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CAPACITY, ENTRY AND FORWARD INDUCTION

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Abstract

We show that when avoidable fixed costs are introduced into the capacity-and-entry model of Dixit (1980) and Ware (1984), there arises a *coordination problem* in selecting among postentry Nash equilibria. Elimination of weakly dominated strategies makes it possible for the entrant to use a *market-capturing strategy*, consisting of a large capacity commitment that selects the entrant's preferred postentry equilibrium and drives the incumbent from the market. Deterring the entrant's market-capturing strategy typically requires the incumbent to reduce its initial capacity choice. As avoidable fixed costs rise, the incumbent must restrict its capacity by a greater amount, and the relative advantage of the entrant rises.

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1. Introduction

In the past twenty years, the merger of Industrial Organization and Game Theory has produced a plethora of theories but few broad conclusions. One conclusion which has been robust to a variety of models, however, is that there is a preemptive advantage to moving first when costs are sunk. This conclusion is well illustrated by Dixit's (1980) model of entry deterrence, wherein the sunk nature of capital expenditures enables an incumbent firm to commit to an aggressive postentry posture, allowing it to lock in a superior market position or to deter entry entirely.

Our fundamental point in this paper is that the first mover advantages associated with incumbency may fail to hold — and indeed may be reversed — when there are multiple equilibria in the postcommitment competition. We frame our analysis in terms of Dixit's sequential capacity choice model, but with two key modifications: first, we assume that along with sunk costs there are significant *avoidable fixed costs*, i.e. fixed costs that are not incurred if the firm shuts down. The presence of avoidable fixed costs leads to a *coordination problem* in choosing postentry outputs: alongside the equilibria in which the firms share the market, there may exist "natural monopoly" equilibria in which one firm produces output so large that the other responds optimally by shutting down.

Second, we modify the notion of rationality that governs the firms' behavior. In past studies of strategic rivalry, first mover advantages have been closely linked to the principle of *backward induction rationality*. As formalized by Selten's (1975) notion of subgame perfect equilibrium, this principle requires that firms base their current decisions on the hypothesis that future behavior will be profit-maximizing. We augment this notion

by requiring in addition that firms believe *past* behavior was profit-maximizing. Since past decisions then give information as to what might be the rational current (or future) decisions, a firm must take account of what its rival did in the past in order to predict the rival's behavior. This basic idea is known as *forward induction rationality*.

These two modifications dramatically alter the strategic balance between the first and second mover. To see why this is true, suppose that the incumbent has made a large capacity commitment in an attempt to deter entry, and suppose further that the entrant responds with a very large capacity investment of its own. The incumbent must then ask itself, what does this mean about postentry competition? If the entrant could not recoup its capacity investment with a postentry market-sharing quantity, the incumbent can only infer that the entrant will respond with a larger output, near its natural monopoly level, as otherwise such a large capacity level would have been an irrational choice by the entrant. More specifically, the incumbent observes the entrant's capacity choice, eliminates capacity and quantity combinations that represent *weakly dominated strategies* for the entrant, and thereby deduces the entrant's possible postentry quantity.¹ Given this inference, the incumbent responds optimally to the large entrant capacity by shutting down.

Thus, the combination of avoidable fixed costs and the incumbent's forward induction inference allows the entrant to play a *market-capturing strategy* involving a high level of capacity investment. Credibility of the market-capturing strategy follows from the fact that the incumbent uses forward induction to form strategic inferences, and thus it cannot escape the logic that leads to the entrant's natural-monopoly equilibrium. Forward induction inferences by the entrant, however, do *not* mandate that a large incumbent capacity commitment communicates that the incumbent will choose a large postentry quantity, because the strategic position of the incumbent is much more ambiguous: the entrant can suppose that the incumbent's past behavior was based on a faulty forecast of the entrant's capacity response, and this allows the entrant to rationalize a much broader

range of possible postentry behavior by the incumbent. The second mover gains the advantage in coordinating among postentry equilibria precisely because its strategic situation is so clear.

The coordination advantage accruing to second movers makes it necessary to rethink the nature of strategic rivalry in environments having multiple postcommitment equilibria. In particular, the fact that the incumbent must move first, and thus must face the prospect of the entrant's market-capturing strategy, alters its incentives to choose capacity. With significant avoidable fixed costs, not only do large capacity commitments lose their preemptive power, they also make the market-capturing strategy easier to employ and more profitable for the entrant. We show that for a large class of cases, the incumbent counteracts the market-capturing strategy by *reducing* its initial capacity investment, in order to lower its endogenous avoidable fixed costs as well as to make the market-sharing equilibrium more attractive to the entrant. Interestingly, the incumbent's commitment to the market is maintained only by taking actions that *restrict* its market share.² As avoidable fixed costs rise, the incumbent must restrict its capacity by a greater amount, and the incumbent must cede the market entirely if avoidable fixed costs are higher still.

Our finding that capacity investment may be an ineffective, and in fact quite dangerous, entry-detering strategy may provide some explanation for why models that predict incumbent excess capacity, such as that of Bulow, Geanakoplos and Klemperer (1985), seem to perform poorly in empirical studies. Lieberman (1987), for example, finds little evidence that incumbents choose excess capacity in order to deter entry. Similarly, product managers responding to the survey conducted by Smiley (1988) indicated that excess capacity was the least frequently chosen among a set of entry deterring strategies in mature-product industries. Incumbents' reluctance to choose large capacity may further account for empirical results of Biggadike (1979) and Robinson (1988) indicating that

incumbents tend to react passively to entry.

Other empirical evidence seems to bear out our finding that large investments by entrants may serve to discourage aggressive postentry behavior by incumbents. Biggadike and especially Robinson find that incumbents typically react more aggressively toward medium-scale than large-scale entrants, even though a large-scale entrant represents a more serious threat to incumbent market share. Apparently, incumbents reason that "a very powerful entrant cannot be stopped or even slowed down" (Robinson, p. 375). Anecdotal evidence from the U.K. potato chip industry (Bevan, 1974) and the U.K. tin can industry (*Business Week*, 1973) suggests that a large-scale entrant can substantially displace a well-entrenched incumbent by demonstrating its willingness to incur sizable losses in the first few years following entry.

This paper builds on a large literature that studies strategic investment by incumbent firms. Two strands of work relate most closely to our analysis. First, the role of avoidable fixed costs in creating coordination problems has been recognized by Dixit (1979) and Arvan (1986). Arvan argues that reputation plays a role in the equilibrium selection process, and he shows that the incumbent may gain the advantage in equilibrium selection by exploiting private information about its costs. Second, Schmalensee (1983) and Fudenberg and Tirole (1984) discuss circumstances under which exercising strategic power may lead an incumbent to choose a less aggressive strategy, in order to exploit strategic complementarity (the "puppy dog ploy"). Our finding that the incumbent must reduce its capacity investment bears a relation to this notion in that in both instances, a reduction in the incumbent's aggressiveness makes it more profitable for the entrant to play a strategy that is beneficial to the incumbent.

Our work has been inspired by the game-theoretic papers of Ben-Porath and Dekel (1992) and van Damme (1989), which resolve the problem of coordination among multiple Nash equilibria by allowing players to engage in "public money burning." Both papers

employ stronger notions of forward induction than that used here: Ben–Porath and Dekel apply multiple rounds of elimination of weakly dominated strategies, while van Damme develops his own concept of forward induction. These authors give examples which show that the last player to burn money is able to select his preferred equilibrium, i.e. strategic communication conveys second mover advantages.³ Our model adds the important new feature that the first mover can use its precommitment ability to offset the communication power of the second mover, in particular by reducing the aggressiveness of its commitment. This effect arises from the fact that precommitment alters the set of postentry equilibria and the second mover’s payoffs in these equilibria.

In the next section, we introduce our basic model and illustrate the coordination problem that confronts firms in choosing their postentry outputs. The notion of forward induction is defined and motivated in Section 3. Next, in Sections 4 and 5, we explore the coordination advantage held by the second mover and the first mover’s countermeasures, respectively. We also analyze in these sections the role of avoidable fixed costs in determining the strategic balance between the first and second mover. Section 6 concludes.

2. The Model

In this section, we present our sequential capacity choice game. The model takes the structure previously developed by Dixit (1980) and Ware (1984) and adds to it the possibility of avoidable fixed costs. As a consequence, firms face a coordination problem in the selection of outputs that follows the choice of capacities.

a. Basic Framework

There are two firms, called Firm 1 and Firm 2. The game consists of three stages:

Stage 1. Firm 1 makes a capacity choice k_1 .

Stage 2. Firm 2 observes the value of k_1 chosen by Firm 1, and makes its own capacity choice k_2 .

Stage 3. Firm 1 observes the value of k_2 chosen by Firm 2, and the firms simultaneously make quantity decisions q_1 and q_2 .

Note that the capacities are chosen sequentially: Firm 1 is the first mover, and Firm 2 is the second mover. Quantities are chosen in a Cournot fashion in the third stage, so that strategic advantage derives solely from the capacity choices.

Strategies for this three-stage game consist of decision plans that indicate what actions are to be chosen in every strategically relevant contingency. A strategy for Firm 1 is a pair $\{\hat{k}_1, \hat{q}_1(k_2)\}$, where $\hat{k}_1$ gives the capacity choice that begins the game, while $\hat{q}_1$ is a function that indicates the quantity to be chosen conditional on every possible capacity choice by Firm 2. Firm 2's strategies are written $\{\hat{k}_2(k_1), \hat{q}_2(k_1)\}$; note that $\hat{k}_2$ depends on k_1 , since Firm 2 is able to observe Firm 1's capacity choice prior to making its own.⁴ We assume that k_1 , k_2 , q_1 and q_2 are drawn from the nonnegative real numbers.

Let $P_i(\hat{k}_1, \hat{q}_1, \hat{k}_2, \hat{q}_2)$ give the payoff for Firm i , $i = 1, 2$, under the indicated strategy profile. These payoff functions are generated from underlying profit functions that depend on the capacity and quantity choices: the strategy profile $(\hat{k}_1, \hat{q}_1, \hat{k}_2, \hat{q}_2)$ determines the outcome $k_1 = \hat{k}_1$, $k_2 = \hat{k}_2(\hat{k}_1)$, $q_1 = \hat{q}_1(\hat{k}_2(\hat{k}_1))$ and $q_2 = \hat{q}_2(\hat{k}_1)$; the profits associated with this outcome determine the payoffs. Let S_1 and S_2 give the sets of strategies for Firms 1 and 2. Elements of S_i are written $s_i = (\hat{k}_i, \hat{q}_i)$. The payoff functions may be written $P_i(s_1, s_2)$.

Additional structure is placed on the payoff functions under the following assumptions on costs and demand. Both firms possess a variable cost function, $C(q_i)$, which satisfies $C' > 0 = C(0)$ and $C'' \geq 0$. In addition, firms face the constraint that production cannot exceed capacity, where capacity imposes a cost of r per unit. The choice

of k_i prior to Stage 3 represents an initial purchase of capacity that can be expanded in Stage 3 if additional production is desired. Finally, the inverse demand function is given by $D(q)$, where $D' < 0 < D$, $D'' \leq 0$ and $C'(0) + r < D(0)$.

Avoidable fixed costs are introduced as follows. We assume that there is an added fixed cost of F that firms incur when they operate at positive scale; F is not paid, however, if a firm chooses to *shut down* by producing zero units in Stage 3. Further, we suppose that a firm can recoup proportion $\alpha < 1$ of the cost of its capacity precommitment by shutting down; however, k_i cannot be reduced should the firm decide to operate at positive scale. Thus by shutting down, firms can avoid fixed costs in the amount of $\alpha r k_i + F$. The payoff function of Firm i is therefore given by:

$$P_i(s_i, s_j) = \begin{cases} D(\hat{q}_i(\hat{k}_j) + \hat{q}_j(\hat{k}_i))\hat{q}_i(\hat{k}_j) - C(\hat{q}_i(\hat{k}_j)) - r \max\{\hat{q}_i(\hat{k}_j), \hat{k}_i\} - F, & \hat{q}_i(\hat{k}_j) > 0 \\ -(1 - \alpha)r\hat{k}_i, & \hat{q}_i(\hat{k}_j) = 0 \end{cases}$$

where the definition suppresses the dependence of $\hat{k}_2$ on $\hat{k}_1$.

b. Equilibrium Coordination in the Postentry Subgame

We turn next to a discussion of the firms' output choices in Stage 3. These choices are most easily characterized with the introduction of some additional notation. Define $q^M(k_i)$ as Firm i 's *monopoly output level*. This is the best (i.e., profit-maximizing) positive output level for Firm i , when Firm j produces zero output and Firm i 's predetermined capacity level is k_i . Two monopoly output levels are of particular interest: $q^M(\infty)$ is the monopoly output level when there is no capacity constraint (i.e., when marginal costs are C'), and $q^M(0)$ is the monopoly output level when capacity must be added in Stage 3 in order to produce any positive output (i.e., when marginal costs are $C' + r$). We refer to $q^M(0)$ as the true or undistorted monopoly output level, since it is the

output level that an incumbent would choose if there were no threat of entry. More generally, it is easy to see that $q^M(x) > q^M(0)$, and that $q^M(k_i)$ may be characterized as follows:

$$q^M(k_i) = \begin{cases} q^M(0), & k_i \leq q^M(0) \\ k_i, & q^M(0) \leq k_i \leq q^M(x) \\ q^M(x), & k_i \geq q^M(x) \end{cases}$$

We assume that F is not so large that monopoly is nonviable:

$$(1) \quad F < \bar{F} = D(q^M(0))q^M(0) - C(q^M(0)) - rq^M(0)$$

where $\bar{F}$ denotes the undistorted monopoly profits (gross of F).

Now consider Firm i 's output choices when Firm j produces positive output. As q^j rises, Firm i 's profits become smaller, and strictly so if Firm i 's total revenue, $D(q^i + q^j)q^i$, is positive. Thus, we may uniquely define the *shutdown point*, $q^X(k_i)$, as the level of q^j that makes Firm i indifferent between its best positive output level and zero output.⁵ It is straightforward to verify that $q^X(k_i)$ is decreasing in α and F ; i.e., higher avoidable fixed costs make shutdown more attractive, and thereby reduce the shutdown point for given k_i .

Finally, we define the *reaction correspondence*, $q^R(q_j | k_i)$, as the set of output levels that maximize Firm i 's profits for given q_j and k_i . Clearly $q^R(0 | k_i) = q^M(k_i)$, and $q^R(q_j | k_i)$ corresponds to Dixit's familiar kinked reaction function for $q_j < q^X(k_i)$, while Firm i best-responds with zero output for $q_j > q^X(k_i)$. $q^R(q_j | k_i)$ is multi-valued only at $q_j = q^X(k_i)$, where there are two best responses, $q_i = 0$ and the limit of the Dixit best responses as q_j approaches $q^X(k_i)$ from below. Figure 1 illustrates these definitions.

In Figure 1 there are three intersections of the reaction correspondences, indicating

the existence of three types of Nash equilibria: (1) *Market-Sharing Equilibrium (MSE)*, at point A, in which both firms choose strictly-positive output levels; (2) *Firm 1 Monopoly Equilibrium (F1ME)*, at point B, in which Firm 2 best-responds with zero output in order to recover its avoidable fixed costs; and (3) *Firm 2 Monopoly Equilibrium (F2ME)*, at point C, in which Firm 1 best-responds with zero output. Under our assumptions, Firm 1's reaction correspondence must have strictly steeper slope than that of Firm 2 at any point of intersection, and this ensures that there can exist at most one MSE for any k_1 and k_2 . Further, there can exist at most one F1ME and at most one F2ME for any k_1 and k_2 , since monopoly output levels are uniquely defined.

The possibility of multiple Nash equilibria, which arises here as a consequence of avoidable fixed costs, confronts firms with a coordination problem in determining their Stage 3 quantities. Further, the firms face a conflict of interest in selecting among postentry equilibria: Firm 1 strictly prefers the F1ME to the MSE, and it strictly prefers the MSE to the F2ME, while the ranking is reversed for Firm 2.

3. Forward Induction

Given the coordination problem that firms confront in the postentry subgame, an application of backward induction as represented in the concept of subgame perfect equilibrium allows for a wide range of equilibrium behavior. We therefore augment the requirement of backward-induction rationality with the requirement that firms "induct forward" from observed capacity choices in forecasting their rivals' quantity choices. Our notion of forward induction is simple and natural: we require only that each firm recognizes that its rival will not choose a dominated strategy. In this section, we define the associated undominated game and describe its key features.

a. The Undominated Game

It is reasonable to suppose that Firm i will choose s_i to maximize its payoff subject to some conjecture as to Firm j 's strategy. This payoff-maximization hypothesis may be excessively weak, however, in the following sense: Firm i may be indifferent between s_i and some other strategy s'_i , because both of these strategies perform equally well when Firm j chooses its strategy from some particular subset of S_j . But s'_i may be strictly superior to s_i for every Firm j strategy that is not in this subset. Unless Firm i is extremely confident that Firm j chooses a strategy in the subset, it seems unreasonable that Firm i would choose s_i rather than the safer strategy s'_i .

This intuition can be formalized as follows. A strategy $s_i \in S_i$ is said to be *weakly dominated* by $s'_i \in S_i$ if, for all $s_j \in S_j$:

$$P_i(s'_i, s_j) \geq P_i(s_i, s_j)$$

and there exists $s'_j \in S_j$ such that:

$$P_i(s'_i, s'_j) > P_i(s_i, s'_j)$$

From the above argument it follows that s_i should not be chosen by Firm i , since s'_i is at least as good for any s_j , and there are s'_j such that s'_i is strictly better. We will henceforth refer to weakly dominated strategies as *dominated*. A strategy s_i is called *undominated* if it is not dominated by any s'_i . Let S_i^u denote the set of undominated strategies for Firm i .

The above argument suggests that a rational firm should not choose a dominated strategy. Further, a firm ought to recognize that its rival is as rational as itself, and thus it should not conjecture that its rival chooses a dominated strategy. To implement this

notion, we replace the strategy sets S_1 and S_2 with the sets S_1^u and S_2^u of undominated strategies: in this *undominated game* firms cannot choose dominated strategies, and neither can they conjecture that their rivals choose dominated strategies. Our goal in the remainder of the paper is to consider how strategic rivalry is affected by requiring that decisions must give subgame perfect equilibria of the undominated game.⁶

b. Communication and Forward Induction

We now consider the manner in which a firm's past capacity choice communicates information about its current quantity selection. Communication of this sort is possible when firms use forward-induction inference in forming conjectures about the behavior of their rivals, since a firm knows that its rival would not select an output level that is dominated in combination with the observed capacity choice. We describe here some of the salient features of the undominated game, paying particular attention to the differences between the first and second movers. A detailed derivation of the undominated game is given in the Appendix.

First of all, it is easy to see that strategies in the undominated game cannot specify extremely large levels of capacity: in this case, capacity costs would be impossible to recover even for a monopolist, so that such strategies would be dominated by simply staying out of the market. More specifically, consider the following inequality:

$$(2) \quad D(q^M(k_i))q^M(k_i) - C(q^M(k_i)) - rk_i - F \leq 0$$

In (2), k_i is so large that Firm i cannot make positive profits even as a monopolist. Thus, no matter what quantity Firm j selects in response to k_i , Firm i would do better by choosing $k_i = q_i = 0$. This implies that no strategy in the undominated game can specify such a k_i .

From (2) we may derive an upper bound to the capacity levels that can appear in undominated strategies. We know that (2) can occur only if $k_i > q^M(0)$, as (1) implies that profits must be strictly positive if $k_i \leq q^M(0)$. Further, concavity of Firm i 's profit function in q_i may be used to show that the left-hand side of (2) is strictly decreasing in k_i . Thus, there exists a capacity level $\bar{k} > q^M(0)$ such that (2) holds if and only if $k_i \geq \bar{k}$, and it follows that capacity choices must lie below $\bar{k}$ in the undominated game.

Dominance rules out large capacity levels for a simple reason: a strategy that specifies entry, but which necessarily yields negative profit in the ensuing subgame, is dominated by a strategy that replaces the entry response with a decision to stay out of the market ($k_i = q_i = 0$). This reasoning in turn makes it possible for Firm 2 to use its capacity choice to communicate an aggressive response in the output subgame. To see how such strategic communication arises, consider Firm 2's responses to some given k_1 . We have seen that Firm 2 cannot choose $\hat{k}_2(k_1) \geq \bar{k}$ in the undominated game, but it is also true that capacities *slightly below* $\bar{k}$ are allowable, so long as Firm 2's subsequent quantity choice is *sufficiently close to the corresponding monopoly output level*. In particular, a response $\hat{k}_2(k_1)$ slightly below $\bar{k}$ is allowable in the undominated game if Firm 2's strategy also specifies $\hat{q}_2(k_1) = q^M(\hat{k}_2(k_1))$, since Firm 2 earns strictly positive profits if $q_1 = 0$, according to (2). On the other hand, a strategy with $\hat{k}_2(k_1)$ slightly below $\bar{k}$, but $\hat{q}_2(k_1)$ *not* close to $q^M(\hat{k}_2(k_1))$, cannot give positive profits even if $q_1 = 0$, since profits are reduced due to Firm 2's failure to choose its monopoly output level; a strategy with such a response is dominated, as Firm 2 would do better to stay out when the given k_1 is observed.

It follows that by choosing k_2 slightly below $\bar{k}$, Firm 2 communicates that q_2 must be chosen close to $q^M(k_2)$ in the ensuing subgame. This reasoning extends to any positive capacity response below $\bar{k}$: any $k_2 > 0$ communicates an intention by Firm 2 to choose q_2 large enough to potentially recover its fixed costs and earn strictly positive profits, else the

associated strategy would be dominated by an alternative strategy that specified staying out when the relevant k_1 is observed. In the Appendix, we define $q^L(k_2)$ as the minimum of the output levels such that Firm 2 could just earn zero profit when $q_1 = 0$; by choosing k_2 , Firm 2 communicates $q_2 > q^L(k_2)$ in the ensuing subgame of the undominated game. It is also shown that $q^L(k_2)$ is nondecreasing and strictly positive for $k_2 > 0$. Figure 2 illustrates $q^L(k_2)$; observe that for capacity levels above k'_2 , Firm 2 communicates a strictly more aggressive quantity choice by raising its capacity.

The general features of the function $q^L(k_2)$ are easily understood. Observe first that Firm 2 cannot select a strictly positive capacity level and then zero output in the undominated game. This is because not all of the capacity expenditures are recoverable when zero output is produced, and so such a response would result in strictly negative profits for Firm 2. It would have been better not to enter at all. Next, note that it is also irrational for Firm 2 to select a strictly positive capacity level and a small output, since then the fixed cost F could not possibly be recovered, and so again profits would be strictly negative (even if $q_1 = 0$). A final observation is that $q^L(k_2)$ is initially constant and then strictly increasing in k_2 . Intuitively, when k_2 is small, strictly positive profits can be earned by Firm 2 only if capacity is eventually expanded; thus, the critical profit-making output level is independent of the initial capacity level. Once the initial capacity level becomes sufficiently large, however, the higher outlay of capacity expenditures necessitates ever more profitable output choices, in order for profits potentially to be strictly positive. This implies that $q^L(k_2)$ becomes strictly increasing. Indeed, and as discussed above, when capacity gets sufficiently large and approaches $\bar{k}$, a rational Firm 2 must choose an output level near the associated monopoly level, $q^M(\bar{k}) = \min\{q^M(x), \bar{k}\}$.

Firm 1 does not have this ability to communicate aggressive quantity choices. The Appendix demonstrates that in the undominated game, the quantities q_1 that Firm 1 may choose in conjunction with k_1 need only guarantee the possible recovery of the *avoidable*

component of fixed costs, rather than the full amount of fixed costs as is the case with Firm 2. Since shutdown allows recovery of the avoidable component, it follows that $q_1 = 0$ is always a possible choice for Firm 1 in any subgame of the undominated game.

Intuitively, a strictly positive capacity choice for Firm 1 does not communicate that Firm 1 will respond to Firm 2's capacity choice with an output sufficiently large to earn strictly positive profits (when $q_2 = 0$), since Firm 1's initial capacity choice may have been based on a faulty forecast as to Firm 2's capacity response. Rather, a strictly positive capacity choice for Firm 1 communicates only that there is *some* Firm 2 capacity response that would induce Firm 2 to select such an output. Once Firm 2's capacity response is observed, moreover, a rational Firm 1 must choose an output that is at least capable of recovering avoidable fixed costs. This requirement is quite weak, though, and does not rule out shutting down.

4. Second-Mover Coordination Advantages

Having seen that it is possible for the second mover to communicate aspects of its quantity choice with its capacity selection, we now analyze the second mover's optimal capacity selection. The second mover chooses capacity strategically, recognizing that it thereby communicates output responses that may eliminate certain Nash equilibria from the postentry subgame. In other words, in the subgame perfect equilibria of the undominated game, the second mover possesses a coordination advantage.

To examine this coordination advantage, we fix k_1 and consider Firm 2's payoff-maximizing choice of k_2 in the undominated game. Firm 1's choice of k_1 will then be analyzed in the next section. Before proceeding, however, we must first be certain that at least one Nash equilibrium exists in Stage 3 for each possible k_1 and k_2 ; as discussed in the Appendix, this requires replacing S_2^U with a slightly-expanded strategy set, denoted by $\bar{S}_2^U$.⁷

To begin with, it is easy to see that Firm 2 can use its capacity choice to eliminate any F1ME that may arise in Stage 3 of the undominated game, by simply choosing $k_2 > 0$: this commits Firm 2 to a strictly positive output level, as shown in Figure 2, so the Nash equilibria of the ensuing subgame must be either MSE or F2ME. Further, by choosing sufficiently large k_2 , Firm 2 may be able to eliminate MSE as well, leaving the F2ME, in which Firm 2 captures the entire market, as the *only possible* Nash equilibrium of the subgame. This *market-capturing strategy* may take two forms: (i) k_2 may be large enough to imply $q^L(k_2) > q^X(k_1)$, in which case Firm 2 communicates a level of q_2 in excess of Firm 1's shutdown point. Firm 1 then prefers to shut down for every level of q_2 that could possibly appear in the subgame. (ii) k_2 may induce a subgame in which, although $q^L(k_2) \leq q^X(k_1)$, MSE fail to exist even in the original version of the subgame.⁸

The scope of Firm 2's market-capturing strategy in subgame-perfect equilibria can be related directly to Firm 1's shutdown point, as the following proposition demonstrates.

Proposition 1. In the undominated game, the following is true.

- (a) For a given level of k_1 , if we have $q^X(k_1) \geq \min\{q^M(x), \bar{k}\}$, then for all k_2 there exists a MSE and/or a F1ME of the (k_1, k_2) -subgame;
- (b) For a given level of k_1 , if we have $q^X(k_1) < \min\{q^M(x), \bar{k}\}$, then there exists $\underline{k}_2(k_1) < \bar{k}$ such that the following is true: if $k_2 > \underline{k}_2(k_1)$, then the unique equilibrium of the (k_1, k_2) -subgame is a F2ME; if $k_2 \leq \underline{k}_2(k_1)$ and $\underline{k}_2(k_1) > 0$, then there exists a MSE and/or a F1ME of the (k_1, k_2) -subgame;⁹
- (c) On the range of k_1 such that $q^X(k_1) < \min\{q^M(x), \bar{k}\}$, we have $\text{sign}\{\underline{k}_2(k'_1) - \underline{k}_2(k_1)\} = \text{sign}\{q^X(k'_1) - q^X(k_1)\}$ when $\underline{k}_2(k'_1) > 0$.

Proposition 1 shows how Firm 1's shutdown point $q^X(k_1)$, which gives the level of output that Firm 2 must choose in order to induce Firm 1 to shut down, affects the

market—capturing strategy. The critical level $q^M(\bar{k}) = \min\{q^M(x), \bar{k}\}$ represents the upper bound of output levels that Firm 2 can possibly choose in Nash equilibria of subgames of the undominated game, so if $q^X(k_1)$ exceeds this level, it is impossible for Firm 2 to commit to sufficient output to induce shutdown. If $q^X(k_1)$ lies below the critical level, then Firm 2 can capture the market by choosing k_2 in excess of the level $\underline{k}_2(k_1)$, since this serves to eliminate all equilibria save the F2ME (for one of the two reasons given above). If $k_2 < \underline{k}_2(k_1)$ is chosen, however, then MSE and/or F1ME exist, and so Firm 2 does not necessarily capture the market. Finally, in part (c) it is shown that the capacity level $\underline{k}_2(k_1)$ that is needed for the market—capturing strategy varies directly with $q^X(k_1)$: as Firm 1's shutdown point rises, Firm 2 must choose a larger level of k_2 to induce a F2ME. For notational convenience, let us henceforth extend the definition of $\underline{k}_2(k_1)$ to all k_1 by setting $\underline{k}_2(k_1) = \bar{k}$ when $q^X(k_1) \geq \min\{q^M(x), \bar{k}\}$.

It is apparent from Proposition 1 that the level of avoidable fixed costs plays a role in determining the scope of the second—mover's coordination advantage, since avoidable fixed costs affect the first mover's shutdown point, which in turn affects the availability of the market—capturing strategy and level of capacity that must be chosen to exercise it. Intuitively, as avoidable fixed costs rise (i.e., α and/or F increase), Firm 1 will find shutdown more attractive (i.e., $q^X(k_1)$ decreases), and so Firm 2 may be able to capture the market with a smaller capacity commitment (i.e., $\underline{k}_2(k_1)$ may decrease). The next proposition formalizes the sense in which this intuition holds.

Proposition 2. For given $k_1 > 0$, there exist functions $\underline{\alpha}: [0, \bar{F}) \rightarrow [0, 1]$ and $\bar{\alpha}: [0, \bar{F}) \rightarrow [0, 1]$, with $\underline{\alpha}(F) \leq \bar{\alpha}(F)$, such that:

- (a) $\alpha < \underline{\alpha}(F)$ implies $\underline{k}_2(k_1) = \bar{k}$, and $\underline{\alpha}(F) > 0$ for F sufficiently small;
- (b) $\alpha > \bar{\alpha}(F)$ implies $\underline{k}_2(k_1) = 0$, and $\bar{\alpha}(F) < 1$ for F sufficiently close to $\bar{F}$; and
- (c) $\underline{\alpha}(F) < \alpha < \bar{\alpha}(F)$ implies $0 < \underline{k}_2(k_1) < \bar{k}$, and $\underline{k}_2(k_1)$ is strictly decreasing in α

and F .¹⁰

Observe that the functions $\underline{\alpha}$ and $\bar{\alpha}$ divide the possible values of F and α into three regions, as illustrated in Figure 3. In the lower region, associated with low avoidable fixed costs, we have $\underline{k}_2(k_1) = \bar{k}$, and there is no level of k_2 that assures Firm 2 of market capture. Here the low values of α and F drive $q^X(k_1)$ above the critical level $\{q^M(x), \bar{k}\}$. In the intermediate region, avoidable fixed costs have risen enough to bring $q^X(k_1)$ below $\{q^M(x), \bar{k}\}$, and the market-capturing strategy is now available to Firm 2. Note that a rise in either α or F on this region, which reduces $q^X(k_1)$, correspondingly lowers the capacity level $\underline{k}_2(k_1)$ that Firm 2 must choose in order to capture the market. Finally, in the upper region, avoidable fixed costs are so high that Firm 2 automatically captures the market at *any* positive k_2 ; the reason for this is that there is no MSE in the original subgame, and so choosing positive k_2 must induce the F2ME since it eliminates the F1ME.¹¹

With Proposition 2 in place, we are now prepared to offer a partial characterization of the set of subgame perfect equilibria for the undominated game.

Corollary 1. In any subgame perfect equilibrium of the undominated game, if avoidable fixed costs are sufficiently large, Firm 1 cedes the market and Firm 2 operates as an undistorted monopolist; that is, if α is sufficiently close to unity and F is sufficiently close to $\bar{F}$, then $\hat{k}_1 = \hat{q}_1(\hat{k}_2(\hat{k}_1)) = 0$ and $\hat{q}_2(\hat{k}_1) = q^M(0)$.

To understand this result, observe first that both F1ME and F2ME exist when avoidable fixed costs are large. When avoidable fixed costs are large and firms employ forward induction inference, however, Firm 2 can overturn the F1ME by selecting an initial capacity level that is even slightly positive. Further, this will always be a preferred strategy for Firm 2, since it can then capture the market with ultimate capacity and

output levels equal to the undistorted monopoly levels. Recognizing that Firm 2 will respond in this fashion, Firm 1 does best by ceding the market.

Corollary 1 cleanly captures the sense in which strategic advantage may fall to the second mover. This corollary, however, addresses only the interesting but special case in which avoidable fixed costs are quite large. We thus proceed to a more general characterization.

5. Leader–Optimal Equilibria and First–Mover Countermeasures

We now allow for general values of α and F and construct a subgame perfect equilibrium of the undominated game that incorporates Firm 2’s market–capturing strategy. Our purpose in constructing such an equilibrium is to examine the countermeasures that the first mover may adopt in Stage 1 in order to offset the second mover’s coordination advantage.

Let us begin by specifying the Nash equilibrium that arises in each of the possible (k_1, k_2) –subgames. As Proposition 1 reveals, we must choose the F2ME in (k_1, k_2) –subgames with $k_2 > \underline{k}_2(k_1)$. For (k_1, k_2) –subgames with $k_2 \leq \underline{k}_2(k_1)$, let the Nash equilibria be chosen as follows: (i) for $k_2 = \underline{k}_2(k_1) < \bar{k}$ we select the F2ME; (ii) MSE are chosen if either $0 < k_2 < \underline{k}_2(k_1)$ or $\underline{k}_2(k_1) = \bar{k}$; and (iii) otherwise for $k_2 = 0$ we choose the Nash equilibria that are most profitable for Firm 1. This construction selects Firm 1’s preferred Nash equilibrium whenever $k_2 < \underline{k}_2(k_1)$, and therefore the resulting subgame–perfect equilibrium is called the *leader–optimal equilibrium*.¹² The construction is completed by specifying Firm 2’s payoff–maximizing capacity responses in a manner that ensures existence of a payoff–maximizing capacity choice for Firm 1. The remaining details are discussed in the Appendix, and we summarize here with the following proposition:

Proposition 3. (a) There exists a leader–optimal equilibrium of the undominated game;
 (b) The leader–optimal equilibria maximize the payoff of Firm 1 over the set of subgame–perfect equilibria of the undominated game, including equilibria in mixed strategies.

Importantly, the proposition establishes that Firm 1’s payoff in leader–optimal equilibria gives an upper bound to the payoff that Firm 1 can achieve in any subgame–perfect equilibrium.

Let us consider Firm 1’s choice of capacity in leader–optimal equilibria. The key new feature introduced by avoidable fixed costs and forward–induction inference is that Firm 1 may have to alter its capacity choice in order to deter Firm 2 from exercising its market–capturing strategy. Firm 1 countermeasures can take two forms. First, Firm 1 may be able to *neutralize* Firm 2’s market–capturing strategy by choosing k_1 so as to raise the shutdown point, $q^X(k_1)$, and thereby to eliminate the market–capturing strategy. A second approach is to choose k_1 so as to *accommodate* Firm 2 by raising Firm 2’s profit in the MSE above that available in the F2ME. In this case, Firm 2 will choose not to exercise the market–capturing strategy, even though it may be feasible. We demonstrate below that the neutralization and accommodation countermeasures both lead Firm 1 to constrain its capacity choice.

To understand how the market–capturing strategy might be neutralized, we explore in the next proposition the manner in which k_1 affects $q^X(k_1)$.

Proposition 4. (a) $q^X(k_1)$ is a quasi–concave function of k_1 , having maximizer k_1^N . If $\alpha > 0$, then quasi–concavity is strict for $q^X(k_1) > 0$, and the maximizer is unique.
 (b) k_1^N is a strictly increasing function of α and F , with $k_1^N \rightarrow 0$ as $(\alpha, F) \rightarrow (0, 0)$, and $k_1^N \rightarrow q^M(0)$ as $(\alpha, F) \rightarrow (1, \bar{F})$.

It follows from part (a) that a most-effective capacity level exists for neutralization: by choosing $k_1 = k_1^N$, Firm 1 maximizes its shutdown point and thereby eliminates Firm 2's market-capturing strategy, if it is at all possible to do so. Part (b) indicates that the neutralization countermeasure involves choosing a low capacity level, as k_1^N lies below the undistorted monopoly output, $q^M(0)$. Observe that neutralization involves even lower capacity choices by Firm 1 as the level of avoidable fixed costs (i.e., α and F) are reduced.

Consider next the possibility that Firm 1 is unable to neutralize Firm 2's market-capturing strategy. In this event, Firm 1 may nevertheless constrain its capacity choice in order better to accommodate Firm 2 in the MSE. A lower level of k_1 commits Firm 1 to a less aggressive reaction function in post-entry competition, which may raise Firm 2's payoff in its preferred MSE. In addition, by lowering k_1 toward $q^M(0)$, Firm 1 makes the market-capturing strategy less attractive for Firm 2, since a higher level of capacity $k_2(k_1)$ is then needed to implement this strategy. Thus, for both of these reasons, a lower value of k_1 can raise Firm 2's profit in the MSE relative to the F2ME, thereby giving Firm 2 a greater incentive not to exercise its market-capturing strategy.

To obtain a more formal expression of the idea that the first mover must reduce its capacity commitment, we strengthen the assumptions as follows. Consider the profits of Firm 1 when Firm 2 chooses $k_2 = 0$, and when the postentry outcome is the point on Firm 2's reaction correspondence at which Firm 1 operates at full capacity:

$$P^W(k_1) = D(k_1 + q^R(k_1|0))k_1 - C(k_1) - rk_1 - F$$

If $P^W(k_1)$ is a strictly quasi-concave function of k_1 , then in the absence of avoidable fixed costs ($\alpha = F = 0$), there is a unique subgame-perfect equilibrium choice of k_1 , which we may denote k_1^W (this is Firm 1's equilibrium capacity choice in Ware's model). The next

proposition gives conditions under which Firm 1's equilibrium capacity in leader–optimal equilibria lies below k_1^W .

Proposition 5. Suppose $P^W(k_1)$ is a strictly quasi–concave function of k_1 , and also $k_1^W \geq q^M(0)$. Let k_1^E be the largest of Firm 1's payoff–maximizing capacity choices in leader–optimal equilibria for given α and F .

- (a) $k_1^E \leq k_1^W$, and $k_1^E < k_1^W$ if (α, F) is sufficiently close to $(1, \bar{F})$;
- (b) k_1^E is decreasing in α , and strictly so if $0 < k_1^E < k_1^W$.

Part (a) indicates that Firm 1's leader–optimal–equilibrium capacity cannot exceed the benchmark level k_1^W , and must be strictly smaller if avoidable fixed costs are sufficiently high. The assumption $k_1^W \geq q^M(0)$ is important for this result, since it then follows from Proposition 4(b) that $k_1^N \leq k_1^W$; thus starting at k_1^W , neutralization and accommodation both involve reduction in Firm 1's capacity level. If we instead have $k_1^W < q^M(0)$, then $k_1^N > k_1^W$ would be possible, and neutralization could lead Firm 1 to choose equilibrium capacity greater than k_1^W for a range of α and F .¹³

According to part (b), Firm 1's equilibrium capacity is reduced as α rises: the quasi–concavity of $P^W(k_1)$ assures that k_1^E must be the highest level of k_1 that does not induce Firm 2 to use its market–capturing strategy; as α rises, deterring Firm 2 places a tighter constraint on Firm 1, and so k_1^E must fall. A similar effect can derive from an increase in F .¹⁴ Overall, a rise in either component of avoidable fixed costs will reduce Firm 1's leader–optimal equilibrium profits, and will tend to lower the equilibrium capacity level chosen by Firm 1.

6. Conclusion

This analysis calls into question the commonly–held view concerning the role of

capacity investment in establishing commitment to the market, which states that expansion of capacity strengthens commitment by increasing the incentive to produce large amounts of output. Our results show that in the presence of avoidable fixed costs and forward-induction inference, capacity investment may turn into a liability if it makes shutdown even more attractive than large output. As demonstrated above, establishing commitment to the market places an upper bound on the feasible investment levels, based on the need to discourage rivals from exploiting the avoidable fixed cost liability.

More broadly speaking, our results suggest that a first mover's ability to exploit its commitment power may be substantially curtailed due to the second mover's superior ability to communicate its strategic intent in subsequent rivalry. While our particular application of forward induction inference relies on details of the model that we consider, our basic point is much more general: as a firm's observable commitments become further removed from ex post rivalry, the scope for clearly communicating strategic intent in the rivalry is diminished. Here the first mover loses communication power as a consequence of being further removed temporally from the ex post stage, but it is reasonable to suppose that geographic proximity or product scope may also influence a firm's ability to use commitments to communicate strategic intent.

We establish further that a key determinant of strategic advantage is the extent to which fixed costs are avoidable. The essential point may be stated briefly: sunk costs convey strategic advantage to first movers, while avoidable costs convey the advantage to second movers. This dichotomy emerges in our model as a consequence of the superior communication power possessed by the second mover. It is interesting to note that a similar dichotomy has emerged elsewhere in the broader literature on strategic rivalry; examples include the entry-deterrence model of Eaton and Lipsey (1980) and the model of contestable markets presented by Baumol and Willig (1981).

Finally, when avoidable fixed costs are sufficiently large to give the strategic

advantage to the second mover, it is reasonable to ask whether the first mover might surrender its position by delaying investment. While this possibility is not explicitly analyzed here, it is possible to embed our model into a game of endogenous entry timing. A straightforward analysis along the lines of Ramey (1988) gives the following result: in symmetric mixed-strategy entry equilibria, second mover advantages are associated with delayed entry, and delay rises with the relative attractiveness of the second position. Further, delay is infinite where the first mover's best strategy is to cede the market, i.e. second mover advantages lead to market failure.¹⁵

Appendix

The undominated game associated with the sequential capacity choice model is derived as follows. Recall that (2) holds if and only if $k_i \geq \bar{k}$. Fix $k_i < \bar{k}$ and consider the equation:

$$(A1) \quad D(q_i)q_i - C(q_i) - r \max\{q_i, k_i\} - F = 0$$

(2) assures that the left-hand side of (A1) is strictly positive at $q_i = q^M(k_i)$. Thus since the left-hand side of (A1) is strictly concave in q_i , (A1) has a single solution $q^L(k_i)$ that lies below $q^M(k_i)$ (along with a single solution that lies above $q^M(k_i)$). Clearly $q^L(k_i) > 0$ and $\lim_{k_i \rightarrow \bar{k}} q^L(k_i) = q^M(\bar{k})$, using the definition of $\bar{k}$. Similarly, consider the equation:

$$(A2) \quad D(q_i)q_i - C(q_i) - r \max\{q_i, k_i\} - F = -(1 - \alpha)rk_i$$

(A2) differs from (A1) only in that the right-hand side of (A2) gives the profits from shutdown, which for $k_i > 0$ will be strictly negative. Let $q^N(k_i)$ denote the single solution to (A2) that lies below $q^M(k_i)$. It follows that $0 < q^N(k_i) < q^L(k_i)$ for $k_i > 0$.

The sets of undominated strategies are characterized as follows.

Lemma A1. S_1^U is exactly the set of all $s_1 = \{\hat{k}_1, \hat{q}_1(k_2)\}$ such that:

- (a) $\hat{k}_1 < \bar{k}$;
- (b) For all k_2 , $\hat{q}_1(k_2) \in (q^N(\hat{k}_1), q^M(\hat{k}_1)]$ whenever $\hat{q}_1(k_2) > 0$; and
- (c) There exists k_2 such that $\hat{q}_1(k_2) \in (q^L(\hat{k}_1), q^M(\hat{k}_1)]$ whenever $\hat{k}_1 > 0$.

Proof. Suppose $s_1 \in S_1^U$. If $\hat{k}_1 \geq \bar{k}$, then (2) implies that $P_1(s_1, s_2) \leq 0$ for any s_2 , and the inequality is strict if $\hat{q}_2(\hat{k}_1) > 0$. Thus s_1 would be dominated by the strategy s'_1 that sets $\hat{k}'_1 = \hat{q}'_1(k_2) = 0$ for all k_2 ; this establishes (a). If $\hat{q}_1(k'_2) \in (0, q^N(\hat{k}_1)]$, then from (A2) it follows that $P_1(s_1, s_2) \leq -(1 - \alpha)\hat{k}_1$ for any s_2 that sets $\hat{k}_2(\hat{k}_1) = k'_2$, and the inequality is strict if $\hat{q}_2(\hat{k}_1) > 0$. Define s'_1 as follows: $\hat{k}'_1 = \hat{k}_1$, $\hat{q}'_1(k_2) = \hat{q}_2(k_2)$ for $k_2 \neq k'_2$, and $\hat{q}'_2(k'_2) = 0$. It follows that $P_1(s'_1, s_2) = P_1(s_1, s_2)$ for any s_2 with $\hat{k}_2(\hat{k}_1) \neq k'_2$, whereas $P_1(s'_1, s_2) = -(1 - \alpha)\hat{k}_1 \geq P_1(s_1, s_2)$ for s_2 with $\hat{k}_2(\hat{k}_1) = k'_2$, where the inequality is strict for $\hat{q}_2(\hat{k}_1) > 0$; thus s'_1 dominates s_1 . If $\hat{q}_1(k'_2) > q^M(\hat{k}_1)$, then we may define s'_1 just as above, except $\hat{q}'_1(k'_2) = q^M(\hat{k}_1)$. Clearly $P_1(s'_1, s_2) > P_1(s_1, s_2)$ for any s_2 with $\hat{k}_2(\hat{k}_1) = k'_2$, since $\hat{q}'_2(k'_2)$ puts Firm 1 closer to its best response for any $\hat{q}_2(\hat{k}_1)$, while $P_1(s'_1, s_2) = P_1(s_1, s_2)$ for all other s_2 ; thus s'_1 dominates s_1 . This establishes (b). If s_1 sets $\hat{k}_1 > 0$ and $\hat{q}_1(k_2) \leq q^L(\hat{k}_1)$ for all k_2 , then using (A1) we have $P_1(s_1, s_2) \leq 0$ for any s_2 , and the inequality is strict if $\hat{q}_2(\hat{k}_1) > 0$. Thus s_1 would be dominated by the strategy s'_1 that sets $\hat{k}'_1 = \hat{q}'_1(k_2) = 0$ for all k_2 ; combining this with (b) establishes (c).

Now we assume s_1 satisfies (a), (b) and (c), and show that no s'_1 dominates s_1 . First consider $\hat{k}_1 > 0$ and $\hat{k}_1 \neq \hat{k}'_1$. By (c) there exists k'_2 such that $\hat{q}_1(k'_2) \in (q^L(\hat{k}_1), q^M(\hat{k}_1)]$. Let s_2 satisfy $\hat{k}_2(\hat{k}_1) = k'_2$, $\hat{q}_2(\hat{k}_1) = 0$, and $\hat{k}_2(\hat{k}'_1) = \hat{q}_2(\hat{k}'_1) > q^X(\hat{k}'_1)$, in which case $P_1(s_1, s_2) > 0 \geq P_1(s'_1, s_2)$. Next consider $\hat{k}_1 = 0$ and $\hat{k}'_1 > 0$, and let s_2 continue to satisfy the preceding. Using (b) we have $P_1(s_1, s_2) \geq 0 > P_1(s'_1, s_2)$. Finally, for $\hat{k}_1 = \hat{k}'_1$ we must have $\hat{q}_1(k'_2) \neq \hat{q}'_1(k'_2)$ for some k'_2 , and since $\hat{q}_1(k'_2) \leq q^M(\hat{k}_1)$ we may find q'_2 such that when Firm 2 chooses $\hat{q}_2(\hat{k}_1) = q'_2$, $\hat{q}_1(k'_2)$ gives strictly higher profits in postentry competition than does $\hat{q}'_1(k'_2)$ (for example, we might have $q^R(q'_2 | \hat{k}_1) \leq \hat{q}_1(k'_2) < \hat{q}'_1(k'_2)$); q'_2 may be used to specify s_2 such that $P_1(s_1, s_2) > P_1(s'_1, s_2)$.

Q.E.D.

Lemma A2. S_2^U is exactly the set of all $s_2 = \{\hat{k}_2(k_1), \hat{q}_2(k_1)\}$ such that, for every k_1 :

- (a) $\hat{k}_2(k_1) < \bar{k}$;
- (b) $\hat{q}_2(k_1) \in (q^L(\hat{k}_2(k_1)), q^M(\hat{k}_2(k_1))]$ whenever $\hat{q}_2(k_1) > 0$; and
- (c) $\hat{k}_2(k_1) = 0$ whenever $\hat{q}_2(k_1) = 0$.

Proof. Suppose $s_2 \in S_2^U$. $\hat{k}_2(k_1) < \bar{k}$ for all k_1 may be established as in the proof of Lemma 1, so it remains to verify that (b) and (c) are satisfied. If there exists k'_1 such that $\hat{q}_2(k'_1) > 0$ and $\hat{q}_2(k'_1) \leq q^L(\hat{k}_2(k'_1))$, then we have $P_2(s_1, s_2) \leq 0$ for all s_1 such that $\hat{k}_1 = k'_1$, with strict inequality if $\hat{q}_1(\hat{k}_2(k'_1)) > 0$. Let s'_2 defined by $\hat{k}'_2(k_1) = \hat{k}_2(k_1)$ and $\hat{q}'_2(k_1) = \hat{q}_2(k_1)$ for $k_1 \neq k'_1$, and $\hat{k}'_2(k'_1) = \hat{q}'_2(k'_1) = 0$. Then $P_2(s_1, s'_2) = P_2(s_1, s_2)$ for any s_1 with $\hat{k}_1 \neq k'_1$, and $P_2(s_1, s'_2) = 0 \geq P_2(s_1, s_2)$ for any s_1 with $\hat{k}_1 = k'_1$, where the inequality is strict if $\hat{q}_1(\hat{k}_2(k'_1)) > 0$; thus s'_2 dominates s_2 . Further, if s_2 sets $\hat{q}_2(k'_1) > q^M(\hat{k}_2(k'_1))$, then we could define s'_2 to agree with s_2 except for putting $\hat{q}'_2(k'_1) = q^M(\hat{k}_2(k'_1))$, and s'_2 would dominate s_2 . This establishes (b). If s_2 puts $\hat{q}_2(k'_1) = 0$, then Firm 2's profits are $-(1 - \alpha)rk_2(k'_1)$ when Firm 1 chooses k'_1 , and s_2 would be dominated unless $\hat{k}_2(k'_1) = 0$; this gives (c).

Now suppose that s_2 satisfies (a), (b) and (c), but $s_2 \notin S_2^U$. Then there must be some s'_2 which dominates s_2 , and for some k'_1 we will have $\{\hat{k}_2(k'_1), \hat{q}_2(k'_1)\} \neq \{\hat{k}'_2(k'_1), \hat{q}'_2(k'_1)\}$. Suppose that $\hat{k}_2(k'_1) \neq \hat{k}'_2(k'_1)$, and consider s_1 which sets $\hat{k}_1 = k'_1$ and:

$$\hat{q}_1(k_2) = \begin{cases} 0, & k_2 = \hat{k}_2(k'_1) \\ q'_1, & k_2 \neq \hat{k}_2(k'_1) \end{cases}$$

If $\hat{k}_2(k'_1) > 0$, then (b) and (c) imply that $\hat{q}_2(k'_1) \in (q^L(\hat{k}_2(k'_1)), q^M(\hat{k}_2(k'_1))]$. Since $\hat{q}_1(\hat{k}_2(k'_1)) = 0$, s_2 gives strictly positive profits as a response to this s_1 , while s'_2 must yield nonpositive profits if q'_1 is sufficiently large; thus, $P_2(s_1, s_2) > P_2(s_1, s'_2)$, and s'_2 cannot dominate s_2 . If instead $\hat{k}_2(k'_1) = 0$, then $\hat{k}'_2(k'_1) > 0$, and so s'_2 again fails to

dominate s_2 , because $P_2(s_1, s_2) \geq 0 > P_2(s_1, s'_2)$ for q'_1 sufficiently large.

Suppose next that $\hat{k}_2(k'_1) = \hat{k}'_2(k'_1)$, so that $\hat{q}_2(k'_1) \neq \hat{q}'_2(k'_1)$. Since $\hat{q}_2(k'_1) \leq q^M(\hat{k}_2(k'_1))$, we can find q'_1 such that when Firm 1 chooses $\hat{q}_1(\hat{k}_2(k'_1)) = q'_1$, $\hat{q}_2(k'_1)$ gives strictly higher profits in postentry competition than does $\hat{q}'_2(k'_1)$. Thus s_2 is strictly better than s'_2 against s_1 which sets $\hat{k}_1 = k'_1$ and $\hat{q}_1(\hat{k}_2(k'_1)) = q'_1$, and s'_2 cannot dominate s_2 .

Q.E.D.

We now consider subgame–perfect equilibria of the undominated game. A technical difficulty must be confronted: in subgames of the undominated game, the allowable quantities associated with given capacity choices typically are not drawn from closed sets, i.e. the intervals $(q^N(k_1), q^M(k_1)]$ and $(q^L(k_2(k_1)), q^M(k_2(k_1))]$ of allowable quantities do not contain their lower endpoints. This creates no problem for Firm 1's choices, as $q^R(q_2|k_1) \in (q^N(k_1), q^M(k_1)]$ for any q_2 such that $q^R(q_2|k_1) > 0$, so that Firm 1's reaction function in any subgame is simply the restriction of $q^R(q_2|k_1)$ over $q_2 \in (q^L(k_2), q^M(k_2)]$. For Firm 2, however, we might have $q^R(q_1|k_2) < q^L(k_2)$ for some q_1 if $k_2 > 0$, in which case $q^L(k_2)$ generates strictly higher profit for Firm 2 in response to k_1 than does any q_2 in the undominated game (this would occur if Firm 2 would not want to enter expecting the response q_1 , but once it has entered it would not want to shut down). In this case, Firm 2's best response function in the (k_1, k_2) –subgame of the undominated game would not be defined.

We remedy this problem by expanding set S_2^U slightly to include responses $q^L(k_2)$; we denote this set by $\bar{S}_2^U$. This allows Firm 2's reaction function to be defined for every subgame, and further, Firm 2's reaction correspondence in the undominated game is continuous in q_1 when $k_2 > 0$, as no strategy in $\bar{S}_2^U$ allows Firm 2 to jump to $q_2 = 0$.

This modification of the undominated game is supportable from two perspectives. First, we can define our subgame–perfect equilibria as limits of outcomes that specify

ϵ –equilibria in every subgame as $\epsilon \rightarrow 0$; this allows us to take $q^L(k_2)$ as the limit of Firm 2 ϵ –best responses that are slightly above $q^L(k_2)$. Second, Firm 2 earns a strictly negative payoff in any subgame in which $k_2 > 0$ and the equilibrium occurs at $\hat{q}_2(k_1) = q^L(k_2)$, whereas by choosing $k_2 = 0$, Firm 2 is assured of a nonnegative payoff in every postentry equilibrium; thus Firm 2 never chooses such a $k_2 > 0$ in any equilibrium. More broadly, whenever Nash equilibria fail to exist for a given subgame in the undominated game, Firm 2 obtains a strictly negative payoff in every Nash equilibrium of the subgame in the original game. Firm 2 would never choose k_2 to induce such a subgame in any subgame–perfect equilibrium of the original game, irrespective of its ability to coordinate equilibrium selection in the subgame; thus it seems reasonable to rule out such subgames based on backward–induction rationality alone.

We now consider the possible equilibria of (k_1, k_2) –subgames of the undominated game, as modified above. Let $q^Z(q_j | k_i)$ denote Firm i ’s reaction correspondence for the case of zero avoidable fixed costs as in the model of Dixit, i.e. $q^Z(q_j | k_i)$ gives the value of $q^R(q_j | k_i)$ when $\alpha = F = 0$. Thus, Firm 2’s best responses in the undominated game are given by $\max\{q^L(k_2), q^Z(q_1 | k_2)\}$. Put $q^I(k_1) = q^Z(q^X(k_1) | k_1)$, i.e. at $q_2 = q^X(k_1)$, Firm 1 is indifferent between $q_1 = 0$ and $q_1 = q^I(k_1)$ (if Firm 1 prefers zero output for all q_2 , put $q^I(k_1) = 0$). Subgame equilibria for $k_2 > 0$ are discussed further in the following lemma.

Lemma A3. Let the strategy sets of the original game be replaced by S_1^U and S_2^U , and consider a (k_1, k_2) –subgame with $k_2 > 0$. If:

$$(A3) \quad \max\{q^L(k_2), q^Z(q^I(k_1) | k_2)\} > q^X(k_1)$$

then the unique Nash equilibrium of the subgame is a F2ME. If (A3) fails to hold and k_2

> 0 , then the subgame possesses a single MSE (F2ME may or may not exist), and the quantity choices in this MSE are continuous functions of k_1 and k_2 .

Proof. For $k_2 > 0$, Firm 2's best responses in the undominated game are bounded away from zero, and so there are no F1ME in such subgames. If $q^L(k_2) > q^X(k_1)$, then $q^R(q_2|k_1) = 0$ for all q_2 that are possible in the undominated game, and the unique intersection of reaction functions is at $q_1 = 0, q_2 = q^M(k_2)$. If $q^Z(q^I(k_1)|k_2) > q^X(k_1)$, then in the (q_1, q_2) plane, Firm 2's reaction correspondence lies above Firm 1's reaction correspondence at $q_1 = q^I(k_1)$; further, Firm 2's reaction correspondence cannot cut Firm 1's from above, since that of Firm 1 must always have the steeper slope at any intersection point, and so there can be no intersection of the reaction correspondences with $q_1 \geq q^I(k_1)$. Thus no MSE exists; a F2ME exists, however, since $q^M(k_2) \geq q^Z(q^I(k_1)|k_2) > q^X(k_1)$.

Now suppose (A3) fails. Existence of a MSE is shown as follows. If $q^Z(q^I(k_1)|k_2) = q^M(k_2)$, then $q^X(k_1) \geq q^M(k_2)$, and we can restrict Firm 1's reaction correspondence to strictly positive responses on $[q^L(k_2), q^M(k_2)]$. Firm 1's reaction correspondence is a continuous function, strictly positive and $q^Z(q_2|k_1) > q^N(k_1)$ on $[q^L(k_2), q^M(k_2)]$, and Firm 2's reaction correspondence is a continuous and strictly positive function on $(q^N(k_1), q^M(k_1)]$, which implies that there exists an intersection of the functions on the restricted domains. If $q^Z(q^I(k_1)|k_2) < q^M(k_2)$, then there are two possibilities. First, if $q^Z(q^I(k_1)|k_2) = q^X(k_1)$, then $(q_1, q_2) = (q^I(k_1), q^X(k_2))$ gives an intersection of the reaction correspondences. Second, if $q^Z(q^I(k_1)|k_2) < q^X(k_1)$, then we have $q^R(q^Z(q^I(k_1)|k_2)|k_1) > q^I(k_1)$, and so the restriction of Firm 1's reaction correspondence to $[q^L(k_2), q'_2]$, where $q'_2 \equiv \max\{q^Z(q^I(k_1)|k_2), q^L(k_2)\}$, is a strictly positive and continuous function. Further, the restriction of Firm 2's reaction correspondence to $[q^R(q'_2|k_1), q^R(q^L(k_2)|k_1)]$ lies in $[q^L(k_2), q'_2]$, since $q_1 \geq q^R(q^Z(q^I(k_1)|k_2)|k_1) > q^I(k_1)$ implies $q^Z(q_1|k_2) \leq q^Z(q^I(k_1)|k_2)$. Again there must be an intersection of the reaction

correspondences in the restricted domains. In each case there exists only one MSE, since for $q_2 < q^X(k_1)$, the reaction correspondences give continuous functions and that of Firm 1 is everywhere steeper than that of Firm 2. These properties, together with the fact that the reaction correspondences and restricted domains shift continuously in k_1 and k_2 , ensure that the MSE quantities are continuous in k_1 and k_2 . Finally, observe that F2ME will exist if and only if $q^M(k_2) \geq q^X(k_1)$. Q.E.D.

Proof of Proposition 1. (a) Suppose $k_2 > 0$. Since $\max\{q^L(k_2), q^Z(q^I(k_1)|k_2)\} \leq q^M(k_2) \leq \min\{q^M(x), \bar{k}\}$, it follows that (A3) can possibly hold for some $k_2 < \bar{k}$ if and only if $q^X(k_1) < \min\{q^M(x), \bar{k}\}$. Thus (A3) fails if $q^X(k_1) \geq \min\{q^M(x), \bar{k}\}$, and Lemma A3 assures existence of a MSE. If $k_2 = 0$ and $q^X(k_1) \geq \min\{q^M(x), \bar{k}\}$, then from the proof of Lemma A3 it follows that MSE fail to exist if and only if Firm 2's reaction correspondence jumps to $q_2 = 0$ for q_1 in an upper subinterval of the domain $(q^N(k_1), q^M(k_1)]$; thus there must exist an intersection of the reaction correspondences at $q_1 = q^M(k_1)$, $q_2 = 0$.

(b) It can be shown that $q^L(k_2) \rightarrow \min\{q^M(x), \bar{k}\}$ as $k_2 \rightarrow \bar{k}$ (as depicted in Figure 2), and so $q^X(k_1) < \min\{q^M(x), \bar{k}\}$ implies $q^L(k_2) > q^X(k_1)$ for $k_2 < \bar{k}$, sufficiently close to $\bar{k}$. It follows that (A3) holds for such k_2 , and we may let $\underline{k}_2(k_1) < \bar{k}$ denote the infimum of values $k_2 \geq 0$ that satisfy (A3). Since the left-hand side of (A3) is a nondecreasing and continuous function of k_2 for $k_2 > 0$, it follows that when $\underline{k}_2(k_1) > 0$, (A3) holds if and only if $k_2 > \underline{k}_2(k_1)$, and thus $k_2 \leq \underline{k}_2(k_1)$ ensures existence of a MSE and/or a F1ME, using Lemma A3 and the proof of part (a) above. (When $\underline{k}_2(k_1) = 0$, (A3) holds for $k_2 > 0$, but it may or may not hold for $k_2 = 0$.)

(c) If $\underline{k}_2(k_1) > 0$, then we have:

$$(A4) \quad \max\{q^L(\underline{k}_2(k_1)), q^Z(q^I(k_1)|\underline{k}_2(k_1))\} = q^X(k_1)$$

Either $q^L(\underline{k}_2(k_1)) = q^X(k_1)$, or else $q^Z(q^I(k_1)|\underline{k}_2(k_1)) = q^X(k_1) > q^L(\underline{k}_2(k_1))$; in the latter case we must have $q^Z(q^I(k_1)|\underline{k}_2(k_1)) = \underline{k}_2(k_1)$, else k_2 could be reduced slightly and (A3) would continue to hold. In either case, a decrease in $q^X(k_1)$ implies that $\underline{k}_2(k_1)$ must fall in order to preserve equality in (A4). Q.E.D.

Proof of Proposition 2. (a) $q^X(k_1)$ is defined by:

$$(A5) \quad \max_{q_1} [D(q_1 + q^X(k_1))q_1 - C(q_1) - r \max\{q_1, k_1\}] - F = -(1 - \alpha)rk_1$$

Clearly $q^X(k_1)$ is continuous and decreasing in α and F , and strictly decreasing in α and F if $q^X(k_1) > 0$ and $D(q^X(k_1)) > 0$. Define $\alpha_a(F)$ by:

$$(A6) \quad \alpha_a(F) = \frac{1}{rk_1} (\max_{q_1} [D(q_1 + q^M(x))q_1 - C(q_1) - r \max\{q_1, k_1\}] - F + rk_1)$$

If $\alpha_a(F) \in [0, 1)$, then we have $q^X(k_1) = q^M(x)$ at $\alpha = \alpha_a(F)$. Further, $q^M(x) > 0$ and $D(q^M(x)) > 0$ hold by definition, so $q^X(k_1)$ is strictly decreasing in α at $\alpha = \alpha_a(F)$, and $\alpha < \alpha_a(F)$ implies $q^X(k_1) > q^M(x)$. If $\alpha_a(F) \geq 1$, then (A5) and (A6) give $q^X(k_1) > q^M(x)$ for all $\alpha \in [0, 1)$.

Next, define $\alpha_b(F)$ by:

$$(A7) \quad \alpha_b(F) = \frac{1}{rk_1} (\max_{q_1} [D(q_1 + \bar{k})q_1 - C(q_1) - r \max\{q_1, k_1\}] - F + rk_1)$$

If $\alpha_b(F) \in [0, 1)$, then we have $q^X(k_1) = \bar{k}$ at $\alpha = \alpha_b(F)$. Further, $\bar{k} > 0$ holds by definition, and $D(\bar{k}) > 0$ is given by $\alpha_b(F) \geq 0$; thus $q^X(k_1)$ is strictly decreasing in α at α

$= \alpha_b(F)$, and $\alpha < \alpha_b(F)$ implies $q^X(k_1) > \bar{k}$. If $\alpha_b(F) \geq 1$, then (A5) and (A7) give $q^X(k_1) > \bar{k}$ for all $\alpha \in [0, 1]$.

Finally, define $\underline{\alpha}(F)$ by:

$$\underline{\alpha}(F) = \begin{cases} \max\{\alpha_a(F), \alpha_b(F)\}, & \alpha_a(F), \alpha_b(F) \in [0, 1] \\ 1, & \max\{\alpha_a(F), \alpha_b(F)\} \geq 1 \\ 0, & \max\{\alpha_a(F), \alpha_b(F)\} < 0 \end{cases}$$

Clearly $\alpha < \underline{\alpha}(F)$ implies $q^X(k_1) > \min\{q^M(x), \bar{k}\}$, so $\underline{k}_2(k_1) = \bar{k}$. If $\alpha = F = 0$, then $q^X(k_1)$ is defined by:

$$(A8) \quad \max_{q_1} [D(q_1 + q^X(k_1))q_1 - C(q_1) - r \max\{q_1, k_1\}] = -rk_1$$

If $D(q^X(k_1)) > C'(0)$, then there exists $q_1 \in (0, k_1)$ that makes the left-hand side of (A8) strictly greater than $-rk_1$; thus $D(q^X(k_1)) = C'(0)$ is necessary. But by definition we have $D(q^M(x)) > C'(q^M(x)) \geq C'(0)$, and $q^M(x) < q^X(k_1)$ is implied. This gives $0 < \alpha_a(0)$, using (A5) and (A6), and continuity of $q^X(k_1)$ in F assures $0 < \alpha_a(F)$ for F sufficiently small, whence $0 < \underline{\alpha}(F)$.

(b) Using Lemma A3, we have that $\underline{k}_2(k_1) = 0$ if:

$$\max\{q^L(0), q^Z(q^I(k_1)|0)\} > q^X(k_1)$$

which may be equivalently expressed as:

$$(A9) \quad \max\{q^L(0) - q^X(k_1), q^Z(q^I(k_1)|0) - q^X(k_1)\} > 0$$

Let $\bar{\alpha}(F)$ denote the infimum over the values of $\alpha \in [0,1)$ at which (A9) holds; if (A9) does not hold for any $\alpha \in [0,1)$, then set $\bar{\alpha}(F) = 1$. Since $\underline{k}_2(k_1) = \bar{k} > 0$ for $\alpha < \underline{\alpha}(F)$, it is immediate that $\bar{\alpha}(F) \geq \underline{\alpha}(F)$. We now show that if (A9) holds at some α' , then (A9) continues to hold for all $\alpha > \alpha'$. This is obviously true if $q^L(0) - q^X(k_1) > 0$ at α' , since $q^L(0)$ is independent of α and $q^X(k_1)$ is decreasing in α . Otherwise we have $q^Z(q^I(k_1)|0) - q^X(k_1) > 0$ at α' , and further:

$$\begin{aligned} \frac{\partial}{\partial \alpha} [q^Z(q^I(k_1)|0) - q^X(k_1)] &= \frac{\partial}{\partial \alpha} [q^Z(q^Z(q^X(k_1)|k_1)|0) - q^X(k_1)] \\ &= \left(\frac{\partial q^Z}{\partial q_1} \frac{\partial q^Z}{\partial q_2} - 1 \right) \frac{\partial q^X}{\partial \alpha} \end{aligned}$$

(where appropriate one-sided derivatives are used at kinked points of q^Z and q^X).

$\partial q^Z / \partial q_j \in (-1, 0]$ is easily verified, so $q^Z(q^I(k_1)|0) - q^X(k_1)$ is increasing in α ; it follows that $q^Z(q^I(k_1)|0) - q^X(k_1) > 0$ for all $\alpha > \alpha'$. Thus (A9) holds for all $\alpha > \bar{\alpha}(F)$. For $(\alpha, F) \rightarrow (1, \bar{F})$, we have $q^L(0) - q^M(0) > 0$ and $q^X(k_1) \rightarrow 0$, and so $q^L(0) - q^X(k_1) > 0$ for (α, F) sufficiently close to $(1, \bar{F})$; this gives $\bar{\alpha}(F) < 1$ for F sufficiently close to $\bar{F}$.

(c) Based on the preceding analysis, it is easy to verify that $\underline{\alpha}(F) < \alpha < \bar{\alpha}(F)$ implies $0 < \underline{k}_2(k_1) < \bar{k}$. Note that $\underline{k}_2(k_1) > 0$ implies $q^X(k_1) > 0$, so $q^X(k_1)$ is strictly decreasing in α and F at such a point, and a slight increase in either leads to lower $\underline{k}_2(k_1)$ according to Proposition 1(c). Q.E.D.

Proof of Proposition 3. (a) Fix k_1 and consider Firm 2's payoff-maximizing choice of k_2 for the selection of (k_1, k_2) -subgame equilibria given in the text. In view of Lemma A3, $k_2 = \underline{k}_2(k_1)$ and $k_2 = 0$ give the only possible points of discontinuity of Firm 2's payoff.

Since the F2ME is selected at $k_2 = \underline{k}_2(k_1)$, Firm 2's payoff has an upward jump at this point. At $k_2 = 0$, Firm 2's payoff may be discontinuous due to selection of the F1ME. Note however that Firm 2's payoff in MSE is independent of k_2 if k_2 is sufficiently small (note from Figure 2 that $q^L(k_2)$ is constant on the range $k_2 < q^L(k_2)$). Thus Firm 2's payoff is constant on $k_2 \in (0, \delta)$ for some small δ , and a downward jump in the payoff at $k_2 = 0$ still does not rule out existence of a payoff-maximizing choice of k_2 . Conclude that for each k_1 , there exists a k_2 that maximizes Firm 2's payoff. If Firm 2 is indifferent between two or more values of k_2 , we specify that it chooses one of those that makes Firm 1's payoff the highest.

Now consider Firm 1's choice of k_1 for the given best responses by Firm 2 and the (k_1, k_2) -subgame equilibria selected in the text. There are four cases in which Firm 1's payoff might be discontinuous at a point k'_1 :

(i) $\underline{k}_2(k'_1) = 0$. Perturbing k'_1 can give $\underline{k}_2(k_1) > 0$, enabling Firm 2 to induce a MSE or F1ME, even though a F2ME must be selected at k'_1 since $k_2 \geq \underline{k}_2(k'_1) = 0$ holds necessarily. A discontinuity of this form emerges only if either $q^L(0) = q^X(k'_1) \geq q^Z(q^I(k'_1)|0)$ or $q^Z(q^I(k'_1)|0) = q^X(k'_1) > q^L(0)$. In the former case, a perturbation of k'_1 can give $q^L(0) < q^X(k_1)$, and an upward jump in k_2 would be required to restore (A3). As can be seen in Figure 2, however, we have $q^L(k_2) > q^X(k_1)$ for $k_2 < q^M(0)$, since $q^X(k_1)$ would be close to $q^X(k'_1)$. Thus, $\underline{k}_2(k_1) < q^M(0)$, and Firm 2 would still choose to induce a F2ME, so that Firm 1's payoff is continuous at k'_1 . As for the latter case, since $q^Z(q^I(k'_1)|0) = q^X(k'_1) > 0$, we have $q^Z(q^I(k'_1)|k_2) = q^Z(q^I(k'_1)|0)$ for sufficiently small $k_2 > 0$, so that $\underline{k}_2(k'_1) > 0$ would be implied.

(ii) $0 < \underline{k}_2(k'_1)$ and a shift in k'_1 eliminates the F1ME. We know that the F1ME still exists at $k_1 = k'_1$, as a consequence of the fact that the reaction correspondences have closed graphs; thus Firm 1's payoff could only jump upward at this point.

(iii) $0 < \underline{k}_2(k'_1) < \min\{q^M(x), \bar{k}\}$ and a perturbation of k'_1 leads Firm 2 to shifts its

capacity response discontinuously. Suppose first that $\underline{k}_2(k_1)$ is continuous at $k_1 = k'_1$. Continuity of Firm 2's F2ME payoffs in k_2 ensures that Firm 2's preferred F2ME payoff is continuous in k_1 . Further, Lemma A3 ensures that Firm 2's payoff in its preferred MSE is continuous in k_1 , and Firm 2's F1ME payoff is constant at zero. Thus, Firm 2 must be indifferent between its limiting capacity responses at k'_1 , and we have specified that at k'_1 Firm 2 induces the equilibrium that is preferred by Firm 1.

If $\underline{k}_2(k_1)$ is discontinuous at $k_1 = k'_1$, then Firm 2 may no longer be indifferent between its limiting capacity responses due to a discontinuous shift in its ability to induce the F2ME. A number of possibilities arise. First, we may have $q^L(\underline{k}_2(k'_1)) = q^X(k'_1) > q^Z(q^I(k'_1)|\underline{k}_2(k'_1))$. Since $\underline{k}_2(k'_1) \in (0, \min\{q^M(x), \bar{k}\})$, $q^L(k_2)$ will be strictly increasing at $k_2 = \underline{k}_2(k'_1)$ (as can be seen in Figure 2), which implies that $\underline{k}_2(k_1)$ is continuous at $k_1 = k'_1$.

Second, we may have $q^Z(q^I(k'_1)|\underline{k}_2(k'_1)) = q^X(k'_1)$. If $q^Z(q^I(k'_1)|\underline{k}_2(k'_1)) = q^Z(q^I(k'_1)|0)$, then $\underline{k}_2(k'_1) = q^Z(q^I(k'_1)|\underline{k}_2(k'_1)) \leq q^M(0)$. Here, a perturbation of k'_1 might lead $\underline{k}_2(k_1)$ to jump to zero, but Firm 2 would induce a F2ME before and after the perturbation. If $q^Z(q^I(k'_1)|0) < q^Z(q^I(k'_1)|\underline{k}_2(k'_1)) < q^Z(q^I(k'_1)|x)$, then we have $q^Z(q^I(k'_1)|k_2) = k_2$ for k_2 in a neighborhood of $\underline{k}_2(k'_1)$, assuring that $\underline{k}_2(k_1)$ is continuous at $k_1 = k'_1$.

Finally, we may have $q^Z(q^I(k'_1)|\underline{k}_2(k'_1)) = q^Z(q^I(k'_1)|x)$. We cannot have $q^Z(q^I(k'_1)|x) > q^L(\underline{k}_2(k'_1))$ in this case, since $q^X(k'_1) = q^Z(q^I(k'_1)|x) > q^L(k_2)$ would hold, and an MSE would exist, for k_2 slightly above $\underline{k}_2(k'_1)$. Thus, $q^Z(q^I(k'_1)|x) = q^L(\underline{k}_2(k'_1))$ is necessary. A slight decrease in $q^X(k'_1)$ to $q^X(k_1)$ can then lead $\underline{k}_2(k'_1)$ to fall discontinuously, to a value k_2 such that $k_2 = q^Z(q^I(k_1)|k_2) = q^X(k_1)$. As long as Firm 2 prefers to induce a F2ME at $k_1 = k'_1$, it will prefer to do so after the perturbation since $\underline{k}_2(k_1)$ falls. If Firm 2 induces a MSE at $k_1 = k'_1$, then it may prefer either a MSE or a F2ME following the perturbation. Firm 1's payoff is continuous at $k_1 = k'_1$ if a MSE is

induced, while it jumps upward at $k_1 = k'_1$ if a F2ME is induced.

(iv) $\underline{k}_2(k'_1) = \min\{q^M(x), \bar{k}\}$. Now perturbing k'_1 can give $q^X(k_1) < \min\{q^M(x), \bar{k}\}$, enabling Firm 2 to induce a F2ME, but Firm 2 cannot do so at k'_1 . In this case a MSE arises when Firm 1 chooses k'_1 , so again Firm 1's payoff can only jump upward at k'_1 .

It follows that Firm 1's profits are upperhemicontinuous in k_1 , and by specifying a profit-maximizing choice of k_1 we complete the construction.

(b) Note that if we make a different specification of postentry equilibria, then Firm 2's capacity choice is affected only if it chooses to induce a postentry equilibrium which gives it greater profits. But this necessarily lowers Firm 1's profits. It follows that the above construction maximizes Firm 1's equilibrium profits over the set of pure-strategy subgame-perfect equilibria.

Finally, there will not be any equilibria with mixed strategies in which Firm 1 earns higher profit. Note first that in the post-entry subgame, neither firm will mix among positive quantities, as a consequence of the strict concavity of the profit function in own quantity. Thus, if $k_2 > 0$ Firm 2 must choose a pure quantity-strategy, and Firm 1 will then mix only if $q_2 = q^X(k_1)$; Firm 1 is indifferent among such outcomes. If $k_2 = 0$, there are two reasons why Firm 2 might mix: (i) $q_1 = q^X(k_2)$. In this case the F1ME exists, and it gives Firm 1 greater profits than any of the mixed outcomes. (ii) Firm 1 mixes. In this case Firm 1's expected postentry profits are $-(1 - \alpha)rk_1$, which are no greater than in the outcome specified above. Thus, replacing the specified postentry outcomes with mixed outcomes can only reduce Firm 1's profits. Further, mixed choices of k_2 will only reduce the incumbent's profits, as we have specified that if Firm 2 is indifferent, it chooses the capacity which makes Firm 1's profits the greatest. *Q.E.D.*

Proof of Proposition 4. (a) Using (A5) and the fact that $q^I(k_1)$ by definition gives the solution to the maximization problem in (A5), the following formulas may be derived

(where appropriate one-sided derivatives are used at kinked points of q^X): (i) for k_1 such that $q^I(k_1) > k_1$:

$$(A10) \quad D'(q^I(k_1) + q^X(k_1))q^I(k_1) + D(q^I(k_1) + q^X(k_1)) - C'(q^I(k_1)) - r = 0$$

and $q^X(k_1)$ is strictly increasing in k_1 ; (ii) for k_1 such that $q^I(k_1) = k_1$:

$$(A11) \quad \frac{\partial q^X(k_1)}{\partial k_1} = \frac{-(D'(k_1 + q^X(k_1))k_1 + D(k_1 + q^X(k_1)) - C'(k_1) - \alpha r)}{D'(k_1 + q^X(k_1))k_1}$$

and (iii) for k_1 such that $q^I(k_1) < k_1$:

$$(A12) \quad D'(q^I(k_1) + q^X(k_1))q^I(k_1) + D(q^I(k_1) + q^X(k_1)) - C'(q^I(k_1)) \leq 0$$

$$(A13) \quad \frac{\partial q^X(k_1)}{\partial k_1} = \frac{\alpha r}{D'(q^I(k_1) + q^X(k_1))q^I(k_1)} \quad \text{for } q^I(k_1) > 0$$

For $F > 0$, it is necessary that $q^I(0) > 0$, and so $q^X(k_1)$ is strictly increasing in k_1 at $k_1 = 0$. Higher $q^X(k_1)$ reduces $q^I(k_1)$, which means that $q^I(k_1) - k_1$ is strictly decreasing in k_1 whenever $q^I(k_1) > k_1$. As k_1 rises, eventually k_1^a is reached such that $q^I(k_1^a) = k_1^a$ and (A10) holds at k_1^a . For $F = 0$, $q^I(0) = 0$ and (A10) holds at $k_1 = 0$; in this case $k_1^a = 0$. Comparing (A10) and (A11) and using the fact that $\alpha < 1$, it follows that $q^X(k_1)$ is strictly increasing at k_1^a .

Because $q^I(k_1) > k_1$ implies that $q^I(k_1) - k_1$ is strictly decreasing in k_1 , it follows that $q^I(k_1) \leq k_1$ for all $k_1 \geq k_1^a$. As k_1 rises, eventually $k_1^b > k_1^a$ is reached such that $q^I(k_1) = k_1$ for all $k_1 \in [k_1^a, k_1^b]$ and (A12) holds at k_1^b (existence of such a k_1^b follows from the

continuity of $q^X(k_1)$ and $q^I(k_1)$ along with the fact that $q^I(k_1) < k_1$ for all $k_1 > q^M(x)$. Comparing (A11) and (A12), and using (A13), it follows that $\partial q^X(k_1)/\partial k_1 = 0$ for all $k_1 \geq k_1^b$ if $\alpha = 0$, and $q^X(k_1)$ is strictly decreasing at k_1^b if $\alpha > 0$. Conclude that $q^X(k_1)$ has at least one extremum in $[k_1^a, k_1^b]$, and further, at any such extremum the value of the derivative in (A11) is zero. Differentiating (A11) with respect to k_1 and using $\partial q^X(k_1)/\partial k_1 = 0$ gives:

$$\frac{\partial^2 q^X(k_1)}{\partial k_1^2} = \frac{-(D''(k_1 + q^X(k_1))k_1 + 2D'(k_1 + q^X(k_1)) - C''(k_1))}{D'(k_1 + q^X(k_1))k_1} < 0$$

Thus any extremum in $[k_1^a, k_1^b]$ must be a local maximum, meaning that there is exactly one extremum, which we denote k_1^N .

If $\alpha > 0$, then $q^X(k_1)$ must be strictly decreasing in k_1 for $k_1 > k_1^b$ and $q^X(k_1) > 0$, which is seen as follows. If $q^I(k_1) < k_1$, or if k_1 is on the boundary of an interval such that $q^I(k_1) < k_1$, then (A11), (A12), (A13) and $\alpha > 0$ assure that $q^X(k_1)$ is strictly decreasing at k_1 . If k_1 is on the interior of an interval such that $q^I(k_1) = k_1$, then from the preceding we know that $q^X(k_1)$ is strictly decreasing at the endpoints of the interval; thus there can be no extremum in the interval since any extremum would have to be a local maximum, as shown above.

(b) From the preceding it follows that k_1^N identically satisfies:

$$(A14) \quad D'(k_1^N + q^X(k_1^N))k_1^N + D(k_1^N + q^X(k_1^N)) - C'(k_1^N) - \alpha r = 0$$

We must have $q^X(k_1^N) > 0$, since $q^X(k_1^N) = 0$ would imply either $q^I(k_1^N) = q^M(0)$ or $q^I(k_1^N) = 0$, while $k_1^N > q^M(0)$ would follow from (A14) and $\alpha < 1$; this contradicts the requirement that $q^I(k_1^N) = k_1^N$. It is direct to establish using (A14) that k_1^N is strictly

increasing in α and F .

For any $k_1 > 0$, we have $q^I(k_1) \rightarrow 0$ as $(\alpha, F) \rightarrow (0, 0)$, and so $q^I(k_1) < k_1$ for sufficiently small α and F ; since $q^X(k_1)$ is strictly decreasing at such a k_1 for $\alpha > 0$, it follows from the strict quasi-concavity of $q^X(k_1)$ that $k_1^N < k_1$, i.e. the maximizer of $q^X(k_1)$ must lie below k_1 . This is true for arbitrarily small $k_1 > 0$ when α and F are taken sufficiently small, and thus $k_1^N \rightarrow 0$ as $(\alpha, F) \rightarrow (0, 0)$. Finally, for any k_1 , we have $q^X(k_1) \rightarrow 0$ as $(\alpha, F) \rightarrow (1, \bar{F})$, and so in the limit (A14) becomes:

$$D'(k_1^N)k_1^N + D(k_1^N) - C'(k_1^N) - r = 0$$

which implies $k_1^N = q^M(0)$.

Q.E.D.

Proof of Proposition 5. (a) Put:

$$P^M(k_1) = D(q^M(\underline{k}_2(k_1)))q^M(\underline{k}_2(k_1)) - C(q^M(\underline{k}_2(k_1))) - r \max\{q^M(\underline{k}_2(k_1)), \underline{k}_2(k_1)\} - F$$

$P^M(k_1)$ gives the maximized payoff of Firm 2 over $k_2 \geq \underline{k}_2(k_1)$. Note that $P^M(k_1) > 0$ if and only if $\underline{k}_2(k_1) < \bar{k}$. Further, let $q_i^S(k_1, k_2)$ denote Firm i 's output in the unique MSE derived in Lemma A3 for $0 < k_2 \leq \underline{k}_2(k_1)$, and put:

$$P^S(k_1) = \max_{k_2 \in (0, \underline{k}_2(k_1)]} [D(q_1^S(k_1, k_2) + q_2^S(k_1, k_2))q_2^S(k_1, k_2) - C(q_2^S(k_1, k_2)) \\ - r \max\{q_2^S(k_1, k_2), k_2\}] - F$$

$P^S(k_1)$ gives the maximized payoff of Firm 2 over $0 < k_2 \leq \underline{k}_2(k_1)$ given that MSE are induced in the (k_1, k_2) –subgames. As in the proof of Proposition 3, existence of a maximizer follows from the fact that $q_2^S(k_1, k_2) = q^L(k_2)$ is constant for k_2 close to zero. If $k_2 = \underline{k}_2(k_1)$ gives the maximizer, then clearly $P^M(k_1) > P^S(k_1)$. Further, $P^S(k_1) > 0$ implies $q_2^S(k_1, k_2) > q^L(k_1)$, and so $(q_1^S(k_1, k_2), q_2^S(k_1, k_2))$ is a point on Firm 2’s reaction correspondence when $\alpha = F = 0$. If $P^S(k_1) \leq 0$, then choosing $k_2 = 0$ induces a F1ME in the (k_1, k_2) –subgame and Firm 2 earns zero in this MSE; in this case set $P^S(k_1) = 0$ instead.

It follows that in the leader–optimal equilibrium, Firm 1 faces the following constraint on its choice of k_1 :

$$(A15) \quad \text{Either } \underline{k}_2(k_1) = \bar{k} \text{ or } \underline{k}_2(k_1) < \bar{k}, P^S(k_1) \geq P^M(k_1)$$

The first part of (A15) indicates that no market–capturing strategy is available to Firm 2, while the second part specifies that Firm 2 must prefer its best market–sharing capacity choice to its best market–capturing one if the latter is available. Observe that $P^S(k_1) > 0$ is necessary in the latter case.

If k_1^W satisfies (A15), then clearly it is chosen by Firm 1. If k_1^W violates (A15), then (A15) is also violated for all $k_1 > k_1^W$: $\underline{k}_2(k_1)$ is nonincreasing in k_1 for $k_1 \geq k_1^W$, using Propositions 1 and 4 along with the assumption $k_1^W \geq q^M(0)$, and so $\underline{k}_2(k_1) < \bar{k}$ is assured for $k_1 > k_1^W$; $P^S(k_1)$ is clearly nonincreasing in k_1 , while $P^M(k_1)$ is nondecreasing in k_1 since lower $\underline{k}_2(k_1)$ raises the profitability of the market–capturing strategy. It follows that Firm 1 will not choose any $k_1 \geq k_1^W$ in a leader–optimal equilibrium. Finally, for (α, F) sufficiently close to $(1, \bar{F})$, we have $\underline{k}_2(k_1^W) = 0$, using Proposition 2(b); clearly $k_1^E = 0$ in this case.

(b) Consider a leader–optimal equilibrium with $0 < k_1^E < k_1^W$; as shown above, (A15)

must be violated for all $k_1 \geq k_1^W$. Further, for k_1 that satisfies (A15), it can be shown that Firm 2's equilibrium capacity responses imply the following: (i) if k_1 exceeds Firm 1's Cournot equilibrium quantity (given by the intersection of the firms' $k_i = 0$ reaction correspondences for $\alpha = F = 0$), then $k_1 = q_1^S(k_1, k_2)$ and $(q_1^S(k_1, k_2), q_2^S(k_1, k_2))$ lies on Firm 2's $k_2 = 0$ reaction correspondence; (ii) if k_1 is less than Firm 1's Cournot equilibrium quantity, but exceeds a certain level $k_1' > 0$, then $k_1 = q_1^S(k_1, k_2)$ and $(q_1^S(k_1, k_2), q_2^S(k_1, k_2))$ lies on Firm 1's $k_1 = 0$ reaction correspondence; (iii) if $k_1 < k_1'$, then $k_1 < q_1^S(k_1, k_2)$ and $(q_1^S(k_1, k_2), q_2^S(k_1, k_2))$ is independent of k_1 . Using the assumption that $P^W(k_1)$ is strictly quasi-concave, it follows that for k_1 that satisfy (A15), Firm 1's payoffs in the continuation game are strictly increasing in k_1 if $k_1 \geq k_1'$, and independent of k_1 if $k_1 < k_1'$. It follows that k_1^E must be the largest value of k_1 that satisfies (A15).

The latter fact implies the following. If $\underline{k}_2(k_1^E) = \bar{k}$, then $q^X(k_1^E) = \min\{q^M(x), \bar{k}\}$ is necessary (else some higher k_1 would satisfy (A15)); thus a rise in α , leaving k_1^E constant, gives $\underline{k}_2(k_1^E) < \bar{k}$. If $P^S(k_1^E) \geq P^M(k_1^E)$, then $P^S(k_1^E) = P^M(k_1^E)$ is necessary (else some higher k_1 would satisfy (A15)); a rise in α , which raises $P^M(k_1^E)$ and leaves $P^S(k_1^E)$ unchanged, gives $P^S(k_1^E) < P^M(k_1^E)$. In either case, k_1^E violates (A15) following the rise in α , so that Firm 1 must choose strictly lower capacity in the new leader-optimal equilibrium.

Q.E.D.

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Footnotes

¹ Throughout this paper, our criterion of forward induction requires only the elimination of weakly dominated strategies. More sophisticated notions of forward induction, such as those discussed by Kohlberg and Mertens (1986), are not required for our analysis.

² This stands in contrast to the usual first-mover tactic of strengthening commitment to the market using strategies that expand postentry output, being more reminiscent of the "judo tactics" used by the entrant in Gelman and Salop's (1983) model.

³ Other sources of second-mover advantages are discussed in Gal-Or (1985,1987), Mailath (1993) and Ramey (1988). The traditional first-mover advantage may also be lost if the first-mover's choice is not perfectly observed, as Bagwell (1995) demonstrates.

⁴ Firm 1's strategy need not indicate what quantities Firm 1 would choose for $k_1 \neq \hat{k}_1$, because given that Firm 1's strategy chooses $\hat{k}_1$, subgames with $k_1 \neq \hat{k}_1$ are never reached no matter what strategy Firm 2 plays. Thus, Firm 1's quantity choices for $k_1 \neq \hat{k}_1$ are strategically irrelevant. Similarly, Firm 2's quantity choices for $k_2 \neq \hat{k}_2(k_1)$ are strategically irrelevant. Eliminating these irrelevant strategies gives the *reduced normal form* representation of the three-stage game.

⁵ If Firm i prefers zero output for *every* q_j , then we set $q^X(k_i) = 0$.

⁶ This application of subgame perfection to the undominated game represents an appropriate combination of forward- and backward-induction inference: in each subgame, players know that rivals are rational in the subgame (backward-induction inference), and also that rivals were rational in all decisions that preceded the subgame (forward-induction inference). Both criteria are satisfied if and only if strategies induce Nash equilibria on each subgame and are restricted to the undominated game.

⁷ The Appendix demonstrates that Nash equilibria may fail to exist in certain subgames of the undominated game because Firm 2's possible quantity choices do not form a closed set: in the undominated game, the infimum of Firm 2's possible choices, $q^L(k_2)$, is not itself a possible choice. Thus $\bar{S}_2^U$ expands Firm 2's available strategies to include those that specify $q_2 = q^L(k_2)$ in conjunction with k_2 . Defenses for this departure from strict forward-induction reasoning are offered in the Appendix.

⁸ The latter form of the market-capturing strategy occurs when, as q_2 rises, Firm 1's reaction correspondence in the original subgame jumps to $q_1 = 0$ before it intersects Firm 2's reaction correspondence.

⁹ For $\underline{k}_2(k_1) = k_2 = 0$, no MSE exists and there may or may not exist an F1ME; a F2ME necessarily exists, however.

¹⁰ It is easy to show that (c) is nontrivial, in the sense that either $\underline{\alpha}(F) \in (0,1)$ or $\bar{\alpha}(F) \in (0,1)$ imply $\underline{\alpha}(F) < \bar{\alpha}(F)$. We also have $\underline{k}_2(k_1) = \bar{k}$ for $\alpha = \underline{\alpha}(F)$ and $\underline{k}_2(k_1) = 0$ for $\alpha = \bar{\alpha}(F)$. A number of cases are possible if $\underline{\alpha}(F) = 0$ or 1, and/or $\bar{\alpha}(F) = 0$ or 1, and these may be analyzed in a straightforward way. For $k_1 = 0$, $\underline{k}_2(k_1)$ becomes independent of α ; it can be shown that $\underline{k}_2(k_1) = 0$ for F sufficiently close to $\bar{F}$, while there are a number of possibilities that may arise for smaller F .

¹¹ Note that while $\underline{k}_2(k_1)$ is decreasing in α and F in the intermediate region, it may happen that a rise in F leads to elimination of the market-capturing strategy, as higher F might reduce $\bar{k}$ at a faster rate than $q^X(k_1)$, i.e. the lower region is entered as F rises. This reflects the fact that the second mover must incur the fixed costs as well as the first mover, so an increase in F narrows the range of k_2 such that the second mover prefers capturing the market to staying out altogether.

¹² Recall that F1ME fail to exist when $k_2 > 0$.

¹³ Results can be obtained for the limiting cases of very low and high avoidable fixed costs, without imposing the assumption $k_1^W \geq q^M(0)$: using Proposition 2, it is straightforward to verify that Firm 1's leader-optimal-equilibrium capacity choice is k_1^W if (α, F) is sufficiently close to $(0,0)$, while Firm 1 chooses zero capacity, and thereby cedes the market to Firm 2, if (α, F) is sufficiently close to $(1, \bar{F})$.

¹⁴ As pointed out above, higher F allows market capture at a lower level of k_2 , but also reduces the attractiveness of the market to Firm 2; either effect can dominate, so that in general a rise in F exerts an ambiguous effect on k_1^E . It can be shown, however, that if $C'' = 0$, then for α sufficiently close to unity, $\underline{k}_2(k_1) < \bar{k}$ holds for all k_1 ; in this case, a rise in F leads to lower k_1^E under the assumptions of Proposition 5.

¹⁵ Consider the following game. In each of the periods $t = 1, 2, \dots$, two firms decide whether to enter or stay out. If both stay out, the game starts again in the next period. If only one firm enters, it observes that the other firm has stayed out and chooses its capacity, while the other firm observes this capacity choice and then chooses its own capacity; this corresponds to the game considered in the present paper. If both enter, then they play some symmetric equilibrium (perhaps in mixed strategies) in which capacities and then quantities are chosen simultaneously. From results in Ramey (1988) we have the following: in symmetric mixed-strategy equilibrium of the timing game, entry is delayed if the first mover's profits are positive but lower than the second mover's, and entry does not occur at all if the first mover is forced to cede the market.

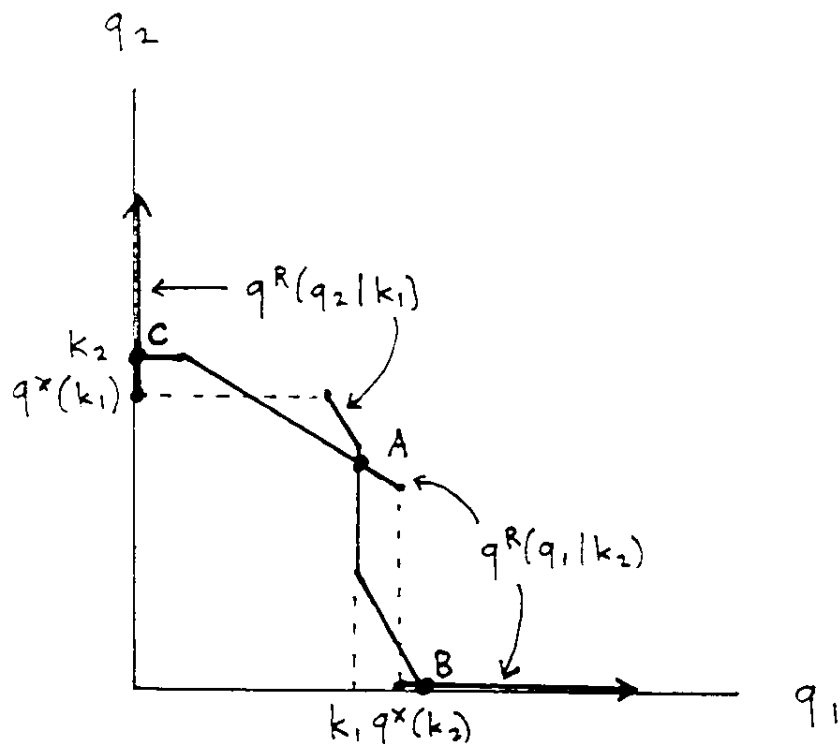


Figure 1
Multiplicity of Subgame Equilibria

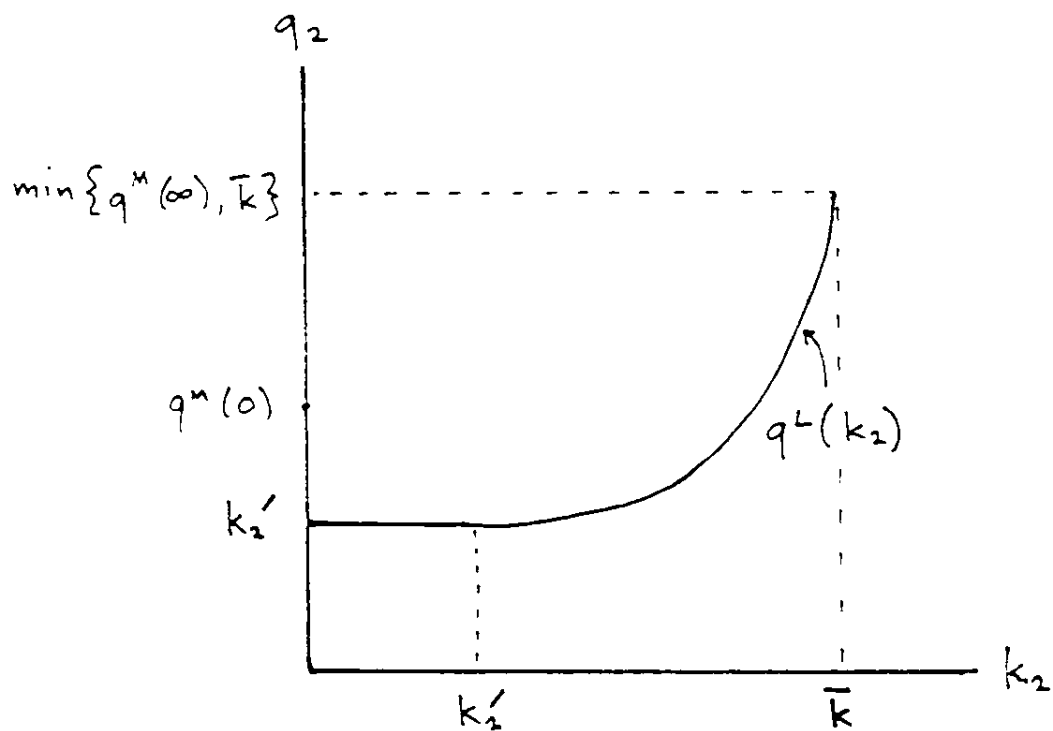


Figure 2
Lower Bound of Firm 2's Undominated Outputs

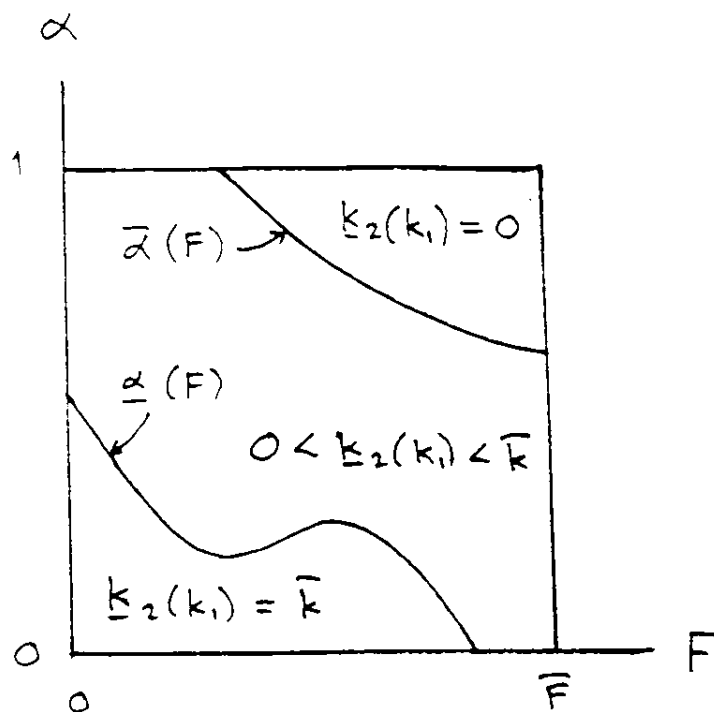


Figure 3

Avoidable Fixed Costs and the
Market-Capturing Strategy