them the exceptional, non-systematic bad (good) circumstances, which render them irrelevant for updating. In other words, the behavior of optimists and pessimists exhibits what is called in the psychology literature a con…rmation bias – the tendency to seek information that con…rms one's own views and overlook evidence that may discon…rm these views. For instance, re‡ecting on many experiments, Wason (1981) reports that once people have a wrong idea they "...evade facts, become inconsistent, or systematically defend themselves against the threat of new information relevant to the

[10] For instance, Lord, Ross and Lepper (1979) found that people who held opposing views on capital punishments and were shown supposedly new research …ndings, one supporting and the other refuting the claim that death penalty deters crime, were more impressed by the study that supported their beliefs, and readily disputed the other study. Anderson, Lepper and Ross (1980) found that people who were told about a risk-taker who was an excellent …re …ghter and a cautious person who was a poor …re …ghter, surmised that risk-takers tend to be better …re …ghters, while other people who were told the opposite concluded that cautious people are better …re …ghters. These beliefs did not change by much even after the researchers revealed that the cases were made up for the experiment, arguably because the participants held on to their explanations for why the their beliefs made sense, even though the evidence was gone.

issue." For experiments that demonstrate the con…rmation bias phenomenon, see e.g. Klayman and Ha (1987) and Skov and Sherman (1986).

Assuming that optimistic (pessimistic) discard each some constant proportion of low (high) realizations of ®; their posterior estimation of p will converge almost surely to some point mass above (below) p: As a result, $®^0$ in the perceived expected payo¤s of optimistic types will converge almost surely to a value above (below) the true average of ®. The higher the percentage of discarded realizations, the further $®^0$ will get from the true average.

Our results show that the con…rmation bias of optimistic types can not only survive evolutionary pressures but also take over the entire population. Using a learning interpretation of selection dynamics, this suggests in turn that there is no reason to believe that over time, individuals will learn to update their beliefs in a Bayesian fashion.

3.2 Larger families of perceived utility functions

We have thus far considered a one-dimensional family of distortions in the perception of one's utility function - those that result from di¤erent evaluations of the impact of one's own actions on payo¤s. What would happen if we were to consider every possible distortion of the utility function?

Dekel, Ely and Yilankaya (1998) show that if the distribution of types (i.e. the perceived payo¤s in our case) ever reaches a stable state in which all the individuals take the same action, then this action must be e¢cient. In the example of the previous section, this action is the joint payo¤ maximizing action $\frac{®}{2(1+b)}$: A population of types who all take a di¤erent action is not immune to an invasion of mutants who perceive their payo¤s di¤erently. To see why, assume for simplicity that all the individuals perceive the payo¤s in the same way, and suppose, by way of contradiction, that when they meet each other they play some ine¢cient $x \neq \frac{®}{2(1+b)}$: Consider mutants that di¤er from the rest of the population only in that they are extremely satis…ed (over and beyond the true payo¤) if both players in the interaction play $\frac{®}{2(1+b)}$: Thus, when a mutant meets an incumbent, she will mimic the incumbent and play x, so that both the incumbent and the mutant will get the same true payo¤. But when a mutant meets another mutant they will recognize each other, play $\frac{®}{2(1+b)}$ and fare better. Hence, such mutants would not tend to disappear.

This result, however, is static in nature. In particular, it does not predict if and when a payo¤ monotonic dynamics would ever converge. Moreover, it does not preclude stable states with a polymorphic distribution of types. But in any case, it does imply that our result need not hold with an extended family of utility distortions, and it leads to ask

what biases are relevant for consideration and in which contexts. Thus, it will certainly be of interest to model more families of biases, and confront the theoretical predictions with empirical …ndings. The results of Dekel et al. (1998) show that the triumph of biased types is not a trivial result in such an exercise, and thus open the door for a challenging process of modeling relevant biases.

4 A General Analysis

In section 2 we analyzed an example of a symmetric two-player game, in which individual i of type $¿^i$ misperceived her true payo¤ function $¦^i(x^i; x^j)$ to be

$$U^i(x^i; x^j) = ¦^i(x^i; x^j) + ¿^i x^i; \quad i = 1; 2; \tag{4.1}$$

where $x^i; x^j$ are the actions of individual i and her opponent j; respectively. Optimistic types had a positive $¿^i$; and thus overestimated the return to each unit of their action, while the converse was the case for pessimistic types. In this section we shall present a much broader class of payo¤ functions $¦^i$; for which some degree of optimism emerges in a dynamic selection process that favors successful biases. The properties of the payo¤ functions $¦^i$ in this class and the induced properties of the "types game" will be detailed in sections 4.1 and 4.2, respectively. In Section 4.3 we bring the precise de…nition of the selection dynamics and analyse conditions for its convergence to a unique selected type.

4.1 The pairwise interactions

We begin by specifying properties of the true payo¤ functions $¦^i ´ ¦(x^i; x^j)$ of individual i; which we assume to be thrice continuously di¤erentiable. For convenience, we shall often denote its partial derivatives by $¦^i_i ´ \frac{@¦(x^i;x^j)}{@x^i}$; $¦^i_j ´ \frac{@¦(x^i;x^j)}{@x^j}$; $¦^i_{ii} ´ \frac{@^2¦^i(x^i;x^j)}{(@x^i)^2}$; and $¦^i_{ij} ´ \frac{@^2¦^i(x^i;x^j)}{@x^i@x^j}$:

Property 1: The true payo¤ function of individual i is strictly concave in x^i, i.e., $¦^i_{ii} < 0$.

The concavity of $¦^i$ in x^i ensures that the best-response of individual i against individual j's action; $BR^i(x^j)$, is implicitly de…ned by the …rst order condition

$$U^i_i(x^i; x^j) = ¦^i_i + ¿^i = 0: \tag{4.2}$$

Property 2: The true payo¤ function of individual i is such that $\left|\frac{¦^i_{ij}}{¦^i_{ii}}\right| < 1 ¡ "$; for some $" > 0$.

Property 2 ensures that the slope $\frac{dBR^i(x^j)}{dx^j}$ of each individual's best-response function is uniformly smaller than 1 in absolute value. This implies in turn that every pair of best-response functions intersect exactly once in the actions space, so that every pairwise interaction has a unique Nash equilibrium.[11]

Let $(\text{¤}^i(¿^i;¿^j);\ \text{¤}^j(¿^i;¿^j))$ be the Nash equilibrium in the interaction between individuals i and j given their types. The equilibrium strategies are implicitly de…ned by the following system of equations:

$$\begin{aligned} \text{¦}_i(\text{¤}^i(¿^i;¿^j);\text{¤}^j(¿^i;¿^j)) + ¿^i &= 0; \\ \text{¦}_j(\text{¤}^i(¿^i;¿^j);\text{¤}^j(¿^i;¿^j)) + ¿^j &= 0: \end{aligned} \tag{4.3}$$

4.2 The arti…cial types game

To study the evolution of optimism we once again consider an arti…cial two-player "types game" in which each player i selects a type $¿^i$. We assume that the set of possible types for each individual is a (compact) interval $T = [\underline{¿};\overline{¿}]$; where $\underline{¿} < 0 < \overline{¿}$, so that both pessimists and optimists are represented in the population. The payo¤ of player i in the "types game" is given by

$$f^i(¿^i;¿^j) \text{ ´ } \text{¦}(\text{¤}^i(¿^i;¿^j);\text{¤}^j(¿^i;¿^j)); \tag{4.4}$$

which is the true equilibrium payo¤ in the interaction between i and j: The uniqueness of the Nash equilibrium in every pairwise interaction implies that the (reduced-form) payo¤ function of each individual in the types game, $f^i(¿^i;¿^j)$; is well-de…ned. Moreover, since all individuals have the same true payo¤ function, the payo¤s in the types game are symmetric in the sense that $f^i(¿^i;¿^j) = f^j(¿^j;¿^i)$. The needed properties of these payo¤ functions are:

Property 3: The payo¤ function f^i of each player i in the arti…cial "types game" is twice di¤erentiable and strictly concave in $¿^i$

Property 4: The payo¤ functions of individuals i and j in the arti…cial "types game" are such that $|f^i_{ii}| < \overline{f^i_{ij}}$:

Property 3 ensures that the types game is well-behaved. Property 4 ensures that the types game has a unique Nash equilibrium because it implies that the slope of each

[11] The uniformity requirement is needed to guarantee that the best-respone functions of the individuals are not mutually asymptotic without intersecting each other. Actually, property 2 ensures that a myopic best-reply process in a repetition of the game (in which each individual plays a best reply to the previous action of the opponent) converges to the unique Nash equilibrium. This may justify the assumption that the individuals essentially play this equilibrium even if initially they do not recognize each other's type, but each encounter between them consists of several rounds.

player's best-response function in the types game is less than 1 in absolute value. Both properties are satis...ed in the example studied in Section 2. Since the payo¤ functions in the types game are symmetric, the Nash equilibrium in the types game is also symmetric. We denote the equilibrium types by $\hat{\theta}$, so that the Nash equilibrium is $(\hat{\theta}; \hat{\theta})$:

To study the properties of the "types game" further, we return for a moment to the underlying pairwise interactions and di¤erentiate (4.3) to obtain the following comparative statics conditions:

$$J \pounds \begin{pmatrix} \hat{x}^i_j \\ \hat{x}^j_i \end{pmatrix} = \begin{pmatrix} ¡1 \\ 0 \end{pmatrix}; \quad J \pounds \begin{pmatrix} \hat{x}^i_j \\ \hat{x}^j_j \end{pmatrix} = \begin{pmatrix} 0 \\ ¡1 \end{pmatrix}; \tag{4.5}$$

where

$$J = \begin{pmatrix} \Delta^i_{ii} & \Delta^i_{ij} \\ \Delta^j_{ji} & \Delta^j_{jj} \end{pmatrix}: \tag{4.6}$$

Using Cramer's rule, this yield

$$\hat{x}^i_i = ¡\frac{\Delta^j_{jj}}{\det J}; \quad \hat{x}^i_j = \frac{\Delta^i_{ji}}{\det J}: \tag{4.7}$$

Property 2 implies $\det J > 0$: Hence, From (4.7) and property 1 it is easy to see that as $¿^i$ increases (individual i becomes more optimistic), the equilibrium action of individual i; $\hat{x}^i$; increases (individual i becomes more aggressive), while the equilibrium action of her rival, $\hat{x}^j$; increases if $\Delta^j_{ij} > 0$ (actions are strategic complements) and decreases if $\Delta^j_{ij} < 0$ (actions are strategic substitutes).

Given Property 3 and using (4.7), an interior Nash equilibrium in the "types game" is de...ned implicitly by the following system of equations:

$$\begin{aligned} f^i_i(¿^i; ¿^j) &= \frac{1}{\det J}\left(¡\Delta^i_i \Delta^j_{jj} + \Delta^i_j \Delta^j_{ji}\right) = 0; \\ f^j_j(¿^i; ¿^j) &= \frac{1}{\det J}\left(¡\Delta^j_j \Delta^i_{ii} + \Delta^j_i \Delta^i_{ij}\right) = 0; \end{aligned} \tag{4.8}$$

where the partial derivative of the actions game are evaluated at the Nash equilibrium actions $(\hat{x}^i(¿^i; ¿^j); \hat{x}^j(¿^i; ¿^j))$: To interpret the equilibrium conditions in the types game, note that (4.8) implies that at an interior Nash equilibrium of the "types" game we have

$$¡\frac{\Delta^i_i}{\Delta^i_j} = ¡\frac{\Delta^j_{ji}}{\Delta^j_{jj}}: \tag{4.9}$$

The left side of (4.9) represents the slope of an iso-payo¤ curve of individual i in the underlying actions game in the $(x^i; x^j)$ space, and the right side of (4.9) represents the

slope of the best response function of individual j in the $(x^i; x^j)$ space. Thus, equation (4.9) says that individual i chooses $¿^i$ optimally in the types game by selecting to be on the "highest" true iso-payo¤ curve, taking as given the best response function of the rival in the action game. The …rst order condition for this constrained maximization problem requires that, holding $¿^j$ constant, the iso-payo¤ curve of individual i will be tangent to the best response function of individual j:[12]

In order to express Properties 3 and 4 directly in terms of the true payo¤ function $¦$; one has to di¤erentiate further (4.8), using (4.7). The resulting complicated expressions are strict inequalities which involve third-order derivatives of $¦$; and they do not have a direct economic interpretation.

Next, we show when it is the case that at the symmetric Nash equilibrium of the types game, $\hat{¿} > 0$: That is, in the equilibrium of the types game, individuals "choose" to become optimistic.

Lemma 1 The Nash equilibrium in the arti…cial "types game" is such that $\hat{¿} > 0$ if and only if $¦^i_j$ and $¦^i_{ij}$ have the same sign, i.e. $¦^i_j ¦^i_{ij} > 0$:

Proof. Substituting from (4.9) into (4.3) and rearranging terms yields

$$\hat{¿} = ¡ \frac{¦^i_j ¦^j_{ji}}{¦^j_{jj}}; \tag{4.10}$$

where the right hand side is evaluated at the symmetric Nash equilibrium $(\hat{x}(\hat{¿}; \hat{¿}); \hat{x}(\hat{¿}; \hat{¿}))$ in the interaction between two individuals who both have the type $\hat{¿}$: By the symmetry of the actions game, it is also the case that

$$\hat{¿} = ¡ \frac{¦^i_j ¦^i_{ij}}{¦^j_{jj}}; \tag{4.11}$$

at $(\hat{x}(\hat{¿}; \hat{¿}); \hat{x}(\hat{¿}; \hat{¿}))$: The denominator of (4.16) is negative by Property 1. Therefore, $\hat{¿} > 0$ if and only if $¦^i_j ¦^i_{ij} > 0$: ■

To interpret Lemma 1, note that $¦^i_j$ captures the externality that individual j's action imposes on individual i's payo¤ in a pairwise interaction between them (the externality is positive if $¦^i_j > 0$; and negative if $¦^i_j < 0$), and the sign of $¦^i_{ij}$ determines whether the actions are strategic substitutes ($¦^i_{ij} < 0$) or strategic of complements ($¦^i_{ij} > 0$). Hence

[12] This is exactly like the behavior of a leader in a Stackelberg duopoly model who chooses the level of output at which its iso-pro…t curve is tangent to the best response function of the follower.

Lemma 1 shows that in a Nash equilibrium of the types game, players will choose to become optimistic either if they impose negative externalities on one another and their actions are strategic substitutes or if they impose positive externalities on one another and their actions are strategic complements. The feature that Φ^i_j and Φ^i_{ij} have the same sign is common in many types of games, including the Cournot duopoly with homogenous or di¤erentiated products, the tragedy of the commons, and tournaments (see footnote 3), where Φ^i_j and Φ^i_{ij} are both negative, and the Bertrand duopoly model with di¤erentiated products or the Cournot duopoly model with complementary products, where Φ^i_j and Φ^i_{ij} are both positive.[13] As we explained above, in either case, the aggressive behavior associated with being an optimist gives individuals a strategic advantage; not surprisingly then, in a Nash equilibrium of the "types game", both players will choose to become optimistic:

Remark: In what follows, we will show that properties 1-4 and the property $\Phi^i_j \Phi^i_{ij} > 0$ in Lemma 1 guarantee the dynamic evolutionary emergence of optimism, in a sense that we shall make precise. The payo¤ functions Φ which satisfy these properties constitute an open set in the space $C^3(R^2_+)$ of thrice continuously di¤erentiable functions[14], because they are de…ned using …nitely many strict inequalities which involve continuous functions of up to third-order derivatives of Φ:[15] Thus, the family of quadratic payo¤ functions explored in section 2, which clearly obey all these properties, is not exceptional: Other payo¤ functions whose derivatives up to the third-order are not too far from one of those quadratic functions admit the same kind of evolution of preferences towards some level of optimism.

The following lemma will be useful for the sequel:

Lemma 2 The arti…cial "types game" is strictly dominance solvable. The unique outcome that survives the process of iterated elimination of strictly dominated strategies is the symmetric Nash equilibrium $(\hat{\rho};\hat{\rho})$, where $\hat{\rho}$ is de…ned implicitly by the …rst order condition, $f^i_i(\hat{\rho};\hat{\rho}) = 0$.

[13] Fershtman and Weiss (1998) show that the same condition implies that social mindedness (enjoying doing what is socialy highly considered) is evolutionary stable. It should be noted however that there are important classes of games in which Φ^i_j and Φ^i_{ij} do not have the same sign. For instance, in arms races, where $\Phi^i = V(x^i - x^j) - C(x^i)$;with $V'(\cdot) > 0 > V''(\cdot)$, $C'(\cdot) > 0$, and $C''(\cdot) \geq 0$; we have $\Phi^i_j < 0 < \Phi^i_{ij}$: Likewise, in voluntary contribution to public goods games, where $\Phi^i = V(x^i + x^j) - C(x^i)$; with $V'(\cdot) > 0 > V''(\cdot)$, and $C'(\cdot) > 0$ and $C''(\cdot) \geq 0$; we have $\Phi^i_j > 0 > \Phi^i_{ij}$:

[14] $C^3(R^2_+)$ is the space of thrice continuously di¤erentiable functions $\Phi : R^2_+ \to R$; with the minimal topology in which Φ_n converges to Φ i¤ $(\Phi_n - \Phi)$ and each of its …rst, second and third derivatives converge to zero uniformly on compact sets on R^2_+.

[15] More precisely, for every …xed $\varepsilon > 0$ in property 2 there are …nitely many strict inequalities involved in the de…nition of this set of payo¤ functions – denote it V_ε – which is therefore open. The actual set of payo¤ functions Φ which obey all the properties is the union of these open V_ε over all the positive ε, which is open as the union of open sets.

Proof. To prove the lemma we invoke Theorem 4 in Moulin (1984) that provides su¢cient conditions for normal form games to be dominance solvable. In the present context, these conditions are:

(i) The strategy set of each player is a one-dimensional compact interval

(ii) The payo¤ function of each player is continuous over the space of outcomes, twice di¤erentiable, and strictly concave with respect to the player's strategy

(iii) The slope of each players' best-response functions is less than 1 in absolute value

Condition (i) is satis…ed in the types game because of the assumption that the set of possible types for each player is a (compact) interval $T = [¿; \bar{¿}]$. Properties 3 and 4 ensure that conditions (ii) and (iii) are satis…ed. Hence, the types game is dominance solvable. ■

4.3 The selection dynamics

In this subsection we turn to the way the population of types evolves over time. To this end, let G_t be the distribution of types in the population at time $t \geq 0$ on the support $T = [¿; \bar{¿}]$: We assume that G_t evolves according to a payo¤ monotonic selection dynamics, where types with higher average payo¤s have higher growth rates. Speci…cally, we shall de…ne the growth rate of types as follows:

De…nition. A continuous growth-rate function $g : T \times ¢(T) \to R$ is payo¤ monotonic and regular if for every $G \in ¢(T)$; the following holds:

(i) Higher payo¤s are associated with higher growth rates:

$$g(¿^i; G) > g(¿^i; G) \quad \Leftrightarrow \quad f(¿^i; G) > f(¿^i; G): \tag{4.12}$$

(ii) The total size of the population is preserved:

$$\int_T g(¢; G)dG(¢) = 0: \tag{4.13}$$

(iii) g can be extended to the domain $T \times X$; where X is the set of signed measures G with variational norm smaller than 2, and on this extended domain, g is bounded and Lipschitz continuous:

$$\sup_{\theta^i \in T, G_t \in X} \left| g(\theta^i; G) \right| < 1; \tag{4.14}$$

$$\sup_{\theta^i \in T} \left| g(\theta^i; G) - g(\theta^i; \tilde{G}) \right| < K \left\| G_t - \tilde{G} \right\|; \qquad G, \tilde{G} \in X;$$

for some constant K; where $\|G\| = \sup_{|h| \leq 1} \left| \int_T h dG \right|$ is the variational norm on signed measures.

Oechssler and Riedel (1999, Lemma 3) proved that property (iii) guarantees that the mapping $G \to \int_T g(\theta; G) dG$ is bounded and Lipschitz continuous in the variational norm, which implies that the differential equation in the space of distributions $\Theta(T)$ defined by

$$\dot{G}_t(S) = \int_S g(\theta; G_t) dG_t(\theta); \qquad S \subseteq T; \tag{4.15}$$

has a unique solution for any initial distribution G_0. A special case of the growth rate that we consider is the familiar replicator dynamics that was introduced by Taylor and Jonker (1978) for distributions with a finite support, and by Oechssler and Riedel (1999) for general distributions. In the case of the replicator dynamics, the distribution of types at time t, G_t; evolves according to the differential equation

$$\dot{G}_t(S) = \int_S \left[f(\theta^i; G_t) - f(G_t; G_t) \right] dG_t(\theta^i); \qquad S \subseteq T; \tag{4.16}$$

where

$$f(\theta^i; G_t) \equiv \int_T f^i(\theta^i; \theta^j) dG_t(\theta^j) \tag{4.17}$$

is the expected true equilibrium payoff of an individual of type θ^i at time t from an interaction with individual j drawn at random from the population, and

$$f(G_t; G_t) \equiv \int_T \int_T f^i(\theta^i; \theta^j) dG_t(\theta^j) dG_t(\theta^i) \tag{4.18}$$

is the expected true equilibrium payoffs when both individuals are drawn at random from the population at time t. That is, if the average performance of a subset of types $S \subseteq T$ is better than the average performance in the population, the relative weight in the population of the types in S increases, at the expense of other sets of types whose performance is below the average. The more general selection dynamics that we consider

may be appropriate when the reproduction process of types is not purely biological, but rather relies on education or imitation (see e.g., Weibull 1995, Section 4.4).

Having de…ned the selection dynamics, we are now interested in the following question: starting from some initial distribution, G_0; how will the distribution of types, G_t; evolve over time with a regular, payo¤ monotonic dynamics? To provide an answer, we now establish the following theorem (the proof of the theorem appears in the Appendix). The theorem, which is of independent interest, generalizes Theorem 1 in Samuelson and Zhang (1992) to the case of games with in…nitely many strategies (Samuelson and Zhang (1992) prove their result for the case of games with …nitely many strategies). For preserving the coherence with our setting, we state the theorem for symmetric two-players games with a compact one-dimensional strategy space; the method of proof works however just as well for more general compact strategy spaces and for asymmetric games.

Theorem 1 Let T = [¿;¿] µ R be a space of strategies, f : T £ T ! R a continuous payo¤ function of a symmetric two-player game, and g : T £ T ! R a regular, payo¤ monotonic growth-rate function. Let G_t be the population dynamics de…ned by the di¤erential equation (4.15) with an initial distribution of strategies G_0 with support T: Suppose that D µ [¿;¿] is the subset of serially dominated strategies (those that do not survive the process of iterated elimination of strictly dominated strategies). Then the strategies in D are asymptotically eliminated from the population: Every iteratively dominated strategy d 2 D has an open neighborhood W_d for which $\lim_{t \to 1} G_t(W_d) = 0$: In particular, if there is only one non-eliminated strategy u 2 T n D; then G_t converges in distribution to the unit mass probability at u.

We are now ready to state our main result:

Theorem 2 Suppose that the payo¤s in the pairwise interactions have properties 1-4 and are such that ¦ $^{i}_{j}$ and ¦ $^{i}_{ij}$ have the same sign. Then given any initial distribution of types with support T; the population of types will converge in distribution to a unit mass on the optimistic type ƀ under any regular, payo¤-monotonic selection dynamics.

Proof. >From Lemma 2 we know that given properties 1-4, the arti…cial "types game" is strictly dominance solvable, with the solution being (ƀ;ƀ). Using Theorem 1 it therefore follows that under a regular, payo¤ monotonic growth-rate function, the population of types will converge in distribution to a unit mass on ƀ. Finally, Lemma 1 ensures that if ¦ $^{i}_{j}$ and ¦ $^{i}_{ij}$ have the same sign, then ƀ > 0: ■

5 Conclusion

We have shown how the pressures of explicit, dynamic evolutionary processes select for moderate optimism rather than for realism, when …tness is gained through interactions of either a competitive nature and strategic substitutes or cooperative nature and strategic complements. According to this insight, the well-documented phenomenon of overcon…-dence and unrealistic high self-esteem of individuals may be due to a bias that "pays" well in many kinds of strategic settings.

Clearly, the way humans evaluate their environment has evolved along the generations via con‡icts with both natural hazards and strategic social interactions with other individuals or groups of individuals. The premises of our model are therefore far from being all-encompassing. And in practice, society is composed of heterogeneous individuals who may di¤er from one another in their degree of optimism/pessimism, unlike the long-run equilibrium in our model where all individuals share the same attitude. Thus, our modest aim was to point at one possible source for the optimism that is so frequently observed in the process of decision making. Searching for competing and complementing evolutionary insights for this and similar behavioral puzzles is a challenge for future research.

6 Appendix

Proof of Theorem 1: Let D_n be the set of strategies that do not survive n or less rounds of iterated elimination of strictly dominated strategies, so $D = [_{n=0}^{1} D_n$: Denote also by $U_n = T \; n \; D_n$ the set of strategies that do survive n rounds of iterated elimination of strictly dominated strategies. We prove by induction on n that U_n is compact, and every eliminated strategy $d \; 2 \; D_n$ has an open neighborhood W_d for which $\lim_{t \; ! \; 1} G_t(W_d) = 0$:

Since $D_0 = \; ;$ and $U_0 = T$; the claim holds for $n = 0$: If D_1 is empty as well, i.e. no strategies are strictly dominated, then the claim holds vacuously. So from now on assume that $D_1 \neq \; ;$: Suppose, by induction, that the claim holds for $n < k$:

We …rst prove that U_k is compact. Indeed, let $d \; 2 \; D_k$ be round-k dominated by the strategy $x \; 2 \; T$; that is for every $y \; 2 \; U_{k \; ¡ \; 1}$

$$f(x;y) \; ¡ \; f(d;y) > 0: \tag{A.1}$$

Since f is continuous, $[f(x;y) \; ¡ \; f(d;y)]$ is continuous in y, and therefore attains its

minimum on U_{k-1}; as this set is compact by the induction hypothesis. Hence by (5.12)

$$\Delta(x;d) \equiv \min_{y \in U_{k-1}} [f(x;y) - f(d;y)] > 0. \tag{A.2}$$

Furthermore, the function $\Delta(x;d)$ is continuous since $[f(x;y) - f(d;y)]$ is. Therefore, for every $\varepsilon > 0$, the set of strategies which are dominated by x by a payoff difference of at least ε

$$\{d : \Delta(x;d) > \varepsilon\} \tag{A.3}$$

is open. Consequently, the set of strategies dominated up to round k

$$D_k = D_{k-1} \cup \bigcup_{x \in U_{k-1}} \bigcup_{\varepsilon > 0} \{d : \Delta(x;d) > \varepsilon\}; \tag{A.4}$$

is open as a union of open sets, and $U_k = T \cap D_k$ is compact, as required.

We now turn to complete the inductive step, and prove that every eliminated strategy $d \in D_k$ has an open neighborhood W_d for which $\lim_{t \to 1} G_t(W_d) = 0$. Indeed, let d be dominated by x in one of the rounds up to k. Then if $k > 1$, not only does x perform better than d against strategies in U_{k-1}, but it also does so against some strategies in an open subset of D_{k-1}. Since f is continuous, the set

$$B = \{y \in T : f(x;y) - f(d;y) \leq 0\} \tag{A.5}$$

is a compact subset of the open set D_{k-1}. Hence B is a proper subset of D_{k-1}, as D_{k-1} is open by the induction hypothesis (except when $k = 1$, in which case $D_{k-1} = \emptyset$, and $B = \emptyset$).

This implies that for some positive number s, we have

$$\lim_{t \to 1} G_t(C) = 0, \tag{A.6}$$

where

$$C = \{y \in D_{k-1} : f(x;y) - f(d;y) \leq s\}. \tag{A.7}$$

Indeed, for $k = 1$, $D_{k-1} = D_0 = \emptyset$, so for whatever positive s chosen we have $G_t(C) = 0$, and (5.16) holds. For $k > 1$, let

$$s = \frac{1}{2} \sup_{y \in D_{k-1}} [f(x;y) - f(d;y)], \tag{A.8}$$

which is positive, by the fact that $B \subset D_{k-1}$, as explained above. With this s, the set C is a closed subset of D_{k-1} (because $f(x;y) - f(d;y)$ is continuous in y), and hence compact.

By the induction hypothesis, every $y \in D_{k-1}$ has an open neighborhood $W_y \ni y$ such that $\lim_{t \to \infty} G_t(W_y) = 0$. Since $C \subseteq \bigcup_{y \in C} W_y$, the compactness of C implies that there exist $y_1, \ldots, y_m \in C$ such that $C \subseteq \bigcup_{i=1}^{m} W_{y_i}$. Therefore, $G_t(C) \leq \sum_{i=1}^{m} G_t(W_{y_i})$ and hence $\lim_{t \to \infty} G_t(C) = 0$, as claimed.

Observe now the following considerations. The continuity of f and ρ imply that there are open neighborhoods $V_x \ni x$ and $W_d \ni d$ such that for every $x' \in \overline{V}_x$, $d' \in \overline{W}_d$

$$\inf_{y \in D_{k-1} \cap C} [f(x', y) - f(d', y)] \geq \frac{\varepsilon}{2} > 0; \tag{A.9}$$

and

$$\min_{y \in U_{k-1}} [f(x', y) - f(d', y)] = \rho(x', d') \geq \frac{\rho(x, d)}{2} > 0. \tag{A.10}$$

Thus, against any strategy $y \notin C$, every strategy $x' \in \overline{V}_x$ outperforms every $d' \in \overline{W}_d$ by at least

$$\varepsilon = \min\left\{\frac{\varepsilon}{2}, \frac{\rho(x, d)}{2}, \frac{1}{2}\right\} > 0, \tag{A.11}$$

i.e.,

$$\inf_{y \in T \cap C} [f(x', y) - f(d', y)] \geq \varepsilon. \tag{A.12}$$

At the same time, since f is continuous on the compact domain T, there exists a bound M such that $|f| \leq M$, and by (A.6), there exists a time $\bar{t}$ such that for $t \geq \bar{t}$ we have $G_t(C) < \frac{\varepsilon}{8M}$ and $G_t(T \cap C) > 1 - \varepsilon$. Altogether this implies that for $x' \in \overline{V}_x$, $d' \in \overline{W}_d$ and $t \geq \bar{t}$

$$\begin{aligned} \int f(x', G_t) - f(d', G_t) &= \int_{\bar{t}}^{\infty} [f(x', \tau) - f(d', \tau)]\, dG_t = \\ \int_C [f(x', \tau) - f(d', \tau)]\, dG_t &+ \int_{T \cap C} [f(x', \tau) - f(d', \tau)]\, dG_t > \\ (-2M)\frac{\varepsilon}{8M} &+ \varepsilon(1 - \varepsilon) \geq -\frac{\varepsilon}{4} + \varepsilon\left(1 - \frac{1}{2}\right) = \frac{\varepsilon}{4}. \end{aligned} \tag{A.13}$$

By the continuity of f, (A.13) holds also when G_t is replaced by any probability measure $\mu \in A \equiv \overline{\{G_t\}_{t \geq \bar{t}}}$, the closure of $\{G_t\}_{t \geq \bar{t}}$ in the topology of weak convergence of probability measures.

Now, by the payoff monotonicity of the growth-rate function g, for every $\mu \in A$, $x' \in \overline{V}_x$ and $d' \in \overline{W}_d$

$$g(x', \mu) - g(d', \mu) > 0. \tag{A.14}$$

The continuous function $[g(x^0; \infty) - g(d^0; \infty)]$ attains its minimum on the compact set $\overline{V}_x \times \overline{W}_d \times A$: Therefore, there is in fact a positive ε for which

$$g(x^0; G_t) - g(d^0; G_t) \geq \varepsilon \tag{A.15}$$

for $x^0 \in \overline{V}_x; d^0 \in \overline{W}_d$ and $t \geq \bar{t}$. A fortiori, this inequality holds also if we replace $g(x^0; G_t)$ and $g(d^0; G_t)$ by their averages in $\overline{V}_x$ and $\overline{W}_d$; respectively. Thus for $t \geq \bar{t}$

$$\frac{\int_{\overline{V}_x} g(\theta; G_t)dG_t}{G_t(\overline{V}_x)} - \frac{\int_{\overline{W}_d} g(\theta; G_t)dG_t}{G_t(\overline{W}_d)} \geq \varepsilon: \tag{A.16}$$

Hence, by (4.15), for $t \geq \bar{t}$

$$\frac{\dot{G}_t(\overline{V}_x)}{G_t(\overline{V}_x)} - \frac{\dot{G}_t(\overline{W}_d)}{G_t(\overline{W}_d)} = \frac{d}{dt}\left[\log \frac{G_t(\overline{V}_x)}{G_t(\overline{W}_d)}\right] \geq \varepsilon; \tag{A.17}$$

so that

$$\frac{G_t(\overline{V}_x)}{G_t(\overline{W}_d)} \geq \frac{G_{\bar{t}}(\overline{V}_x)}{G_{\bar{t}}(\overline{W}_d)} \exp[\varepsilon(t - \bar{t})] \to_{t \to \infty} \infty: \tag{A.18}$$

Therefore, $\lim_{t \to \infty} G_t(W_d) = 0$; as required. ¥

7 References

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