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# Financial Market Globalization and Endogenous Inequality of Nations

By Kiminori Matsuyama<sup>1</sup>

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## Abstract

This paper analyzes the effects of financial market globalization on the cross-country pattern of development in the world economy. To this end, it develops a dynamic macroeconomic model of imperfect credit markets, in which the investment becomes borrowing-constrained at the lower stage of development. In the absence of the international financial market, the world economy converges to the symmetric steady state, and the cross-country difference disappears in the long run. In the presence of the international financial market, the symmetric steady state could lose its stability, in which case the cross-country distribution of the capital stocks is concentrated into two mass points in every stable steady state. The symmetry-breaking caused by unrestricted flows of financial capital leads to a polarization of the world economy into the rich and the poor. The model thus demonstrates the possibility that financial market globalization may cause, or at least magnify, inequality of nations, and the international financial market is a mechanism through which some countries become rich at the expense of others. The model suggests that the poor countries cannot jointly escape from the poverty trap by merely cutting their links to rich countries. Nor would foreign aids to the poor eliminate the inequalities; as in a game of musical chairs, some countries must be excluded from being rich.

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*Keywords:* imperfect credit markets, borrowing-constraint, convergence versus divergence, the international financial market, symmetry-breaking, poverty traps

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## 1. Introduction

The role of the international financial market in economic development is one of the most controversial issues in macroeconomics. The standard, neoclassical view suggests that an integration of financial markets leads to the efficient allocation of the world saving by facilitating the flows of financial capital from rich to poor countries. This accelerates economic development in poor countries. Without borrowing from abroad, poor countries would have to finance their investment entirely by the domestic saving, which would slow down their development. According to this view, financial market globalization is an equalizing force, which will bring about a greater, faster convergence of economic performances across countries.

As many have pointed out, however, even casual observations seem to refute this standard, textbook view. In reality, many poor countries receive little private credit from abroad.<sup>2</sup> They are indeed more concerned that the access to the international financial market might lead to an outflow of domestic funds, and continue to impose restrictions in their efforts to channel more domestic saving into domestic investment. These restrictive policies did not seem to prevent some former developing countries, such as Korea and Taiwan, from achieving rapid growth; some even argue that these policies were essential elements of their successful development strategies. Furthermore, a greater integration of financial markets after the W.W.II seems to have done little to reduce the cross-country difference in per capita income in the world economy. On the contrary, the evidence, reported by Quah (1993, 1997) and others, suggests that the world economy is increasingly polarized into the rich and the poor.

There is indeed the popular view that the international financial market magnifies the gap between the rich and the poor. According to this view, financial market globalization is an unequalizing force. The believers of this view often advocate that poor countries should impose more controls to stem the outflows of the domestic saving and that official aids from rich countries are needed for the development of poor countries. Some even hold a radical view that the World Bank and the IMF, which promote financial market globalization, are agents of the global corporate capitalism that exploits

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<sup>2</sup>This prompted Lucas (1990) to pose now the famous question, “Why doesn’t capital flow from rich to poor countries?”, which led to the huge literature on the subject.

developing countries. These radical economists often suggest that the poor countries should jointly cut their links to the rich countries and unite among themselves to escape the poverty.

The standard neoclassical framework is simply inadequate to deal with these issues. What is needed is an alternative theoretical framework, which is consistent with the lack of the private financial capital flows from the rich to the poor, and allows for the possibility that the international financial market may be as a cause of the inequalities of nations. Only within such a framework could one examine the validity of policy proposals offered by the radical economists.

To this end, this paper proposes a dynamic macroeconomic model with imperfect credit markets. The imperfection arises due to potential defaults by the borrowers (and imperfect sanction against them). Due to the imperfection, the borrowers need to have enough wealth to start the project, which makes the domestic investment be borrowing-constrained at the lower stage of development. The model is examined in three different environments: i) the autarky; ii) the small open economy that faces the exogenously given world interest rate; and iii) the world economy consisting of a continuum of inherently identical economies.

In autarky, the dynamics of capital formation is shown to be independent of the imperfection, and the economy reaches to the unique steady state in the long run. Even though the borrowing constraint is binding at the lower stage of development, the effect is entirely offset by the lower interest rate. In the small economy case, without offsetting changes in the interest rate, the borrowing constraint at the lower stage of development has the effect of reducing the investment, thereby slowing down the development process. Under some conditions, this effect is strong enough to generate multiple steady states in the dynamics of capital formation, which suggests the possibility of a poverty trap. It is also shown how even a small, temporary change in the (exogenous) world interest rate could have significant permanent effects.

Having examined the autarky and small open economy cases, the paper turns to the analysis of the world economy. Without the international financial market, the world economy is simply a collection of autarky economies, and hence converges to the unique symmetric steady state, in which all the countries have the same level of the capital stock. In other words, the cross-country difference will

disappear in the long run. In the presence of the international financial market, however, the world economy is a collection of small open economies (with the interest rate being endogenously determined in the international financial market). Under some conditions, the symmetric steady state loses its stability, and the cross-country distribution of the capital stock is concentrated into two mass points in every stable steady state. The symmetry-breaking caused by unrestricted flows of financial capital thus leads to a polarization of the world economy into the rich and the poor. In any stable steady state, the rich countries are richer and the poor countries are poorer than in the autarky steady state. Therefore, the model demonstrates not only the possibility that financial market globalization may cause or at least magnify inequality of nations, but also offers a theoretical justification for the view that the international financial market is a mechanism through which some countries become rich at the expense of others. At the same time, the model suggests that poor countries cannot jointly escape from the poverty trap by merely cutting their links to rich countries and that official aids from the rich would not eliminate the inequalities. Just as in a game of musical chairs, some countries must be excluded from being rich.

It should also be emphasized that financial market globalization does not necessarily lead to the symmetry-breaking and the polarization. Under different conditions, the model predicts the convergence, even though the speed of convergence may be smaller in the presence of the international financial market. One major advantage of the present model is that it is capable of generating the two alternative scenarios, convergence and divergence, thereby providing theoretical justifications for the two conflicting views of the world. More importantly, which of these two alternative scenarios will materialize depends on a few key parameters in an interesting way. The present model thus serves as an organizing principle on these controversial and seemingly intractable issues.

Before proceeding, mention should be made of the title. The term, “financial market globalization” is chosen instead of “capital mobility” to emphasize two points. First, the perspective adopted in this paper is global. We are not so much interested in the effects of capital mobility on poor countries, but in the effects of the international financial market on the *cross-country* pattern of development in the *world economy*. And, as will be discussed later, the global perspective offers different policy implications. Second, the paper is concerned with the effect of international mobility of

*financial* capital, or the possibility of international *lending* and *borrowing*, which is modeled as *intertemporal* trade in the final good. Throughout the paper, it is assumed that physical capital, i.e., the capital good used in the production of the final good, is *nontradeable*.<sup>3</sup> The use of the term, “capital mobility,” is avoided, because it could mean, to many, the tradeability of the capital good.<sup>4</sup>

The rest of the paper is organized as follows. Section 2 develops the building blocks of the model. The three alternative environments, --autarky, a small open economy, and the world economy-- , are examined in sections 3, 4, and 5, respectively. Section 6 considers an extension that allows for heterogeneous agents. Section 7 discusses the related work in the literature. Section 8 concludes.

## 2. The Model

The model comes in three versions: the autarky, the small open economy, and the world economy consisting of a continuum of inherently identical economies. This section explains the common elements.

Time is discrete and extends from zero to infinity ( $t = 0, 1, 2, \dots$ ). There are two goods, a final good and physical capital, and one primary factor of production, labor. The final good produced in period  $t$  may be consumed in period  $t$  or may be invested in the production of physical capital, which become available in period  $t+1$ . The technology of the final goods sector is given by a linear homogeneous production function,  $Y_t = F(K_t, L_t)$ , where  $K_t$  and  $L_t$  are aggregate domestic supplies of physical capital and labor in period  $t$ . (Both factors of production, physical capital and labor, are assumed to be immobile across countries. Only the final good can be traded.) Let  $y_t \equiv Y_t/L_t = F(K_t/L_t, 1) \equiv f(k_t)$  where  $k_t \equiv K_t/L_t$  and  $f(k)$  is  $C^2$  and satisfies  $f'(k) > 0 > f''(k)$ ,  $f(0) = 0$ , and  $f'(\infty) = \infty$ . The factor markets are competitive, and the factor rewards for physical capital and for labor are

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<sup>3</sup> Here, the adjective “physical” is used as opposed to “financial,” not as opposed to “human.” What truly matters in the following analysis is that some accumulable forms of factor inputs have nontradeable components. Human capital could equally play the same role, and hence “physical capital” may be broadly interpreted to include human capital as well.

<sup>4</sup> The distinction between the international mobility of financial capital and the tradeability of the capital good would not be important if the credit market were perfect. In a world of the perfect credit market, nontradeable capital goods would become effectively tradeable with the access to the international financial market, because the economy could finance the production of the nontradeable capital good by borrowing from abroad. This distinction is critical, however, when the credit market is imperfect.

equal to  $\rho_t = f'(k_t)$  and  $w_t = f(k_t) - k_t f'(k_t) \equiv W(k_t)$ , which are both paid in the final good. For simplicity, physical capital is assumed to depreciate fully in one period.

There are overlapping generations of two-period lived agents. Each generation consists a continuum of homogenous agents with unit mass. Each agent supplies one unit of labor inelastically to the final goods sector only in the first period, and consumes only in the second.<sup>5</sup> Thus,  $L_t = 1$ , and the wage income,  $w_t$ , is also equal to the level of wealth held by the young agents at the end of period  $t$ . They allocate their wealth,  $w_t$ , in order to finance their consumption in period  $t+1$ . They have two options. First, they may lend it in the competitive credit market, which earns the gross return equal to  $r_{t+1}$  per unit. If they lend the entire wealth, their second-period consumption is equal to  $r_{t+1} w_t$ . Second, they may become an entrepreneur and start a project. The project comes in discrete, nondivisible units and each young agent can manage only one project. It transforms one unit of the final good in period  $t$  into  $R > 0$  units of physical capital in period  $t+1$ .

In what follows, it is assumed

$$(A1) \quad W(R) < 1$$

or equivalently  $R \in (0, R^+)$ , where  $R^+$  is defined by  $W(R^+) = 1$ . As seen later, (A1) ensures that  $w_t < 1$ , so that the agent needs to borrow  $1 - w_t > 0$  in the competitive credit market, in order to start the project. The second period consumption, if the agent starts the project, is equal to  $\rho_{t+1} R - r_{t+1} (1 - w_t)$ . This is greater than or equal to  $r_{t+1} w_t$  (the second period consumption if the agent lends the entire wage income) when the net present discounted value of the project,  $\rho_{t+1} R / r_{t+1} - 1$ , is nonnegative. This condition can be expressed as

$$(1) \quad R f'(k_{t+1}) \geq r_{t+1}.$$

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<sup>5</sup> It is straightforward to allow the agent to work also in the second period. Such an extension would be desirable if we want to interpret physical capital more broadly to include human capital.

The young agents are willing to borrow and to start the project, when (1) holds. We shall call (1) *the profitability constraint*.

The credit market is competitive in the sense that both lenders and borrowers take the equilibrium rate,  $r_{t+1}$ , given. It is not competitive, however, in the sense that one cannot borrow any amount at the equilibrium rate. The borrowing limit exists because of the enforcement problem: the payment is made only when it is the borrower's interest to do so. More specifically, after having borrowed  $1-w_t$ , and the project being completed, the entrepreneur would refuse to honor its payment obligation,  $r_{t+1}(1-w_t)$ , if it is greater than the cost of default, which is taken to be a fraction of the project revenue,  $\lambda \rho_{t+1} R$ .<sup>6</sup> Knowing this, the lender would allow the entrepreneur to borrow only up to  $\lambda \rho_{t+1} R / r_{t+1}$ . Thus, the agent can start the project only if  $1-w_t \leq \lambda \rho_{t+1} R / r_{t+1}$ , or

$$(2) \quad \lambda R f'(k_{t+1}) \geq r_{t+1}(1-W(k_t)).$$

We shall call (2) *the borrowing constraint*. The parameter,  $0 < \lambda \leq 1$ , can be naturally taken to be the degree of the efficiency of the credit market. Note that there is no default in equilibrium. It is the possibility of default that makes the credit market imperfect. It should also be noted that the same enforcement problem rules out the possibility that different agents may pool their wealth to overcome the borrowing constraint.

In order for the young agents in period  $t$  to start the project, both the profitability constraint (1) and the borrowing constraint (2) must be satisfied. In other words, they must be both willing and able to borrow. These constraints can be summarized as

$$(3) \quad R \geq R_t \equiv \begin{cases} (r_{t+1}/f'(k_{t+1}))(1-W(k_t))/\lambda & \text{if } k_t < K(\lambda), \\ r_{t+1}/f'(k_{t+1}) & \text{if } k_t \geq K(\lambda), \end{cases}$$



where  $R_t$  may be interpreted as the project productivity *required* in order for the project to be undertaken in period  $t$ , and  $K(\lambda)$  is defined implicitly by  $W(K(\lambda)) = 1 - \lambda$ . Note that which of the two constraints is binding depends entirely on  $k_t$ . The borrowing constraint (2) is binding if  $k_t < K(\lambda)$ ; the profitability constraint (1) is binding if  $k_t > K(\lambda)$ . The critical value of  $k$ , which separates these two regimes,  $K(\lambda)$ , is decreasing in  $\lambda$ , and satisfies  $K(0) = R^+$ , and  $K(1) = 0$ .

### 3. The Autarky Case.

In autarky, there is no possibility of intertemporal trade in the final good with the rest of the world, which precludes international lending and borrowing. Thus, the domestic investment (by the young) must be equal to the domestic saving (by the young) in equilibrium.<sup>7</sup> This condition is illustrated in Figure 1. The domestic saving is equal to  $W(k_t)$ , given by the vertical line. The domestic investment is equal to zero if  $R_t > R$ , and to one, if  $R_t < R$ . If  $k_t < R$ , (A1) implies that the equilibrium holds at the horizontal segment of the investment schedule, where  $R_t = R$ . In equilibrium, the aggregate investment is made equal to  $W(k_t)$ . Thus, the fraction of the young agents who become borrowers/entrepreneurs is equal to  $W(k_t)$ , while the rest,  $1 - W(k_t)$ , become lenders. If  $k_t \geq K(\lambda)$ , the young agents are indifferent between borrowing and lending. When  $k_t < K(\lambda)$ , on the other hand, they strictly prefer borrowing to lending. Therefore, the equilibrium allocation necessarily involves credit rationing, where the fraction  $1 - W(k_t)$  of the young agents are denied the credit. Those who are denied the credit cannot entice the potential lenders by raising the interest rate, because the lenders would know that the borrowers would default at a higher rate. (In the present model, credit rationing is an inevitable feature of the equilibrium

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<sup>6</sup>A natural interpretation of the cost is that the creditor seizes a fraction  $\lambda$  of the project revenue in the event of default. One may also interpret that this fraction of the revenue will be dissipated in the borrower's attempt to default. This makes no difference, because the default does not occur in equilibrium.

<sup>7</sup>The GNP accounting of a closed economy, of course, implies that the saving by all the residents is equal to the investment by all the residents, including not only the young but also the old. However, in this model, the old is never engaged in the investment activity and the old consumes all their income, so that their saving is zero. Hence, the equality of the saving and the investment by the young is indeed the equilibrium condition when the economy is in autarky. In what follows, we shall simply call the domestic saving and the domestic investment, without specifically mentioning "by the young."

whenever the borrowing constraint is binding. As will be explained in section 6, however, what is essential is the borrowing constraint, not credit rationing.<sup>8)</sup>

Therefore, regardless of  $k_t < K(\lambda)$  or  $k_t \geq K(\lambda)$ , the measure of the young agents who start the project is equal to  $W(k_t)$ . Since every one of them produces  $R$  units of capital in period  $t+1$ ,

$$(4) \quad k_{t+1} = RW(k_t).$$

Eq. (4) completely describes the dynamics of capital formation. Note that, if  $k_t < R$ ,  $k_{t+1} = RW(k_t) < RW(R) < R$ . Therefore,  $k_0 < R$  implies  $k_t < R$  and  $w_t = W(k_t) < 1$  for all  $t > 0$ , as has been assumed.

Notably, the dynamics of  $k$ , (4), is entirely independent of  $\lambda$ ; the credit market imperfection has no effect on the capital formation in the autarky case. The borrowing constraint, which is binding at the lower level of economic development,  $k_t < K(\lambda)$ , creates the gap between the return to investment and the opportunity cost of capital,  $Rf'(k_{t+1}) > r_{t+1}$ . In autarky, the former is determined by the domestic saving, so that the effect of the imperfection appears entirely on the interest rate. From (3), (4), and  $R = R_t$ , the equilibrium interest rate is

$$(5) \quad r_{t+1} = \begin{cases} \lambda Rf'(RW(k_t))/(1-W(k_t)) < Rf'(RW(k_t)) & \text{if } k_t < K(\lambda), \\ Rf'(RW(k_t)) & \text{if } k_t \geq K(\lambda). \end{cases}$$

Note that a greater imperfection in the credit market (a smaller  $\lambda$ ) manifests itself in the reduction of the interest rate.

Clearly, the result that the dynamics of capital formation in autarky is unaffected by the credit market imperfection is not a robust feature of the model. In particular, it critically depends on the fact

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<sup>8</sup> While some authors use the term, “credit-rationing,” whenever some credit limits exist, here it is used to describe the situation that the aggregate supply of credit falls short of the aggregate demand, so that some borrowers cannot borrow up to their credit limit. In other words, there is no credit rationing if every borrower can borrow up to its limit. In such a situation, their borrowing may be constraint by their wealth, which affects the credit limit, but not because they are credit-rationed. This is consistent with the following definition of credit rationing by Freixas and Rochet

that the supply of the domestic saving is inelastic. Nevertheless, this feature of the model makes the autarky case a useful benchmark for examining the effects of financial market globalization in the presence of the imperfection.

The dynamics of capital formation in autarky, given by eq. (4), even though it is independent of  $\lambda$  and unaffected by credit markets imperfection, may still have multiple steady states. It is well-known (see, e.g. Azariadis 1993) that the overlapping agents model imposes less restrictions on the equilibrium dynamics than the infinitely-lived representative agent model. This is a nuisance that has nothing to do with the credit market imperfection. To avoid any unnecessary complications that arise from this feature of overlapping generations model, we impose the following assumption:

$$(A2) \quad W'(0) = \infty \text{ and } W''(k) < 0.$$

Many standard production functions imply (A2). For example, if  $y = f(k) = A(k)^\alpha$  with  $0 < \alpha < 1$ ,  $W(k) = (1-\alpha)A(k)^\alpha$ , which satisfies (A2).

Under (A2), for any  $R \in (0, R^+)$ , eq. (4) has the unique steady state,  $k^* = K^*(R) \in (0, R)$ , defined implicitly by  $k^* = RW(k^*)$ , and for  $k_0 \in (0, R)$ ,  $k_t$  converges monotonically to  $k^* = K^*(R)$ , as shown in Figure 2a. The function,  $K^*(R)$ , is increasing and satisfies  $K^*(0) = 0$  and  $K^*(R^+) = R^+$ . It is worth emphasizing that  $K^*(R)$ , the steady state level of  $k$ , is independent of  $\lambda$ , and  $K(\lambda)$ , the critical level of  $k$ , below which the borrowing constraint is binding, is independent of  $R$ . Therefore, the borrowing constraint may or may not be binding in the steady state.

To summarize,

**Proposition 1.** In autarky, the dynamics of  $k$  is given by  $k_{t+1} = RW(k_t)$ , which is independent of  $\lambda$ , and converges monotonically to the unique steady state,  $K^*(R)$ , where  $K^*(R)$  is increasing in  $R$  and satisfies  $K^*(0) = 0$  and  $K^*(R^+) = R^+$ . If  $K^*(R) < K(\lambda)$ , the borrowing constraint is binding in the steady state. If  $K^*(R) > K(\lambda)$ , the profitability constraint is binding in the steady state.

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(1997, Ch.5), who attributed it to Baltensperger: “some borrower’s demand for credit is turned down, even if this borrower is willing to pay all the price and nonprice elements of the loan contract.”

Figures 2a and Figure 2b illustrate Proposition 1. The downward-sloping curve in Figure 2b is given by  $K^*(R) = K(\lambda)$ , which connects  $(\lambda, R) = (0, R^+)$  and  $(\lambda, R) = (1, 0)$ .

#### 4. Financial Market Globalization: The Small Open Economy

The goal of this section is twofold. First, it examines the effect of financial market globalization on the capital formation of the small open economy. Second, it offers a preliminary step for the analysis of the world economy in the presence of the international financial market.

The agents in the small open economy are allowed to trade intertemporally the final good with the rest of the world at exogenously given prices. In other words, international lending and borrowing is allowed. The interest rate, the intertemporal price of the final good, is exogenously given in the international financial market and assumed to be invariant over time:  $r_{t+1} = r$ .

In what follows, we will focus on the case  $Rf'(R) < r$  for the ease of exposition.<sup>9</sup> Then, the equilibrium is given by  $R_t = R$  and from (3),  $Rf'(k_{t+1}) = r(1 - W(k_t))/\lambda$ , if  $k_t < K(\lambda)$ , and  $Rf'(k_{t+1}) = r$  if  $k_t \geq K(\lambda)$ . This can be further rewritten as

$$(6) \quad k_{t+1} = \Psi(k_t) \equiv \begin{cases} \Phi(r(1 - W(k_t))/\lambda R) & \text{if } k_t < K(\lambda), \\ \Phi(r/R) & \text{if } k_t \geq K(\lambda), \end{cases}$$

where  $\Phi$  is the inverse of  $f'$ , which is a decreasing function and satisfies  $\Phi(\infty) = 0$ .

Eq. (6) governs the dynamics of the small open economy. Unlike in the autarky case, the domestic investment is no longer equal to the domestic saving. Instead, the investment is determined entirely by the profitability and borrowing constraints. If the credit market were perfect ( $\lambda = 1$  and  $K(1) = 0$ ), the economy would immediately jumps to  $\Phi(r/R)$ , from any initial condition. In the presence of

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<sup>9</sup> If  $Rf'(R) \geq r$ , the dynamics is given by  $k_{t+1} = \min\{R, \Psi(k_t)\}$ , where  $\Psi(k_t)$  is defined as in eq. (6). Assuming  $Rf'(R) < r$  ensures  $k_{t+1} = \Psi(k_t) < R$ , and hence the equilibrium is never at the corner. This restriction helps to reduce the notational burden significantly, but the result can be easily extended to the case where  $Rf'(R) \geq r$  as well. This

the imperfection, this occurs only when the economy is at the higher level of development ( $k_t \geq K(\lambda)$ ), where the profitability of the project is the only binding constraint. At the lower level of development ( $k_t < K(\lambda)$ ), the borrowing constraint is binding, which creates the gap between the return to investment and the interest rate. In this range,  $k_{t+1}$  is increasing in  $\lambda R/r$  and in  $k_t$ . In the autarky case, the investment was determined by the saving, so that a greater imperfection reduced the equilibrium interest rate. In the small open economy case, on the other hand, the interest rate is given exogenously in the international financial market. Therefore, a greater imperfection has the effect of reducing the domestic investment (and channeling more of the domestic saving into investment abroad). A higher level of the capital stock in one period leads to a higher level of the capital stock in the next, because the higher wage income/wealth level of the young agents alleviate their borrowing constraint.

The steady states of the small open economy are given by the fixed points of the map (6), satisfying  $k = \Psi(k)$ . The following lemma summarizes some properties of the set of the fixed points. While elementary, they turn out to be quite useful, and will be evoked repeatedly in the subsequent discussion.

#### Lemma.

- a) Eq. (6) has at least one steady state.
- b) Eq. (6) has at most one steady state above  $K(\lambda)$ . If it exists, it is stable and equal to  $\Phi(r/R)$ .
- c) Eq. (6) has at most two steady states below  $K(\lambda)$ . If there is only one,  $k_L$ , either it satisfies  $0 < k_L < \lambda R/r$  and is stable, or,  $k_L = \lambda R/r$  at which  $\Psi$  is tangent to the 45° line. If there are two,  $k_L$  and  $k_M$ , they satisfy  $0 < k_L < \lambda R/r < k_M < K(\lambda)$ , and  $k_L$  is stable and  $k_M$  is unstable.

Proof. See Appendix.

One immediate implication of Lemma is that there are only three generic cases of the dynamics generated by (6). They are illustrated in Figures 3a-3c. In Figure 3a, the unique fixed point,  $k_L$ , is located below  $K(\lambda)$ , to which  $k_t$  converges from any  $k_0 \in (0, R)$ . In Figure 3c, the unique fixed point,

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restriction can alternatively be justified on the ground that, in the world economy version of the model developed later, the world interest rate prevailing in any steady state satisfies  $Rf'(R) < r$ .

$k_H = \Phi(r/R)$ , is located above  $K(\lambda)$ , to which  $k_t$  converges from any  $k_0 \in (0, R)$ . In Figure 3b, there are three fixed points; two stable steady states,  $k_L$  and  $k_H$ , are separated by the third (unstable) steady state,  $k_M$ , which is located between  $k_L$  and  $K(\lambda)$ , and  $k_t$  converges to  $k_L$  if  $k_0 < k_M$  and to  $k_H$  if  $k_0 > k_M$ .

The following proposition provides the exact condition for each of the three cases.

Proposition 2. Let  $\lambda_c \in (0, 1)$  be defined by  $f(K(\lambda_c)) = 1$ . Then,

- a) If  $Rf'(K(\lambda)) < r$ , there exists a unique steady state,  $k_L$ . It is stable and satisfies  $k_L < K(\lambda)$ .
- b) If  $Rf'(K(\lambda)) > r$ ,  $f(\lambda R/r) < 1$ , and  $\lambda < \lambda_c$ , there exist three steady states,  $k_L$ ,  $k_M$ , and  $k_H$ . They satisfy  $k_L < k_M < K(\lambda) < k_H$ , and  $k_L$  and  $k_H$  are stable and  $k_M$  is unstable.
- c) If  $Rf'(K(\lambda)) > r$  and either  $f(\lambda R/r) > 1$  or  $\lambda > \lambda_c$ , there exists a unique steady state,  $k_H$ . It is stable and satisfies  $k_H > K(\lambda)$ .

Proof. See Appendix.

Proposition 2 is also illustrated by Figure 4. The conditions for Proposition 2a), 2b) and 2c) are satisfied in Region A, B, and C, respectively. The outer limit of Region A is given by  $Rf'(K(\lambda)) = r$ , and the border between Regions B and C are given by  $f(\lambda R/r) = 1$ . These two downward-sloping curves meet tangentially at  $\lambda = \lambda_c$ .

Proposition 2 states that the dynamics of capital formation in the small open economy differ drastically from the autarky case. The difference is most significant when the world interest rate is such that the parameters lie in Region B, as illustrated by point P in Figure 4. In this case, an integration of this economy to the international financial market creates multiple steady states, as shown in Figure 3b. If the integration occurs at the lower stage of development ( $k_t < k_M$ ), the economy will gravitate toward the lower stable steady state,  $k_L$ , in which the borrowing constraint is binding. On the other hand, if the integration takes place at the higher level ( $k_t > k_M$ ), the economy converges to the higher stable steady state,  $k_H$ , in which the borrowing constraint is no longer binding. This case thus suggests that the timing of the integration has significant permanent effects on the capital formation.

This does not mean, however, that the integration would have negligible effects on the capital formation in other cases. For example, suppose that the world interest rate is such that the parameters lie in Region C. In this case, the economy will eventually converge to the unique steady state, in which the borrowing constraint is not binding. The convergence could take long time, however, because the economy must go through the “narrow corridor” between the map and the 45° line, as illustrated in Figure 3c.

More generally, a comparison between the shapes of the two maps,  $k_{t+1} = RW(k_t)$  for the autarky case and  $k_{t+1} = \Psi(k_t)$  for the small open economy case, suggests that the integration have the effect of slowing down the growth process of middle-income economies.

Let us now consider the effect of a change in the world interest rate on the capital formation of the small open economy. We focus on the case, where the parameters lie in Region B, depicted by P in Figure 4, and the dynamics is hence illustrated by Figure 3b. Suppose that the economy is trapped in  $k_L$ . A decline in the world interest rate, illustrated in Figure 4 as the vertical move from point P in Region B to point P' in Region C eliminates  $k_L$  and the dynamics is now illustrated by Figure 3c. The decline in the interest rate thus helps the economy to escape from the trap and to start a (perhaps long and slow) process of growth toward  $k_H$ . Furthermore, even a temporary decline in the interest rate could have similar long run effects. Once the economy accumulates enough capital, the economy will not fall back to the trap, when the interest rate returns to the original level. Therefore, even a small, temporary decline in the interest rate could have a significant permanent effect.<sup>10</sup> Similarly, one could show that even a small, temporary rise in the world interest rate could lead to a permanent stagnation of the economy, if it is initially located at  $k_H$  in Figure 3b.

One might be tempted to argue that Region B of Figure 4, which gives rise to the dynamics illustrated in Figure 3b with multiple stable steady states, can be used to explain the divergence (or the lack of convergence) of economic performance across the countries. Imagine that there are two small open countries, called N and S, which share the same technology, the same demographic structure, etc. Furthermore, both countries are fully integrated into the international financial market and face the same

world interest rate. The only difference is that the capital stock in N is equal to  $k_H$  and the capital stock in S is equal to  $k_L$ . The model does explain why this situation can persist, because both  $k_H$  and  $k_L$  are stable steady states of the dynamics, if the parameters lie in Region B of Figure 4.

While suggestive, this argument explains only *the possibility* that we may not observe the convergence of the two otherwise identical countries, but does not predict the *inevitability* of the divergence. This is because the model also allows for the possibility of convergence. Indeed, the situation in which the capital stocks are both equal to  $k_H$  in N and S, and the situation in which they are both equal to  $k_L$  in N and S, (as well as the situation in which it is equal to  $k_H$  in S and  $k_L$  in N) are also stable steady states under the same condition. The argument does not offer any reason why one should believe that the divergence is a more plausible outcome than the convergence. In other words, the small open economy version of the model cannot impose any restriction on the degree of inequality that might be observed in the world economy. It therefore fails to *predict* the divergence, or the empirical finding reported in Quah (1993, 1997) that the distribution of the per capita income tends to converge to a bimodal, or “twin-peaked,” distribution in the long run. The small open economy version of the model imposes no restriction on the cross-country difference because it takes into account no interaction between the dynamics of different countries.<sup>11</sup>

To resolve this problem, therefore, one must move beyond the small open economy framework, and analyze the model from a global perspective. In the next section, the world economy version of the model is analyzed. This helps not only to endogenize the world interest rate, but also to address the issue of divergence versus convergence in a more satisfactory manner.

Analyzing the model from a global perspective is also important for the policy analysis. From the prospective of an individual country, escaping from the poverty trap may appear simple. One might be tempted to argue that the poor countries should temporarily cut their financial links or that foreign

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<sup>10</sup>Of course, how small the decline can be in order to have the permanent effect depends on the distance between point P and the border between Regions B and Region C. Furthermore, the larger the decline, the shorter it can be to have the permanent effect.

<sup>11</sup> This drawback is not limited to the use of small open economy models with multiple steady states. Any attempt to explain the divergence by using closed economy models with multiple steady states, like those in Azariadis and Drazen (1990), Ciccone and Matsuyama (1996) and others, may be criticized on the same ground.



aids from the rich countries should solve the problem. The global perspective will show, however, why these measures may not be able to eliminate the poverty trap.

### 5. Financial Market Globalization: The World Economy

In the world economy version of the model, there is a continuum of inherently identical countries with unit mass. In the absence of the international financial market, this is merely a collection of the autarky economies analyzed in section 3. Hence one can immediately conclude that the world economy would converge to the symmetric steady state, in which each country holds  $K^*(R)$  units of the capital stock.

In what follows, let us assume that all the countries are fully integrated in the international financial market, where each country faces the same interest rate. The world economy hence can be viewed as a collection of inherently identical small open economies of the type analyzed in section 4. Since the world as a whole is a closed economy, the interest rate is now endogenously determined to equate the world saving and the world investment.

The presence of the international financial market does not change the fact that the state in which every country has the capital stock equal to  $K^*(R)$  is a steady state. However, it may change the stability property of the symmetric steady state, in which case the world economy cannot be expected to converge to it in the long run. Furthermore, it may create other steady states. What we need to do is to characterize the *entire* set of *stable* steady states of the world economy.

In any stable steady state of the world economy, each country must be at a stable steady state of the small open economy. As stated in Lemma, there are at most two stable steady states in which each small open economy can be located. This means that a stable steady state of the world economy must be one of the following two types. The first type is the case of perfect equality, or the case of convergence. In such a steady state, all the countries have the same level of capital,  $k^*$ . The second type is the case of endogenous inequality, or the case of divergence. In such a steady state, the world economy is polarized into the rich and the poor, in which the poor (rich) countries have the same level of

capital stock, given by  $k_L$  ( $k_H$ ), which satisfies  $k_L < K(\lambda) < k_H$ . Let us derive the condition for the existence of these two types of stable steady states.

### 5.1. The Steady State with Equality of Nations: The Case of Convergence.

Suppose that all the countries have the same level of capital stock,  $k^*$ , in a steady state. Then, the world saving is equal to  $W(k^*)$ . Since the world economy as a whole is closed, the measure of the young agents that invest in this steady state must be equal to  $W(k^*)$ . Since every one of them produces  $R$  units of capital, the steady state capital must satisfy  $k^* = RW(k^*)$ , or equivalently,  $k^* = K^*(R)$ . If  $k^* = K^*(R) > K(\lambda)$ , the borrowing constraint is not binding, hence the world interest rate in this steady state is  $r = Rf'(K^*(R)) < Rf'(K(\lambda))$ . This inequality can be rewritten as  $\Phi(r/R) > K(\lambda)$ , which is exactly the condition under which a small open economy has a stable steady state,  $k_H = \Phi(r/R) = K^*(R) = k^*$ . (See also Proposition 2b)-2c.) This proves that  $K^*(R) > K(\lambda)$  is the condition under which there exists a stable steady state in which all the countries have the same level of capital stock,  $k^* = K^*(R) > K(\lambda)$ .

If  $k^* = K^*(R) < K(\lambda)$ , the borrowing constraint is binding, hence the world interest rate in this steady state is  $r = \lambda Rf'(K^*(R))/[1-W(K^*(R))]$ . From c) of Lemma,  $k^* = K^*(R) < K(\lambda)$  is a stable steady state for each small open economy, if and only if it satisfies  $k^* = K^*(R) < \lambda R/r = [1-W(K^*(R))]/f'(K^*(R))$ . This condition can be rewritten to  $K^*(R)f'(K^*(R)) + W(K^*(R)) = f(K^*(R)) < 1$ . This proves that  $K^*(R) < K(\lambda)$  and  $f(K^*(R)) < 1$  are the condition under which there exists a stable steady state in which all the countries have the same level of capital stock,  $k^* = K^*(R) < K(\lambda)$ .

The above argument also shows that, if  $K^*(R) < K(\lambda)$  and  $f(K^*(R)) > 1$ , a symmetric steady state, in which all the countries have the same level of capital stock, is unstable. To see this, in such a steady state, the capital stock in each country must be equal to  $k^* = K^*(R) < K(\lambda)$ , which means that the borrowing constraint is binding. Therefore, the world interest rate is equal to  $r = \lambda Rf'(K^*(R))/[1-W(K^*(R))]$ . When  $f(K^*(R)) > 1$ , this implies  $k^* = K^*(R) > \lambda R/r$ , which means that  $k^* = k_M$  from Lemma c). Thus, it is unstable. Figure 5 illustrates this situation. Suppose that there is no international financial market at the beginning. Then, the dynamics of every country follows  $k_{t+1} =$

$RW(k_t)$ , which converges to  $K^*(R)$ . In this steady state, the interest rates are equal across countries, even though there is no international lending and borrowing. If the international financial market is open at this point, the dynamics of each country is now governed by  $k_{t+1} = \Psi(k_t)$ , which cut the  $45^\circ$  line from below at  $K^*(R)$ . This situation is unstable, even though it is still a steady state. With occasional disturbances, the world economy will move away from it.

To summarize the above,

Proposition 3. Let  $R_c \in (0, R^+)$  be defined by  $f(K^*(R_c)) = 1$ . Then,

- a) If  $K^*(R) < K(\lambda)$  and  $R < R_c$ , the state in which all the countries have  $k^* = K^*(R)$ , is a stable steady state of the world economy.
- b) If  $K^*(R) < K(\lambda)$  and  $R > R_c$ , there exists no stable steady state in which all the countries have the same level of capital stock.
- c) If  $K^*(R) > K(\lambda)$ , the state in which all the countries have  $k^* = K^*(R)$ , is a stable steady state of the world economy.

Note  $R_c$  satisfies  $K^*(R_c) = K(\lambda_c)$ ; it is well-defined in  $(0, R^+)$ , since  $f(K^*(0)) = 0 < 1 = W(R^+) < f(K^*(R^+))$  and  $f(K^*(R))$  is strictly increasing and continuous in  $R$ .

Figure 6 illustrates Proposition 3. In Regions A and AB, the condition in Proposition 3a) is satisfied. In Region B, the condition in Proposition 3b) is satisfied. In Regions BC and C, the condition in Proposition 3c) is satisfied. The border between Regions AB and B is given by  $f(K^*(R)) = 1$ , or  $R = R_c$ . The border between Regions B and BC (as well as the border between A and C) is given by  $K^*(R) = K(\lambda)$ .

What happens when the condition in Proposition 3b) holds, so that the steady state with perfect equality is unstable? The next subsection provides the answer.

## 5.2. Steady States with Endogenous Inequality of Nations: The Case of Divergence.

Suppose now that the world economy is a stable steady state, in which a fraction  $X$  of the countries have the capital stock equal to  $k_L < K(\lambda)$ , and a fraction  $1-X$  of the countries have the capital stock equal to  $k_H > K(\lambda)$ . Since all the countries face the same world interest rate,  $k_L$  and  $k_H$  must satisfy  $Rf'(k_H) = r = \lambda Rf'(k_L)/(1-W(k_L))$ , or

$$(7) \quad f'(k_H) = \lambda f'(k_L)/(1-W(k_L)),$$

in addition to

$$(8) \quad k_L < K(\lambda) < k_H.$$

From Lemma b),  $k_t = k_H$  is a stable steady state for each small open economy. From Lemma c), the stability of  $k_t = k_L$  requires  $k_L < \lambda R/r = [1-W(k_L)]/f'(k_L)$ , which can be rewritten to  $k_L f'(k_L) + W(k_L) = f(k_L) < 1$ , or

$$(9) \quad k_L < K^*(R_c) = K(\lambda_c).$$

Since the young agents in the fraction  $X$  of the countries earn  $W(k_L)$  and those in the fraction  $1-X$  earn  $W(k_H)$ , the world saving is given by  $XW(k_L) + (1-X)W(k_H)$ , which is equal to the world investment, which produces  $R$  units of capital per unit. Hence, the total capital stock must satisfy

$$(10) \quad Xk_L + (1-X)k_H = XRW(k_L) + (1-X)RW(k_H).$$

A stable steady state with endogenous inequality exists if there are  $k_L$  and  $k_H$  that solve (7)-(10).

Proposition 4. Let  $R_c \in (0, R^+)$  and  $\lambda_c \in (0, 1)$  be defined by  $f(K^*(R_c)) = f(K(\lambda_c)) = 1$ . The world economy has a continuum of stable steady states, in which a fraction  $X \in (X^-, X^+) \subset (0, 1)$  of the

countries have the capital stock,  $k_L < K(\lambda)$ , and a fraction  $1 - X$  of the countries have the capital stock equal to  $k_H > K(\lambda)$ , if and only if  $\lambda < \lambda_c$ ,  $f'(K(\lambda)) > \lambda f'(K^*(R)) / [1 - W(K^*(R))]$  where  $R < R_c$ , and  $\lambda < f'(K^*(R))K(\lambda_c)$ . Furthermore,  $X^- > 0$  if  $R > R_c$  and  $X^+ < 1$  if  $K^*(R) < K(\lambda)$ .

Proof. See Appendix.

Figure 6 illustrates Proposition 4, whose condition is satisfied in Regions, AB, B, and BC. The border between Regions A and AB is given by  $f'(K(\lambda)) = \lambda f'(K^*(R)) / [1 - W(K^*(R))]$  with  $R < R_c$  and  $\lambda < \lambda_c$ . It is upward-sloping and connecting  $(\lambda, R) = (0, 0)$  and  $(\lambda, R) = (\lambda_c, R_c)$ . The border between Regions BC and C is given by  $f'(K^*(R))K(\lambda_c) = \lambda$ . This curve is downward-sloping, and stays above  $K^*(R) = K(\lambda)$  for  $\lambda < \lambda_c$ , and tangent to it at  $(\lambda, R) = (\lambda_c, R_c)$ .<sup>12</sup> Combined with Proposition 3, we can conclude the following. In Regions A and C, there is a unique stable steady state, which is symmetric. In both cases, the model predicts the convergence of economic performances across countries. In Region A, the investment is borrowing-constrained in all the countries. In Region C, the borrowing constraint is not binding in any country. In Region B, there is no stable steady state with perfect equality. Even though there are a continuum of stable steady states, they all show that the long-run distribution of the capital stock, and hence those of the income, the wage, etc, have two mass points. In Region B, therefore, the co-existence of rich and poor nations is an inevitable feature of the world economy. In other words, the model predicts that financial market globalization *causes* inequality of nations. In Region AB, and Region BC, these two types of the steady states co-exist.

The prediction of the model is most stark when the parameters lie in Region B of Figure 6. In this case,  $K^*(R) < K(\lambda)$  so that that, in autarky, each country would converge to the same steady state, in which the borrowing constraint is binding. In the presence of the international financial market, the symmetry-breaking caused by unrestricted flows of financial capital leads to the polarization of the world economy into the rich and the poor. In any such stable steady state, the rich countries accumulate enough capital that the borrowing constraint is no longer binding for the entrepreneurs in the rich

<sup>12</sup> To see this, let  $\Theta(\lambda) \equiv f'(K(\lambda))K(\lambda_c) - \lambda$ . Then,  $\Theta(\lambda_c) = f'(K(\lambda_c))K(\lambda_c) - \lambda_c = f'(K(\lambda_c))K(\lambda_c) - f(K(\lambda_c)) + (1 - \lambda_c) = (1 - \lambda_c) - W(K(\lambda_c)) = 0$ , and  $\Theta'(\lambda) \equiv f'(K(\lambda))K(\lambda_c)K'(\lambda) - 1 = K(\lambda_c)/K(\lambda) - 1 < 0$  for  $\lambda < \lambda_c$ , since  $K'(\lambda) = 1/f'(K(\lambda))K(\lambda)$  by differentiating  $W(K(\lambda)) = (1 - \lambda)$ . Therefore,  $\Theta(\lambda) > \Theta(\lambda_c) = 0$  for  $\lambda < \lambda_c$ . Thus,  $\lambda = f'(K^*(R))K(\lambda_c)$  implies  $f'(K(\lambda))K(\lambda_c) > \lambda = f'(K^*(R))K(\lambda_c)$  or  $K^*(R) > K(\lambda)$  for  $\lambda < \lambda_c$ . The tangency follows from  $\Theta'(\lambda_c) = 0$ .

countries, while it is binding for those in the poor countries ( $k_L < K(\lambda) < k_H$ ). Furthermore, one can show that, from (A2) and (10),  $k_L < K^*(R) < k_H$  in these steady states. That is to say, the rich countries become richer and the poor become poorer than in autarky. Therefore, this case provides a theoretical justification for the view that the international financial market is a mechanism through which rich countries become richer at the expense of poor countries.

When the world economy is polarized, the countries that became poor find themselves in the stable steady state with the binding borrowing constraint,  $k_L$  in Figure 3b. From a perspective of an individual country, the problems of poor countries may seem easy to solve. It may appear that, in order to escape the poverty trap and to join the club of rich countries, all the government has to do is to cut its link to the international financial market temporarily. The global perspective, however, offers a different view. Such temporary isolationist policy cannot work if it is attempted by all the countries. This is because, once the restriction is removed, a positive measure of countries must find themselves in the lower steady state. (Note that, in Region B, a fraction of the countries that become poor is bounded away from zero.) Similar points can be made for a joint attempt for the poor countries to cut their links to the rich countries and to unite among themselves. It is impossible for all of them to escape from the poverty trap. Nor would the official aids from the rich countries eliminate the inequalities. As illustrated in Figure 5, one of the reasons why the symmetric steady state is unstable is that there is not enough world saving to finance the investment required to make all the countries rich. As long as the parameters lie in Region B of Figure 6, some countries must be excluded from being rich, just as in a game of musical chairs.

## 6. Heterogeneous Agents

In the models presented above, the agents are assumed to be homogeneous. This assumption, while simplifying the analysis significantly, implies that the agents are equally willing and equally credit-worthy as an entrepreneur. This means that the saving and the investment can be equalized in the autarky case only by means of credit-rationing, when the borrowing constraint is binding. This section briefly sketches a model with heterogeneous agents, and demonstrates that, even though equilibrium

credit rationing does not occur, much of the results obtained above carry over. This should help to convince the reader that what matters in the analysis is the borrowing constraint, not the presence of equilibrium credit rationing.

Let us assume that the agents are heterogeneous in terms of their productivity as an entrepreneur. More specifically,  $R$  is now an agent-specific, and its cumulative distribution is given by  $G(R)$ , without any mass point, with the density function,  $g(R) = G'(R) > 0$ . In period  $t$ , only the young agents whose productivity satisfies  $R \geq R_t$  are willing to borrow and credit-worthy. If  $k_t \geq K(\lambda)$  so that  $R_t = r_{t+1}/f'(k_{t+1})$ , the agents with  $R < R_t$  are not willing to start the projects, because they are not profitable. If  $k_t < K(\lambda)$  so that  $R_t = r_{t+1}(1-W(k_t))/\lambda f'(k_{t+1})$ , the agents with  $R < R_t$  want to borrow but they are not denied credit, because they are not as credit-worthy as the agents with  $R \geq R_t$ . Thus, the domestic investment in period  $t$  is equal to  $1-G(R_t)$ , which is a well-defined function, and decreasing in  $R_t$ . The capital stock in period  $t+1$  is now given by

$$(11) \quad k_{t+1} = \int_{R_t}^{\infty} R g(R) dR \equiv H(R_t),$$

where  $H$  is decreasing in  $R_t$  with  $H'(R_t) = -R_t g(R_t) < 0$ .

In the autarky case, the domestic investment is equal to the domestic saving:

$$(12) \quad W(k_t) = 1 - G(R_t)$$

Since the RHS is a well-defined decreasing function in  $R_t$ , eq. (12) determines  $R_t$  uniquely as a decreasing function of  $k_t$ . Since  $R_t$  adjusts to ensure the saving-investment balances, there is no equilibrium credit rationing. The dynamics is described entirely by (11) and (12), or

$$(13) \quad k_{t+1} = H(G^{-1}(1-W(k_t))) \equiv \Lambda(k_t)$$

which is independent of  $\lambda$ . When  $k_t < K(\lambda)$ , a greater credit market imperfection reduces the interest rate, but not the dynamics of capital formation. Some algebra verifies  $\Lambda'(k_t) = R_t W'(k_t) = G^{-1}(1 - W(k_t))W'(k_t) > 0$ , and that (A2) ensures that  $\Lambda'(0) = \infty$  and  $\Lambda''(k_t) = R_t W''(k_t) - (W'(k_t))^2/g(R_t) < 0$ . Therefore, for any distribution  $G$ ,  $k_t$  converges to the unique steady state,  $K^*(G) > 0$ . In the steady state, the borrowing constraint is binding if and only if  $K^*(G) < K(\lambda)$ .

In the small open economy, eq. (3) with  $r_{t+1} = r$  and (11) yield

$$H^{-1}(k_{t+1})f'(k_{t+1}) = \begin{cases} r(1 - W(k_t))/\lambda, & \text{if } k_t < K(\lambda), \\ r & \text{if } k_t \geq K(\lambda), \end{cases}$$

where the LHS is strictly decreasing in  $k_{t+1}$ . By denoting the inverse function of  $H^{-1}(k)f'(k)$  by  $\Omega$ , the dynamics of the small open economy is described by

$$(14) \quad k_{t+1} = \begin{cases} \Omega(r(1 - W(k_t))/\lambda) & \text{if } k_t < K(\lambda), \\ \Omega(r) & \text{if } k_t \geq K(\lambda), \end{cases}$$

which defines a map from  $k_t$  to  $k_{t+1}$ , which is continuous, increasing in  $k_t < K(\lambda)$ , and constant in  $k_t \geq K(\lambda)$ . A greater credit market imperfection reduces the rate of capital formation, when  $k_t < K(\lambda)$ , but not when  $k_t \geq K(\lambda)$ . For a sufficiently high  $r$  or a sufficiently low  $\lambda$ ,  $\Omega(r) < K(\lambda)$  holds, and there are, generically speaking,  $m$  stable and  $m - 1$  unstable steady states ( $m = 1, 2, \dots$ ), all of which are located below  $K(\lambda)$ . If  $\Omega(r) > K(\lambda)$ , there is one and only stable steady state above  $K(\lambda)$ , in addition to the same number of stable and unstable steady states below  $K(\lambda)$ .

Qualitatively, (14) differs from (6) only in that there may be additional pair of stable and unstable steady states below  $K(\lambda)$ . Thus, (14) is capable of generating any qualitative feature of the dynamics generated by (6), which is nothing but a special case of (14).



A characterization of the steady states in the world economy case is hopelessly complicated.

This is only because that there may be more than two stable steady states of the small open economy, which dramatically increases the number of the types of the steady states of the world economy. If the existence of  $m$  stable steady states of the small open economy cannot be ruled out,  $2^m - 1$  types of the steady states of the world economy need to be distinguished. It should be obvious, however, that a sufficiently small heterogeneity, which makes  $H^{-1}(k)$  almost constant, would not change the nature of the model. This can be verified, for example, by letting  $G$  is the uniform distribution around a fixed parameter,  $R$ , and making the support increasingly small.

## 7. Related Work in the Literature.

Starting with Bernanke and Gertler (1989), many recent studies have examined the implications of imperfect credit markets on the aggregate investment behavior. The critical feature of the present model, --an increase in the entrepreneur's wealth eases the borrowing constraint--, is common in this literature. Many of these studies assume the presence of an alternative storage technology that helps to pin down the interest rate, which makes their models effectively partial equilibrium ones, as in the small open economy case above. What is crucial in this paper is that the extent to which the interest rate can adjust endogenously changes with financial market globalization.

Most of these studies introduce imperfect credit markets through moral hazard, adverse selection, and costly state verification models. Among macroeconomic studies that introduce the imperfect credit markets through the threat of potential defaults are Kiyotaki and Moore (1997), Obstfeld and Rogoff (1996, Ch.6.1 and Ch.6.2), and Aghion, Banerjee and Piketty (1999). The specification here follows Matsuyama (forthcoming). The main advantage of using potential defaults as a source of imperfection is its simplicity and tractability.

A large number of recent studies examine imperfect credit markets in an open economy context. They mostly focus on the issue of short-run volatility, motivated by recent economic crises in emerging markets. Only a few studies have addressed the role of the international financial market on the cross-country pattern of development in the presence of imperfect credit markets. The seminal work is

Gertler and Rogoff (1990). Their model is static, with the entrepreneur's wealth being exogenous, so that the dynamic effect of financial market globalization cannot be addressed. Furthermore, in the simple version of their model, presented in Obstfeld and Rogoff (1996, Ch.6.4), the credit market imperfection only reduces the flow of financial capital from the rich to the poor, but does not generate a reverse flow. This can happen in their model only when the poor initially has sufficiently high external debt. Boyd and Smith (1997) succeeded in eliminating these limitations of the Gertler-Rogoff model. They developed an overlapping generations model of a two-country world economy, with the imperfect credit market arising from a costly-state-verification problem. However, their model is so complicated that they had to assume that the borrowing constraint is always binding for both countries, both in and out of the steady states, and even then, they had to rely on the numerical simulation to prove the stability of asymmetric steady states. They also restricted their parameters in such a way the symmetric steady state is always unstable. The model presented in this paper has advantage of being tractable, which makes it possible to characterize all the stable steady states for a full set of the parameter values, without making any auxiliary assumption. In other words, the present model allows one to derive the analytical conditions for the stability of the symmetric and asymmetric steady states and for the borrowing constraint to be binding in these steady states. This in turn makes it possible to examine the effects of changing the parameter values, making the model useful as an intuition-building device on the issue of convergence versus divergence. Furthermore, the analysis have shown that the rich are not borrowing-constrained and the poor remain borrowing-constrained in all asymmetric stable steady states, which exist whenever the symmetric steady state is unstable.

Matsuyama (forthcoming) also developed a model of endogenous inequality based on credit market imperfections that arise from the potential risk of defaults. The model differs from the present one in two crucial respects. First, it is a closed economy model. Second, its intertemporal transmission mechanism operates through bequests. These features played useful roles for addressing the issue of inequality across households within an economy. In the present model, the agents have no bequest motives and intertemporal transmission operates through nontraded factor markets, which helps to focus on the issue of inequality across nations.

The present paper is concerned with globalization of financial markets. Some studies considered the effects of globalization in goods markets on the pattern of economic development. Matsuyama (1991) showed that the properties of an endogenous growth could change drastically in the presence of the world trade. Both Krugman and Venables (1995) and Matsuyama (1996) have shown, in static models of the world economy that consists of identical economies, that the symmetry-breaking caused by international trade leads to a polarization of the world economy into the rich and the poor.<sup>13</sup> In these models, however, the countries that become poorer than others may or may not be poorer than in autarky. In contrast, the poor countries are always poorer than in autarky in the present model.

## 8. Concluding Remarks

This paper analyzed the effects of financial market globalization on the cross-country pattern of development in the world economy. To this end, it developed a dynamic macroeconomic model of imperfect credit markets, in which the investment becomes borrowing-constrained at the lower stage of development. In the absence of the international financial market, the world economy converges to the symmetric steady state, and the cross-country difference disappears in the long run. In the presence of the international financial market, the symmetric steady state could lose its stability, in which case the cross-country distribution of the capital stocks is concentrated into two mass points in all the stable steady states. The symmetry-breaking caused by unrestricted flows of financial capital leads to a polarization of the world economy into the rich and the poor. The model thus demonstrates the possibility that financial market globalization may cause, or at least magnify, inequality of nations, and the international financial market is a mechanism through which some countries become rich at the expense of others. At the same time, the model also suggests that poor countries cannot jointly escape from the poverty trap by merely cutting their links to rich countries.

Needless to say, the model presented is highly stylized, and can be extended in a number of ways. Let us suggest four extensions that seem particularly important.

First, in the analysis above, the effects of financial market globalization were examined by comparing the two extreme cases, autarky and full financial market integration. It would be more

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<sup>13</sup> As the reader might have noticed, the title of this paper was inspired by that of Krugman and Venables (1995).

satisfactory to introduce some parameters that may be interpreted as a measure of financial market globalization. Any empirical assessment of the impacts of the international financial market on the cross-country pattern of development would require such an extension.

Second, the model assumes that globalization has no effect on the penalty for defaults, and hence the efficiency of the credit markets. This assumption may be justified as a benchmark case, because it is not obvious in which direction globalization might affect the operation of credit markets.<sup>14</sup> Yet, the reader should keep in mind that the results of this paper is conditional on this assumption. At the same time, it is highly desirable to combine the present macroeconomic model with a variety of microeconomics of credit markets in such a way that one could examine full impacts of globalization of financial markets.

Third, the model assumes that the project started by an agent produces the capital good that can be used only in his/her country. This assumption seems reasonable when the capital good is interpreted as a form of human capital. For the physical capital good, this assumption may be too restrictive. If starting a project in a foreign country is possible (with some costs), the agents in rich countries have an incentive to a project in poor countries, where the rental rate of capital is high. Such an extension would be useful for examining the effects of foreign direct investment.

Fourth, the model does not allow for sustainable growth of the world economy as a whole. It would be interesting to examine the condition under which endogenous inequality of nations occurs in a growing global economy. This would require the model to be extended in such a way that the minimum requirement for the project investment would increase with the growth of the world economy.

It is hoped that the model presented in this paper serves as a useful first step toward these extensions.

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<sup>14</sup> On one hand, one might argue that, the lower the cost of international financial transactions is, the harder it becomes to catch the borrowers that defaulted. If so, globalization has the effect of reducing the efficiency of credit markets. On the other hand, one might also argue that the globalization and resulting competition for the world saving provide a greater incentive for an individual country to improve legal and other protections for both domestic and foreign creditors. If so, globalization has the effect of enhancing the efficiency of credit markets.

## Appendix

Proof of Lemma.

- a) This follows from that  $\Psi$  is a continuous map on  $[0, R]$  into itself.
- b) This is trivial, because the map,  $\Psi$ , is constant above  $K(\lambda)$  and equal to  $\Phi(r/R)$ .
- c) Differentiating (6) yields  $\Psi'(k_t) = k_t[f''(k_t)/f'(\Psi(k_t))](r/\lambda R)$  for  $k_t < K(\lambda)$ . By setting  $k_t = \Psi(k_t) = k$ , the slope of the map at a steady state,  $k < K(\lambda)$ , is equal to  $\Psi'(k) = k(r/\lambda R)$ , which is increasing in  $k$ . Also,  $\Psi(0) = \Phi(r/\lambda R) > 0$ . Therefore, at the smallest steady state,  $0 < k_L < K(\lambda)$ , if there is one, either  $\Psi$  is tangent to the 45° line (i.e.,  $\Psi'(k_L) = k_L(r/\lambda R) = 1$  or  $k_L = \lambda R/r$ ), in which case it is the only intersection below  $K(\lambda)$ , or  $\Psi$  cuts the 45° line from above (i.e.,  $\Psi'(k_L) = k_L(r/\lambda R) < 1$  or  $k_L < \lambda R/r$ ), in which case it is stable. At the second smallest steady state,  $k_M$ , if it exists,  $\Psi$  cuts the 45° line from below (i.e.,  $\Psi'(k_M) = k_M(r/\lambda R) > 1$ , or  $k_M > \lambda R/r$ ) and hence it is unstable, which also implies that  $\Psi$  cannot cut the 45° line from above between  $k_M$  and  $K(\lambda)$ , ruling out the existence of a third steady state below  $K(\lambda)$ . This completes the proof.

Proof of Proposition 2.

The proof consists of four steps.

*Step 1.* Since  $f(K(\lambda))$  is strictly decreasing and continuous in  $\lambda$  and  $f(K(1)) = f(0) = 0 < 1 = W(R^+) < f(R^+) = f(K(0))$ ,  $\lambda_c \in (0, 1)$  is well-defined and  $f(K(\lambda)) > (<) 1$  if and only if  $\lambda < (>) \lambda_c$ .

*Step 2.* Consider the nongeneric case of  $Rf'(K(\lambda)) = r$ . Then,  $K(\lambda) = \Phi(r/R)$  and hence  $K(\lambda)$  is a fixed point of the map,  $\Psi$ . Because  $f(K(\lambda)) - 1 = K(\lambda)f'(K(\lambda)) + W(K(\lambda)) - 1 = K(\lambda)r/R - \lambda = \lambda[\lim_{k \uparrow K(\lambda)} \Psi'(k) - 1]$ , the left derivative of the map at  $K(\lambda)$  is greater (less) than one if and only if  $f(K(\lambda)) > (<) 1$  or  $\lambda < (>) \lambda_c$ . These properties are illustrated in Figure A1 for  $\lambda < \lambda_c$  and Figure A2 for  $\lambda \geq \lambda_c$ . Note that, from Lemma,  $\Psi$  has another intersection,  $0 < k_L < K(\lambda)$ , in Figure A1, and has no other intersection in Figure A2.

*Step 3.* Consider the case where  $Rf'(K(\lambda)) < r$ . This case can be studied by reducing  $R$ , starting from the case,  $Rf'(K(\lambda)) = r$ , while fixing  $\lambda$  and  $r$ . This change is captured by a downward shift of the map,  $\Psi$ , in Figures A1 and A2. Clearly, with any downward shift,  $\Psi$  has the unique stable fixed point, which satisfies  $k_L < K(\lambda)$ . This proves Proposition 2a).

*Step 4.* Consider the case where  $Rf'(K(\lambda)) > r$ , which can be studied by increasing  $R$ , starting from the case,  $Rf'(K(\lambda)) = r$ , while fixing  $\lambda$  and  $r$ . This change is captured by an upward shift of the map  $\Psi$  in Figures A1 and A2. In Figure A2, i.e., if  $f(K(\lambda)) \leq 1$ ,  $\Psi$  has the stable unique fixed point,  $k_H = \Phi(r/R) > K(\lambda)$ , after any upward shift. In Figure A1, i.e., if  $f(K(\lambda)) > 1$ , there is a critical value of  $R$ ,  $R'$ , such that, if  $r/f'(K(\lambda)) < R < R'$ , there are three fixed points,  $k_L < k_M < K(\lambda) < k_H$ , and, if  $R > R'$ , there is the unique fixed point,  $k_H = \Phi(r/R) > K(\lambda)$ . In the borderline case,  $R = R'$ ,  $\Psi$  is tangent to the  $45^\circ$  line below  $K(\lambda)$ . From Lemma c), the value of  $k$  at the tangency is equal to  $\lambda R'/r$ , and hence  $\Psi(\lambda R'/r) = \lambda R'/r$ , which can be rewritten as  $(\lambda R'/r)f'(\lambda R'/r) = 1 - W(\lambda R'/r)$ , or  $f(\lambda R'/r) = 1$ . Thus,  $f(\lambda R/r) < 1$  implies the three fixed points and  $f(\lambda R/r) > 1$  implies the unique steady state,  $k_H = \Phi(r/R) > K(\lambda)$ . This proves Proposition 2b) and 2c).

#### Proof of Proposition 4.

First, note that (7) defines  $k_H$  as a function of  $k_L$ . Differentiating (7) shows that this function, denoted by  $k_H = \phi(k_L)$ , is increasing if and only if  $f(k_L) < 1$  or equivalently  $k_L < K^*(R_c) = K(\lambda_c)$ . Furthermore, it satisfies  $\phi(0) = 0$  and  $\phi(K(\lambda)) = K(\lambda)$ . If  $\lambda \geq \lambda_c$ ,  $K(\lambda) \leq K(\lambda_c)$  and hence  $k_L < K(\lambda)$  implies  $k_H = \phi(k_L) < \phi(K(\lambda)) = K(\lambda)$ , which violates (8). If  $\lambda < \lambda_c$ , the set of  $(k_L, k_M)$  that satisfies (7), (8), and (9) is nonempty, and illustrated by the solid curve in Figure A3.

Second, (A2) and (8) imply that (10) has a solution,  $X \in (X^-, X^+) \subset (0, 1)$ , if and only if  $k_L < K^*(R) < k_H$ . This condition is illustrated by the shaded area in Figure A3. Therefore, a stable steady state,  $(k_L, k_H, X)$ , exists if and only if the solid curve (the segment of  $k_H = \phi(k_L)$  that satisfies (8) and (9)), overlaps with the shaded area, or equivalently, if and only if  $K(\lambda) < \phi(K^*(R))$  and  $\phi(K(\lambda_c)) = \phi(K^*(R_c)) > K^*(R)$ . The first condition can be rewritten to  $f'(K(\lambda)) > \lambda f'(K^*(R)) / [1 - W(K^*(R))]$  where  $R < R_c$  and the second to  $\lambda < f'(K^*(R))K(\lambda_c) = f'(K^*(R))K^*(R_c)$ .

That  $X^- > 0$  requires that the upper-right end of the solid curve is strictly inside the shaded area, or equivalently  $R > R_c$ . Similarly,  $X^+ < 1$  if and only if the lower-left end of the solid curve is strictly inside the shaded area, or equivalently,  $K(\lambda) > K^*(R)$ .

This completes the proof.

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