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Discussion Paper No. 780

PUBLIC UTILITY PRICING AND CAPACITY
CHOICE UNDER RISK: A RATIONAL
EXPECTATIONS APPROACH

by

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Public Utility Pricing and Capacity Choice Under Risk: A Rational Expectations Approach

ABSTRACT

Over the last two decades there has developed an extensive literature on the theory of public enterprise pricing and capacity choice under uncertainty. A major concern has been the analysis of the rationing of consumers in states in which demand exceeds available system capacity. An issue that has been largely ignored however is the effect that consumers' probability of being rationed (*system reliability*) has on their demand for the service. In this paper we develop a model that reflects the intuitive notion that a more reliable service is a higher quality service, so that an increase in system reliability shifts consumers' demand curves outward. We then incorporate this effect into our analysis of the utility's optimal pricing and investment rules. Finally, we demonstrate how the 'value of reliability' can, in principle, be estimated from generally available demand data.

Over the last two decades there has been considerable analysis of the problem faced by a public utility that must set its price and capacity before demand and/or supply uncertainties are resolved. Such is the world that most utilities find themselves in, since, in general, neither 'spot pricing' nor state-contingent contracts are possible mechanisms for the sale of their (nonstorable) services. The first paper on this topic was that of Gardner Brown and M. Bruce Johnson (1969). Their analysis generated a great deal of controversy (and many comments) because of their conclusion that the socially optimal price for the utility was equal to marginal operating cost, leaving the entire cost of capacity as a loss to the firm. Ralph Turvey (1970) argued that this result was dependent on Brown and Johnson's unrealistic assumption that, in the event of excess demand, the utility's available capacity would be rationed according to greatest willingness to pay. If available capacity would always be allocated efficiently without recourse to the price system, price should be set equal to marginal cost, so that no one willing to pay the marginal operating cost of a unit of the service would be discouraged from entering the market. This intuition was confirmed by Michael Visscher (1973) and Dennis Carlton (1977) who demonstrated that changing this key assumption about how demand is rationed leads to significant changes in the results of the model.¹

A further criticism of Brown and Johnson's approach was that it did not take account of the costs of system unreliability.² In response to this criticism, Robert Meyer (1975) introduced an exogenously specified reliability constraint into the problem. This also led to an optimal price greater than marginal operating cost, even with rationing by greatest willingness to pay. Michael Crew and Paul Kleindorfer (1978) criticized this approach on the grounds that system reliability should be determined endogenously, inside the model. In

order to accomplish this, they assumed that the costs of rationing were an increasing convex function of excess demand. Their analysis provided a method for calculating optimal reliability levels once a rationing cost function is specified.³

Interestingly enough, however, this focus on reliability did not extend to a consideration of its effect on consumers' demand for the service in question.⁴ Since reliability is an important quality variable one would expect it to influence consumer demand. If this is so, normative calculations should take account of the fact that the utility's pricing and investment decisions will not only affect consumer welfare directly, but also indirectly, via their impact on system reliability.⁵

In this paper we address both the theoretical and policy problems posed above by developing a rational expectations approach to modelling consumers' behavior under risk that makes precise the intuitive notion that the reliability of service perceived by consumers will influence market demand. We then derive optimal pricing and investment rules and explain how the 'value of reliability' can, in principle, be estimated from consumers' behavior.

At the outset it should be stressed that our objective is to make as clear as possible the basic approach rather than attempt to achieve maximum generality. As a consequence we have used a very simple model. We consider a public utility producing electricity. The sole source of uncertainty is assumed to be the random availability of the utility's capacity⁶ and the only users of electricity in the economy are industrial consumers (i.e., firms). Following Visscher and Carlton, we assume that if demand ever exceeds available capacity, the available capacity is rationed *randomly*. It should be noted, however, that the model can be extended to incorporate different types

of consumers, demand uncertainty and more complicated rationing schemes, albeit at the cost of greater analytical complexity.

The remainder of the paper is organized as follows. The model is presented in Section I. In Section II the optimal pricing and investment rules are derived. Section III discusses the problem of estimating the value of reliability, and Section IV offers some suggestions for further research. An Appendix contains the proof of one of the results stated in the text.

I. The Model

Consider a region in which electricity is produced by a public utility and used as an input by firms. The public utility is assumed to produce electricity using a single generation technology with costs given by

$$C = bE + \beta K$$

where E is the amount of electricity it produces (kilowatt hours) and K is the level of installed capacity (kilowatts). The level of capacity actually available is given by the random variable $\tilde{K} = \epsilon K$ where ϵ is the realization of a random variable with range $[0,1]$, continuous positive density $f(\epsilon)$ and distribution function $F(\epsilon)$. The utility must choose the price of electricity p and its installed capacity before the state of nature is realized. It is possible therefore, that aggregate electricity demand may exceed available capacity. In this situation, we assume that the available capacity is rationed randomly; that is, firms are blacked out randomly until demand equals available capacity.

Let n denote the number of firms in the downstream industry. Each firm produces an identical output y according to the neoclassical production function $y = g(e, x, z)$ where e represents electricity, x is a vector of other variable

inputs, and z is the amount of capital. Since we assume that this downstream industry is perfectly competitive, it is convenient to use the restricted profit function representation of this technology.⁷ Thus the level of profits, gross of capital equipment costs, earned by each firm is given by $V(p_0, p, w, z)$, where p_0 is the price of the competitive firms' output, and w the prices of variable inputs other than electricity. Since p_0 and w play no part in our analysis, they will be suppressed.⁸ We assume that capital equipment and electricity are complementary inputs ($V_{zp} < 0$) and that electricity is essential for production ($g(0, x, z) = 0$), so that no profits can be earned if electricity is unavailable ($V(\infty, z) = 0$). Using Hotelling's Lemma, each firm's demand for electricity, given its level of capital, is $e(p, z) = -V_p$.

While firms may delay their purchase of other variable inputs until they know whether or not electricity is available, we assume that they must choose their levels of capital before the state of nature is realized. How much capital will each firm employ? Suppose that a firm were to employ z units. There are two possibilities. The first is that the firm is not blacked out. In this case it will demand $e(p, z)$ units of electricity and will obtain profits of $V(p, z) - rz$, where r is the price of capital equipment. It is straightforward to verify that, given our assumptions, $e_p \leq 0$, $e_z > 0$. The second possibility is that the firm is blacked out. In this case, since output cannot be produced without electricity, the firm will obtain a profit level of $-rz$. The firm's expected profits, if it assigns a probability ρ to being blacked out, will therefore be given by

$$\pi(p, r, z, \rho) = (1 - \rho)V(p, z) - rz \quad (1)$$

Given (p, r, ρ) , the firm will choose to employ $z^*(p, r, \rho)$ units of capital where

$$z^*(p,r,\rho) = \operatorname{argmax}\{\pi(p,r,z,\rho) : z \in \mathbb{R}_+\}$$

At a regular interior maximum, it is easy to verify that

$$z_p^* = -V_{zp}/V_{zz} < 0 \quad (2)$$

$$z_r^* = 1/(1-\rho)V_{zz} < 0 \quad (3)$$

and

$$z_\rho^* = r/(1-\rho)^2 V_{zz} < 0 \quad (4)$$

Thus, as one would expect, the firm's demand for capital is decreasing in the price of electricity, the price of capital, and the probability of being blacked out.

Having derived the firm's capital demand function $z^*(p,r,\rho)$ we can now write its electricity demand function as

$$e^*(p,r,\rho) \equiv e(p,z^*(p,r,\rho)), \quad (5)$$

and its expected profit function as

$$\pi^*(p,r,\rho) \equiv \pi(p,r,z^*(\cdot),\rho).$$

Note here that, by the envelope theorem,

$$\pi_p^* = -(1-\rho)e^*(p,r,\rho) \quad (6)$$

and

$$\pi_{\rho}^* = -V \quad (7)$$

Assuming all firms in the region share the same belief about the probability of being blacked out, aggregate demand for electricity in this region will be $E(p,r,\rho)$ where

$$E(p,r,\rho) \equiv ne^*(p,r,\rho) \quad (8)$$

As indicated, it is a function of the price of electricity, the price of capital, and firms' perception of the reliability of the system, as measured by the probability of being blacked out. It is easy to verify that $E_p < 0$, $E_r < 0$, and $E_{\rho} < 0$; i.e., as one would expect, aggregate demand is decreasing in all its arguments.

To complete the analysis, we need to discuss how firms' expectations of the probability of being blacked out determine the actual black out probability. Let (p,r,ρ) and K be given, and let $m(\epsilon)$ denote the number of firms who would have to be blacked out if the realization of the random variable is ϵ . It is easy to verify that

$$m(\epsilon) = \begin{cases} 0 & \text{if } E(p,r,\rho) \leq \epsilon K \\ (E(p,r,\rho) - \epsilon K)/e^*(p,r,\rho) & \text{otherwise} \end{cases}$$

Then the probability that an individual firm will be blacked out is obtained by integrating over ϵ and dividing by the number of firms. Thus, given (p,r,ρ)

and K , firms' maximizing behavior results in an *actual* interruption probability

$$h(p, r, \rho, K) = \int_0^1 (m(\epsilon)/n) f(\epsilon) d\epsilon$$

or, equivalently,

$$h(p, r, \rho, K) = \begin{cases} 0 & \text{if } E(p, r, \rho) = 0 \\ E(p, r, \rho)/K \\ \int_0^1 (1 - \epsilon K / E(p, r, \rho)) f(\epsilon) d\epsilon & \text{otherwise} \end{cases}$$

To close the model we assume that firms' expectations are rational in the sense that ρ is equal to the actual probability of being blacked out. That is, we assume that given information on the level of installed capacity K and the prices p and r , firms form an estimate of the probability of being blacked out which is validated by their resulting behavior.

Definition: The expectation of $\hat{\rho}$ is said to be *rational* if

$$\hat{\rho} = h(p, r, \hat{\rho}, K)$$

We now have the following result

Theorem: For all $(p,r,K) \in \mathbb{R}_{++}^3$ there exists a unique rational expectation $\hat{\rho}(p,r,K)$.

Proof: See Appendix.

Firms' (rational) expectation of the probability of being blacked out will obviously vary with the prices of electricity and capital and the utility's level of installed capacity. By the implicit function theorem, if $E(p,r,\hat{\rho}) > 0$,

$$\hat{\rho}_p = \sigma E_p / (E - \sigma E_p) < 0 \quad (9)$$

$$\hat{\rho}_r = \sigma E_r / (E - \sigma E_p) < 0 \quad (10)$$

and

$$\hat{\rho}_K = -\sigma E / K(E - \sigma E_p) < 0 \quad (11)$$

where σ is the average fraction of demand served in the event of a shortfall:

$$\sigma = \frac{E/K}{\int_0^{E/K} (K/E) \epsilon f(\epsilon) d\epsilon}$$

As one would expect, since increases in the prices of electricity or capital repress the demand for electricity, they also lower firms' equilibrium expectation of the probability of being blacked out. An increase in installed capacity also reduces the expected probability of being blacked out.

II. Optimal Pricing and Investment Rules

Having completed the model we can now derive the optimal pricing and investment rules. Suppose the public utility were to choose a price p and a level of capacity K . The resulting level of reliability would be $\hat{\rho}(p, K)$.⁹ Firms' total expected profits would be $n\pi^*(p, \hat{\rho})$ and the utility's expected profits would be

$$U(p, K, \hat{\rho}) = (1 - \hat{\rho})(p - b)E(p, \hat{\rho}) - \beta K \quad (12)$$

Total expected surplus would therefore be

$$S(p, K, \hat{\rho}) = n\pi^*(p, \hat{\rho}) + U(p, K, \hat{\rho}) \quad (13)$$

Notice that, since $\pi^*(p, \rho)$ will be zero for sufficiently large p , we may write

$$\pi^*(p, \rho) = \int_{\infty}^p \pi_p^*(\tilde{p}, \rho) d\tilde{p}$$

or, using (6),

$$\pi^*(p, \rho) = (1 - \rho) \int_p^{\infty} e^*(\tilde{p}, \rho) d\tilde{p}$$

Thus we may write

$$S(p, K, \hat{\rho}) = (1 - \hat{\rho}) \int_p^{\infty} E(\tilde{p}, \hat{\rho}) d\tilde{p} + U(p, K, \hat{\rho})$$

This corresponds to the usual measure of surplus employed in the literature; that is, expected consumer surplus - as measured by the area under the demand curve - plus the utility's expected profits. In contrast to the traditional formulation, however, the price and capacity levels chosen by the utility will also affect consumer surplus through their impact on the perceived reliability of the system $\hat{\rho}$.

The socially optimal price-capacity pair is, of course, that which maximizes total expected surplus. Assuming that it is optimal to produce at least some electricity, we know that the optimal price-capacity pair must satisfy the following first order conditions:

$$S_p + S_{\hat{\rho}} \hat{\rho}_p = 0 \quad (14)$$

and

$$S_K + S_{\hat{\rho}} \hat{\rho}_K = 0 \quad (15)$$

Partially differentiating (12) and (13) yields $S_p = (1 - \hat{\rho})(p - b)E_p$ and $S_K = -\beta$. Using (7), we also obtain

$$S_{\hat{\rho}} = -nV - (p - b)E + (1 - \hat{\rho})(p - b)E_{\rho} \quad (16)$$

Equations (14) and (15) are the optimal pricing and investment rules. Together they implicitly define the socially optimal price and level of capacity. Since the responsiveness of demand to changes in the perceived reliability of the system (E_{ρ}) shows up in equation (16), it will influence the optimal price

and capacity choices. Failure to take into account the effect of reliability on demand will therefore result in the selection of sub-optimal levels of price and capacity.

The relationship between price, demand and capacity at the optimum is simple and intuitive. Note, from (9) and (11), that $\hat{\rho}_K/\hat{\rho}_p = -E/KE_p$. Then dividing equation (15) by equation (14) and rearranging yields

$$p = b + \beta(K/(1-\hat{\rho})E) \quad (17)$$

This optimal pricing rule has a natural interpretation. Price is set equal to the marginal cost of providing another unit of electricity *without* degrading the quality of service; i.e., without reducing the reliability of the system. In our model, this means the price paid must equal marginal operating cost plus the cost of adding enough capacity to serve an additional unit of demand without increasing the probability of curtailment. The additional increment in capacity required is greater than one unit, because to add only one unit would clearly reduce system reliability. Consequently, the optimal price exceeds the long run marginal production cost.¹⁰ Equation (17) also reveals that the utility earns zero expected profits. Thus, as one would expect under constant returns, at the optimum, price is set equal to marginal cost, appropriately defined, and the firm exactly breaks even.

III. On Estimating the Value of Reliability

The reader will have noticed that the optimal pricing and investment rules derived in the previous section require that the analyst possess complete information about cost and demand conditions. It is commonplace to assume that the required information about the price responsiveness of

demand can, in principle, be obtained from econometric studies. However, in order to implement the rules developed above, it is also necessary to know something about the effect of increased reliability on demand. Unfortunately, direct estimates of the effects of reliability on demand will be impossible to obtain, even in principle, unless there is sufficient variation in reliability in the data. Reliability levels in the U.S. have tended to be very high, with little change over time or across jurisdictions.¹¹ Fortunately, it is possible to derive relationships between the effects in question and more readily observable magnitudes. In this section we illustrate this important point.

Suppose that in our model price and capacity have, in the past, been chosen to maintain reliability at a constant level, so that while the planner is able to estimate the responsiveness of electricity demand to changes in the prices of electricity and capital, i.e., E_p and E_r , he is unable to directly estimate E_p . We claim that this lack of information about the responsiveness of demand to changes in reliability need not prevent him from implementing the optimal rules in equations (14) and (15). Notice from (5) and (8) that

$$E_p = ne_z z_p^*$$

and

$$E_r = ne_z z_r^*$$

But from (3) and (4)

$$z_{\rho}^* = rz_r^* / (1-\rho)$$

and hence

$$E_{\rho} = rE_r / (1-\rho) \quad (18)$$

Thus the planner can calculate the responsiveness of electricity demand to changes in reliability indirectly using equation (18) and his estimate of E_r .

This point can be made more concrete by means of a specific example. Thus let $V(p,z) = z^{\alpha} p^{-\gamma}$, where $1 > \alpha > 0$ and $\gamma \geq 0$. From Hotelling's Lemma, each firm's demand for electricity is given by

$$e(p,z) = -V_p = \gamma z^{\alpha} p^{-\gamma-1} \quad (19)$$

Using (1), expected profits can be written as

$$\pi(p,r,z,\rho) = (1-\rho)z^{\alpha} p^{-\gamma} - rz$$

Maximizing this with respect to z and solving yields

$$z^*(p,r,\rho) = [rp^{\gamma} / \alpha(1-\rho)]^{\tau} \quad (20)$$

where $\tau = 1/(\alpha-1)$. Combining equations (19) and (20), we obtain the industry electricity demand function

$$E(p,r,\rho) = ne(p,z^*) = n\gamma\alpha^{-\alpha\tau} r^{\alpha\tau} p^{(\gamma+1-\alpha)\tau} (1-\rho)^{-\alpha\tau}$$

This leads naturally to the log linear estimating equation

$$\ln E = A + \alpha\tau \ln r + (\gamma+1-\alpha)\tau \ln p - \alpha\tau \ln (1-\rho) \quad (21)$$

where $A = \ln[n\gamma\alpha^{-\alpha\tau}]$.

Equation (21) illustrates the solution to the empirical analyst's problem. In this simple example, it is clear that all that is needed to forecast electricity demand are estimates of the parameters α and γ . Clearly, it is not possible to calculate these values from the estimated price elasticity alone, and if there is no variation in reliability, its elasticity estimate cannot be obtained. However, if there is variation in r over the sample, its elasticity estimate, combined with the price elasticity estimate, can be used by the analyst to recover all the parameters of interest.¹² Thus, even if there is no sample variation in reliability, it still may be possible to estimate a demand system that fully takes account of the effect of reliability on the demand for electricity. In effect, variation in the price of complementary factors can "substitute" for lack of observed variation in reliability.

This example can also be used to illustrate how generally available demand data can be used to obtain estimates of the impact of reliability on electricity demand when reliability itself is not observed. Consider the "short-run" industry demand for electricity, $ne(p, z^*)$. That is, suppose the analyst could observe the industry demand for electricity and the per firm level of capital equipment employed, but was unable to observe system reliability. In terms of the example, he would be faced with the log linear estimating equation

$$\ln E = \ln n\gamma + \alpha \ln z^* - (1+\gamma) \ln p \quad (22)$$

For fixed z , the firm's demand for electricity is independent of system reliability. Thus the analyst can obtain the required estimates of α and γ from estimating equation (22), so long as he recognizes that z is an endogenous variable. And what variable is available to serve as an instrument? Why r , of course!

IV. Concluding Remarks

This paper makes two main contributions. First, it shows how service reliability can be formally incorporated into the analysis of public utility pricing under uncertainty. Second, it suggests how the value of reliability might be estimated empirically, even in the absence of suitable reliability data. The modeling approach was that of rational expectations equilibrium. This seems to us essential, once it is recognized that loss of service, in itself, is economically significant only if the customer can take actions, *ex ante*, which affect his valuation of the service. Such *reliance investments* require that the customer make some assessment of the reliability of the system. It is then natural to assume that such assessments will coincide with experience in long-run equilibrium.

It is important that the reader bear in mind that the particular rational expectations equilibrium model developed in the paper captures only one aspect of this "feed-back" effect. For example, consumers' investments could be in a substitute technology, rationing could be by greatest (or least) willingness to pay, or the utility could be engaged in priority service pricing, as in Hung-po Chao and Robert Wilson (1987). All of these extensions are important and challenging subjects for further research.

Appendix

Proof of Theorem: Let $(p,r,K) \in \mathbb{R}_{++}^3$ be given. Define the function $\varphi: [0,1] \rightarrow \mathbb{R}$ as follows:

$$\varphi(\rho) = \rho - h(p,r,\rho,K) \quad (\text{A.1})$$

Clearly, $\hat{\rho}$ is a rational expectation if and only if $\varphi(\hat{\rho}) = 0$. It suffices to show, therefore, that there exists one and only one $\hat{\rho}$ such that $\varphi(\hat{\rho}) = 0$.

We begin by establishing three properties of the function φ .

Property 1: $\varphi(0) \leq 0 \leq \varphi(1)$

This is immediate from (A.1).

Property 2: φ is continuous on $[0,1]$

Establishing this property requires a little more work. First define the scalar ρ^* as follows:

$$\rho^* = \min \{ \rho \in [0,1] : E(p,r,\rho) = 0 \}$$

Since E is nonincreasing in ρ , $\rho \geq \rho^*$ implies that $E(p,r,\rho) = 0$ and $\rho < \rho^*$ implies $E(p,r,\rho) > 0$. It follows that

$$\varphi(\rho) = \begin{cases} \rho - \frac{E(p,r,\rho)/K}{\int_0^{E(p,r,\rho)/K} (1-\varepsilon K/E(p,r,\rho))f(\varepsilon)d\varepsilon} & \text{if } \rho < \rho^* \\ \rho & \text{if } \rho \geq \rho^* \end{cases} \quad (\text{A.2})$$

It is clear from (A.2) that φ is continuous on $[0, \rho^*)$ and $(\rho^*, 1]$. It therefore suffices to show that φ is continuous at ρ^* . Note first, from (A.2), that

$$\lim_{\rho \rightarrow \rho^*} \varphi(\rho) = \rho^* = \varphi(\rho^*) \quad (\text{A.3})$$

Now note that

$$\begin{aligned} \lim_{\rho \rightarrow \rho^*} \varphi(\rho) &= \rho^* - \lim_{\rho \rightarrow \rho^*} \left[\frac{E(p,r,\rho)/K}{\int_0^{E(p,r,\rho)/K} (1-\varepsilon K/E(p,r,\rho))f(\varepsilon)d\varepsilon} \right] \\ &= \rho^* - \lim_{\rho \rightarrow \rho^*} \left[\frac{E(p,r,\rho)/K}{F(E(p,r,\rho)/K) - K \int_0^{E(p,r,\rho)/K} \varepsilon f(\varepsilon)d\varepsilon / E(p,r,\rho)} \right] \\ &= \rho^* + \lim_{\rho \rightarrow \rho^*} K \frac{E(p,r,\rho)/K}{\int_0^{E(p,r,\rho)/K} \varepsilon f(\varepsilon)d\varepsilon / E(p,r,\rho)} \end{aligned}$$

Since

$$\lim_{\rho \rightarrow \rho^*} K \frac{E(p,r,\rho)/K}{\int_0^{E(p,r,\rho)/K} \varepsilon f(\varepsilon)d\varepsilon / E(p,r,\rho)} = \lim_{\rho \rightarrow \rho^*} E(p,r,\rho) = 0$$

it follows from L'Hôpital's Rule that

$$\lim_{\rho \rightarrow \rho_-^*} K \int_0^{E(p,r,\rho)/K} \epsilon f(\epsilon) d\epsilon / E(p,r,\rho) = \lim_{\rho \rightarrow \rho_-^*} E(p,r,\rho) f(E/K) / K = 0$$

Thus

$$\lim_{\rho \rightarrow \rho_-^*} \varphi(\rho) = \rho^* = \varphi(\rho^*) \quad (\text{A.4})$$

It follows from (A.3) and (A.4) that φ is continuous at ρ^* .

Property 3: φ is increasing on $[0,1]$

It is clear from (A.2) that φ is increasing on $[\rho^*, 1]$ and that if $\rho < \rho^* \leq \rho'$, $\varphi(\rho) < \varphi(\rho')$. Thus, to establish Property 3 it is enough to show that φ is increasing on $[0, \rho^*)$. To see this note that φ is differentiable on $[0, \rho^*)$ and that

$$\varphi'(\rho) = 1 - \int_0^{E(p,r,\rho)/K} \epsilon f(\epsilon) d\epsilon K E_\rho / E^2 > 0$$

The Theorem now follows easily. From Property 1, Property 2, and the intermediate value theorem, it follows that there must exist at least one $\hat{\rho}$ such that $\varphi(\hat{\rho}) = 0$. Property 3 implies there can exist no more than one such $\hat{\rho}$. $\square$

Notes

¹ Visscher considered two different rationing technologies: random rationing and rationing according to least willingness to pay. Under the assumption of additive demand uncertainty, he showed that the optimal price was above marginal operating cost in both cases. Carlton showed that, with multiplicative demand uncertainty, these rationing assumptions lead to an optimal price greater than long-run marginal cost.

² See, for example, Turvey.

³ In the 'self-rationing' model of John Panzar and David Sibley (1978), individual consumers controlled the reliability of their own service through the amount of capacity to which they subscribed. System reliability was perfect in their model because the utility constructed the level of capacity subscribed to by consumers, and there was no supply-side uncertainty.

⁴An exception is the model of John Tschirhart and Frank Jen (1979). Their analysis of a profit maximizing monopoly's pricing of interruptible service allows for the demand of each priority class to depend upon the reliability with which it is served. However, unlike the model we present below, the customers' valuations of reliability are not derived from more basic considerations. This makes it impossible for Tschirhart and Jen to analyze welfare optimal pricing rules. Nevertheless, by taking explicit account of the effect of reliability on consumer demand and constraining the monopolist to install sufficient capacity to fulfill reliability promises, they have incorporated important aspects of the rational expectations equilibrium approach that we develop and analyze below.

⁵ This point has also been made by Arthur De Vany (1976).

⁶ As Hung-po Chao (1983) has pointed out, this is an important source of uncertainty in practice.

⁷See Hal Varian (1984), pp. 21.

⁸We are assuming that the price of the industry's output and inputs are determined outside the region: i.e., they are unaffected by changes in the price or reliability of electric power in the region under study.

⁹To avoid notational clutter, r will be suppressed for the remainder of this section.

¹⁰ The fact that $K/(1-\hat{\rho})E$ is greater than one, can easily be verified.

¹¹ See the discussion in Michael Telson (1975).

¹²If n is unknown, it can be calculated using the two price elasticities and the estimated intercept.

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