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Wiseman, Thomas; Yilankaya, Okan

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Cooperation, Secret Handshakes, and Imitation in the Prisoners' Dilemma

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Discussion Paper No. 1248

Cooperation, Secret Handshakes, and Imitation in the Prisoners' Dilemma

By

Thomas Wiseman
Northwestern University
Department of Economics
twiseman@nwu.edu

and

Okan Yilankaya
Northwestern University
Department of Economics
okan-y@nwu.edu

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Cooperation, Secret Handshakes, and Imitation in the Prisoners' Dilemma*

Thomas Wiseman and Okan Yilankaya**

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Abstract

In the prisoners' dilemma game, the only evolutionarily stable strategy is defection, even though mutual cooperation yields a higher payoff. Building on a paper by Robson (1990), we introduce mutants who have the ability to send a (costly) signal, i.e., the "secret handshake," before each round of the game and to condition their actions on whether or not they observe the same signal from their opponent. A population playing the strategy "always defect" is vulnerable to secret handshake mutants who cooperate when they meet other secret handshakers and defect against other opponents. However, these secret handshakers are in turn vulnerable to a second round of mutants who imitate the secret handshake and then defect against all opponents. But now a new group of secret handshakers with a different secret handshake can arise. Thus, play can cycle between cooperation and defection. We study the dynamics of that cycling. We show that in the limit, as the probability of mutation goes to zero, cooperation occurs on average half the time. Using simulations to study the implications of our model when the mutation probability is larger than zero, we find that it is possible for cooperation to be sustained for long periods. In general, cooperation is favored when mutual cooperation has a large payoff advantage over mutual defection, and when the payoff advantage of unilateral defection is small. Surprisingly, however, there are cases where an increased payoff to unilateral defection actually raises the level of cooperation. *Journal of Economic Literature* Classification Numbers: C70, C72, D60.

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** Department of Economics, Northwestern University, 2003 Sheridan Road, Evanston, IL 60208; twiseman@nwu.edu, okan-y@nwu.edu.

1. Introduction

In the context of evolutionary games, an evolutionarily stable strategy (ESS) is one that has a higher average payoff than any other strategy against any population where most of the population plays the ESS, and the rest plays the other strategy. That is, a population playing an ESS can repel "invaders" playing any other strategy, since the invaders will achieve a lower payoff and be driven to extinction under any payoff monotone dynamics. Evolutionary stability does not guarantee efficiency. A game may have multiple ESS's, each with a different payoff. If a low payoff ESS is played, the definition of ESS means that play will stay there — players are trapped in an inefficient equilibrium.

Signaling can provide a way out of this dilemma. Robson (1990) demonstrates that if one ESS of a game yields a higher payoff than another, then a population playing the inefficient ESS can be successfully invaded by a mutant who has the ability to send a signal (the "secret handshake") before the game is played. (Signaling of this kind that is costless is usually referred to as "cheap talk.") When a mutant observes the signal from its opponent, it knows that it is facing another mutant. The mutant plays the high payoff ESS when it meets another mutant; against the original population, it matches the low payoff ESS played by its opponent. Thus, the mutant gets a higher average payoff than the original population, so any ESS that is not Pareto efficient cannot survive.

Robson (1990) also shows that if the type of mutation that he proposes is allowed, any payoff outcome that does not come from an ESS cannot survive. In the prisoners' dilemma, for example, mutual cooperation is not sustainable, because mutual defection (even though it gives a lower payoff than mutual cooperation) is the only ESS. The argument is as follows: Although a population playing the strategy "always defect" is vulnerable to a round of signaling mutants who cooperate with each other and defect against the rest of the population, those mutants are themselves vulnerable to a second round of mutants. The new mutants imitate the secret handshake of the old mutants, but defect against every opponent. When matched with a new mutant, an old mutant is tricked into cooperating while its opponent defects and receives the worst possible payoff. Selection then drives out the first round of mutants, leaving a mixture of the original population and the second round of mutants. Some of the players send the phony

signal, but every player defects every time, so everyone gets the ESS payoff. Thus, cooperation, the efficient outcome, can be attained only temporarily.

Robson (1990) ends his analysis there. But what if a third mutant is introduced, one that has a new secret handshake that the second mutant cannot imitate? Then the newest mutant would thrive, until a fourth mutant that imitates the new secret handshake arises and drives out the third mutant. Now, though, the population is vulnerable to yet another mutant with yet another secret handshake, and so on. Play can cycle between defecting and cooperating, never reaching a stable state. In this paper, we study the dynamics of that constant cycling. We show that in the limit, as the probability of mutation goes to zero, cooperation occurs on average half the time. We use simulations to study the dynamics when the mutation probability is larger than zero, with an emphasis on determining what values of the parameters lead to increased cooperation. We find that in the framework of our model it is possible for cooperation to be sustained for long periods. For a wide range of parameter values, on average (over time) a significant fraction of the population cooperates. In general, cooperation is favored when mutual cooperation has a large payoff advantage over mutual defection, and when the payoff advantage of unilateral defection is small. Surprisingly, however, we show that in some cases an increased payoff to unilateral defection can actually raise the level of cooperation.

Robson (1990) examines biological evolution, but the model can just as easily be used to describe a social process. In a population where people are engaged in an interaction that can be modeled as a prisoners' dilemma, a group of them can find a way to cooperate among themselves when everyone else is defecting. They identify themselves to each other by giving a "secret handshake."[1] If members of another group learn to imitate the secret handshake, though, and trick the secret handshakers into cooperating while they defect, then the handshakers are at a disadvantage. But a new and different secret handshake will again allow cooperation to thrive.

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[1] Frank (1987) suggests that facial expression, tone of voice, posture, or other physical characteristics can serve as signals.

There are examples of signaling and imitation in repeated interactions in both biology and economics. The North American monarch butterfly feeds on milkweeds, which contain chemicals that are poisonous to birds. Any bird that makes the mistake of eating such a butterfly vomits. The monarch uses the brightly colored patterns on its wings to signal potential predators that it is unpalatable. But other, perfectly edible butterflies, like the viceroy butterfly, live in the same habitats and exhibit very similar patterns on their own wings. By imitating the appearance of the monarch, the viceroy increases its own chances of survival (Owen, 1980). A variety of other insects, as well as certain mammals, plants, birds, snakes, fish, and amphibians, are also natural signal mimics (Wickler, 1968).

In the realm of human economic behavior, the wearing of particular fashions of clothing can act as a signal to other members of the population. New fashion signals are repeatedly developed as the old ones become widely imitated. Pesendorfer (1995) models people as engaging in a "dating game," in which they strive to match themselves with high-quality mates. He argues that the upper stratum of society wears the latest fashions to differentiate itself from the lower classes. They abandon the fashion for a new one as soon as the lower strata adopt the old one. To illustrate his point, he quotes an observer of 18th century Paris: "nothing makes noble persons despise the gilded costume so much as to see it on the bodies of the lowest men in the world... So the upper classes had to invent new 'gilded costumes,' or new distinctive signs, whatever they might be, every time complaining that 'things have changed indeed, and the new clothes being worn by the bourgeois, both men and women, cannot be distinguished from those of persons of quality' " (M. de Paulmy, 1774 p. 220, as quoted in Pesendorfer (1995)).

Another example of signaling and imitation in human interaction is the counterfeiting of paper money. The intricate designs on a bill are intended to signal economic agents that the bill represents real wealth and is not just a scrap of paper. Counterfeiters learn to copy the designs, though, and manufacture fake money that is indistinguishable from the real thing. Now the issuers of the money must invent a new design, with microfilaments or color-changing inks or other features that are more difficult to copy. The goal of the issuers is to continually come up with new designs for their money before the old ones are too widely counterfeited.

The problem faced by professors in trying to write questions for an exam is another example of signaling and imitation in human interaction. Utility-maximizing professors are tempted to reuse questions from exams given in previous years by changing a few words or numbers, either to save time and effort or because the old questions are the best for measuring students' comprehension. But some students can prepare by memorizing how to solve problems that have shown up on earlier exams, rather than by studying and learning the material. Then, when they recognize a similar question on their own exam, they can write down the solution without thinking. The students who memorize have an advantage over the students who study and try to solve the problem during the exam period, and professors cannot use the exam to accurately evaluate students. However, professors can counter by writing questions that seem similar to ones that the students have memorized, but in fact have a different solution. That is, modified questions mimic the characteristic form of the memorized question. The students who memorized copy down the old, wrong answer, and the ones who studied work to solve the new problem. The memorizers are thwarted. Eventually, though, students will learn to recognize the modified question, and the process starts again.

None of these examples fit *exactly* the model that is described in this paper. They do demonstrate, however, that similar cycles of signaling and imitation actually take place, and that examining the dynamics of the cycling can provide insight into real-life interactions.

We focus on the prisoners' dilemma because of its broad applications in economic modeling. Versions of the prisoners' dilemma have been used to model interactions as diverse as international politics (Muller-Furstenberger and Stephan, 1997; Cohen, 1994; Hillman, 1989; Kimenyi, 1988; Majeski, 1984), development of social norms (Bicchieri et al, 1997), the provision of public goods (Okada and Sakakibara, 1991; Hillier, 1989; Hirshleifer, 1986), family interaction (Linster, 1998), duopoly (Rosenthal and Spady, 1989; Conybeare, 1987), animal behavior (Tullock, 1994; Maynard Smith, 1982), and, of course, criminal legal strategy (Ashenfelter and Bloom, 1990). In addition, there is a large literature on evolution in the repeated prisoners' dilemma. The goal of much of that literature (Guttman, 1996; Ockenfels, 1993; Bergstrom and Stark, 1993; Nachbar, 1992;

Fudenberg and Maskin, 1990; Axelrod 1984) has been to find ways that cooperation can survive, despite not being an ESS. Fudenberg and Maskin (1990), for example, show that if players use strategies of finite complexity and make mistakes with positive probability, then in the repeated prisoners' dilemma cooperation actually is evolutionarily stable. The literature on cheap talk in games has also focused on equilibrium selection. Using arguments similar to those of Robson (1990), papers by Kim and Sobel (1995) and Matsui (1991) have shown that pre-play communication leads to the efficient equilibrium, if one exists, because inefficient outcomes can be destabilized by drift. Bhaskar (1998) obtains a similar result by relying on noisy communication rather than drift.

The rest of the paper is organized as follows: Section 2 presents the model, Section 3 describes the cycle of play, Section 4 reports the analytic and simulation results, and Section 5 concludes.

2. The Model

There is an infinite population of players. In each period, players are randomly paired to play the symmetric 2×2 game G, below. G is a prisoners' dilemma where, without loss of generality, the payoff to cooperating (strategy C) when one's opponent defects (strategy D) is normalized to one:

G :

	C	D
C	x	1
D	y	z

where $y > x > z > 1$. Defection is the only ESS of the prisoners' dilemma, and (D,D) is also the only Nash Equilibrium.

In the standard evolutionary model, the players are divided into different types. Each type plays a different strategy, so a player's type is synonymous with the strategy that it plays. The growth rate of each type depends on its evolutionary fitness, which is measured by its average payoff against the population. Under the replicator dynamics (in fact, under any payoff monotone dynamics), the population share of the type that defects

converges to 100 percent, as long as there are some defectors in the initial population. Cooperating types are always driven out of the population by defectors.

However, if the game is modified to allow for a mutant with the ability to give a secret handshake after being matched with an opponent but before playing G, and if the mutant can condition its play on whether or not it observes the secret handshake from its opponent, then an allowable strategy is to cooperate against other mutants and to defect against the rest of the population. Call that strategy S. We assume that there is a small cost, $c > 0$, incurred by a player giving the secret handshake. The second round of mutants is the imitators. They give the secret handshake before playing G (also at cost c), but then defect regardless of their opponent.[2] Call that strategy M. The next round of mutants will use a new secret handshake and will not be fooled by the imitators. But the next round will imitate that new secret handshake, and so on. Rather than adding infinite numbers of S', M', S'', M'', ... , it is simpler to set up the game with play switching between two payoff regimes. In one, the imitators successfully mimic the secret handshake. In the other, there is a new secret handshake, and the imitators are still using the old one. To model the game this way, we make the simplifying assumption that there is at most one group of secret handshakers at a time, all using the same secret handshake. Similarly, not more than one secret handshake is being imitated in each period. Allowing multiple secret handshakes to exist at the same time would not change the limiting results, and we believe that it would have little effect on the game's dynamics when the mutation probability is greater than zero.

The first payoff regime represents the situation where both the secret handshakers and the imitators have entered. Two type S players matched together achieve the payoff to mutual cooperation, x, at a cost of c, for a net payoff of $x - c$. Against an imitator, a type S player is tricked into cooperating in the face of defection, and gets payoff $1 - c$; the imitator receives payoff $y - c$. Two imitators matched together get $z - c$ from mutual defection, as does an imitator or secret handshake paired with a defector. Defectors receive payoff z against any type of opponent. Call this regime GI.

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[2] We make the cost of imitating equal to the cost of signaling in order to simplify the exposition. The qualitative results do not change if the costs are allowed to differ. Frank (1987) discusses the relative costs of signaling and imitation.

In *GI*, the imitators will drive out the secret handshakers. But then a new round of mutants with a different secret handshake will arise. They will not be fooled by the imitators. That situation is represented by the second payoff regime, *GII*. In *GII*, secret handshakers still get payoff $x - c$ from cooperating with each other, but now they defect and get $z - c$ against both defectors and imitators. Defectors receive payoff z against every opponent, and imitators get $z - c$ against every opponent. Now the imitators are at a disadvantage, since in regime *GII* strategy *D* dominates strategy *M*.

The two regimes are shown below:

GI :

	D	S	M
D	z	z	z
S	z-c	x-c	1-c
M	z-c	y-c	z-c

GII :

	D	S	M
D	z	z	z
S	z-c	x-c	z-c
M	z-c	z-c	z-c

The strategy *C* has been removed, since it is driven out by *D* under any monotone dynamics.[3]

Fitness and Dynamics Within Regimes:

At any time there are potentially three types of players in the population, corresponding to the strategies that they play: *D*, *S*, and *M*. Let the fraction of the population made up of players of type i in period t be denoted σ^i_t, for $i \in \{D, S, M\}$. Let the average payoff of strategy i at time t be denoted π^i_t, and let the average payoff of the whole population be denoted π_t. Natural selection proceeds according to the replicator dynamics. Under the replicator dynamics, the rate of growth of each type is proportional to the ratio of its average payoff to the population average payoff. In terms of population shares, that relationship implies that $\sigma^i_{t+1} = (\pi^i_t / \pi_t)\sigma^i_t$.

[3] For the same reason, we set the probability that a mutant who plays the strategy *C* re-enters the population equal to zero. That assumption should have no qualitative effects on the dynamics of the model.

In regime *GI*, the average payoffs (π^i_t) to the types are as follows:

$$\pi^D_t = z,$$
$$\pi^S_t = \sigma^D_t(z-c) + \sigma^S_t(x-c) + \sigma^M_t(1-c)$$
$$= \sigma^D_t z + \sigma^S_t x + \sigma^M_t - c,$$
$$\pi^M_t = (\sigma^D_t + \sigma^M_t)(z-c) + \sigma^S_t(y-c)$$
$$= (\sigma^D_t + \sigma^M_t)z + \sigma^S_t y - c, \text{ and}$$
$$\pi_t = \sigma^D_t\pi^D_t + \sigma^S_t\pi^S_t + \sigma^M_t\pi^M_t.$$

The resulting population dynamics are

$$\sigma^D_{t+1} = (z/\pi_t)\sigma^D_t,$$
$$\sigma^S_{t+1} = [(\sigma^D_t z + \sigma^S_t x + \sigma^M_t - c)/\pi_t]\sigma^S_t, \text{ and}$$
$$\sigma^M_{t+1} = [((\sigma^D_t + \sigma^M_t)z + \sigma^S_t y - c)/\pi_t]\sigma^M_t$$
$$(= 1 - \sigma^D_{t+1} - \sigma^S_{t+1}).$$

In regime *GII*, the average payoffs are

$$\pi^D_t = z,$$
$$\pi^S_t = (\sigma^D_t + \sigma^M_t)(z-c) + \sigma^S_t(x-c)$$
$$= (\sigma^D_t + \sigma^M_t)z + \sigma^S_t x - c,$$
$$\pi^M_t = z - c, \text{ and}$$
$$\pi_t = \sigma^D_t\pi^D_t + \sigma^S_t\pi^S_t + \sigma^M_t\pi^M_t,$$

and the dynamics are

$$\sigma^D_{t+1} = (z/\pi_t)\sigma^D_t,$$
$$\sigma^S_{t+1} = [((\sigma^D_t + \sigma^M_t)z + \sigma^S_t x - c)/\pi_t]\sigma^S_t, \text{ and}$$
$$\sigma^M_{t+1} = [(z-c)/\pi_t]\sigma^M_t$$
$$(= 1 - \sigma^D_{t+1} - \sigma^S_{t+1}).$$

In regime *GI*, the imitators mimic the right secret handshake. Consequently, the secret handshakers are at a disadvantage. In fact, strategy M dominates strategy S — the imitators do better against secret handshakers and other imitators, and get the same payoff against defectors. How well the secret handshakers fare relative to defectors depends on how many imitators and secret handshakers are in the population. The secret handshake does better against itself, getting the mutual cooperation payoff x instead of the mutual defection payoff z, but it does worse against imitators, because it cooperates against the imitators' defection and gets payoff 1 instead of defecting and getting z. Also, the secret handshakers pay a cost c against every opponent. Thus, the higher the fraction of type S in the population and the lower the fraction of type M, the better the secret handshakers do compared to the defectors. A large gain from mutual cooperation ($x - z$), a small payoff to mutual defection (z), and a low cost of signaling (c) also favor the secret handshakers. Overall, the fitness of type S is higher than the fitness of type D if and only if $(x-z)\sigma^S_t > (z-1)\sigma^M_t + c$. The imitators' fitness relative to the defectors also depends on the population share of the secret handshakers. Against defectors and other imitators, imitators get the same mutual defection payoff z as the defectors, but against secret handshakers they trick their opponents and receive the reward of betrayal, y. Balancing that advantage is the fact that the imitators must also pay cost c against every opponent. Thus, a large fraction of type S in the population, a high value of a defector's bonus from having a trusting opponent ($y - z$), and a low cost of signaling (c) favor the imitators. Overall, the imitators have a higher fitness than the defectors if and only if $(y-z)\sigma^S_t > c$.

In regime *GII*, the secret handshakers have the upper hand. Now the imitators are using the wrong secret handshake, and strategy S dominates strategy M — the imitators' advantage came from tricking the secret handshakers, which they can no longer do. Now they get the same payoff z as the defectors against every opponent while having to pay the cost c. The success of secret handshakers relative to defectors depends on the population share of the handshakers, as mentioned before. The secret handshakers' average payoff is higher than the defectors' if and only if $(x-z)\sigma^S_t > c$.

Since all the payoffs are greater than zero, $\sigma^i_{t+1} = 0$ only if $\sigma^i_t = 0$. That is, once a type is in the population, it can never die out. Even the population share of a strategy that

is strictly dominated will only approach zero asymptotically. However, extinction of types is crucial in the way that evolution is modeled here. This problem is avoided with the assumption that if $\sigma^i_t < k$, then σ^i_t drops immediately to zero, where the threshold value k is between zero and one. Once the proportion of a type in the population drops below the threshold value, the type dies out.

Mutation and Switching Between Regimes:

The mutation process and the rules for switching between payoff regimes are as follows: Play begins in regime *GI* with a population made up entirely of defectors. In any period when the population share of one of the strategies is zero, there is a probability p that a mutant group arises that plays that strategy.[4] Imitator mutants can enter only if there are secret handshakers to imitate. For ease of exposition, we let the initial population share of an entering mutant be equal to the threshold value k. After a secret handshake mutation, play shifts to regime *GII*, where the new secret handshake cannot be imitated by any type M players that may be in the population[5]. (The first time the secret handshakers enter, of course, there will not be any type M players. Later on, there may be.) After an imitation mutation, play switches to regime *GI* — the new imitators are copying the right secret handshake. The introduction of defectors will not affect whether the imitators are mimicking the correct secret handshake or an obsolete one, so play remains in whichever regime it was in before the defection mutation.

Mutation probabilities for the different types are independent of each other. A secret handshake mutation and an imitation mutation cannot occur in the same period,

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[4] We assume that the probability of a mutation is the same for each type: p. We also assume that p is constant across periods — mimicking secret handshakes or inventing new ones becomes neither easier nor harder. Extensions of this paper might relax those assumptions. For example, a type's mutation probability in a given period might depend on the payoff that it would receive if it entered the current population. Alternatively, the size of a mutation might be random, so the probability of a sustainable secret handshake mutation would depend on the threshold value k.

[5] Immediately after the type S mutation, the secret handshakers' population share is k, so their fitness as measured by average payoff is $kx + (1-k)z - c$. To survive, they must have a higher fitness than the Nash equilibrium payoff z earned by the defectors. That means that the threshold value k must be greater than $c/(x-z)$ — the greater the proportion of type S in the population, the more often secret handshakers are paired together and enjoy the higher payoff of mutual cooperation. If k is less than this minimum size, then each wave of type S mutants will immediately be driven out by the defectors, who do not have to pay the cost c. In the rest of the paper, therefore, we assume that k is large enough (or c is small enough) that $k > c/(x-z)$.

since secret handshakers must be in the population for an imitator mutant to arise. However, it is possible for secret handshakers and defectors to enter at the same time if imitators make up the whole population, or for defectors and imitators to do so if secret handshakers have a 100 percent population share. If secret handshakers and defectors enter together, each gets an initial population share of k, leaving $1 - 2k$ for the imitators. Play switches to regime *GII* with a new secret handshake. When defector and imitator mutants arise at the same time, they both get share k, and play switches to regime *GI*.

The mutation and regime-switching processes are summarized here:

- If $\sigma^S_t = 0$, then with probability p a secret handshake (S) mutation occurs in the next period. If such a mutation does occur, then $\sigma^S_{t+1} = k$, $\sigma^D_{t+1} = (1 - k)\sigma^D_t$, and $\sigma^M_{t+1} = (1 - k)\sigma^M_t$. After the mutation, play is in regime *GII*.
- If $\sigma^M_t = 0$ and $\sigma^S_t > k$, then with probability p an imitation (M) mutation occurs in the next period. If such a mutation does occur, then $\sigma^M_{t+1} = k$, $\sigma^D_{t+1} = (1 - k)\sigma^D_t$, and $\sigma^S_{t+1} = (1 - k)\sigma^S_t$. After the mutation, play is in regime *GI*.
- If $\sigma^D_t = 0$, then with probability p a defection (D) mutation occurs in the next period. If such a mutation does occur, then $\sigma^D_{t+1} = k$, $\sigma^S_{t+1} = (1 - k)\sigma^S_t$, and $\sigma^M_{t+1} = (1 - k)\sigma^M_t$. The regime does not change after a defection mutation.[6]

Note that when a mutant enters, its share k of the population is taken from the other types in proportion to their own population shares.

3. Cycle of Play

Play begins in payoff regime *GI* with a population made up entirely of defectors. Every period there is a probability p that a secret handshake mutant will enter. The

[6] If a secret handshake mutation and a defection mutation both occur after period t, then $\sigma^S_{t+1} = k$, $\sigma^D_{t+1} = k$, and $\sigma^M_{t+1} = 1 - 2k$. Play switches to regime *GII*. If a defection mutation and an imitation mutation both occur after period t, then $\sigma^D_{t+1} = k$, $\sigma^M_{t+1} = k$, and $\sigma^S_{t+1} = 1 - 2k$. Play switches to regime *GI*. It is not possible for a secret handshake mutation and an imitation mutation to occur in the same period.

expected time until the mutation occurs is $1/p$. When it does occur, play shifts to regime *GII*, and the secret handshakers begin to drive out the defectors. As the population share of type *S* increases, so does its fitness advantage over the defectors.

In every period after the secret handshakers enter, there is a probability p that an imitator mutant emerges to take advantage of them. The expected time from the *S* mutation to the *M* mutation is again $1/p$. If p is small (and $1/p$ is large), the imitation mutation will most likely occur after the defectors have been driven to extinction. (It is possible, in fact, that the defectors will die out and arise again via mutation before the imitator mutant enters. However, a defector mutant in a population of secret handshakers will immediately be driven out again.) After the mutation, play shifts back to regime *GI*, and the imitators will grow at the expense of the tricked secret handshakers.

When p is small, the population share of the secret handshakers will probably shrink to zero before the next mutant enters. If it does, then the entering mutant can be either a defector or a secret handshake. (The probability that both types enter at the same time can be ignored for small values of p.) If a secret handshake emerges first, with a new secret handshake, play will switch to regime *GII*. Now the handshakers have the advantage and will drive out the imitators. The situation will be the same as before the imitators entered — a population of secret handshakers in payoff regime *GII*. On the other hand, if a defector mutant enters before the secret handshakers, then play will remain in regime *GI*. With no secret handshakers in the population, the defectors will drive out the imitators. That process will lead to a population of defectors in regime *GI*, which is just the way the game began.

To sum up: Assume that p is small, so that the expected time between mutations is large relative to the time required for natural selection in each payoff regime to reach a steady state. (Of course, that state is not a steady state of the game, since mutants can enter, but only of the regime.) Then secret handshakers will enter the initial population of defectors and drive them out. Defector mutants may reemerge, but they will immediately die out again. Eventually (after about $1/p$ periods), imitators will arise and

drive out the secret handshakers. The population of imitators is vulnerable to two kinds of mutants, so the expected time until the imitators are overthrown is only about $1/(2p)$.[7]

If a defection mutation occurs first, the imitators will be driven to extinction, and the cycle of play starts again from the beginning. If the secret handshakers emerge first, they will drive out the imitators, and play will pick up at the point in the cycle just before the imitation mutation. Thus, each type spends part of the time with a 100 percent population share and most of the rest of the time with a zero share. The secret handshakers spend twice as much time as the imitators with a 100 percent share, because the expected time until the secret handshakers get driven out ($1/p$) is twice as long as the expected time until the imitators are replaced ($1/(2p)$). The secret handshakers also have twice the time of the defectors at 100 percent — half the time the cycle of play skips over the defectors from the imitators directly back to the secret handshakers. When p is small, therefore, the average population share over time of type S will be roughly 1/2, as the secret handshakers have a 100 percent population share half of the time and a zero percent share the other half. The average shares of types D and M will be roughly 1/4 each. That result is formalized in Proposition 1.

Let τ^i_T denote the average population share over time after T periods of strategy i, for $i \in \{D, S, M\}$. Let τ^i denote the limit of τ^i_T as T goes to infinity.

Proposition 1. Let the mutation probability p converge to zero with everything else fixed, and let k be greater than $c/(x - z)$. Then τ^S converges to 1/2, τ^D converges to 1/4, and τ^M converges to 1/4.

Proof: After a mutant enters, it takes time for the dynamics of the regime to reach a new steady state. As $p \to 0$, the probability that another mutant enters before a steady state has been reached falls to zero, and the ratio of the number of periods spent getting to regime steady states to the number spent in regime steady states also goes to zero. It is therefore possible to treat transitions between steady states as if they took place instantaneously.

[7] Precisely, the limiting expected time as p shrinks relative to the speed of natural selection is $1/(2p - p^2)$.

Play cycles among three regime steady states: a population of defectors in payoff regime *GI* (State *D*), a population of secret handshakers in regime *GII* (State *S*), and a population of imitators in regime *GI* (State *M*). In State *D*, there is no probability of an imitation mutation, since there are no secret handshakers to imitate. Only type *S* can enter, which it does with probability p in each period. Once the mutation occurs, play switches to regime *GII*, the secret handshakers drive out the defectors, and State *S* is reached. In State *S*, either an imitation mutation or a defection mutation can occur, each with probability p per period. If defectors enter, they are immediately driven out, so play stays in State *S*. When imitators enter, play switches to regime *GI*, the imitators drive out the secret handshakers, and State *M* is reached. In State *M*, either a type *S* or a type *D* mutant may enter, each with independent probability p of entry per period. The probability that a defector mutant enters in a given period without a secret handshake entering is $p(1-p)$. If it does, play stays in regime *GI*. The defectors will drive out the imitators, and State *D* is reached again. The probability in each period that a secret handshake enters, with or without a defector mutant, is p. If it does, play switches to regime *GII*. The secret handshakers drive out the other types, and play reaches State *S*.

That cycle of play can be modeled as a Markov chain, with transition matrix

$$R = \begin{array}{c} \\ D \\ S \\ M \end{array} \begin{array}{c} \begin{array}{ccc} D & S & M \end{array} \\ \begin{bmatrix} 1-p & p & 0 \\ 0 & 1-p & p \\ p(1-p) & p & (1-p)^2 \end{bmatrix} \end{array}.$$

The entry in column i of row j is the probability that play will move to State i in the next period, starting from State j, for $i, j \in \{D, S, M\}$. Each state is accessible from every other state, and the probability of staying in the same state from one period to the next is positive for all states; that is, the Markov chain is indecomposable and aperiodic. Therefore, the chain is ergodic. (See, for example, Shiryaev (1996).)

A stationary distribution $\phi(p) = [\sigma^D(p) \quad \sigma^S(p) \quad \sigma^M(p)]$ of the chain is a relative frequency distribution such that $\phi(p)R = \phi(p)$. An ergodic Markov chain has a unique stationary distribution, and the stationary distribution is the limit distribution. That is, $\sigma^i(p)$ is the limiting fraction of time spent in State i, which is also the limiting average

population share of type i, τ^i. For this chain, $\phi(p) = \left[\frac{1-p}{2(2-p)}, \frac{1}{2}, \frac{1}{2(2-p)}\right]$. As $p \to 0$, therefore, $\tau^D \to 1/4$, $\tau^S \to 1/2$, and $\tau^M \to 1/4$. ■

As c approaches zero, the fitness cost of sending a signal (true or imitated) becomes negligible. The fitness of defectors declines relative to the fitness of the other two types. When c is very small, the defectors can drive out imitators in payoff regime GI only very slowly. Instead, when a defection mutation occurs in a population of imitators, the two type still have almost the same fitness. The share of type D remains close to the entry share k until a new secret handshake emerges and drives out both defectors and imitators. Then the secret handshakers will thrive until another imitation mutation occurs. In this case, the imitators are vulnerable only to secret handshake mutations, and the defectors never get a turn to have a 100 percent population share (except at the beginning of the game). Thus, approximately half of the time will be spent with imitators as the whole population, and half with secret handshakers as the whole population, as Proposition 2 demonstrates.

Proposition 2. Let the mutation probability p be close to zero, and let the cost of signaling c converge to zero with everything else fixed. Then τ^D converges to $k/4$, τ^S converges to $1/2$, and τ^M converges to $1/2 - k/4$. More precisely, $\lim_{p \to 0}\left[\lim_{c \to 0}\left(\tau^D\right)\right] = \frac{k}{4}$, $\lim_{p \to 0}\left[\lim_{c \to 0}\left(\tau^S\right)\right] = \frac{1}{2}$, and $\lim_{p \to 0}\left[\lim_{c \to 0}\left(\tau^M\right)\right] = \frac{1}{2} - \frac{k}{4}$. If both p and the threshold value k converge to zero, then $\lim_{p,k \to 0}\left[\lim_{c \to 0}\left(\tau^D\right)\right] = 0$, $\lim_{p,k \to 0}\left[\lim_{c \to 0}\left(\tau^S\right)\right] = \frac{1}{2}$, and $\lim_{p,k \to 0}\left[\lim_{c \to 0}\left(\tau^M\right)\right] = \frac{1}{2}$.

Proof: Let p be small enough that the ratio of time spent getting to regime steady states to time spent in regime steady states is close to zero. When c is close enough to zero, k is greater than $c/(x - z)$, and cooperation is viable. As c decreases, the defectors drive out imitators in payoff regime GI more and more slowly. As $c \to 0$ with p fixed, they drive out the imitators so slowly that the expected share of defectors is still very close to k

when the secret handshake mutation occurs and play moves to State S. Play still cycles among three states, as in the proof of Proposition 1, but State D is no longer a steady state with a population of defectors. Instead, it is an almost-steady state in regime GI where the population share of the defectors is k and the share of the imitators is $1 - k$.

Now the limiting time average population share of the imitators is the limiting fraction of time spent in State M, plus $(1 - k)$ times the limiting fraction of time spent in State D: $\tau^M = \sigma^M(p) + (1 - k)\sigma^D(p) = \dfrac{2-k-(1-k)p}{2(2-p)}$. The limiting average population share of the defectors is k times the limiting proportion of periods spent in State D: $\tau^M = k\sigma^D(p) = \dfrac{k(1-p)}{2(2-p)}$. The limiting average share of the secret handshakers is the same as before: $\tau^S = \sigma^S(p) = 1/2$.

When $p \to 0$, the limit of (τ^D, τ^S, τ^M) as $c \to 0$ is $(k/4, 1/2, 1/2 - k/4)$. When both $p \to 0$ and $k \to 0$, the limit of (τ^D, τ^S, τ^M) as $c \to 0$ is $(0, 1/2, 1/2)$. ■

When p is not small, the cycle of play is more complicated. As the expected time between mutations shrinks, a greater portion of time is spent away from the steady states of the regimes. Consequently, the rates of convergence to regime steady states and the dynamics away from the steady states, which depend on the parameters of the game, become more important. A large p also means that the chance of two mutants entering together is significant. That event further complicates the cycle of play. Calculating the average population shares over time of the different types analytically becomes difficult. For that reason, we use computer simulations to explore what parameter values lead to cooperation — that is, a large population share for the secret handshakers.[8] Anything that increases the payoff of the secret handshakers relative to the defectors and imitators will increase the average population share of the secret handshakers. The greater the gain from mutual cooperation $(x - z)$ is, and the smaller the gain from betrayal $(y - x)$ and the mutual defection payoff (z) are, all else equal, the higher will be the average share of the

[8] The computer code is available from the authors upon request.

handshakers. The effect of reducing the cost of signaling c is ambiguous. A lower cost improves the fitness of the secret handshakers, but it also improves the imitators' fitness.

4. Simulation Results

Computer simulations were run using different values of the parameters of the game. Each simulation ran for ten million periods. The initial conditions are a population of defectors playing in payoff regime GI. (The number of periods is so great, though, that the initial conditions should have very little effect on the time average population shares.) Table A1 in the Appendix presents the average populations shares of the three types from simulations using every combination of the following parameter values: $(x - z, y - x, z - 1) \in \{0.1, 1, 10\}^3$, $c \in \{0.1, 0.01, 0.001\}$, $k \in \{0.05, 0.011\}$, and $p \in \{0.1, 0.001, 0.00001\}$. Those combinations yield 27 different parameterizations of the prisoners' dilemma G. Combinations where the threshold level k is less than the minimum value for cooperation to be viable, $c/(x - z)$, are omitted. The results are summarized below.

Proposition 1 predicts that as the mutation probability p shrinks to zero, the average population shares of defectors, secret handshakers, and imitators tend toward 1/4, 1/2, and 1/4, respectively. For each parameterization of G, the simulations suggest that a value of 0.00001 for p is near enough to zero for the population shares to be very close to their limiting values. Table 1 shows the convergence toward 1/4, 1/2, 1/4 as p shrinks for two representative games.

Average Percent Population Shares over Time of D, S, M

G:	3	1
	4	2

x - z = 1 y - x = 1 z - 1 = 1

G:	1.2	1
	11.2	1.1

x - z = 0.1 y - x = 10 z - 1 = 0.1

k = 0.05, **c** = 0.001

p=0.1	1,37,62	p=0.1	3,26,71
p=0.01	1,46,53	p=0.01	2,35,64
p=0.001	2,48,49	p=0.001	4,46,50
p=0.0001	11,50,39	p=0.0001	19,50,32
p=0.00001	23,50,27	p=0.00001	25,51,24

Table 1

For larger values of p, factors that increase the relative payoff of the secret handshakers should also increase their average population share. The simulations indicate that they do. In the first prisoners' dilemma in Table 2, the gain from mutual cooperation is large ($x - z = 10$) compared to the gain from betrayal ($y - x = 0.1$) and the mutual defection payoff ($z = 1.1$). The secret handshakers have a big payoff advantage, enabling them to drive out other types quickly in payoff regime *GII* and to be driven out slowly in regime *GI*. Thus, their average population share is high. In the next two prisoners' dilemmas in Table 2, the payoff advantage of the secret handshakers declines, and so do their average population shares.

Average Percent Population Shares over Time of D, S, M

k = 0.05, **c** = 0.001

G:		
	11.1	1
	11.2	1.1

x - z = 10 y - x = 0.1 z - 1 = 0.1

p=0.1	0,65,35
p=0.001	3,52,45
p=0.00001	26,49,25

G:		
	3	1
	4	2

x - z = 1 y - x = 1 z - 1 = 1

p=0.1	1,37,62
p=0.001	2,48,49
p=0.00001	23,50,27

G:		
	11.1	1
	21.1	11

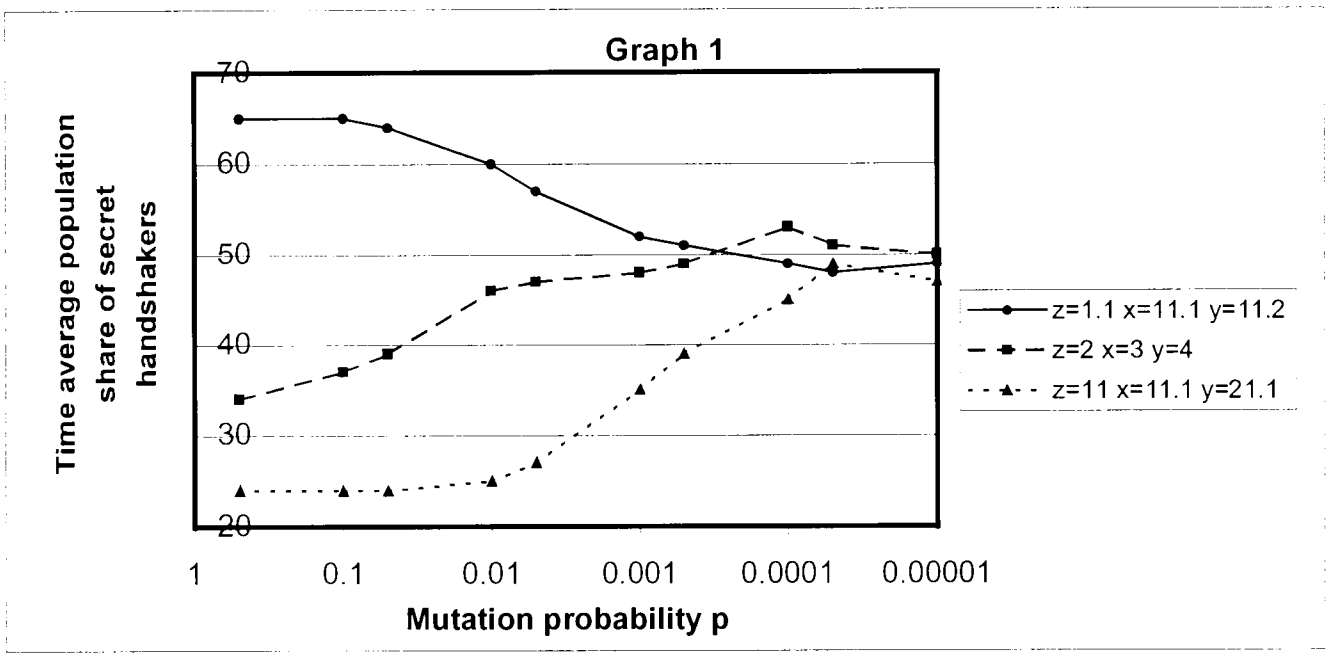
x - z = 0.1 y - x = 10 z - 1 = 10

p=0.1	4,24,73
p=0.001	2,35,63
p=0.00001	12,47,41

Table 2

Table 2 also demonstrates that the effect of a falling mutation probability on the prevalence of cooperation varies with the relative payoffs; an expanded version of Table 2 is presented in Graph 1[9]. When the secret handshakers' payoff advantage is large, they do well away from steady states. The result is that for small values of p, the secret handshakers' average population share is greater than 1/2. As p shrinks, an increasing proportion of time is spent in steady states, and their average share declines toward 1/2. When their payoff advantage is small, on the other hand, the secret handshakers do poorly away from steady states, so the average population share of type S increases from less than 1/2 up to 1/2 as p falls to zero. Thus, whether a change in mutation probability affects the level of cooperation positively or negatively depends on the payoff matrix.

[9] The small non-monotonicities in Graph 1 are the result of sampling variation, which can be a factor even with ten million periods when the mutation probability p is very close to zero.



One surprising result is that in a few cases an increase in the gain from betrayal, which lowers the payoff advantage of the secret handshakers, actually raises their average population share. The share of type S when $p = 0.1$ in the first parameterization of G in Table 3, where $y - x = 1$, is 1 percent. When $y - x$ increases to 10 in the second parameterization, the share of the secret handshakers rises to 10 percent. The reason for this counterintuitive result lies in the dynamics of payoff regime GI. In GI, the imitators' fitness edge over the defectors comes from taking advantage of fooled secret handshakers. For a given set of parameter values, if the share of type S in the population is too small, then the defectors have a higher fitness than the imitators, as explained in "Fitness and Dynamics Within Regimes" in Section 2. In the first two parameterizations in Table 3, the gain from mutual cooperation ($x - z$) is only 0.1, so when secret handshakers enter the initial population of defectors, their share grows slowly. When the mutation probability p is high, then, the imitator mutants enter when there are still very few secret handshakers in the population. Consequently, the defectors drive out the other two types, and their average population share is close to one. If the gain from betrayal increases, as in the second parameterization in Table 3, the payoff advantage of the imitators grows. Now the imitator mutants will triumph, and the defectors' lock on the population is broken. A decrease in p (which increases the expected time between mutations) or an increase in the gain from mutual cooperation (as in the third

parameterization) will increase the expected share of the secret handshakers when the imitators enter, and thus also keep the defectors from dominating the population forever.

Average Percent Population Shares over Time of D, S, M

k = 0.011, **c** = 0.001

G:		
	2.1	1
	3.1	2

x - z = 0.1 y - x = 0.1 z - 1 = 1

p=0.1	99,1,1
p=0.001	2,29,69
p=0.00001	25,43,32

G:		
	2.1	1
	12.1	2

x - z = 0.1 y - x = 10 z - 1 = 1

p=0.1	2,10,89
p=0.001	1,29,69
p=0.00001	23,48,30

G:		
	3	1
	4	2

x - z = 1 y - x = 1 z - 1 = 1

p=0.1	1,18,81
p=0.001	1,46,54
p=0.00001	24,46,29

Table 3

When the signaling cost c shrinks to zero and the value of p is small, Proposition 2 predicts that the time average population shares of types D, S, and M will be approximately $k/4$, $1/2$, and $1/2 - k/4$, respectively. Table 4 demonstrates the result, using the same two representative parameterizations of the prisoners' dilemma as Table 1.

Average Percent Population Shares over Time of D, S, M

G:		
	3	1
	4	2

x - z = 1 y - x = 1 z - 1 = 1

G:		
	1.2	1
	11.2	1.1

x - z = 0.1 y - x = 10 z - 1 = 0.1

k = 0.05, p = 0.00001

c=0.001	23,50,27
c=0.0001	6,47,47
c=0.00001	2,48,51
c=0.000001	1,48,51
c=0.0000001	1,45,53

c=0.001	25,51,24
c=0.0001	12,49,39
c=0.00001	2,49,50
c=0.000001	2,49,50
c=0.0000001	1,49,49

Table 4

Finally, Table 5 shows the relationship between average population shares over time and the proportion of periods in which each type makes up more than 90 percent of the population, for a representative parameterization of G. The results for other parameterizations are similar. The two fractions are positively related. As the mutation probability p declines, the fraction of time spent in steady states increases, and the two

numbers become almost identical. That result illustrates the conclusion that as p goes to zero, time spent in regime steady states determines average population shares.

Average Percent Population Shares over Time of D, S, M
(Percent of Time Spent with Over 90% of the Population for D, S, M)

G:	3	1
	4	2

x - z = 1 y - x = 1 z - 1 = 1

	k = 0.05		k = 0.011	
c:	0.01	0.001	0.01	0.001
p=0.1	2,37,61	1,37,62	99,1,0	1,18,81
	(0,17,22)	(0,17,35)	(100,0,0)	(0,10,73)
p=0.001	15,48,37	2,48,49	17,44,39	1,46,54
	(9,47,29)	(0,48,44)	(0,45,53)	(12,43,33)
p=0.00001	24,51,24	23,50,27	22,51,27	24,46,29
	(24,51,24)	(20,50,24)	(21,51,27)	(22,46,27)

Table 5

5. Conclusion

In our model, cooperation cannot be sustained forever. A population of secret handshakers is always vulnerable to invasion by mutants who imitate the handshake and then defect. But just as the aristocrats found new "gilded costumes" when the old ones were imitated, new generations of mutants with new secret handshakes will rise up. The prevalent strategy in the population alternates endlessly between cooperation and defection, and extended periods of cooperation recur forever.

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Appendix Table A1

Average Percent Population Shares over Time of D, S, M

1) G: 1.2 | 1 / 1.3 | 1.1 x - z = 0.1 y - x = 0.1 z - 1 = 0.1

k		c: 0.1	0.01	0.001
	p=0.1	*	*	3,30,67
0.05	p=0.001	*	*	4,47,49
	p=0.00001	*	*	19,53,28
	p=0.1	*	*	99,1,1
0.011	p=0.001	*	*	2,35,63
	p=0.00001	*	*	19,53,28

2) G: 2.1 | 1 / 2.2 | 1.1 x - z = 1 y - x = 0.1 z - 1 = 0.1

k		c: 0.1	0.01	0.001
	p=0.1	*	1,49,50	0,50,50
0.05	p=0.001	*	18,50,32	3,51,46
	p=0.00001	*	24,51,24	20,51,29
	p=0.1	*	1,39,60	0,39,61
0.011	p=0.001	*	18,45,36	1,48,51
	p=0.00001	*	27,49,24	26,44,30

3) G: 11.1 | 1 / 11.2 | 1.1 x - z = 10 y - x = 0.1 z - 1 = 0.1

k		c: 0.1	0.01	0.001
	p=0.1	0,65,35	0,65,35	0,65,35
0.05	p=0.001	21,52,27	15,53,32	3,52,45
	p=0.00001	26,48,26	25,51,24	26,49,25
	p=0.1	0,71,29	0,71,29	0,71,29
0.011	p=0.001	19,54,27	13,55,33	1,55,44
	p=0.00001	25,50,25	22,55,23	22,50,28

4) G: 1.2 | 1 / 2.2 | 1.1 x - z = 0.1 y - x = 1 z - 1 = 0.1

k		c: 0.1	0.01	0.001
	p=0.1	*	*	3,26,71
0.05	p=0.001	*	*	4,46,50
	p=0.00001	*	*	22,52,26
	p=0.1	*	*	2,11,88
0.011	p=0.001	*	*	2,34,63
	p=0.00001	*	*	23,47,30

5) G: 2.1 | 1 / 3.1 | 1.1 x - z = 1 y - x = 1 z - 1 = 0.1

k		c: 0.1	0.01	0.001
	p=0.1	*	1,38,61,	1,38,62
0.05	p=0.001	*	18,49,33	3,49,47
	p=0.00001	*	27,48,25	22,49,29
	p=0.1	*	1,21,78	1,21,79
0.011	p=0.001	*	21,44,35	1,48,51
	p=0.00001	*	21,56,24	21,46,33

6) G: 11.1 | 1 / 12.1 | 1.1 x - z = 10 y - x = 1 z - 1 = 0.1

k		c: 0.1	0.01	0.001
	p=0.1	1,50,49	0,50,50	0,50,50
0.05	p=0.001	24,50,26	18,50,32	3,50,47
	p=0.00001	30,50,20	19,57,24	22,50,28
	p=0.1	1,48,51	0,47,53	0,47,53
0.011	p=0.001	23,49,27	15,50,35	1,50,49
	p=0.00001	22,50,27	28,48,24	23,48,29

7) G: 1.2 | 1 / 11.2 | 1.1 x - z = 0.1 y - x = 10 z - 1 = 0.1

k		c: 0.1	0.01	0.001
	p=0.1	*	*	3,26,71
0.05	p=0.001	*	*	4,46,50
	p=0.00001	*	*	25,51,24
	p=0.1	*	*	2,10,88
0.011	p=0.001	*	*	2,36,62
	p=0.00001	*	*	21,52,27

8) G: 2.1 | 1 / 12.1 | 1.1 x - z = 1 y - x = 10 z - 1 = 0.1

k		c: 0.1	0.01	0.001
	p=0.1	*	2,37,61	1,37,62
0.05	p=0.001	*	18,50,32	4,49,47
	p=0.00001	*	27,49,24	20,54,26
	p=0.1	*	1,18,81	1,18,81
0.011	p=0.001	*	18,47,35	1,48,51
	p=0.00001	*	24,51,25	24,51,25

9) G: 11.1 | 1 / 21.1 | 1.1 x - z = 10 y - x = 10 z - 1 = 0.1

k		c: 0.1	0.01	0.001
	p=0.1	3,47,50	1,47,52	1,47,52
0.05	p=0.001	25,49,26	19,50,32	4,51,46
	p=0.00001	29,49,23	30,43,27	27,47,26
	p=0.1	1,38,61	0,37,63	0,37,63
0.011	p=0.001	24,49,27	16,49,35	1,50,49
	p=0.00001	25,50,25	23,54,23	25,49,25

G: x | 1 / y | z

Notes: Number of iterations is 10,000,000.
Cases where k < (x - z)/c are omitted and marked with a *.

Table A1 continued

Average Percent Population Shares over Time of D, S, M

10) G:

2.1	1
2.2	2

x - z = 0.1 y - x = 0.1 z - 1 = 1

k	c:	0.1	0.01	0.001
	p=0.1	*	*	8,27,65
0.05	p=0.001	*	*	3,45,56
	p=0.00001	*	*	24,51,25
	p=0.1	*	*	99,1,1
0.011	p=0.001	*	*	2,30,68
	p=0.00001	*	*	19,52,30

11) G:

3	1
3.1	2

x - z = 1 y - x = 0.1 z - 1 = 1

k	c:	0.1	0.01	0.001
	p=0.1	*	2,47,52	1,47,52
0.05	p=0.001	*	15,50,35	2,50,48
	p=0.00001	*	27,45,28	15,57,28
	p=0.1	*	99,1,0	1,32,67
0.011	p=0.001	*	17,45,38	1,48,52
	p=0.00001	*	27,49,24	16,53,31

12) G:

12	1
12.1	2

x - z = 10 y - x = 0.1 z - 1 = 1

k	c:	0.1	0.01	0.001
	p=0.1	1,72,27	0,72,28	0,72,27
0.05	p=0.001	23,51,26	13,51,35	2,52,46
	p=0.00001	22,52,26	19,51,30	21,47,32
	p=0.1	0,80,20	0,80,20	0,80,20
0.011	p=0.001	22,53,26	10,55,36	0,55,45
	p=0.00001	28,49,23	24,47,29	19,51,30

13) G:

2.1	1
3.1	2

x - z = 0.1 y - x = 1 z - 1 = 1

k	c:	0.1	0.01	0.001
	p=0.1	*	*	3,25,71
0.05	p=0.001	*	*	2,45,53
	p=0.00001	*	*	22,45,33
	p=0.1	*	*	99,1,1
0.011	p=0.001	*	*	2,29,69
	p=0.00001	*	*	25,43,32

14) G:

3	1
4	2

x - z = 1 y - x = 1 z - 1 = 1

k	c:	0.1	0.01	0.001
	p=0.1	*	2,37,61	1,37,62
0.05	p=0.001	*	15,48,37	2,48,49
	p=0.00001	*	24,51,24	23,50,27
	p=0.1	*	99,1,0	1,18,81
0.011	p=0.001	*	17,44,39	1,46,54
	p=0.00001	*	22,51,27	24,46,29

15) G:

12	1
13	2

x - z = 10 y - x = 1 z - 1 = 1

k	c:	0.1	0.01	0.001
	p=0.1	1,57,42	1,57,42	0,57,43
0.05	p=0.001	24,50,26	14,51,36	2,51,47
	p=0.00001	25,54,22	28,47,25	24,47,29
	p=0.1	1,56,44	0,55,45	0,55,45
0.011	p=0.001	24,48,27	10,51,39	1,51,49
	p=0.00001	23,52,25	28,45,27	19,48,33

16) G:

2.1	1
12.1	2

x - z = 0.1 y - x = 10 z - 1 = 1

k	c:	0.1	0.01	0.001
	p=0.1	*	*	3,25,72
0.05	p=0.001	*	*	3,45,53
	p=0.00001	*	*	25,47,28
	p=0.1	*	*	2,10,89
0.011	p=0.001	*	*	1,29,69
	p=0.00001	*	*	23,48,30

17) G:

3	1
13	2

x - z = 1 y - x = 10 z - 1 = 1

k	c:	0.1	0.01	0.001
	p=0.1	*	2,33,64	1,34,65
0.05	p=0.001	*	14,50,36	2,49,48
	p=0.00001	*	27,46,27	17,50,33
	p=0.1	*	1,15,84	1,15,84
0.011	p=0.001	*	17,43,40	1,46,53
	p=0.00001	*	24,50,26	21,49,31

18) G:

12	1
22	2

x - z = 10 y - x = 10 z - 1 = 1

k	c:	0.1	0.01	0.001
	p=0.1	2,48,50	1,48,51	1,48,52
0.05	p=0.001	24,51,26	15,49,35	2,50,47
	p=0.00001	28,47,25	27,49,24	20,51,28
	p=0.1	1,37,62	0,37,63	0,37,63
0.011	p=0.001	24,49,27	11,50,39	1,50,50
	p=0.00001	26,49,25	23,55,22	19,51,31

Table A1 continued

Average Percent Population Shares over Time of D, S, M

19) G:

11.1	1
11.2	11

x - z = 0.1 y - x = 0.1 z - 1 = 10

k	c:	0.1	0.01	0.001
	p=0.1	*	*	94,3,3
0.05	p=0.001	*	*	2,35,63
	p=0.00001	*	*	13,48,38
	p=0.1	*	*	99,1,1
0.011	p=0.001	*	*	99,1,0
	p=0.00001	*	*	20,37,43

20) G:

12	1
12.1	11

x - z = 1 y - x = 0.1 z - 1 = 10

k	c:	0.1	0.01	0.001
	p=0.1	*	72,10,18	8,30,63
0.05	p=0.001	*	4,46,49	1,47,51
	p=0.00001	*	19,53,29	11,48,41
	p=0.1	*	99,1,0	3,16,82
0.011	p=0.001	*	2,37,61	0,37,63
	p=0.00001	*	22,50,28	9,45,46

21) G:

21	1
21.1	11

x - z = 10 y - x = 0.1 z - 1 = 10

k	c:	0.1	0.01	0.001
	p=0.1	1,56,43	1,56,43	1,56,43
0.05	p=0.001	19,50,32	4,49,47	1,51,48
	p=0.00001	33,48,19	24,50,27	11,51,39
	p=0.1	99,1,0	0,55,44	0,55,45
0.011	p=0.001	20,48,32	1,50,49	0,51,49
	p=0.00001	26,50,25	22,54,24	6,48,46

22) G:

11.1	1
12.1	11

x - z = 0.1 y - x = 1 z - 1 = 10

k	c:	0.1	0.01	0.001
	p=0.1	*	*	91,4,4
0.05	p=0.001	*	*	2,34,64
	p=0.00001	*	*	7,50,44
	p=0.1	*	*	99,1,1
0.011	p=0.001	*	*	99,1,0
	p=0.00001	*	*	12,43,45

23) G:

12	1
13	11

x - z = 1 y - x = 1 z - 1 = 10

k	c:	0.1	0.01	0.001
	p=0.1	*	6,28,65	5,28,67
0.05	p=0.001	*	4,47,49	1,47,52
	p=0.00001	*	27,47,26	14,49,37
	p=0.1	*	99,1,0	2,12,86
0.011	p=0.001	*	2,35,62	0,36,63
	p=0.00001	*	23,51,26	5,47,48

24) G:

21	1
22	11

x - z = 10 y - x = 1 z - 1 = 10

k	c:	0.1	0.01	0.001
	p=0.1	1,50,49	1,50,49	1,50,49
0.05	p=0.001	19,50,32	4,50,47	1,50,49
	p=0.00001	25,52,23	24,49,27	10,49,42
	p=0.1	99,1,0	1,37,63	0,37,63
0.011	p=0.001	20,47,33	1,49,50	0,48,51
	p=0.00001	28,49,24	23,49,27	7,50,43

25) G:

11.1	1
21.1	11

x - z = 0.1 y - x = 10 z - 1 = 10

k	c:	0.1	0.01	0.001
	p=0.1	*	*	4,24,73
0.05	p=0.001	*	*	2,35,63
	p=0.00001	*	*	12,47,41
	p=0.1	*	*	98,1,1
0.011	p=0.001	*	*	99,1,0
	p=0.00001	*	*	11,41,48

26) G:

12	1
22	11

x - z = 1 y - x = 10 z - 1 = 10

k	c:	0.1	0.01	0.001
	p=0.1	*	3,26,71	1,27,73
0.05	p=0.001	*	4,47,49	1,47,52
	p=0.00001	*	24,50,25	13,49,38
	p=0.1	*	99,1,1	1,11,89
0.011	p=0.001	*	2,36,62	0,35,64
	p=0.00001	*	17,53,30	4,53,43

27) G:

21	1
31	11

x - z = 10 y - x = 10 z - 1 = 10

k	c:	0.1	0.01	0.001
	p=0.1	2,41,58	1,41,58	1,41,58
0.05	p=0.001	19,49,32	4,49,47	1,49,49
	p=0.00001	22,53,25	24,53,23	10,50,39
	p=0.1	99,1,0	1,22,77	0,22,77
0.011	p=0.001	21,46,33	1,48,50	0,48,51
	p=0.00001	24,51,25	22,51,27	7,52,41