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## The Potential Disadvantage of Becoming Too Large In Duopolistic Industry

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Discussion Paper No. 794

THE POTENTIAL DISADVANTAGE OF BECOMING  
TOO LARGE IN DUOPOLISTIC INDUSTRY

by

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The Potential Disadvantage of Becoming Too Large  
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A b s t r a c t

The paper considers a duopolistic market in which consumers are not necessarily aware of the firms' existence. Thus there are four segments in the market; each firm has a captive market segment, there is also a duopolistic segment and there is a set of consumers who are unaware of either firm. By advertising, firms control the proportion of consumers who are aware of their existence. Thus, if a firm advertises, it causes an increase in its own captive segment, an increase in the duopolistic segment and a decrease in the captive segment of its competitor. Clearly the relative sizes of these segments affect the equilibrium of the duopolistic pricing game. We elaborate on this pricing game and on its relation to market segmentation. We show that being large might sometimes be disadvantageous (the fat-cat effect). Moreover, we show that even if gaining awareness is costless firms might wish to be "small", i.e. to have less than one hundred percentage awareness level. The intuitive explanation of this result is that if the firm becomes too big it steps on its rivals' toes and makes its rival more "aggressive".



## The potential disadvantage of becoming too large in duopolistic industry

### 1. Introduction

The standard assumption in the economic analysis of markets is that although consumers might have some incomplete information regarding prices, products' quality or other attributes, they are always aware of the existence of the different firms operating in the market. Such an assumption simplifies the analysis, but it seems to us that assuming that consumers are always aware of all the different firms and brands in the market is too restrictive.

Analyzing consumers' awareness gives rise to many conceptual difficulties. For example, how does someone become aware of a particular firm? Is it obvious that once a consumer is aware of the existence of a firm he will always consider this firm in making his brand choice or is there a possibility that he will forget about the firm, given enough time? In particular what is the specific relationship between the degree of awareness and the demand facing the firm?

In a recent paper, Fudenberg and Tirole [1984]<sup>1</sup> discuss the entry deterrence problem under the assumption that consumers become aware of a firm by being exposed to the firm's advertisement. In such a model the concept of goodwill that firms have receives an immediate interpretation as to the number

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<sup>1</sup>See also their survey of dynamic oligopoly (Fudenberg and Tirole (1987)) and the survey on oligopoly behavior by Shapiro (1988).

of potential customers that are aware of the firm. This goodwill can be interpreted as an asset of the firm. If we consider a monopolistic firm and assume that gaining awareness (goodwill) is costless it is obvious that the firm benefits from any increase in its goodwill and thus it wishes to have the highest possible goodwill, namely that all potential clients will be aware of its existence.

As demonstrated by Fudenberg and Tirole, this intuitive result cannot be directly extended to duopolistic markets. (See also Bulow, Geanakoplos and Klemperer (1985) for similar arguments). In discussing the choice of awareness of an incumbent firm they show that under some circumstances a high level of capital (or awareness) can be disadvantageous for the incumbent firm since it reduces its incentives to respond aggressively to its competitor (the fat-cat effect). This result contradicts previous results in which it was argued that the incumbent firm overinvest in order to deter entry. If gaining awareness is costless, however, the optimal choice of the incumbent is still one hundred percentage awareness since by choosing this level entry becomes impossible.<sup>2</sup>

In this paper we consider a duopolistic market in which consumers are not necessarily aware of the firms' existence. Thus we can think of four different market segments. Each firm has a captive market segment, there is a duopolistic segment that consists of consumers who are aware of both firms,

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<sup>2</sup>Fudenberg and Tirole (1984) assumed that after the incumbent chooses its awareness level, at the second stage of the game all active firms will choose to cover the remaining market.

and there is also a set of consumers who are not aware of either firm. At the first stage of the game firms determine, by advertising, the level of awareness to their product. If a firm advertises it causes a change in the size of all the four segments. The size of its own captive market increases, the dupolistic segment increases but the captive market of its competitor as well as the segment of consumers who are not aware of either firms decrease. Thus, in determining its advertising policy firms should take into account the effect of all these changes on the second-stage pricing game and on the final profits. In such a framework for example, if a firm increases its investment in awareness it makes its rival more aggressive in the pricing game since it reduces the size of its rival's captive market segment and increases the dupolistic segment. Thus, we show that even if gaining awareness is costless firms might wish to remain "small". By being too big the firm steps on its rival's toes and makes him more aggressive. Since both firms eventually might lose from aggressive behavior it is in the interest of each firm to accommodate its rival so that it will remain a "fat" and "satisfied cat".

The paper is organized as follows: in section 2 we present the model and elaborate on the relationship between awareness levels and the segmentation of the market. In section 3 we discuss the pricing game. In section 4 we show that even when awareness is costless firms might choose not to be at its maximum level. We discuss a simple example in section 5 and in section 6 we discuss the homogenous good case. Clearly, in the homogenous good case the existence of captive segment implies that the Bertrand equilibrium is not an equilibrium in our model. We thus discuss the mixed price equilibrium and

show that in such an equilibrium the firms' profits are equal to the maximum that they can gain from their captive segment without any contribution from the duopolistic segment.

## 2. Awareness and duopolistic market

Consider a duopolistic industry in which firms sell a differentiated product to  $N$  identical consumers. Let us assume that these consumers are not necessarily aware of the existence of the two firms. Thus, although we let consumers be identical in terms of their preferences and endowments, they can differ in terms of the information they have. They are different in that they are aware of different sets of firms. As in Fudenberg and Tirole, we assume that consumers become aware of the firm as they are exposed to the firm's advertising.

The firms in our model are competing along two dimensions: Awareness and prices. We denote the number of consumers that are aware of the first firm as  $B_1$  and similarly  $B_2$  is the number of consumers that are aware of the second firm. Clearly  $B_i \leq N$ . Consumers can now be categorized into four groups:

- (i)  $Z_1$  - number of consumers who are aware only of firm 1, (captive market of firm 1)
- (ii)  $Z_2$  - number of consumers who are aware only of firm 2, (captive market of firm 2)
- (iii)  $Z_3$  - number of consumers who are aware of both firms, (duopolistic segment)
- (iv)  $Z_4$  - number of consumers who are not aware of either firm.



Clearly  $Z_1, \dots, Z_4$  are mutually exclusive and exhaustive. A consumer can belong only to one of these groups. Before proceeding, we would like to establish the relationship between  $B_i$  and  $Z_i$ . We assume that potential customers are exposed to advertising at random so that the probability of a customer being aware of firm 2 is independent of his awareness of firm 1. Clearly, this is a restrictive assumption. We might think of a negative correlation as follows: a consumer that is aware of one firm will not bother to read advertising in newspapers and magazines that specialize in the specific product. On the other hand, we might justify an assumption of positive correlation.

The probability that a randomly selected consumer is aware of firm 1 is  $B_1/N$ . Given the independence assumption, the probability that a random consumer is exposed to the two firms is  $B_1 B_2 / N^2$ . Similarly the probability that a randomly selected consumer is not aware of any firm is  $(1 - \frac{B_1}{N})(1 - \frac{B_2}{N})$ . Thus, given that there is a large number of consumers, it is evident that

$$Z_1 = \frac{B_1 (N - B_2)}{N} \quad ; \quad Z_2 = \frac{(N - B_1) B_2}{N} \quad ; \quad Z_3 = \frac{B_1 B_2}{N} \quad ; \quad Z_4 = \frac{(N - B_1)(N - B_2)}{N}$$

Notice that  $Z_1$  is not directly under the control of one particular firm. Each firm can affect the level of awareness to its own product, i.e.,  $B_i$ , the different segments are determined jointly by the level of awareness to both firms.

When firm 1 advertises and increases the level of awareness to its product, i.e.,  $B_1$  rises, affecting the distribution of consumers among the four segments. An increase in  $B_1$  causes an increase in  $Z_1$  and  $Z_3$  and a decrease in  $Z_2$  and  $Z_4$ . We can describe this process as follows. Consumers who were in  $Z_4$ , by being exposed to the new advertising, become aware of firm 1 and thus become part of  $Z_1$ . Consumers that were captive consumers of firm 2 now become aware of firm 1 and become part of  $Z_3$  which is the duopolistic segment. Intuitively these shifts look favorable for firm 1 since its exposure increases and in particular the size of its captive market increases. Thus, it seems that such a change is favorable to firm 1. In particular, if we consider the extreme case in which advertising is costless, intuition may suggest that firms wish to increase their awareness as much as possible.

Since the consumers in the first group are captive customers of firm 1, this firm has a complete monopolistic power with respect to these customers. Let  $p_1$  be the price charged by the first firm. We assume that each customer in the first segment will buy  $a(p_1)$  units of the product where  $a(p_1)$  is a standard downward sloping demand function. The total demand of this segment is  $Z_1 a(p_1) = \frac{B_1(N-B_2)}{N} a(p_1)$ . Similarly for firm two, given its price  $p_2$ , its captive customers buy  $Z_2 a(p_2) = \frac{B_2(N-B_1)}{N} a(p_2)$ . The consumers in the third segment are aware of both firms. Given the firms' prices, we assume that the demand facing each firm is  $\frac{B_1 B_2}{N} b_i(p_1, p_2)$ .<sup>3</sup> Thus, given  $(B_1, B_2)$

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<sup>3</sup> Clearly there is a relation between  $a(p_i)$  and  $b_i(p_1, p_2)$ . For example when  $p_1 = p_2$ , then  $b_1(p_1, p_2) + b_2(p_1, p_2) = a(p_1)$ . This relationship will be

and the firms' prices, the demand facing each firm is

$$(1) \quad D_i(B_1, B_2, p_1, p_2) = \frac{B_i(N-B_j)}{N} a(p_i) + \frac{B_i B_j}{N} b_i(p_1, p_2).$$

We model a two-stage game in which, in the first stage the firms set the level of awareness, and in the second - prices. We will start the analysis as is customary in the second stage assuming the respective levels have already been set. In this way we gain subgame perfection.

### 3. The pricing game

Given awareness levels of  $(B_1, B_2)$  and assuming for convenience zero production cost, the firm's profit function is:

$$(2) \quad \Pi_i(B_1, B_2, p_1, p_2) = p_i \frac{B_i(N-B_j)}{N} a(p_i) + p_i \frac{B_i B_j}{N} b_i(p_i, p_j).$$

The firm can thus be thought of as having two types of markets. In one - its captive market - it has a complete monopoly power while in the other segment the industry is duopolistic. If it is possible to identify the consumers of each group and price discriminate, the firm would charge its monopolistic price  $p_1^m = \text{Argmax } p_1 a(p_1)$  in its captive customers and  $p_1^d(p_j) = \text{Arg max}_{p_1} p_1 b_i(p_1, p_j)$  in its duopolistic segment.<sup>4</sup> However, we assume that

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evident in our example in section 6.

<sup>4</sup>Notice that according to the assumption of our model  $p_1^m$  as well as the

such a strategy is not feasible either because the firm cannot identify the consumers in each segment or because it cannot eliminate a resale market or because of legal constraints..

We assume concavity of the revenue functions  $p_i a(p_i)$  and  $p_i b_i(p_i, p_j)$ . This guarantees concavity of the profit function (2). The first-order conditions are therefore:

$$(3) \quad (N-B_j)[a(p_i) + p_i a'_i(p_i)] + B_j[b_i(p_i, p_j) + p_i \frac{\partial b_i(p_i, p_j)}{\partial p_i}] = 0.$$

Equation (3) can be regarded as the  $i$ 'th firm reaction function and it is a linear combination of the monopolist reaction function (i.e., its f.o.c.) and the reaction function of a regular duopolist. Observe that firm  $j$  by determining  $B_j$  affects the reaction function of firm  $i$  in the duopolistic pricing game.<sup>5</sup> Thus, using (3), let  $\phi_1(p_2, B_2) = \text{Arg max}_{p_1} \Pi_1(B_1, B_2, p_1, p_2)$  be the reaction function of firm 1 as a function of the awareness of firm 2. When  $B_2 = 0$ , the dupolistic segment disappears and the firm faces only its captive customers. The size of this captive segment depends on  $B_1$  and the optimal price is the monopolistic price, i.e.,  $\phi_1(p_2, 0) = p_1^M$ .

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optimal response function  $p_i^d(p_j)$  do not depend on the size of each segment.

<sup>5</sup>Note that unlike some other two-stage games (see, for example, Fershtman and Judd (1987)) the firm's strategy at the first stage of the game affects its rival second stage reaction function and not its own reaction function.

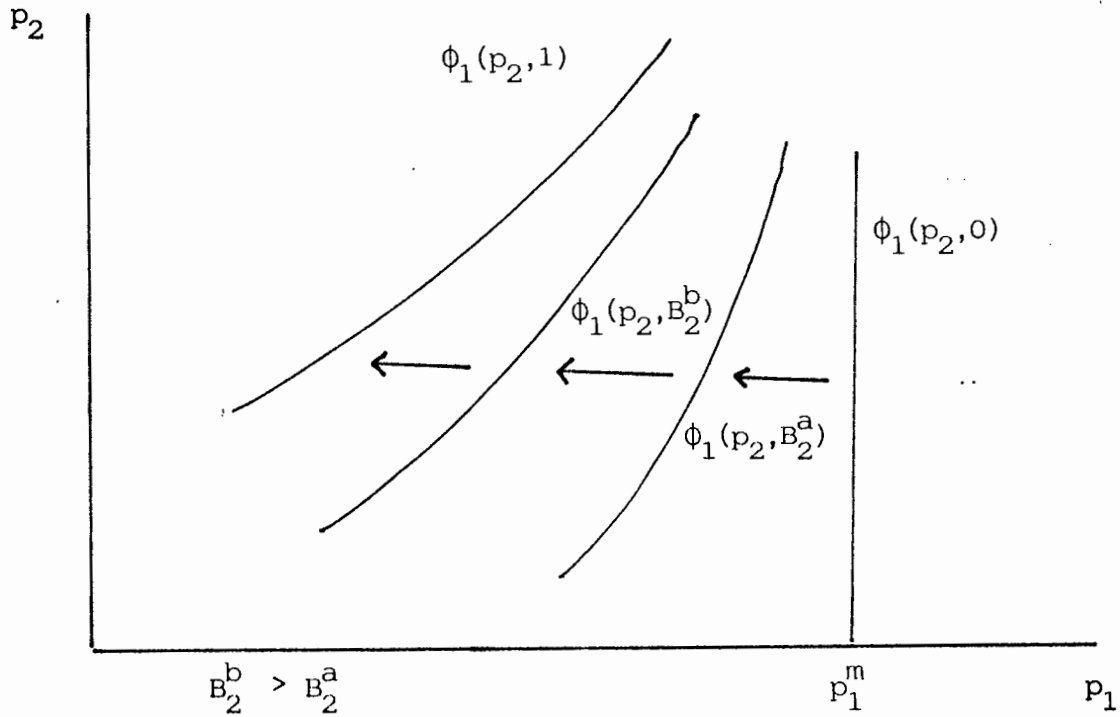


Figure 1

For  $B_2 = 1$ , the captive segment disappears. The firm is facing a duopolistic market and its reaction function,  $\phi_1(p_2, 1)$  is identical to that of a regular duopolistic firm. As  $B_2$  increases we can see from equation (3) that the emphasis of firm 1 shifts from its captive market segment to its duopolistic segment, and thus an increase in  $B_2$  pushes the reaction function of firm 1 leftward towards the duopolistic reaction function  $\phi_1(p_2, 1)$ .

The possibility of each firm to control the reaction function of its competitor introduces a new and interesting dimension to the strategic

interaction among the firms. In determining the degree of awareness of consumers to its product, the firm not only affects the different segments it faces but also affects the strategic behavior of its competitor in the second stage of the game.

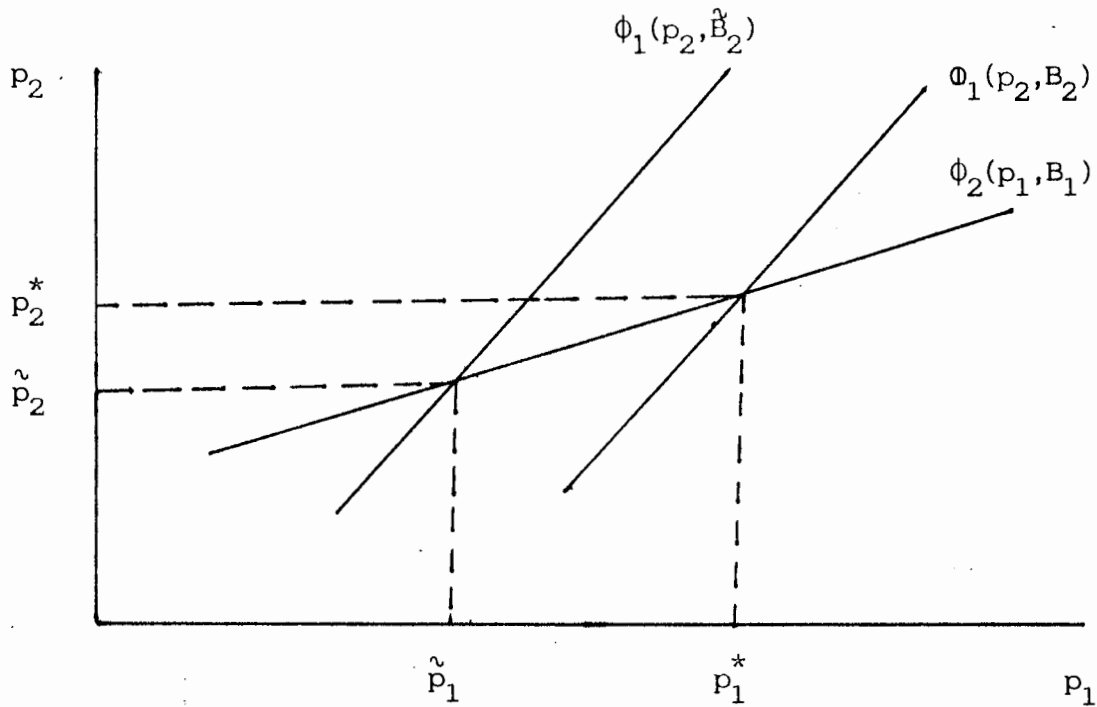


Figure 2.

Let  $\phi_2(p_1, B_1)$  be the reaction function of the second firm. In Figure 2 we depict the two reaction functions. Clearly without further assumptions on the profit function there is a possibility of multiple equilibria. Let  $\Pi_i^{ij} = \frac{\partial^2 \Pi_i}{\partial p_i \partial p_j}$ . Assuming that  $\Pi_1^{11} \Pi_2^{22} > \Pi_1^{12} \Pi_2^{21}$  guarantees that there is a unique intersection point of the two reaction functions. This intersection point

defines the equilibrium prices. For a given  $B_1, B_2$ , we let  $p_1^* = h^1(B_1, B_2)$ ,  $p_2^* = h^2(B_1, B_2)$  be the unique equilibrium prices when the awareness levels are  $(B_1, B_2)$ . When  $(B_1, B_2) = (1, 1)$  there is a standard duopolistic competition in which no firm has a captive segment and the equilibrium is identical to the standard Nash equilibrium of a price competition in non-homogenous good duopoly.

**Proposition 1:** The equilibrium prices  $(p_1^*, p_2^*)$  decrease as the levels of awareness  $B_1$  or  $B_2$  increase.

**Proof:** Assume that the second firm increases the awareness of its product. Let us denote the new awareness level as  $\tilde{B}_2 > B_2$ . An increase in  $B_2$  implies that the first firm emphasises more the duopolistic segment and thus lowers its price which implies (see figure 2) that its reaction function shifts leftwards. The new equilibrium prices are thus  $(\tilde{p}_1, \tilde{p}_2)$  and it is evident that  $\tilde{p}_1 < p_1^*$ , i.e., the equilibrium prices are lower.

**Proposition 2:**  $B_2 > B_1$  implies that  $p_2^* > p_1^*$ , i.e., the large firm charges a higher price at equilibrium.

**Proof:** The proof is straightforward using the above reaction function analysis.

#### 4. Investing in awareness

Since the equilibrium prices are determined by the levels of awareness we are now able to express the firms' profit as a function of  $B_1$  and  $B_2$ .

Let  $D_i^*(B_1, B_2)$  be the quantity sold by firm  $i$  as a function of the firm's awareness level, i.e.  $D_i^*(B_1, B_2) = D_i(B_1, B_2, h_1(B_1, B_2), h_2(B_1, B_2))$ .

$$R_i(B_1, B_2) = \Pi_i(B_1, B_2, h^1(B_1, B_2), h^2(B_1, B_2)) = h^i(B_1, B_2) D_i^*(B_1, B_2) -$$

(4)

$$h^i(B_1, B_2) \left[ \frac{B_i(1-B_j)}{N} a(h^i(B_1, B_2)) + \frac{B_i B_j}{N} b_i(h^1(B_1, B_2), h^2(B_1, B_2)) \right].$$

The first stage of the game is now well defined. The players choose the awareness levels  $(B_1, B_2)$  and their payoffs are specified by equation (4). Clearly the existence of an equilibrium in such a game is not guaranteed. Although the concavity of  $R_i(B_1, B_2)$  with respect to  $B_i$  is sufficient for existence, it is not guaranteed by our previous assumptions regarding the demand and the cost functions.

Using equation (4) we are now able to discuss the optimal awareness level of the firm. Notice that in defining the profit function  $R_i(B_1, B_2)$  we assume that gaining awareness is costless. This assumption is not made for simplicity sake, but to emphasize the fact that firms choose partial awareness levels not because creating awareness is costly, but because of strategic reasons.



**Theorem 1:** (i) Full awareness (i.e.  $B_1 = N$ ) is not necessarily the optimal strategy of a firm in a duopolistic market even if creating awareness is costless, i.e. given an awareness level of firm 2,  $B_2$ , the optimal response of firm 1 in the awareness game is not necessarily  $B_1 = N$ . (ii) Depending on the specific structure of the demand functions it is possible to find an equilibrium  $(B_1^*, B_2^*)$  which differs from  $(N, N)$ .

Clearly in the above theorem (i) and (ii) are related. We choose to present the theorem in this manner so as to clarify the driving force behind our result. We prove this theorem by demonstrating such a situation in the example provided in section 5, where we show that whether or not  $B_1 = N$  is optimal highly depends on the cross price elasticity of demand.

Intuitively under the assumption of costless awareness one would expect that  $\frac{\partial R_1(B_1, \bar{B}_2)}{\partial B_1}$  would be positive. The level of awareness is regarded as some form of an asset owned by the firm and an increase in such an asset should induce higher profits. Analyzing, however, equation (4) yields that intuition might be misleading. An increase in  $B_1$  benefits the first firm by providing it with a larger potential market. The captive consumers segment is larger and so is the duopolistic segment. But the increase in  $B_1$  causes different changes in the market segmentation faced by the second firm. Its captive segment decreases while its duopolistic segment increases. Using the second firm reaction function of the pricing stage of the game (i.e. equation 3), an increase of  $B_1$  implies that the second firm will put more emphasis on the

duopolistic segment. Its reaction function  $\phi_2$  of Figure 2 will move downwards and in order to gain a larger share from the duopolistic segment the second firm is charging a lower price. Firm 1 responds by lowering its own price and as it is demonstrated in figure 2 the intersection of the two reaction functions is at lower prices. The reduction in prices may offset the increase in the firm captive and duopolistic segments.

### 5. An Example

In this section we present a detailed example that demonstrates our main theorem.

Let  $a(p_i)$  and  $b_i(p_1, p_2)$  be defined as follows:

$$(5) \quad a(p_i) = 2 - kp_i$$

$$(6) \quad b_i(p_1, p_2) = (1/2)(1 - kp_i + (p_j/p_i)^\gamma)$$

Clearly when  $p_1 = p_2 = p$  we have:

$$(7) \quad b_1(p_1, p_2) + b_2(p_1, p_2) = a(p).$$

The function  $a(p_i)$  is a standard linear demand function. In defining  $b_i(p_1, p_2)$  we deviate from the linear case and let the cross effect term of the demand be  $(p_j/p_i)^\gamma$ . The parameter  $\gamma$  plays a crucial role in our result as will be shortly demonstrated. It is possible to use the same procedure,

**mutatis mutandis**, to show that in the linear case, i.e. when the cross term effect is  $\gamma(p_j - p_i)$ , the equilibrium is  $B_1 = B_2 = N$ . If the firms could price discriminate, they would choose a price of  $p^m = 1/k$  for the monopoly part of the market and a price of  $p^c = (2-\gamma)/2k$  for the common, duopoly part of the market. To ensure nonnegativity of the duopoly price we restrict the parameter  $\gamma$  as follows:  $0 \leq \gamma \leq 2$ . Equation (3), i.e. the first-order condition for the pricing stage of the game is now the following

$$(8) \quad \frac{\partial \Pi_i}{\partial p_i} = 2(N-B_j)(1-kp_i) + (B_j/2)(1-2kp_i + (1-\gamma)(p_j/p_i)^\gamma) = 0$$

Let  $\eta$  be the price elasticity of demand for the duopolistic segment of the market, i.e.,  $\eta = (\partial b_i / \partial p_i)(p_i / b_i)$ .

Define  $\gamma^*$  and  $\gamma^{**}$  as follows. Let  $\gamma^*$  be such that  $\gamma > \gamma^*$  if and only if  $\eta > 1$  and let  $\gamma^{**}$  be such that for all  $\gamma < \gamma^{**}$  the following inequality holds:  $\Pi_1^{11} \Pi_2^{22} > \Pi_1^{12} \Pi_2^{12}$  where  $\Pi_i^{ij} = \partial^2 \Pi_i / \partial p_i \partial p_j$  and  $\Pi_i$  is defined in equation 2. Note that the condition  $\Pi_1^{11} \Pi_2^{22} > \Pi_1^{12} \Pi_2^{12}$  is the standard condition that guarantees the uniqueness of the equilibrium in the pricing game.

**Claim i:** The two stage game with equations (6) and (7) defining the demand functions is such that for all  $\gamma^* < \gamma < \gamma^{**}$   $(B_1, B_2) = (N, N)$  is not an equilibrium of the game.

**Proof:** We show in Appendix A that if  $\gamma^* < \gamma < \gamma^{**}$  when  $B_1 = B_2 = N$ ,  $\partial R_i / \partial B_i < 0$ . Thus, under our assumption  $B_1 = B_2 = N$  is not an equilibrium.

The intuition of this claim is as follows:  $\gamma$  measures the elasticity of the cross-term effect in the duopolistic segment. If this elasticity is large enough, so that the elasticity of the demand of the duopolistic segment is large (larger than unitary elasticity), then any increase in firm's 1 awareness ( $B_1$ ) will cause the duopolistic market to expand. This will induce the rival (firm 2) to lower its own price since its price is a convex combination of its monopolist and oligopolist prices. Since the weight of the duopolistic market is now larger, firm 2 lowers its price. Since the cross elasticity is large this reduction in the rival's price causes such reduction in profit in the duopolistic segment of firm 1 that is not covered by the increase in profits in its captive segment.

Thus, indeed firm 1, by increasing its awareness level, put too much pressure on its rival. Its reaction, coupled with the fact that the cross elasticity is large, causes this increase in awareness to be unprofitable.

**Claim ii:** The two-stage game with equations (6) and (7) defining the demand function is such that for  $\gamma = 1$  there exists a unique equilibrium with  $B_1 < N$ .

**Proof:** See Appendix B.

## 6. Homogenous Good Duopoly

Assuming the same market segmentation as in our previous analysis let us consider the case of homogenous good. Each firm has a complete monopolistic

power in its captive market and thus the demand of this segment continues to be  $z_1 a(p_1)$  for the first firm and  $z_2 a(p_2)$  for the second firm. The assumption of homogenous good affects the duopolistic segment, i.e.  $z_3$ . The firm that has the lower price sells to the whole market, i.e.

$$(10) \quad D_1(B_1, B_2, p_1, p_2) = \begin{cases} (z_1 + z_3)a(p_1) & p_1 < p_2 \\ z_1 a(p_1) & p_1 > p_2 \\ (z_1 + \frac{z_3}{2})a(p_1) & p_1 = p_2 \end{cases}$$

We assume that if both firms charge the same price they split the duopolistic segment of the market.<sup>6</sup> The firms payoff function is  $\Pi_i(B_1, B_2, p_1, p_2) = p_i D_i(B_1, B_2, p_1, p_2)$ .

The above homogenous good duopoly game does not have a Nash equilibrium with pure strategies. The Bertrand equilibrium price of the duopolistic segment  $p_1 = p_2 = 0$  is not an equilibrium since each firm can increase its payoffs by increasing its price. Although by increasing its price the firm loses its duopolistic segment, it gains positive profits from its captive consumers. If  $p^m \geq p_1 = p_2 > 0$  each firm can gain by a small reduction in its price. If  $p^m > p_1 > p_2 > 0$ , the first firm can gain by increasing its price to  $p^m$ . If its price already equals  $p^m$  the second firm can gain by increasing its price so that it will be above  $p_2$  but still below  $p_1$ .

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<sup>6</sup>As in similar works our result is not sensitive to the specific assumption regarding the sharing rule.

The difference between our model and the Bertrand price competitor is that each firm in our model also has captive segments. Thus, by lowering its price the firm may gain consumers in its duopolistic segment but it will lose in its captive market. Clearly at any equilibrium of the above pricing game the equilibrium payoff of firm  $i$  must be at least  $z_i p^m a(p^m)$ . Each firm can guarantee to itself such profits by asking the monopolistic price and selling the good to its captive customers.

Let  $\hat{p}_1$  and  $\hat{p}_2$  be defined by the following equations:

$$(11a) \quad (z_2 + z_3)\hat{p}_1 a(\hat{p}_1) - z_2 p^m a(p^m) = 0$$

$$(11b) \quad (z_1 + z_3)\hat{p}_2 a(\hat{p}_2) - z_1 p^m a(p^m) = 0.$$

Let  $\hat{p} = \max(\hat{p}_1, \hat{p}_2)$ .

**Proposition 3:** When  $z_1 \geq z_2$  the following mixed strategies constitute a Nash equilibrium in the duopolistic pricing game:

$$(12) \quad F(p) = \begin{cases} 1 & p \geq p_m \\ 0 & p < \hat{p} \\ \frac{(z_2+z_3)pa(p) - z_2 p^m a(p^m)}{z_3 p a(p)} & \text{otherwise} \end{cases}$$

$$(13) \quad G(p) = \begin{cases} 1 & p \geq p^m \\ 0 & p < \hat{p} \\ \frac{(z_1+z_3)pa(p) - z_1p^m a(p^m)}{z_3 pa(p)} & \text{otherwise} \end{cases}$$

Similar equilibrium strategies can be defined for the case  $z_2 > z_1$ .

**Proof:** First note that  $F(p)$  and  $G(p)$  are proper distribution functions. The support of  $F(p)$  and  $G(p)$  is  $[\hat{p}, p^m]$ . We will show that given the strategy of the first firm i.e.,  $F(p)$ , the second firm cannot gain by deviating from  $G(p)$ . More specifically we will show that any price in the interval  $[\hat{p}, p^m]$  yields identical profits for the second firm and the firm cannot benefit by stating a price outside this interval. Note that  $z_1 \geq z_2$  implies that  $\hat{p}_2 \geq \hat{p}_1$  and thus  $\hat{p} = \hat{p}_2$ . Given  $F(p)$  the expected profits of the second firm when it states the price  $p_2$  is:

$$(14) \quad \Pi_2(p_2, F(p)) = F(p_2) z_2 p_2 a(p_2) + (1 - F(p_2))(z_2 + z_3) p_2 a(p_2)$$

Rearranging (14) and using (12) to substitute for  $F(p_2)$  yields that for  $p_2 \in [\hat{p}, p^m]$

$$(15) \quad \Pi_2(p_2, F(p)) = z_2 p^m a(p^m).$$

Thus,  $\Pi_2(p_2, F(p))$  yields the same expected profits for every  $p_2$  in  $[\hat{p}, p^m]$ . For  $p_2 < \hat{p}$ , substituting  $F(p_2)$  into (14) yields that

$$(16) \quad \Pi_2(p_2, F(p)) = (z_2 + z_3)p_2 a(p_2) \quad p_2 < \hat{p}.$$

Since  $\hat{p}_2 = \hat{p}$  and since  $p_2 a(p_2)$  is an increasing function in the interval  $[0, \hat{p}^m]$  it is evident that the second firm cannot benefit from stating a price below  $\hat{p}$ .

Similar arguments imply that the first firm cannot benefit from deviating from its own strategy and thus the above strategies constitute a Nash equilibrium for the investigated pricing game.  $\square$

Note that  $F(p)$  has a mass at  $p_1 = \hat{p}$ . This mass is a result of the asymmetric assumption  $z_1 > z_2$ . When  $z_1 = z_2$  this mass disappears since in the symmetric case  $\hat{p}_1 = \hat{p}_2$ . However, since, according to our definitions this mass can occur only in the strategy of one of the firms and never in both we do not have to worry about the case in which both firms charge the same price. Thus the breaking tie rule that we adopt does not affect the above equilibrium.

**Remark:** The above equilibrium can be regarded as the equivalent of the standard Bertrand equilibrium. The expected profits of each firm are  $z_1 p^m a(p^m)$  which are exactly what the firm can get from its captive market. The existence of the duopolistic segment does not contribute anything to the firm's payoffs.

We are now ready to examine the effect of changing the awareness levels  $B_1$  and  $B_2$  on the above equilibrium prices. We carry on our assumption that  $z_1 > z_2$  which implies that  $B_1 > B_2$ . An increase in  $B_1$  implies that  $z_1$  increases even more while  $z_2$  decreases. Examining the equilibrium



strategies (12) and (13) indicates that the increase of  $B_1$  implies that  $\hat{p}_2$  goes up while  $\hat{p}_1$  decreases. Since in our case  $\hat{p}_2 \geq \hat{p}_1$  and  $\hat{p} = \hat{p}_2$ , the higher  $B_1$  implies that the support of  $F(p)$  and  $G(p)$  is now at a higher range of prices.

Concerning the incentives to invest in awareness the analysis is now much simpler than before. At the above equilibrium the expected profits of each firm are  $z_1 p^m a(p^m)$ . Thus the firm wishes to increase its captive segment. Since an increase in  $B_1$  implies an increase of  $z_1$  (as long as  $B_j < N$ ), each firm wishes to increase  $B_1$  as much as possible. If we consider now the awareness game and assume that gaining awareness is costless the equilibrium of the awareness game is  $B_1 = B_2 = N$ . Specifically at the equilibrium all the consumers are aware of both firms. There are no captive segments, i.e.,  $z_1 = z_2 = 0$  which implies that we are back in a regular Bertrand pricing game and the equilibrium is a regular Bertrand equilibrium  $p_1 = p_2 = 0$  which implies zero profits.

### Concluding Remarks

The main objective of this paper is to demonstrate that it is not always optimal to be a "big" firm. The intuitive explanation that if a firm is too big it may step on its rivals' toes does not have any cooperative flavor in it. Each firm wishes to make its rival a "fat cat" and thus allows it to have a captive market segment. The analysis, however, was carried out under the assumption that the size of the firm does not affect its feasible strategy set. This is a straightforward assumption when we interpret size as the

degree of awareness. Size, however, can describe other characteristics of the organization some of which can be advantageous to the firm and some of them not. One can imagine a situation in which the size of a firm affects its feasible strategy space or the speed it can process information and react to new information. The relationship between firm's size and its performance in different market structures is of major importance as it combined the analysis of internal organization and industry equilibrium.

APPENDIX A

What we show in this Appendix is that in the symmetric game presented in section 5, when  $\gamma^* \leq \gamma \leq \gamma^{**}$  and  $B_1 = B_2 = N$ , then  $\partial R_1 / \partial B_1 < 0$ .

Differentiate  $R_1$ , set  $B_1 = B_2 = N$ , and recall that the first-order condition in prices implies that  $\partial R_1 / \partial h^1 = 0$ , to achieve the following:

$$(A.1) \quad \partial R_1 / \partial B_1 = N h^i \frac{1}{2} [2 - k h^i] + (\gamma N^2 / 2) (h^j)^{\gamma-1} (h^i)^{1-\gamma} (\partial h^j / \partial B_1).$$

From the first-order condition it is evident that when  $B_1 = B_2 = N$ , equilibrium prices are given by the duopolistic prices, i.e.,  $p_1^* = p_2^* = (2-\gamma)/(2k)$ . A straightforward calculation then reveals the following:

$$(A.2) \quad \partial R_1 / \partial B_1 = (N/2) \{ (2-\gamma)(2+\gamma)/4k + \gamma N \partial h^j / \partial B_1 \}$$

In order to calculate  $\partial h^j / \partial B_1$ , differentiate equation (9) with respect to  $B_1$ , then substitute  $B_1 = B_2 = N$  to achieve the following two equations:

$$(A.3) \quad -(2-\gamma^2) \partial h^1 / \partial B_1 + \gamma(1-\gamma) \partial h^2 / \partial B_1 = 0$$

$$(A.4) \quad \gamma(1-\gamma) \partial h^1 / \partial B_1 - (2 - \gamma^2) \partial h^2 / \partial B_1 = \gamma(2-\gamma)/kN.$$

Solving this system of two equations with two unknowns yields

$$\partial h^1 / \partial B_j = \gamma(2-\gamma)(2-\gamma^2) / (kN)(5\gamma^2 - 2\gamma^3 - 4).$$

Since  $0 \leq \gamma \leq 2$ , in order to show that  $\partial R_1 / \partial B_1 \leq 0$ , we have to show the following inequality for  $\gamma^* \leq \gamma < \gamma^{**}$ .

$$(A.5) \quad \frac{4\gamma^2(2-\gamma^2)}{4+2\gamma^3-5\gamma^2} > \gamma + 2.$$

Checking the condition  $\Pi_1^{11} \Pi_2^{22} > \Pi_1^{12} \Pi_2^{12}$ , we find that  $\gamma$  has to be smaller than  $\gamma^{**}$  which is the solution to the following quadratic equation

$$(A.6) \quad -2\gamma^2 + \gamma + 2 = 0.$$

Thus  $\gamma^{**} = 1/4 + \sqrt{1 + 1/16}$ . Obviously  $\gamma^{**} < 2$ . Thus we can restrict the rest of the analysis to values of  $\gamma$  that are less than  $\gamma^{**}$ .

A straightforward inspection reveals that both numerator and denominator of the l.h.s. of inequality (A.5) are positive at this range of  $\gamma$ . Thus we have to show:

$$(A.7) \quad 4\gamma^2(2-\gamma^2) > (2+\gamma)(4+2\gamma^3 - 5\gamma^2).$$

Let  $\gamma^*$  be the solution of the equation  $4\gamma^2(2-\gamma^2) = (2+\gamma)(4+2\gamma^3 - 5\gamma^2)$ . IN the range  $(0, \gamma^{**})$  there is a unique solution to the above equation (0.9173). Thus in order to show that inequality (A.7) holds for  $\gamma^* < \gamma < \gamma^{**}$ ,

let  $L = 4\gamma^2(2-\gamma^2)$  and  $R = (\gamma + 2)(4 + 2\gamma^3 - 5\gamma^2)$ . At  $\gamma = \gamma^*$ ,  $L(\gamma^*) = R(\gamma^*)$ . Observe that  $R$  vanishes exactly at  $\gamma^{**}$ , and that  $\partial L/\partial\gamma > \partial R/\partial\gamma$  at the relevant range. This is true since this last inequality holds at  $\gamma = \gamma^*$  and  $\partial^2 L/\partial\gamma^2 > \partial^2 R/\partial\gamma^2$  at the relevant range.

In order to show that when  $\gamma > \gamma^*$ ,  $\eta = 1$ , observe that if an equilibrium in awareness exists it is less than  $N$ , and calculate  $\eta$ .

APPENDIX B

What we show in this appendix is that the symmetric game presented in section 5, with  $\gamma = 1$  has a unique equilibrium in which  $B_1^* < N$ .

From equation (9) with  $\gamma = 1$  we achieve the following:

$$(B.1) \quad h^i = (1 - B_j / (4N - 2B_j)) / k$$

Differentiate  $R_i$  to get:

$$(B.2) \quad \partial R_i / \partial B_i = R_i / B_i + \frac{1}{2} B_1 B_2 \partial h^j / \partial B_i$$

Differentiation of equation (B.2) yields

$$(B.3) \quad \partial^2 R_i / \partial B_i^2 = B_j \partial h^i / \partial B_i + \frac{1}{2} B_1 B_2 \partial^2 h^i / \partial B_i^2.$$

Substituting equation (B.1) (and its respective derivatives) into (B.3) reveals that  $\partial^2 R_i / \partial B_i^2 < 0$ .

Let  $x = B/N$ . Substitution of equation (B.1) into (B.2) yields the following equation for the equilibrium  $x^* = B^*/N$ :

$$(B.4) \quad F(x) = (2-x)(2-x/2)(2 - 3x/2) - x^2 = 0.$$

In order to prove that  $F(x)$  has a unique solution between zero and one, observe that:  $F(0) = 8$ ,  $F(1) = -1/4$ ,  $dF/dx$  is negative, and  $F$  is continuous for  $0 \leq x \leq 1$ .

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