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Relationship Admitting Families of Candidates

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Discussion Paper No. 823

RELATIONSHIP ADMITTING FAMILIES OF CANDIDATES

by

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Abstract

A central theme in social choice is to determine when must there be a relationship among a group's sincere election rankings of several different subsets of candidates. This issue is completely resolved here for the positional voting methods. Namely, necessary and sufficient conditions are derived for a family of subsets of candidates to determine when there is a choice of positional voting methods so that there are relationships among the election rankings. The same issue is resolved for a related family of social choice mappings. Then, in part, these necessary and sufficient conditions are used i) to analyze sequential voting procedures, ii) to show how to create new classes of axiomatic representations for social choice mappings that uniquely characterize the Borda Count, and iii) to determine the limits of indeterminacy for positional voting election outcomes.

Theorem 1. For $N \geq 3$, a family F, and a system voting vector $\mathbf{w}_F$, $D(\mathbf{w}_F)$ is a subset [illegible] of $(\mathbf{w}_F)$. [illegible] for $\mathbf{w} \in D(\mathbf{w}_F)$, there is a profile so that for any choice of $\mathbf{w}_F$, the listing of election outcomes over the sets in F is $\mathbf{w}$.

Definition [illegible]

Proposal 1. [illegible]

Theorem 1. (a) If no subfamily of F satisfies the RIP, then F is not relative admissible. Namely, for any choice of w^n_F, $\Pi(w^n_F) = U^n_F$.

(b) Example (1). Assume that $n \geq 5$. Let F be a family of nested sets $\{S_1, S_2, \ldots, S_n\}$ where S_i is a proper subset of S_{i+1} for $i = 1, \ldots, n-1$. Then family F is not relative admissible.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Theorem 4. The family $\mathcal{F} = \{S_1, \ldots, S_n\}$ is 2-intersection supersaturated if [illegible] $\dim(\mathcal{F}) > n(n-1)/2$.

In particular, if inequality [illegible] is satisfied, then $\dim(\mathcal{F})$ is a strict upper bound on $f^{\mathcal{F}}_2$, the number of independent relationships among the of [illegible] [illegible] $n(n-1)/2$.

[illegible]

[illegible]

Example 5. [illegible]

[illegible]

[illegible]

$$[illegible] = [illegible]$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

... ranking is the (1,1,1/2,...) of outcomes ...

... among these rankings. ■

[illegible]

Theorem 7. Let F be any family of subsets. For almost all choices of w^F, F is not relation admitting. In particular, if the plurality vote is used to rank each subset of candidates, then there does not exist a relation admitting family of subsets.

[illegible] be a collection of positional voting methods used to tally the

[illegible] to all subsets of coalitions. [illegible] includes the Borda count. Thus the [illegible] admissible families for [illegible] is characterized by Theorem 2. [illegible] does not include the Borda count. Thus the position admitting families for [illegible] form a proper subset of the strict pairwise admissible families for the [illegible].

[illegible]

[illegible]

Theorem 3. For a given family F, consider the set of all mappings φ_F that consists of monotonic, neutral social choice functions satisfying IIA that are

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] choice function satisfies [illegible] and consistent over each of the sets in F. [illegible] is necessary for the four selection [illegible] $S_3(p)=m_1$, and $S_4(p)=m_1$. [illegible]

[illegible]

[illegible]

[illegible]

[illegible]